Winter school project: Non-rigid Structure from Motion using Semi-Definite Programming (SDP) Relaxation

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Problem description: Non-Rigid Structure-from-Motion (NRSfM) is the problem of finding the 3D shape of a deforming object given a set of monocular images. NRSfM has been long treated in the literature as separate (different) problems from rigid structure-from-motion. When the scene structure is rigid the problem is generally well defined and has been much studied, with rigidity at the heart of almost all vision-based 3D reconstruction theories and methods. The NRSfM problem is naturally under constrained because there can be many different deformations that produce the same images. The common constraints, which include low dimensionality, local rigidity, and isometry, have been exploited to limit the set of solutions. As an open problem, the existing approaches for the NRSfM problem tend to be inaccurate, ill-posed, and unrobust. Existing NRSfM methods can be grouped into two main categories, Low-rank based methods and Physical based methods, depending on how deformations are modeled. A method belonging to Physical based methods will be introduced. As an example, the following image Fig. 1 shows the inputs and outputs of this problem:

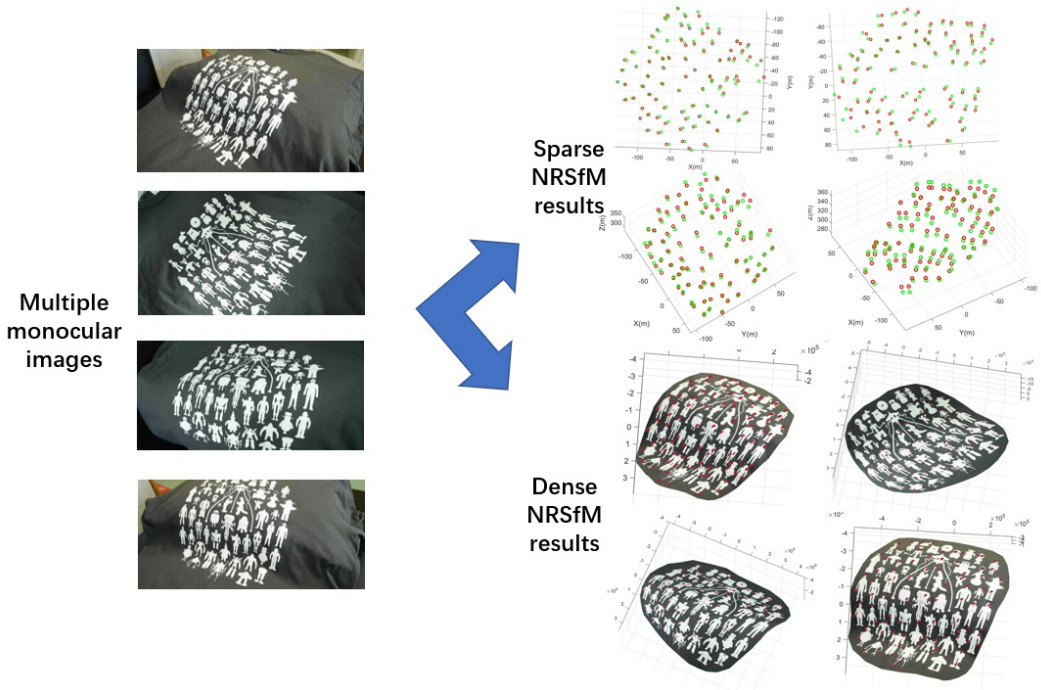

Figure 1: An example to show the inputs and outputs of NRSfM

Purpose: This project aims to introduce a Semi-Definite Programming (SDP) Relaxation based method, which is a convex formulation without requiring very good initialization, to solve the NRSfM problem. Based on this project, we would like to introduce the basic operations and solutions for this problem and further promote its application in the deformable SLAM.

Main reference: Paper: Ji P, Li H, Dai Y, et al. " Maximizing rigidity" revisited: a convex programming approach for generic 3D shape reconstruction from multiple perspective views[C]//Proceedings of the IEEE International Conference on Computer Vision. 2017: 929-937.

Code: Our code is based on MATLAB and CVX toolbox. https://github.com/cyb1212/Non-Rigid-Structure-from-Motion-based-on-SDP-Winter-school.git

Dataset: The datasets, including the flag, hulk, and T-shirt datasets, are shown in the code.

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Solution framework: The introduced framework is based on the cosine law and the Semi-Definite Programming (SDP) relaxation formulation. Based on the perspective projection and the nearest-neighbourhood graph (NNG), the camera center and two connected 3D features can be parameterized based on the triangle edges. The length of the edges are set as variables, including two depth values corresponding to two features and a connected edge between two features. Based on these triangles, naturally, we have a qadratically constrained quadratic program (QCQP) problem with limited scale for the NRSfM problem. In order to apply the inner method to solve this problem, this formulation is converted to a convex program using SDP relaxation. The formulation changing are shown in the following image Fig. 2 and the readers can find their sepecific meaning in the main reference.

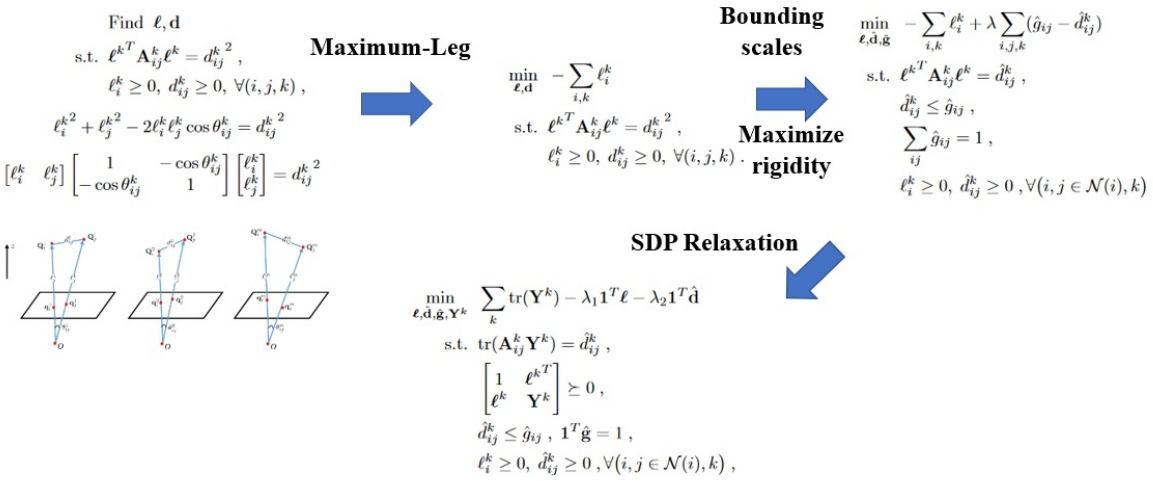

Figure 2: Model changing

Project step: The participants will be asked to complete the following exploration steps based on the provided code, including:

- **Adding constaints** In our provided code, the constraints, including scale limitation, SDP constraint, and positive limitation, have been deleted as a question for participants. You need to configure environment, adjust parameters, and fill this gap. The used codes need to follow the format of the CVX toolbox. This step is the basic and most difficult step in this project.

- **Testing more datasets** In our provided code, only the reduced T-shirt dataset with 4 scenes is offered. The participants are asked to add some other datasets in this project. The potential datasets include: (1) Flag dataset with 30 images, (2) full T-shirt dataset with 10 scenes; (3) Hulk dataset wiht 10 scenes. Because this formulation is relatively slow and the full datasets with too many images are time-consuming, it is totally fine to pick out partial dataset or divide the full one into multiple sub-parts.

- **Dealing with missing data** The case with some missing data is very common in the NRSfM problem and its later applications. The provided code is only suitable for the result with full dataset. If some of the feature correspondences are missing, the inextensibility constaints connected with them need to be delete. The readers is encourage to consider this sitation.

- **Speeding up codes (optional)** As menton before, this formulaion is time-consuming. Any ways to improve the running ability of the code is welcome.

Final evluation: The marks for these four steps are respectively: Adding constaints (60), Testing more datasets (20), Dealing with missing data (10), and Speeding up codes (10). The total marks are 100. Each part will be evaluated by combining the code and the final report.