6.3 A

4.(3) $\iiint\limits_{(V)} \frac{e^z}{\sqrt{x^2+y^2}}dV$ (V)由 $z=\sqrt{x^2+y^2}$, $z=1$ $z=2$ 围成的区域

$$\iiint\limits_V \frac{e^z}{\sqrt{x^2+y^2}}dV = \iiint\limits_V \frac{e^z}{\rho}\rho d\rho d\theta = \int_0^{2\pi}d\theta\int_0^z d\rho\int_1^2 e^z dz$$

$$= \int_0^{2\pi}d\theta\int_1^2 ze^z dz = 2\pi e^2$$

4.(10) $\iiint\limits_{(V)} z^2 dV$ V为 $x^2+y^2+z^2\le R^2$ 与 $x^2+y^2+z^2\le 2Rz$ 的公共部分

$x^2+y^2+z^2\le 2Rz$ 即 $x^2+y^2+(z-R)^2\le R$

取柱坐标 则公共部分为

$$\begin{cases} 0\le\theta\le 2\pi. \\ 0\le\rho\le\sqrt{R^2-z^2} \\ \frac{R}{2}\le z\le R \end{cases} \quad 与 \quad \begin{cases} 0\le\theta\le 2\pi. \\ 0\le\rho\le\sqrt{2Rz-z^2} \\ 0\le z\le\frac{R}{2}. \end{cases}$$

$$\iiint\limits_V z^2 dV = \int_0^{2\pi}d\theta\int_0^{\sqrt{R^2-z^2}}\rho d\rho\int_{\frac{R}{2}}^{R} z^2 dz + \int_0^{2\pi}d\theta\int_0^{\sqrt{2Rz-z^2}}\rho d\rho\int_0^{\frac{R}{2}} z^2 dz$$

$$= \left(\frac{R^5}{3}-\frac{1}{5}R^5-\frac{R^5}{24}+\frac{R^5}{32}\right)\pi = \frac{59}{480}\pi R^5$$

4.(11) $\iiint\limits_V xyz\,dV$, V为 $x^2+y^2+z^2=1$ 位于第一卦限中的闭区域

取球坐标,则

$$\iiint\limits_V xyz\,dV = \iiint\limits_V r^5 \sin^3\varphi\cos\varphi\sin\theta\cos\theta\, d\theta d\varphi dr$$

$$= \int_0^1 r^5 dr\int_0^{\frac{\pi}{2}}\sin^3\varphi\cos\varphi d\varphi\int_0^{\frac{\pi}{2}}\sin\theta\cos\theta d\theta$$

$$= \frac{1}{6}\times\frac{1}{4}\times\frac{1}{2} = \frac{1}{48}$$

5.(2) $\int_{-3}^{3}dx\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}dy\int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz$

由 $x^2+y^2+z^2\le 9$ 的上半部分组成

原式 $= \int_0^{2\pi}d\theta\int_0^{\frac{\pi}{2}}\sin\varphi\cos\varphi d\varphi\int_0^3\rho^4 d\rho$

$= \frac{243}{5}\pi$