Homework (16)

1. [Continued from Problem 3 of Homework 15] With $f(\mathbf{x}) := x_1^2 + x_2^2$ for $\mathbf{x} \in \mathbb{R}^2$ consider

$$(P)\begin{cases} \min\limits_{\mathbf{x}} f(\mathbf{x}) \\ -x_2 \leq 0 \\ x_1^3 - x_2 \leq 0 \\ x_1^3(x_2 - x_1^3) \leq 0 \end{cases}.$$

a) Check whether SCQ holds.

b) Check whether GCQ and ACQ hold at its KKT points.

c) Find its dual function, with the domain specified.

Homework (16)

2. Find the point $\mathbf{x} \in \mathbb{R}^2$ that lies closest to the point $\mathbf{p} := (2, 3)^T$ under the constraints $g_1(\mathbf{x}) := x_1 + x_2 \leq 0$ and $g_2(\mathbf{x}) := x_1^2 - 4 \leq 0$.

   a) Illustrate the problem graphically.

   b) Verify that the problem is convex and fulfills SCQ.

   c) Determine the KKT points by differentiating between three cases: none is active, exactly the first one is active, exactly the second one is active.

   d) Find its dual function, with the domain specified.

Homework (16)

3. Given a support vector machine:

$$
\begin{aligned}
&\min_{\mathbf{w}} \frac{1}{2}\|\mathbf{w}\|^2, \\
&s.t.\ y_i(\langle \mathbf{w}, \mathbf{x}_i \rangle + \beta) \geq 1, (i = 1, \cdots, m).
\end{aligned}
$$

   a) Check whether the problem fulfills SCQ. What does SCQ mean in this scenario?

   b) Find its dual function, with the domain specified.

Homework (16)

4. Find the dual functions of the following problems:

a)

$$\min_{x} \; x^2 + 1$$
$$s.t. \; (x-2)(x-4) \leq 0,$$

b)

$$\min_{\mathbf{x}} \; \mathbf{c}^T \mathbf{x}$$
$$s.t. \; f(\mathbf{x}) \leq 0,$$

with $\mathbf{c} \neq \mathbf{0}$.