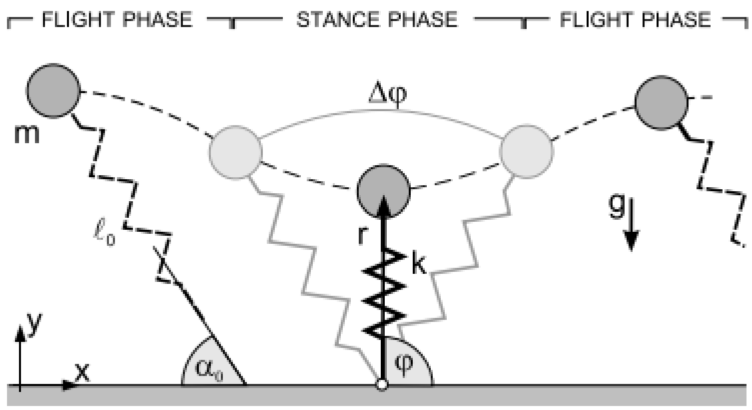
Question 1

Following the techniques outlined in [1], I will model walking locomotion for myself as a **planar spring-mass model**.

This model cycles through 2 modes: ‘flight’ and ‘stance’. Each phase has different dynamics and therefore different equations of motion, and solving for the system trajectory requires different sets of states and differential equations.

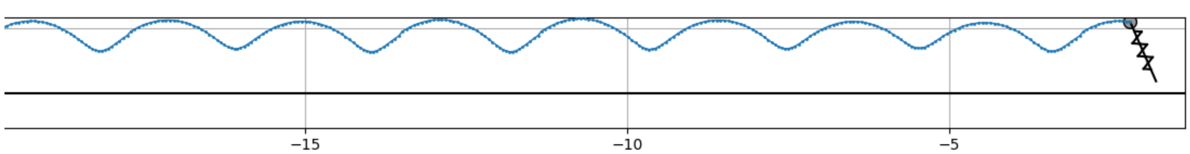



1. The simplifying assumptions:
    a. Legs are massless, and all joints and components are negligible
    b. Two legs have the same stiffness
    c. Knees are simplified as compressing springs
    d. Everything above the waist is simplified as a single point mass, and the angle at the waist is negligible
    e. The leg can be controlled to any position instantaneously (since leg is massless), so the control input u = $\theta$
2. The model that I picked is one found in literature. As I am walking, one can consider one of my legs in the ‘flight’ phase and the other in the ‘stance’ phase, and with every step the two legs switch phases. My ‘apex’ y term in the model mirrors the movement of my torso ‘center of mass’ during my walk.
3. The system parameters, as seen in the diagram above:
    a. The spring constant k
    b. The length of the fully extended leg $l_0$
    c. The angle of attack of the ‘foot’ $\alpha$
    d. The mass of the body m
    e. Gravity g
    f. The stance angle range $\phi$
4. There are 2 phases to this model, each with a state and dynamics
    a. Flight:
        i. The state in this phase is s = [x, y, x*, y*]
        ii. The dynamics are based on the fact that gravity is the only force acting the system:

        1. $s^* = [x^*, y^*, x^{**}, y^{**}] = [x^*, y^*, 0, -g]$
    iii. We transition out of this phase when the foot **collides into the ground**, or mathematically when $y <= 0$

b. Stance:
    i. The state in this phase is s = [r, theta, r*, theta*]. These are the polar coordinates of the system
    ii. The forces involved are the kinetic energy *T* (angular and translational velocities), and potential energy *U* from the spring force and gravity. The leg by definition is **compressed** in this phase, so we use r instead of $l_0$
    iii. The Lagrangian yields:
        1. $T = m / 2 (r^{*2} + r^{*2}\theta^{*2})$
        2. U = gravitational potential energy + spring energy = $mg (r\cos\theta) + k / 2 (l_0 - r)^2$
        3. $L = T - U = m / 2 (r^{*2} + r^{*2}\theta^{*2}) - mg (r\cos\theta) - k / 2 (l_0 - r)^2$
    iv. We can substitute L to a known expression to obtain the **Euler-Lagrange equation of motion** to get the stance dynamics
        1. $mr^{**} - mr\theta^{*2} + mg \cos\theta - k(l_0 - r) = 0$
        2. $mr^2\theta^{**} + 2mrr^*\theta^* - mgr\sin\theta = 0$
    v. Note that the right hand side for these expressions is 0, the system is **unforced**
        1. We introduce energy into the system via controlling the leg angle at takeoff instantaneously
    vi. We transition out of the stance phase when the leg extends / relaxes to its uncompressed length $r >= l_0$

5. The following is a plot of the SLIP model after controller iterating / tuning. It is a P controller on the xdot / ydot terms and the output is routed to the takeoff angle.




Main references:

https://www.cs.cmu.edu/~hgeyer/Publications/Geyer05PhDThesis.pdf

http://underactuated.mit.edu/simple_legs.html#example5

Question 2

Following the techniques outlined in [1], I will model walking locomotion for my charging dog as **2 planar spring-mass models** that are coupled with a torso defined as 1 mass-less link connected to either leg and each other, similar to in the figure below.

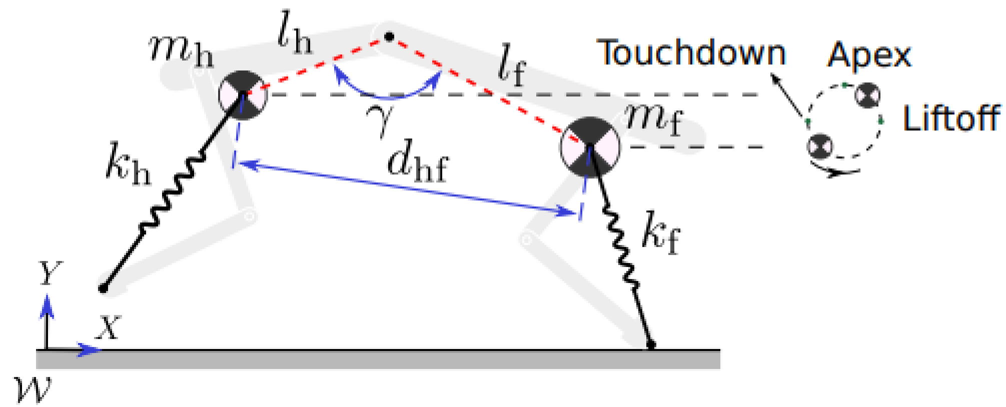



Fig. 1: Dual-SLIP template model for quadrupedal running.

This model cycles through 2 modes: ‘flight’ and ‘stance’. Each phase has different dynamics and therefore different equations of motion, and solving for the system trajectory requires different sets of states and differential equations.

1. Simplifying assumptions:
    a. All of the assumptions from Question 1 hold still for the front and hind legs, and they have the **same static parameters**
        i. Legs are massless, and all joints and components are negligible
        ii. Two legs have the same stiffness
        iii. Knees are simplified as compressing springs
        iv. Everything above the waist is simplified as a single point mass, and the angle at the waist is negligible
        v. The leg can be controlled to any position instantaneously (since leg is massless), so the control input u = $\theta$
    b. The torso is **massless** and does **not bend**
    c. When the hind leg is in stance, the front of the body **in flight**, and vice-versa.
    d. The two springs have the **same** stiffness k
    e. The two masses have the **same mass**
2. When my dog is charging / running, he largely propels forward using his hind legs and then again using his front legs. His apex, head motion, follows an apex map similar to the one proposed by this dual-slip model
3. Like in Question 1, this model cycles through multiple states. The sequence of states is as follows: hind SLIP, fore SLIP, hind SLIP, etc. Unlike Question 1, however, because the first SLIP is coupled to a linkage, the dynamics when $m_1$ is ‘flying’ are those of a

pendulum swinging along the torso link about $m_2$, and vice versa. The system parameters are as follows:

   a. mass of mass 1 and mass 2 hip and shoulder **m**
   b. Extended link of both hind and front leg **l0**
   c. Stiffness of hind front legs **k**
   d. Length of the torso **torsol**

4. During the two states of the system, the hind and front legs **alternate between flight / stance phases**
   a. When hind is stance, front is flying
      i. The hind leg equations of motion are solved in Question 1 for the stance phase
      ii. For the front leg, the hind leg to torso angle phih is changing and driving the motion here
         1. Kinetic energy T = m / 2 ($x_{hind}^2$ + $y_{hind}^2$) + m / 2 (torsol$^{*2}$ + torsol$^{*2}$phi$_{hind}^{*2}$), this is the sum of the hind mass (a constant w.r.t. phi) and the polar translational velocity of phi about mass$_{hind}$
         2. Potential energy U = m g $y_{mass2}$ = m g ($y_{hind}$ + torsol * sin(phi))
      iii. The Lagrangian yields:
         1. L = T - U
      iv. We can substitute L to a known expression to obtain the **Euler-Lagrange equation of motion** to get the stance dynamics for **phi**
         1. el1 = L.diff(phih) - (L.diff(phih.diff(t))).diff(t)
         2. $-gmtorsol \cos \phi_{hind} - mtorsol^2 \ddot{\phi}_{hind} = 0$, note that this is **unforced**
         3. $\ddot{\phi}_{hind} = \frac{gcos\phi_{hind}}{torsol}$
         4. So for the hind leg we integrate as in Question 1, and we integrate $\phi_{hind}$ and front leg moves as a function of hind but remains static otherwise
      v. Transition conditions still hold from Question 1, so when the hind legs r >= l0, it ‘takes off’ and when the front leg y <= 0, it ‘touches down’
   b. Note that when the front leg is stancing and hind is flying, the same equation of motion applies for $\phi_{front}$, and this drives the ‘flying’ hind leg to fall back down.