Homework 1

(Due on: Tue, April 18 by 8:00PM)

A simple nonlinear model of pendulum dynamics is given by:

$$mL\ddot{x} = -mg\sin x \tag{1}$$

This second order differential equation can be re-written as a system of differential equations

$$\begin{aligned} \dot{x}_1 &= x_2 && (2)\\ \dot{x}_2 &= -\frac{g}{L}\sin x_1, \qquad g/L = 0.2 && (3) \end{aligned}$$

a) Use the MATLAB function ode45 (or ode23) to numerically solve the system of differential equations for the initial condition $x_1 = \pi/6$, $x_2 = 0$ and plot diagrams for $x_1(t)$, $x_2(t)$ and $x_1(t)$ vs. $x_2(t)$.

b) Let us consider the pendulum with a random acceleration term $f(t)$

$$\begin{aligned} \dot{x}_1 &= x_2 && (4)\\ \dot{x}_2 &= -\frac{g}{L}\sin x_1 + f(t), \qquad g/L = 0.2 && (5) \end{aligned}$$

where the random acceleration term $f(t)$ is described as follows: t_1, t_2, $...t_k$ are time points of random events such that the probability density function of time intervals $t_k - t_{k-1}$ is

$$p(t_k - t_{k-1}) = \lambda e^{-\lambda(t_k - t_{k-1})}, \quad \lambda = 0.025, t_k > t_{k-1}, t_0 = 0 \tag{6}$$

and $f(t) = \sum_k f(t_k)\delta(t - t_k)$, $t_k > 0$

$$f(t_k) = \begin{cases} 0, & t \neq t_k \\ \varepsilon \sim \mathcal{N}(0, \sigma = 0.1), & t = t_k \end{cases} \tag{7}$$

Solve (4) and (5) numerically for the initial condition $x_1 = 0$, $x_2 = 0$ and plot diagrams for $x_1(t)$, $x_2(t)$ and $x_1(t)$ vs. $x_2(t)$. The plots should show that the angle $x_1 \in (-\pi, \pi]$.

Submission instructions: Send your computer-typed solution(s) as a single pdf document to dmilutin@ucsc.edu. To the same e-mail, also attach a .zip file that includes all your Matlab code(s). Your document should include the title “Homework 1” and your name. Please DO NOT include any additional personal information such as you student ID.