Homework (15)

1. Give examples, which are different from the ones in the lecture note and the classroom, to show that $\mathcal{C}_{fd}(\mathbf{x}_0) \neq \mathcal{C}_t(\mathbf{x}_0) \neq \mathcal{C}_l(\mathbf{x}_0)$.

2.

a) Solve the optimization problem

$$\min_{\mathbf{x}} f(x_1, x_2) := 2x_1 + 3x_2, \quad s.t. \quad \sqrt{x_1} + \sqrt{x_2} = 5,$$

using Lagrange multipliers.

b) Visualize the contour lines of f as well as the set of feasible points, and mark the optimal solution $\mathbf{x}^*$.

c) Find all its KKT points. Do they all correspond to local minima?

Homework (15)

3. With $f(\mathbf{x}) := x_1^2 + x_2^2$ for $\mathbf{x} \in \mathbb{R}^2$ consider

$$(P)\begin{cases} \min\limits_{\mathbf{x}} f(\mathbf{x}) \\ -x_2 \leq 0 \\ x_1^3 - x_2 \leq 0 \\ x_1^3(x_2 - x_1^3) \leq 0 \end{cases}.$$

a) Determine the linearizing cone, the tangent cone and the feasible direction cones at the (strict global) minimal point $\mathbf{x}_0 := (0,0)^T$.

b) Find all its KKT points. Do they all correspond to local minima?

Homework (15)

4. Determine a triangle with minimal area containing two disjoint disks with radius 1. Without loss of generalization, let $(0,0)$, $(x_1, 0)$ and (x_2, x_3) with $x_1, x_3 \geq 0$ be the vertices of the triangle; (x_4, x_5) and (x_6, x_7) denote the centers of the disks.

a) Formulate this problem as a minimization problem in terms of seven variables and nine constraints.

b) $\mathbf{x}^* = (4 + 2\sqrt{2}, 2 + \sqrt{2}, 2 + \sqrt{2}, 1 + \sqrt{2}, 1, 3 + \sqrt{2}, 1)^T$ is a solution of this problem; calculate the corresponding Lagrange multipliers $\boldsymbol{\lambda}^*$, such that the KKT conditions are fulfilled.