

6.6 A

1. (6) $\oint_C |y| ds$ C 为 $x^2 + y^2 + z^2 = 2$ 与 $x = y$ 的交线.令 $x = y = \cos \theta$ 则 $z = \sqrt{2} \sin \theta$.

$$\oint_C |y| ds = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta \cdot \sqrt{2} d\theta + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} (-\cos \theta) \sqrt{2} d\theta = 4\sqrt{2}.$$

3. (2) $\oint_C \sqrt{x^2 + y^2} ds$ C 为 $x^2 + y^2 = ax$ ($a > 0$)令 $x = \frac{a}{2} + \frac{a}{2} \cos \theta$ $y = \frac{a}{2} \sin \theta$.

$$\begin{aligned} \oint_C \sqrt{x^2 + y^2} ds &= \int_0^{2\pi} \sqrt{\frac{a^2}{4}(1 + \cos \theta)} \cdot \sqrt{\frac{a^2}{4}} d\theta \\ &= \int_0^{2\pi} \frac{\sqrt{a^2}}{4} \sqrt{2 \cos \frac{\theta}{2}} d\theta \\ &= 2a^2 \end{aligned}$$

10. (1) $\iint_S (2x + \frac{4}{3}y + z) ds$ S 为 $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ ($x \geq 0, y \geq 0, z \geq 0$)则 $z = 4 - 2x - \frac{4}{3}y$ $0 \leq x \leq 2$ $0 \leq y \leq 3$

$$\begin{aligned} \iint_S (2x + \frac{4}{3}y + z) ds &= 4 \iint_D \sqrt{1 + z_x^2 + z_y^2} d\sigma \\ &= \frac{4}{3} \sqrt{61} \int_0^2 dx \int_0^{3-2x} dy \\ &= 4 \sqrt{61} \end{aligned}$$

10. (8) $\oint_{(S)} (xy + yz + zx) ds$ S 为 $z = \sqrt{x^2 + y^2}$ 被 $x^2 + y^2 = 2ax$ ($a > 0$) 所截部分令 $z = \rho$ 则 $x = \rho \cos \theta$, $y = \rho \sin \theta$. $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ $0 \leq \rho = 2a \cos \theta$

$$\oint_{(S)} (xy + yz + zx) ds = \iint_{(S)} \rho (\sin \theta \cos \theta + \cos \theta + \sin \theta) \rho d\rho d\theta$$

$$= \int_0^{2a \cos \theta} \rho^3 d\rho \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin \theta \cos \theta + \cos \theta + \sin \theta) d\theta = \frac{64}{15} \sqrt{2} a^4$$