A

(1) 错，没有取曲线的正方向

$\int_{\widehat{OB} \cup \overline{BA} \cup \overline{AO}} y\,dx = -\iint -1\,d\sigma = \frac{\pi}{4}$

由于 $\int_{\overline{BA}} y\,dx = 0$，$\int_{\overline{AO}} y\,dx = 0$ 故 $\int_{(c)} y\,dx = \frac{\pi}{4}$

(2) 解法一错误 由于 (c) 的内部含原点，不可直接计算

取 $\varepsilon > 0$ 足够小，以 O 为中心，半径为 ε 的圆周 (C_ε) 使 (C_ε) 全位于 (c) 内部

则 $\int_{(+C)} \frac{x\,dy - y\,dx}{x^2+y^2} + \int_{(-C_\varepsilon)} \frac{x\,dy - y\,dx}{x^2+y^2} = \iint \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) d\sigma = 0$

$\therefore \int_{(+C)} \frac{x\,dy - y\,dx}{x^2+y^2} = \int_{(+C_\varepsilon)} \frac{x\,dy - y\,dx}{x^2+y^2}$

$\int_{(+C_\varepsilon)} \frac{x\,dy - y\,dx}{x^2+y^2} = \int_0^{2\pi} \frac{\varepsilon^2(\cos^2 t + \sin^2 t)}{\varepsilon^2} dt = 2\pi$

$\therefore I = -\int_{(+C_\varepsilon)} \frac{x\,dy - y\,dx}{x^2+y^2} - \int_{\widehat{OB'A}} \frac{x\,dy - y\,dx}{x^2+y^2} = -\pi$

(3) 解法二 正确

(5) $\int_{(c)} (e^x \sin y - my)\,dx + (e^x \cos y - m)\,dy$ (c) 为 $A(a,0)$ $O(0,0)$ 的上半圆周 $x^2+y^2 = ax\ (a>0)$

$\int_{\widehat{AO} + \overline{OA}} (e^x \sin y - my)\,dx + (e^x \cos y - m)\,dy = \iint_{(\sigma)} m\,d\sigma = m \cdot \frac{1}{2} \cdot \pi \left(\frac{a}{2}\right)^2 = \frac{1}{8} m\pi a^2$

又 $\because \int_{\overline{OA}} (e^x \sin y - my)\,dx + (e^x \cos y - m)\,dy = 0$

$\therefore \int_{(c)} (e^x \sin y - my)\,dx + (e^x \cos y - m)\,dy = \frac{1}{8} m\pi a^2$

(1) $A = (2x\cos y - y^2 \sin x)\,i + (2y\cos x - x^2 \sin y)\,j$;

$\because \frac{\partial}{\partial x}(2y\cos x - x^2 \sin y) = -2y\sin x - 2x\sin y$

$\frac{\partial}{\partial y}(2x\cos y - y^2 \sin x) = -2x\sin y - 2y\sin x$

故 A 为有势场

令 $u = x^2\cos y + y^2\cos x + \varphi(y)$

$\frac{\partial u}{\partial y} = 2y\cos x - \cancel{2x} \cdot x^2 \sin y) + \varphi'(y)$

$\therefore \varphi(x) = C$

$\therefore u = x^2\cos y + y^2\cos x + C$

16.(5) $\iint_{(S)} x\,dy \wedge dz + y\,dz \wedge dx + (x+y+z+1)\,dx \wedge dy$ (S) 为 $z = c\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}$ 的上侧

$= \iiint_{(V)} 3\,dv - [\iint_{(S_1)} (x+y+z+1)\,dx\,dy]$

$= 2\pi abc - ab\int_0^{2\pi} d\theta \int_0^1 (\rho a\cos\theta + \rho b\sin\theta)\rho\,d\rho$

$= \pi ab(2c+1)$