Homework (11)

1. Consider the problem for computing the analytic center of the set of linear inequalities:

$$\min_{\mathbf{x}} f(\mathbf{x}) = -\sum_{i=1}^{m} \log(1 - \mathbf{a}_i^T \mathbf{x}) - \sum_{i=1}^{n} \log(1 - x_i^2),$$

with variable $\mathbf{x} \in \mathbb{R}^n$, and dom $f = \{\mathbf{x} | \mathbf{a}_i^T \mathbf{x} < 1, i = 1, \cdots, m, |x_i| < 1, i = 1, \cdots, n\}$.

We can choose $\mathbf{x}^{(0)} = \mathbf{0}$ as our initial point. You can generate instances of this problem by choosing $\mathbf{a}^i$ from some distribution on $\mathbb{R}^n$. Use the gradient method to solve the problem, using reasonable choices for the backtracking parameters, and a stopping criterion of the form $\|\nabla f(\mathbf{x})\|_2 \leq \eta$. Plot the objective function and step length versus iteration number. (Once you have determined p^* to high accuracy, you can also plot $f - p^*$ versus iteration.) Experiment with the backtracking parameters α and β to see their effect on the total number of iterations required. Hand in your code and a report showing how the parameters are chosen and the figures.

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2. Suppose f is strongly convex with $m\mathbf{I} \preceq \nabla^2 f(\mathbf{x}) \preceq M\mathbf{I}$. Let $\Delta\mathbf{x}$ be a descent direction at $\mathbf{x}$. Show that the backtracking stopping condition holds for

$$0 < t \leq -\frac{\nabla f(\mathbf{x})^T \Delta\mathbf{x}}{M\|\Delta\mathbf{x}\|_2^2}.$$

Use this to give an upper bound on the number of backtracking iterations.

3. Explain how to find a steepest descent direction in the ℓ_∞-norm, and write down its pseudo-code. Apply it to solve Problem 1 (i.e., redo Problem 1 by replacing the gradient descent with your steepest descent).