3.13 Truth tables

Truth table basics

A Boolean function can be represented in various ways, like an equation, a circuit, or a truth table. A ***truth table*** lists all poss value combinations on the left, and lists the function's value for each combination on the right. Each row corresponds to a minterm. Generating all combinations is done by counting up in binary.

Note: Minterms are sometimes written as m0, m1, ..., indicating their row's decimal equivalent: a'b'c' is 000 or m0, a'b'c is 0(

A function with N variables will have a truth table with 2^N rows:

- 2 variables yields $\mathbf{2^2 = 4}$ rows
- 3 variables yields $\mathbf{2^3 = 8}$ rows
- 4 variables yields $\mathbf{2^4 = 16}$ rows
- (And so on)

PARTICIPATION ACTIVITY 3.13.1: Truth table basics.

Start 2x speed

		a	b	f(a, b)
m0	a'b'	0	0	1
m1	a'b	0	1	0
m2	ab'	1	0	0
m3	ab	1	1	1

	a b c	f(a, b, c)
m0	0 0 0	
m1	0 0 1	
	0 1 0	
	0 1 1	
...	1 0 0	
	1 0 1	
	1 1 0	
m7	1 1 1	

PARTICIPATION ACTIVITY 3.13.2: 2-input truth table.

Consider the following incomplete truth table.

a	b	f(a, b)
0	0	M
K	J	
1	0	N
1	L	

1) What should J be?
   - ○ 1
   - ○ 0

2) What should K be?
   - ○ 1
   - ○ 0

3) What should L be?
   - ○ 1
   - ○ 0

4) Row 00 corresponds to what possible minterm?
   - ○ a'b'
   - ○ ab

5) Row 10 corresponds to what possible

minterm?

- a'b
- ab'

6) Function f(a, b) = ab' is to be represented on the above table. What value should be written for N?

- 1
- 0

7) Function f(a, b) = ab' is to be represented on the above table. What value should be written for M?

- 1
- 0

8) A function f(a, b, c, d, e) has 5 variables. How many rows will the function's truth table have?

- 5
- 32

Converting a truth table to an equation

A function captured as a truth table can be transformed to a sum-of-minterms equation by summing the minterms in rows equation can then be converted to a circuit.

PARTICIPATION ACTIVITY 3.13.3: Converting a truth table to an equation, and then a circuit.

Start 2x speed

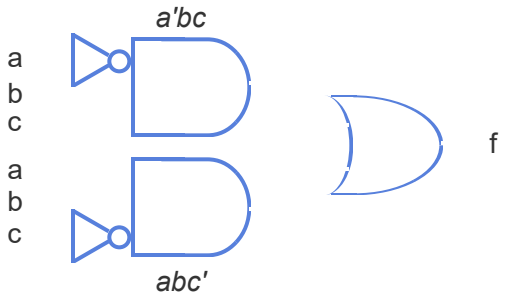
Truth table

a	b	c	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

Equation

f = a'bc + abc'

Circuit



PARTICIPATION ACTIVITY 3.13.4: Converting a truth table to an equation.

Consider the following truth table:

e	f	y	
0	0	0	
0	1	1	(a)
1	0	0	
1	1	1	(b)

1) Type the minterm corresponding to row (a).

Check Show answer

2) Type the minterm corresponding to row

(b).

Check Show answer

3) y = ?

Check Show answer

PARTICIPATION ACTIVITY 3.13.5: Converting a truth table to an equation and then circuit.

Consider the following truth table. An equation will be y = ___ + ___.

a b	y
0 0	0
0 1	1
1 0	1
1 1	0

1) Which is one of the minterms in the equation?

- a'b'
- a'b

2) Which is one of the minterms in the equation?

- ab'
- ab

3) Considering y's equation, how many AND gates will exist in a circuit derived directly from that equation?
   - 1
   - 2
   - 3

4) Considering y's truth table, how many AND gates will exist in a circuit derived directly from the equation derived from that table?
   - 2
   - 4

5) A 3-input function's truth table has 5 1's. How many AND gates will exist in a circuit derived directly from the equation derived from that table?
   - 3
   - 5
   - 8

Capturing behavior as a truth table

Some functions are more easily captured as a truth table, others as an equation.

PARTICIPATION ACTIVITY 3.13.6: Some functions are more easily captured as a truth table, others as an

ACTIVITY equation.

Start 2x speed

Three parking spaces exist (a, b, c).
A parked car causes the space's variable to be 1.

a	b	c
0	1	0

y is 1 if two cars are parked adjacently

z is 1 if a car is parked on an edge

a b c	y
0 0 0	0
0 0 1	0
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	0
1 1 0	1
1 1 1	1

y = a'bc + abc' + abc

z = a + c

PARTICIPATION ACTIVITY 3.13.7: Capturing behavior as a truth table or equation.

Consider the above parking space example.

1) Function y could have been captured directly as an equation.
   - ○ True
   - ○ False

2) Converting a truth table to a sum-of-minterms equation involves much thought and tradeoffs.

- ○ True
- ○ False

3) A function has 12 inputs. Which is a designer more likely to try first when capturing the function's behavior?
   - ○ Truth table
   - ○ Equation

4) Function z could have been captured directly as a truth table.
   - ○ True
   - ○ False

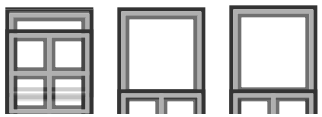
5) If function z was captured as a truth table, how many rows would have an output 1 for z?
   - ○ 2
   - ○ 6

Example: Windows-open detector

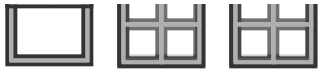
PARTICIPATION ACTIVITY 3.13.8: Windows-open detector: A truth table easily captures the behavior.

Start ☐ 2x speed

Three windows, each with sensor a, b, c. 1 means open.
Sound alarm (set y = 1) if two or more windows are open



a	b	c	y
0	0	0	0



0	0	1	0	
0	1	0	0	
0	1	1	1	Two windows are open
1	0	0	0	
1	0	1	1	Two windows are open
1	1	0	1	Two windows are open
1	1	1	1	Three windows are open

y = a'bc + ab'c + abc' + abc

PARTICIPATION ACTIVITY 3.13.9: Windows-open example.

Consider the above windows-open example.

1) Could the behavior have been captured directly as an equation?
   - Yes
   - No

2) How many truth table rows have 1's in the output column?
   - 4
   - 8

3) The equation also includes specific minterms for truth table rows with 0's.
   - True
   - False

4) The functionality of y differs depending on whether the designer captured behavior using a truth table or equation.

- ○ True
- ○ False

Converting an equation to a truth table

Sometimes a designer wants to convert an equation to a truth table. Such conversion can be achieved by first transforming the equation to sum-of-minterms (discussed in an earlier section). Then, the designer can simply place a 1 in each minterm's row in the truth table.

Like sum-of-minterms form, a truth table is a canonical representation (discussed earlier) of a function.

CHALLENGE ACTIVITY 3.13.1: Convert the table to a sum-of-minterms.

Start

a	b	y
0	0	0
0	1	0
1	0	1
1	1	0

1 2 3 4 5

y = Ex: ab'+a'b

1 2 3 4 5

Check Next

Provide feedback on this section