Construction and Building Materials 173 (2018) 786–800

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Assessing concrete strength variability in existing structures based on the results of NDTs

Nuno Pereira, Xavier Romão *

Civil Engineering Department, Faculty of Engineering of the University of Porto, Rua Dr. Roberto Frias, 4200-465 Porto, Portugal

HIGHLIGHTS

- Correlation between the variability of RN and concrete strength data is analysed.
- Correlation between the variability of UPV and concrete strength data is analysed.
- Expressions estimating in situ concrete strength variability using UPV are proposed.
- Expressions estimating in situ concrete strength variability using RN are proposed.

ARTICLE INFO

Article history:
Received 11 August 2017
Received in revised form 2 April 2018
Accepted 6 April 2018
Available online 24 April 2018

Keywords:
Concrete strength
Concrete variability
In situ assessment
NDT techniques
Uncertainty

ABSTRACT

One of the main factors affecting the survey of concrete strength in existing structures is the inherent material variability, which can only be fully characterized when destructive tests are performed, especially in older structures. Therefore, having a preliminary estimate of the concrete strength variability facilitates the planning of destructive testing campaigns. In light of this, the proposed study presents the development of general empirical expressions estimating the in situ concrete strength variability using non-destructive test (NDT) results. These expressions are defined by examining the correlation between statistical parameters of datasets of concrete core strength, rebound hammer and ultrasonic pulse velocity test results using several correlation models that are based on common conversion models and on a generalization of the bi-objective approach. Based on these analyses, empirical models that are able to provide a reliable estimate of concrete strength variability using NDT results are proposed.

1. Introduction

Over the past few years, repairing and upgrading existing reinforced concrete (RC) structures has been recognized as an important priority ([1,2]). A crucial part of assessing the conservation state and the structural performance of these structures involves evaluating their actual mechanical properties [3]. Among these, evaluating the concrete compressive strength (f_c) is particularly important given its impact in the structural performance and the known issues associated with its assessment (e.g. see [4]).

Characterizing f_c in existing buildings usually involves determining two specific parameters: a location parameter, often the mean value of concrete strength μ, and a variability parameter, usually either the standard deviation σ or the coefficient of variation (*CoV*) of the data [5]. Given the properties of these parameters, the uncertainty associated with estimating μ can be seen to be related with the uncertainty associated with estimating the variability ([6]). Thus, estimating μ requires a reliable estimate of the inherent variability (i.e. σ or *CoV*) of concrete strength, which can attain very large values (e.g. see [7]) due to the effect of workmanship ([8,9]) among other factors. Other authors (e.g. [10,11]) highlighted the importance of the uncertainty associated with core testing and within-member variability when assessing in-situ concrete strength. Furthermore, previous research [5] has also shown the effect of sampling uncertainty (mainly focusing on member-to-member variability) associated with the use of samples of small size. The authors highlighted that even when adopting a finite population strategy to control the statistical uncertainty, large size samples of concrete core strength test results are required to get a reliable estimate of the variability.

The need for a large number of concrete core strength test results to accurately estimate concrete strength variability has led to the use of alternative methods involving additional sources of information. Bayesian methods have been proposed as possible approaches to incorporate the information of different sources

* Corresponding author.
E-mail address: xnr@fe.up.pt (X. Romão).

https://doi.org/10.1016/j.conbuildmat.2018.04.055
0950-0618/

when estimating the σ or *CoV* of concrete strength ([1,2,12]) or to quantify material safety factors ([13,14]). In some cases, prior information can also be established using data about σ or *CoV* based on past studies. For example, Caspeele and Taerwe [1] proposed a set of informative priors for different concrete classes based on concrete production data from Germany. Although their strategy can be adapted to different countries, its applicability to older RC structures for which there is no information regarding the expected concrete class may be difficult without preliminary in situ testing to estimate the concrete variability. In another case, Jalayer et al. [12] used a prior concrete strength distribution defined by a lognormal distribution with a median of 16.18 MPa and a *CoV* of 0.15 to represent typical values found in post-world war II construction in Italy. Alternatively, prior information can account for the results provided by non-destructive test (NDT) results. Giannini et al. [2] proposed a systematic framework combining concrete core and NDT results that requires a given number of cores to develop a case-specific regression model to convert NDT results into f_c estimates.

It has been shown that NDTs can be used to reduce the epistemic uncertainty, despite having as a main drawback the fact that they require the use of a conversion model [2]. Recently, Alwash et al. [15] analysed the uncertainties associated with destructive tests, NDT results and with the models that are used to convert NDT results into concrete strength estimates. In terms of conversion models, these authors analysed the efficiency of specific regressions, calibrating prior models (such as those in [16]) and the bi-objective approach [5]. They concluded that all the approaches can efficiently (i.e. using a low number of core strength test results) provide adequate estimates for the mean, but only the bi-objective approach was seen as a reliable method to estimate the variability. The bi-objective approach is a method proposed by Alwash et al. [5] where the first and second statistical moments of the in situ distribution of f_c are directly related to those obtained from the sample of NDT results. Therefore, this method provides an alternative estimate of the conversion model parameters based on aggregated data instead of using the classical approach based on individual test results.

Despite the significance of NDTs towards reducing the uncertainty in the concrete strength assessment process and reducing the number of destructive tests that need to be performed, no universal conversion model can be defined between the test results of a certain type of NDT and f_c, [17,20]. However, the possibility of developing empirical expressions that are able to provide estimates of the in situ concrete strength variability using NDT results has not been analysed so far. Therefore, the present paper addresses this issue by combining the main rationale behind the prior distributions proposed in [12] and the principles of the bi-objective approach. In particular, this paper analyses if empirical models correlating the statistical parameters of a population of concrete core strength test results and those of a population of rebound hammer test results (*RN*) or ultrasonic pulse velocity test results (*UPV*) can be used to establish initial estimates for the variability of the in situ concrete strength. Furthermore, the results of the study also provide information that can be used to improve the selection of conversion models for the bi-objective approach or for specific regression methods.

2. Determining the statistical parameters of the concrete strength distribution based on NDTs

2.1. Brief review of existing conversion models

The variability of concrete strength in existing RC structures, particularly in older RC buildings, can be associated with multiple factors. Some of the factors affect not only the concrete strength but also the NDT results ([15,17]). As such, the conversion models that are established between NDT results and f_c are also significantly affected by those factors. Therefore, as referred by Breysse et al. [18], an adequate conversion model can only be developed when based on data collected in situ. Among others, [19,20] present a thorough review of different types of conversion models that are available to correlate *RN* test results or *UPV* test results with f_c. Fig. 1 shows the distribution of the type of conversion models adopted in past studies based on the surveys in [17] and [20].

As shown in Fig. 1, linear models correlating f_c with *RN* or *UPV* are often adopted. These models usually establish a linear conversion function between the NDT results (T_i) and the core strength test results $f_{c,i}$ similar to:

$$f_{c,i} = a \cdot T_i + b. \tag{1}$$

According to the data presented in Fig. 1, the number of studies using linear conversion models is approximately the same as the number of cases that consider a power model instead. The performance of this type of model was analysed by Breysse & Fernández-Martinez [20] who considered the use of a power correlation model between *RN* and f_c with regression coefficients *c* and *d* such as:

$$f_{c,i} = c \cdot RN_i^d, \tag{2}$$

which is equivalent to the following linear correlation model on a log-log space:

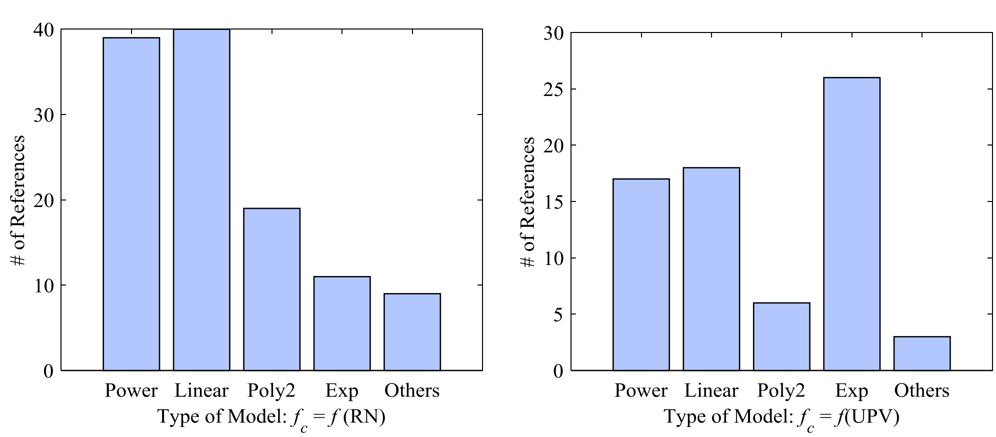

Fig. 1. Variety of models adopted to correlate f_c with *RN* or *UPV* based on the surveys in [17] and [20] (Power, Linear, Poly2 and Exp stand for power, linear, polynomial and exponential model, respectively).

$$\ln(f_{c,i}) = \ln(c) + d \cdot \ln(RN_i). \tag{3}$$

After analysing several models, these authors also found there is a correlation between the values of the regression coefficients c and d, with coefficient d being able to be defined as a function of c by $d = 1.031–0.259\cdot\ln(c)$. Furthermore, in a different study, Breysse [17] observed that the coefficient d of a power model correlating UPV and f_c such as:

$$f_{c,i} = c \cdot UPV_i^d, \tag{4}$$

could be approximated by the relation $d = \alpha - \delta\cdot\ln(c)$, with $\alpha = 2.393$ and $\delta = 0.684$. The range of d values for the power models indicated by Breysse ([17]; Tables 7 & 9) are 2.22–6.29 and 1.16–2.47 for UPV and RN test results, respectively.

Fig. 1 also shows that among the strategies adopted to correlate UPV and f_c, it is also common to select an exponential model expressed by:

$$f_{c,i} = r \cdot e^{q \cdot UPV_i}, \tag{5}$$

which is equivalent to the following linear correlation model on a log-log space:

$$\ln(f_{c,i}) = \ln(r) + q \cdot UPV_i. \tag{6}$$

Apart from the previous models that individually use RN or UPV to establish a correlation with f_c, other models can also be found in the literature where combinations of these NDTs are used. Among those, the model combining both RN and UPV, usually known as the SonReb method, is one of the most popular [17]. According to [17], the most commonly found models that combine NDTs can be seen to either follow a bilinear model defined by:

$$f_{c,i} = \alpha + \beta \cdot RN_i + \chi \cdot UPV_i. \tag{7}$$

or a double power model defined by:

$$f_{c,i} = \delta \cdot RN_i^{\varepsilon} \cdot UPV_i^{\phi}. \tag{8}$$

which is equivalent to the following linear form:

$$\ln(f_{c,i}) = \ln(\delta) + \varepsilon \cdot \ln(RN_i) + \phi \cdot \ln(UPV_i). \tag{9}$$

Irrespective of the selected type of model or of the number of NDTs that are used, the coefficients of the model have to be determined by regression analysis using in situ data. After determining the regression parameters, the model can be used to obtain pointwise (e.g. in a member of the structure) estimates of concrete strength values based on additional NDT results. These estimated f_c values are then used to estimate the mean and the dispersion of the concrete strength, which, in turn, can be used to assess the characteristic value of the concrete strength ([16,21]). These were the main principles adopted in [5] to establish the bi-objective approach which defines a conversion model between f_c and the NDTs not as a function of their pointwise values, but as a function of their mean and standard deviation instead. This approach was shown to be more effective than using classical pointwise conversion models to obtain reliable estimates of the mean and standard deviation of concrete strength since it requires less data. As a consequence, this correlation approach also enables the analysis of the correlation problem in terms of random variables and statistical distributions.

2.2. Using the bi-objective approach to establish the statistical parameters of the concrete strength distribution

The bi-objective approach developed in [5] can be interpreted as a method that establishes the regression parameters as a function of random variables defined by the concrete strength f_c and the NDT results T ([22]; *pp.*180). For the case where a linear correlation like Eq. (1) is assumed between variables T and f_c, the bi-objective approach establishes the regression parameters as:

$$b = \mu_{f_c} - a \cdot \mu_T \tag{10}$$

$$a = \frac{s_{f_c}}{s_T} \tag{11}$$

where μ_{f_c} and s_{f_c} represent the mean and the standard deviation of f_c, respectively, and μ_T and s_T represent the mean and the standard deviation of T, respectively. Hence, if the random variable f_c is defined by a general function $f_c = g(T)$, the expected value μ_{f_c} and the variance $s_{f_c}^2$ of f_c can be obtained using a Taylor series expansion which, by using only the first order terms, simplifies to ([22]; *pp.*183):

$$\mu_{f_c} \approx g(\mu_T) \tag{12}$$

$$s_{f_c}^2 \approx s_T^2 \cdot \left(\frac{\partial g(\mu_T)}{\partial T}\right)^2. \tag{13}$$

Eqs. (12) and (13) become Eqs. (10) and (11) when function g is assumed to be linear. If a power function similar to Eq. (2) is assumed instead, Eqs. (12) and (13) can be re-writhen as:

$$d = \frac{\ln\left(\frac{\mu_{f_c}}{c}\right)}{\ln(\mu_T)} \tag{14}$$

$$c = \frac{s_{f_c}}{s_T} \cdot \frac{1}{d \cdot \mu_T^{d-1}} \tag{15}$$

Alternatively, by considering the linear version of the power model on a log-log space, function g then takes a form similar to Eq. (3) which yields:

$$\ln(c^*) = \mu_{\ln f_c} - d^* \cdot \mu_{\ln T} \tag{16}$$

$$d^* = \frac{s_{\ln f_c}}{s_{\ln T}} \tag{17}$$

where $\mu_{\ln f_c}$ and $s_{\ln f_c}$ represent the mean and the standard deviation of the natural logarithm of f_c, respectively, and $\mu_{\ln T}$ and $s_{\ln T}$ represent the mean and the standard deviation of the natural logarithm of T, respectively. Additionally, if both random variables T and f_c are assumed to follow a lognormal distribution, it is possible to re-write Eqs. (16) and (17) as ([22]; *pp.*102):

$$\ln(c^*) = \mu_{\ln f_c} - d^* \cdot \mu_{\ln T} \approx \left(\ln \mu_{f_c} - \frac{1}{2} CoV_{f_c}^2\right) - d^* \cdot \left(\ln \mu_T - \frac{1}{2} CoV_T^2\right) \tag{18}$$

$$d^* \approx \frac{CoV_{f_c}}{CoV_T} \tag{19}$$

The approximations involved in Eqs. (18) and (19) consider that $s_{\ln f_c} = \sqrt{\ln(1 + CoV_{f_c}^2)} \approx CoV_{f_c}$ and $s_{\ln T} = \sqrt{\ln(1 + CoV_T^2)} \approx CoV_T$. These approximations can be shown to lead to an error below 7% as long as the standard deviation of the natural logarithm of the data is smaller than 0.5.

Finally, in case of adopting an exponential correlation model, the corresponding function g is similar to Eq. (5) and leads to a bi-objective approach that yields the following regression parameters:

$$q = \frac{\ln\left(\frac{\mu_{f_c}}{r}\right)}{\mu_T} \tag{20}$$

$$r = \frac{s_{f_c}}{s_T} \cdot \frac{1}{q \cdot e^{(q \cdot \bar{x}_T)}}, \tag{21}$$

By considering a linearized version of function g similar to Eq. (6) instead, and simultaneously assuming that f_c follows a lognormal distribution and that T follows a normal distribution, the following regression parameters are obtained:

$$\ln(r^*) = \mu_{\ln f_c} - q^* \cdot \mu_T \tag{22}$$

$$q^* = \frac{s_{\ln f_c}}{s_T} \approx \frac{CoV_{f_c}}{s_T}, \tag{23}$$

which also considers that $s_{\ln f_c} \approx CoV_{f_c}$. Eqs. (10)–(23) can be seen to represent simplified statistical moment-based bi-objective conditions that are compatible with commonly adopted models defining the relation between concrete strength and NDT results. The possibility of correlating statistical descriptors of the data in a way that is consistent with typical conversion models is one of the advantages of considering the bi-objective approach. Simultaneously, these descriptors are also estimators of the parameters of probabilistic distributions that are commonly considered for the material properties in later stages of the safety assessment of a structure. This aspect is particularly relevant when trying to extend the bi-objective approach to conversion models that involve more than two parameters, such as the SonReb approach (see Eqs. (7)–(9)). In these cases, deriving a multi-objective approach would require information about the third statistical moment. However, within the scope of a concrete strength assessment framework similar to the one proposed in [5] in which samples of data with relatively small sizes are normally involved, deriving such multi-objective approach may be inadequate. Estimates of third order or higher order statistical moments are known to be highly dependent on the sample size ([23]) and reliable estimates can only be obtained with sample sizes that will seldom be compatible with typical concrete strength assessment practice. In light of these arguments, a multi-objective approach is not developed herein for the SonReb approach. Nevertheless, regression models involving the variability of concrete strength and NDT results using a SonReb-like approach can be developed and tested, as seen in the following.

2.3. Development of general models for the concrete strength variability based on NDTs

Given that developing an adequate survey plan to characterize the concrete strength of an existing building requires information about the variability of the concrete strength, it is important to have methods capable of providing a preliminary estimate of this property.

By following principles similar to those attempting to establish generic strength-NDT laws, general variability relations compatible with the strength-NDT laws presented in the previous section are developed herein to provide preliminary estimates of the concrete strength variability. The importance of these general models, as referred before, lies in their ability to provide information for defining the minimum number of destructive tests necessary to evaluate concrete strength ([4]) based on an estimate of the concrete strength variability. The functional form of the candidate models that are developed based on the previously analysed strength-NDT laws and the corresponding terminology that was considered to reference them hereon are presented in Table 1.

Parameters μ_j, s_j and CoV_j stand for the mean, standard deviation and coefficient of variation of a given data j, respectively, where j can be defined by f_c, RN or UPV test results. Parameters a, c, d, d^*, r, q and q^* are the model coefficients obtained by regression analysis. It is noted that second order polynomial models are not among the considered candidate approaches given their limitations in modelling the physical phenomena that are involved (i.e. they don't provide fully monotonic relations between the dependent and independent variables of the model, in this case the variability of f_c and that of the selected NDT). Additionally to the models presented in Table 1, general relations involving SonReb-like approaches were also developed considering that the most common SonReb-like models involve a linear combination of RN and UPV or a double power model, as referred in [17]. Following the principles that were considered for the case where a single NDT is used, the general form of Eq. (13) can be used to write a first order variance estimate given by ([22]; *pp.*186):

$$s^2_{f_c} \approx s^2_{RN} \cdot \left(\frac{\partial g(\mu_{RN})}{\partial RN}\right)^2 + s^2_{UPV} \cdot \left(\frac{\partial g(\mu_{UPV})}{\partial UPV}\right)^2 + \rho_{RN,UPV} \cdot s_{RN} \cdot s_{UPV} \left(\frac{\partial^2 g(\mu_{RN}, \mu_{UPV})}{\partial RN \cdot \partial UPV}\right). \tag{24}$$

As can be seen, this expression requires the correlation factor $\rho_{RN,UPV}$ between RN and UPV to be known. However, it must be noted that this parameter is rarely (if ever) available from studies involving the development of a regression model by the SonReb method. Table 2 shows the candidate models that can be developed using Eq. (24) based on the different g functions presented before (Eqs. (7)–(9)). M1-SonReb is the first order Taylor approximation for the variance using a linear combination similar to Eq. (7) which can be seen to be independent of the value of $\rho_{RN,UPV}$. On the other hand, M2-SonReb is the first order approximation obtained using a double power model and depends on $\rho_{RN,UPV}$. The case where f_c, UPV and RN are uncorrelated is represented by

Table 1
Candidate models selected to evaluate the potential correlation between the variability estimators of RN, UPV and f_c test results.

Reference	Correlation functions compatible with typical regression models	Based on	Hypothesis
M1-RN M1-UPV	$s_{f_c} = a \cdot s_{RN}$ $s_{f_c} = a \cdot s_{UPV}$	Eq. (11)	Linear regression between NDT and f_c
M2-RN M2-UPV	$\frac{s_{f_c}}{s_{RN}} = c \cdot d \cdot \mu_{RN}^{d-1}$ $\frac{s_{f_c}}{s_{UPV}} = c \cdot d \cdot \mu_{UPV}^{d-1}$	Eq. (15)	Power regression between NDT and f_c
M3-RN M3-UPV	$CoV_{f_c} = d^* \cdot CoV_{RN}$ $CoV_{f_c} = d^* \cdot CoV_{UPV}$	Eq. (19)	Power regression between NDT and f_c and both variables are assumed to follow a lognormal distribution.
M4-RN M4-UPV	$\ln\left(\frac{s_{f_c}}{s_{RN}}\right) = \ln(r) + \ln(q) + q \cdot \mu_{RN}$ $\ln\left(\frac{s_{f_c}}{s_{UPV}}\right) = \ln(r) + \ln(q) + q \cdot \mu_{UPV}$	Eq. (21)	Exponential regression between NDT and f_c
M5-RN M5-UPV	$CoV_{f_c} = q^* \cdot s_{RN}$ $CoV_{f_c} = q^* \cdot s_{UPV}$	Eq. (23)	Exponential regression between NDT and f_c where f_c and the NDT are assumed to follow a lognormal and a normal distribution, respectively

Table 2
Candidate models selected to evaluate the potential correlation between the variability estimators of RN, UPV and f_c test results using SonReb-like approaches.

Ref.	Correlation functions compatible with typical regression models	Hypothesis
M1-SonReb	$s_{f_c}^2 = (\beta \cdot s_{RN})^2 + (\chi \cdot s_{UPV})^2$	Variance approximation for the SonReb model according to Eq. (7).
M2-SonReb	$s_{f_c}^2 = \left(\varepsilon \cdot \delta \cdot s_{RN} \cdot \mu_{RN}^{(\varepsilon-1)} \cdot \mu_{UPV}^{\phi}\right)^2 + \left(\phi \cdot \delta \cdot s_{UPV} \cdot \mu_{RN}^{\varepsilon} \cdot \mu_{RN}^{(\phi-1)}\right)^2$ $+\rho_{RN,UPV} \cdot s_{RN} \cdot s_{UPV} \cdot \left(\varepsilon \cdot \delta \cdot \phi \cdot \mu_{RN}^{(\varepsilon-1)} \cdot \mu_{RN}^{(\phi-1)}\right)$	Variance approximation for the SonReb model according to Eq. (8).
M2*-SonReb	$s_{f_c}^2 \approx \left(\varepsilon \cdot \delta \cdot s_{RN} \cdot \mu_{RN}^{(\varepsilon-1)} \cdot \mu_{UPV}^{\phi}\right)^2 + \left(\phi \cdot \delta \cdot s_{UPV} \cdot \mu_{RN}^{\varepsilon} \cdot \mu_{RN}^{(\phi-1)}\right)^2$	Similar to M2-SonReb but assuming that *UPV* and *RN* are uncorrelated.
M3-SonReb	$CoV_{f_c}^2 \approx (\varepsilon^* \cdot CoV_{RN})^2 + (\phi^* \cdot CoV_{UPV})^2$	CoV approximation for the SonReb model according to Eq. (9) assuming that all variables are lognormally distributed.

the expression of model M2*-SonReb (a special case of M2-SonReb). Finally, model M3-SonReb extends the assumptions of M2*-SonReb by also assuming that f_c, *UPV* and *RN* follow lognormal distributions.

3. Methodology adopted to evaluate the generalized estimators for the concrete strength variability

3.1. Selected datasets of test results

The validity of the selected candidate models for establishing preliminary estimates of the concrete strength variability was analysed using a series of datasets comprising test results of core strength f_c, *RN* and *UPV* obtained from different in situ and laboratory tests. The database of selected results involves statistical parameters (μ, *s*, *CoV* and sample size) extracted from the test campaign data obtained from [4,2,24–51], some of which have also been used in the study conducted in [52]. In total, the database contains 78 sets of data, where 68 sets have statistical data from f_c and *RN* test results, and 50 sets have statistical data from f_c and *UPV* test results. Among these datasets, a total of 40 have test results of f_c and simultaneously of *RN* and *UPV*. Since information regarding the correlation between the *RN* and *UPV* test results was not available for these 40 datasets, only models not involving information about this parameter were analysed herein (M1-SonReb, M2*-SonReb and M3-SonReb). The full database of statistical parameters adopted, along with the corresponding references can be found in the Appendix.

3.2. Regression analysis

The models estimating concrete strength variability were defined based on the three-step ROUT (robust regression and outlier removal) procedure proposed in [53]. The first step of the procedure involves fitting a robust curve to the data. In the second step, the residuals of the robust fit are analysed to determine if one or more values are trend outliers. In the third step, the data identified as trend outliers in the second step are removed and an ordinary least squares regression is performed on the remaining data. The detection process considered in the second step is based on an outlier identification test adapted from the False Discovery Rate approach for testing multiple comparisons, as proposed in [53]. To analyse the sensitivity of the results of this second step to the type of weight function considered in the robust fitting, different functions were tested (e.g. see [54,55]). These preliminary analyses indicated that the outlier identification process was not sensitive to the selected weight function. Therefore, the robust fits were all performed using Tukey's bisquare function. Furthermore, the outlier identification process also depends on the value selected for the false discovery rate. This value was set as 10% based on the discussion presented in [53] and to account for the uncertainty of the measured data (e.g. repeatability and reproducibility issues, variability due to environmental conditions). By selecting a 10% threshold, the final regression analyses will be more clearly focussed on fitting the bulk of the data, thus emphasizing the average character of the models that are envisioned. The adjusted coefficient of determination (adj-R^2) was used as the goodness-of-fit measure of the model obtained from the final regression. To emphasize these results, the corresponding fits that would be obtained without removing the outlying data (i.e. without applying the ROUT procedure) are also presented, along with their value of adj-R^2. For the more relevant cases, this goodness-of-fit analysis is also complemented by examining the root mean squared error (RMSE) and by examining the ratios between the predictions made by the model and the corresponding real values of the concrete strength variability. For the M2* SonReb model, a nonlinear fitting procedure was adopted to adjust the multi-parameter curve.

4. Results obtained from the correlation analyses

4.1. Overview

The following sections present the results obtained by fitting the models identified in Section 2 to the experimental data defined in Section 3 using regression analysis with and without removing the outlying data. In the following plots, the outlying data are represented by squares while the data considered in the regression analysis are represented by circles. In total, 68 pairs of f_c and *RN* data were used in the results shown in Section 4.2, 50 pairs of f_c and *UPV* data in those shown in Section 4.3, and 40 triplets of f_c, *RN* and *UPV* data in those presented in Section 4.4. Details of the adopted datasets are available in the Appendix, as referred before. Section 4.4 presents the overall analysis of the regression results using relations between predicted and real values due to difficulties in representing the surface plots that are obtained from the regression analyses. Still, the goodness of the regression results is discussed using the principles adopted in Sections 4.2 and 4.3.

4.2. Results obtained for the correlation between the variability of f_c and RN

The regression results obtained with model M1-RN are presented in Fig. 2. The quality of the regression that is obtained without the ROUT procedure (Fig. 2a) shows there is no correlation between s_{f_c} and s_{RN} (the value of adj-R^2 is negative, which means that the fit provides results that are worse than a horizontal line equal to the mean value of the data). When applying the ROUT method, one outlying value is excluded from the final regression (Fig. 2b) but the properties of the correlation remain similar to the previous case.

Fig. 3 shows the regression results obtained using the power-based correlation defined by model M2-RN. In this case, the level of correlation found between the compound variable s_{f_c}/s_{RN} and

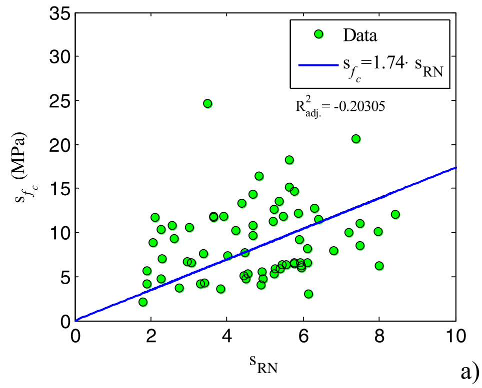

Fig. 2. Results of the regression analysis for model M1-RN without the ROUT procedure (a) and with the ROUT procedure (b).

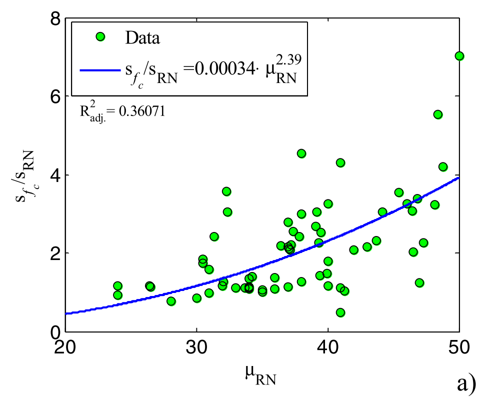

Fig. 3. Results of the regression analysis for model M2-RN without the ROUT procedure (a) and with the ROUT procedure (b).

the mean of the *RN* test results μ_{RN} without the ROUT procedure (Fig. 3a) is larger than the one observed when using the linear regression model (adj-R^2 is 0.36). However, when applying the ROUT procedure, five outlying values are excluded from the final regression (Fig. 3b) and only a minor change is observed in the correlation level (adj-R^2 is now 0.35).

When analysing model M3-RN (which considers that both f_c and *RN* follow a lognormal distribution) the results shown in Fig. 4 are obtained. As can be seen, there is a noticeable linear correlation between CoV_{f_c} and CoV_{RN}. On average, it can be seen that CoV_{f_c} is approximately two times the estimated value of CoV_{RN}. Still, the fit that was obtained without the ROUT procedure (Fig. 4a) has a value of adj-R^2 which is low (0.17). However, after applying the ROUT procedure (Fig. 4b), four outlying values are excluded from the regression and the value of adj-R^2 increases up to 0.50. For this case, parameter d^* was found to have an expected value of 1.94 with a 95% confidence interval ($CI_{95\%}$) of [1.84, 2.04]. The value of the RMSE for this regression is 0.056, which is lower than the value that was obtained without the ROUT procedure (0.090).

When fitting an exponential-based correlation between f_c and *RN* defined by model M4-RN, the regression results shown in Fig. 5 are obtained.

The correlation was established on a semi-log space in order to reduce the nonlinearity of the regression model and try to improve the quality of the fit. In this case, the ROUT procedure identified one outlying value, which improved the correlation level and lead to an increase of the adj-R^2 from 0.33 to 0.38.

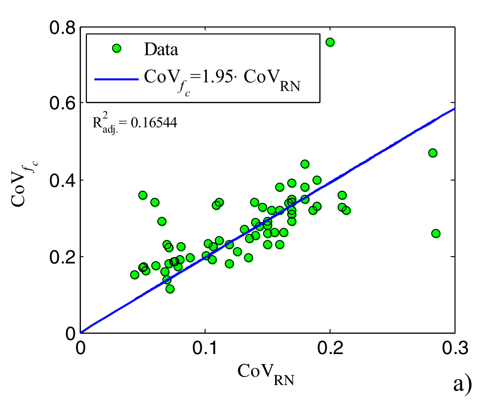

Fig. 4. Results of the regression analysis for model M3-RN without the ROUT procedure (a) and with the ROUT procedure (b).

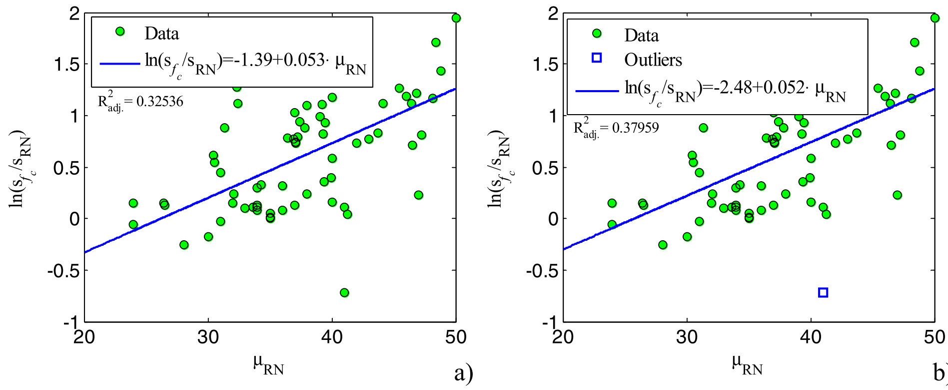

Fig. 5. Results of the regression analysis for model M4-RN without the ROUT procedure (a) and with the ROUT procedure (b).

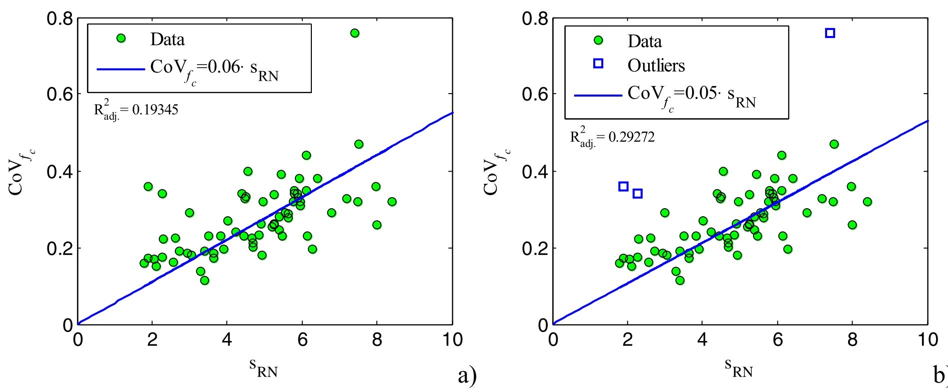

Fig. 6. Results of the regression analysis for model M5-RN without the ROUT procedure (a) and with the ROUT procedure (b).

Fig. 6 shows the results obtained by fitting model M5-RN derived from an exponential-based correlation between f_c and *RN*, and considering also that f_c and *RN* follow a lognormal distribution and a normal distribution, respectively. The level of correlation improves after applying the ROUT procedure. When the regression is performed without applying the ROUT procedure (Fig. 6a), adj-R^2 is 0.19. After applying the procedure (Fig. 6b), three outlying values are excluded and the correlation level of the new fit has now an adj-R^2 value of 0.29.

4.3. Results obtained for the correlation between the variability of f_c and UPV

The regression results obtained with model M1-UPV are shown in Fig. 7. The trends that were found in the regressions obtained with and without applying the ROUT procedure show there is no correlation between s_{f_c} and s_{UPV} (negative values of adj-R^2s were obtained).

Unlike for the previous model, a significant level of correlation was observed when analysing the power-based model M2-UPV, as shown in Fig. 8. The correlation level found without applying the ROUT procedure (Fig. 8a) yielded a value of adj-R^2 of 0.54 and a RMSE of 18.61, which indicates that s_{f_c} and s_{UPV} are moderately correlated when using model M2-UPV. This correlation improved further by applying the ROUT procedure and excluding three trend outliers, leading to an adj-R^2 of 0.66 and a RMSE of 9.20. The model that best fitted the data yielded an expected value for parameter c of 0.063 (with a $CI_{95\%}$ of [−0.0192, 0.146]) and an expected value for parameter d of 4.35 (with a $CI_{95\%}$ of [3.574, 5.117]).

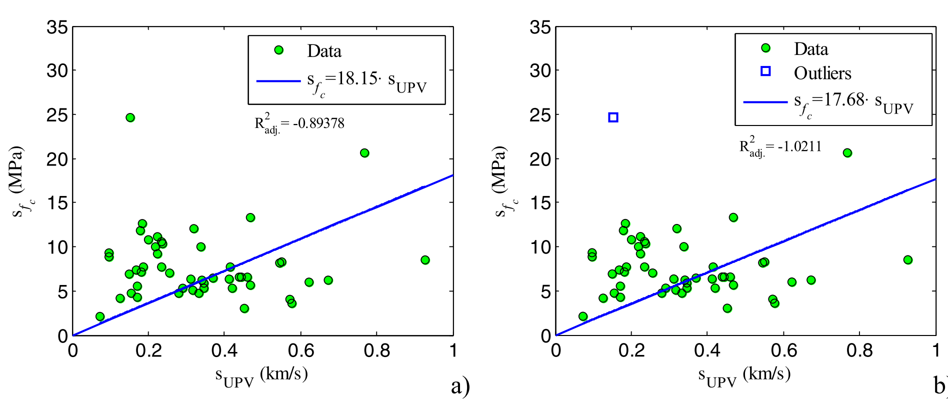

Fig. 7. Results of the regression analysis for model M1-UPV without the ROUT procedure (a) and with the ROUT procedure (b).

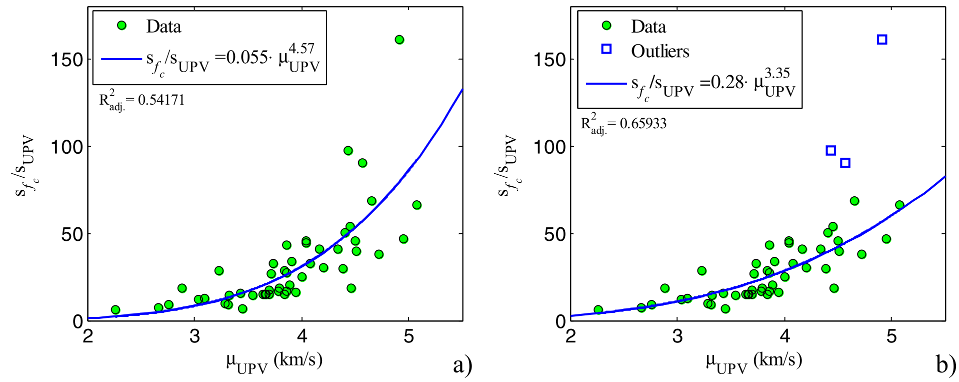

Fig. 8. Results of the regression analysis for model M2-UPV without the ROUT procedure (a) and with the ROUT procedure (b).

When analysing model M3-UPV, which also considers a power-based correlation but assumes that f_c and *UPV* follow a lognormal distribution, the results shown in Fig. 9 are obtained. In this case, the ROUT procedure identified two trend outlier data. Contrary to what was seen for model M3-RN, no meaningful correlation was observed when using model M3-UPV (the value of adj-R^2 is negative), even after excluding the identified outliers.

Similar to what was found for model M2-UPV, a better correlation was obtained when analysing the exponential-based relation considered by model M4-UPV. In this case, the ROUT procedure was unable to identify outlying data. Therefore, the final regression presented in Fig. 10 corresponds to the ordinary least squares fit obtained for the entire data. As can be seen, model M4-UPV leads to a high correlation level (the adj-R^2 is 0.68 and the RMSE is 0.40). The model fit yielded an expected value for parameter q of 0.981 (with a $CI_{95\%}$ of [0.790, 1.171]) and an expected value for parameter r of 0.563 (with a $CI_{95\%}$ of [0.220, 1.442]).

When analysing model M5-UPV, which considers that f_c follows a lognormal distribution and that *UPV* follows a normal distribution, the regression results presented in Fig. 11 do not show an improvement in the level of correlation when compared to the results shown in Fig. 10. The results of Fig. 11 are only better than those shown in Fig. 9 for model M3-UPV that also involves distribution assumptions. The correlation level that is found without applying the ROUT procedure (Fig. 11a) is very low (the value of adj-R^2 is 0.08). After applying the ROUT procedure, no outlier value was found.

4.4. Results obtained using the SonReb-like approaches

As mentioned before, regression results for the SonReb-like approaches are presented using plots displaying predicted *vs* real values. It is also noted that RMSE values appearing in those plots correspond to variance or CoV^2 RMSE values, depending on the regression model under analysis. Fig. 12 presents the results obtained for model M1-SonReb in terms of the predicted standard deviation against its real value. As seen in Fig. 12b, the ROUT procedure excluded four outlying values. The comparison of both regression results highlights the larger sensitivity of this model to variations of the standard deviation of *RN* than to those of the standard deviation of *UPV* (β is considerably larger than χ). Although similar coefficients were found before and after applying the ROUT procedure, the latter leads to a smaller RMSE but also to a lower R^2_{adj}.

Fig. 13 shows the results obtained using model M2*-SonReb, assuming that *UPV* and *RN* are uncorrelated. The results obtained

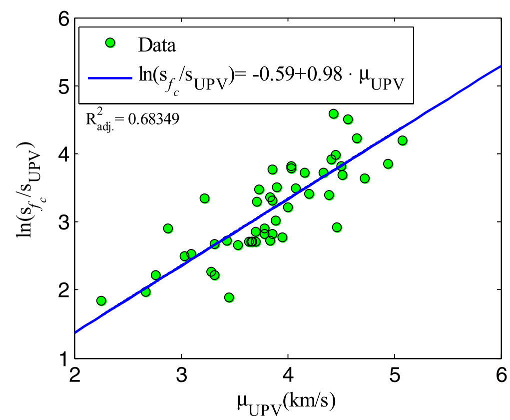

Fig. 10. Results of the regression analysis for model M4-UPV without the ROUT procedure since no outliers were identified.

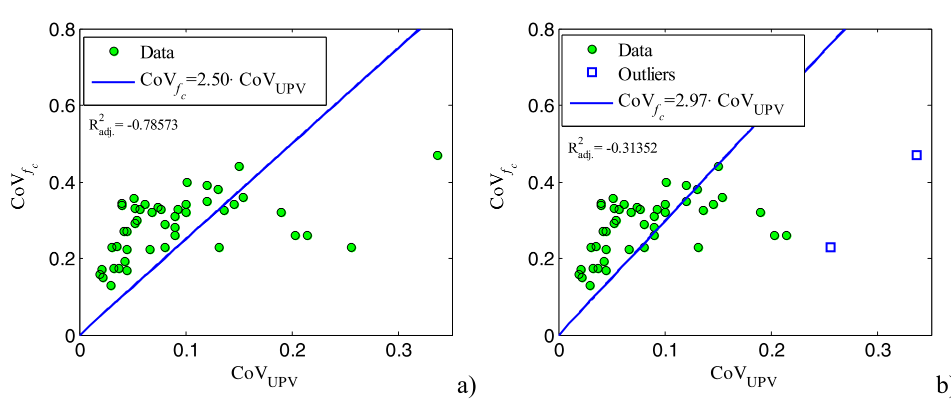

Fig. 9. Results of the regression analysis for model M3-UPV without the ROUT procedure (a) and with the ROUT procedure (b).

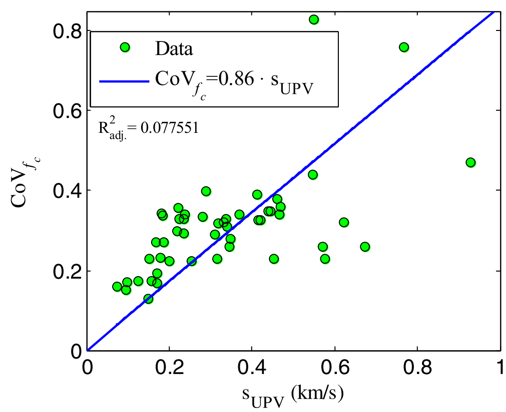

Fig. 11. Results of the regression analysis for model M5-UPV without the ROUT procedure since no outliers were identified.

after excluding four outliers (Fig. 13b) identified using the ROUT procedure indicate that concrete strength variability estimated using M2*-SonReb is more sensitive to the *UPV* data (the power coefficient of *UPV* is higher than that of *RN*). After applying the ROUT procedure, the regression results show a similar coefficient of determination but lead to a reduction of the RMSE to about one third.

Finally, Fig. 14 shows the results obtained for model M3-SonReb with and without applying the ROUT procedure. This model considers the same hypothesis of M2*-SonReb while also considering that *UPV*, *RN* and f_c follow lognormal distributions.

As seen in Fig. 14, the *RN* coefficient ε* only changes slightly between the cases where outliers are excluded or not. Conversely, the *UPV* coefficient ϕ* indicates the contribution of *UPV* to be irrelevant when estimating $CoV^2_{f_c}$ using this model. Applying the ROUT procedure leads to regression results that have a lower coefficient of determination but also a lower RMSE. It can be seen that a reduction of the RMSE of $CoV^2_{f_c}$ from 0.049 to 0.040 is equivalent to a reduction of the RMSE of CoV_{f_c} from 0.22 to 0.20.

5. Discussion of the results

5.1. Results obtained for the correlation between the variability of f_c and RN

Fig. 2 has shown that the regression coefficient *a* (that has a value close to 1.70) obtained for model M1-RN is within the range of expected values, according to [16]. However, the correlation level found in the analysis does not support the adoption of a linear model to correlate the variability of f_c and *RN*. Although the use of linear conversion models can be found in previous studies available in the literature, such models are only expected to provide adequate results if the fitting range is chosen to be narrow [19].

Using the power-based model M2-RN improves the level of correlation found between the variability of f_c and *RN*. This result

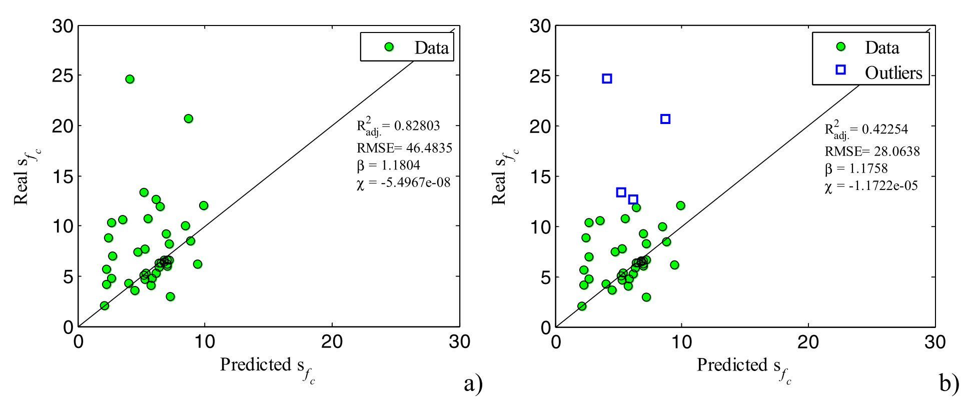

Fig. 12. Results of the regression analysis for model M1-SonReb without the ROUT procedure (a) and with the ROUT procedure (b).

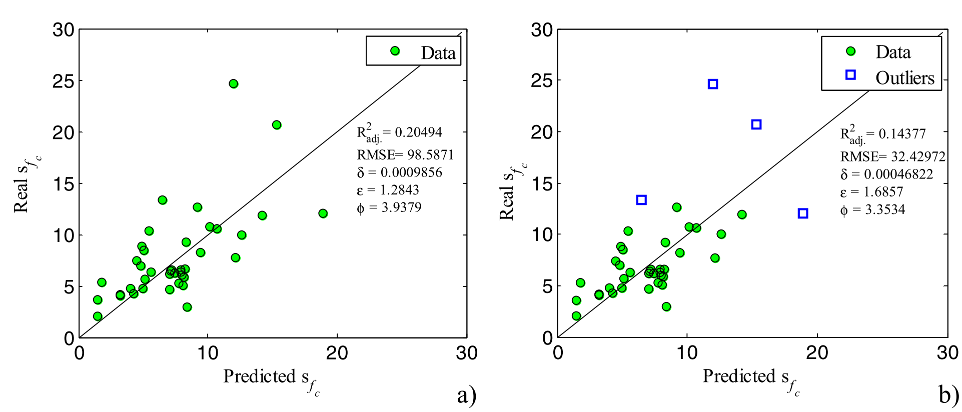

Fig. 13. Results of the regression analysis for model M2*-SonReb without the ROUT procedure (a) and with the ROUT procedure (b).

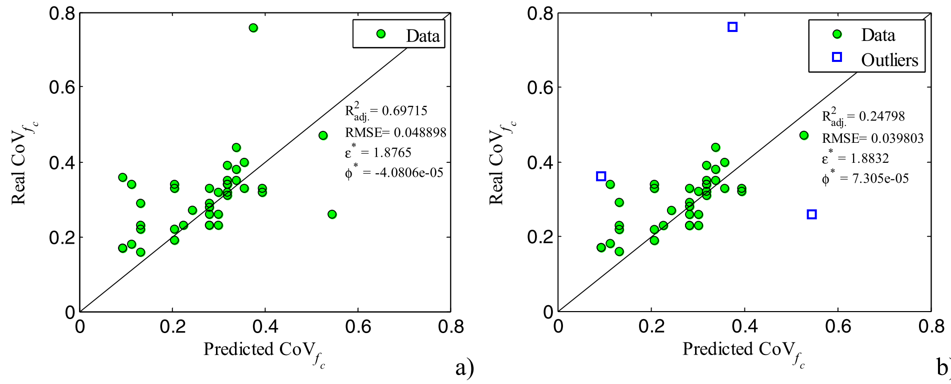

Fig. 14. Results of the regression analysis for model M3-SonReb without the ROUT procedure (a) and with the ROUT procedure (b).

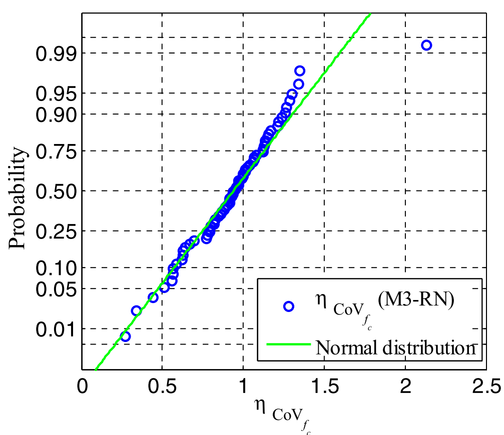

Fig. 15. Probability plot of the ratio between the estimates made with model M3-RN and the real values of CoV_{f_c}.

reflects the main issues reported in [20] where a systematic review of conversion models suggested the use of power models in this case. However, a stronger correlation level was found for this type of model when f_c and *RN* are assumed to follow a lognormal distribution, i.e. for model M3-RN. The value of adj-R^2 that was obtained is 0.50 which corresponds to a Pearson correlation coefficient of 71%. Furthermore, the expected value of the coefficient d^* that was found to be 1.94 is seen to be in line with values available from previous studies addressing conversion models of this type and that range from 1.0 to 4.0, with an average value of 2.10 [17]. It can therefore be concluded that model M3-RN provides an adequate indicator for the variability of f_c based on the variability of *RN*.

Although the use of an exponential-based conversion model is not as common as the power-based approach, the former was found to be used in several cases according to the review in [19]. Still, given that the level of correlation that was obtained with model M3-RN is larger than the one observed for M4-RN, M3-RN is considered to be more adequate to estimate concrete variability. For the case of model MR5-RN, the regression coefficient that was found for the correlation between CoV_{f_c} and s_{RN} when *RN* and f_c are assumed to follow a normal and a lognormal distribution, respectively, ($q^* = 0.05$) is compatible with the values found in [19], which range between 0.06 and 0.08. However, the quality of the fit obtained with this model is lower than the one obtained with model M4-RN, making it also less reliable than M3-RN to estimate concrete variability.

In the overall, model M3-RN is seen as an adequate relation to establish a preliminary estimate for CoV_{f_c} due to the relevant correlation level that was found. The estimates obtained with this model are associated with the assumption that both f_c and *RN* follow a lognormal distribution. Since *RN* test results usually have a lower bound limit of 20, the likelihood of *RN* having an asymmetric distribution is high, particularly when lower values of f_c are involved.

To complement the analysis presented for model M3-RN, its reliability was also examined by analysing the distribution of the ratios $\eta_{CoV_{f_c}}$ between the predictions made for CoV_{f_c} and the corresponding real values of the concrete strength variability. Fig. 15 shows the probability plot of the $\eta_{CoV_{f_c}}$ values that were obtained using model M3-RN and considering all the data (i.e. without removing the trend outliers).

The plot assumes the $\eta_{CoV_{f_c}}$ values follow a normal distribution but a visual assessment of the plot indicates there is large deviation in the upper tail of the data due to one $\eta_{CoV_{f_c}}$ value that invalidates this assumption. The Shapiro-Wilk goodness-of-fit test [56] was also applied to the $\eta_{CoV_{f_c}}$ ratios and the result showed that the normality assumption was rejected for a confidence level of 95%. Nevertheless, the normal distribution fits the majority of the data, exhibiting a mean value of 0.94 and a standard deviation of 0.28. Given this mean value, it can be seen that model M3-RN is able to provide relatively unbiased estimates of CoV_{f_c}.

5.2. Results obtained for the correlation between the variability of f_c and UPV

The results presented in Fig. 7 (model M1-UPV) confirmed the inadequacy of using a linear model to correlate the variability of f_c with that of *UPV*, given the lack of trend that was observed. This observation is in line with the expected relation between the physical properties of *UPV* and f_c. As pointed out by Breysse [17], the modification of concrete strength with time is not captured by a linear relation involving *UPV* test results. As such, in matured concrete, a change in concrete strength is not followed by a significant change in the *UPV* values. Therefore, a linear correlation between the standard deviations of f_c and *UPV* is not expected to provide a significant result. As mentioned also in [17], the non-proportionality between the increase of *UPV* with f_c implies that most studies define the relation between these parameters using power models with a large power coefficient or exponential models.

With respect to the performance of the power model, the results obtained with model M2-UPV provided the best fit among the multiple alternatives analysed for both NDTs. The power coefficient ($d = 4.35$) can be seen to be within the range of values identified by Breysse [17] (i.e. 1.7447–12.809) and corroborates the high level of nonlinearity of the relation between f_c and *UPV*. However, unlike the results found for *RN*, considering the power model relation and assuming that both *UPV* and f_c follow a lognormal distribution (model M3-UPV) was not seen as an adequate approach.

The adequacy of using an exponential model instead of a power model to correlate the variability of f_c with that of *UPV* was also confirmed, as seen in Fig. 10 (model M4-UPV). Even though the correlation level found for this model (adj-$R^2 = 0.68$) is similar to that of M2-UPV (adj-$R^2 = 0.66$), the correlation level of model M4-UPV was obtained without removing any outlier. Hence, model M4-UPV is preferred instead. Furthermore, introducing the additional assumptions that f_c follows a lognormal distribution and that *UPV* follows a normal distribution (model M5-UPV) was not seen to yield an adequate level of correlation (Fig. 11). Although the values of q^* that were obtained are in line with the range of expected values referred in [17] (i.e. 0.60–2.27 s/km), the fitting results led to an inadequate value of adj-R^2 due to the very large variation of the data with respect to the trend line. This variation of the data is further illustrated in Fig. 16a where the mean value of the *UPV* dataset corresponding to each s_{UPV} value is also represented, providing an additional scale to assess the nonlinearity issues previously discussed. Together with these data points, three linear correlation ranges are also shown. As can be seen, when accounting for the mean value of the *UPV* datasets μ_{UPV}, the ratio between CoV_{f_c} and s_{UPV} follows a different trend depending on the value of μ_{UPV}. As such, approximate trend lines can be defined for three ranges of μ_{UPV}. For μ_{UPV} values larger than 4.0 km/s, the trend line proposed by Turgut [57] with a coefficient q of 1.29 s/km is suggested. For μ_{UPV} values lower than 3.5 km/s, a trend line with a coefficient q close to the minimum value defined by Breysse [17] is considered. For μ_{UPV} values between 3.5 km/s and 4.0 km/s, a trend line with a coefficient q of 0.91 km/s was defined which corresponds to the robust average trend found for the data that is not covered by the two other trend lines. The multi-linear correlation model between CoV_{f_c} and s_{UPV} is represented in Fig. 16b and summarized in Eq. (25).

$$CoV_{f_c} = \begin{cases} 1.29 \times s_{UPV} & \mu_{UPV} > 4 \text{ km/s} \\ 0.91 \times s_{UPV} & 4 \text{ km/s} \geqslant \mu_{UPV} \geqslant 3.5 \text{ km/s} \\ 0.55 \times s_{UPV} & \mu_{UPV} < 3.5 \text{ km/s} \end{cases} \tag{25}$$

As carried out for model M3-RN, a complementary analysis was performed to quantify the relation between the model predictions and the real values of the concrete strength variability. Therefore, for model M4-UPV, the ratios $\eta_{s_{f_c}}$ between the predictions made for s_{f_c} and the corresponding real values were analysed (Fig. 17a). For the model defined by Eq. (25), the ratios $\eta_{CoV_{f_c}}$

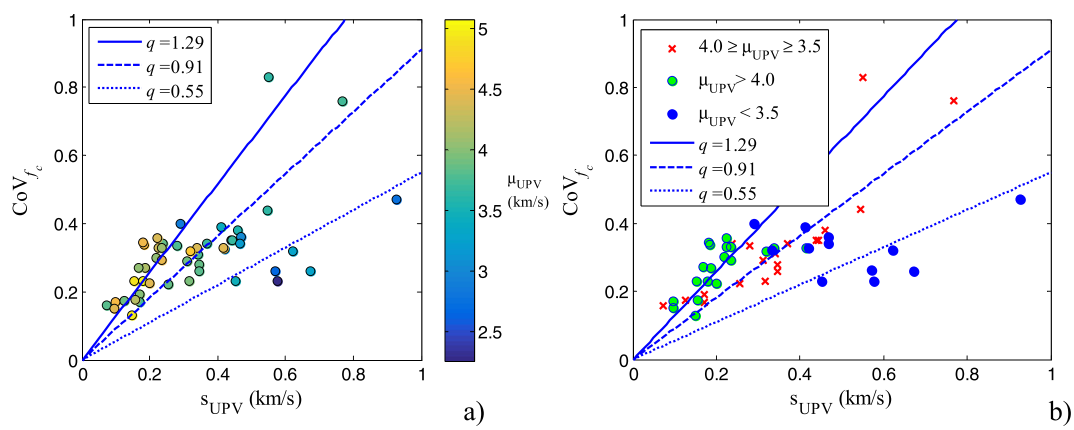

Fig. 16. Disaggregation of the s_{UPV} data according to the mean value of the corresponding dataset (a) and representation of the proposed multi-linear correlation model (b).

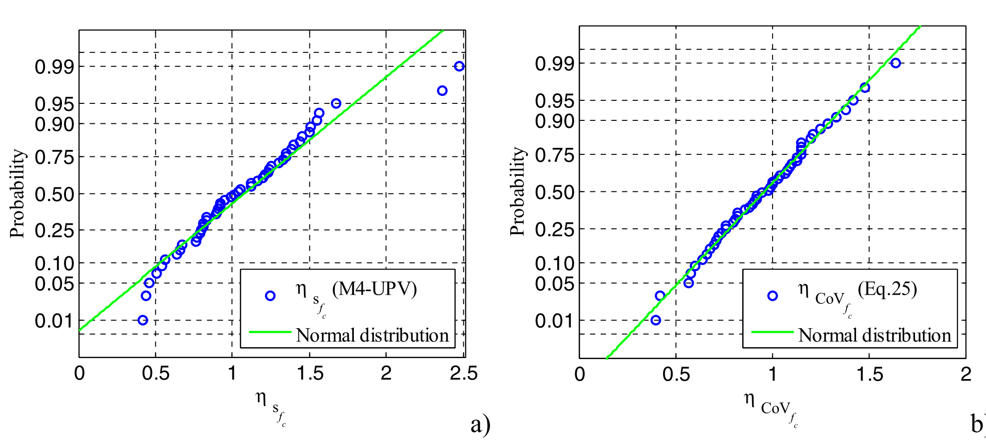

Fig. 17. Probability plots of the ratio between the estimates made with model M2-UPV and the real values of s_{f_c} (a), and with Eq. (25) and the real values of CoV_{f_c} (b).

between the predictions made for CoV_{f_c} and the corresponding real values were also analysed (Fig. 17b). As for the corresponding plot involving *RN* test results, the plots of Fig. 17 were also obtained considering all the data (i.e. without removing the trend outliers).

A visual assessment of the plots indicates there are small deviations between the probability plots of the reference normal distributions and those of $\eta_{s_{f_c}}$ and $\eta_{CoV_{f_c}}$. After applying the Shapiro-Wilk goodness-of-fit test [56] to the $\eta_{s_{f_c}}$ and $\eta_{CoV_{f_c}}$ ratios, the results indicate that the normality assumption could not be rejected for a confidence level of 95%. The normal distribution fitted to the $\eta_{s_{f_c}}$ ratios for model M4-UPV has a mean value of 1.08 and a standard deviation of 0.43, while for the model defined by Eq. (25) the mean value of $\eta_{CoV_{f_c}}$ is 0.96 and the standard deviation is 0.27. Given the mean values that were obtained, both models are seen to lead to relatively unbiased estimates of the concrete strength variability.

5.3. Results obtained for the SonReb-like approaches approach

Given the larger complexity of the SonReb-like approaches (i.e. the correlation models now involve three variables instead of two), the number of data triplets considered in the regressions analyses (i.e. 40) may be insufficient to perform a detailed analysis of their adequacy. Therefore, this limitation must be accounted for when examining the trends that are discussed herein. Nevertheless, some of the results that were obtained show a considerable level of consistency with the correlation models analysed in the previous sections and with the main observations in the review performed by Breysse [17]. Regarding the linear correlation model M1-SonReb, Fig. 12 shows that a linear correlation between s_{f_c} and s_{UPV} is unable to be established, given the large sensitivity of the model to s_{RN}. The value of coefficient β (1.18, see Fig. 12) is of the same order of magnitude as the regression coefficient that was obtained with model M1-RN (1.70, see Fig. 2) while coefficient χ is almost zero (see Fig. 7 and the results of model M1-UPV). Thus, the results obtained with M1-SonReb are in line with those observed in Sections 4.2 and 4.3.

A similar level of consistency is observed when analysing the results obtained using model M3-SonReb. In this case, the correlation between CoV_{f_c} and CoV_{RN} is clearly dominant, and the value of the regression coefficient ε^* that is found (1.88, see Fig. 14) is also of the same order of magnitude as the regression coefficient that was obtained with model M3-RN that involves the same conditions (1.94, see Fig. 4). This connection between CoV_{f_c} and CoV_{RN} is further highlighted given the value of the regression coefficient ϕ^* (close to zero, see Fig. 14), which implies that model M3-SonReb behaves like M3-RN, irrespective of the *UPV* data being involved.

The most commonly found SonReb-like conversion models have the functional form of model M2*-SonReb (e.g. see [17] and references therein). Still, the general form of this model (i.e. M2-SonReb) was unable to be analysed given the lack of information regarding the correlation between the *RN* and *UPV* data in each of the considered cases. As such, only the particular case where both parameters are assumed to be uncorrelated was evaluated (i.e. model M2*-SonReb). Unlike for the results of M1-SonReb and M3-SonReb, the results obtained with M2*-SonReb show that a general model of this type has the potential to be developed, namely given the significant values that were obtained for the *RN* and *UPV* regression coefficients. Despite the limitations surrounding the use of this model, it is nevertheless interesting to notice that the regression coefficients obtained for M2*-SonReb are within the average of twelve models analysed by Breysse [17]. Still, further studies with an extensive number of datasets have to be carried out to evaluate consistently the advantages of model M2*-SonReb.

6. Conclusions

The presented study analysed the possibility of developing generalized empirical expressions to obtain an estimate of the in situ concrete strength variability using NDT results. The study analysed the performance of several correlation models between different estimators of the variability of concrete strength and NDT results. In addition, the models were also associated with assumptions regarding the statistical distributions of f_c and of the NDT results.

By analysing the performance of the selected correlation models using several sets of variability data for f_c and NDT results, it was concluded that several general empirical expressions can be established. When *RN* test results are available, a preliminary estimate for the variability of concrete strength can be obtained by the expression $CoV_{f_c} = 1.94 \cdot CoV_{RN}$ which assumes that both f_c and *RN* follow a lognormal distribution. However, no reliable correlation was found that would allow estimating the standard deviation of f_c, s_{f_c}, using *RN* test results.

When using *UPV* test results, expressions were obtained to define preliminary estimates of both s_{f_c} and CoV_{f_c}. The expression $s_{f_c} = s_{UPV} \cdot \exp(0.98 \cdot \mu_{UPV} - 0.59)$ was seen to provide reliable estimates of s_{f_c} but needs to also involve the mean value of the *UPV* test results in order to capture the effect of the nonlinear relation between *UPV* and f_c. Since no reliable expression was found to correlate CoV_{f_c} with a measure of the variability of *UPV* without involving μ_{UPV}, a multi-linear model was proposed instead. This multi-linear model estimates CoV_{f_c} as a function of s_{UPV} and three different ranges of μ_{UPV} (Eq. (25)). This model is seen to provide a level of reliability similar to that provided by the model involving s_{f_c}. Results obtained using SonReb-like models confirmed some of the trends that were identified by the single NDT models. Still, no statistically significant SonReb-like model was identified. This conclusion is mostly related to limitations associated to the available data. As such, further research needs to address the development of SonReb-like models such as those analysed herein, namely to assess, among other factors, the sensitivity of the models to the correlation between *RN* and *UPV*.

Finally, the results obtained from the study that was performed highlight several aspects regarding the selection of adequate conversion models between NDTs and f_c results when using specific regression methods or a bi-objective approach. The results indicate that an adequate conversion model between NDTs and f_c results should involve a power or an exponential model, especially in existing structures where the CoV_{f_c} is expected to be above 10%. In particular, the results that were obtained confirm the conclusions in [17] that suggest the use of exponential or power models with large power coefficients for *UPV*-f_c conversion and the use of power models for *RN*-f_c conversion.

Conflicts of Interest

The authors declares that they have no conflict of interest.

Acknowledgements

The authors would like to thank Professor Denys Breysse (University of Bordeaux, France) for kindly providing some of the datasets that were used in this research.

Appendix

Database adopted for the study of general models for the concrete strength variability

N	$f_{c,mean}$	UPV_{mean}	RN_{mean}	s_{fc}	s_{UPV}	s_{RN}	CoV_{fc}	CoV_{UPV}	CoV_{RN}	Ref.
19	27.46	–	40	7.96	–	6.80	0.29	–	0.17	[4]
27	28.11	–	38	10.12	–	7.98	0.36	–	0.21	[4]
20	30.14	–	40	11.45	–	6.40	0.38	–	0.16	[4]
25	35.99	–	42	12.24	–	5.88	0.34	–	0.14	[4]
20	30.66	–	41	5.52	–	4.92	0.18	–	0.12	[4]
21	19.74	–	36	3.75	–	2.74	0.19	–	0.08	[25]
145	18.80	3.67	34	6.58	0.44	5.78	0.35	0.12	0.17	[24]
83	17.30	3.54	33	6.57	0.46	5.94	0.38	0.13	0.18	[24]
62	20.90	3.86	36	5.85	0.35	5.40	0.28	0.09	0.15	[24]
27	18.70	3.64	34	8.23	0.55	6.12	0.44	0.15	0.18	[24]
26	18.90	3.70	34	6.43	0.37	5.78	0.34	0.10	0.17	[24]
30	15.00	3.32	31	4.80	0.33	4.96	0.32	0.10	0.16	[24]
32	20.10	3.79	35	6.23	0.34	5.95	0.31	0.09	0.17	[24]
30	21.90	3.95	37	5.04	0.32	4.44	0.23	0.08	0.12	[24]
24	18.90	3.70	34	6.62	0.44	6.12	0.35	0.12	0.18	[24]
59	16.20	3.43	32	6.32	0.41	5.44	0.39	0.12	0.17	[24]
46	21.80	3.89	37	6.32	0.31	5.55	0.29	0.08	0.15	[24]
16	20.30	3.84	35	5.28	0.35	5.25	0.26	0.09	0.15	[24]
9	30.06	–	47	4.12	–	3.29	0.14	–	0.07	[26]
48	13.30	2.88	24	5.32	0.29	4.56	0.40	0.10	0.19	[39]
67	27.20	3.72	37	20.67	0.77	7.40	0.76	0.21	0.20	[39]
8	30.40	3.86	38	10.34	0.24	2.28	0.34	0.06	0.06	[39]
8	52.00	4.57	41	8.84	0.10	2.05	0.17	0.02	0.05	[39]
21	28.00	4.34	31	9.24	0.23	5.89	0.33	0.05	0.19	[39]
6	15.80	3.04	38	5.69	0.47	1.90	0.36	0.15	0.05	[39]
16	15.70	2.26	24	3.61	0.58	3.84	0.23	0.25	0.16	[39]
13	13.00	3.45	41	2.99	0.45	6.15	0.23	0.13	0.15	[39]
14	18.80	3.29	35	6.02	0.62	5.95	0.32	0.19	0.17	[39]
21	107.00	4.91	50	24.61	0.15	3.50	0.23	0.03	0.07	[39]
40	10.00	3.66	–	8.30	0.55	–	0.83	0.15	–	[39]
26	28.60	4.17	–	7.72	0.19	–	0.27	0.05	–	[39]
13	53.60	4.95	–	6.97	0.15	–	0.13	0.03	–	[39]
7	33.30	4.04	–	9.99	0.22	–	0.30	0.05	–	[39]
207	23.40	4.08	–	7.70	0.23	–	0.33	0.06	–	[39]
144	35.95	–	39	6.69	–	2.95	0.19	–	0.08	[27]
118	63.30	–	48	11.74	–	3.64	0.19	–	0.08	[27]
114	41.73	–	32	9.35	–	2.62	0.22	–	0.08	[27]
144	77.41	–	48	11.70	–	2.12	0.15	–	0.04	[27]
100	67.19	–	47	7.65	–	3.39	0.11	–	0.07	[27]
136	45.78	–	37	9.71	–	4.68	0.21	–	0.13	[27]
120	44.50	–	37	11.29	–	5.21	0.25	–	0.14	[27]
120	42.62	–	38	10.26	–	4.24	0.24	–	0.11	[27]
118	65.35	–	47	12.77	–	6.29	0.20	–	0.14	[27]
172	69.20	–	46	11.88	–	3.64	0.17	–	0.08	[27]
208	71.12	–	46	14.34	–	4.68	0.20	–	0.10	[27]
216	70.25	–	47	16.38	–	4.84	0.23	–	0.10	[27]
160	60.72	–	44	11.87	–	3.90	0.20	–	0.09	[27]
212	54.71	–	39	15.13	–	5.64	0.28	–	0.14	[27]
212	45.82	–	37	14.71	–	5.77	0.32	–	0.15	[27]
204	63.55	–	40	18.30	–	5.64	0.29	–	0.14	[27]
136	55.16	–	39	13.57	–	5.38	0.25	–	0.14	[27]
167	34.69	–	40	11.08	–	7.49	0.32	–	0.19	[27]
130	66.74	–	49	10.78	–	2.57	0.16	–	0.05	[27]
13	36.30	4.50	45	10.60	0.23	3.00	0.29	0.05	0.07	[28]
10	15.71	2.67	30	4.10	0.57	4.90	0.26	0.21	0.16	[29]
18	18.09	2.76	27	8.50	0.93	7.50	0.47	0.34	0.28	[30]
25	31.39	3.86	32	7.00	0.26	2.30	0.22	0.07	0.07	[31]
20	22.40	4.00	32	4.30	0.17	3.40	0.19	0.04	0.11	[32]

Appendix (*continued*)

N	$f_{c,mean}$	UPV_{mean}	RN_{mean}	s_{fc}	s_{UPV}	s_{RN}	CoV_{fc}	CoV_{UPV}	CoV_{RN}	Ref.
19	24.28	3.91	37	4.20	0.13	1.90	0.17	0.03	0.05	[33]
23	14.07	3.78	41	4.70	0.28	4.50	0.33	0.07	0.11	[34]
14	23.85	3.32	28	6.20	0.67	8.00	0.26	0.20	0.29	[35]
32	30.49	4.39	34	10.00	0.34	7.20	0.33	0.08	0.21	[36]
19	13.21	3.84	26	2.10	0.07	1.80	0.16	0.02	0.07	[37]
31	36.78	–	43	6.58	–	3.05	0.18	–	0.07	[38]
22	16.21	3.10	–	5.29	0.42	–	0.33	0.14	–	[2]
21	32.57	3.74	–	5.51	0.17	–	0.17	0.05	–	[2]
16	26.26	3.23	39	13.33	0.47	4.39	0.34	0.14	0.11	[40]
80	51.53	5.08	36	11.89	0.18	5.47	0.23	0.04	0.15	[41]
63	27.17	4.21	37	4.76	0.16	2.26	0.18	0.04	0.06	[42]
60	23.61	4.46	31	7.73	0.42	4.47	0.33	0.09	0.15	[43]
40	37.70	4.72	39	12.04	0.32	8.41	0.32	0.07	0.21	[44]
30	47.94	4.45	44	10.76	0.20	4.67	0.22	0.04	0.11	[45]
20	27.35	4.04	30	7.42	0.17	4.01	0.27	0.04	0.13	[46]
16	37.33	4.66	31	12.63	0.18	5.25	0.34	0.04	0.17	[47]
120	31.37	4.41	–	11.16	0.22	–	0.36	0.05	–	[48]
60	61.83	4.44	–	9.36	0.10	–	0.15	0.02	–	[49]
24	20.96	4.51	–	7.20	0.18	–	0.34	0.04	–	[50]
120	22.45	–	34	5.88	–	5.26	0.26	–	0.16	[51]

References

[1] R. Caspeele, L. Taerwe, Bayesian assessment of the characteristic concrete compressive strength using combined vague-informative priors, Constr. Build. Mater. 28 (2012) 342–350.

[2] R. Giannini, L. Sguerri, F. Paolacci, S. Alessandri, Assessment of concrete strength combining direct and NDT measures via Bayesian inference, Eng. Struct. 64 (2014) 68–77.

[3] A. Fiore, F. Porco, G. Uva, M. Mezzina, On the dispersion of data collected by in situ diagnostic of the existing concrete, Constr. Build. Mater. 47 (2013) 208–217.

[4] N. Pereira, X. Romão, Assessment of the concrete strength in existing buildings using finite population approach, Constr. Build. Mater. 110 (2016) 106–116.

[5] M. Alwash, Z.M. Sbartaï, D. Breysse, Non-destructive assessment of both mean strength and variability of concrete: a new bi-objective approach, Constr. Build. Mater. 113 (2016) 880–889.

[6] N. Pereira, X. Romão, Material strength safety factors for the seismic safety assessment of existing RC buildings, Constr. Build. Mater. 119 (2016) 319–328.

[7] M.T. Cristofaro, A. D'Ambrisi, M. De Stefano, R. Pucinotti, M. Tanganelli, Studio sulla Dispersione dei Valori di Resistenza a Compressione del Calcestruzzo di Edifici Esistenti. Il Giornale delle Prove non Distruttive Monitoraggio e Diagnostica 2012; 2 (in Italian).

[8] M.G. Stewart, Workmanship and its influence on probabilistic models of concrete compressive strength, ACI Mater. J. 92 (4) (1995) 361–372.

[9] F.M. Bartlett, J.G. MacGregor, Variation of in-place concrete strength in structures, ACI Mater. J. 96 (2) (1999) 261–270.

[10] G. Uva, F. Porco, A. Fiore, M. Mezzina, Proposal of a methodology of in situ concrete tests and improving the estimate of the compressive strength, Constr. Build. Mater. 38 (1) (2013) 72–83.

[11] A. Masi, L. Chiauzzi, An experimental study on the within-member variability of in situ concrete strength in RC building structures, Constr. Build. Mater. 47 (2013) 951–961.

[12] F. Jalayer, L. Elefante, I. Iervolino, G. Manfredi, Knowledge-based performance assessment of existing RC buildings, J. Earthq. Eng. 15 (2011) 362–389.

[13] G. Monti, S. Alessandri, Application of Bayesian techniques to material strength evaluation and calibration of confidence factors, in: E. Cosenza (Ed.), Eurocode 8 Perspectives From the Italian Standpoint Workshop, Doppiavoce, Naples, Italy, 2009, pp. 67–77.

[14] X. Romão, R. Gonçalves, A. Costa, R. Delgado, Evaluation of the EC8-3 confidence factors for the characterization of concrete strength in existing structures, Mater. Struct. 45 (2012).

[15] M. Alwash, D. Breysse, Z.M. Sbartaï, K. Szilágyi, A. Borosnyói, Factors affecting the reliability of assessing the concrete strength by rebound hammer and cores, Constr. Build. Mater. 140 (1) (2017) 354–363.

[16] CEN, EN 13791-Assessment of in situ compressive strength in structures and precast concrete. CEN, Brussels (2007).

[17] D. Breysse, Nondestructive evaluation of concrete strength: an historical review and new perspective by combining NDT methods, Constr. Build. Mater. 33 (2012) 139–163.

[18] D. Breysse, M. Soutsos, A. Moczko, S. Laurens, Quantitative non-destructive assessment of in situ concrete properties: the key question of calibration, Structural faults and repair, Edinburgh, 15–17, June 2010.

[19] K. Szilágyi, A. Borosnyói, 50 years of experience with the Schmidt Rebound hammer, Concr. Struct. 10 (2009) 46–56.

[20] D. Breysse, J. Martinez-Fernandez, Assessing concrete strength with rebound hammer: review of key issues and ideas for more reliable conclusions, Mater. Struct. 47 (2014) 1589–1604.

[21] ACI 228.1R-03, In-Place Methods to Estimate Concrete Strength, American Concrete Institute, USA, 2003.

[22] A.H.S. Ang, W.H. Tang, Probability Concepts in Engineering: Emphasis on Applications to Civil and Environmental Engineering (vol 1), second ed., John Wiley and Sons, 2006.

[23] Y. Bao, On sample skewness and kurtosis, Econometric Rev. 32 (4) (2013) 415–448.

[24] K. Ali-Benyahiaa, Z.M. Sbartaï, D. Breysse, S. Kenaid, M. Ghricia, Analysis of the single and combined non-destructive test approaches for on-site concrete strength assessment: general statements based on a real case-study, Case Stud. Constr. Mater. 6 (2017) 109–119.

[25] A. Monteiro, A. Gonçalves, Assessment of characteristic strength in structures by the rebound hammer test according to EN 13791:2007, Proceedings of NDTCE'09 Conference, Nantes, France, 2009.

[26] M. Soutsos, D. Breysse, V. Garnier, A. Goncalves, A. Monteiro, Estimation of on-site compressive strength of concrete. In Non-Destructive Assessment of Concrete Structures: Reliability and Limits of Single and Combined Techniques. Springer Netherlands, 2012.

[27] K. Szilágyi K, Rebound surface hardness and related properties of concrete. Ph. D. thesis, University of Technology and Economics Budapest, Hungary, 2013.

[28] A. Gennaro-Santori, NDT measurements on the concrete elements of the bridges – Technical Report n. 74.05, CND Controlli NonDistruttivi s.r.l., ANAS – SS761 Val Seriana; 2005.

[29] M. Brognoli, Prove e controlli non distruttivi per la verifica degli edifici esistenti secondo la normativa sismica e le norme tecniche per le costruzioni, Brescia, June 15, 2007.

[30] A. Masi, M. Vona, La Stima della Resistenza del Calcestruzzo In-Situ: Impostazione delle Indagini ed Eleborazione dei Risultati, Progettazione sismica, n° 1/2009, IUSS Press, ISSN 1973-7432, 2009.

[31] A. Masi, M. Dolce, M. Vona, D. Nigro, G. Pace, M. Ferrini, Indagini sperimentali su elementi strutturali estratti da una scuola esistente in c.a., XII ANIDS conf., Pisa, June 10–14, 2007.

[32] M. Dolce, A. Masi, M. Ferrini, Estimation of the actual in-place concrete strength in assessing existing RC structures, 2nd Int. FIB Conference, Napoli, Italy, June 5–8, 2006.

[33] H.S. Han, Estimation of In-situ concrete strength by combined Non-Destructive Method, Mag. Korea Concr. Inst. 4 (4) (1992) 57–66 (in Korean).

[34] R. Pucinotti, Reinforced concrete structure: non-destructive in situ strength assessment of concrete, Constr. Build. Mater. 75 (2015) 331–341.

[35] A. Masi, L. Chiauzzi, V. Manfredi, Criteria for identifying concrete homogeneous areas for the estimation of in-situ strength in RC buildings, Constr. Build. Mater. 121 (2016) 576–587.

[36] F. Nada Mahdi, I.S. Abd Muttalib, J. Ali Khalid, Prediction of compressive strength of reinforced concrete structural elements by using combined non-destructive tests, J. Eng. 19 (10) (2013) 1189–1211.
[37] S. Hannachi, N. Guetteche, Le contrôle non destructif des ouvrages en béton – évaluation de la résistance à la compression du béton sur site – utilisation de la méthode combinée, 29èmes Rencontres AUGC (2011), Tlemcen, 28-30.
[38] A. Scanlon, L. Mikhailovsky, Strength evaluation of an existing concrete bridge based on core and non-destructive test data, Can. J. Civ. Eng. 14 (1987) 145–154.
[39] G. Fabbrocino, A. Di Fusco, G. Manfredi, In situ evaluation of concrete strength for existing constructions: critical issues and perspectives of NDT methods, fib symposium, keep attractive concrete. Budapest (2005).
[40] L. Nobile, Prediction of concrete compressive strength by combined non-destructive methods, Meccanica 50 (2) (2015) 411–417.
[41] F. Cianfrone, I. Facaoaru, Study on the introduction into Italy on the combined nondestructive method, for the determination of in situ concrete strength, Matér. Constr. 12 (71) (1979) 413–424.
[42] P. Knaze, P. Beno, The use of combined non-destructive testing methods to determine the compressive strength of concrete, Matér. Constr. 17 (3) (1984) 207–210.
[43] O. Oktar, H. Moral, M. Taşdemir, Sensitivity of concrete properties to the pore structure of hardened cement paste, Cem. Concr. Res. 26 (11) (1996) 1619–1627.
[44] A. Jain, A. Kathuria, A. Kumar, Y. Verma, K. Murari, Combined use of non-destructive tests for assessment of strength of concrete in structure, Procedia Eng. 54 (2013) 241–251.
[45] L. Rojas-Henao, J. Fernández-Gómez, J. López-Agüí, Rebound hammer, pulse velocity, and core tests in self-consolidating concrete, ACI Mater. J. 109 (2) (2012) 235–243.
[46] V. Nikhil, R. Balki Minal, S. Deep, D. Vijay, S. Vishal, S. Patil, The use of combined non-destructive testing in the concrete strength assessment from laboratory specimens and existing buildings, Int. J. Curr. Eng. Scientific Res. 2 (5) (2015) 55–59.
[47] Z. Sbartaï, D. Breysse, M. Larget, J.-P. Balayssac, Combining NDT techniques for improved evaluation of concrete properties, Cem. Concr. Compos. 34 (2012) 725–733.
[48] M. Musmar, N. Abedalhadi, Relationship between ultrasonic pulse velocity and standard concrete cube crushing strength, J. Eng. Sci., Assiut Univ. 36 (1) (2008) 51–59.
[49] A. El Mir, S. Nehme, A comparative study on ultrasonic pulse velocity for normally vibrated and self-compacting concretes, Concr. Struct. 17 (2016) (2016) 8–12.
[50] L. del Rio, A. Jiménez, F. Lopez, F. Rosa, M. Rufo, J. Paniagua, Characterization and hardening of concrete with ultrasonic testing, Ultrasonics 42 (2004) 527–530.
[51] H. Hajjeh, Correlation between destructive and non-destructive strengths of concrete cubes using regression analysis, Contemporary Eng. Sci. 5 (10) (2012) 493–509.
[52] M. Alwash, Assessment of concrete strength in existing structures using non-destructive tests and cores: analysis of current methodology and recommendations for more reliable assessment. PhD. Dissertation, University of Bordeaux, 2017.
[53] H. Motulsky, R. Brown, Detecting outliers when fitting data with nonlinear regression – a new method based on robust nonlinear regression and the false discovery rate, BMC Bioinformatics 7 (1) (2006) 123–143.
[54] D. O'Leary, Robust regression computation using iteratively reweighted least squares, SIAM J. Matrix Anal. Appl. 11 (3) (1990) 466–480.
[55] R. Wilcox, Introduction to robust estimation and hypothesis testing, Elsevier Academic Press, London, UK, 2005.
[56] S. Shapiro, M. Wilk, An analysis of variance test for normality (complete samples), Biometrika 52 (3/4) (1965) 591–611.
[57] P. Turgut, Research into the correlation between concrete strength and UPV value. NDT.net 12 (2004) (12).