3.16 Basic properties of Boolean algebra

Some first properties

The benefit of building circuits from logic gates, rather than directly from transistors, becomes clear after learning some ba
Boolean algebra.

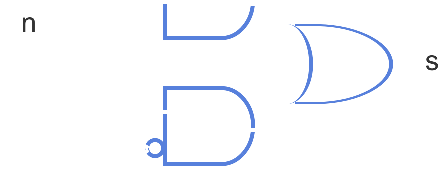
Table 3.16.1: A few basic properties of Boolean algebra.

Property	Name	Description
a(b + c) = ab + ac	***Distributive (for AND)***	Same as multiplication in regular algebra
a + a' = 1	***Complement***	Clearly one of a, a' must be 1 1 + 0 = 1 0 + 1 = 1
a · 1 = a	***Identity***	Result of a · 1 is always a's value *0* · 1 = *0* *1* · 1 = *1*

PARTICIPATION ACTIVITY 3.16.1: The properties of Boolean algebra are useful to simplify an equation, yielding a simpler circuit: Out-of-bed alarm system.

Start 2x speed

s = un + un'



Inputs: u: person up from bed,
n: nurse call button pressed
Outputs: s: sound alarm

Goal behavior: Sound alarm if person up and button pressed, or person up and button not pressed.

s = u

s = un + un'	
s = u(n + n')	Distributive (in reverse)
s = u(1)	Complement
s = u	Identity

u s

Applying Boolean algebra properties led to a simpler expression and thus a simpler circuit. Simplifying expressions is a cor Boolean algebra.

PARTICIPATION ACTIVITY 3.16.2: Simplifying an expression using Boolean algebra.

Original expression: (d')(e + f)(d + d')

(d')(e + f) (d')(e + f)(d + d') d'e + d'f (d')(e + f)(1)

Original expression

Complement

Identity

Distributive

Reset

PARTICIPATION ACTIVITY 3.16.3: Simplify the equation using Boolean algebra properties.

Video: How to use this activity

Start

Properties

Distributive

ab+ac = a(b+c)

Identity

a·1 = a

Complement

a+a' = 1

Select next term Apply Undo

More properties

Below are more properties of Boolean algebra.

Table 3.16.2: More properties.

Property	Name	Description
ab = ba	***Commutative (for AND)***	Same as multiplication for regular algebra
a + b = b + a	***Commutative (for OR)***	Same as addition for regular algebra

a + 1 = *1*	***Null elements***	OR only needs one 1 to evaluate to 1 a = 0 0 + 1 = *1* a = 1 1 + 1 = *1*
a + a = a aa = a	***Idempotent***	0 + 0 = 0 1 + 1 = 1 0 · 0 = 0 1 · 1 = 1

PARTICIPATION ACTIVITY 3.16.4: Simplifying an expression using more Boolean algebra properties.

Original expression: (e + 1)(e'f + fe' + d')

e'f + d' **(1)(e'f + fe' + d')** **e'f + e'f + d'** **e'f + fe' + d'** **(e + 1)(e'f + fe' + d')**

Original expression

Null elements

Identity

Commutative (for AND)

Idempotent

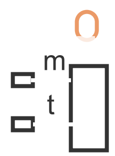
Reset

Example: Motion-sensing light equation

A designer may initially write an equation that matches his/her natural thinking of desired behavior, as below. The designer Boolean algebra properties to obtain a simpler equation (and thus a simpler eventual circuit).

PARTICIPATION ACTIVITY 3.16.5: Simplifying an equation using Boolean algebra properties: Motion-sensing light.

Start 2x speed



Inputs: m: motion sensed
t: test mode
Outputs: i: illuminate lamp
Goal: Illuminate lamp if motion and not test mode,
or if test mode and no motion, or if test mode and motion

$i = mt' + tm' + tm$	Original equation
$i = mt' + m't + mt$	Commutative (for AND)
$i = mt' + m't + mt + mt$	Idempotent
$i = mt' + mt + m't + mt$	Commutative (for OR)
$i = m(t' + t) + (m' + m)t$	Distributive (twice)
$i = m(1) + (1)t$	Complement (twice)
$i = m(1) + t(1)$	Commutative (for AND)
$i = m + t$	Identity (twice)

PARTICIPATION ACTIVITY 3.16.6: Motion-sensing light example.

Consider the above motion-sensing light example.

1) The designer captured the desired behavior little-by-little as an equation, resulting in _____ terms on the right side.
   - 1
   - 2
   - 3

2) The first modification (commutative) just _____ literals within terms.
   - rearranged
   - eliminated

3) The next modification (idempotent) _____ the number of terms.
   - decreased
   - did not change
   - increased

4) Subsequent modifications resulted in a final equation having _____ terms on the right side.
   - 2
   - 3

Summary of common Boolean algebra properties

The following table summarizes commonly-used basic properties of Boolean algebra.

Table 3.16.3: Commonly-used basic properties of Boolean algebra.

Property	Name	Description
$a(b + c) = ab + ac$ $a + (bc) = (a + b)(a + c)$	Distributive (AND) Distributive (OR)	(AND) Same as multiplication in regular algebra (OR) Not at all like regular algebra
$ab = ba$ $a + b = b + a$	Commutative	Variable order does not matter. Good practice is to sort variables alphabetically.
$(ab)c = a(bc)$ $(a + b) + c = a + (b + c)$	Associative	Same as regular algebra
$aa' = 0$ $a + a' = 1$	Complement (AND) Complement (OR)	(AND) Clearly one of a, a' must be 0 $1 \cdot 0 = 0 \cdot 1 = 0$ (OR) Clearly one of a, a' must be 1 $1 + 0 = 0 + 1 = 1$
$a \cdot 1 = a$ $a + 0 = a$	Identity (AND) Identity (OR)	(AND) Result of $a \cdot 1$ is always a's value $0 \cdot 1 = 0$ $1 \cdot 1 = 1$ (OR) Result of $a + 0$ is always a's value $0 + 0 = 0$ $1 + 0 = 1$
$a \cdot 0 = 0$ $a + 1 = 1$	Null elements	Result doesn't depend on the value of a.
$a \cdot a = a$ $a + a = a$	Idempotent	Duplicate values can be removed.
$(a')' = a$	Involution	$(0')' = (1)' = 0$ $(1')' = (0)' = 1$
$(ab)' = a' + b'$ $(a + b)' = a'b'$	DeMorgan's Law	*Discussed in another section*

PARTICIPATION ACTIVITY 3.16.7: Basic properties of Boolean algebra.

1) Which property allows one to change zxy into xyz?

- Associative
- Commutative
- Identity

2) Which property allows one to change a + a into just a?

- Identity
- Idempotent
- Complement

3) Which property allows transforming xy + xy' into x(y + y')

- Complement
- Distributive

4) Which property allows transforming x(y + y') into x(1)?

- Complement
- Identity

5) Which property allows transforming x(1) into x?

- Complement
- Identity

PARTICIPATION ACTIVITY 3.16.8: Simplify the equation using Boolean algebra properties.

Video: How to use this activity

Start

Properties

Distributive

ab+ac	=	a(b+c)

Identity

a·1	=	a

Identity

a+0	=	a

Complement

a+a'	=	1

Idempotence

a+a	=	a

Complement

aa'	=	0

Null elements

a·0	=	0

Null elements

a+1	=	1

Select next term Apply Undo

Provide feedback on this section