Proceedings of the 2018 IEEE
International Conference on Robotics and Biomimetics
December 12-15, 2018, Kuala Lumpur, Malaysia

Humanoid Robot Gait Planning Based on Virtual Supporting Point

Xueheng Zhang, Mingguo Zhao

***Abstract*—Linear Inverted Pendulum model（LIPM）is a classical theory of biped robot gait planning. However, the fixed Zero Moment Point（ZMP）of each step does not conform to the laws of human motion. In this paper, we proposed a VSP-based gait planning algorithm to make the ZMP trajectory comply the law of Heel to Toe and smooth the velocity of the center of mass（CoM）. We found it remarkable that combining Virtual Supporting Point（VSP）with classical LIPM increases gait stability. The simulation and hardware experimental results on THU-Walker Platform strongly verified our algorithm.**

I. Introduction

Research on bipedal robots began in the late 1960s[1]. Although there is less than sixty years of research history, the research of biped robots still attracts the attention of many scholars at home and abroad. Over the years, Researchers focus on gait planning, gait control, energy efficiency and walking stability of bipedal locomotion. Among these fields, we mainly focus on the gait planning, which is the most fundamental one.

In terms of bipedal robots, the two most fundamental concepts are widely used to reveal the essence of gait planning. In static walking, the projection of the center of mass (CoM) is kept in the supporting polygon. In dynamic walking, the Zero Momentum Point (ZMP), introduced by Vukobratovic and Stepanenko[2], is kept in the supporting polygon.

Kajita[3] came up with the LIPM, which is a linear physics equation to simplify the gait generator, to relate CoM to ZMP by approximating a robot to a point mass held at a constant height z_0. Pratt[4] proposed Capture Point in 2006, which is an effective way to solve the problem about when and where to step, when robot has to take a step to comply the disturbance. Based on the assumption of point foot, Capture Point studies the relationship between ZMP and CoM. When people want the ZMP at each step not to be fixed, their algorithm needs further expansion. Englsberger[5] proposed Divergent Component of Motion (DCM) in 2013. DCM theory is to try to consider how to design a reasonable ZMP trajectory to suppress the divergence component.

Despite the many algorithms mentioned above, LIPM is the most classic and widely used gait planning algorithm. Among the many assumptions of LIPM, one assumption deserves our special attention, which is that each step of ZMP is a fixed point (Fig. 1). This assumption simplifies model calculations, but it does not conform to the law of human motion. Physiological studies have shown that, when human

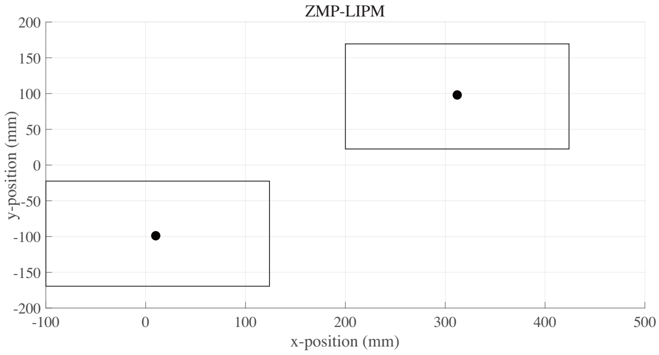

Figure 1. Fixed ZMP based LIPM

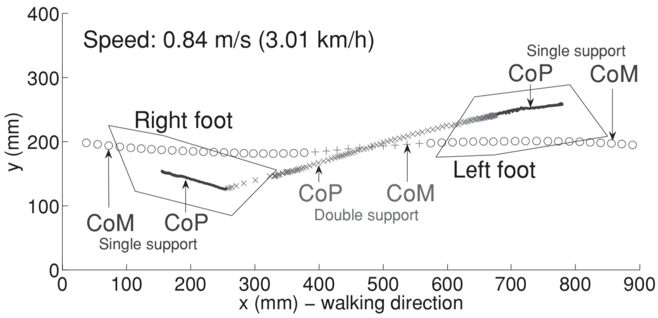

Figure 2. Human ZMP law of Heel to Toe[9]

walk, ZMP has the law of Heel to Toe[6]-[9] (Fig. 2). There are some improvements as follows. In gait planning, we can plan the ZMP trajectory in advance and calculate the CoM trajectory using cart-table model[10]. In gait control, we can design gait controller to achieve closed loop control. But the gait planning improvements mentioned above have some drawbacks. First of all, the physical meaning of the improved algorithm is not clear enough. What is more, the cost of calculation is large and cumbersome.

Based on inverted pendulum theory, Miff[11] proposed the concept of virtual walking surface from the perspective of exploring the energy change of inverted pendulum. Later, Tsuji[12] proposed the concept of virtual support point and explored the CoM motion law between the LIPM based on virtual support point and the classical LIPM. Waki[13] explored lateral sway motion generation for biped robots using characteristic of LIPM with VSP that a supporting point can be flexibly moved is presented. Kameta[14] proposed a new approach to generate a walking pattern based on a Spherical Inverted Pendulum model with an Underfloor Pivot. However, none of these studies discuss the impact of VSP on ZMP

*Research supported by UBTECH Robotics, Inc.

Authors are with the Department of Automation, Tsinghua University, & Center for Brain-Inspired Computing Research (CBICR), Beijing Innovation Center for Future Chip.
Email: zhangxueheng510@gmail.com, mgzhao@mail.tsinghua.edu.cn

978-1-7281-0377-8/18/$31.00 © 2018 IEEE

theoretically and there is no systematic explanation of the impact of VSP on CoM.

In this paper, the ZMP law of Heel to Toe implementation for biped robots using LIPM with VSP that the length of pendulum can be flexibly changed is presented. Section II presents the VSP-based gait planning. Section III demonstrates the experiment results and compare our proposed gait planning algorithm with classical LIPM. Section IV discusses the some assumptions and meaning of the proposed algorithm. Section V concludes the paper and introduces our future work.

II. VSP-BASED GAIT PLANNING

A. VLIP Model

Kajita[3] proposed the Linear Inverted Pendulum model (LIPM) to relate CoM to ZMP (Fig. 3). It has a simple equation of motion.

$$\ddot{c} = w^2(c - p) \tag{1}$$

Where $c = [c_x, c_y]$ is horizontal position of CoM, $p = [p_x, p_y]$ is the position of ZMP, $w = \sqrt{g/z_0}$ is the constant parameter, where z_0 equals to a constant height of CoM and g is the gravitational acceleration.

Based on the LIPM assumption, the ZMP for each step is fixed. The biped change to the next step by changing the ZMP point. If we want to implement ZMP law of Heel to Toe with LIPM, we must replace ZMP points every frame, with unclear physical meaning and troublesome calculations. In addition, the body structure of the biped robot cannot match the real structure of the human anyway. The ZMP obtained by the accurate measurement of human walking does not produce good results when used directly in gait planning.

By replacing ZMP p with VSP v (Fig. 4), We get the dynamic expression of a Virtual Linear Inverted Pendulum model (VLIPM)

$$\ddot{c} = w_v^2(c - v) \tag{2}$$

Where $v = [v_x, v_y]$ is the position of VSP, $w_v = \sqrt{g/(alpha * z_0)}$ is the constant parameter. $alpha$ means the multiple of VLIP length, which equals to the length of VLIPM divided by the length of LIPM.

A straight line passes through VSP and CoM, and its intersection point with the ground is ZMP. During the gait planning at each step, the VSP point is kept fixed, and the ZMP position changes accordingly with the change of the CoM position of each frame. Therefore, with the change of CoM position at each step, ZMP will form a continuous curve on the ground, that is, realizing the ZMP law of Heel to Toe. The ZMP curve formula of the VLIPM can be represented by Formula (3).

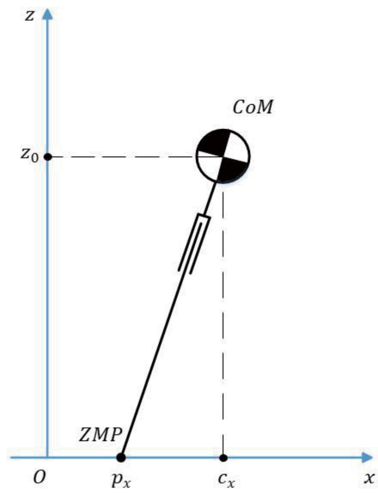

Figure 3. Model of LIP

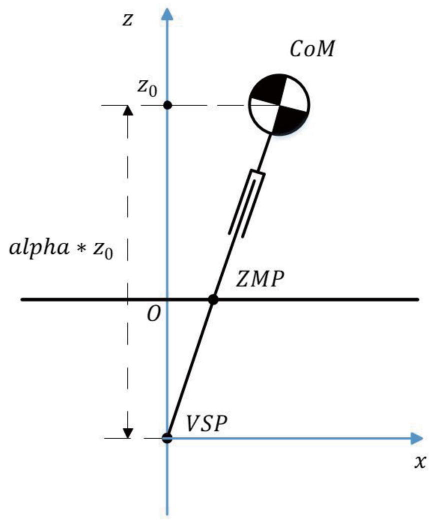

Figure 4. Model of VLIP

$$p = (c - v) * \left(\frac{alpha-1}{alpha}\right) + v \tag{3}$$

By solving the differential equation (2), we get the analytical expression of position (4), velocity (5), and acceleration (6).

$$c(t) = (c(0) - v)\cosh(w_v t) + \frac{1}{w_v}\dot{c}(0)\sinh(w_v t) + v \tag{4}$$

$$\dot{c}(t) = w_v(c(0) - v)\sinh(w_v t) + \dot{c}(0)\cosh(w_v t) \tag{5}$$

$$\ddot{c}(t) = w_v^2(c(0) - v)\cosh(w_v t) + w_v \sinh(w_v t) \tag{6}$$

Where $c(0)$ is the initial position of CoM. $\dot{c}(0)$ is the initial velocity of CoM. It can be seen that as long as the VSP and the initial value of the CoM are known, the analytical expression of the CoM state can be obtained.

Comparing LIPM and VLIPM, we can see that VLIPM is equivalent to increasing the pendulum length in disguise without changing the height of CoM. Under the same step length and fixed VSP, intuitively, longer pendulum length can lead to longer length of ZMP curves, smaller CoM acceleration, and more gentle CoM speed changes. Consistently, Formula (3)-(5) can prove the above conclusion.

B. Effect of Alpha Value on CoM and ZMP

It can be seen from the formula (4)-(6) that the change of alpha value will result in a corresponding change in w_v, causing changes in CoM status.

In MATLAB simulation, we select the alpha value from 1 to 5, observing the effect on the trajectory of ZMP when alpha takes different values. In particular, the ZMP mentioned below refers to ZMP with single support phase.

The black box in the Fig. 5 represents the footprint. In the simulation, the footprint size is 22.5cm*17cm. From Fig. 5, we can see when value of alpha takes 1, which is the classic LIPM, the ZMP keeps fixed point in the middle of footprint. When value of alpha increases, the change of ZMP conforms the law of Heel to Toe and the length of ZMP trajectory also grows. At the same time, we should pay attention to the scope limitation of ZMP. During the walking process, ZMP trajectory should be in the footprint. Consequently, the value of alpha should not be too large. In summary, as the alpha value increases, the ZMP law of Heel to Toe will become more and more obvious, but the ZMP trajectory also gets closer and closer to the edge of the footprint, making the noise immunity worse.

Next, we also select the alpha value from 1 to 5, observing the effect on the trajectory of CoM velocity, when alpha takes different values (Fig. 6, Fig. 7).

The main goal of using the VLIP model is to implement ZMP law of Heel to Toe. At the same time, we find that the increase of the alpha value can smooth the velocity of CoM, which is an important indicator to evaluate the gait planning algorithm. Fig. 6 is the Y direction - CoM velocity-time image. Fig. 7 is the X direction - CoM velocity-time image. It can be seen from Fig. 6 that the volatility of CoM velocity in y direction is maximum when the alpha value is 1. Similar to the trend in Fig. 6, we can see that the volatility of CoM velocity in x direction is maximum when the alpha value is 1. As the alpha value gradually increases, volatility of CoM velocity decreases. From the perspective of volatility of CoM velocity, the larger the value of alpha, the better.

Fig. 8 is the CoM two-dimensional top view. The overall shape of Fig. 8 is different from Fig. 6 and Fig. 7. Although different alpha values have a great impact on volatility of CoM velocity, the average velocity of the CoM is approximately the same. Therefore, the alpha value has little effect on the CoM trajectory. But we can observe that as the alpha value increases, the lateral shift of the CoM has a slight decrease, which is exactly what we want.

From the above results and analysis, it can be seen that as the alpha value increases, the ZMP law of Heel to Toe will become more and more obvious and the volatility of CoM velocity will decrease continually. But when the alpha value is too large, the length of ZMP trajectory will be too long, which

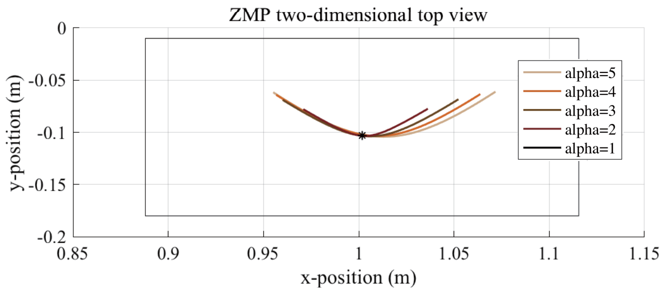

Figure 5. ZMP two-dimensional top view

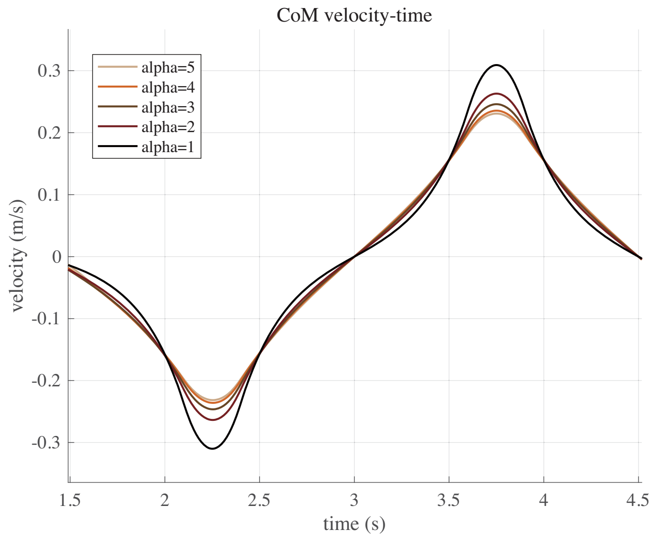

Figure 6. Y direction - CoM velocity-time image

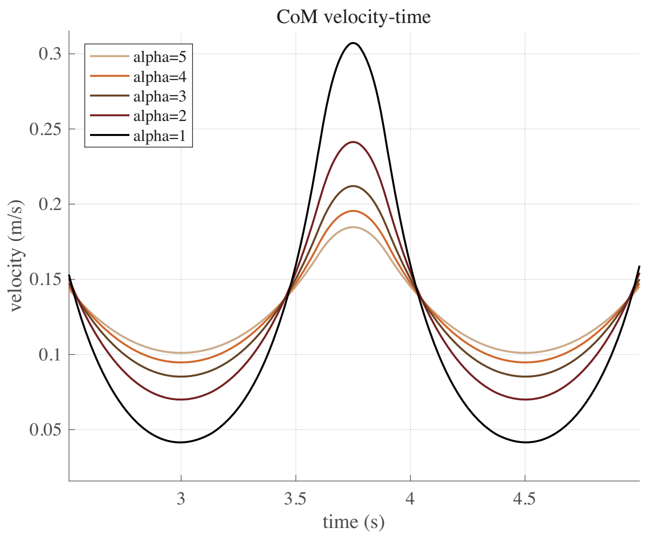

Figure 7. X direction - CoM velocity-time image

is an important limitation on the value of alpha. In conclusion, the alpha value must be taken into account in its effect on CoM velocity and ZMP position. From the simulation results, the result is moderate when the alpha value equals to 3.

III. Experiment

A. Experimental Platform

Biped robot THU-Walker developed in our laboratory as shown in Fig. 9. is used as an experimental platform. THU-Walker is the autonomous biped robot in which all electronic devices, force sensor, G sensor and gyro sensor are built in. It has 6 joints on each leg and 12-DoF in total. Its weight is about 46kg. Its body height and CoM height are respectively 1.35m and 0.76m. The foot size is 22.5cm*17cm.

B. Experimental Results

Considering the step length is 15cm, gait cycle is 1.2s, single support phase is 0.96s, double support phase is 0.24s, we recorded and analyzed the CoM and ZMP data, when alpha value took 1 and 3.

Fig. 10 and Fig. 11 respectively show the ZMP two-dimensional top view trajectory measured actually by proposed VLIPM, when alpha values equal to 1 and 3. The red line and blue line respectively represent the ZMP curve actually measured when in Single Support Phase (SSP) and Double Support Phase (DSP). From the figure, we can see that the proposed algorithm realized the ZMP law of Heel to Toe basically.

When alpha value was 1, we found that the ZMP curve in SSP exceeded the footprint, which made the gait unstable. In fact, when the ZMP was beyond the footprint, we needed to help to continue walking. When alpha value was 3, we found that the ZMP curve in SSP exceeded the footprint suddenly and regularly, which was the result of feet hitting the ground (We will talk about the phenomenon in DISCUSSION). Without considering the influence of this factor, the ZMP complied the law of Heel to Toe, which proved the effectiveness of our proposed algorithm.

Fig. 12, which has four lines, respectively shows that the CoM velocity-time trajectory in y direction planned and measured actually by proposed VLIPM, when alpha value takes 1 and 3. In this experiment, when alpha took 1, the maximum velocity error was 0.0647m/s for feet hitting the ground. Without considering the influence of this factor, the maximum velocity error was 0.0517m/s. When alpha took 3, the maximum velocity error was 0.145m/s for feet hitting the ground. Without considering the influence of this factor, the maximum velocity error was 0.0415m/s. The mean and standard deviation of planning and measure data are shown in TABLE I. According to the above analysis, increase in alpha value can reduce the volatility of CoM velocity.

Fig. 13, which has four lines, respectively shows that the CoM velocity-time trajectory in x direction planned and measured actually by proposed VLIPM, when alpha value takes 1 and 3. In this experiment, when alpha took 1, the maximum velocity error was 0.119m/s for feet hitting the ground. Without considering the influence of this factor, the maximum velocity error was 0.0499m/s. When alpha took 3, the maximum velocity error was 0.145m/s for feet hitting the ground. Without considering the influence of this factor, the maximum velocity error was 0.0415m/s. The mean and standard deviation of planning and measure data are shown in TABLE II. According to the above analysis, increase in alpha value can reduce the volatility of CoM velocity. From the above results and analysis, we can conclude that the proposed algorithm is better than the original algorithm.

TABLE I. Velocity Error in Y Direction

Alpha	Velocity Error (m)			
	Mean of planning data	***Mean of measured data***	***Standard deviation of planning data***	***Standard deviation of measured data***
1	8.6084e-05	1.1342e-04	0.1475	0.1428
3	6.9916e-05	3.6848e-05	0.1277	0.1235

TABLE II. Velocity Error in X Direction

Alpha	Velocity Error (m)			
	Mean of planning data	***Mean of measured data***	***Standard deviation of planning data***	***Standard deviation of measured data***
1	0.0827	0.0828	0.0788	0.0828
3	0.0766	0.0767	0.0637	0.0663

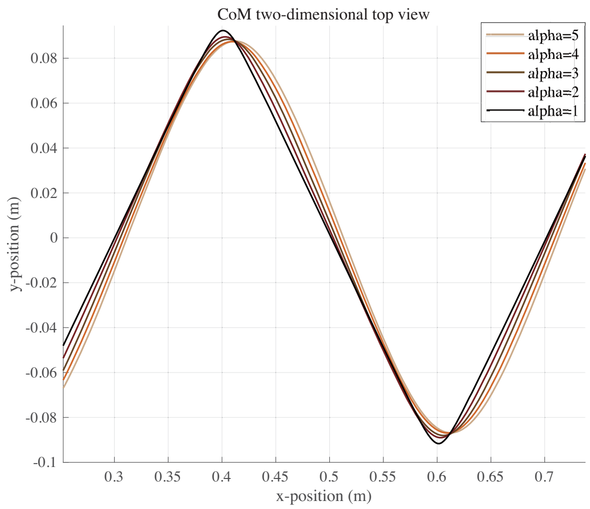

Figure 8. CoM two-dimensional top view

Figure 9. Biped robot THU-Walker

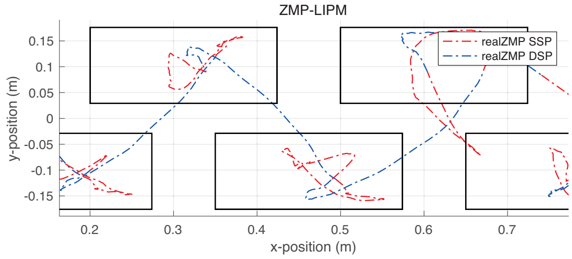

Figure 10. ZMP 2D top view when alpha equal to 1

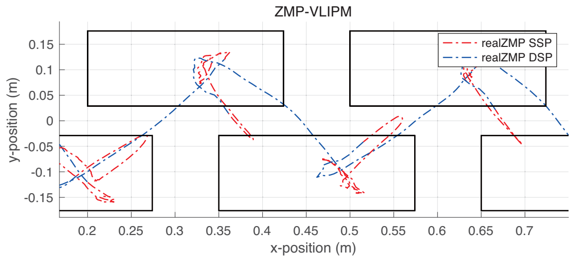

Figure 11. ZMP 2D top view when alpha equal to 3

Fig. 14, which has four lines, respectively shows that CoM two-dimensional top view planned and measured actually by proposed VLIPM, when alpha value takes 1 and 3. The overall shape of measured data is similar to simulation. From the measured data, we can observe that as the alpha value increases, the lateral shift of the CoM has a slight decrease, which is exactly what we want.

IV. Discussion

A. Some assumptions in the original algorithm

The assumptions of the improved algorithm are the same as the original algorithm. There is no new assumption. The original algorithm assumes that the mass of the robot is concentrated in the CoM and the two legs have no mass. But on our THU-Walker, the mass of the two legs cannot be ignored. We take the mass of the upper body and two legs as input, input them to the three-mass model, and get the position of the waist. After the inverse kinematics calculation, the joint angle and joint angular velocity are obtained.

The original algorithm does not mention how to plan swing legs. In our algorithm, we perform cubic spline interpolation on the three directions respectively. The trajectory of the swinging leg is completely up to us. We can determine the trajectory of the swing leg based on the position and speed of the support leg.

In LIPM, each step is a walk primitive, and the trajectory of the CoM in each walk primitive is a hyperbolic. However, for the particularity of the first step in robot gait planning, it is necessary to make minor adjustments to the planned ZMP points at each step. The ZMP adjustment method of the improved algorithm is based least squares method, which is the same as the original algorithm.

B. The rationality of the data

From Fig. 10 and Fig. 11, we can see that the motion of the ZMP is not always directed forward, which has a circular motion. In fact, when human step lengths are small, ZMP will have the same phenomenon. Erbatur[16] had studied a similar problem and measured ZMP data at a human step size of 6 cm. The data shows that ZMP has a circular motion, which is similar to our phenomenon.

Throughout the experiment, there has always been a phenomenon of robot feet hitting the ground, causing by hardware real-time problem. The actual execution time of the hardware is slightly different from the time of gait planning.

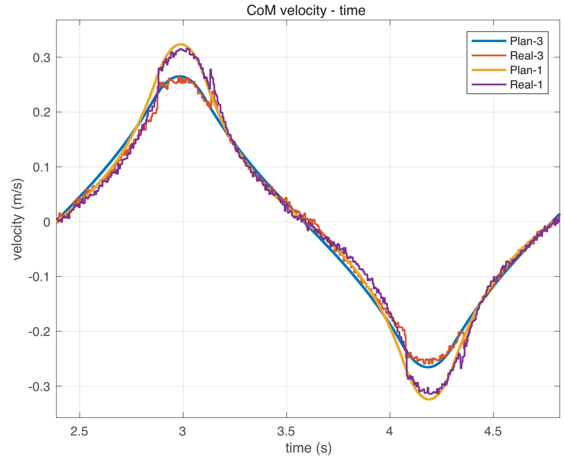

Figure 12. Y direction - CoM velocity-time image

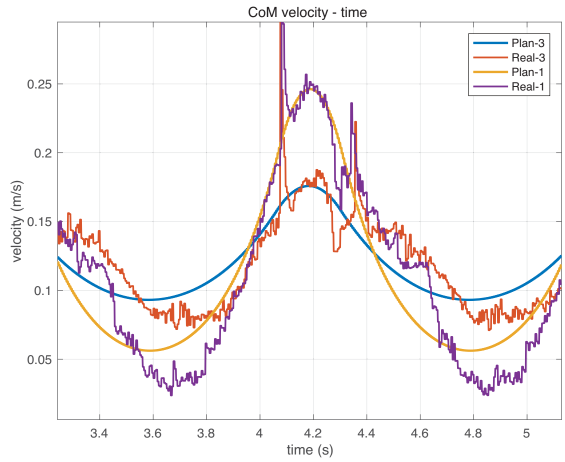

Figure 13. X direction - CoM velocity-time image

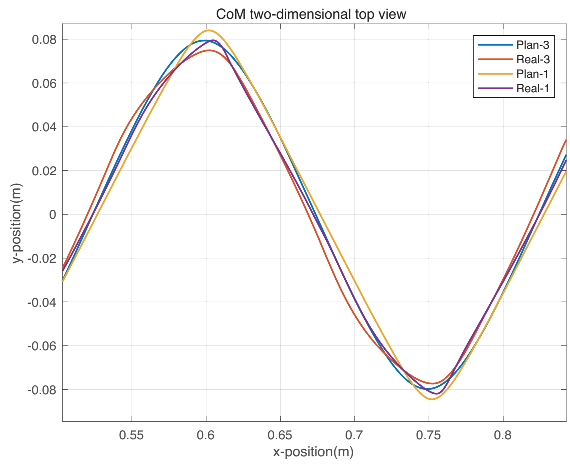

Figure 14. CoM two-dimensional top view

We can only try to reduce the impact of real-time problem on the performance. Therefore, the actual measurement error in this experiment is acceptable.

C. Extended meaning of the proposed algorithm

The VLIP model presented in this paper is all when the alpha is greater than 1. Actually, when alpha is less than 1, we can use VLIP model to plan the double support phase of the biped robot. In fact, Shibuya[15] have already realized this idea, but he assumed that VLIP model is in a two-dimensional plane to solve the problem of DSP gait planning, limiting the application of the model. We further explored the three-dimensional plane and concluded that the model is not decoupled in both x and y directions, so it cannot be applied in the 3D plane.

And we are exploring whether human walking model are similar to VLIP, and whether exists a virtual support point underground. If external conditions of human walking change, whether only the VSP needs to be adjusted. Waki[13] made some attempts in this field to explore how VSP changes affect the CoM trajectory, but it was not combined with human physiology. Next, our research may combine physiology and explore the basic laws of human movement.

V. Conclusion and Future Work

A. Conclusion

In this paper, we proposed a gait planning algorithm for generating walking gait whose ZMP meets the law of human walking – Heel to Toe. We have explored the effect of different parameter values on the state of ZMP and CoM. We found that appropriate parameter value can satisfy ZMP law of Heel to Toe and ensure that the lateral shift of CoM is reduced. In our algorithms, we have expanded the original algorithms to a larger field so that the performance of biped robots can be improved as well. The proposed algorithms can significantly improve bipedal walking stability, which was also proven in experiments.

B. Future Work

1) ZMP Controller: All the above improvements to the original algorithm are based on the gait planning open-loop level. Next, we will shift the focus of research to gait control and enhance the stability of the gait algorithm through closed-loop control

2) Event trigger LIP: The current research is based on time triggering. The biped robots perform fixed actions at given time. If encounter obstacles while walking, the robot cannot adjust in time. After changing to the event trigger, the start and end of each step are determined by the contact force instead of the time, which greatly enhances the robustness of the algorithm.

3) Gait Planning in Double Support Phase: In our improved algorithm, the gait planning algorithm in the Double Support Phase (DSP) is the same as the original algorithm. The original algorithm[10] uses cubic spline interpolation on the velocity, which guarantees smooth transition of the speed curve at the highest point. But the acceleration curve is not smooth. Next, we will try to use five spline interpolation on the velocity.

Acknowledgment

This work was partially supported by UBTECH Robotics, Inc. and Tsinghua University. The authors want to thank the UBTECH's staff from Beijing Research Institute for providing the humanoid robot THU-Walker. We also want to thank the Tsinghua lab members: Haitao Wang, Rongge Zhang, Zhongyuan Tian, Qilun Wang and Wenbin Hu. We acknowledge all the reviewers' comments, which are valuable to us.

References

[1] Noritake K, Kato S, Yamakita T, et al. A motion generation system for humanoid robots-Tai Chi motion[C]//Micromechatronics and Human Science, 2003. MHS 2003. Proceedings of 2003 International Symposium on. IEEE, 2003: 265-269.

[2] Vukobratovic M, Juricic D. Contribution to the synthesis of biped gait[J]. IEEE Transactions on Biomedical Engineering, 1969 (1): 1-6.

[3] Kajita S, Tani K. Study of dynamic biped locomotion on rugged terrain-derivation and application of the linear inverted pendulum mode[C]//Robotics and Automation, 1991. Proceedings., 1991 IEEE International Conference on. IEEE, 1991: 1405-1411.

[4] Pratt J, Carff J, Drakunov S, et al. Capture point: A step toward humanoid push recovery[C]//Humanoid Robots, 2006 6th IEEE-RAS International Conference on. IEEE, 2006: 200-207.

[5] Englsberger J, Ott C, Albu-Schäffer A. Three-dimensional bipedal walking control using divergent component of motion[C]//Intelligent Robots and Systems (IROS), 2013 IEEE/RSJ International Conference on. IEEE, 2013: 2600-2607.

[6] Dasgupta A, Nakamura Y. Making feasible walking motion of humanoid robots from human motion capture data[C]//Robotics and Automation, 1999. Proceedings. 1999 IEEE International Conference on. IEEE, 1999, 2: 1044-1049.

[7] Zhu C, Tomizawa Y, Luo X, et al. Biped walking with variable ZMP, frictional constraint, and inverted pendulum model[C]//Robotics and Biomimetics, 2004. ROBIO 2004. IEEE International Conference on. IEEE, 2004: 425-430.

[8] Kurt O, Erbatur K. Biped robot reference generation with natural ZMP trajectories[C]//Advanced Motion Control, 2006. 9th IEEE International Workshop on. IEEE, 2006: 403-410.

[9] Boutin L, Eon A, Zeghloul S, et al. From human motion capture to humanoid locomotion imitation Application to the robots HRP-2 and HOAP-3[J]. Robotica, 2011, 29(2): 325-334.

[10] Kajita S, Hirukawa H, Harada K, et al. Introduction to humanoid robotics[M]. Springer Berlin Heidelberg, 2014.

[11] Miff S C, Childress D S, Gard S A. The effect of step length and cadence on the instantaneous forward velocity of walking[C]//The 4 Annual Meeting of the American Society of Biomechanics. 2000: 19-22.

[12] Tsuji T, Ohnishi K. A control of biped robot which applies inverted pendulum mode with virtual supporting point[C]//Advanced Motion Control, 2002. 7th International Workshop on. IEEE, 2002: 478-483.

[13] Waki N, Matsumoto K, Kawamura A. Lateral sway motion generation for biped robots using virtual supporting point[C]//Advanced Motion Control, 2010 11th IEEE International Workshop on. IEEE, 2010: 124-128.

[14] Kameta K, Sekiguchi A, Tsumaki Y, et al. Walking control around singularity using a spherical inverted pendulum with an underfloor pivot[C]//Humanoid Robots, 2007 7th IEEE-RAS International Conference on. IEEE, 2007: 210-215.

[15] Shibuya M, Suzuki T, Ohnishi K. Trajectory planning of biped robot using linear pendulum mode for double support phase[C]//IEEE Industrial Electronics, IECON 2006-32nd Annual Conference on. IEEE, 2006: 4094-4099.

[16] Erbatur K, Okazaki A, Obiya K, et al. A study on the zero moment point measurement for biped walking robots[C]//Advanced Motion Control, 2002. 7th International Workshop on. IEEE, 2002: 431-436.