

3.18 Two-level combinational logic minimization

Simplifying a sum-of-products expression

During design, simplifying a sum-of-products expression before converting to a circuit can result in a smaller circuit.

PARTICIPATION ACTIVITY

3.18.1: Simplifying an expression before converting to a circuit.

Start ☐ 2x speed

A store's entry system should activate ringer (output $r = 1$) if motion is sensed (input $a = 1$) and daylight is detected (input $b = 1$) and clerk is not present (input $c = 0$), OR if motion sensed and no daylight detected and clerk not present.

$$r = a b c' + a b' c'$$

$$r = a c' (b + b')$$

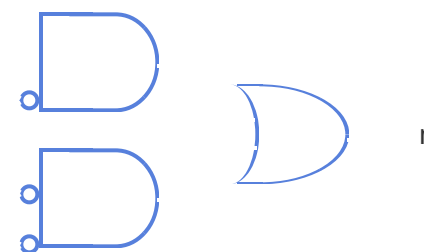
$$r = a c' (1)$$

$$r = a c'$$

Distributive
Complement
Identity

a
b
c

$$r = a b c' + a b' c'$$



a
b
c

$$r = a c'$$



**PARTICIPATION
ACTIVITY**

3.18.2: Simplifying a sum-of-products expression.

Consider the example above.

- 1) How many literals exist in the original expression?
☐ 3
☐ 6
- 2) If each AND or OR gate input requires two transistors, how many transistors does the original expression's circuit require? Ignore NOT gates.
☐ 8
☐ 16
- 3) If each AND or OR gate input requires two transistors, how many transistors does the simplified expression's circuit require? Ignore NOT gates.
☐ 4
☐ 16

Seeking $i(j + j')$ opportunities

Given a sum-of-products expression, knowing what to simplify can be hard. To make simplification opportunities more obvious algebraic simplification process is to:

- Convert to sum-of-minterms
- Seek $i(j + j')$ opportunities: $ij + ij' = i(j + j') = i$

**PARTICIPATION
ACTIVITY**3.18.3: Seeking $i(j + j')$ opportunities.Start ☐ 2x speedOriginal expression: $ac + ab'c$

$$ac(1) + ab'c$$

$$ac(b + b') + ab'c$$

$$abc + ab'c + ab'c$$

$$abc + ab'c$$

$$ac(b + b')$$

$$ac(1)$$

$$ac$$

**PARTICIPATION
ACTIVITY**3.18.4: Seeking $i(j + j')$ simplification opportunities.Only type the ? part. Type answers as: ab'

1) $y = cd + cd'$

$y = c(?)$

Check

[Show answer](#)

2) $y = c(d + d')$

$y = c(?)$

Check

[Show answer](#)

3) $y = c(1)$

$y = ?$

Check**Show answer**

4) $y = efg + ef'g$

$y = eg(?)$

Check**Show answer**

5) $y = cd' + cd$

$y = c(?)$

Check**Show answer**

6) $y = dc + d'c$

$y = c(?)$

Check**Show answer****PARTICIPATION
ACTIVITY**

3.18.5: First translating to sum-of-minterms, then seeking simplification opportunities.

Simplify. Only type the ? part. Type answers as: ab'

1) $y = cd + c$
 $y = cd + c(d + ?)$

Check[Show answer](#)

2) $y = cd + c$
 $y = cd + c(d + d')$
 $y = cd + cd + ?$

Check[Show answer](#)

3) $y = cd + cd + cd'$
 $y = ? + cd'$

Check[Show answer](#)

4) $y = cd + cd'$
 $y = c(?)$

Check[Show answer](#)

5) $y = c(d + d')$
 $y = ?$

Check[Show answer](#)

Algebraic simplification by hand can be hard

The algebraic simplification process can be hard to do by hand.

PARTICIPATION ACTIVITY

3.18.6: Algebraic simplification can be hard to do by hand.

Start ☐ 2x speed

Original expression: $ab + a'$
 $ab + a'(b + b')$
 $ab + a'b + a'b'$
 $\underline{ab + a'b} + \underline{a'b + a'b'}$
 $(a + a')b + a'(b + b')$
 $(1)b + a'(1)$
 $a' + b$

PARTICIPATION ACTIVITY

3.18.7: Simplifying algebraically can be hard.

Consider the example above.

1) What expression came after: $ab + a'b + a'b'$

- ☐ $(a + a')b + a'b'$
- ☐ $ab + a'(b + b')$
- ☐ $ab + a'b + a'b + a'b'$

2) How many equations were written

during the simplification process?

- ☐ 3
- ☐ 7
- ☐ 21

 **Provide feedback on this section**