Local Average Treatment Effect without Monotonicity

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INTRODUCTION

- Framework: Local Average Treatment Effect (hereafter LATE)

$$\begin{cases} Y = Y_1 D + Y_0(1-D) \\ D = D_1 Z + D_0(1-Z) \end{cases}$$

where D, Z are binary

- Definition: four strata of people

D_0	D_1	Type
0	0	Never-takers
0	1	Compliers
1	0	Defiers
1	1	Always-takers

- Motivation

LATE Key Assumption: **Monotonicity**
(Imbens and Angrist, 1994; Angrist et al., 1996)

$$D_1 \geq D_0 \ \ or \ \ D_0 \geq D_1$$

Under independence, monotonicity and exclusion restriction assumptions, $\theta_{IV} = \frac{E[Y|Z=1]-E[Y|Z=0]}{E[D|Z=1]-E[D|Z=0]}$ identifies LATE for compliers

- Research Question

What if monotonicity assumption fails to hold?
Existence of defiers?

- Existence of defiers (Empirical Evidence)
  - Angrist and Evans (1998)

Key variables

Treatment: D=1 if the household has a third child
IV: Z=1 if the first two children are of the same sex

  - Card (1995)

Key variables

Treatment: D=1 if if the individual has a college degree
IV: Z=1 there exists a four-year college in the local labor market where the individual was born

- Related Literature
  - LATE framework and relaxations of assumptions
    Imbens and Angrist (1994), Angrist et al. (1996) and Vytlacil (2002)
    Test the validity of IV: Huber and Mellace (2015), Kitagawa (2015), Mourifié and Wan (2017)
    Violation of independence: Kédagni (2021)
    Violation of exclusion restriction: Kédagni and Wu (2023)
    Monotonicity (related): Huber et al. (2017), de Chaisemartin (2017), Noack (2021), Dahl, Huber and Mellace (2023)
  - Sensitivity analysis in the LATE framework
    Breakdown points: Horowitz and Manski (1995)
    Breakdown frontiers: Masten and Poirier (2020, 2021), Noack (2021)
  - Empirical literature: Card (1995), Angrist and Evans (1998)

MODEL AND RESULTS

Assumptions

Random Assignment, Relevance, Exclusion Restriction

- For any Borel set $A \in \mathcal{Y}$

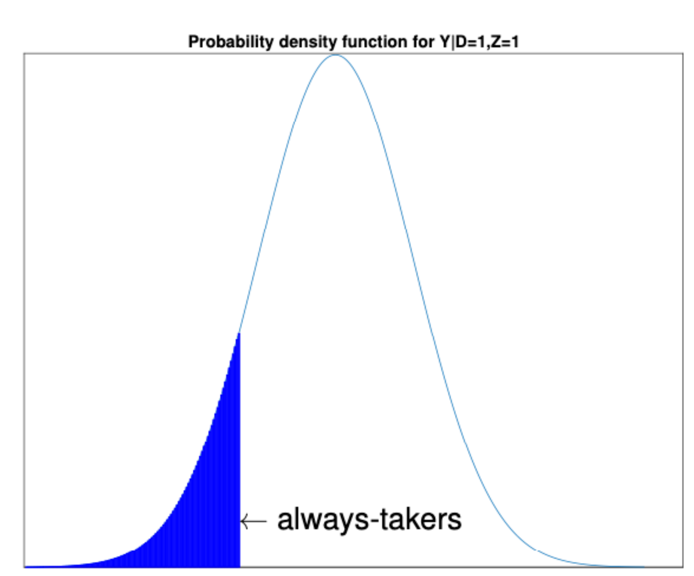
$$P(Y \in A, D=1|Z=1) = p_c P(Y_1 \in A|T=c) + p_a P(Y_1 \in A|T=a)$$
$$P(Y \in A, D=1|Z=0) = p_{df} P(Y_1 \in A|T=df) + p_a P(Y_1 \in A|T=a)$$
$$P(Y \in A, D=0|Z=1) = p_{df} P(Y_0 \in A|T=df) + p_n P(Y_0 \in A|T=n)$$
$$P(Y \in A, D=0|Z=0) = p_c P(Y_0 \in A|T=c) + p_n P(Y_0 \in A|T=n)$$

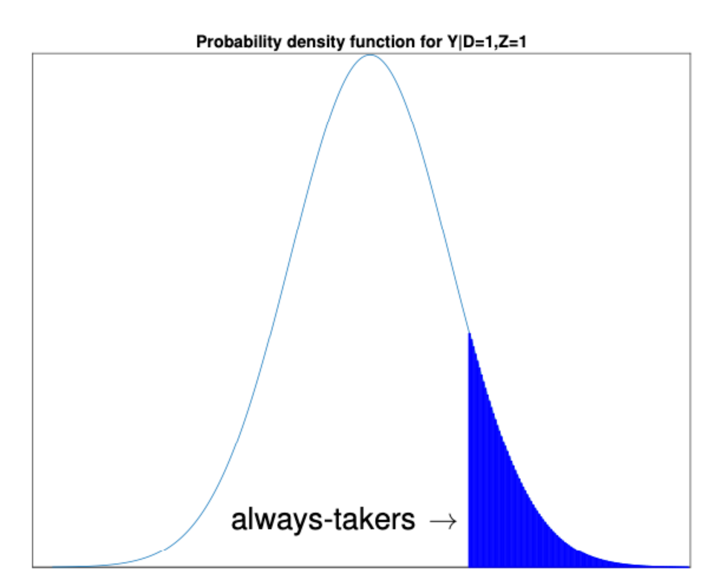
Proposition 1: Sharp bounds for p_{df} (Noack, 2021)

$$\max\left\{\max_{s\in\{0,1\}}\left\{\begin{matrix} sup_A\{P(Y\in A, D=s|Z=1-s) \\ -P(Y \in A, D=s|Z=s)\}\end{matrix}\right\}, 0\right\}$$
$$\leq p_{df} \leq min\{E[D|Z=0], E[1-d|Z=1]\}$$

Lemma 1: Mixture Representation

$$E[Y|D=1,Z=1] = \frac{p_a}{E[D|Z=1]}\mu_{1a} + \frac{E[D|Z=1]-p_a}{E[D|Z=1]}\mu_{1c}$$
$$E[Y|D=1,Z=0] = \underbrace{\frac{p_a}{E[D|Z=0]}}_{p_1}\mu_{1a} + \underbrace{\frac{E[D|Z=0]-p_a}{E[D|Z=0]}}_{1-p_1}\mu_{1df}$$





Proposition 2: Sharp bounds for $\mu_{1a}(p_a)$

$$max\{LB^1_{1a}(p_a), LB^0_{1a}(p_a)\} \leq \mu_{1a}(p_a) \leq min\{UB^1_{1a}(p_a), UB^0_{1a}(p_a)\}$$

Theorem 1: Sharp bounds for μ_{1c} and μ_{1df}

$$\mu_{1c} \epsilon \left[inf_{p_a \epsilon \Upsilon, \mu_{1a} \epsilon \Gamma_1(p_a)}\left\{\frac{E[YD|Z=1]-p_a\mu_{1a}}{E[D|Z=1]-p_a}\right\}, sup_{p_a \epsilon \Upsilon, \mu_{1a} \epsilon \Gamma_1(p_a)}\left\{\frac{E[YD|Z=1]-p_a\mu_{1a}}{E[D|Z=1]-p_a}\right\}\right]$$

$$\mu_{1df} \epsilon \left[inf_{p_a \epsilon \Upsilon, \mu_{1a} \epsilon \Gamma_1(p_a)}\left\{\frac{E[YD|Z=0]-p_a\mu_{1a}}{E[D|Z=0]-p_a}\right\}, sup_{p_a \epsilon \Upsilon, \mu_{1a} \epsilon \Gamma_1(p_a)}\left\{\frac{E[YD|Z=0]-p_a\mu_{1a}}{E[D|Z=0]-p_a}\right\}\right]$$

ILLUSTRATIONS

- DGP 1

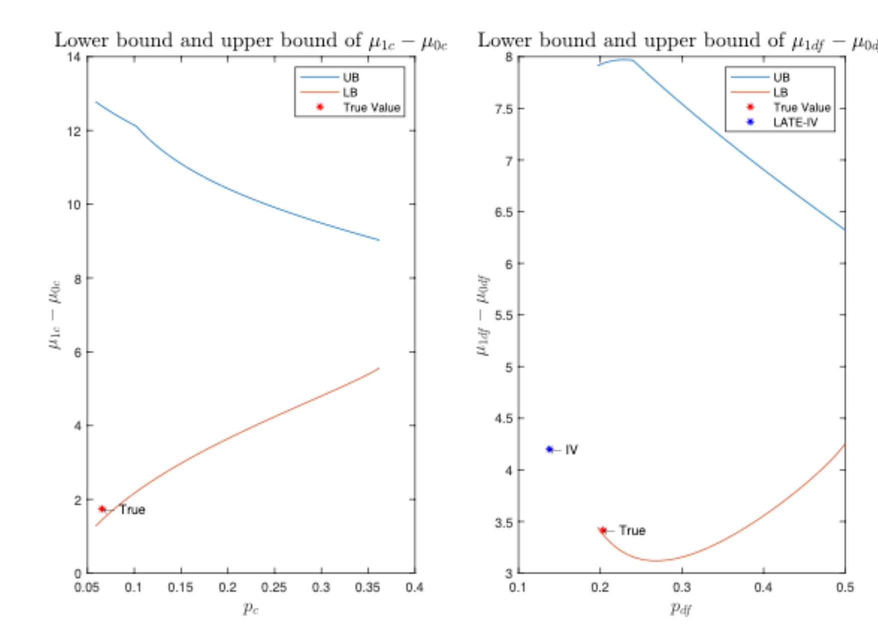
$$\begin{cases} Y = \beta D + U \\ D = 1\{V > \delta Z\} \\ Z = 1\{\varepsilon > 0\} \end{cases}$$

where $\beta = 5\Phi(\theta), U = 2\theta, V = \theta, (\theta, \delta, \varepsilon)' \sim N(0, [0, 0.5, 0]'), \Phi(\cdot)$ is the standard normal CDF.

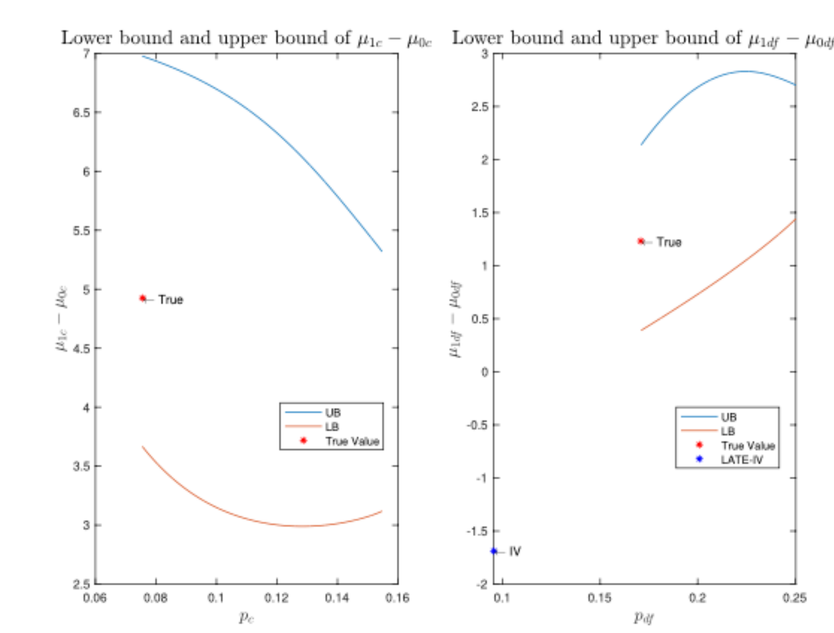
- DGP 2

$$\begin{cases} Y = \beta D + U \\ D = 1\{V_1 > 2Z, V_2 > Z\}, \\ Z = 1\{\varepsilon > 0\} \end{cases}$$

where $\beta = 5\Phi(2V_1 + V_1), U = \frac{1}{2}(V_1 + V_2), (V_1, V_2, \varepsilon)' \sim N(0, I), \Phi(\cdot)$ is the standard normal CDF.

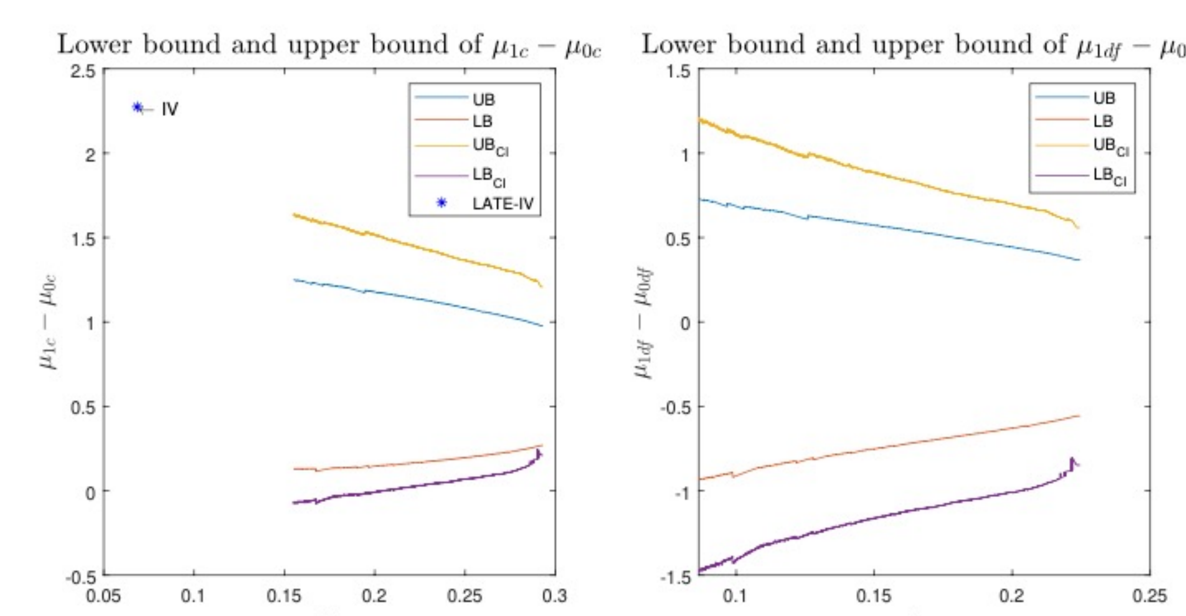


DGP 1: Bounds for LATE with IV and true value



DGP 2: Bounds for LATE with IV and true value

- Empirical examples
  - Card (1995)



  - Angrist and Evans (1998)