

## 3.12 Sum-of-products form

### Sum-of-products

Circuits are commonly designed by creating a simplified expression in sum-of-products form, then converting to a simple c

A **product term** is an ANDing of (one or more) variables, like  $ab'c$ . A product term is sometimes called just a *product* or just expression in **sum-of-products** form consists solely of an ORing of product terms, like  $ab'c + ab$ .

Due to similarities with regular algebra, convention uses the word "product" for AND and "sum" for OR; hence the name "sum-of-products". But AND is not really multiplication (product), and OR is not really addition (sum).

#### PARTICIPATION ACTIVITY

#### 3.12.1: Products.

Choose Yes if the item is a product term.

1)  $abc$

☐ Yes

☐ No

2)  $a'b'cd$

☐ Yes

☐ No

3)  $a + bc$

☐ Yes

☐ No

4) a

- ☐ Yes  
☐ No

**PARTICIPATION  
ACTIVITY**

## 3.12.2: Sum-of-products form.

Choose Yes if the expression is in sum-of-products form.

1)  $abc' + abc + ab'c$ 

- ☐ Yes  
☐ No

2)  $abc' + c$ 

- ☐ Yes  
☐ No

3)  $a + c$ 

- ☐ Yes  
☐ No

4)  $ab$ 

- ☐ Yes  
☐ No

5) a

- ☐ Yes  
☐ No

6)  $a(b + c)$

☐ Yes

☐ No
7)  $(a + b)(b' + c)$ 
☐ Yes

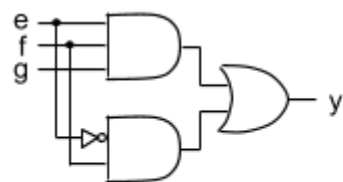
☐ No

## Converting sum-of-products to a circuit

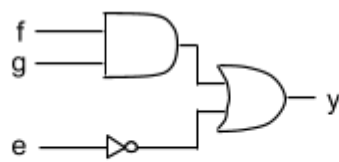
A sum-of-products equation can be easily converted to a circuit, consisting of a column of AND gates (one gate per product) by an OR gate, which is known as a **two-level circuit**. (The NOT gates preceding the AND gates aren't considered a level).

### PARTICIPATION ACTIVITY

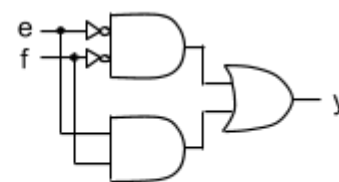
3.12.3: Converting a sum-of-products equation into a two-level circuit.



(a)



(b)



(c)



(d)

(a)

(d)

(c)

(b)

$$y = fg + e'$$

$$y = efg + e'f$$

$$y = e'f' + ef$$

$$y = f + g$$

Reset

## Converting to sum-of-products before creating a circuit

Circuits are commonly created by multiplying out an initial expression into sum-of-products form, then creating a two-level

### PARTICIPATION ACTIVITY

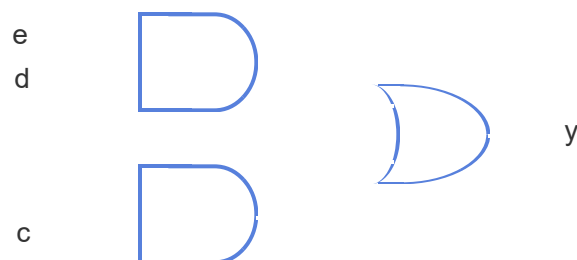
3.12.4: Multiplying out an expression to sum-of-products, before creating a circuit.

Start ☐ 2x speed

Goal: Sound alarm (set output  $y = 1$ ) if alarm is enabled ( $e = 1$ )  
AND (door is open ( $d = 1$ ) OR window is open ( $c = 1$ )).

$$y = e(d + c)$$

$$y = ed + ec$$



### PARTICIPATION ACTIVITY

3.12.5: Multiplying out expressions to convert to sum-of-products form.

Transform to sum-of-products form. Simplify when possible.  
Type only the ? part.

1)  $y = a(b + b'c)$

$y = ab + ?$

**Check**

[Show answer](#)

2)  $y = c(a + b)$

$y = ac + ?$

**Check**

[Show answer](#)

3)  $y = ab(c + d)$

$y = abc + ?$

**Check**

[Show answer](#)

4)  $y = ac(b + a)$

$y = abc + ?$

**Check**

[Show answer](#)

5)  $y = a + c(b + ab')$

$y = ? + bc + ab'c$

Check

Show answer

6)  $y = a'(b + b'c)$

$y = a'b + ?$

Check

Show answer

7)  $y = (a' + b)(c + d)$

$y = a'c + a'd + ?$

Check

Show answer

### Example: Interrupt logic component

A processor executes computer programs. Various devices (like keyboards or USB ports) surrounding a processor may request the processor to execute a sub-program on behalf of that device, a request known as an **interrupt**. Devices may be in two categories: low-priority and high-priority, and the processor may disable either category or both.

- Low-priority: keyboard (k = 1), mouse (m = 1), USB port (u = 1). Disable all: p = 1.
- High-priority: network interface (n = 1), battery backup (b = 1). Disable all: q = 1.

An interrupt logic component determines whether interrupt requests from devices result in an actual interrupt to the processor.

#### PARTICIPATION ACTIVITY

3.12.6: Multiplying out an expression into sum-of-products form before creating a circuit: An interrupt logic example.

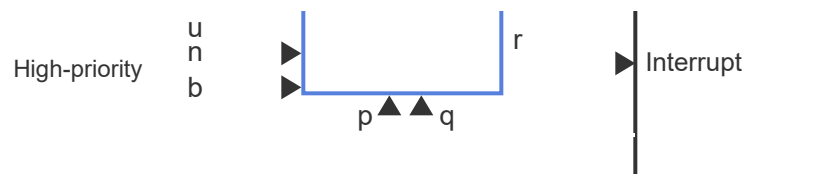
Start ☐ 2x speed

Low-priority

k  
m

Processor

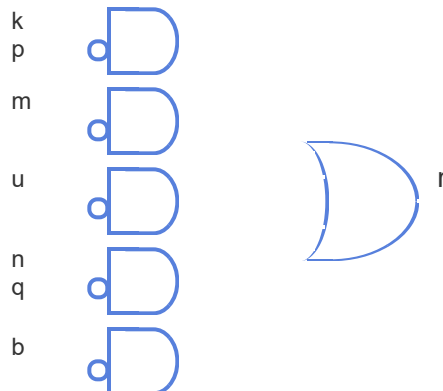
## 3.12. Sum-of-products form



## Interrupt logic

$$r = (k + m + u)p' + (n + b)q'$$

$$r = kp' + mp' + up' + nq' + bq'$$

PARTICIPATION  
ACTIVITY

## 3.12.7: Interrupt logic example.

Consider the above interrupt logic example.

- 1) How many inputs does the interrupt logic component have?

Check

Show answer

- 2) How many outputs does the interrupt

logic component have?

**Check**

**Show answer**

- 3) Did the designer originally capture the interrupt logic's behavior as a sum-of-products equation? Type yes or no.

**Check**

**Show answer**

- 4) How many product terms are in the sum-of-products form of the interrupt logic's equation?

**Check**

**Show answer**

- 5) How many AND gates are in the interrupt logic's circuit?

**Check**

**Show answer**

 **Provide feedback on this section**