Part 3
Introduction to Legged Locomotion

Reading:

1. Please download the following book from the University Library

   Sharbafi, M. A., & Seyfarth, A. (Eds.). (2017). Bioinspired legged locomotion: models, concepts, control and applications.

Scale effect:

- Locomotion emerges from complex interactions between animals' **neural**, **sensory** and **motor** systems. Between their muscle/body dynamics and their environments.

- Observation: Smaller faster animals take longer step length compared to their leg length.

- Goal: develop a locomotion strategy, which can apply for different types of legged robots.

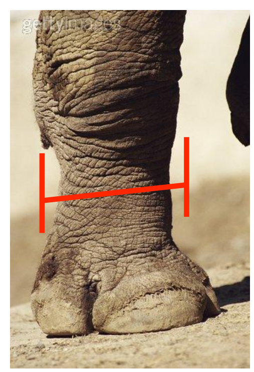
- Despite the **energetic cost per gram per stride** is almost constant along different sized animals running at equivalent speed, smaller animals are generally **less efficient** than bigger ones. This because the specific energetic cost is proportional to **stride frequency**

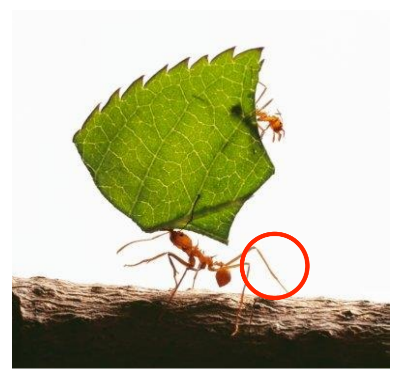
It's all relative ...

- Volume related forces and surface forces scales differently, the former proportional to l^3 the latter to l^2. This influence the bone diameter, the posture and many other gait characteristics, such as **maximum exerted force** respect to body weight (BW)

- As both the **maximum yield stress** and **safety factor** in bones are about constant for different animals, in small animals supporting and moving the body is not a critical issue. Thus they can easily achieve high relative running speed. This performance would be impossible for an elephant due to structural bone and muscles limitations.

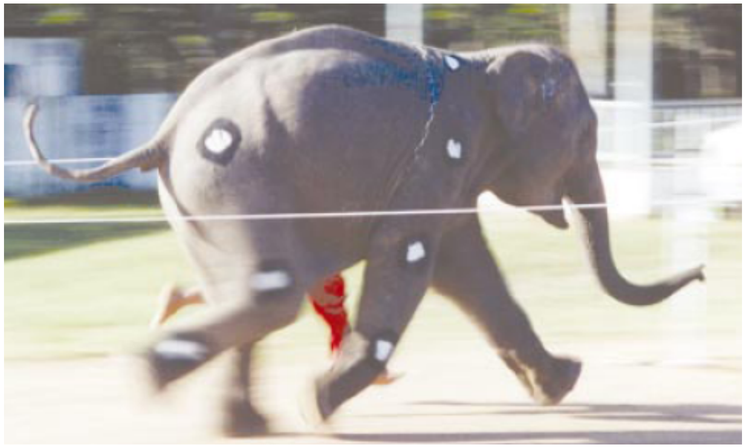
Relative running speed [3]	[body length s^{-1}]
Elephant (*Loxodonta Africana*)	1.40
Rodent (*Dipodomys merriami*)	80.31





- A force comparable to body weight means small variation in vertical momentum, and thus a short airborne phase. Elephants can reach speeds of 6 m/s, without engaging an airborne phase, while rodents difficultly move at low speeds.

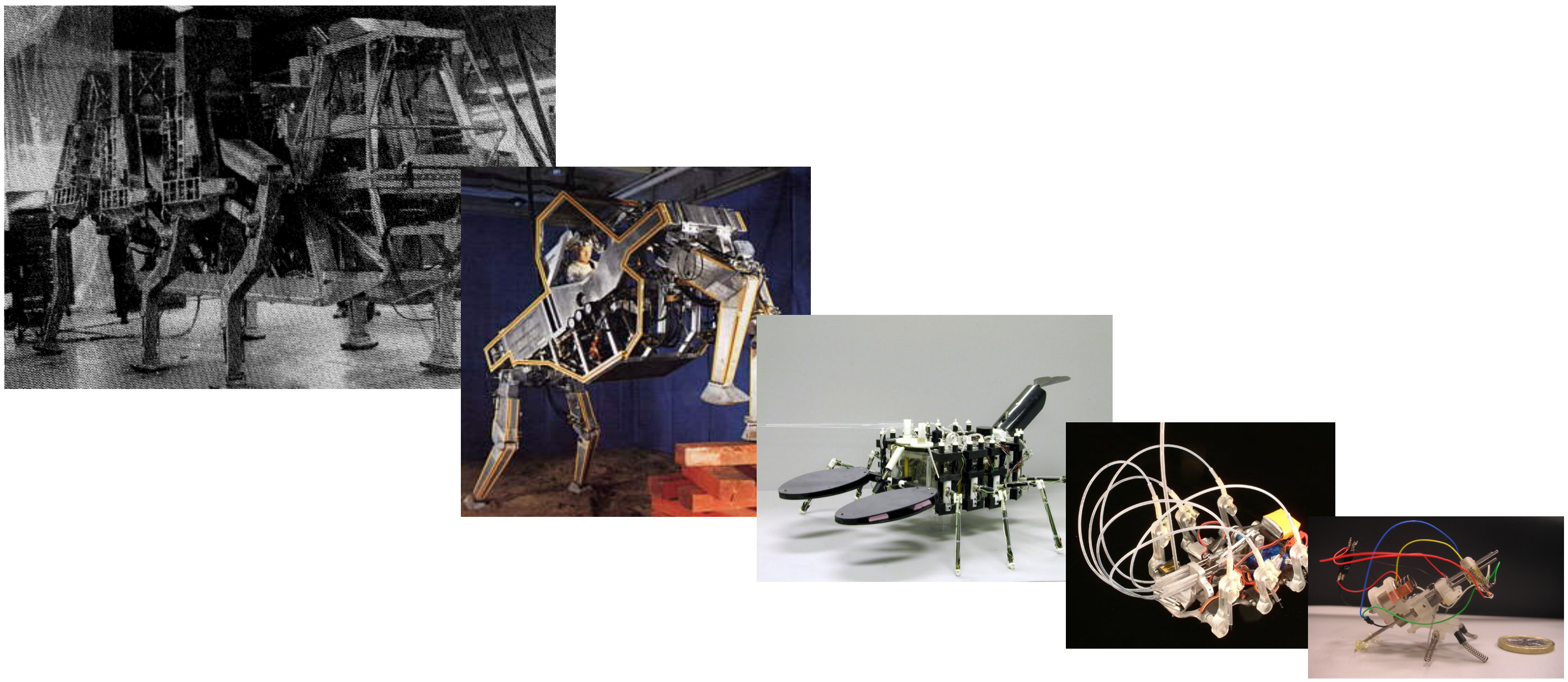
- Gait frequency is triggered by **gravity**: relatively high vertical forces naturally makes it easier for small animals to jump several body length within one step



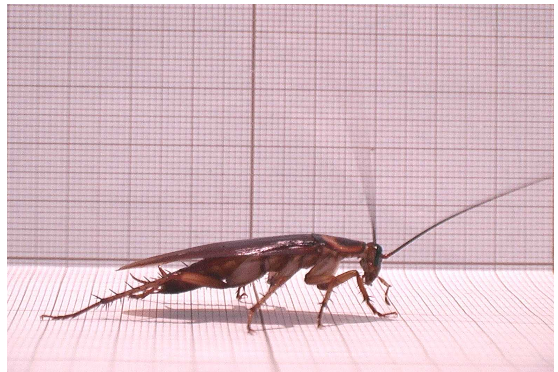
Animal	**Maximum force in *body weight*** [5]
Human	***3***
Kangaroo rat	***8***
Locust	***13***
Froghoppers	***400***
Fleas	***135***

Bio-inspiration and scale effect.

- Taking into account scale effects, small robots would have a longer airborne phase respect to bigger ones, leading to a jumping gait as a possible efficient solution for locomotion



Different approaches: Many legs high frequency



RHex
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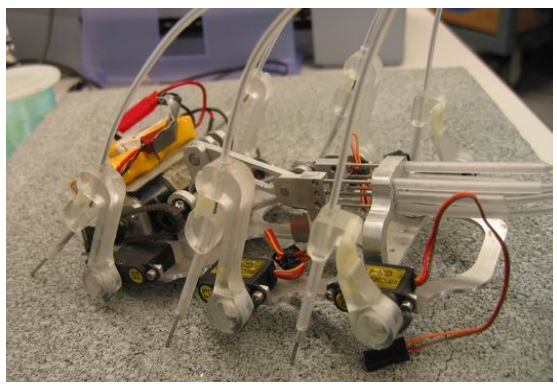
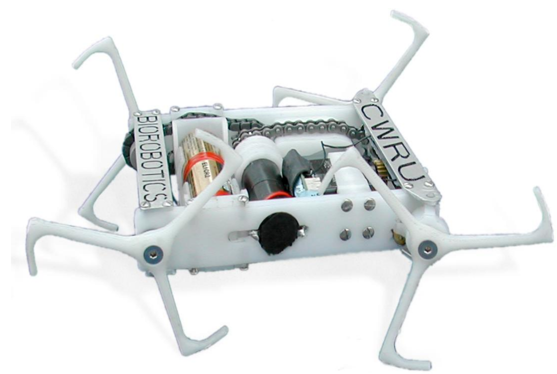


iSprawl
Stanford University



Whegs
Case western University

Locomotion results from **complex**, **high-dimensional**, **non-linear**, **dynamically coupled** interactions between an *organism* and its *environment*

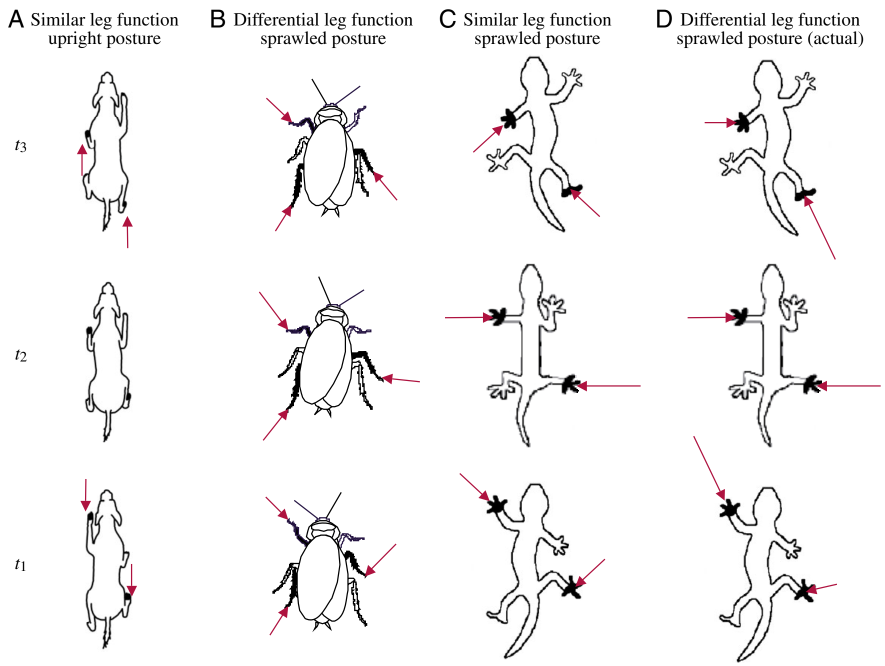


Fig. 1. Horizontal plane GRFs in upright- and sprawled-posture trotters during a step when running at a constant average speed. (A) Upright-posture quadruped running with similar leg function. Fore- and hindlegs first (t_1) both generate decelerating fore–aft forces (arrows) followed by an accelerating force later in the step (t_3). No lateral GRFs are present. (B) Sprawled-posture hexapod running with differential leg function. Fore- and middle legs first (t_1) both generate decelerating forces, while the hindleg generates an accelerating force (Full et al., 1991). All legs develop large lateral forces directed toward the midline. At midstep (t_2) forelegs continue to generate decelerating forces and the hindleg an accelerating force. The middle leg only develops a lateral force. At the end of the step (t_3), the foreleg generates a decelerating force. Hind- and middle legs both generate accelerating forces. (C) Hypothetical sprawled-posture quadruped running with similar leg function resulting from adding opposing lateral forces to the upright posture pattern in A. Fore- and hindlegs first (t_1) both generate decelerating forces followed by an accelerating force later in the step (t_3). Lateral GRFs were added assuming sprawled-posture animals tend to produce them. Horizontal forces sum to produce a clockwise yaw throughout the step. (D) Sprawled-posture quadruped running with differential leg function. GRFs approximate those measured in the present study on geckos. The foreleg first (t_1) generates the majority of fore–aft decelerating force. At midstep (t_2), fore- and hindlegs only generate lateral forces directed toward the midline. Later in the step (t_3) hindlegs generate all of the fore–aft accelerating force. The major decelerating force by the foreleg (t_1) and accelerating force by the hindleg (t_3) are directed to the animal's COM, reducing yaw, and are aligned axially along the leg, reducing joint moments.

JJ Chen, AM Pettie, K. Autumn and R.J. Full "Differential leg function in a sprawled-posture quadrupedal trotter" The Journal of Experimental Biology 209, 249-259

There are 3 broad approaches:

1. **Neurobiology approach:** studies of central pattern generators (CPGs):
   - Networks of neurons in spinal cords of vertebrates and invertebrate thoracic ganglia, capable of generating muscular activity in the absence of sensory feedback
   - CPGs are typically studied in preparations isolated in vitro, with sensory inputs and higher brain "commands" removed and sometimes in neonatal animals.
2. **Reflex-driven approach** concentrates on the role of feedback and inter and intra limb coordination in shaping locomotory patterns.
3. **Biomechanical approach:** focuses on body-limb environment dynamics and usually ignores neural detail.

Conclusion: although each has amassed vast amounts of data, no single approach can encompass the whole problem.

Modeling:

- A mathematical modeling approach (at various levels and complexities), can play a critical role in synthesizing parts of these data by developing unified descriptions of locomotive behavior

- This approach can guide the modeling and understanding of other biological systems, as well as bio-inspired robots.

However: while biomechanical and neurobiological models of varying complexity are individually relatively well developed, their integration remains largely open.

Hierarchical control loops:

- It is believed that a successful locomotion depends upon a hierarchical family of control loops.

1. At the lowest level of the neuro-mechanical hierarchy there are very basic mechanical feedback loops (preflexes):

   - neural clock-excited and tuned muscles acting through chosen skeletal postures.
   - biomechanical models provide the basic description,
   - simple models in which legs are represented as passively sprung, massless links.

2. Acting above the preflexive level, but together with it, there are:

   a) **feedforward** muscle activation loops from the CPG,

   b) above that, sensory, **feedback**-driven reflexes that further increase an animal's stability and dexterity by suitably adjusting CPG and motoneuron outputs.

   Here modeling of neurons, neural circuitry, and muscles is central.

preflex: “zero-delay intrinsic response of a neuro-musculoskeletal system to a perturbation" - they note that they are programmable via preselection of muscle activation.

Hierarchical control loops (2):

3. At the highest level, **goal-oriented behaviors** such as foraging or predator-avoidance employ environmental sensing and operate on a step-to-step timescale to "direct" the animal's path.

   - This is the least level to be investigated

   - It is still very early for a well developed mathematical model.

   - More abstract notions of connectionist neural networks and information and learning theory are appropriate at this level.

Template:

- A model, which containing the smallest number of variables and parameters that exhibits a behavior of interest, is (sometime) called **template.**

- This is a **simplification of the biological system**, which preserves the basic principles of the modeled creature.

- In robotics applications, the template is an attracting invariant on which the restricted dynamics takes a form prescribed to solve the specific task at hand.

- In both robots and animals, templates are composed to solve different tasks in various ways by a supervisory controller in the central nervous system (CNS).

- An example of template is the **spring loaded inverted pendulum (SLIP)**. This is a classical locomotion template that describes the center of mass behavior of different legged animals.

- The SLIP represents the animal's body as a point mass bouncing along on a single elastic leg that models the action of the legs supporting each stance phase (muscles, neurons, and sensing are excluded)

Anchor

- An anchor model represents the neural circuitry including the motoneurons, muscles, individual limb segments and joints, and ground contact effects

- However, in spite of such complexity, it can be shown that, under suitable conditions, animals with diverse morphologies, leg numbers, many mechanical and neural degrees of freedom, run as if their mass centers were following SLIP dynamics.

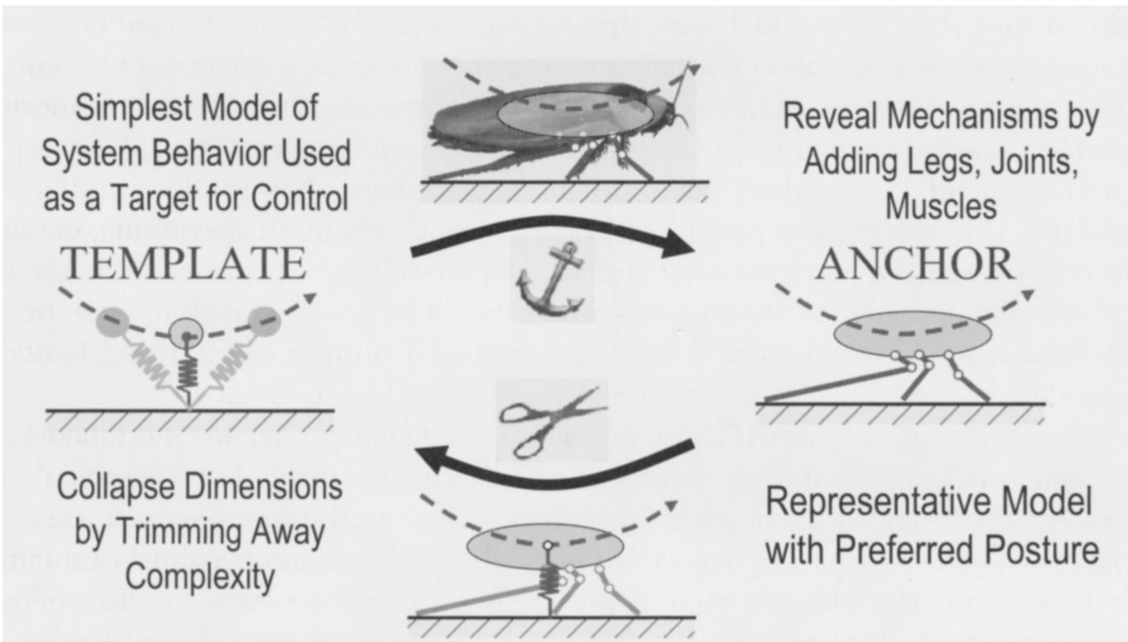


Fig. 1 *Templates and anchors: a preferred posture leads to collapse of dimension. The SLIP is shown at left, and a multilegged and jointed model at right. Both share the mass center dynamics of the insect (top). Reprinted from* [208] *with permission from Elsevier.*

Mechanical Models and Lagged Machines

- Diverse species that differ in leg number and posture, while running fast, exhibit motion approximating that of SLIP in a vertical plane

- The same mode also describes the gross dynamics of lagged machines

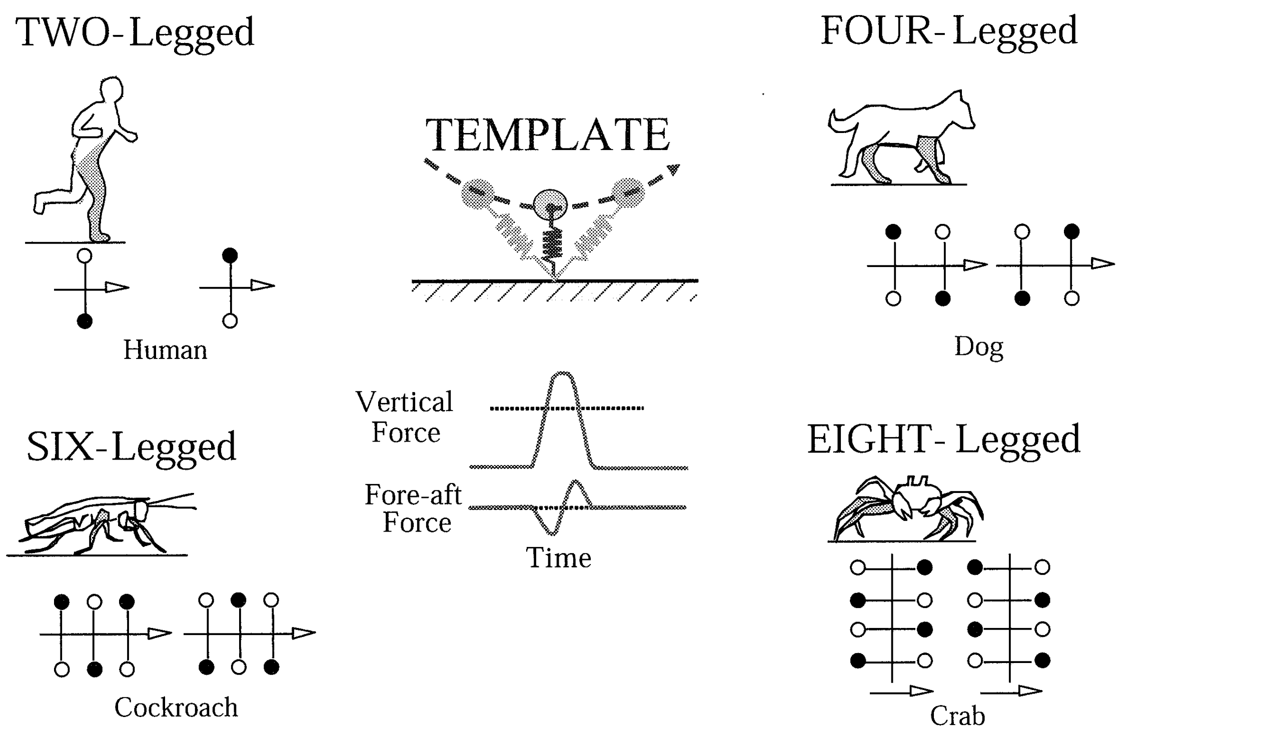
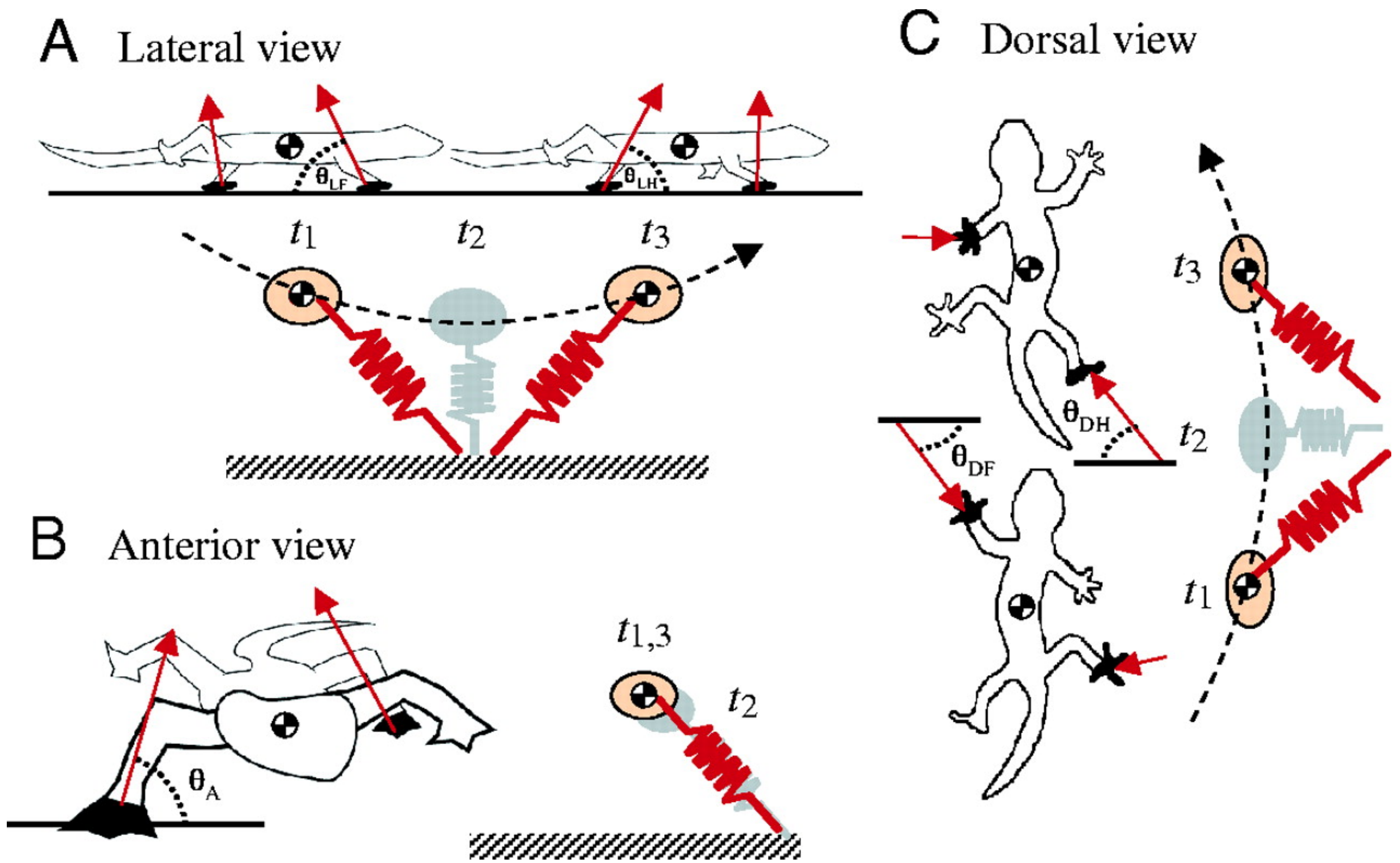


Fig. 4 *COM dynamics for running animals with two to eight legs. Groups of legs act in concert so that the runner is an effective biped, and mass center falls to its lowest point at midstride. Stance legs are shown shaded, with qualitative vertical and fore-aft force patterns through a single stance phase at bottom center. The SLIP, which describes these dynamics, is shown in the center of the figure.*

Single leg GRFs compared with the hypothesized templates for whole body dynamics.



Chen J J et al. J Exp Biol 2006;209:249-259

The Journal of Experimental Biology

Single leg GRFs compared with the hypothesized templates for whole body dynamics. (A) Lateral or sagittal view. Individual leg GRFs represented by red arrows at the beginning of the step (t1) and toward its end (t3). Below is the corresponding spring-loaded inverted pendulum representing the COM dynamics. (B) Anterior view. Peak forces are represented by red vectors at midstep (t2). To the right is a simple mass on top of a spring that represents the summed action of both legs as the animal's bounces down and to its left. (C) Dorsal view. Individual leg GRFs represented by red arrows at the beginning of the step (t1) and toward its end (t3). To the right is the corresponding lateral spring representing the COM dynamics.

- Locomotion results from **complex**, **high-dimensional**, **non-linear**, **dynamically coupled** interactions between an *organism* and its *environment*

- FACT: In the strict engineering sense, animals appear to be **over-built**, or **over-complete**

  Animals seam to have:

  - **kinematic redundancy** for locomotion - they have far more joint degrees of freedom than their three body positions and three body orientations
  - **actuator redundancy** for locomotion - they often have at least twice as many muscles as joint degrees of freedom.
  - **neuronal redundancy** for locomotion - they have more participating interneurons than necessary to generate the observed motor neuron signals.

- Conclusion: We ***need*** simplifying models.

R.J. Full and D.E. Koditschek "Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land" The Journal of Experimental Biology 202, 3325–3332 (1999)

Dynamics models for locomotion

- There are two types of dynamics models for locomotion: **templates** and **anchors**

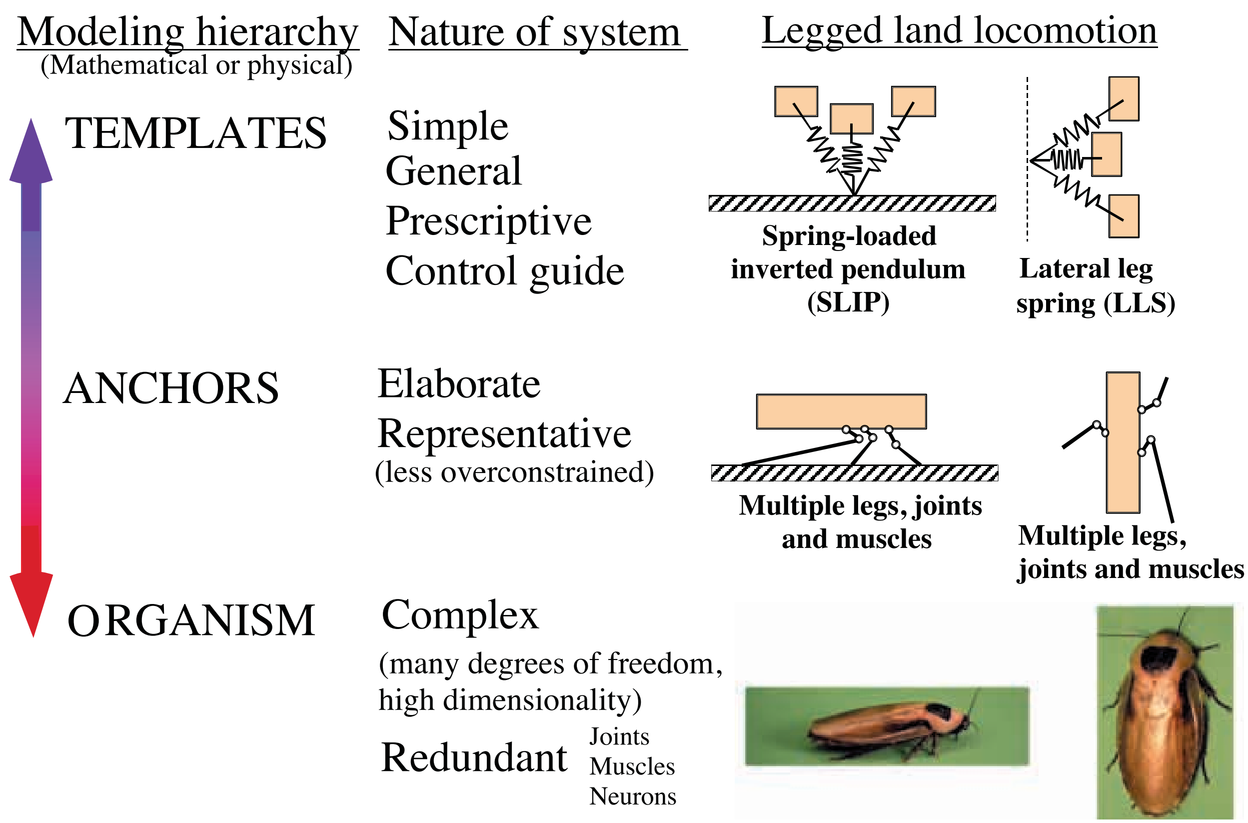


Fig. 1. Modeling hierarchy of legged land locomotion. Organisms such as the cockroach shown are complex and redundant in the engineering sense. The simplest models, termed templates, encode the task-level behavior of the system. Two templates are shown: the often-used sagittal-plane spring-loaded inverted pendulum (SLIP) and a new lateral leg spring template (LLS) operating in the horizontal plane as it bounces from side to side (J. Schmitt and P. Holmes, in preparation). Elaborate models termed anchors are more representative of the animal. Anchored models can reveal the mechanisms by which legs, joints and muscles function to produce the behavior of the template.

R.J. Full and D.E. Koditschek "Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land" The Journal of Experimental Biology 202, 3325–3332 (1999)

<u>Templates</u> - Definitions

- A **<u>template</u>** is a pattern that describes and predicts the behavior of the body in pursuit of a goal

  - this is **the simplest model of a robot that exhibits a targeted behavior** operating with **the least number** of variables and parameters.
  - **Diverse species that differ in skeletal type**, leg number and posture **run in a stable manner like** sagittal- and horizontal-plane **spring-mass systems**.
  - Therefore: templates suggest control strategies that can be tested against empirical data.
  - Templates must be grounded in more detailed morphological and physiological models to ask specific questions about multiple legs, the joint torques that actuate them, the recruitment of muscles that produce those torques and the neural networks that activate the ensemble.

A template is a <u>low dimensional model</u> of a robot <u>operating within a specified environment</u> that is <u>capable of expressing a specific task</u> as the limit set of a suitably tuned dynamical system involving some controlled (robot) and un-controlled (environment) degrees of freedom.

R.J. Full and D.E. Koditschek "Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land" The Journal of Experimental Biology 202, 3325–3332 (1999)

Templates - Why?

- **How** to get a template?

  ‘trim away’ all the incidental complexities:

  - joints
  - muscles
  - neurons.

- **Why** need a template?

  - general behavior (e.g the relative speed at a gait change), can be found by using a template on animals that differ in leg number, posture or skeletal type.
  - A template is not only a simple model but also serves as a guide or **target for the control** of locomotion.

- **Drawback**: Templates do not incorporate detailed mechanisms, which are often required in the study.

R.J. Full and D.E. Koditschek “Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land” The Journal of Experimental Biology 202, 3325–3332 (1999)

Anchors - Definitions

- An **anchor** is a more elaborate model, which introduces representations of specific biological details whose mechanism of coordination is of interest.

  - this is a more realistic model **fixed firmly** or grounded **in the morphology** and physiology of an animal.
  - incorporates specific hypotheses concerning the manner in which unnecessary motion or energy from legs, joints and muscles is removed, leaving behind the behavior of the body in the low-degree-of-freedom template.
  - This is a physically realistic higher degree of freedom representation of the robot and its environment,
  - An anchor is not only a more elaborate dynamic system, but **must have embedded within it the behavior of its templates**

R.J. Full and D.E. Koditschek "Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land" The Journal of Experimental Biology 202, 3325–3332 (1999)

Anchors - Definitions

- **When** do we need an anchor?
  - when we also seek models of how legs, joints, multiple muscles and neural networks work together to produce locomotion
- What is the main **challenge**?
  - The next challenge is true integration - Fundamental advances in understanding have been made using simple models of the body or isolated preparations of its parts.
- What are the **advantages**?
  - exploring the relationships between templates and anchors will allow the generation of neuromechanical hypotheses that span levels of organization from neurons to whole-body locomotion.

R.J. Full and D.E. Koditschek “Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land” The Journal of Experimental Biology 202, 3325–3332 (1999)

<u>Templates and Anchors</u> - Control

- <u>Locating the origin of control is a challenge</u> because **neural** and **mechanical** systems are **dynamically coupled** and both play a role.

- The control

  - of **slow**, variable-frequency locomotion *appears to be* dominated by the **nervous system**,
  - whereas during **rapid**, rhythmic locomotion, the control *may reside* more within the **mechanical system**.

- Anchored templates of many-legged, sprawled-postured animals suggest that passive, dynamic self-stabilization from a feedforward, tuned mechanical system can reject rapid perturbations and simplify control.

- <u>Solution</u>: creation of a field embracing comparative neuromechanics - *not here yet*

R.J. Full and D.E. Koditschek “Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land” The Journal of Experimental Biology 202, 3325–3332 (1999)

Hypotheses:

Building a template needs empirical hypothesis

Examples of hypotheses of diverse, legged, locomotor systems:

- H1: Locomotion results from complex, high-dimensional non-linear, dynamically coupled interactions between an organism and its environment
- H2: Diverse species that differ in skeletal type, leg number and posture walk stably like sagittal-plane inverted-pendulum systems
- H3: Diverse species that differ in skeletal type, leg number and posture run stably like sagittal-plane spring-mass systems
- H4: Diverse species that differ in skeletal type, leg number and posture run stably like horizontal-plane, laterally directed, spring-mass systems
- H5: Maneuvers require minor neuromechanical alterations to straight-ahead running
- H6: Control strategy is dependent on the precision, rhythmicity and speed of locomotion
- H7: Joints moments are minimized by ground reaction force vectors aligning along the leg axially
- H8: Differential leg structure and function in sprawled-posture runners permits greater stability and maneuverability
- H9: Passive, dynamic feedback from a 'tuned' mechanical system allows rapid response to perturbations and can simplify control
- H10: Feedforward control, as opposed to continuous neural feedback, sets the basic patterns during rapid locomotion
- H11: Neural feedback may function more in a state-event-dependent manner during rapid locomotion
- H12: Distributed, preflexive mechanisms at the level of the muscle and skeleton can allow rejection of rapid perturbations and simplify control

R.J. Full and D.E. Koditschek "Templates and ANchors: Neuromechanical Hypothesis of Legged Locomotion on Land" The Journal of Experimental Biology 202, 3325–3332 (1999)

Creating a template:

- Redundancies in legged locomotion can be resolved in large part by searching for

  - **symmetries** - correspondence of parts on opposite sides of a plane through the body
  - **synergies** - parts working together in combined action or operation. Groups of neurons, muscles, joints and legs can work in concert as if they were one

Walking by vaulting – inverted pendulum template

Assumptions:

- Body:
    - point mass *m* at the COM
- Legs
    - massless, independent, compressive springs
    - linear springs of equal rest length
    - stiffness k

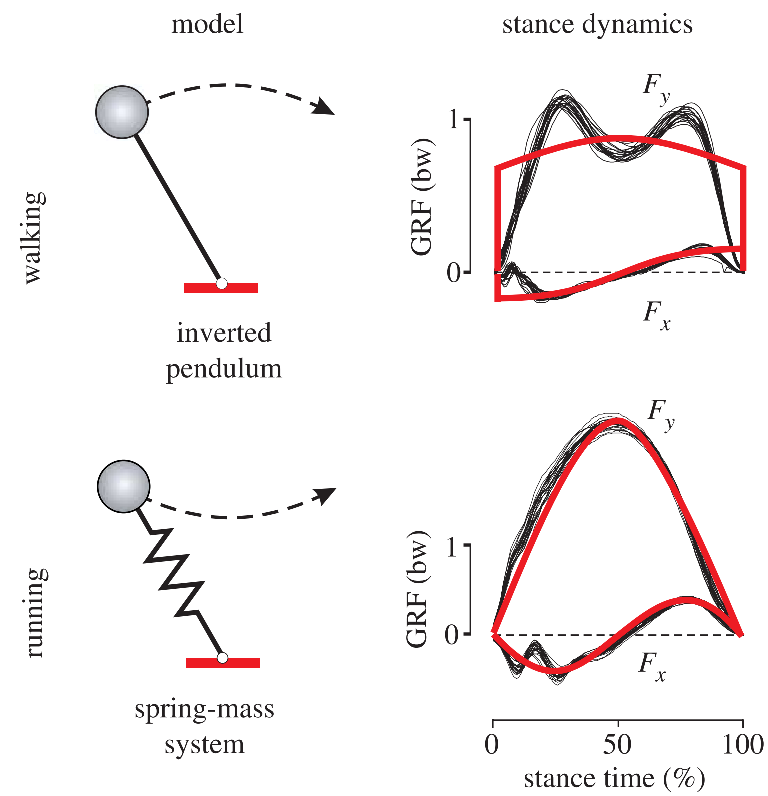


Figure 1. Standard conceptual models of legged locomotion and their predictive power with respect to walking and running dynamics. The inverted pendulum and the spring–mass system are the standard models for walking and running. The model-predicted stance dynamics (red lines) fit experimental data (black traces recorded from human treadmill walking at 1.2 m s^{-1} and running at 4.0 m s^{-1}) only for the spring–mass model for running. Note that, in the inverted pendulum dynamics, delta functions appear at 0 and 100% stance time if one adds collision and push-off models imitating double support. $F_{x,y}$, horizontal and vertical ground reaction force (GRF) normalized to body weight (bw).

H. Gayer, A. Seyfarth and R. Blikhan “leg behavior explains basic dynamics of walking and running”Proc. R. Soc. B (2006) 273, 2861–2867

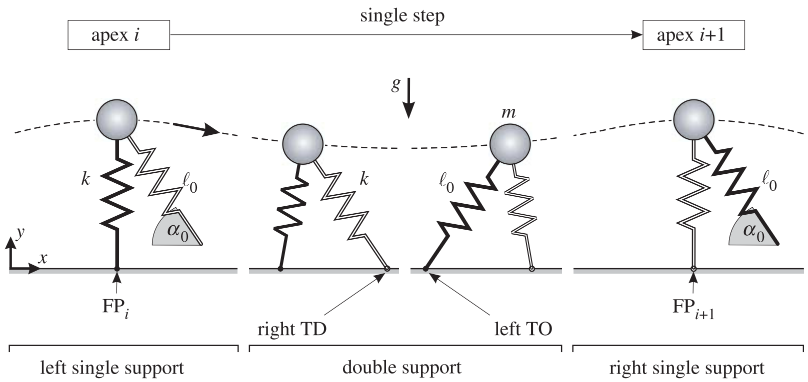


Figure 2. The bipedal spring–mass model. The model has two independent, massless spring legs attached to a point mass m. Both springs have stiffness k, rest length ℓ_0 and, in their swing phases, a constant orientation α_0 with respect to gravity (g, gravitational acceleration). A single step is shown that starts at the highest COM position in left leg single support (apex i), includes the double support ranging from right leg touchdown (right TD) to left leg take-off (left TO), and ends at the next apex in right leg single support (apex $i+1$). FP, foot point position in single support.

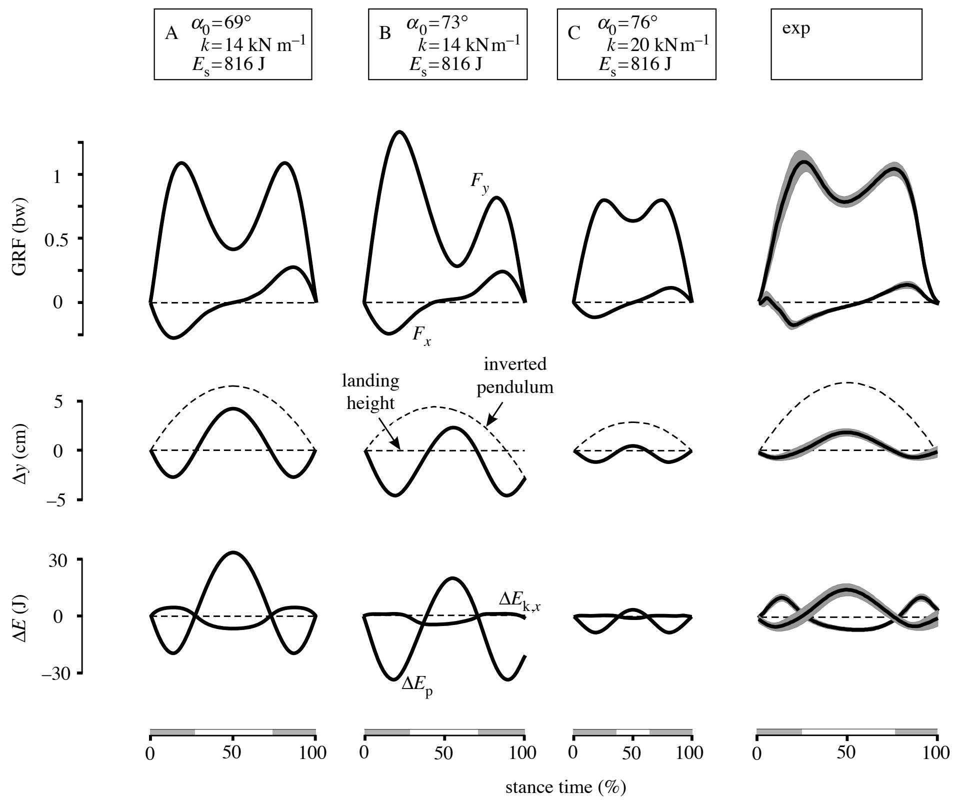


Figure 3. Stance-phase patterns of walking at about 1.2 m s^{-1}. (A–C) Examples of three characteristic steady-state solutions of the bipedal spring–mass model are compared with (exp) experimental results (mean and s.d. shown as line and shaded area) of five subjects (mean ± s.d. of mass: 81 ± 3.5 kg, leg length: 1.07 ± 0.03 m) walking on a treadmill (Adal3D, TecMachine, France; with force sensors recording horizontal and vertical GRFs). The subplots show horizontal and vertical GRFs, F_x and F_y; vertical displacement, Δy; and changes in forward kinetic and gravitational potential energies, $\Delta E_{k,x}$ and ΔE_p. The vertical displacement is compared with that of an inverted pendulum (dashed line). The shaded segments at the time-scales denote double supports. The depicted lengths of the time-scales reflect the absolute stance times.

Model:

- Governing equations for a single step:

$$m\ddot{x} = Px$$
$$m\ddot{y} = Py - mg$$

- Double support

$$m\ddot{x} = Px - Q(d - x)$$
$$m\ddot{y} = Py + Q\,y - mg$$

- Final leg

$$m\ddot{x} = -Q(d - x)$$
$$m\ddot{y} = Q\,y - mg$$

$$P = k(\ell_0/\sqrt{x^2 + y^2} - 1), \qquad Q = k(\ell_0/\sqrt{(d - x)^2 + y^2} - 1) \qquad d = \mathrm{FP}_{i+1,x} - \mathrm{FP}_{i,x}$$

Results

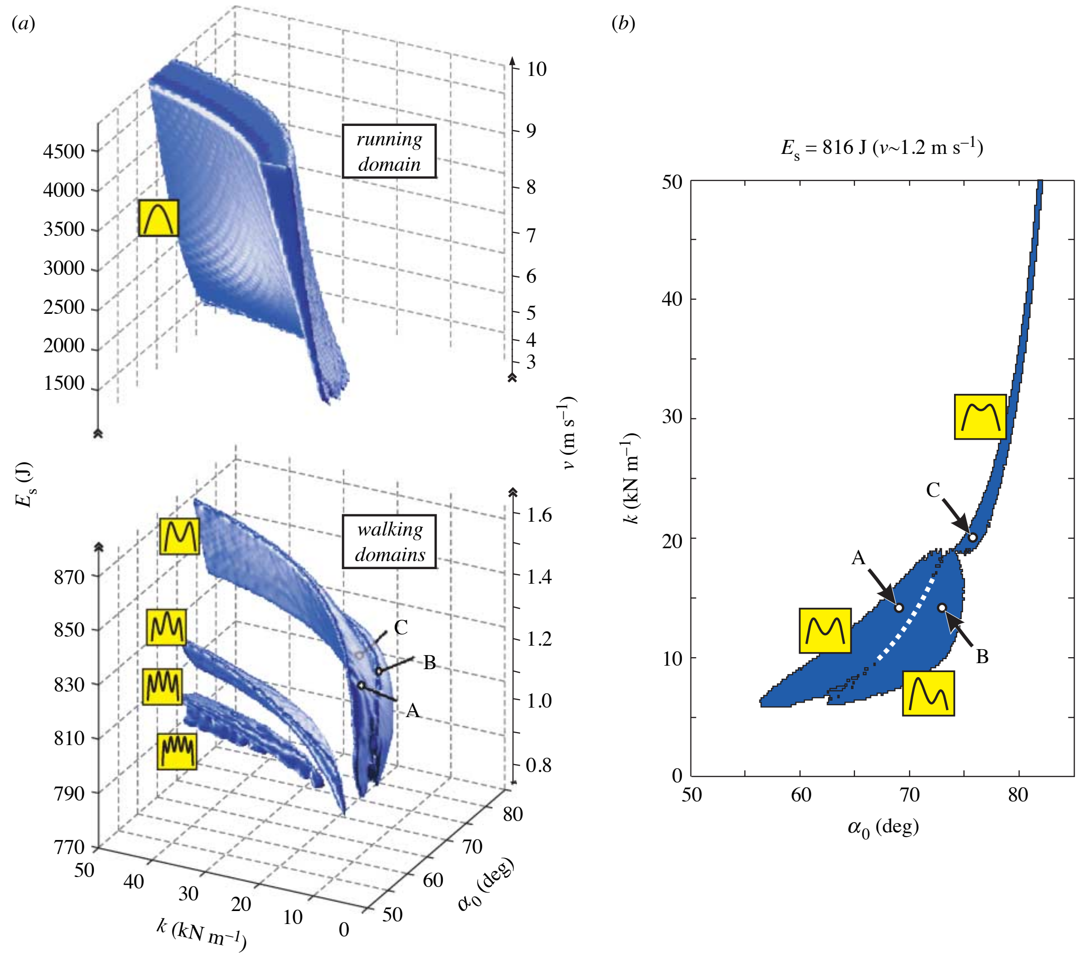


Figure 4. Parameter domains for stable walking and running. (*a*) Combinations of angle of attack α_0, spring stiffness k and system energy E_s leading to stable locomotion are shown. Related to E_s, the locomotion speed v is shown, which is the average speed of all solutions that belong to one system energy (maximum deviation 0.1 m s^{-1} at E_s=800 J). The model finds stable walking at low energies or slow speeds (walking domains): next to the domain with double-peak patterns of the vertical GRF, domains with multi-peak patterns exist (small icons). Owing to the limited scan resolution, only domains with up to five peaks are resolved, and the four- and five-peak domains seem to overlap. Circles indicate the parameter sets of the examples A–C shown in figure 3. In addition to walking, the model finds stable running with single-peak vertical GRF above an energy or speed gap of about 500 J or 1.5 m s^{-1} (running domain). Note the different scales of system energy at the walking domains and the running domain. (*b*) A slice at E_s=816 J ($v\sim$1.2 m s^{-1}) through the walking domain with double-peak patterns is shown. Three sub-domains of parameters exist that lead to three qualitatively different steady-state patterns (small icons) exemplified by the three solutions A–C (compare figure 3).

Example: A Simply Stabilized Running Model

- (a) shows the parametrization of the SLIP model as a schematic representation for the stance phase of a running (or hopping) biped with at most one foot on the ground at any time.

- The model incorporates a

  - rigid body of mass *m*
  - moment of inertia *I*
  - massless sprung leg attached at a hip joint
  - *H* hip joint
  - *G* is the distance to center of mass

- The compressed leg-spring length (*O*-*H*):

$$\eta = \sqrt{d^2 + \zeta^2 + 2d\zeta \cos\left(\psi + \theta\right)}$$

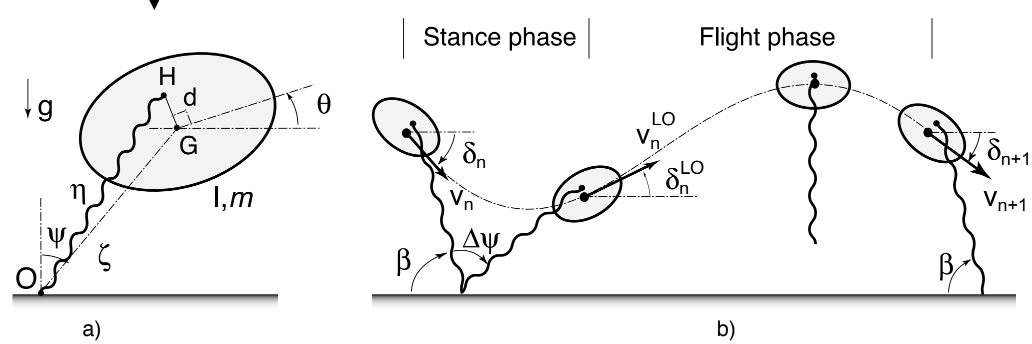


Figure 1. *The hopping rigid body* (a), *and the stance and flight phases comprising a full stride* (b).

R. M. Ghigliazza, R. Altendorfer, P. Holmes and D. Koditschek "A Simply Stabilized Running Model" SIAM J. APPLIED DYNAMICAL SYSTEMS 2,2,187-218

- The body is assumed to remain in the vertical (sagittal) plane, and its state at any point in time is defined by
  - the position of *G.* (*x*G,*y*G) - Cartesian inertial frame
  - pitch angle θ
  - generalized polar coordinates ζ, ψ (Note that ψ increases clockwise, while θ increases counterclockwise.)
- Extension to “classical” SLIP - the model could consider a rigid body with distributed mass and allow pitching motions. (non zero *d* and θ)
- The kinetic energy of the body

$$T = \frac{1}{2}m(\dot{\zeta}^2 + \zeta^2\dot{\psi}^2) + \frac{1}{2}I\dot{\theta}^2,$$

- The potential energy of the body:

$$V_{tot} = mg\zeta\cos\psi + V\left(\eta\left(\zeta, \psi, \theta\right)\right),$$

- where $V = V_{spr}$ is the spring potential.

- We form the Lagrangian *L=T-V* and write : $\partial V/\partial \eta = V_\eta,$

- **<u>The equation of motion for the stance phase</u>** can be written as:

$$\ddot{\zeta} = \zeta\dot{\psi}^2 - g\cos\psi - \frac{V_\eta(\eta)}{m\eta}\left(\zeta + d\cos(\psi+\theta)\right),$$
$$\zeta\ddot{\psi} = -2\dot{\zeta}\dot{\psi} + g\sin\psi + d\frac{V_\eta(\eta)}{m\eta}\left(\sin(\psi+\theta)\right),$$
$$\ddot{\theta} = d\zeta\frac{V_\eta(\eta)}{\eta I}\sin(\psi+\theta).$$

- **<u>The equations of motion during the flight phase</u>** are simply the ballistic COM translation and torque-free rotation equations, which may be integrated to yield:

$$x_G(t) = x^{LO} + \dot{x}^{LO}t, \quad y_G(t) = y^{LO} + \dot{y}^{LO}t - \frac{1}{2}gt^2, \quad \theta(t) = \theta^{LO} + \dot{\theta}^{LO}t,$$

where the superscripts LO refer to the system state at liftoff

The case d = 0 neglecting gravitational effects in stance

- If the leg is attached at the COM (H ≡ G), then d = 0, ζ ≡ η, the stance phase dynamics simplifies to the "classical" SLIP, and the pitching equation decouples

- The third equation describes the conservation of angular momentum of the body about its COM:

$$\ddot{\zeta} = \zeta\dot{\psi}^2 - g\cos\psi - \frac{V_\zeta(\zeta)}{m}\ ,\quad \zeta\ddot{\psi} = -2\dot{\zeta}\dot{\psi} + g\sin\psi,$$
$$\ddot{\theta} = 0 \Rightarrow \theta(t) = \theta(0) + \dot{\theta}(0)\,t.$$

- Neglecting gravity, the first two equations of simplify to:

$$\ddot{\zeta} = \zeta\dot{\psi}^2 - \frac{V_\zeta(\zeta)}{m}\ ,\ \zeta\ddot{\psi} = -2\dot{\zeta}\dot{\psi}.$$

- The second of these equations expresses the conservation of the moment of linear momentum of the COM about the foot. The first equation is, therefore, integrable:
- $$\ddot{\zeta} = \frac{p_\psi^2}{m^2\zeta^3} - \frac{V_\zeta(\zeta)}{m} \;\Rightarrow\; m\ddot{\zeta}\dot{\zeta} = \frac{p_\psi^2}{m\zeta^3}\dot{\zeta} - V_\zeta(\zeta)\,\dot{\zeta} \;\Rightarrow$$
$$H \triangleq \left(\frac{m\dot{\zeta}^2}{2} + \frac{p_\psi^2}{2m\zeta^2} + V(\zeta)\right) = \text{const.}$$

Results

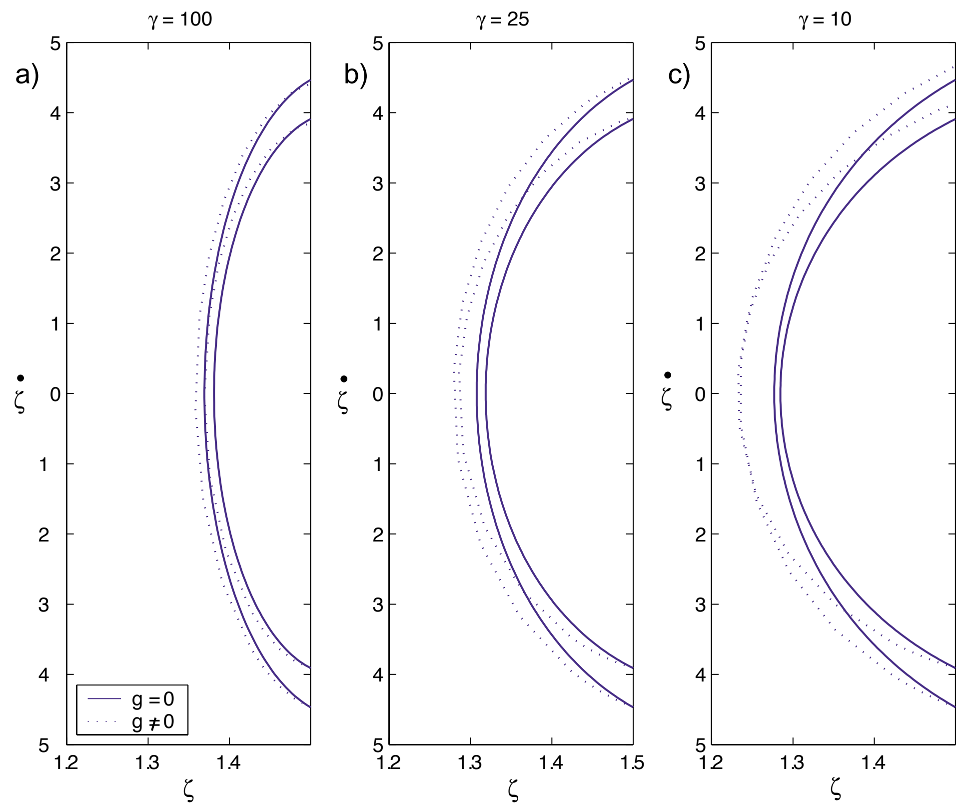


Figure 2. *Solutions of the integrable system* (2.7) *(solid) and the full* $(d = 0)$ *system (dashed) in the stance phase:* $m = 1, \eta_0 = 1.5, \beta = \pi/4$ *with linear spring stiffnesses* $k = 654$ $(\gamma = 100)$ (a), $k = 163.5$ $(\gamma = 25)$ (b), *and* $k = 65.4$ $(\gamma = 10)$ (c). *Dimensional units, unless otherwise stated, are MKS.*

Results

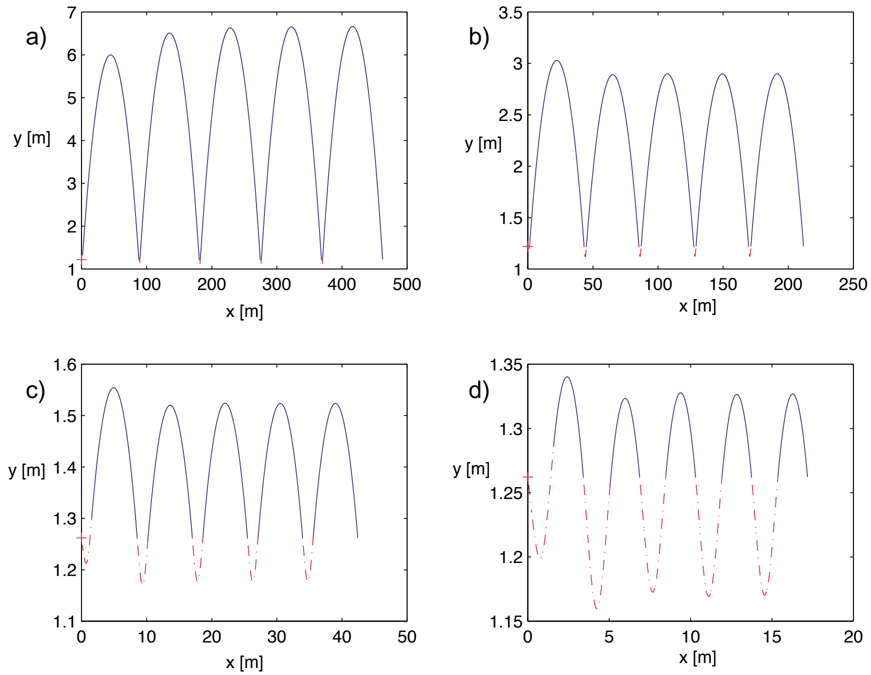


Figure 4. *Stable gaits with gravity included in the stance phase, showing the effect of spring stiffness.* $m = 1, \eta_0 = 1.5$, $k = 2000$ ($\gamma = 306$) (a); $k = 1000$ ($\gamma = 153$) (b); $k = 250$ ($\gamma = 38.2$) (c); *and* $k = 100$ ($\gamma = 15.3$) (d). *Here* $\beta = 0.95$ *for the upper graphs, and* $\beta = 1.0$ *for the lower ones.* v_n *was* $45, 35, 15, 8$, *respectively. Stance phases are shown chain dotted, and flight phases are shown solid; note the differing vertical scales.*

Results

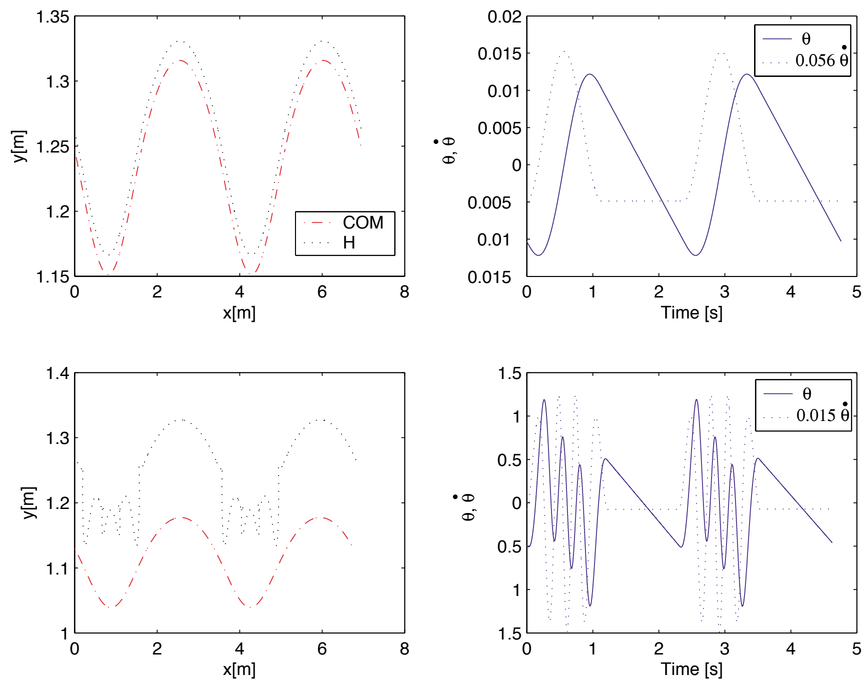


Figure 5. *Periodic gaits of the model with pitching included* $(d \neq 0)$. *The left-hand panels show COM and hip paths in physical space, and the right-hand panels show pitch angle and angular velocity. Computed for parameter values* $k = 100, m = 1, \eta_0 = 1.5, \beta = 1$, *and* (a) $d = 0.015, I = 2.25 \times 10^{-2}, \bar{v} = 7.93$; (b) $d = 0.15, I = 2.25 \times 10^{-4}, \bar{v} = 8.26$. *Note that* $\dot{\theta}$ *scales differ in right-hand panels.*

Results

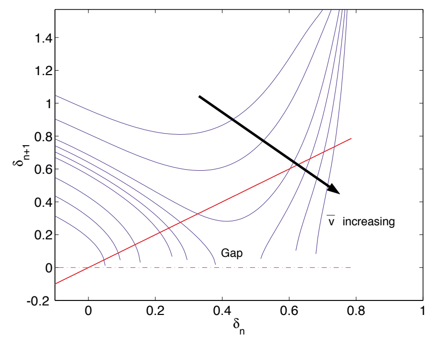


Figure 7. *The Poincaré map P_2 for a linear spring hopper with $k = 10, m = 1, \eta_0 = 1.5, \beta = \pi/4$, and speeds $\bar{v}$ ranging from 3.2 to 8. Note how the two fixed points appear in a saddle-node bifurcation, and a gap then opens as $\bar{v}$ increases. For very high speeds, only one fixed point exists.*

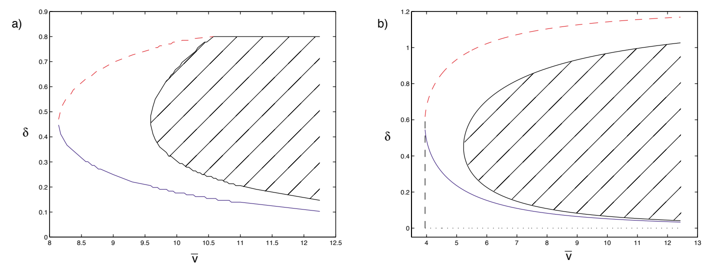


Figure 8. *Bifurcation diagrams for the linear spring hopper with $m = 1, k = 50, \eta_0 = 1.5$, and touchdown angle $\beta = 0.8$ in* (a) *and for the air spring hopper with $m = 1, c = 23, \eta_0 = 1.5$, and $\beta = 1.25$ in* (b). *Stable branches of fixed points are shown solid, unstable branches are dashed, and cross-hatching identifies the region in which the map is not defined. Saddle-node bifurcations occur at $\bar{v}_{SN} = 8.12$ in* (a) *and $\bar{v}_{SN} = 3.95$ in* (b)*; below these no periodic gaits exist.*

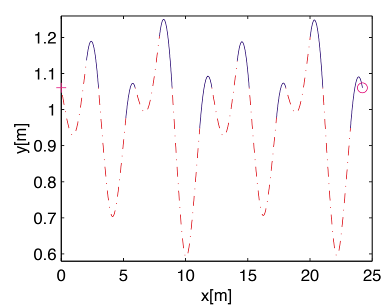


Figure 9. *A period two gait of the linear spring hopper with $k = 10, m = 1, \eta_0 = 1.5, \beta = \pi/4$, and $\bar{v} = 3.95$.*

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Example

ECE 216
Bio-inspired legged locomotion

Robots as controlled multi-body dynamics systems

Figure 1.1: A serial manipulator (left), the ABB IRB1400, and a parallel manipulator (right), the ABB IRB940Tricept. Photos courtesy of ABB Robotics.

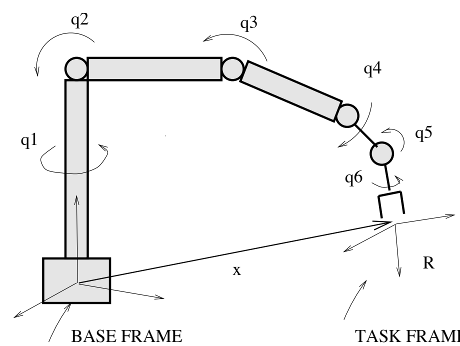

Figure 1.2: A Serial Link Manipulator showing the attached Base Frame, Task Frame, and configuration variables.

Robot Dynamics and Control
Mark W. Spong
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Fig. 1. Natural impedance of human legs: a) leg muscles enable running by changing their strength characteristics, b) leg muscles and ligaments as natural body "actuators" and natural "shock-absorbers".

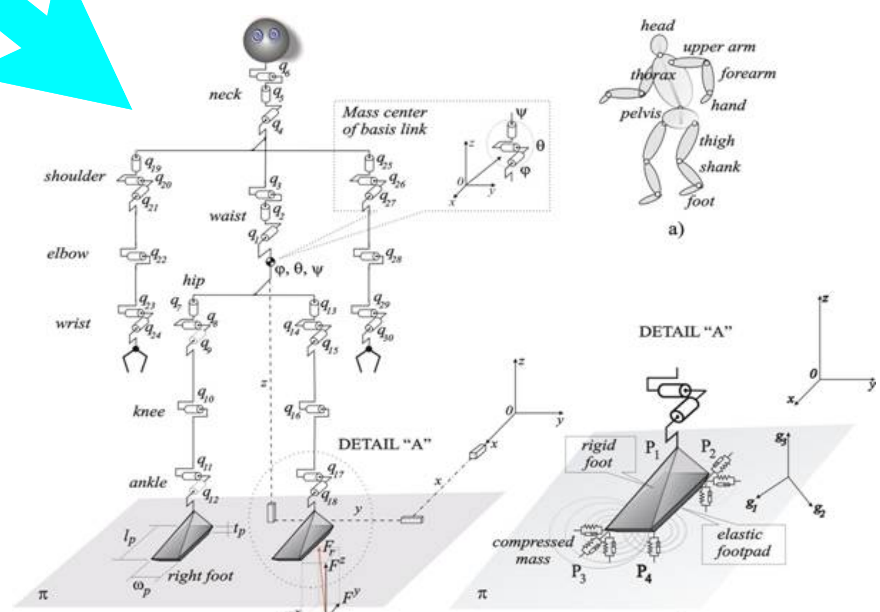

Fig. 3. a) Spinal robot model with two-segments trunk considered in the project, b) Spatial model of a biped robot mechanism interacting with dynamic environement used for verification of the adaptive leg imepdance modulation algorithms, c) compliance model of the ground support used in simulation.

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Two approaches:

There are two approaches, which predict the behavior of a dynamic system:

1. Analytic:- fast, but not always applicable

2. Numerical: - slower, but significantly more versatile.

Equivalent System

- For matter in any form the quantity of energy must be conserved during physical processes.

- In physics of motion, and under idealized conditions (i.e. no losses in the form of heat, sound, etc) the principle of conservation of energy translates to:

$$K + U = \text{constant} \Rightarrow \frac{d}{dt}(K + U) = 0 \Rightarrow K_{\max} = U_{\max}$$

$$K = \text{kinetic enery, } U = \text{potential energy}$$

- **Kinetic energy** is stored in the mass by the virtue of its velocity. Thus, the familiar Einstein equation: *E=mc^2*.

- **Potential energy** is stored in the form of work done in either elastic deformation or effort expended against a force field, such as gravity. Thus, the familiar equation: *E=mgh*,

- For any system with distributed masses such as beams, rods, and springs with non-negligible masses, Lord Rayleigh has shown that the deflection of the system can be used to evaluate kinetic and potential energies of the system

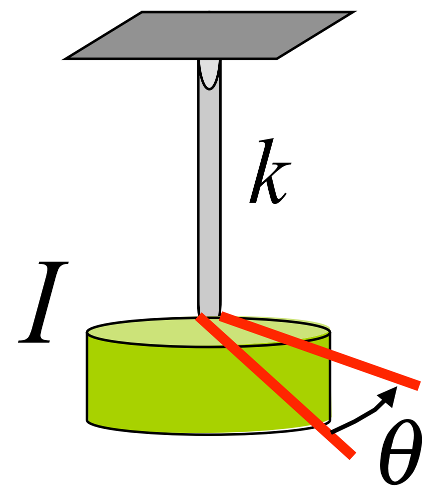
Torsional Stiffness:

- Rods, rotors and shafts are often subject to torsional oscillations, see figure below, for a torsional pendulum.

- Remember the swinging pendulum from your basic dynamics class.

- *I* is the mass moment of inertia in *kgm^2*.

- **Mass moment of inertia** is obtained by taking moments f the element about a given position, such as its centre of grav example, for a disk about is centre of gravity: *I=mr^2*



Use of Energy Method:

Let us consider an example of torsional vibration.

Assuming that the oscillatory motion is harmonic, then:

$$\theta = \hat{\theta} \sin \omega_n t$$

$$\dot{\theta} = \hat{\theta}\omega_n \cos \omega_n t \ , \ \dot{\theta}_{\max} = \hat{\theta}\omega_n$$

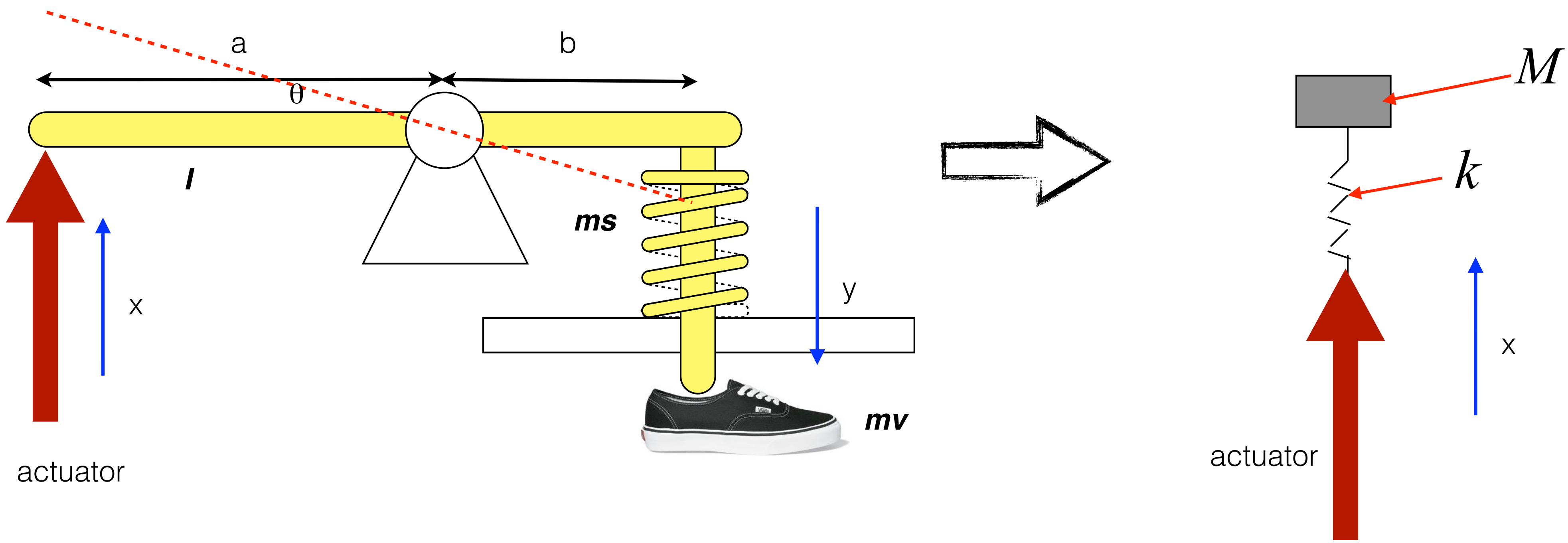
$$\theta_{\max} = \hat{\theta} = \text{amplitude of oscillation.}$$

$$\text{Max. kinetic energy, } K_{\max} = \frac{1}{2} I \dot{\theta}_{\max}^2 = \frac{1}{2} I \omega_n^2 \hat{\theta}^2$$

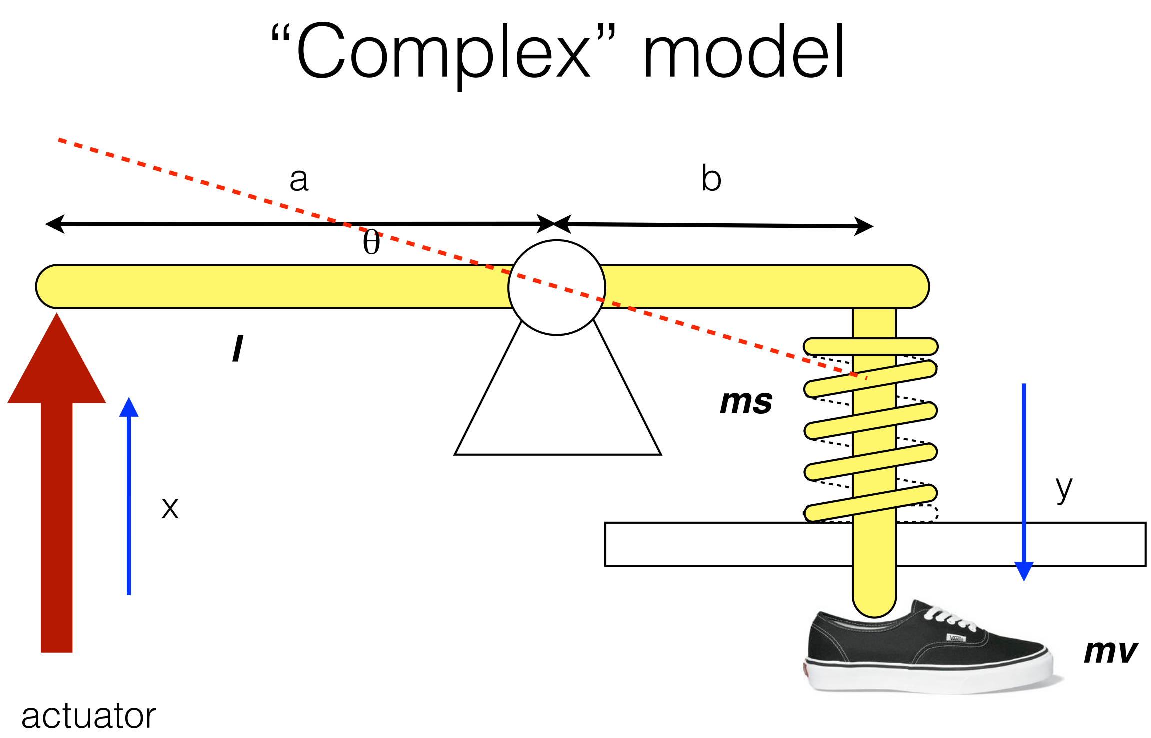
$$\text{Max. potential energy, } U_{\max} = (k\theta_{\max}) \frac{\theta_{\max}}{2} = \frac{1}{2} k \theta_{\max}^2 = \frac{1}{2} k \hat{\theta}^2$$

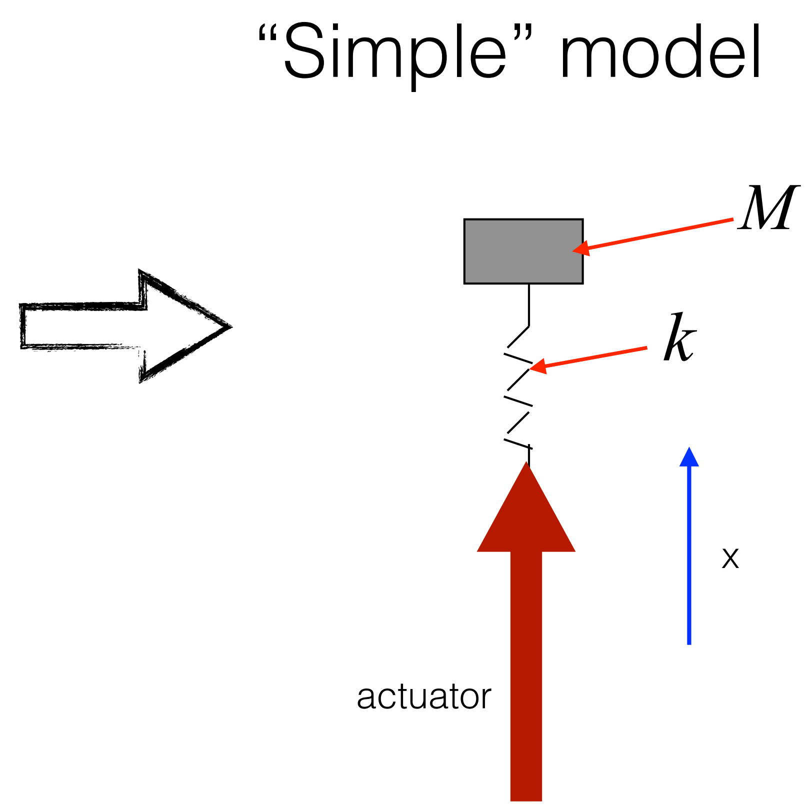
$$K_{\max} = U_{\max}, \text{ Thus: } \omega_n = \sqrt{\frac{k}{I}}$$

“Complex” model   “Simple” model



- In most mechanisms the mass of the spring can be ignored. Most theoretical studies also assume negligible spring mass.
- However, in practice you will meet many problems, where the mass of spring cannot be ignored (e.g. valve spring, see figure).
- The system has 2 degrees of freedom - we are interested in the translation motion





$$\text{K.E. System: } K_{\max} = \frac{1}{2} M \dot{x}^2$$

$$K_{\max} = \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} m_v \dot{y}^2$$

Equating the above and: $\dot{y} = b\dot{\theta},\ \dot{x} = a\dot{\theta}\ \therefore\ \dot{\theta} = \dfrac{\dot{x}}{a}$

Then: $\boxed{M = \dfrac{I + m_v b^2}{a^2}}$

Spring with non-negligible mass:

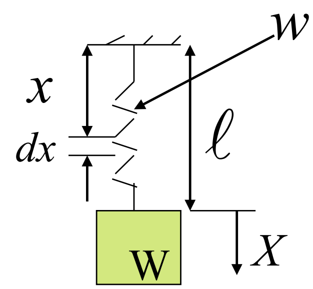
The translational DOF can be represented by a simple mass-spring model, but with the mass of spring being significant

$X =$ displacement of bottom of spring

$\frac{w}{g} =$ mass per unit length of spring

$\frac{x}{\ell} X =$ displacement at any point x from fixed end

$\ell =$ length of spring at static equilibrium.



Each element of spring has the mass: $\frac{w}{g} dx$

Harmonic motion at position x: $\frac{x}{\ell} X \sin \omega_n t$

Velocity at position x: $\frac{x}{\ell} \omega_n X \cos \omega_n t$

K.E. of element o spring, $dK_{\max} = \frac{1}{2} \frac{w}{g} dx \left(\frac{x}{\ell} \omega_n X \right)^2$

K.E. for the spring: $K_{\max} = \frac{w}{2g} \left(\frac{\omega_n X}{\ell} \right)^2 \int_0^{\ell} x^2 dx = \frac{1}{2} \left(\frac{w\ell}{3g} \right) (\omega_n X)^2$

Add K.E. of mass $\frac{W}{g}$ or: $\frac{W}{g} \omega_n^2 X^2$

Thus: $K_{\max} = \frac{1}{2} \left(\frac{W + \frac{1}{3} w\ell}{g} \right) \omega_n^2 X^2$

The potential energy of the system is calculated as:

$$U_{max} = \int_0^X k(\delta + x)dx - WX = \frac{1}{2}kX^2$$

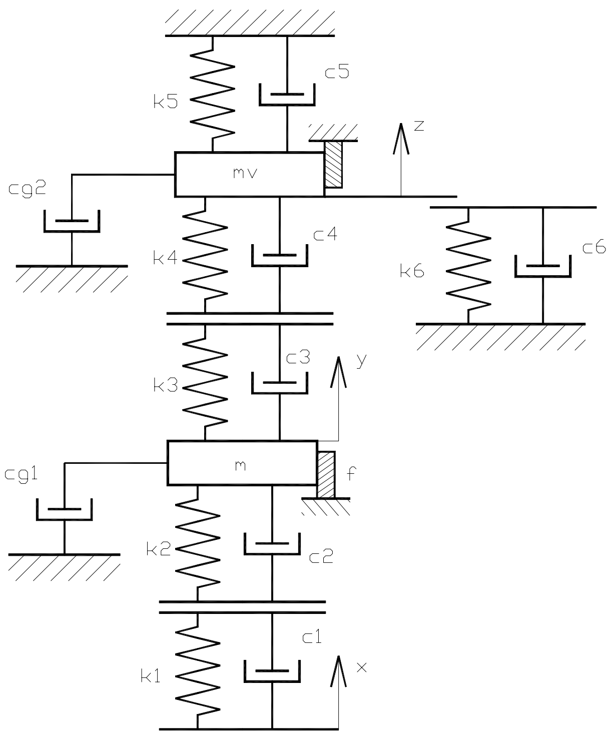
where: $\delta = \frac{W}{g} = \text{static deflection.}$

Now: $K_{max} = U_{max} \Rightarrow \frac{1}{2}\left(\frac{W + \frac{1}{3}w\ell}{g}\right)\omega_n^2 X^2 = \frac{1}{2}kX^2$

$$\Rightarrow \omega_n = \sqrt{\frac{kg}{W + \frac{1}{3}w\ell}}$$

Therefore, one third of mass of spring is added to the suspended mass to find the equivalent mass of the system.

Method of solution: Newton-Euler formulation



$$\begin{pmatrix} L_i\ddot{\xi}_i + (c_{e_i} + c_{e_{i+1}})\dot{\xi}_i - c_{e_i}\dot{\xi}_{i-1} - c_{e_{i+1}}\dot{\xi}_{i+1} + \\ +(k_{e_i} + k_{e_{i+1}})\xi_i - k_{e_i}\xi_{i-1} - k_{e_{i+1}}\xi_{i+1} + \Phi_i \end{pmatrix} = 0$$

where:

$$\xi_i \in \{x_i, \varphi_i\} \quad , \quad L_i \in \{m_i, I_i\} \quad \text{and} \quad \Phi_i \in \{F_i, M_i\}$$

$$k_{e_i} = \frac{k_i k_{i+1}}{k_i + k_{i+1}} \qquad c_{e_i} = \frac{c_i c_{i+1}}{c_i + c_{i+1}}$$

- Li is a generic concentrated inertial element (mass or moment of inertia),
- ξi is a generic deflection (displacement or angle of twist),
- Φi is a generic excitation (force or moment).