Fuzzy Regression Discontinuity Design without Monotonicity

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Introduction

AER publication (2005-2025)

Study	Mono	Indep	IV	FRD
Allcott et al. (2020)		✓	✓	
Andrews (2016)	✓	✓		✓
Autor et al. (2019)	✓	✓	✓	
Dinkelman (2011)		✓	✓	
...	...	...	...	...
Simcoe (2012)			✓	

Table: Summary statistics of AER using IV/FRD (2005-2025)[1]

1. Note: Only 19 of these 39 articles (50%) that explicitly identify a local average treatment effect using either an IV or FRD approach include the word "monotonicity"!

Definition

Framework (FRD and LATE)

Let $(\Omega, \mathcal{F}, P)$ be the probability space
$D(\cdot,\cdot) : \mathcal{R} \times \Omega \to \{0, 1\}$ is the observed binary treatment assignment,
$Y(\cdot,\cdot) : \mathcal{R} \times \Omega \to \mathcal{Y}$ is the observed outcome of interest,
$R(\cdot) : \Omega \to \mathcal{R}$ is a continuous running variable with a known cut-off r_0.
For the outcome $Y(R)$, we have $Y(R) := Y_1(R)D(R) + Y_0(R)(1 - D(R))$.
(D is binary.)

Motivation

Key Assumption: Local Monotonicity (Arai et al., 2022 and Hsu et al., 2024)

There exists a small $\epsilon > 0$ such that $T_\epsilon \in \{a, c, n\}$ almost surely.

- Also called no-defier assumption
- Under local continuity and local monotonicity,
  - $\theta_{FRD} = \frac{\mathbb{E}[Y|R=r_0^+]-\mathbb{E}[Y|R=r_0^-]}{\mathbb{E}[D|R=r_0^+]-\mathbb{E}[D|R=r_0^-]}$ identifies LATE for compliers (Hahn et al, 2001), where R is the running variable and r_0 is the cutoff point.
- Question
  - Existence of defiers?
  - What if local monotonicity fails to hold?

Existence of defiers (Kirkeboen et al, 2016)

Key variables (Kirkeboen et al, 2016)

Treatment : D=1 if student graduates with a degree in science rather than a degree in humanities

R : student's performance score for college admission, the cut-off is an admission threshold to a competitive major (say, science) rather than less competitive majors (say, humanities)

- Existence of defiers (Arai et al., 2022)
  - if some students, who tend to be attracted by nonmajored subjects and/or change their minds about their career choices, always switch from their assigned major to the other based on revisions of their beliefs or preferences

Literature Review

- LATE framework literature and relaxations of assumptions
  - Imbens and Angrist (1994), Angrist et al. (1996) and Vytlacil (2002)
  - Test the validity of IV: Huber and Mellace (2015), Kitagawa (2015), Mourifié and Wan (2017)
  - Independence: Kédagni (2023)
  - Exclusion restriction: Cui et al. (2024)
  - Monotonicity (related): Huber et al. (2017), de Chaisemartin (2017), Noack (2021), Fiorini and Stevens (2021), Cui et al. (2024)
- RD framework literature (SRD and FRD)
  - Thistlethwaite and Campbell (1960): SRD
  - RD Application: Angrist and Lavy (1999), van der Klaauw (2002), Lee et al. (2004), DiNardo and Lee (2011), Choi and Lee (2023)
  - FRD framework: Arai et al. (2022), Hsu et al. (2024)
    - Local continuity and local monotonicity

Model and Identification

Definition (Four strata of people add "i")

$$T_\epsilon = \begin{cases} a, & \text{if } D(r) = 1, \text{ for } r \in B_\epsilon, \\ n, & \text{if } D(r) = 0, \text{ for } r \in B_\epsilon, \\ c, & \text{if } D(r) = 1\{r \geq r_0\}, \text{ for } r \in B_\epsilon, \\ df, & \text{if } D(r) = 1\{r < r_0\}, \text{ for } r \in B_\epsilon, \\ i, & \text{otherwise,} \end{cases} \tag{1}$$

Existence Example

where $B_\epsilon = \{r \in \mathcal{R} : |r - r_0| \leq \epsilon\}$ of the cutoffs.

Model Setting

Assumption 1. Local continuity in distributions

For $d \in \{0,1\}, t \in \{a, c, n, df\}$, and all measurable subset $A \subseteq \mathcal{Y}$,

$$\lim_{\varepsilon \to 0} \mathbb{P}(Y_d \in A, T_\varepsilon = t \mid R = r_0 - \varepsilon) = \lim_{\varepsilon \to 0} \mathbb{P}(Y_d \in A, T_\varepsilon = t \mid R = r_0 + \varepsilon). \tag{2}$$

Assumption 2: Local no indefinite type

For $t = i$,

$$\lim_{\varepsilon \to 0} P(T_\varepsilon = t \mid Z = c - \varepsilon) = \lim_{\varepsilon \to 0} P(T_\varepsilon = t \mid Z = c + \varepsilon) = 0 \tag{3}$$

Model Setting

For any Borel set $A \in \mathcal{Y}$,

$$\begin{aligned}
\mathbb{P}(Y \in A, D = 1 | R = r_0^+) &= \mathbb{P}(T = c | R = r_0^+) \mathbb{P}(Y_1(r_0) \in A | T = c, R = r_0^+) \\
&\quad + \mathbb{P}(T = a | R = r_0^+) \mathbb{P}(Y_1(r_0) \in A | T = a, R = r_0^+) \\
\mathbb{P}(Y \in A, D = 1 | R = r_0^-) &= \mathbb{P}(T = df | R = r_0^-) \mathbb{P}(Y_1(r_0) \in A | T = df, R = r_0^-) \\
&\quad + \mathbb{P}(T = a | R = r_0^-) \mathbb{P}(Y_1(r_0) \in A | T = a, R = r_0^-)
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}(Y \in A, D = 1 | R = r_0) = &\quad \mathbb{P}(T = a | R = r_0) \cdot \mathbb{P}(Y_1(r_0) \in A | T = a, R = r_0) \\
&+ \mathbb{P}(T = c | R = r_0) \cdot \mathbb{P}(Y_1(r_0) \in A | T = c, R = r_0)
\end{aligned}$$

Similar results hold for $D = 0$. Visualization

LATE

Mean Potential Outcomes: $\mathbb{E}[Y_1(r)|T = c, R = r]$

$$\mathbb{E}[Y_1(r)|T = c, R = r] = \lambda_c \times \lim_{r \to r_0} \mathbb{E}[Y_1(r)|T_{0<|r-r_0|\leq\epsilon} = c, R \in B_\epsilon \setminus r_0] + (1-\lambda_c) \times \mathbb{E}[Y_1(r)|T_{r=r_0} = c, R = r_0],$$

where

$$\lambda_c = \frac{\mathbb{P}\left(R \in B_\epsilon \setminus \{r_0\}, T = c\right)}{\mathbb{P}\left(R \in B_\epsilon \setminus \{r_0\}, T = c\right) + \mathbb{P}\left(R = r_0, T = c, D = 1\right)}$$

and

$$\underline{\lambda_c} = \frac{\mathbb{P}\left(R \in B_\epsilon \setminus \{r_0\}, T = c\right)}{\mathbb{P}\left(R \in B_\epsilon \setminus \{r_0\}, T = c\right) + \mathbb{P}\left(R = r_0, D = 1\right)}.$$

Conclusion

Main contributions and strengths:

1. Extend the LATE framework without monotonicity to FRD
2. Derive the sharp bounds for LATE for compliers and defiers

Future work:

1. Finish the empirical examples
2. Finish the inference part (Chernozhukov et al. (2011) precision-corrected estimator)

Thanks!

Appendix

Definition (cont')

For the existence of type "indefinite", we can consider the single-threshold crossing specification of potential treatment like Hsu et al. (2024),

$$D(r) = 1\{r + 1\{r \geq r_0\} + X < 0\}, r \in [r_0 - \epsilon, r_0 + \epsilon]$$

where $X \sim N(0, 1)$. For simplicity, we fix $\epsilon > 0$ and $r_0 = 0$, the support of X can be divided into the groups from figure below. back

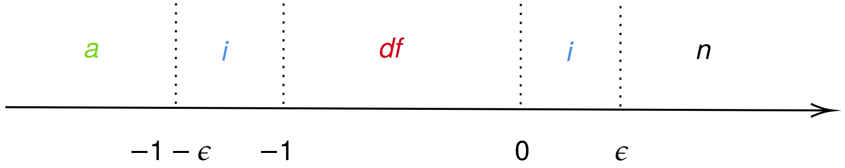

Figure: The distribution of types under the support of X

Visualization of Types

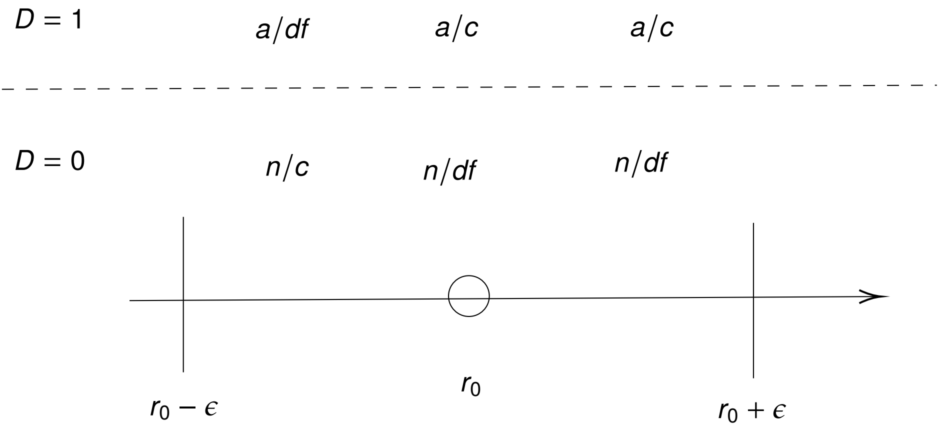

Figure: Visualization of types back