5.3A.

31.(1) $\ln\sqrt{x^2+y^2} = \arctan\frac{y}{x}$

$F(x,y) = \ln\sqrt{x^2+y^2} - \arctan\frac{y}{x} = 0$

$F_x = \frac{x-y}{x^2+y^2}$

$F_y = \frac{y-x}{x^2+y^2}$

$\frac{dy}{dx} = -\frac{F_x}{F_y} = \frac{x+y}{x-y}$

$\frac{d^2y}{dx^2} = \frac{2x^2-2y^2}{(x-y)^3}$

32.(2). $x^2-2y^2+z^2-4x+2z-5=0$

$F(x,y,z) = x^2-2y^2+z^2-4x+2z-5$

$F_x = 2x-4 \quad F_y = -4y \quad F_z = 2z+2$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{2-x}{z+1} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{2y}{z+1}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right) = \frac{-(z+1)^2-(2-x)^2}{(z+1)^3} \quad \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{2(z+1)^2+4y^2}{(z+1)^3}$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{2y(x-2)}{(z+1)^3} \quad \frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{2y(x-2)}{(z+1)^3}$

36.(2) $x^2+y^2+z^2 = yf\left(\frac{z}{y}\right)$

$F_x = x^2+y^2+z^2 - yf\left(\frac{z}{y}\right)$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{2x}{yf'\left(\frac{z}{y}\right)-2z} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{2y-f\left(\frac{z}{y}\right)+\frac{z}{y}f'\left(\frac{z}{y}\right)}{yf'\left(\frac{z}{y}\right)-2z}$

$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$

$= \frac{2x}{f'\left(\frac{z}{y}\right)-2z}dx + \frac{2y-f\left(\frac{z}{y}\right)+\frac{z}{y}f'\left(\frac{z}{y}\right)}{yf'\left(\frac{z}{y}\right)-2z}dy$