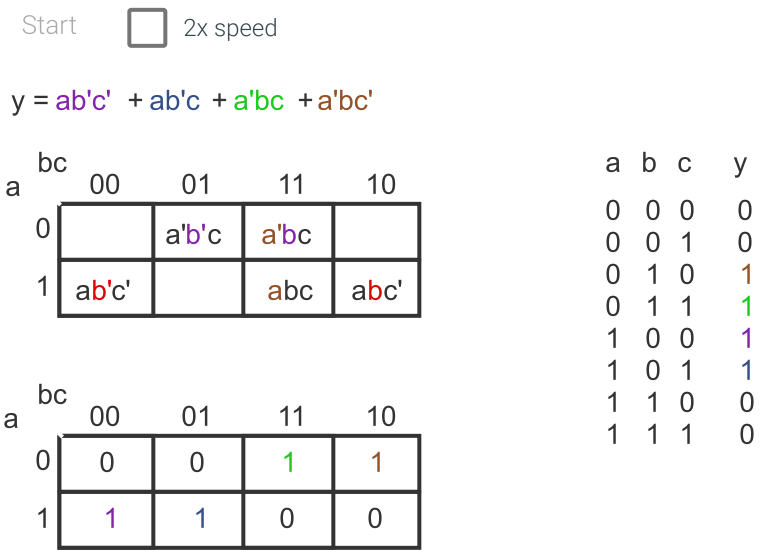
3.21 3- and 4-variable K-maps

3-variable K-map

A K-map for three variables has two variables across the top. For adjacent columns to differ in only one variable, note that t don't count up in binary (00, 01, 10, 11), but rather are 00, 01, 11, 10. Cells on the far left and far right also differ by one varia "adjacent"; the K-map wraps like a bracelet.

PARTICIPATION ACTIVITY 3.21.1: 3-variable K-map basics.



Start □ 2x speed

y = ab'c' + ab'c + a'bc + a'bc'

a \ bc	00	01	11	10
0		a'b'c	a'bc	
1	ab'c'		abc	abc'

a \ bc	00	01	11	10
0	0	0	1	1
1	1	1	0	0

a	b	c	y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

PARTICIPATION ACTIVITY 3.21.2: Click a cell to show/hide the corresponding minterm.

bc a	00	01	11	10
0				
1				

PARTICIPATION ACTIVITY 3.21.3: 3-variable K-map.

bc a	00	01	11	10
0	1(J)	0	(L)	(M)
1	0	0	1(K)	0

Given function y = a'b'c' + abc + a'bc, represented in the above figure's K-map.

1) (J) corresponds to which minterm?

Check Show answer

2) (K) corresponds to which minterm?

Check Show answer

3) (L) should have what value (0 or 1)?

Check Show answer

4) (M) should have what value (0 or 1)?

Check Show answer

5) Cells (L) and (K) differ in what variable: a, b, or c?

Check Show answer

6) Cells (L) and (M) differ in what variable: a, b, or c?

Check Show answer

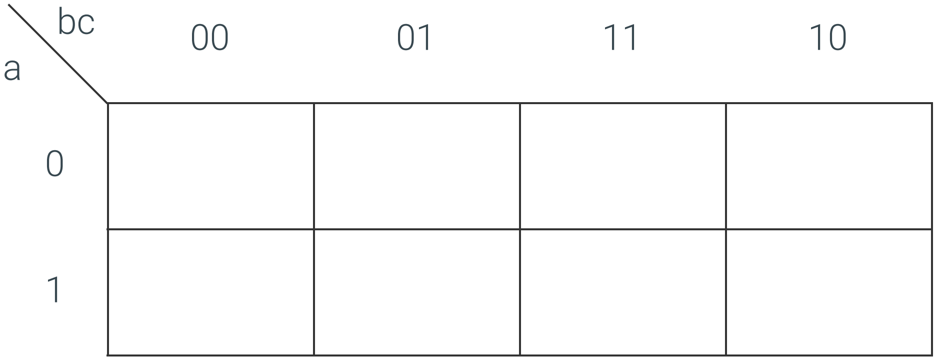
7) Cells (L) and (J) differ in how many variables?

Check Show answer

CHALLENGE ACTIVITY 3.21.1: Select the shown minterm(s).

Start

a' b' c



a \ bc	00	01	11	10
0				
1				

1	2	3	4	5	6

Check Next

1 2 3 4 5 6

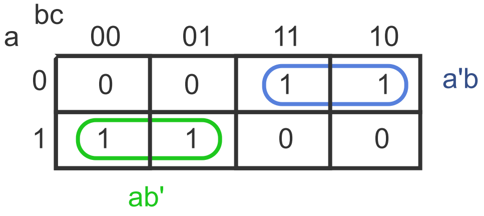
Simplification with a 3-variable K-map

PARTICIPATION ACTIVITY 3.21.4: Simplification with a 3-variable K-map: i(j + j') opportunities are obvious.

Start □ 2x speed

$y = ab'c' + ab'c + a'bc + a'bc'$

$y = ab' + a'b$

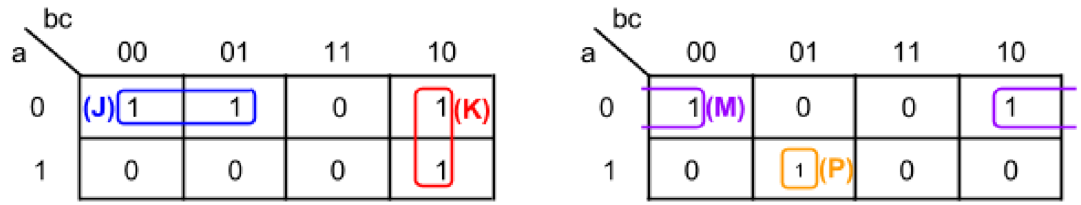


a \ bc	00	01	11	10
0	0	0	1	1
1	1	1	0	0

ab'c' + ab'c	a'bc + a'bc'
ab'(c' + c) ab'(1) ab'	a'b(c + c') a'b(1) a'b

PARTICIPATION ACTIVITY

3.21.5: Simplification with 3-variable K-maps.



a \ bc	00	01	11	10
0	1	1	0	1
1	0	0	0	1

a \ bc	00	01	11	10
0	1	0	0	1
1	0	1	0	0

1) Circle (J) corresponds to what simplified term?

Check Show answer

2) Circle (K) corresponds to what simplified term?

Check Show answer

3) Circle (M) corresponds to what simplified term?

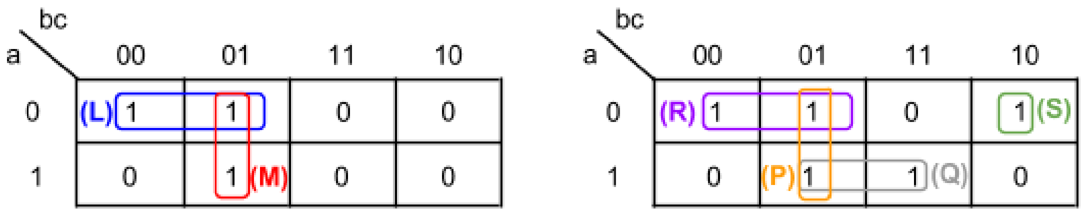
Check Show answer

4) Circle (P) is what term?

Check Show answer

PARTICIPATION ACTIVITY

3.21.6: More simplification with 3-variable K-maps.



a \ bc	00	01	11	10
0	1	1	0	0
1	0	1	0	0

a \ bc	00	01	11	10
0	1	1	0	1
1	0	1	1	0

1) Circle (L) is what simplified term?

Check Show answer

2) Circle (M) is what simplified term?

Check Show answer

3) Is circle (P) necessary? Type: yes or no

Check Show answer

4) Is circle (Q) necessary? Type: yes or no

Check Show answer

5) Is circle (S) a good circle? Type: yes or no

Check Show answer

CHALLENGE ACTIVITY 3.21.2: Add fewest and largest circles to cover all the 1s.

Start

a \ bc	00	01	11	10
0	0	0	0	1
1	0	0	0	0

Add circle Undo

1

2

3

4

5

6

7

1 2 3 4 5 6 7

Check Next

CHALLENGE ACTIVITY 3.21.3: Write the simplified term or terms.

1 2 3 4 5 6 7

Start

a \ bc	00	01	11	10
0	0	0	0	0
1	0	1	0	0

1 2 3 4 5 6 7

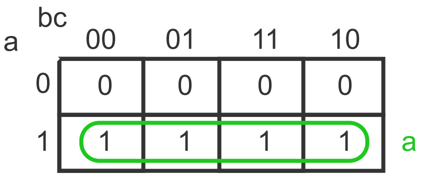
Check Next

Larger circles

A circle may encompass four adjacent 1's, which removes two variables rather than just one.

PARTICIPATION ACTIVITY 3.21.7: Circling four 1's removes two variables.

Start □ 2x speed

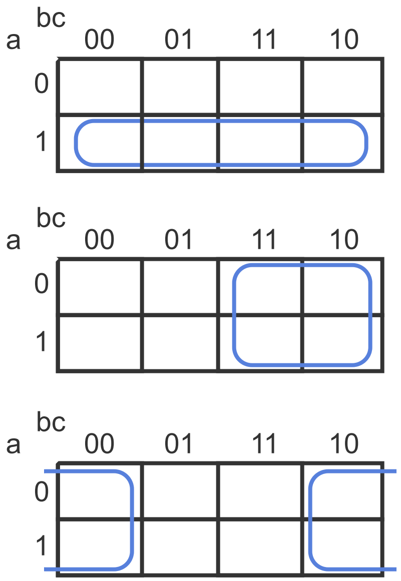


a bc	00	01	11	10
0	0	0	0	0
1	1	1	1	1

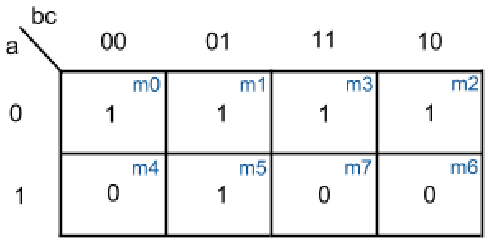
$y = ab'c' + ab'c + abc + abc'$

$y = a\,(b'c' \;+\; b'c \;+\; bc \;+ bc')$

$y = a(b'(c' + c) + b(c + c'))$
$y = a(b'(1) + b(1))$
$y = a(b + b')$
$y = a(1)$
$y = a$



PARTICIPATION ACTIVITY 3.21.8: Drawing largest circle with 3-variable K-maps.



a bc	00	01	11	10
0	m0 1	m1 1	m3 1	m2 1
1	m4 0	m5 1	m7 0	m6 0

1) A circle covering which cells should be drawn?

○ m0, m1

○ m3, m2

○ m0, m1, m3, m2

2) A circle covering which cells should be drawn?

○ m1, m5

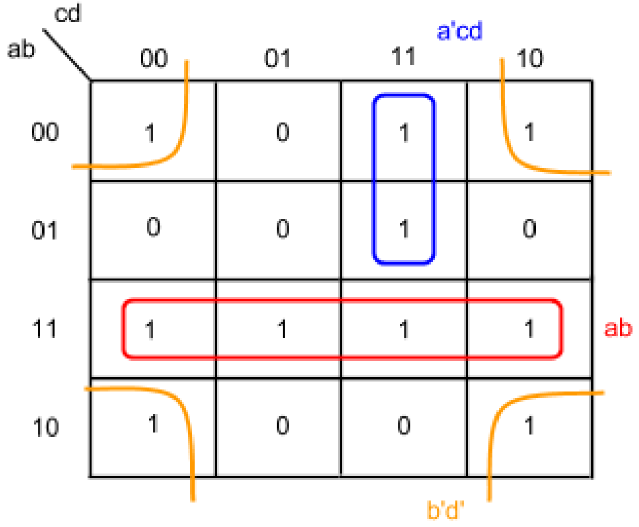
○ m5

○ m5, m7

4-variable K-map

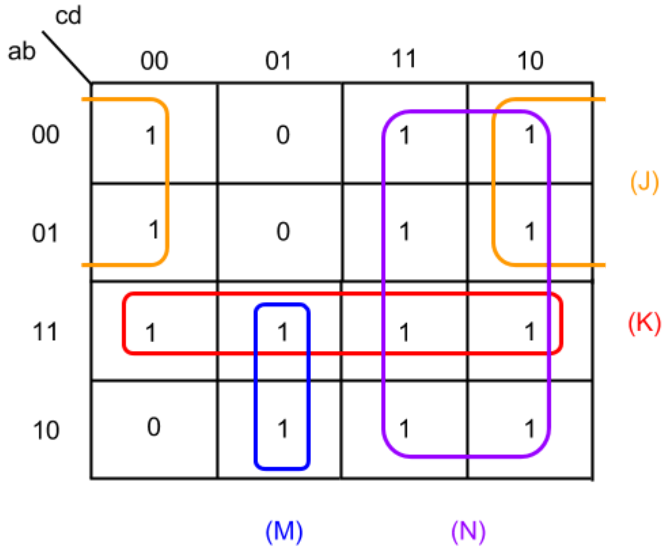
A K-map can be drawn for four variables. Top and bottom rows are adjacent (as are left and right columns). Valid circle size
or 16 cells.

Figure 3.21.1: Four-variable K-map example.



ab \ cd	00	01	11	10
00	1	0	1	1
01	0	0	1	0
11	1	1	1	1
10	1	0	0	1

PARTICIPATION ACTIVITY 3.21.9: Simplification with 4-variable K-maps.



ab \ cd	00	01	11	10
00	1	0	1	1
01	1	0	1	1
11	1	1	1	1
10	0	1	1	1

Match the simplified term.

(J) (N) (K) (M)

a'd'

ab

ac'd

c

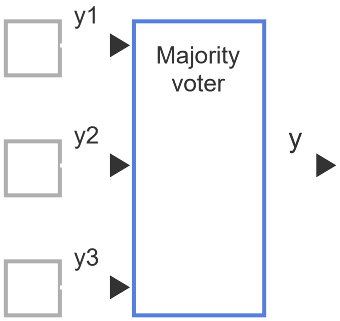
Reset

Example: Majority voter circuit

An earlier section captured a majority voter circuit's behavior as a truth table. The resulting equation can be simplified using map before creating a circuit, yielding a smaller circuit than for the original unsimplified equation.

PARTICIPATION ACTIVITY 3.21.10: Majority voter circuit, simplified using a K-map.

Start □ 2x speed

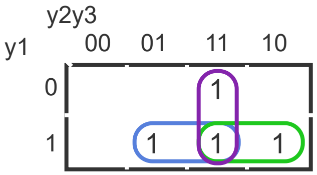


Capture

y1	y2	y3	y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

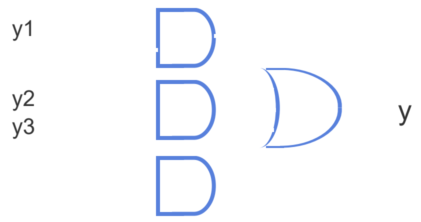
Convert

y = y1'y2y3 + y1y2'y3 + y1y2y3' + y1y2y3



y1 \ y2y3	00	01	11	10
0			1	
1		1	1	1

$y = y1y3 + y2y3 + y1y2$



PARTICIPATION ACTIVITY 3.21.11: Majority voter circuit simplified using a K-map.

Consider the example above. Type numbers as 2, not two.

1) The K-map had _____ 1's.

Check Show answer

2) The designer drew _____ circles.

Check Show answer

3) _____ circles included the cell for y1y2y3.

Check Show answer

4)

If each AND and OR gate input is two transistors, the final circuit requires _____ transistors.

Check Show answer

Expression simplification in programming

K-maps usage is not restricted to just digital design. Computer programmers sometimes simplify expressions using K-maps. Ex: A program may make a decision based on an expression: If ((isRed AND !isBlue) OR (!isRed AND isBlue) OR (isRed AND isBlue) then take action X. (! means NOT).

The expression represents a function with two variables. Using a two-variable K-map, the programmer simplifies the expression, yielding: If (isRed OR isBlue) then take action X. The resulting program is easier to read.

Exploring further:

- Online K-map tool (T. Thormaehlen)

! Provide feedback on this section