6.7A

2.(4) $\int_{(c)} x dx + y dy + (x+y-1) dz$ (c)为 (1,1,1) 到 (1,3,4) 的直线段.

$t \in (0,1)$ 则直线为 $\begin{cases} x=1 \\ y=1+2t \\ z=1+3t \end{cases}$

$\therefore \int_{(c)} x dx + y dy + (x+y-1) dz$

$= \int_0^1 1 \cdot 0 dt + \int_0^1 (1+2t) 2 dt + \int_0^1 (1+2t) 3 dt = 10$

(6) $\oint_{(c)} (z-y) dx + (x-z) dy + (x-y) dz$ (c)为 $\begin{cases} x^2+y^2=1 \\ x-y+z=2 \end{cases}$ 从z正→z负, (c)取顺时针

令 $x=\cos\theta$ $y=\sin\theta$, 则 $z=2+\sin\theta-\cos\theta$ $\theta \in (0, 2\pi)$

$\oint_{(c)} (z-y) dx + (x-z) dy + (x-y) dz$

$= \int_0^{2\pi} (2-\cos\theta) \sin\theta d\theta + \int_0^{2\pi} (2\cos\theta - \sin\theta - 2) \cos\theta d\theta + \int_0^{-2\pi} (\cos\theta - \sin\theta)(\sin\theta + \cos\theta) d\theta$

$= -2\pi$

11. $\vec{F} = (y, -x, z^2)$ (S)是 $z=\sqrt{x^2+y^2}$ 上 $0 \le x \le 1$, $0 \le y \le 1$ 部分的下侧

$\iint_{(S)} \vec{F} \cdot d\vec{S} = \iint_{(S)} y\, dy \wedge dz - x\, dx \wedge dz + z^2 dx \wedge dy$

$= \iint_{yOz} y\, dy dz - \iint_{xOz} x\, dx dz - \iint_{xOy} (x^2+y^2) dx dy$

$= \int_0^z y dy \cdot \int_0^1 dz - \int_0^z x dx \int_0^1 dz - \int_0^{\frac{\pi}{4}} d\theta \int_0^1 \rho^2 d\rho$

$= -\frac{\pi}{16}$

12.(2) $\oiint_{(S)} xy\, dy \wedge dz + yz\, dz \wedge dx + zx\, dx \wedge dy$ $x \ge 0, y \ge 0, z \ge 0$, $x+y+z \le 1$ 的外侧.

$\oiint_{(S)} xy\, dy \wedge dz + yz\, dz \wedge dx + zx\, dx \wedge dy$

$= \int_0^1 dz \int_0^{1-z} (1-y-z) y dy + \int_0^1 dz \int_0^{1-z} (1-x-z) dx + \int_0^1 dx \int_0^{1-x} x(1-x-y) dy$

$= \frac{1}{24} + \frac{1}{24} + \frac{1}{24}$

$= \frac{1}{8}$

12(6) $\oiint_S z^2 dx dy = \iint_{xOy} (1+\sqrt{1-x^2-y^2})^2 dx dy - \iint_{xOy} (1-\sqrt{1-x^2-y^2})^2 dx dy$

$= 4 \iint_{xOy} \sqrt{1-x^2-y^2} dx dy$

$= 4 \int_0^{2\pi} d\theta \int_0^1 \sqrt{1-\rho^2} \rho d\rho$

$= \frac{8}{3}\pi$