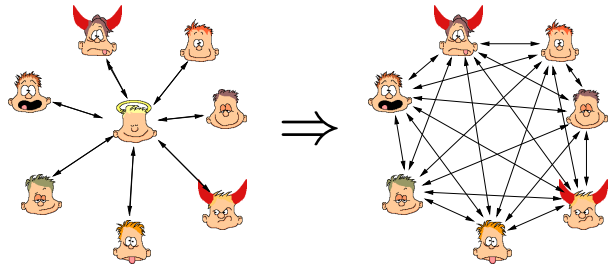
Cryptographic Protocols

Spring 2019

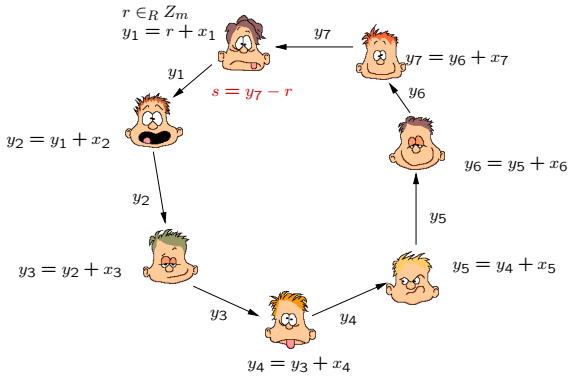
Part 6

Multi-Party Computation: Goal



Specification Protocol

Sum Protocol



Sum Protocol II

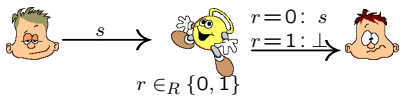
						$\cdots$	
x_1	x_{11}	x_{12}	x_{13}	x_{14}	$\cdots$	x_{1n}	
x_2	x_{21}	x_{22}	x_{23}	x_{24}	$\cdots$	x_{2n}	
x_3	x_{31}	x_{32}	x_{33}	x_{34}	$\cdots$	x_{3n}	
x_4	x_{41}	x_{42}	x_{43}	x_{44}	$\cdots$	x_{4n}	
$\vdots$	$\vdots$			$\vdots$			
x_n	x_{n1}	x_{n2}	x_{n3}	x_{n4}	$\cdots$	x_{nn}	
	y_1	y_2	y_3	y_4	$\cdots$	y_n	$y = \sum_{i=1}^{n} y_i$

Known Results

Setting	Adv. Type	Condition	Literature
cryptographic	passive	$t < n$	[GMW87]
cryptographic	active	$t < n/2$	[GMW87]
information-theoretic	passive	$t < n/2$	[BGW88,CCD88]
information-theoretic	active	$t < n/3$	[BGW88,CCD88]
i.-t. with broadcast	active	$t < n/2$	[RB89,Bea91]

Oblivious Transfer

OT Sender Receiver



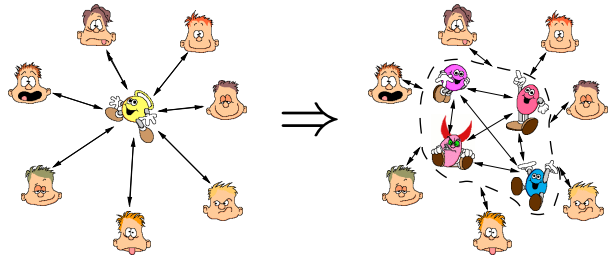
1-2-OT Sender Receiver

1-k-OT Sender Receiver

1-2-OST based on RSA and DES

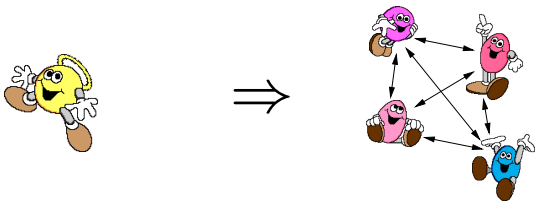
Sender		Receiver
messages s_0, s_1		selector $b \in \{0,1\}$
generate RSA-Keys n_0, e_0, d_0 and n_1, e_1, d_1 with $n_0 \approx n_1$	$\xrightarrow{n_0, e_0, n_1, e_1}$	
	$\xleftarrow{u}$	k at random, $u = k^{e_b} \pmod{n_b}$
$k_0 = u^{d_0} \pmod{n_0}$		
$k_1 = u^{d_1} \pmod{n_1}$		
$y_0 = \mathrm{DES}_{k_0}(s_0)$		
$y_1 = \mathrm{DES}_{k_1}(s_1)$	$\xrightarrow{y_0, y_1}$	$s_b = \mathrm{DES}_k^{-1}(y_b)$

Multi-Party Computation: Goal III



Specification ⇒ Protocol

Multi-Party Computation: Goal IV



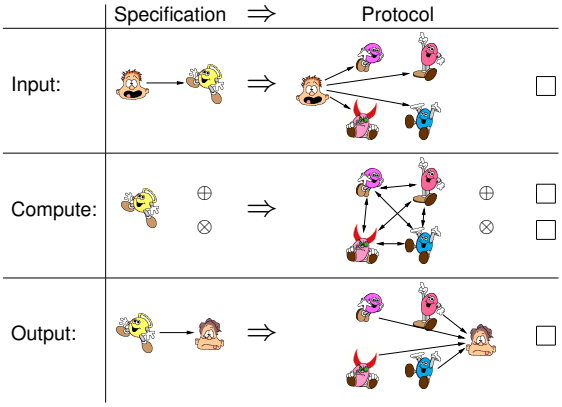
Trusted party

- receive input
- $\oplus$ and $\otimes$ over finite field $\mathbb{F}$
- give output

Simulating players ...

- n players: $\mathcal{P} = \{P_1, \ldots, P_n\}$
- players can $\oplus$ and $\otimes$ in $\mathbb{F}$
- players can communicate

Operations



Sum Protocol III

x_1	x_{11}	x_{12}	x_{13}	x_{14}	$\cdots$	x_{1n}	
x_2	x_{21}	x_{22}	x_{23}	x_{24}	$\cdots$	x_{2n}	
x_3	x_{31}	x_{32}	x_{33}	x_{34}	$\cdots$	x_{3n}	
x_4	x_{41}	x_{42}	x_{43}	x_{44}	$\cdots$	x_{4n}	
$\vdots$			$\vdots$				
x_n	x_{n1}	x_{n2}	x_{n3}	x_{n4}	$\cdots$	x_{nn}	
	y_1	y_2	y_3	y_4	$\cdots$	y_n	$y = \sum_{i=1}^{n} y_i$

Passive Protocol

Share input

- P_i has input s.
- P_i selects $r_1, ..., r_t$ at random.
- P_i comp. $\begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = A \begin{pmatrix} s \\ r_1 \\ \vdots \\ r_t \end{pmatrix}$.
- P_i sends s_j to every P_j.

Reconstruct Output

- a is shared by $a_1, ..., a_n$.
- every P_j sends a_j to P_i.
- P_i comp. $a = \mathcal{L}(a_1, ..., a_n)$.

Addition and linear functions $\mathcal{L}$

- $a, b, \ldots$ shared by $a_1, ..., a_n,\ b_1, ..., b_n$, etc.
- every P_i computes $c_i = \mathcal{L}(a_i, b_i, \ldots)$.

Multiplication

- a, b are shared by $a_1, ..., a_n, b_1, ..., b_n$.
- every P_i computes $d_i = a_i b_i$.
- every P_i shares $d_i \rightarrow d_{i1}, \ldots, d_{in}$.
- every P_j computes $c_j = \mathcal{L}(d_{1j}, \ldots, d_{nj})$.