Homework (20)

1. Look at Example 638 of the lecture note. It is proven that $\mathbf{x}_\gamma$ is an eigenvector of $\mathbf{Q}$. Which eigenvalue is $\mathbf{x}_\gamma$ assocaited to? Prove your claim.
2. Consider the problem

$$\min_{\mathbf{x}} \ \frac{1}{2}\|\mathbf{x}\|_2, \quad s.t. \quad \mathbf{A}\mathbf{x} = \mathbf{b}, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{m\times n}, \mathbf{b} \in \mathbb{R}^m, m \leq n$, and rank $\mathbf{A} = m$. Let $\mathbf{x}^*$ be the solution and $\mathbf{x}_\gamma^* = \operatorname*{argmin}_{\mathbf{x}} \ \frac{1}{2}\|\mathbf{x}\|_2 + \gamma\|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2$.

   a. Suppose that

$$\mathbf{A} = [1 \quad 1], \quad \mathbf{b} = [1]. \tag{2}$$

   Verify that $\mathbf{x}_\gamma^*$ converges to the solution $\mathbf{x}^*$ of the original constrained problem as $\gamma \to \infty$.

   b. Prove that $\mathbf{x}_\gamma^* \to \mathbf{x}^*$ as $\gamma \to \infty$ holds in general. Try not to directly apply Theorem 640. Does γ really need to approach ∞ in order to achieve the solution to (1)?

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c. Randomly generate $\mathbf{A} \in \mathbb{R}^{200\times 300}$ and $\mathbf{b} \in \mathbb{R}^{200}$ and solve problem (1) numerically by the penalty method. Hand in your code and report.

3. Consider:

$$\min_{\mathbf{x}\in\mathbb{R}^N} \|\mathbf{y} - \mathbf{D}\mathbf{x}\|_2^2, \quad s.t. \quad \|\mathbf{x}\|_? \leq 1,$$

where $\|\mathbf{x}\|_?$ can be either the ℓ_1 norm or the ℓ_∞ norm. Randomly generate $\mathbf{D} \in \mathbb{R}^{200\times 300}$ and $\mathbf{y} \in \mathbb{R}^{200}$ and use Frank-Wolfe algorithm to solve it, for both ℓ_1 norm and ℓ_∞ norm. Further compare F-W algorithm with the projected gradient descent in convergence speed (objective function value vs. iteration number). Hand in your code and report.