

6.6 A

1.(6) $\oint_C |y| ds$ (C) $x^2+y^2+z^2=2$ 且 $x=y$ 且 $y>0$

$$\therefore x^2y = \cos\theta \quad \text{且} \quad z = \sqrt{2}\sin\theta$$

$$\oint_C |y| ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\cos\theta| \cdot \sqrt{2} d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos\theta) \sqrt{2} d\theta = 4\sqrt{2}.$$

3.(2) $\oint_C \sqrt{x^2+y^2} ds$ (C) $x^2+y^2=a^2$ ($a>0$)

$$\therefore x = \frac{a}{2} + \frac{a}{2}\cos\theta \quad y = \frac{a}{2}\sin\theta$$

$$\begin{aligned} \oint_C \sqrt{x^2+y^2} ds &= \int_0^{2\pi} \sqrt{\frac{a^2}{4}(1+\cos\theta)} \cdot \sqrt{\frac{a^2}{4}} d\theta \\ &= \int_0^{2\pi} \frac{\sqrt{2}a^2}{4} \sqrt{2\cos\frac{\theta}{2}} d\theta \\ &= 2a^2 \end{aligned}$$

10.(1) $\iint_S (2x+\frac{4}{3}y+z) ds$ (S) 为 $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$. ($x \geq 0, y \geq 0, z \geq 0$)

$$\text{且 } z = 4 - 2x - \frac{4}{3}y \quad 0 \leq x \leq 2 \quad 0 \leq y \leq 3$$

$$\begin{aligned} \iint_S (2x+\frac{4}{3}y+z) ds &= 4 \int_0^2 \int_0^{3-\frac{3}{2}x} (4 - 2x - \frac{4}{3}y) dy dx \\ &= \frac{2}{3} \sqrt{16} \int_0^2 dx \int_0^{3-\frac{3}{2}x} dy \\ &= 4\sqrt{16} \end{aligned}$$

10.(8) $\iint_S (xy+yz+zx) ds$ (S) $z=\sqrt{x^2+y^2}$ 且 $x^2+y^2=2ax$ ($a>0$) 阶梯部分

$$\therefore z=p \quad \text{且} \quad x=p\cos\theta, y=p\sin\theta, -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, 0 \leq p \leq 2a\cos\theta$$

$$\iint_S (xy+yz+zx) ds = \iint_{(S)} p^2 (\sin\theta\cos\theta + \cos\theta + \sin\theta) p dp d\theta$$

$$= \int_0^{2a\cos\theta} \sqrt{2} p^3 dp \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin\theta\cos\theta + \cos\theta + \sin\theta) d\theta = \frac{64}{15} \sqrt{2} a^4$$