Homework (14)

1. Solve $\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2 + \lambda\|\mathbf{x}\|_1$ using the following majorant minimization methods:

   a. Use the Lipschitz gradient majorant function of $\frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2$;

   b. Use the variational majorant function $\min_{\mathbf{d}>\mathbf{0}} \frac{1}{2}\left(\mathbf{x}^T \mathbf{D}\mathbf{x} + \mathbf{1}^T \mathbf{D}^{-1}\mathbf{1}\right)$ of $\|\mathbf{x}\|_1$, where $\mathbf{1}$ is an all-one vector and $\mathbf{D}$ is a diagonal matrix with $\mathbf{d}$ on the diagonal.

Compare their perfomances.

2. Find a Lipschitz gradient surrogate of $f(\mathbf{x}) = \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2$. The tighter the better.

3. Can you design variational surrogates for $\|\mathbf{x}\|_\infty$ of vectors and $\|\mathbf{X}\|_{2,1}$ (sum of the ℓ_2-norms of columns) of matrices?

For coding problems, please write reports and hand in both codes and reports. Remember to restart your computer before running your codes.