# Active SLAM for Mobile Robots with Area Coverage and Obstacle Avoidance
## Supplementary Material

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This document provides supplementary material to the paper [1]. Therefore, it should not be considered a self-contained document, but instead regarded as an appendix of [1], and cited as:

*"Y. Chen, S. Huang, R. Fitch, D. Yang and J. Yu, Active SLAM for Mobile Robots with Area Coverage and Obstacle Avoidance, (Supplementary Material), 2018."*

Throughout this report, standard notations are used to refer to equations from [1]. This document is organized as follows: Appendices A, B, C, D and E provide proofs for some preliminaries, Conclusion 1, Conclusion 2 and the whole algorithm summary in [1] respectively.

# 1 Appendix A: Preliminaries

In this part, we give some preliminary knowledge about the variational description of eigenvalues and the Fischer's min-max theorem

**Theorem 1** *All eignvalues of a block diagonal matrix are the eignvalues of all block matrixes on the diagonal line.*

**Theorem 2 (Variational description of eignvalues)** *([2], p. 232) Let $\boldsymbol{\mathcal{A}}$ be a real symmetric $n \times n$ matrix with eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Let $\boldsymbol{S} = (\boldsymbol{s}_1, \boldsymbol{s}_2, \cdots, \boldsymbol{s}_n)$ be an orthogonal $n \times n$ matrix which diagonalizes $\boldsymbol{\mathcal{A}}$, so that:*

$$\boldsymbol{S}^T \boldsymbol{\mathcal{A}} \boldsymbol{S} = diag(\lambda_1, \lambda_2, \cdots, \lambda_n) \tag{1}$$

*Then, for $k = 1, 2, \cdots, n$, we have:*

$$\lambda_k = \min_{\boldsymbol{\mathcal{R}}_{k-1}^T \boldsymbol{x}=0} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{A}} \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}} = \max_{\boldsymbol{\mathcal{T}}_{k+1}^\top \boldsymbol{x}=0} \frac{\boldsymbol{x}^T \boldsymbol{\mathcal{A}} \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}} \tag{2}$$

*where,*

$$\begin{aligned} \boldsymbol{\mathcal{R}}_k &= (\boldsymbol{s}_1, \boldsymbol{s}_2, \cdots, \boldsymbol{s}_k), \\ \boldsymbol{\mathcal{T}}_k &= (\boldsymbol{s}_k, \boldsymbol{s}_{k+1}, \cdots, \boldsymbol{s}_n) \end{aligned} \tag{3}$$

**Theorem 3 (The Fischer's min-max theorem)** *([2], p. 233) Let $\boldsymbol{\mathcal{A}}$ be a real symmetric $n \times n$ matrix with eigenvalues $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$. Let $1 \leq k \leq n$. Then, for every $n \times (k-1)$ matrix $\boldsymbol{\mathcal{B}}$,*

$$\min_{\boldsymbol{\mathcal{B}}^T \boldsymbol{x}=0} \frac{\boldsymbol{x}^T \boldsymbol{\mathcal{A}} \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}} \leq \lambda_k \tag{4}$$

*for every $n \times (n-k)$ matrix $\boldsymbol{\mathcal{C}}$,*

$$\max_{\boldsymbol{\mathcal{C}}^T \boldsymbol{x}=0} \frac{\boldsymbol{x}^T \boldsymbol{\mathcal{A}} \boldsymbol{x}}{\boldsymbol{x}^T \boldsymbol{x}} \geq \lambda_k \tag{5}$$

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# 2 Appendix B: Proof of the lower bound in feature-based SLAM

**Proof:** Introduce the Jacobian matrix, we have:

$$\begin{aligned}
&\boldsymbol{\mathcal{I}}(\boldsymbol{X}) = \boldsymbol{J}(\boldsymbol{X})^{\top}\boldsymbol{\Sigma}^{-1}\boldsymbol{J}(\boldsymbol{X}) \\
&= \begin{bmatrix} \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{p}}^{\top} & \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{p}}^{\top} \\ \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{\theta}}^{\top} & \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{\theta}}^{\top} \end{bmatrix} \begin{bmatrix} \boldsymbol{\Sigma_p}^{-1} \otimes \boldsymbol{I}^{2\times 2} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Sigma_\theta}^{-1} \end{bmatrix} \begin{bmatrix} \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{p}} & \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{\theta}} \\ \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{p}} & \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{\theta}} \end{bmatrix}
\end{aligned} \tag{6}$$

For every part, we have:

$$\begin{aligned}
&\frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{p}}^{\top} \boldsymbol{\Sigma}_p^{-1} \otimes \boldsymbol{I}^{2\times 2} \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{p}} = \left(\boldsymbol{A_g} \boldsymbol{R} \boldsymbol{\Sigma_p}^{-1} \boldsymbol{R}^{\top} \boldsymbol{A_g}^{\top}\right) \otimes \boldsymbol{I}^{2\times 2} \\
&\frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{p}}^{\top} \boldsymbol{\Sigma_p}^{-1} \otimes \boldsymbol{I}^{2\times 2} \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{\theta}} = \left(\boldsymbol{A_g} \otimes \boldsymbol{I}^{2\times 2}\right) \boldsymbol{R} \boldsymbol{\Sigma_p}^{-1} \otimes \boldsymbol{I}^{2\times 2} \boldsymbol{\Gamma} \boldsymbol{R}^{\top} \triangle_{\boldsymbol{w_p}} \\
&\frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{\theta}}^{\top} \boldsymbol{\Sigma_p}^{-1} \otimes \boldsymbol{I}^{2\times 2} \frac{\partial \boldsymbol{h_p}}{\partial \boldsymbol{\theta}} + \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{\theta}}^{\top} \boldsymbol{\Sigma_\theta}^{-1} \frac{\partial \boldsymbol{h_\theta}}{\partial \boldsymbol{\theta}} \\
&= \triangle^{\top} \boldsymbol{R} \boldsymbol{\Gamma}^{\top} (\boldsymbol{\Sigma_p}^{-1} \otimes \boldsymbol{I}^{2\times 2}) \boldsymbol{\Gamma} \boldsymbol{R}^{\top} \triangle + \boldsymbol{A_p} \boldsymbol{\Sigma_\theta}^{-1} \boldsymbol{A}_p^{\top}
\end{aligned} \tag{7}$$

Based on $\boldsymbol{R}\boldsymbol{\Sigma_p}^{-1}\boldsymbol{R}^{\top} = \boldsymbol{\Sigma_p}^{-1}$, $\boldsymbol{\Gamma}\boldsymbol{R}^{\top} = \boldsymbol{R}^{\top}\boldsymbol{\Gamma}$, $\boldsymbol{\Gamma}^{\top}\boldsymbol{\Gamma} = \boldsymbol{I}^{2m\times 2m}$ and the FIM is a symmetrical matrix, we can get the FIM $\boldsymbol{\mathcal{I}}(\boldsymbol{X})$.

Based on the D-optimality criterion, we have:

$$\log(\det \begin{bmatrix} \boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}} \otimes \boldsymbol{I}^{2\times 2} & \boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}} \otimes \boldsymbol{I}^{2\times 2} \boldsymbol{\Gamma} \triangle_{\boldsymbol{w_p}} \\ (\boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}} \otimes \boldsymbol{I}^{2\times 2} \boldsymbol{\Gamma} \triangle_{\boldsymbol{w_p}})^{\top} & \triangle^{\top}_{\boldsymbol{w_p}} \triangle_{\boldsymbol{w_p}} + \boldsymbol{L}^{\boldsymbol{p}}_{\boldsymbol{w_\theta}} \end{bmatrix}), \tag{8}$$

Schur's Determinant Formula: if $\widetilde{\boldsymbol{A}}^{-1}$ exists,

$$\det \begin{bmatrix} \widetilde{\boldsymbol{A}} & \widetilde{\boldsymbol{B}} \\ \widetilde{\boldsymbol{C}} & \widetilde{\boldsymbol{D}} \end{bmatrix} = \det(\widetilde{\boldsymbol{A}}) \det(\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{C}} \widetilde{\boldsymbol{A}}^{-1} \widetilde{\boldsymbol{B}}) \tag{9}$$

Using the Schur's Determinant Formula, we can re-write the Eq.(46) into:

$$\det(\widetilde{\boldsymbol{A}}) = \det(\boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}} \otimes \boldsymbol{I}^{2\times 2}) = \det(\boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}})^2 \tag{10}$$

$$\begin{aligned}
&\det\left(\widetilde{\boldsymbol{D}} - \widetilde{\boldsymbol{C}} \widetilde{\boldsymbol{A}}^{-1} \widetilde{\boldsymbol{B}}\right) = \\
&\det( \triangle^{\top}_{\boldsymbol{w_p}} \triangle_{\boldsymbol{w_p}} + \boldsymbol{L}^{\boldsymbol{p}}_{\boldsymbol{w_\theta}} - \triangle^{\top}_{\boldsymbol{w_p}} \boldsymbol{\Gamma}^{\top} (\boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}}{}^{\top} \boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}}{}^{-1} \boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}}) \otimes \boldsymbol{I}^{2\times 2} \boldsymbol{\Gamma} \triangle_{\boldsymbol{w_p}})
\end{aligned} \tag{11}$$

Eq.(11) can be writed as:

$$\begin{aligned}
\text{Eq.(11)} &= 2\log\det(\boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}}) + \log\det(\triangle^{\top}_{\boldsymbol{w_p}} \boldsymbol{P}^{\boldsymbol{g}}_{\boldsymbol{w_p}} \triangle_{\boldsymbol{w_p}} + \boldsymbol{L}^{\boldsymbol{p}}_{\boldsymbol{w_\theta}}) \\
\boldsymbol{P}^{\boldsymbol{g}}_{\boldsymbol{w_p}} &= \boldsymbol{I}^{2m\times 2m} - \boldsymbol{\Gamma}^{\top} (\boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}}{}^{\top} \boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}}{}^{-1} \boldsymbol{A}^{\boldsymbol{g}}_{\boldsymbol{w_p}}) \otimes \boldsymbol{I}^{2\times 2} \boldsymbol{\Gamma}
\end{aligned} \tag{12}$$

For any two matrixes $\boldsymbol{N}$ and $\boldsymbol{M}$, meeting $\boldsymbol{M} \succeq \boldsymbol{0}$ and $\boldsymbol{N} \succeq \boldsymbol{0}$, we have:

$$\det(\boldsymbol{M} + \boldsymbol{N}) = \det(\boldsymbol{M} + \boldsymbol{N}) \geq \det(\boldsymbol{M}) \tag{13}$$

Because $\boldsymbol{P}^{g}_{w_p}$ is the orthogonal projection matrices, So we have:

$$\log(\det(\boldsymbol{\mathcal{I}}(\boldsymbol{X}))) \geq 2\log(\det(\boldsymbol{L}^{\boldsymbol{g}}_{\boldsymbol{w_p}})) + \log\left(\det\left(\boldsymbol{L}^{\boldsymbol{p}}_{\boldsymbol{w_\theta}}\right)\right) \tag{14}$$

It is proved.

# 3 Appendix C: Proof of Conclusion 1

**Proof:** The coefficient matrix $\boldsymbol{A_Z}$ has a speical struction. In fact, it shows the corresponding relationship between the state vectors of the submap and the joining global map. So it is a block matrix with multiple identity matrix and column full rank. At the same time, in every row, there is only one non-zero element, like:

$$\boldsymbol{A_Z} = \begin{bmatrix} & & & \vdots & & & \\ 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \\ & & & \vdots & & & \end{bmatrix}^{r_1 \times r_2}, \tag{15}$$

where $r_1 \geq r_2$ So we can get:

$$(\boldsymbol{A_Z^\top} * \boldsymbol{A_Z})_{i,j} = \begin{cases} zero, & i \neq j, \\ non-zero, & i = j. \end{cases} \tag{16}$$

So it is a diagonal matrix.

# 4 Appendix D: Proof of Conclusion 2

**Proof:** (1) Based on (2), we have:

$$\lambda_i(\boldsymbol{\mathcal{I}_Z}) = \min_{\boldsymbol{\mathcal{R}}_{i-1}^\top \boldsymbol{x}=0} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z} \boldsymbol{x}}{\boldsymbol{x}^\top \boldsymbol{x}}. \tag{17}$$

When $\boldsymbol{x} = \boldsymbol{A_Z y}$, which is a special solution for (17), we have:

$$\min_{\boldsymbol{\mathcal{R}}_{k-1}^\top \boldsymbol{x}=0} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z} \boldsymbol{x}}{\boldsymbol{x}^\top \boldsymbol{x}} \leq \min_{\substack{\boldsymbol{\mathcal{R}}_{k-1}^\top \boldsymbol{x} = 0 \\ \boldsymbol{x} = \boldsymbol{A_Z y}}} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z} \boldsymbol{x}}{\boldsymbol{x}^\top \boldsymbol{x}}. \tag{18}$$

Let $\boldsymbol{\mathcal{B}} = \boldsymbol{\mathcal{R}}_{k-1}^\top \boldsymbol{A_Z}$, we have:

$$Eq.(18) = \min_{\boldsymbol{\mathcal{B}y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top I_Z A_Z y}}{\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y}}. \tag{19}$$

Because $\boldsymbol{A_Z^\top A_Z}$ are the diagonal matrix, defined as:

$$\boldsymbol{A_Z^\top A_Z} = diag(\lambda_1(\boldsymbol{A_Z}), \lambda_2(\boldsymbol{A_Z}), \cdots, \lambda_k(\boldsymbol{A_Z})). \tag{20}$$

We can re-written the denominator $\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y}$ as: $\lambda_1(\boldsymbol{A_Z})y_1^2 + \lambda_2(\boldsymbol{A_Z})y_2^2 + \cdots \lambda_k(\boldsymbol{A_Z})y_k^2$. Then, we have:

$$\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y} = \lambda_1(\boldsymbol{A_Z})y_1^2 + \cdots \lambda_k(\boldsymbol{A_Z})y_k^2 \geq \widehat{\lambda}(\boldsymbol{A_Z})(\boldsymbol{y}^\top \boldsymbol{y}). \tag{21}$$

So, we obtain:

$$\min_{\boldsymbol{\mathcal{B}y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y}} \leq \min_{\boldsymbol{\mathcal{B}y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\widehat{\lambda}(\boldsymbol{A_Z})\boldsymbol{y}^\top \boldsymbol{y}}. \tag{22}$$

Based on **The Fischer's min-max theorem** ([2], p. 233) and $\widehat{\lambda}(A_Z) > 0$, we can get:

$$\lambda_i(\boldsymbol{\mathcal{I}_Z}) \leq \min_{\boldsymbol{\mathcal{B}}y=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\widehat{\lambda}(\boldsymbol{A_Z})\boldsymbol{y}^\top \boldsymbol{y}} \leq \frac{\lambda_i(\boldsymbol{\mathcal{I}_{all}})}{\widehat{\lambda}(\boldsymbol{A_Z})}. \tag{23}$$

$$\Longrightarrow \widehat{\lambda}(\boldsymbol{A_Z})\lambda_i(\boldsymbol{\mathcal{I}_Z}) \leq \lambda_i(\boldsymbol{\mathcal{I}_{all}}). \tag{24}$$

In short, we has proved the left part of **Conclusion 3**.
(2) Then, let's prove the other one:
For $n - k + 1 \leq j \leq n$

$$\lambda_j(\boldsymbol{\mathcal{I}_Z}) = \max_{\mathcal{T}_{k+1}^\top \boldsymbol{x}=0} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z} \boldsymbol{x}}{\boldsymbol{x}^\top \boldsymbol{x}}. \tag{25}$$

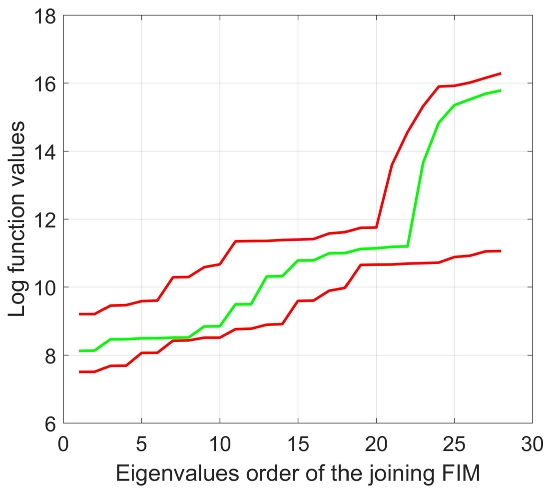


Figure 1: log function of eignvalues and their bounds

When $\boldsymbol{x} = \boldsymbol{A_Z y}$, which is a special solution for (25), we have:

$$\max_{\boldsymbol{\mathcal{T}}_{k+1}^\top \boldsymbol{x}=0} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z x}}{\boldsymbol{x}^\top \boldsymbol{x}} \geq \max_{\substack{\boldsymbol{\mathcal{T}}_{k+1}^\top \boldsymbol{x}=0 \\ \boldsymbol{x}=\boldsymbol{A_Z y}}} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z x}}{\boldsymbol{x}^\top \boldsymbol{x}}. \tag{26}$$

Let $\boldsymbol{\mathcal{B}_1} = \mathcal{T}_{k+1}^\top \boldsymbol{A_Z}$, we have:

$$\max_{\substack{\mathcal{T}_{k+1}^\top \boldsymbol{x}=0 \\ \boldsymbol{x}=\boldsymbol{A_Z y}}} \frac{\boldsymbol{x}^\top \boldsymbol{\mathcal{I}_Z x}}{\boldsymbol{x}^\top \boldsymbol{x}} = \max_{\boldsymbol{\mathcal{B}_1 y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y}}. \tag{27}$$

Because $\boldsymbol{A_Z^\top A_Z}$ are the diagonal matrix, we have:

$$\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y} = \lambda_1(\boldsymbol{A_Z}) y_1^2 + \cdots \lambda_k(\boldsymbol{A_Z}) y_k^2 \leq \widetilde{\lambda}(\boldsymbol{A_Z})(\boldsymbol{y}^\top \boldsymbol{y}). \tag{28}$$

So we can get:

$$\max_{\boldsymbol{\mathcal{B}_1 y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\boldsymbol{y}^\top \boldsymbol{A_Z^\top A_Z y}} \geq \max_{\boldsymbol{\mathcal{B}_1 y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\widetilde{\lambda}(\boldsymbol{A_Z})(\boldsymbol{y}^\top \boldsymbol{y})}. \tag{29}$$

Based on **The Fischer's min-max theorem** ([2], p. 233) and $\widetilde{\lambda}(\boldsymbol{A_Z}) > 0$, we can get:

$$\lambda_j(\boldsymbol{\mathcal{I}_Z}) \geq \max_{\mathcal{B}_1 \boldsymbol{y}=0} \frac{\boldsymbol{y}^\top \boldsymbol{A_Z^\top \mathcal{I}_Z A_Z y}}{\widetilde{\lambda}(\boldsymbol{A_Z})(\boldsymbol{y}^\top \boldsymbol{y})} \geq \frac{\lambda_{k-n+j}(\boldsymbol{\mathcal{I}_{all}})}{\widetilde{\lambda}(\boldsymbol{A_Z})}. \tag{30}$$

$$\Longrightarrow \widehat{\lambda}(\boldsymbol{A_Z})\lambda_j(\boldsymbol{\mathcal{I}_Z}) \geq \lambda_{k-n+j}(\boldsymbol{\mathcal{I}_{all}}). \tag{31}$$

Let's choose $j = n - k + i (1 \leq i \leq k)$:

$$\Longrightarrow \widehat{\lambda}(\boldsymbol{A_Z})\lambda_{n-k+i}(\boldsymbol{\mathcal{I}_Z}) \geq \lambda_i(\boldsymbol{\mathcal{I}_{all}}). \tag{32}$$

Finally, based on (24) and (32), we have: For $i = 1, 2, \cdots, k$

$$\lambda_i(\boldsymbol{\mathcal{I}_Z})\widehat{\lambda}(\boldsymbol{A_Z}) \leq \lambda_i(\boldsymbol{\mathcal{I}_{all}}) \leq \lambda_{n-k+i}(\boldsymbol{\mathcal{I}_Z})\widetilde{\lambda}(\boldsymbol{A_Z}). \tag{33}$$

It is proved.

In order to numerically verify Conclusion 2, we finish a small feature-based Linear SLAM with 6 poses and 5 features based on two small submaps, and then compute the *log* function of the eigenvalues $\lambda_i(\boldsymbol{\mathcal{I}_{all}})$ of the global joining matrix and their lower $\lambda_i(\boldsymbol{\mathcal{I}_Z})\widehat{\lambda}(\boldsymbol{A_Z})$ and upper bounds $\lambda_{n-k+i}(\boldsymbol{\mathcal{I}_Z})\widetilde{\lambda}(\boldsymbol{A_Z})$, shown in Fig. 1 .

We can see the log function (Green line) of the eigenvalues of joining FIM is bigger than their corresponding lower bound $\lambda_i(\boldsymbol{\mathcal{I}_Z})\widehat{\lambda}(\boldsymbol{A_Z})$ and smaller than their corresponding upper bound $\lambda_{n-k+i}(\boldsymbol{\mathcal{I}_Z})\widetilde{\lambda}(\boldsymbol{A_Z})$ (Red line).

# 5 Appendix E: The pseudocode of the whole algorithm

**Algorithm 1** Active SLAM based on submap joining, convex optimization and graph toplogy

**Require:** Area to be covered $Space$; Vehicle parameters: Velocity $v$, Sensor range $R_s$, Control limitation $C_u$; No-fly zone size $R_h^n$ and position $\boldsymbol{x_h^n}$; Other setting parameters.

**Ensure:** Estimated poses and mapped features (SLAM results).

1: **repeat**
2:     //Get model and measurement
3:     Move the robot based on dynamic model with noises
4:     Get the measurements from sensor model
5:     //SLAM
6:     Date association
7:     Solve SLAM problem by GN by MATLAB-Graph-Optimization package
8:     //Switching mechanism
9:     **if** $Index_1 \geq C_2^{index}$ **then**
10:         //Active SLAM task
11:         Build weighted Laplacian matrix $\widehat{\boldsymbol{L}}_{\boldsymbol{w_p}}^{\boldsymbol{g}}$
12:         Identify the mapped feature with a good accuracy in other submaps and mark them
13:         Compute coefficient $c_j$ by Algorithm 1 in [1]
14:         Built convex optimization problem by Eq. (25) in [1], solve and round it
15:     **elseif** $Index_1 \geq C_2^{index}$ **then**
16:         //Coverage task
17:         Built coverage optimization by Eq. (16) in [3] and solve it by SQP
18:     **else**
19:         //Both tasks
20:         Solve active SLAM by line 10-14 and coverage task by line 15-16 both
21:     **end if**
22:     //Submap joining
23:     Whether to open a new submap by current pose
24:     Whether to use Linear SLAM to finish map joining
25:     $t \leftarrow t + \triangle t$
26: **until** Believable covered area is large enough

# References


[1] Y. Chen, S. Huang, L. Zhao, J. Zhao and G. Dissanayake, Active SLAM for Mobile Robots with Area Coverage and Obstacle Avoidance, IEEE Transactions on Mechatronics, 2018, submitted.

[2] J. R. Magnus and H. Neudecker. Matrix differential calculus with applications in statistics and econometrics. Wiley series in probability and mathematical statistics, 1988.

[3] Y. Chen, S. Huang, R. Fitch, and J. Yu, Efficient Active SLAM based on Submap Joining, in 2017 Australasian Conference on Robotics and Automation (ACRA 2017), 2017.