Homework (1)

1. There is at least one photo of the people in "History of optimization" which is incorrect (not accounting for the ages and poses). Find them, provide your correct photos and give justifications.
2. The no-free-lunch (NFL) theorem is actually quite general. Give more instances that the NFL theorem applies.

Rethink whether to take this course if your math is not good enough

Homework (1.5)

1. Judge the properties of the following sets (openess, closeness, boundedness, compactness) and give their interiors, closures, boundaries, and accumulation points:

   a. $\mathcal{C}_1 = \emptyset$.

   b. $\mathcal{C}_2 = \mathbb{R}^n$.

   c. $\mathcal{C}_3 = \{x|0 \leq x < 1\} \cup \{x|2 \leq x \leq 3\} \cup \{x|4 < x \leq 5\}$.

   d. $\mathcal{C}_4 = \{(x, y)^T|x \geq 0, y > 0\}$.

   e. $\mathcal{C}_5 = \{k|k \in \mathbb{Z}\}$.

   f. $\mathcal{C}_6 = \{k^{-1}|k \in \mathbb{Z}\}$.

   g. $\mathcal{C}_7 = \{(1/k, \sin k)^T|k \in \mathbb{Z}\}$.

2. Prove that a set $\mathcal{C} \subseteq \mathbb{R}^n$ is closed iff (aka. if and only if) it contains the limit point of every convergent sequence in it.

Homework (1.5)

3. Prove that a point $\mathbf{x}$ is a boundary point of $\mathcal{C} \subseteq \mathbb{R}^n$ iff for $\forall \epsilon > 0$, there exists $\mathbf{y} \in \mathcal{C}$ and $\mathbf{z} \notin \mathcal{C}$ such that

$$\|\mathbf{y} - \mathbf{x}\|_2 \leq \epsilon, \quad \|\mathbf{z} - \mathbf{x}\|_2 \leq \epsilon.$$

4. Prove that $\mathcal{C} \subseteq \mathbb{R}^n$ is closed iff it contains its boundary, and is open iff it contains no boundary points.

5. Prove the following:

   a. $\overline{\mathcal{A} \cup \mathcal{B}} = \overline{\mathcal{A}} \cup \overline{\mathcal{B}}$; $\overline{\mathcal{A} \cap \mathcal{B}} \subseteq \overline{\mathcal{A}} \cap \overline{\mathcal{B}}$. Give an example showing that $\overline{\mathcal{A} \cap \mathcal{B}} \neq \overline{\mathcal{A}} \cap \overline{\mathcal{B}}$.

   b. $(\overline{\mathcal{A} \cap \mathcal{B}})^\circ = \mathcal{A}^\circ \cap \mathcal{B}^\circ$; $(\overline{\mathcal{A} \cup \mathcal{B}})^\circ \supseteq \mathcal{A}^\circ \cup \mathcal{B}^\circ$. Give an example showing that $(\overline{\mathcal{A} \cup \mathcal{B}})^\circ \neq \mathcal{A}^\circ \cup \mathcal{B}^\circ$.