

3.19 K-maps: Introduction

K-maps

A **K-map** is a graphical function representation that eases the simplification process for expressions involving a few variables by placing minterms that differ in exactly one variable. K-map is short for **Karnaugh map**. "Map" is used like how a country map is used next to each other. A K-map lays out minterms instead.

A K-map lays out possible minterms as adjacent cells (boxes). Adjacent minterm cells differ by exactly one variable. Each cell gets a 1; other cells get 0.

A K-map is a reoriented truth table.

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3.19.1: A 2-variable K-map: Adjacent cells differ in exactly one variable.

Start ☐ 2x speed

$$y = a'b' + ab'$$

	b	0	1
a	0		a'b
	1	ab'	ab

a	b	y
0	0	1
0	1	0
1	0	1
1	1	0

	b	0	1
a	0	1	0
	1	1	0

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3.19.2: 2-variable K-map basics.

		b	
		0	1
a	0	0	1 (J)
	1	(L)	1 (K)

Given function $y = ab + a'b$, represented in the above figure's K-map.

1) (J) corresponds to which minterm?

Check**Show answer**

2) (K) corresponds to which minterm?

Check**Show answer**

3) (L) should have what value (0 or 1)?

Check**Show answer**

4) Cells (J) and (K) differ in what variable:
a, or b?

Check**Show answer**

- 5) Cells (L) and (K) differ in what variable:
a, or b?

Check**Show answer**

- 6) Cells (L) and (J) differ in how many
variables?

Check**Show answer**

Simplifying an expression with a K-map

Because adjacent minterm cells differ in exactly one variable, a K-map's key benefit is to make $i(j + j')$ simplification opportunities. Adjacent 1's are an $i(j + j')$ opportunity. Circling two adjacent 1's graphically represents the algebraic simplification $i(j + j') = i$. Drawing such a circle, a designer can write a product term with the differing variable omitted.

PARTICIPATION ACTIVITY

3.19.3: Simplification with a 2-variable K-map: $i(j + j')$ opportunities are obvious.

Start ☐ 2x speed

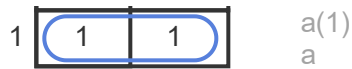
$$y = ab' + ab$$

$$y = a$$

		b	
a	0	1	
	0	0	

$$ab' + ab$$

$$a(b' + b)$$



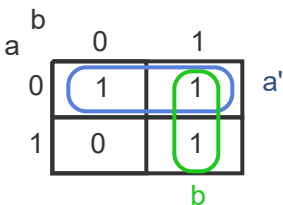
A powerful feature of a K-map is how easily replicating a minterm is achieved (recall an earlier section's example), merely b twice.

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3.19.4: Circling a 1 twice is like replicating a minterm to create $i(j + j')$ opportunities.

Start ☐ 2x speed

$y = ab + a'b + a'b'$
 $y = a' + b$



$a'b' + a'b$
 $a'(b' + b)$
 a'

 $a'b + ab$
 $b(a' + a)$
 b

Table 3.19.1: Rules for simplifying a sum-of-minterms expression with a K-map.

Rule 1:	Cover every 1 at least once using circles. Add circle's term to expression.
Rule 2:	Use fewest and largest circles possible, to achieve simplest expression.

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3.19.5: Basic 2-variable K-map.

		b	
		0	1
a	0	1	0
	1	1	0

		b	
		0	1
a	0	1	0
	1	1	1

		b	
		0	1
a	0	1	1
	1	0	1

Consider the K-maps in the figure above.

1) Circle (L) is what simplified term?

Check

Show answer

2) Is circle (M) necessary? Type: yes or no

Check

Show answer

3) Is circle (P) a good circle? Type: yes or no

Check

Show answer

Example: Out-of-bed alarm

An example in an earlier section involved sounding an alarm ($s = 1$) if a person was up from bed ($u = 1$) and a button pressed person was up and button was not pressed. The captured equation was $s = ub + ub'$. A K-map can be used to simplify the e

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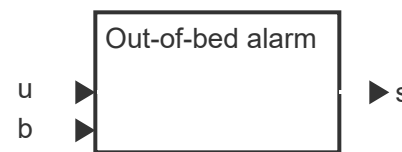
3.19.6: Simplifying with a K-map: Out-of-bed alarm.

Start ☐ 2x speed

$$s = ub + ub'$$

$$s = u$$

u	b	0	1
0		0	0
1		1	1



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3.19.7: Out-of-bed alarm system.

Consider the example above.

- 1) The designer captured behavior as $s = ub + ub'$, but simplification yielded $s = u$. Thus, the designer incorrectly captured the original behavior.
 - ☐ True
 - ☐ False
- 2) The simplification on the K-map was quite obvious.

☒ True

☐ False

Example: Motion-sensing light

An earlier section captured a motion-sensing lamp's behavior and then simplified algebraically. That example can more-easily be done by a K-map instead.

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3.19.8: Simplifying with a K-map: Motion-sensing light.

Start ☐ 2x speed

Inputs: m: motion sensed

t: test mode

Outputs: i: illuminate lamp

Goal: Illuminate lamp if motion and not test mode, or if test mode and no motion, or if test mode and motion

Algebraic simplification

$$\begin{aligned}
 i &= mt' + tm' + tm \\
 i &= mt' + m't + mt \\
 i &= mt' + m't + mt + mt \\
 i &= mt' + mt + m't + mt \\
 i &= m(t' + t) + (m' + m)t \\
 i &= m(1) + (1)t \\
 i &= m(1) + t(1) \\
 i &= m + t
 \end{aligned}$$

K-map simplification

$$i = mt' + m't + mt$$

$$i = m + t$$

	t	0	1
m			
0			1
1	1		1



PARTICIPATION ACTIVITY

3.19.9: Motion-sensing light system.

Consider the example above.

- 1) How many equations were involved using algebraic simplification?
 - ☐ 2
 - ☐ 8
- 2) How many circles were drawn using K-map simplification?
 - ☐ 2
 - ☐ 3
- 3) K-maps help with simplification by not obeying algebraic properties.
 - ☐ True
 - ☐ False

 **Provide feedback on this section**