

1.4 General number bases

Bases

Numbers can use nearly any base, such as base 5 or base 17. Popular bases are:

- 2: Binary, due to computers using 0's and 1's
- 8: Octal, which can compactly represent binary (each octal digit represents three bits)
- 16: Hexadecimal (hex), which can compactly represent binary (each hex digit represents four bits).
- 10: Decimal, due to humans having ten fingers.

When a number's base is unclear, the base is written as a subscript, as in 930_{15} .

Each digit in a base B is weighted by a power of B , as below. Each digit can be 0 to $B - 1$.

Figure 1.4.1: Each digit in a base B is weighted by a power of B .

$$\begin{array}{cccc}
 \overline{} & \overline{} & \overline{} & \overline{} \\
 B^3 & B^2 & B^1 & B^0
 \end{array}
 \qquad
 \begin{array}{cccc}
 v & w & y & z \\
 \overline{} & \overline{} & \overline{} & \overline{} \\
 B^3 & B^2 & B^1 & B^0
 \end{array}$$

$$v \times B^3 + w \times B^2 + y \times B^1 + z \times B^0$$

PARTICIPATION ACTIVITY

1.4.1: General bases.

- 1) For base 5, what are the possible values for each digit?

☐ 0 to 4☐ 0 to 5

2) 104 base 5 is ____ in decimal.

☐ 29☐ 135

3) 128 is a valid base 8 number.

☐ True☐ False

From any base to decimal

Converting from any base to decimal is straightforward: Each digit's decimal value is multiplied by each digit's decimal weight and summed.

PARTICIPATION ACTIVITY

1.4.2: Various bases to decimal tool: Try selecting different bases.

Base-2 ▼

Reset

0

0

0

0

0

0

0

0

 2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

128

64

32

16

8

4

2

1

$$0 \cdot 128 + 0 \cdot 64 + 0 \cdot 32 + 0 \cdot 16 + 0 \cdot 8 + 0 \cdot 4 + 0 \cdot 2 + 0 \cdot 1 = 0$$

(decimal value)

PARTICIPATION

ACTIVITY

1.4.3: General bases to decimal.

1) What is 111 base 5 in decimal?

- ☐ 31
- ☐ 111

2) What is 200 base 30 in decimal?

- ☐ 60
- ☐ 1800

From decimal to any base

Converting from decimal to any base can be done using a simple algorithm. The decimal is divided by the base, and remainder rightmost digit. The process repeats with the quotient and the next digit, until the quotient is 0.

PARTICIPATION
ACTIVITY

1.4.4: Converting from decimal to any base.

Start ☐ 2x speed $2B3_{15}$ Any base
to decimal

$$\begin{aligned}
 &= 2 \times 15^2 + 11 \times 15^1 + 3 \times 15^0 \\
 &= 450 + 165 + 3 \\
 &= 618_{10}
 \end{aligned}$$

Decimal to
any base

2	14	73_{10}
/ 5	/ 5	/ 5
0 rem 2	2 rem 4	14 rem 3
243_5		

$$\begin{aligned}
 &2 \times 5^2 + 4 \times 5^1 + 3 \times 5^0 \\
 &= 50 + 20 + 3 \\
 &= 73
 \end{aligned}$$

**PARTICIPATION
ACTIVITY**

1.4.5: Converting decimal to/from other bases.

1) What is 34 base 6 in decimal?

Check[Show answer](#)

2) What is 102 base 3 in decimal?

Check[Show answer](#)

3) When converting 29 decimal to base 3,
what is the rightmost digit in base 3?

Check[Show answer](#)

4) When converting 29 decimal to base 3,
29 / 3 yields quotient 9. What is the
second digit in base 3?

Check[Show answer](#)

5) What is 29 decimal in base 3?

[Check](#)[Show answer](#)

Any base to any base

To convert a number in any base B directly to any other base C, a straightforward approach first converts the base B number to decimal, then converts that decimal number to base C. Ex: To convert 320_5 to base 8, one starts with $320_5 = 3 \times 25 + 2 \times 5 + 0 \times 1 = 85_{10}$. Converting to base 8: $85/8 = 10$ remainder 5, $10/8 = 1$ remainder 2, $1/8 = 0$ remainder 1, so concatenating yields 125_8 .

PARTICIPATION ACTIVITY

1.4.6: Any base to any base.

Order the steps for converting 200_3 to base 9.

 200_3
 $18 / 9 = 2 \text{ rem } 0$
 $2 / 9 = 0 \text{ rem } 2$
 $2 \times 9 + 0 \times 3 + 0 \times 1$ 20_9 18_{10}

1

2

3

4

5

[Reset](#)

CHALLENGE ACTIVITY

1.4.1: Converting between bases.

Start

Convert from base 8 to base 10

$13_8 =$ $_{10}$

1	2	3	4	5	6
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Check

Next

 [Provide feedback on this section](#)

1

2

3

4

5

6