

Local Zeta Functions

From Calabi-Yau Differential Equations

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Abstract

It was observed by Dwork that the ζ -function of a manifold is closely related to and sometimes can be calculated completely in terms of its periods. We report here on a practical and computationally rapid implementation of this procedure for families of Calabi-Yau manifolds with one complex structure parameter φ . Although partly conjectural, it turns out to be possible to compute the matrix of the Frobenius map on the third cohomology group of X_φ directly from the Picard-Fuchs differential operator of the family. To illustrate our method, we compute tables of the quartic numerators of the ζ -functions for six manifolds of increasing complexity as the parameter φ varies in \mathbb{F}_p . For four of these manifolds, we do this for the 500 primes $p = 5, 7, \dots, 3583$, while for two manifolds we extend the calculation to 1000 primes. The tables for $5 \leq p \leq 97$ are part of this article while the remaining tables are attached in electronic form. Interest attaches to the cases for which the numerators factorise. Some of these factorisations can be associated with parameter values for which the underlying manifold becomes singular. For the cases we consider here, the singularities are all of conifold or hyperconifold type. In these cases the numerator degenerates to a cubic and this factorises into the product of a linear and a quadratic factor. As has been noted elsewhere, the quadratic term contains a coefficient that is the p 'th coefficient of a modular form. Some of our examples have singularities when the parameter satisfies a polynomial equation that does not factorise over \mathbb{Q} . When this happens, the corresponding forms are modular forms with neben type or Hilbert modular forms. The numerator can also factorise into two quadrics. This happens when the Hodge structure of the manifold splits, sometimes this happens for algebraic values of the parameter and we identify, in this way, attractor points of rank two of the parameter space.

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1. Introduction

1.1. Preamble

Consider a Calabi-Yau threefold X , defined over \mathbb{Q} ; by this is meant that X is defined by polynomials with rational coefficients. By multiplying out denominators, we may take the polynomials to have coefficients in \mathbb{Z} . It now makes sense to reduce these coefficients modulo a prime p and to consider the manifold over the field \mathbb{F}_p . The fundamental quantities of interest are the numbers, N_r , of points of X , with coordinates in the fields \mathbb{F}_{p^r} , $r = 1, 2, \dots$ and the zeta-function, which is a generating function for these

$$N_r = \#X(\mathbb{F}_{p^r}) ; \quad \zeta_X(T) = \exp \left(\sum_{r=1}^{\infty} \frac{N_r T^r}{r} \right) .$$

In the case that the Picard group is generated by divisors defined over the ground field \mathbb{F}_p , the general form of ζ_X is dictated by the Weil conjectures

$$\zeta_X = \frac{R(T)}{(1-T)(1-pT)^{h^{11}}(1-p^2T)^{h^{11}}(1-p^3T)} . \quad (1.1)$$

We will be concerned with one-parameter families of such manifolds, X_φ , and use φ to denote the parameter. Since the manifold depends on φ , so do the N_r . However, this dependence is generally suppressed in the notation.

The function ζ_X has remarkable properties in virtue of the Weil conjectures. Among these is the fact that it is a rational function, as a function of T , with the degrees of the numerator and denominator governed by the Betti numbers of X . Since it is a rational function, ζ_X may be found from a knowledge of the first few numbers N_r , and these may be found, in principle, by counting points, as per the definition. In practice, however, it is frequently difficult to count the points directly, even for low values of r . A case when this is so is the frequent case when the manifold is defined as a quotient. It is therefore both a remarkable and useful fact that the N_r , and so ζ_X , can be computed in terms of the periods of X . The periods vary with φ and satisfy certain differential equations, the Picard-Fuchs equations. Thus we come to a fact that is well known although surprising on first acquaintance: that ζ_X may be found by solving differential equations. This observation was exploited to great effect by Dwork in his researches relating to the ζ -function and the Weil conjectures.

We will here be concerned with the computation of the numerator $R(T)$. When X is smooth, R has the form

$$R = 1 + aT + bpT^2 + ap^3T^3 + p^6T^4 ,$$

and is determined by two integers a and b that depend on φ . This quantity may be calculated as a determinant

$$R(\varphi, T) = \det (\mathbb{1} - TU(\varphi)) \quad \text{where} \quad U(\varphi) = E^{-1}(\varphi^p)U(0)E(\varphi) \quad (1.2)$$

and E and U are 4×4 matrices, with E a wronskian of periods of X and U the matrix that represents the inverse of the Frobenius map. We will use expansions around a point 0 of

maximal unipotent monodromy and a priori it is not clear what matrix $U(0)$ to use, as the corresponding Calabi-Yau space is very degenerate. We conjecture a precise form for $U(0)$ that worked in all cases considered.

When X is singular, the degree of R is reduced. For a conifold or hyperconifold singularity, for example, R reduces to a cubic and this factorises into a linear factor and a quadric, and these have interesting forms that are dictated by a modular group. We will also be interested in the circumstance that R factorises over \mathbb{Z} into two quadrics, when such a factorisation is determined by algebraic values of the parameter, independently of the prime p .

This paper is largely concerned with the practical evaluation of the matrix $U(\varphi)$ and so of $R(\varphi, T)$. A first remark is that, while the matrix E has logarithms, these cancel in the process of forming U , which can therefore be understood as a matrix of power series in φ with rational coefficients. We wish to evaluate U for, say, φ an integer in the range $1 \leq \varphi \leq p-1$. As series of real numbers, these do not converge. But the series *do* converge, if they are regarded as p -adic series. However, although the series converge, in this sense, they do so only slowly. Naive procedures for summing the series typically require summing to p^6 terms in order to identify the integers a and b that appear in R , and this becomes impractical already for moderate values of p . It is a key observation of A. Lauder [1] that the convergence of the series for U is greatly improved by regarding them as a limit of a sequence of rational functions of the parameter φ .

In practice this means the following: we fix good prime p and a p -adic accuracy, say $\mathcal{O}(p^4)$, since this order is sufficient to identify the integers a and b for $p \geq 5$. To indicate the process, start by taking $p < 100$, say, and expand $U(\varphi)$ as a matrix of series to 1000 terms. The series in φ have coefficients that are p -adic integers, mod p^4 , hence also integers mod p^4 .

The Picard-Fuchs operator takes the form

$$\mathcal{L} = S_4 \vartheta^4 + S_3 \vartheta^3 + S_2 \vartheta^2 + S_1 \vartheta + S_0 ; \quad \vartheta = \varphi \frac{d}{d\varphi} . \quad (1.3)$$

with coefficient functions S_j that are polynomials in φ . The coefficient S_4 we call the discriminant of the differential equation and plays a special role. If we multiply the matrix of power series $U(\varphi)$ by successive powers of $S_4(\varphi)$, we get a succession of such series. However when we multiply by $S_4(\varphi)^p$ we find that the series, that consisted previously of 1000 terms, shorten dramatically to polynomials of degree a small multiple of p . In other words $U(\varphi)$ has the form, mod p^4 , of a matrix of rational functions of the parameter φ .

$$U(\varphi) = \frac{\mathcal{U}(\varphi)}{S_4(\varphi)^p} + \mathcal{O}(p^4) . \quad (1.4)$$

Given this form, one may immediately evaluate $U(\varphi)$ for values of the parameter other than the singular values of the differential equation, which are the zeros of the polynomial $S_4(\varphi)$. It is an important observation, however, that the behaviour of the series for $U(\varphi)$ is even better behaved, in many cases, owing to cancellations between the numerator and denominator in the above expression.

A first remark is that there seems to be an important distinction depending on whether or not the differential equation has *apparent singularities*, which are singularities of the differential equation that are not singularities of the manifold. Their occurrence is quite frequent and, when they occur, they turn out to play a significant role in the calculation of the zeta-function. We denote by Δ the discriminant of the manifold, so for the case that there is no apparent singularity, this can be taken to be equal to the discriminant of the equation,

$$S_4(\varphi) = \Delta(\varphi) .$$

For the cases that there is a generic apparent singularity at $\varphi = \varphi_0$, we have instead the relation

$$S_4(\varphi) = (\varphi - \varphi_0)^2 \Delta(\varphi) .$$

If there are no apparent singularities, then the zeros of S_4 correspond to the genuine singularities of the manifold, the mirror of the quintic threefold being the prime example. For the cases of conifold, or hyperconifold, singularities, which are the most common form of singularities, there is complete cancellation of the denominator in (1.4), at least for the examples considered here. In these cases, we are left with

$$U(\varphi) = \tilde{\mathcal{U}}(\varphi) + \mathcal{O}(p^4) , \quad (1.5)$$

where $\tilde{\mathcal{U}}$ is a matrix of polynomials of degree of order of magnitude p . We will be more precise about the degree of $\tilde{\mathcal{U}}$ in the following. Now we may use this form to evaluate $U(\varphi)$ and do this also for values of φ for which X have hyper conifold singularities.

If, on the other hand, the differential equation has an apparent singularity φ_0 (it can have more than one apparent singularity, but the examples we study here have at most one) then, surprisingly, there is a cancellation of the true singularities and we are left with

$$U(\varphi) = \frac{\hat{\mathcal{U}}(\varphi)}{(\varphi - \varphi_0)^{2p}} + \mathcal{O}(p^4) , \quad (1.6)$$

where $\hat{\mathcal{U}}$ denotes another matrix of polynomials of degree of order of magnitude p . We may use this expression to evaluate $U(\varphi)$ for all values of φ , apart from the apparently singular value φ_0 . In particular, this expression allows us to evaluate the factor $R(\varphi, T)$ for the values of φ for which the manifold X is genuinely singular. It has been observed that for conifold or hyperconifold points, which are the singularities that we encounter here, the factor R degenerates to the form

$$R = (1 - p\chi T)(1 - \beta_p T + p^3 T^2) , \quad (1.7)$$

where $\chi = \pm 1$ is a character. Moreover, for the case that the discriminant $\Delta(\varphi)$ factors over \mathbb{Q} , the quantity β_p is the p 'th coefficient in the q -expansion of a weight 4 modular form for a principle congruence subgroup $\Gamma_0(N) \subset \mathrm{SL}(2, \mathbb{Z})$, for an integer N , whose prime factors are a subset of the bad primes.

Two of our examples, the mirror of a complete intersection in the Grassmannian $G(2, 5)$ and the Pfaffian Calabi-Yau in \mathbb{P}^6 considered by Rødland, have discriminants that do not factor

over \mathbb{Q} . For the case of the mirror of a hypersurface in $G(2, 5)$, the roots of the discriminant are $-\frac{11}{32} \pm \frac{5}{32}\sqrt{5}$ and so only exist in \mathbb{F}_p if 5 is a square mod p . By quadratic reciprocity, this is when $p = \pm 1 \pmod{5}$. For these manifolds we observe that, when the discriminant factors in \mathbb{F}_p , the quantity R again factors as in (1.7). The coefficients α_p , when this happens, have been identified, in the thesis of A. Thorne [2], with coefficients of a Hilbert modular form.

Another interesting factorisation over the integers arises, for suitable values of φ , in all our examples:

$$R = (1 - p\alpha_p T + p^3 T^2)(1 - \beta_p T + p^3 T^2) . \quad (1.8)$$

Note the ‘extra’ factor of p associated with α_p in this expression, so the first factor can be considered to be

$$1 - \alpha_p(pT) + p(pT)^2$$

which has the form of the numerator of the ζ -function of an elliptic curve.

The frequency of such factorisations varies significantly between manifolds, as is evident from Figures 1 and 2 which compares these frequencies for the manifold AESZ34 with the mirror quintic. Many of these factorisations occur for unexplained, or perhaps random reasons. However sometimes, and this is the case for AESZ34, they correspond to a *persistent factorization*: one that occurs for φ the root of a polynomial $G(\varphi)$ with integer coefficients. When this happens, it makes sense to reduce $G(\varphi) \pmod{p}$ and the polynomial will have roots in \mathbb{F}_p for infinitely many, and in fact a nonzero proportion of all primes. Such values of φ correspond to a splitting of the Hodge structure of the manifold and the roots of $G(\varphi)$ are attractor points of rank two, in the sense of type II supergravity. This phenomenon arises with respect to the manifold AESZ34 and we have examined this in detail in a parallel publication [3].

1.2. Utility of the tables

There is interesting information in the tables of Appendix C, that are the principal output of our calculations. Each of the examples that follow have conifold, or hyperconifold, singularities for certain values of the parameters, that are defined by algebraic equations. For these values of the parameters, the polynomial $R(\varphi, T)$ becomes a cubic and factors as in Eq (1.7). The coefficients β_p can be read off from the tables and these identify a weight-four modular form. This is, as we have indicated, a well-known story, but our tables provide many examples of this modular behaviour and the method can generate many more.

Plots such as those of Figure 1 and Figure 2 are a crude summary of aspects of the full tables, yet even here there is additional information about the possible existence of rank two attractor points. If we assume that, for a given manifold, there are finitely many rank two attractor points and that these correspond to values of the parameter that are the roots of a polynomial $G(\varphi)$, with rational, and so integer, coefficients. The polynomial $G(\varphi)$ may be reducible over \mathbb{Q} , if so, let us suppose that it has k irreducible factors. The following result, which is a known, but perhaps not widely known, consequence of the Cebotarëv density theorem is relevant. A proof is reviewed in Appendix A.

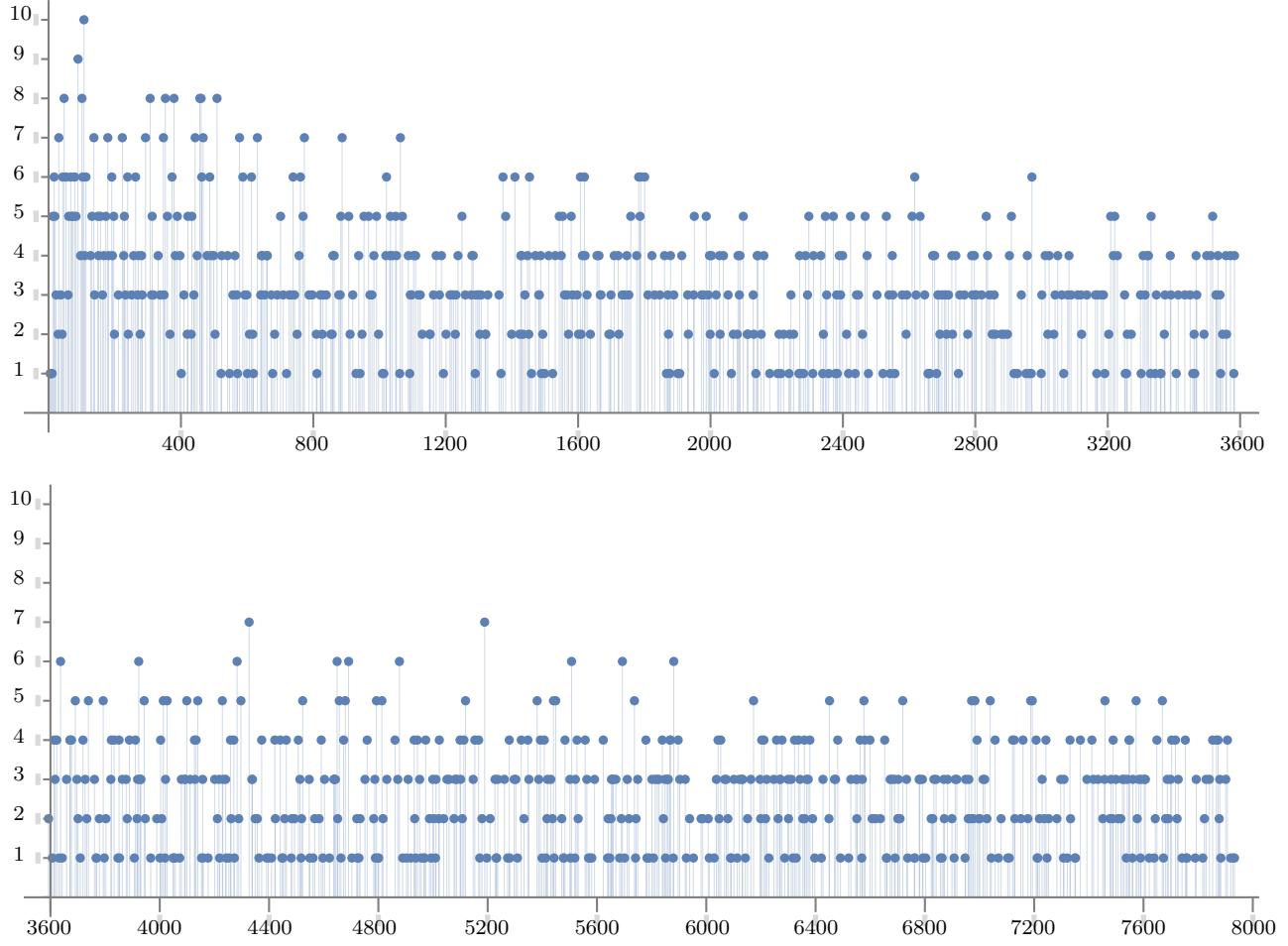


Figure 1: These plots show the number of factorisations into two quadratics, for AESZ34, as φ varies over each \mathbb{F}_p , for the 1000 primes $5 \leq p \leq 3583$ and $3593 \leq p \leq 7933$.

Theorem: Let $f(\varphi)$ be a polynomial with integer coefficients, that is irreducible over \mathbb{Q} , and let ν_p be the number of roots of $f(\varphi)$ over \mathbb{F}_p . Further let S be a large set of primes. Then, subject to some mild assumptions about the set¹ S ,

$$\lim_{|S| \rightarrow \infty} \frac{1}{|S|} \sum_{p \in S} \nu_p = 1 .$$

Applying this theorem to each of the irreducible factors of G , we see that the expected number of roots of G over \mathbb{F}_p is k . If now n_p is the number of factorisations over \mathbb{F}_p then $n_p \geq \nu_p$ and so

$$\lim_{|S| \rightarrow \infty} \frac{1}{|S|} \sum_{p \in S} n_p \geq k .$$

If the inequality remains valid for S large, but not infinite, then, since the average is easily computed from the data of plots such as those of Figures 1 and 2, we obtain bounds on k .

¹For example, S could be the set of all $p \leq p_{\max}$.

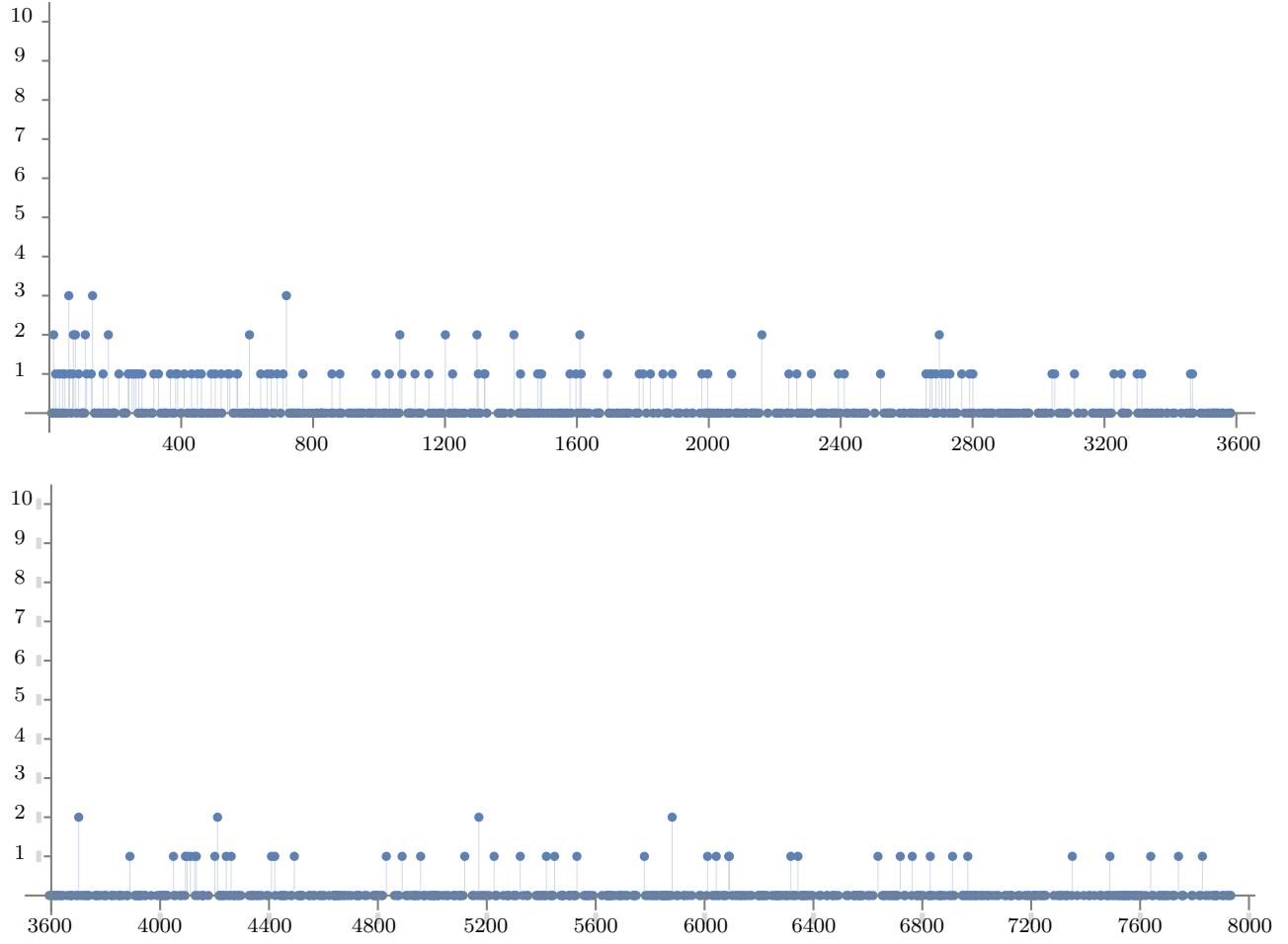


Figure 2: These plots shows the number of factorisations into two quadrics, for the mirror quintic, as φ varies over each \mathbb{F}_p , for the 999 primes $7 \leq p \leq 3583$ and $3593 \leq p \leq 7793$.

For the mirror quintic we see from the second plot, even by eye, that the average of n_p is less than one. If we compute the average for primes in bins of 125 primes we find averages that are given in Figure 4. The case is strong that $k = 0$, and that there are no rank two attractor points for the mirror quintic.

For the manifold AESZ34 the situation, with regard to the existence of rank two attractor points, is more interesting. We plot the running averages of the n_p , for the 1000 primes $5 \leq p \leq 7933$ in Figure 3. The data for 500 primes corresponds to the first four bars, which might have been consistent with $k = 3$, but by including the extra primes it is compelling that in fact $k \leq 2$. In [3] it is shown that G has at least two irreducible factors, so there is a compelling statistical case that $k = 2$ for this manifold.

1.3. Layout of the article

The principal concern of this article is not directly with the utility of the ζ -function so much as with how to compute it as a practical matter. We compute this function from

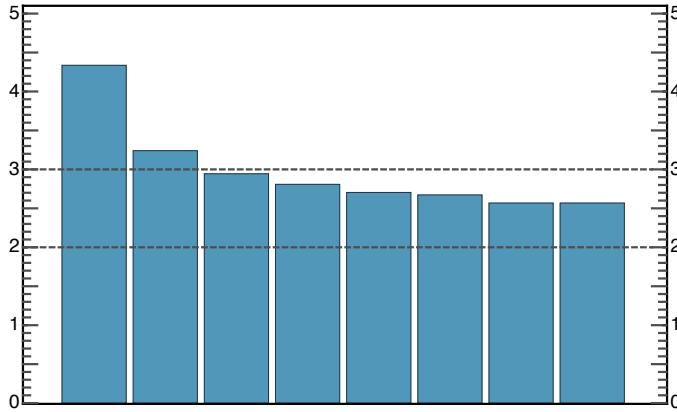


Figure 3: *Running averages for the data of AESZ34 from Figure 1. The averages are taken for bins of 125 primes for the 1000 primes $5 \leq p \leq 7933$.*

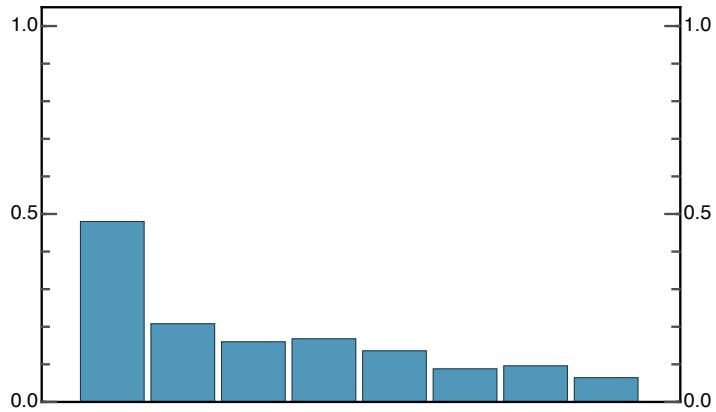


Figure 4: *Running averages for the mirror quintic data of Figure 2. The averages are taken for bins of 125 primes for the 999 primes $7 \leq p \leq 7933$. Note that the vertical scale on this bar chart is very different from that of Figure 3.*

the periods and these are determined by the Picard-Fuchs operator. We therefore start in §2 with a discussion of the method of Frobenius and a basis of the periods. From the periods, we can calculate the matrix E_i^j , that appears in (1.2). We discuss here also certain number-theoretic aspects of the Picard-Fuchs equation. The equation has a discriminant that vanishes for parameter values for which the equation is singular. It can happen that two singular values φ_i and φ_j , that are generically distinct, coincide mod p for certain primes p and this confluence of singularities affects the form of the ζ -function. The study of this phenomenon devolves upon the study of the discriminant of the discriminant of the Picard-Fuchs equation, as a polynomial, and we call this the *hyperdiscriminant*. This has, generically, simple zeros, however, owing to the confluence of roots for certain bad primes, the hyperdiscriminant can develop repeated roots when considered mod p .

The observation of Dwork that the Frobenius polynomial R can be calculated in terms of a matrix $U(\varphi)$ that corresponds to the inverse of the Frobenius map as in (1.2) is central

to our calculation. The computation of the Frobenius matrix $F(\varphi) = U^{-1}(\varphi)$ proceeds in two stages. First, the matrix is calculated for a particular value φ_0 of φ and then $F(\varphi)$ is extended to a region containing φ_0 by means of the relation

$$F(\varphi) = E^{-1}(\varphi)F(\varphi_0)E(\varphi^p),$$

where $E(\varphi)$ is a matrix constructed from the periods of X . We examine this process for the case that φ_0 is a value of the parameter for which X is smooth in §3 and for the case that $\varphi_0 = 0$ is the point of maximal unipotent monodromy, for which X is highly singular, but is the case we actually use in calculation, in §4. The Frobenius matrix has an analogue for the infinite prime and this is the matrix of complex conjugation $C(\varphi)$. We compute this matrix and see that it is indeed closely analogous to $F(\varphi)$.

In §4 we calculate the Frobenius matrix taking the point of maximal unipotent monodromy as the point about which we expand. Although clearly inspired by the procedure for the case that φ_0 is a point for which X is smooth, expanding about $\varphi = 0$ is nevertheless a significant adaptation. We discuss in this section also the form of the central matrix $U(0)$. A notable fact is that while the periods contain logarithms, as can be seen in (2.2), nevertheless these logarithms cancel from the matrix $U(\varphi)$, so this is a matrix of power series. We find also that the $U(0)$ that we conjecture, is given by a p -adic gamma class. So the conjectured form, which is supported by the calculations that produce our zeta-function tables, amounts to a p -adic gamma-class conjecture. We compute also the charge conjugation matrix and the closely related matrix corresponding to the natural hermitian form on X and observe that these are closely analogous to $U(0)$.

In §5 we consider the series expansion of $U(\varphi)$. These series converge, but do so only slowly. Summing the series is rendered practical by the observation of Lauder [1] that, after fixing a p -adic accuracy, the series can be resummed to rational functions, thereby permitting their rapid evaluation. We review also, in this section, the ‘unit root method’. It was observed by Dwork that one of the roots of the Frobenius polynomial is a p -adic unit and this root can, moreover, be computed as a ratio of partial sums of the fundamental period. A knowledge of one root can be used to fix the Frobenius polynomial, when this is irreducible, and to fix one of the quadratic factors, when the Frobenius polynomial factorises. We discuss the utility and the limitations of this process. Finally, in this section, we discuss the bounds on the a, b coefficients required by the Weil conjectures. These coefficients have a natural parametrisation that we also discuss.

In §6 we give a telegraphic account of the manifolds that are our examples. These are all taken from the AESZ list and are chosen to exhibit increasing complication, by which we mean that the respective Picard-Fuchs equations become more complicated. These operators all have the form (1.3) but the coefficient polynomials S_j become of increasingly higher degree. The first manifold is the mirror quintic, which is AESZ1 and has coefficient polynomials that are linear in the parameter φ , while our last two examples have coefficient polynomials that are of degree 8 and 7, respectively. As the degree of the S_j increases, two phenomena begin to arise. The first is that the discriminants S_4 have increasing numbers of roots, so

the corresponding manifolds have increasing numbers of singularities, though, it turns out that not all the singularities of the differential operator correspond to singularities of the manifold. The singularities of the operator, that are not singularities of the manifold, are termed apparent singularities. These apparent singularities play an important, though still somewhat mysterious, role with respect to the computation of the rational functions of the parameter that lead to the identification of the Frobenius polynomial.

Our first example is, as we have observed, the mirror quintic. This must be the most studied of all Calabi-Yau manifolds. We use this manifold to set out the procedure that we will apply to all our examples. We turn next to a manifold defined by the complete intersection of three polynomials of degrees $(1, 2, 2)$ in the Grassmannian $G(2, 5)$. This space has a Picard Fuchs equation with coefficient polynomials of degree 2. There is no apparent singularity, but the discriminant does not factor over \mathbb{Q} . The roots of the discriminant exist in \mathbb{F}_p only when 5 is a square mod p . A consequence of this is that while the conifold singularities of the manifold exhibit modular behaviour, the modular forms are not classical modular forms for $\Gamma_0(N)$, for some N , but rather modular forms with nebentype and Hilbert modular forms.

The third example is the Hulek-Verrill manifold. The coefficient polynomials for the corresponding Picard-Fuchs operator are of degree three. A striking feature of this example is the remarkable frequency with which the Frobenius polynomial factorises into two quadrics as in (1.8). This can be appreciated from Figures 1 and 2, the latter being much closer to the general case.

The fourth example is the Rødland manifold. This is a manifold whose definition does not involve toric geometry, and which has many remarkable properties. Among these is the fact that the moduli space of this manifold contains two large complex structure points and so can be said to correspond to two different mirror manifolds. The coefficient polynomials of the Picard-Fuchs equation are of degree five and the discriminant has the form

$$S_4 = (\varphi - 3)^2 (\varphi^3 - 289\varphi^2 - 57\varphi + 1) .$$

This exhibits two complications: the factor $(\varphi - 3)^2$ corresponds to an apparent singularity, the moduli space is smooth at $\varphi = 3$ and the monodromy about this point is trivial; and the second factor of the discriminant does not factor over \mathbb{Q} . The roots of this cubic factor are conifold points and, at these points the Frobenius polynomial factors in the form (1.7) with a quadratic factor that we believe to be modular, in the sense of corresponding to a Hilbert modular form, though we have not yet identified the modular form.

Our last two examples are somewhat similar. The first of these is a manifold which arises as the manifold of a three generation heterotic model of string theory. For this case the coefficient polynomials have degree eight, and $S_4(\varphi) = (\varphi - 3/2)^2 \Delta(\varphi)$, with $\varphi = 3/2$ corresponding to an apparent singularity. The parameter space for this manifold has six conifold points.

The last example concerns the remarkable manifolds, discovered by V. Braun [4], that have Hodge numbers $h^{1,1} = h^{2,1} = 1$. In many ways this example is similar to the previous one. The coefficient polynomials are, in this case, of degree seven and there is also an apparent

singularity. The parameter space corresponding to this manifold also has six conifold points. In §5.5 we return to (1.5) and (1.6) and enquire as to the degree of the polynomials in the matrices $\tilde{\mathcal{U}}$ and $\hat{\mathcal{U}}$, for our examples. We find that these degrees are determined in a simple manner by the indices corresponding to ∞ that appear in the Riemann symbols for the differential equations.

In §5.6 we indicate briefly how the process of expressing $U(\varphi)$ as a rational function is modified if a higher p-adic accuracy is specified.

Two appendices deal with ancillary matters: in Appendix A we indicate a proof of a corollary to the Frobenius-Cebotarëv theorem and in Appendix B we give a telegraphic review of the theory of the p -adic Γ and ζ -functions.

2. The Picard-Fuchs Equation

2.1. The large p-adic structure expansion

The Picard-Fuchs operators listed in the AESZ database all take the general form (1.3) with coefficient functions S_k that are polynomials in φ with integer coefficients. Our examples differ in the precise form of the coefficient polynomials S_k and, in particular, in the degrees of these polynomials. All the operators have a point of large p-adic structure by which we mean a point of maximal unipotent monodromy. The parameter φ is chosen such that this point corresponds to $\varphi = 0$.

We take a Frobenius solution of the form

$$\varpi(\varphi, \epsilon) = \sum_{n=0}^{\infty} A_n(\epsilon) \varphi^{n+\epsilon}; \quad A_0(\epsilon) = 1. \quad (2.1)$$

It is a consequence of $\varphi = 0$ being a point of maximum unipotent monodromy that the indicial equation is $\epsilon^4 = 0$. Periods $\varpi_k(\varphi)$ are obtained by expanding the Frobenius period in powers of ϵ

$$\varpi(\varphi, \epsilon) = \sum_{k=0}^3 \frac{\epsilon^k}{k!} \varpi_k(\varphi),$$

with the periods ϖ_k , $k = 0, \dots, 3$, themselves given by expressions of the form

$$\begin{aligned} \varpi_0(\varphi) &= f_0(\varphi) \\ \varpi_1(\varphi) &= f_0(\varphi) \log \varphi + f_1(\varphi) \\ \varpi_2(\varphi) &= f_0(\varphi) \log^2 \varphi + 2f_1(\varphi) \log \varphi + f_2(\varphi) \\ \varpi_3(\varphi) &= f_0(\varphi) \log^3 \varphi + 3f_1(\varphi) \log^2 \varphi + 3f_2(\varphi) \log \varphi + f_3(\varphi), \end{aligned} \quad (2.2)$$

where the $f_j(\varphi)$ are regular series in φ . A consequence of taking the coefficient $A_0(\epsilon) = 1$ is that $f_0(0) = 1$ and $f_j(0) = 0$ for $j = 1, 2, 3$, and this fixes this basis uniquely. We will term this basis of periods the *arithmetic Frobenius basis*. There are other bases of periods that are useful and commonly used, so we pause to list these and indicate the relations between them. These are mostly elementary, but it is worth being explicit about the relations between different sets of conventions.

First we give two bases that are very simple variations of the arithmetic Frobenius basis.

- *The complex Frobenius basis:*

$$\widehat{\varpi}_j = \frac{\varpi_j}{(2\pi i)^j}$$

This is used in relating the arithmetic basis with the integral basis.

- A widely used basis that, for want of a better name, we can term *the sophisticated basis*:

$$\varpi_j^\sharp = \frac{1}{j!} \varpi_j.$$

- *The integral basis:* a particular choice within $H^3(X, \mathbb{Z})$ that is adapted to mirror symmetry and the large complex structure limit, see, for example [3] and references cited therein. Briefly put: a symplectic basis $\{\alpha_a, \beta^b\}$, $a, b = 1, 2$, is chosen for $H^3(X, \mathbb{Z})$ and the corresponding periods are defined by writing the holomorphic three form in terms of this basis

$$\Omega = z^a \alpha_a - \mathcal{F}_b(z) \beta^b$$

and it can be shown that there is a function $\mathcal{F}(z)$, called the prepotential, such that $\mathcal{F}_b = \partial \mathcal{F} / \partial z^b$.

In a matrix notation, in which the four integral periods are gathered into a vector

$$\Pi = \begin{pmatrix} \mathcal{F}_a \\ z^b \end{pmatrix}$$

and the four ϖ_j are denoted by a vector ϖ , we have

$$\Pi = \hat{\rho} \hat{\varpi}, \quad (2.3)$$

with

$$\hat{\rho} = \begin{pmatrix} -\frac{1}{3}Y_{000} & -\frac{1}{2}Y_{001} & 0 & \frac{1}{6}Y_{111} \\ -\frac{1}{2}Y_{001} & -Y_{011} & -\frac{1}{2}Y_{111} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

The quantities Y_{abc} are constants that appear in the prepotential, which has an expansion

$$\mathcal{F} = -\frac{1}{3!} \frac{Y_{abc} z^a z^b z^c}{z^0} + \dots$$

where the terms indicated by the ellipsis are of order $\mathcal{O}(e^{2\pi i z^1/z^0})$.

By choice of symplectic basis, the quantities Y_{abc} are related to invariants of the mirror manifold \tilde{X} . It is believed that here is a choice of basis such that

$$\begin{aligned} Y_{111} &= \int_{\tilde{X}} e^3 \\ Y_{011} &\in \left\{ 0, \frac{1}{2} \right\} \\ Y_{001} &= -\frac{1}{12} \int_{\tilde{X}} c_2 e \\ Y_{000} &= -3 \frac{\zeta(3)}{(2\pi i)^3} \chi(\tilde{X}) \end{aligned}$$

where e is the generator of $H^2(\tilde{X}, \mathbb{Z})$. It is perhaps intuitive that the coefficients Y_{011} should be given by the integral of $c_1 e^2$ and so vanish. However, this is not quite true. Rather Y_{011} can, by choice of basis, be made to take either the value 0 or $\frac{1}{2}$. For the

case of one parameter, the rule is simple and depends on whether Y_{111} is even or odd. If Y_{111} is even, then Y_{011} can be taken to vanish, and if Y_{111} is odd, it can be taken to be $\frac{1}{2}$.

- Note that the matrix $\hat{\rho}$ has rational elements, apart from the element Y_{000} . We can remove this element by a further change of basis. This brings us to *the modified complex Frobenius basis* or, in the slightly shorter form, the *modified complex basis*. This basis differs from $\tilde{\varpi}_j$ only when $j = 3$

$$\tilde{\tilde{\varpi}}_j = \begin{cases} \tilde{\varpi}_j, & \text{for } j = 0, 1, 2, \\ \tilde{\varpi}_3 - 2\frac{Y_{000}}{Y_{111}}\tilde{\varpi}_0, & \text{for } j = 3. \end{cases}$$

This basis is related to the integral basis Π by a matrix $\tilde{\tilde{\rho}}$

$$\Pi = \tilde{\tilde{\rho}} \tilde{\tilde{\varpi}}$$

with

$$\tilde{\tilde{\rho}} = \begin{pmatrix} 0 & -\frac{1}{2}Y_{001} & 0 & \frac{1}{6}Y_{111} \\ -\frac{1}{2}Y_{001} & -Y_{011} & -\frac{1}{2}Y_{111} & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

- Finally, we will require also a *modified arithmetic Frobenius basis*, or for short a *modified arithmetic basis*, which we shall denote by $\tilde{\varpi}_j$ with

$$\tilde{\varpi}_j = \begin{cases} \varpi_j, & \text{for } j = 0, 1, 2, \\ \varpi_3 + 6\zeta(3)\frac{\chi(\tilde{X})}{y}, & \text{for } j = 3. \end{cases}$$

where here and in the following we write y for the topological Yukawa coupling Y_{111} .

2.2. Discriminants and hyperdiscriminants

Let us denote the roots of Δ by φ_i , $i = 1, \dots, k$, say. For the cases considered here, these are simple zeros. It is important to observe that there are bad primes for the manifold and for the Picard-Fuchs operator, owing to confluence of the roots φ_i . This arises because, for some p , two roots φ_i and φ_j , that are generically distinct, will coincide mod p . Also, if a root φ_i is rational, and p divides the numerator of φ_i , then φ_i coincides, mod p , with the point of maximal unipotent monodromy, $\varphi = 0$. Furthermore, if p divides the denominator of a root, the root moves to ∞ where it may undergo confluence with other roots, or with a preexisting singularity. It seems best to take the parameter space of φ mod p to be the projective space $\mathbb{P}\mathbb{F}_p$, so ∞ is an allowed value.

Thus we will be interested in the discriminant of Δ as a polynomial. For a polynomial

$$h_n\varphi^n + h_{n-1}\varphi^{n-1} + \dots + h_0 ,$$

with roots φ_i this is understood to be

$$h_n^{2n-2} \prod_{i < j} (\varphi_i - \varphi_j)^2 , \quad (2.4)$$

which vanishes whenever two roots coincide. The overall constant is conventional and, with this choice, the discriminant of a quadratic equation is the familiar $h_1^2 - 4h_0h_2$. Since we are now dealing with the discriminant of the discriminant, we shall refer to this quantity as the *hyperdiscriminant of the manifold*. We will abuse notation by redefining Δ , and related quantities in relation to thinking of $\varphi = (u:v)$ in terms of projective coordinates. We now understand Δ as the homogeneous polynomial

$$\Delta = h_n u^n + h_{n-1} u^{n-1} v + \dots + h_0 v^n ,$$

of course, setting $u = \varphi$ and $v = 1$ returns us to the previous definition. The new definition however has the advantage that the value of the hyperdiscriminant is invariant under the action of the Möbius group on the coordinates $(u:v)$. We will also understand by *hyperdiscriminant* the discriminant of the new definition of Δ and we will denote this quantity by Δ .

For the case that there is an apparent singularity, it is of interest to know when φ_0 coincides, mod p , with one of the genuine singularities. To this end we consider the homogenised discriminant of $(\varphi - \varphi_0)\Delta$, note that the first factor is linear, so this is not the same as $S_4 = (\varphi - \varphi_0)^2\Delta$. We denote this quantity by \mathbb{S}_4

$$\mathbb{S}_4 = \text{discr}\left((\varphi - \varphi_0)\Delta\right)$$

and refer to it as the *hyperdiscriminant of the Picard-Fuchs equation*. This hyperdiscriminant corresponds to a product analogous to (2.4), but where the index i also runs over the value 0.

The quantities Δ and \mathbb{S}_4 do not reveal the whole story of the bad primes of the differential equation since it can also happen that a prime can be bad for all parameter values, we will refer to such primes as *notorious primes*. The mirror quintic furnishes an example of this phenomenon. For this case the discriminant is linear

$$\Delta = v - 5^5 u$$

and $\Delta = 1$, independent of $\varphi = (u:v)$. Nevertheless we want to take 5 to be a bad prime for the following reason. Take the defining equation of the mirror quintic to be $F = 0$, with

$$F(x) = \sum_{i=1}^5 x_i^5 - \tilde{\psi} x_1 x_2 x_3 x_4 x_5 .$$

Then the five derivatives

$$\frac{\partial F}{\partial x_i} = 5x_i^4 - \tilde{\psi} \prod_{j \neq i} x_j$$

all vanish, mod 5, at points where any two coordinates vanish. So the manifold is singular mod 5, irrespective of the value of the parameter.

It is notable and indicative of the fact that the Picard-Fuchs equations have an important number-theoretic significance that, in the examples we study here, no new bad primes are introduced by passing from Δ to \mathbb{S}_4 .

3. The Frobenius and Complex Conjugation Matrices I

In this section we set up the basic formalism related to Dwork's deformation method. We examine the formulation of the Frobenius map and the complex conjugation matrix for parameter values for which the variety X_φ is smooth. The Frobenius map is an automorphism of every manifold defined over \mathbb{Q}_p . For the infinite prime, so for manifolds defined over \mathbb{R} , there is an analogous map which is complex conjugation. For this reason it is appropriate to discuss these maps together. The discussion proceeds by picking a point $\varphi = \varphi_0$ for which X_{φ_0} is smooth and then studying the maps for φ in a neighbourhood of φ_0 .

In the following section we extend the technique to parameter values of φ in the neighbourhood of the point of maximal unipotent monodromy (MUM) $\varphi = 0$, for which the variety is very singular. It is the technique developed in that section the one used to compute the data for the ζ -function.

3.1. Complex conjugation and the Frobenius map

The field \mathbb{Q} of rational numbers carries, for each $r \in \mathbb{Q}$ and prime number p , a p -adic absolute value $\|r\|_p$ as well as the usual absolute value $|r| = \|r\|_\infty$, which is sometimes said to belong to the *infinite prime*. The completions with respect to these absolute values give the field \mathbb{Q}_p of p -adic numbers and the field \mathbb{R} of real numbers. For each $r \in \mathbb{Q}$ the *product formula*

$$\|r\|_\infty \prod_{p \text{ prime}} \|r\|_p = 1$$

holds, which underlines that all primes, including the infinite prime, have an equal footing. The absolute values $\|*\|_p$ is *non-archimedean*

$$\|x + y\|_p \leq \max(\|x\|_p, \|y\|_p).$$

As a result, the set of $x \in \mathbb{Q}_p$ for which $\|x\|_p \leq 1$ is closed under multiplication and addition and forms the ring \mathbb{Z}_p of p -adic integers and computing mod p in \mathbb{Z}_p brings us to the finite field \mathbb{F}_p . The analogous set $|x| \leq 1$, in \mathbb{R} , is closed under multiplication, but not under addition, so there is no complete analog of the ring \mathbb{Z}_p for the infinite prime.

Consider now an algebraic variety X . For sake of concreteness, let us suppose it is defined by a polynomial $g(x) := g(x_1, x_2, \dots, x_n)$ with coefficients in \mathbb{Z} . We then can look for the solution set $X(\mathcal{R})$ to the equation

$$g(x_1, x_2, \dots, x_n) = 0, \tag{3.1}$$

where all the coordinates x_i belong to a ring \mathcal{R} . The equation (3.1) may not admit any real solutions, but there is always a non-empty manifold of complex solutions $X(\mathbb{C})$. As the coefficients of g are in \mathbb{Z} , so in particular real, one has

$$\overline{g(x)} = g(\bar{x}),$$

so that *complex conjugation* defines a map

$$\text{fr}_\infty : X(\mathbb{C}) \rightarrow X(\mathbb{C}) ; \quad x \mapsto \bar{x}$$

and the fixed point set is nothing but $X(\mathbb{R})$. We can try to do something similar for each prime p . We can look for solutions to (3.1) in \mathbb{F}_p , the field with p elements, or in extension fields \mathbb{F}_{p^k} , or even its algebraic closure $\mathbb{F}_p^{\text{alg}}$. By Fermat's little theorem, we have

$$g(x)^p = g(x^p) \pmod{p} ,$$

so that taking the p 'th power of the coordinates defines the *Frobenius map*

$$\text{fr}_p : X(\mathbb{F}_p^{\text{alg}}) \rightarrow X(\mathbb{F}_p^{\text{alg}}) ; \quad x \mapsto x^p .$$

The fixed points of this map fr_p form precisely the set $X(\mathbb{F}_p)$, and similarly, the fixed point set of $(\text{fr}_p)^k$ is $X(\mathbb{F}_{p^k})$.

The maps fr_∞ and fr_p act on the cohomology groups of the corresponding varieties. For the complex manifold $X(\mathbb{C})$ one can consider singular cohomology or the deRham cohomology with real coefficients. As the precise choice is of no importance here, we will just denote them by $H^k(X(\mathbb{C}))$ and get linear maps induced by complex conjugation:

$$\text{Fr}_\infty : H^k(X(\mathbb{C})) \rightarrow H^k(X(\mathbb{C})) .$$

To do something similar for $X(\mathbb{F}_p^{\text{alg}})$, one has to invent a good cohomology theory for varieties over a finite field. Dwork [5], Washnitzer-Monsky (see [6]), Grothendieck, Berthelot (see [7]) and others constructed p -adic cohomology spaces, which are finite dimensional vector spaces over \mathbb{Q}_p . Details will not be important here, but all constructions involve *lifting* the variety over \mathbb{F}_p to \mathbb{Z}_p and considering the result over the field \mathbb{Q}_p and taking the deRham complex of this lift, the cohomology of which is essentially independent of the choices of the lifts that are made. We will denote the cohomology groups simply by $H^k(X(\mathbb{F}_p^{\text{alg}}))$. The action of Frobenius can be lifted and induces Frobenius maps acting on the p -adic cohomology

$$\text{Fr}_p : H^k(X(\mathbb{F}_p^{\text{alg}})) \rightarrow H^k(X(\mathbb{F}_p^{\text{alg}})) .$$

If $\psi : X \rightarrow X$ is a self-map of a topological space X , then the *Lefschetz trace formula* expresses the alternating sum of cohomological traces

$$\sum_{k=0}^{2d} (-1)^k \text{Tr} (\psi | H^k(X))$$

in terms of the fixed point set of ψ . Weil famously imagined this to be applicable to the Frobenius map acting on appropriate cohomology groups, which were constructed subsequently, leading to the relation

$$\sum_{k=0}^{2d} (-1)^k \text{Tr} (\text{Fr}_{p^n} | H^k(X)) = \#(X(\mathbb{F}_{p^n})) .$$

It was shown by Dwork that this also holds for his p-adic cohomology theory. By standard linear algebra, this leads to the representation of the zeta-function in cohomological terms

$$\exp \left(\sum_{r=1}^{\infty} \#X(\mathbb{F}_{p^r}) \frac{T^r}{r} \right) = \frac{R_1(T)R_3(T)\dots R_{2d-1}(T)}{R_0(T)R_2(T)\dots R_{2d}(T)}$$

where

$$R_k(T) := \det(1 - T \text{Fr}_p^{-1}|H^k(X)) .$$

We refer to [8] for a nice introduction, [9] for a physics oriented account, [10] for a computationally oriented introduction and [11, 12] for more details. In the situation we consider in this paper $d = 3$, and the Betti numbers b^1 and b^5 vanish. Therefore, the factors R_1 and R_5 are trivial and we are left with R_3 and we henceforth omit the index. If the variety is smooth the form of the ζ -function is given by equation (1.1) whenever the Picard group is generated by divisors which are defined over \mathbb{F}_p . In the contrary case that the Picard group is not generated by divisors defined over \mathbb{F}_p , but rather over an extension \mathbb{F}_{p^k} , then the factor $(1 - pT)^{h^{11}}(1 - p^2T)^{h^{11}}$ in the denominator of the ζ -function assumes the more general form

$$(1 - pT)^{h^{11}-m_p}(1 + pT)^{m_p}(1 - p^2T)^{h^{11}-m_p}(1 + p^2T)^{m_p} ,$$

where m_p is an integer, $0 \leq m_p \leq h^{11}$ that can depend on p .

3.2. The deformation method

We will also consider families of varieties X_φ , parametrised by φ , defined by a polynomial equation

$$g(x, \varphi) = 0 ,$$

where we assume, as before, that the coefficients of g are in \mathbb{Z} . Note that

$$\overline{g(x, \varphi)} = g(\bar{x}, \bar{\varphi}) ,$$

and this gives rise to a map

$$\text{fr}_\infty(\varphi) : X_\varphi(\mathbb{C}) \rightarrow X_{\bar{\varphi}}(\mathbb{C}) ,$$

which is a self-map only for real φ .

Analogously, since

$$g(x, \varphi)^p = g(x^p, \varphi^p) \pmod{p} ,$$

we obtain maps

$$\text{fr}_p(\varphi) : X_\varphi(\mathbb{F}_p^{\text{alg}}) \rightarrow X_{\varphi^p}(\mathbb{F}_p^{\text{alg}}) ,$$

which is a self-map only for φ in the ground field \mathbb{F}_p . We assume the manifolds X_φ are smooth for $\varphi \in S$, where S is \mathbb{P}^1 , minus a finite set of singular values. Then the cohomology groups $H^n(X_\varphi(\mathbb{C}))$ form a vector bundle \mathcal{H} on S , such that

$$\mathcal{H}_\varphi^n = H^n(X_\varphi(\mathbb{C})) .$$

It carries a canonical *Gauss-Manin* connection: for each vector field ϑ on the base we obtain a covariant derivative in the direction ϑ , giving a map

$$\nabla_\vartheta : \mathcal{H} \rightarrow \mathcal{H} .$$

In the following, we will always take the logarithmic derivative

$$\vartheta = \varphi \frac{\partial}{\partial \varphi}$$

as vector field and write simply ∇ for ∇_ϑ .

If we fix a frame e^k for \mathcal{H} , the information of the connection is given by a connection matrix $B = (B_j{}^k)$

$$\nabla e^k = e^j B_j{}^k .$$

The reason for the ‘transposed’ convention associated with the connection matrix B is that, if we think of the periods ϖ_j as forming a column vector, then a relation such as the following, that we shall come across in the next subsection,

$$\Omega = \varpi_j^\sharp f^j ,$$

with Ω the holomorphic three-form, the ϖ_j^\sharp the ‘sophisticated’ periods from §2.1 and the f^j a certain cohomology basis, shows that the f^j should be thought of as forming a row vector. The entries of the matrix B are rational functions of φ , with rational coefficients, that have poles in the complement of S . The matrix B can be computed effectively, [1, 5, 13–15], in fact we shall do this presently.

A similar construction can be done in p -adic cohomology for a prime of good reduction. This then leads to the ‘same’ matrix B , only now the coefficients are interpreted as belonging to \mathbb{Q}_p . In this situation, it was realised by Dwork [16] that the Frobenius transformation $\text{Fr}(\varphi) : \mathcal{H}_\varphi \rightarrow \mathcal{H}_{\varphi^p}$ is *compatible with the Gauss-Manin connection*. The map $\text{Fr} = \text{Fr}(\varphi)$ has the fundamental commutation relation

$$\nabla \text{Fr} = p \text{Fr} \nabla . \quad (3.2)$$

In this way we end up with a mathematical structure that is commonly referred to as a *crystal* [17]; it consists of a free module \mathcal{H} over a certain p -adic ring $\mathcal{R} \subset \mathbb{Z}_p[[\varphi]]$, together with two operations $\nabla : \mathcal{H} \rightarrow \mathcal{H}$ and $\text{Fr} : \mathcal{H} \rightarrow \mathcal{H}$ that satisfy (3.2). For the coefficient ring \mathcal{R} one has two similar operations $\mathcal{R} \rightarrow \mathcal{R}$

$$\vartheta : c(\varphi) \rightarrow \varphi \frac{\partial c(\varphi)}{\partial \varphi} \quad \text{and} \quad \text{fr} : c(\varphi) \rightarrow c(\varphi^p)$$

that satisfy the similar relation

$$\vartheta \text{fr} = p \text{fr} \vartheta .$$

The operators ∇ and Fr are compatible with these: ∇ satisfies the appropriate Leibniz rule, whereas Fr is Frobenius-linear. For example

$$\text{Fr}(c(\varphi)e^k) = c(\varphi^p) \text{Fr}(e^k) .$$

If we write out the action of Fr on the frame e^k as

$$\text{Fr}(\varphi)(e^k) = e^j F_j^k(\varphi)$$

and consider, say, the quantity $\nabla \text{Fr}(e^k)$ and use the identity (3.2) we find that

$$\vartheta(F(\varphi)) = pF(\varphi)B(\varphi^p) - B(\varphi)F(\varphi) , \quad (3.3)$$

which can be interpreted as a differential equation for $F(\varphi)$. The upshot is that if we know the Frobenius matrix $F(\varphi)$ at some point $\varphi_0 \in S$, then we can, in principle, use the above differential equation to determine F at other points. This is the core of the *deformation method*, which was worked out by Dwork in special cases. In [5], Dwork took initial points φ_0 corresponding to Fermat-varieties, for which the Frobenius matrix can be written down in closed form, as was first done by Weil [18].

It is easy to solve the differential equation (3.3) in terms of the *fundamental solution* $E(\varphi)$ for the matrix differential equation defined by B . The matrix E is the unique matrix series solution to the equation

$$\vartheta E(\varphi) = E(\varphi) B(\varphi)$$

with the property that

$$E(\varphi_0) = \mathbb{1} .$$

Then the series

$$F(\varphi) = E(\varphi)^{-1} F(\varphi_0) E(\varphi^p) \quad (3.4)$$

solves (3.3). Indeed,

$$\begin{aligned} \vartheta(F(\varphi)) &= -E(\varphi)^{-1} E(\varphi) B(\varphi) E(\varphi)^{-1} F(\varphi_0) E(\varphi^p) + pE(\varphi)^{-1} F(\varphi_0) E(\varphi^p) B(\varphi^p) \\ &= -B(\varphi)F(\varphi) + pF(\varphi)B(\varphi^p) , \end{aligned}$$

which is again (3.3).

The crystals arising from the cohomology of smooth d -dimensional projective varieties have also additional structure. There is the Poincaré intersection pairing $(*, *) : H^d \times H^d \rightarrow H^{2d}$, symmetric for d even and alternating for d odd. The connection ∇ and Frobenius map Fr are also compatible with respect to this pairing. In fact one has

$$\vartheta(\xi, \eta) = (\nabla\xi, \eta) + (\xi, \nabla\eta) \quad (3.5a)$$

$$(\text{Fr}\xi, \text{Fr}\eta) = p^d \text{Fr}((\xi, \eta)) . \quad (3.5b)$$

Furthermore, one can introduce a *Hodge-filtration*

$$\text{Fil}^d \subset \text{Fil}^{d-1} \subset \dots \subset \text{Fil}^0 = \mathcal{H} , \quad \text{with } \nabla(\text{Fil}^i) \subset \text{Fil}^{i-1} \text{ and } \text{Fr}(\text{Fil}^i) \subset p^i \mathcal{H} . \quad (3.5c)$$

Augmenting the crystal structure by requiring also the compatibility (3.5 a) and (3.5 b) yields a structure that may be called a *polarised F-crystal*, and if the filtration is also included,

then we have a *polarised divisible Hodge F-crystal* also known as a *Fontaine-Lafaille crystal*, we will call it a *CY-crystal* for short.

Let us now compute the matrix of complex conjugation, which we will denote by $C(\varphi)$, for a basis e^k which is assumed to be holomorphic. This matrix is defined by

$$\overline{e^k} = e^j C_j^k(\varphi) .$$

Note that the matrix C cannot be holomorphic. A first relation follows from the identity $\nabla \overline{e^k} = 0$. On writing out the derivative, we have

$$\nabla \overline{e^k} = \nabla (e^j C_j^k) = e^j (B_j^i C_i^k + \vartheta C_j^k) .$$

So

$$\vartheta C = -BC .$$

The antiholomorphic derivative of C is determined by the identity $\overline{\nabla e^k} = \overline{\nabla} \overline{e^k}$, which yields the relation

$$\overline{\vartheta} C = C \overline{B} .$$

These equations are solved by writing

$$C(\varphi) = E^{-1}(\varphi) C(\varphi_0) E(\overline{\varphi}) \quad (3.6)$$

and the condition $C(\varphi) \overline{C(\varphi)} = \mathbb{1}$ is satisfied for all φ , if it is satisfied for $\varphi = \varphi_0$. The complete analogy between (3.6) and (3.4) is apparent.

4. The Frobenius and Complex Conjugation Matrices II

4.1. Structure near a MUM-point

It is of great interest to adapt the discussion of the previous section to deform about singular points. We will now study these structures for a family of Calabi-Yau threefolds near a MUM-point, that is a point of *maximal unipotent monodromy*, also known, depending on context, as a large complex structure limit, or a large p -adic structure limit. We will always assume that $h^{2,1} = 1$, so that the cohomology group H^3 is four-dimensional.

Our aim is to obtain the power-series expansions for the inverse matrix of the Frobenius action on $H^3(X_\varphi)$ and the charge conjugation matrix about this point, something that was already explored by Dwork [19] and Lauder [20] for the Frobenius matrix.

We follow Dwork and Lauder in conjecturing that this matrix can be written in an analogous form to that of the previous section

$$U(\varphi) = E^{-1}(\varphi^p)U(0)E(\varphi). \quad (4.1)$$

The MUM-point $\varphi = 0$ corresponds, however, to a variety X_0 that is highly singular. Owing to the singularity the matrix $E(\varphi)$ contains logarithms and is itself singular at $\varphi = 0$. So while $U(\varphi)$ can be written in the form above for some matrix $U(0)$ as a consequence of the differential equation (3.3) it is not clear, a priori, that $U(\varphi)$ has a finite limit as $\varphi \rightarrow 0$ and that $U(\varphi)$ tends to the matrix that we have denoted by $U(0)$. However, if we take $U(0)$ to be consistent with the CY-crystal structure then, as we will see, the logarithms that are present in $E(\varphi)$ cancel and $U(\varphi)$ has an expansion about $\varphi = 0$ as a matrix of power series and so tends to a well defined matrix $U(0)$ as $\varphi \rightarrow 0$. We are led to further conjecture the precise form of the matrix $U(0)$ in terms of the p -adic Γ -function. In fact it is compelling to conjecture that the Frobenius matrix is governed by a p -adic version of the Γ -conjectures. We set this out in §4.4.

Consider the sophisticated basis of periods ϖ_j^\sharp defined in §2.1 and its Wronskian matrix E

$$\varpi_j^\sharp = \frac{1}{j!} \varpi_j, \quad E_j^k = \vartheta^k \varpi_j^\sharp.$$

The PF equation in first order form is

$$\vartheta E = EB \quad \text{with} \quad B = \begin{pmatrix} 0 & 0 & 0 & -S_0/S_4 \\ 1 & 0 & 0 & -S_1/S_4 \\ 0 & 1 & 0 & -S_2/S_4 \\ 0 & 0 & 1 & -S_3/S_4 \end{pmatrix}. \quad (4.2)$$

We will show presently that these matrices $E(\varphi)$ and $B(\varphi)$ thus defined, are the appropriate analogues to the matrices $E(\varphi)$ and $B(\varphi)$ of the previous section. As has been noted, owing to the presence of logarithms, $E(\varphi)$ is singular at $\varphi = 0$.

Let Ω be the holomorphic three-form and define a basis in cohomology f^j by

$$\Omega = \varpi_j^\sharp f^j.$$

Note that the f^j are related to the integral basis by a constant matrix, which we will calculate later explicitly, so $\nabla f^j = 0$. Let us define also a matrix $F(0)$ by

$$\text{Fr } f^k = f^j F_j^k(0) .$$

Now take a moving cohomology basis

$$e^k = \nabla^k \Omega = f^j E_j^k .$$

It would be better to take a basis that transforms covariantly under gauge transformations and so adapt crystalline cohomology to special geometry [21, 22], but we leave aside this interesting topic, to which we hope to return elsewhere.

To continue, we define also a matrix $F(\varphi)$ by

$$\text{Fr } e^k = e^j F_j^k(\varphi) .$$

By the previous relation, we have

$$\text{Fr } e^k = f^i F_i^j(0) E_j^k(\varphi^p) = e^\ell E^{-1}(\varphi)_\ell^i F_i^j(0) E_j^k(\varphi^p) .$$

Thus

$$F(\varphi) = E^{-1}(\varphi) F(0) E(\varphi^p) \quad \text{and} \quad U(\varphi) = F^{-1}(\varphi) = E^{-1}(\varphi^p) U(0) E(\varphi) .$$

It remains to show that $F(\varphi)$, as just defined, tends to $F(0)$ as $\varphi \rightarrow 0$. This is true, but not obvious a priori owing to the fact that the E -matrices contain logarithms of φ and are singular as $\varphi \rightarrow 0$. We will show presently that the logarithms cancel in the product and that $F(\varphi)$ is indeed continuous at $\varphi = 0$.

The calculation of the charge conjugation matrix is somewhat more delicate than that for the Frobenius matrix, so is deferred until after we discuss the form of the wronskian matrix.

4.2. The form of the matrix E

Consider the matrix E^0 , corresponding to the leading terms of E as $\varphi \rightarrow 0$

$$E^0{}_j{}^k = \frac{1}{j!} \vartheta^k \log^j \varphi = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \log \varphi & 1 & 0 & 0 \\ \frac{1}{2} \log^2 \varphi & \log \varphi & 1 & 0 \\ \frac{1}{6} \log^3 \varphi & \frac{1}{2} \log^2 \varphi & \log \varphi & 1 \end{pmatrix} .$$

and in terms of this define

$$\tilde{E} = (E^0)^{-1} E .$$

The prefactor $(E^0)^{-1}$ removes the logarithms from E and the result is formally the same as performing the differentiations to compute E and then setting $\log \varphi = 0$

$$\tilde{E}_j^k = \frac{1}{j!} \vartheta^k \varpi_j \Big|_{\log \varphi=0} = \sum_{s=0}^{\min(j,k)} \frac{k!}{(j-s)! (k-s)! s!} \vartheta^{k-s} f_{j-s} .$$

Note that, when $\varphi = 0$, the only terms to survive in the sum above are those for which $s = k = j$ and that these surviving terms are unity. So we see that

$$\tilde{E}(0) = \mathbb{1}.$$

Return now to the Picard-Fuchs equation (4.2). Note that it is a general property that the terms S_j vanish at $\varphi = 0$ for $0 \leq j \leq 3$, while $S_4(0)$ is non-zero. Also, let us denote the matrix $B(0)$ by ϵ .

$$\epsilon = B(0) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

Notice that $\epsilon^4 = 0$ and also that $B(0)$ generates the monodromy of E around $\varphi = 0$. Furthermore, E and the vector ϖ^\sharp have the same monodromy. Thus

$$\varpi^\sharp \rightarrow e^{2\pi i \epsilon} \varpi^\sharp$$

and we see that we can identify the matrix ϵ with the ϵ that appears in the Frobenius period.

We can now show that, in virtue of the CY-crystal relations, we may replace E by \tilde{E} , for the purpose of computing $U(\varphi)$, in (1.2). Note first that setting $\varphi = 0$ in (3.3) yields an identity that we conjecture holds for $U(0)$

$$p \epsilon U(0) = U(0) \epsilon. \quad (4.3)$$

We observe also that

$$E^0(\varphi) = \varphi^\epsilon.$$

Thus we have

$$U(0) E^0(\varphi) = \varphi^{p\epsilon} U(0) = E^0(\varphi^p) U(0).$$

Hence

$$(E^0(\varphi^p))^{-1} U(0) E^0(\varphi) = U(0)$$

and it follows that

$$E^{-1}(\varphi^p) U(0) E(\varphi) = \tilde{E}^{-1}(\varphi^p) U(0) \tilde{E}(\varphi).$$

It is now immediate, since $\tilde{E}(\varphi)$ is smooth at $\varphi = 0$ and $\tilde{E}(0) = \mathbb{1}$, that $F(\varphi) \rightarrow F(0)$ and $U(\varphi) \rightarrow U(0)$ as $\varphi \rightarrow 0$.

4.3. The form of $U(0)$

Let us now seek to write down the explicit form of $U(0)$. The commutation relation (4.3) restricts $U(0)$ to the form

$$U(0) = \begin{pmatrix} u & 0 & 0 & 0 \\ a & pu & 0 & 0 \\ b & pa & p^2u & 0 \\ c & pb & p^2a & p^3u \end{pmatrix}.$$

There are further restrictions to be made on this matrix. If we impose the condition (3.5 c) we find that the i 'th row of $U(0)$ should be divisible by p^i , $0 \leq i \leq 3$ so that we can write

$$a = p u \alpha, \quad b = p^2 u \beta, \quad c = p^3 u \gamma.$$

Now we have

$$U(0) = u \begin{pmatrix} 1 & 0 & 0 & 0 \\ p\alpha & p & 0 & 0 \\ p^2\beta & p^2\alpha & p^2 & 0 \\ p^3\gamma & p^3\beta & p^3\alpha & p^3 \end{pmatrix}.$$

In order to constrain the form of $U(0)$ yet further, we should pause to examine the anti-symmetric and hermitian forms that exist for the case of immediate interest for which the complex dimension is three.

There is a natural antisymmetric form $(*, *) : H^3 \times H^3 \rightarrow H^6$

$$(\xi, \eta) = \int_X \xi \wedge \eta$$

There is also an associated hermitean form, that is sesquilinear on the first argument

$$\langle \xi, \eta \rangle = i(\bar{\xi}, \eta).$$

It is useful to have explicit matrices corresponding to these quadratic forms. For the co-homology group $H^3(X)$ we have already introduced the integral symplectic basis $\{\alpha_a, \beta^b\}$, which we can take together and denote by $\{\alpha^m\}$. Thus we can write

$$\Omega = \varpi_k^\sharp f^k = \Pi_m \alpha^m. \quad (4.4)$$

If we take a matrix

$$\lambda = \text{diag} \left(1, \frac{1!}{2\pi i}, \frac{2!}{(2\pi i)^2}, \frac{3!}{(2\pi i)^3} \right),$$

then

$$\Pi = \rho^\sharp \varpi^\sharp; \quad \rho^\sharp = \hat{\rho} \lambda,$$

where $\hat{\rho}$ is the matrix defined in (2.3). The f^k are related to the α^m by the matrix ρ^\sharp which is the “matrix of constants” alluded to earlier

$$f^k = \alpha^m \rho_m^\sharp{}^k. \quad (4.5)$$

We have

$$(\alpha^m, \alpha^n) = \Sigma^{mn}; \quad \Sigma = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}.$$

Let us denote the matrix corresponding to the antisymmetric product in the $\{f^j\}$ -basis by σ . Then

$$\sigma^{jk} = (f^j, f^k) ; \quad \sigma = -\frac{y}{(2\pi i)^3} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix} .$$

Now from (3.5c) we see that we should have

$$U(0) \sigma U(0)^T = p^3 \sigma$$

and this has the consequence

$$u^2 = 1 \quad \text{and} \quad \beta = \frac{1}{2}\alpha^2 .$$

We have that $u = \pm 1$ and we come to the form

$$U(0) = u \Lambda (\mathbb{1} + \alpha \epsilon + \beta \epsilon^2 + \gamma \epsilon^3) , \quad (4.6)$$

where $\beta = \alpha^2/2$ and

$$\Lambda = \text{diag}(1, p, p^2, p^3) .$$

4.4. Conjectures relating to the form of $U(0)$

Based on many calculations, we make the following

Conjecture: *For Calabi-Yau threefolds in a one parameter family,*

- *the matrix $U(0)$ is as in (4.6), with $u = 1$, $\alpha = \beta = 0$ and*

$$\gamma = \frac{\chi(X)}{y} \zeta_p(3) ,$$

where χ is the Euler number of the manifold, y denotes the large complex structure value of the Yukawa coupling, so the triple intersection value of the generator of H^2 , for the mirror manifold, and $\zeta_p(3)$ denotes the p -adic analog of $\zeta(3)$, which we calculate in terms of the p -adic Γ -function by means of the formula

$$\zeta_p(3) = -\frac{1}{2} \left(\Gamma_p'''(0) - \Gamma_p'(0)^3 \right) . \quad (4.7)$$

- *Moreover, for good primes, $U(\varphi) = E^{-1}(\varphi^p) U(0) E(\varphi)$ is a matrix of power series in φ with coefficients that are p -adically integral.*

Some special cases of these conjectures have been proved. The statement that $u = 1$ is proved, for the 14 hypergeometric, one-parameter families of Calabi-Yau threefolds, in the

dissertation of K. Samol [23] and the expression (4.7) has been derived, for the case of the mirror quintic, by Shapiro [24] and by Thorne [2].

In (4.7) Γ_p denotes the Γ -function that is named for Morita [25]. Though, on the matter of attribution, Dwork, writing under the pseudonym of Boyarski [26], pointed out that this function was known to him much earlier [5].

There is a degree of choice in the definition of the p -adic zeta function and so in the definition of $\zeta_p(3)$. We give a derivation of the formula above for $\zeta_p(3)$, and discuss these matters further, in Appendix B.

One can consider our conjecture as a manifestation of a p -adic version of the Γ -conjectures (see [27–32]), which are still to be formulated in full generality. Let us nevertheless pursue these matters a little further.

Having taken $\alpha = 0$ the form for $U(0)$ is

$$U(0) = \Lambda \left(\mathbb{1} + \frac{\chi(X)}{y(\tilde{X})} \zeta_p(3) \epsilon^3 \right).$$

Consider the following identity

$$e^{-\lambda \Gamma_p'(0) \epsilon} \Gamma_p(\lambda \epsilon) = \mathbb{1} - \frac{1}{3} \lambda^3 \zeta_p(3) \epsilon^3$$

This follows easily by expanding the left hand side in powers of ϵ and using the identity $\Gamma_p''(0) = \Gamma_p'(0)^2$ as well as (4.7). It is also an immediate consequence of (B.8) from Appendix B.

In order to perform a characteristic class calculation we write λ_j , $j = 1, 2, 3$ for the eigenvalues of the quantity Θ/ω , where Θ denotes the curvature matrix and ω denotes the Kähler form of \tilde{X} . On taking the product of three instances of the expression above we find

$$\prod_{j=1}^3 e^{-\lambda_j \Gamma_p'(0) \epsilon} \Gamma_p(\lambda_j \epsilon) = \mathbb{1} - \frac{1}{3} \left(\sum_{j=1}^3 \lambda_j^3 \right) \zeta_p(3) \epsilon^3,$$

Writing the power sum on the right in terms of the symmetric polynomials we have

$$\sum_{j=1}^3 \lambda_j^3 = \sigma_1^3 - 3\sigma_1\sigma_2 + 3\sigma_3.$$

We have $\sigma_1 = c_1/\omega = 0$ and

$$\sigma_3 = \frac{c_3(\tilde{X})}{y(\tilde{X})} = -\frac{\chi(X)}{y(\tilde{X})}$$

The upshot is that

$$U(0) = \Lambda \prod_{j=1}^3 e^{-\lambda_j \Gamma_p'(0) \epsilon} \Gamma_p(\lambda_j \epsilon).$$

Note that

$$\prod_{j=1}^3 e^{-\lambda_j \Gamma_p'(0) \epsilon} = e^{-\sigma_1 \Gamma_p'(0) \epsilon}$$

and that σ_1 is set to zero. Thus the factors $e^{-\lambda_j \Gamma_p'(0) \epsilon}$, while useful for simplifying intermediate expressions, can in fact be omitted. Hence we may write

$$U(0) = \Lambda \widehat{\Gamma}_p \quad \text{with} \quad \widehat{\Gamma}_p = \prod_{j=1}^3 \Gamma_p(\lambda_j \epsilon) .$$

Our interest is primarily with threefolds, but since we have the expansion (B.8) to hand, we may write out the expansion of $\widehat{\Gamma}_p$ in a form that allows us to pick out the terms appropriate to higher dimensions.

$$\begin{aligned} \prod_j \Gamma_p(\lambda_j \epsilon) &= 1 - \zeta_p(3) \sigma_3 \epsilon^3 - \zeta_p(5) (\sigma_5 - \sigma_2 \sigma_3) \epsilon^5 + \frac{1}{2} \zeta_p(3)^2 \sigma_3^2 \epsilon^6 \\ &\quad - \zeta_p(7) \left(\sigma_3 (\sigma_2^2 - \sigma_4) - \sigma_2 \sigma_5 + \sigma_7 \right) \epsilon^7 + \zeta_p(3) \zeta_p(5) \sigma_3 (\sigma_5 - \sigma_2 \sigma_3) \epsilon^8 \\ &\quad - \left[\frac{1}{6} \zeta_p(3)^3 \sigma_3^3 + \zeta_p(9) \left(\frac{1}{3} \sigma_3^2 - \sigma_3 \sigma_2^3 + \sigma_5 \sigma_2^2 + (2\sigma_3 \sigma_4 - \sigma_7) \sigma_2 - \sigma_4 \sigma_5 - \sigma_3 \sigma_6 + \sigma_9 \right) \right] \epsilon^9 \\ &\quad + \dots . \end{aligned}$$

4.5. The matrix of complex conjugation

As we have indicated, complex conjugation can be considered as an analogue of the Frobenius maps, corresponding to the infinite prime. A complication is that while the Frobenius matrix $F(\varphi)$ is regular at the MUM-point, the same is not true of the complex conjugation matrix $C(\varphi)$. Nevertheless we may proceed as follows.

We define a matrix \mathcal{C}^0 relative to the cohomology basis $\{f^k\}$ of (4.4)

$$\overline{f^k} = f^j \mathcal{C}^0_j{}^k .$$

As before, we also have the relations

$$e^k = f^j E_j{}^k(\varphi) \quad \text{and} \quad \overline{e^k} = e^j C_j{}^k(\varphi) .$$

We also have

$$\begin{aligned} \overline{e^k} &= \overline{f^j} E_j{}^k(\overline{\varphi}) \\ &= f^i \mathcal{C}^0_i{}^j E_j{}^k(\overline{\varphi}) \\ &= e^\ell E^{-1} \ell^i(\varphi) \mathcal{C}^0_i{}^j E_j{}^k(\overline{\varphi}) . \end{aligned}$$

So

$$C(\varphi) = E^{-1}(\varphi) \mathcal{C}^0 E(\overline{\varphi}) .$$

But it is not asserted that $C(\varphi) \rightarrow \mathcal{C}^0$ as $\varphi \rightarrow 0$.

To compute \mathcal{C}^0 , note that by means of (4.5) we can relate the $\{f^k\}$ -basis to the real $\{\alpha^m\}$ -basis. So we may write

$$\overline{f^k} = \alpha^m \overline{\rho_m^k} = f^j ((\rho^\sharp)^{-1})_j{}^m \overline{\rho_m^k}.$$

So

$$\mathcal{C}^0 = (\rho^\sharp)^{-1} \overline{\rho^\sharp} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{2x}{y}\zeta(3) & 0 & 0 & -1 \end{pmatrix}.$$

Note the consonance with the observed form for the matrix $U(0)$. The diagonal entries show that for the infinite prime one has to replace p by -1 . The extra factor of 2, associated with the lower left corner of the matrix, however, remains currently unexplained.

As for the behaviour of $C(\varphi)$ as φ tends to zero. Note that

$$\mathcal{C}^0 \epsilon = -\epsilon \mathcal{C}^0.$$

Thus

$$C(\varphi) \sim \varphi^{-\epsilon} \mathcal{C}^0 (\overline{\varphi})^\epsilon = \mathcal{C}^0 |\varphi|^{2\epsilon} = \mathcal{C}^0 + \begin{pmatrix} 0 & 0 & 0 & 0 \\ -L & 0 & 0 & 0 \\ \frac{1}{2}L^2 & L & 0 & 0 \\ -\frac{1}{6}L^3 & \frac{1}{2}L^2 & -L & 0 \end{pmatrix},$$

where, in the last expression, $L = \log |\varphi|^2$. We see that, in some sense, \mathcal{C}^0 is the regular part of $C(\varphi)$ at $\varphi = 0$.

Finally, in this subsection, let us write down the matrix h corresponding to the hermitean form. We have

$$h^{jk} = \langle f^j, f^k \rangle = i(f^\ell \mathcal{C}^0 {}_{\ell}{}^j, f^k) = -i\sigma^{k\ell} \mathcal{C}^0 {}_{\ell}{}^j.$$

Hence

$$h = -i(\sigma \mathcal{C}^0)^T = \frac{y}{(2\pi)^3} \begin{pmatrix} \frac{2x}{y}\zeta(3) & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}.$$

We could also have come to this same expression for h by noting that

$$h = i(\rho^\sharp)^\dagger \Sigma \rho^\sharp.$$

5. Computing $U(\varphi)$

We wish to calculate the determinant $R(\varphi, T)$ as a function of the parameter φ . Which amounts to computing the coefficients a and b . We will discuss two methods of evaluating these coefficients, which differ in important ways with regard to their practical application.

5.1. Evaluation of $U(\varphi)$ at Teichmüller points

It is our aim to compute

$$R(\varphi, T) = \det(\mathbb{1} - TU(\varphi))$$

for values $\phi \in \mathbb{F}_p$. We have seen that the matrix can be expressed in the form

$$U(\phi) = E^{-1}(\phi^p)U(0)E(\phi) = \tilde{E}^{-1}(\phi^p)U(0)\tilde{E}(\phi).$$

If we want to evaluate this series at some point $\phi \in \mathbb{F}_p$, we first form the Teichmüller lift $\phi := \text{Teich}(\varphi) \in \mathbb{Z}_p$ and the issue of the convergence of the series $U(\phi)$ at this point becomes relevant. Note that by definition $\phi = \phi^p$, so $\|\phi\|_p = 1$ if $\phi \neq 0$. The periods and therefore the matrix \tilde{E} converge in the disk $\|\phi\|_p < 1$, so it is not permissible to try to substitute the Teichmüller value directly into \tilde{E} . Indeed, if it were permissible to do this, then the matrix $U(0)$ would be merely conjugated by $E(\phi)$ and $R(\phi, T)$ would be independent of ϕ . Instead, we have to expand $U(\varphi)$ as a matrix of p -adic series. These series converge on the larger disk $\|\varphi\|_p \leq 1 + \delta$ for some $\delta > 0$: the series for the U -matrix are said to be *overconvergent*. This important property, discovered by Dwork and proved in great generality by Berthelot, implies that it *is* permissible to substitute the Teichmüller value into these series and that the corresponding limit is correct.

5.2. $U(\varphi)$ as a sequence of rational functions

The matrix $U(\varphi)$ is, as we have observed, a power series in φ and so also are the coefficients a and b . The problem that arises, however, is that, while the series for $U(\varphi)$ converge, they do so only slowly. The aim, when working with these series is therefore to rearrange the terms so as to achieve faster convergence.

Let us fix a p -adic accuracy. We will see in §5.4, that a knowledge of a and $b \pmod{p^3}$ is sufficient to identify these coefficients exactly, for $p \geq 5$. However, we lose a power of p when passing from U to b , owing to the second relation in (5.2), so, to start, we work $\pmod{p^4}$ with U .

We find that, $\pmod{p^4}$, the matrix U is a matrix of rational functions of φ of the form stated in (1.4). This much seems to be true in all cases and already permits us to calculate R for the values of φ for which X is smooth. We have stated already in §5, that in many cases the above relation performs better than advertised owing to cancellations between the numerator and denominator. For the examples that we examine here, we are able to calculate the Frobenius polynomial R for all $\varphi \neq 0, \infty$, apart from apparent singularities. The difficulty that we find with apparent singularities is puzzling. Let us pause to explain the nature of the difficulty.

The form of the expression (1.6) suggests that $U(\varphi)$ has a singularity at an apparent singularity φ_0 . However this cannot be so, since the manifold is smooth at φ_0 . Indeed one can check that the numerator $\widehat{U}(\varphi)$ is $\mathcal{O}(p^4)$ at $\varphi = \varphi_0$. This being so, it is natural to try to evaluate $U(\varphi_0)$ by taking a limit as $\varphi \rightarrow \varphi_0$. In order to discuss this, let us first note that, instead of (1.6), we can write

$$U(\varphi) = \frac{\mathcal{U}^\sharp(\varphi)}{(\varphi^p - \varphi_0^p)^2} + \mathcal{O}(p^4),$$

This form does not extend to higher p-adic accuracy, for which we have to proceed as in §5.6 but it does work mod p^4 for the apparent singularities that we study here. The numerator $\mathcal{U}^\sharp(\varphi)$ is again $\mathcal{O}(p^4)$ when $\varphi = \varphi_0$. The problem, however, is now clear: if we take two derivatives of both the numerator and denominator, to compute a limit as $\varphi \rightarrow \varphi_0$, then a factor of p^2 appears in the denominator and the accuracy with which we can calculate the coefficients is correspondingly reduced. The upshot is that we can only calculate a mod p and b mod 1. This is disappointing, we have no information about b and there are easier ways of calculating a mod p . One such is the formula

$$a = - \sum_{n=0}^{p-1} a_n \varphi^n \pmod{p},$$

which is the reduction mod p of the unit root, which we discuss in the following subsection.

5.3. Calculation of the unit root

It has already been noted that, as a consequence of the Weil conjectures, the eigenvalues λ_i of the matrix U have norm $p^{\frac{3}{2}}$, as complex numbers. Since U is computed as a matrix of p-adic numbers, the meaning of this statement is perhaps not immediately obvious. The polynomial $R(\varphi, T)$ however is the characteristic polynomial of U and this is a polynomial in T with coefficients that are rational integers and the eigenvalues λ_i are the (inverse) roots of this polynomial, so are complex numbers. We may also solve for the inverse roots of R as p-adic numbers and we assume that the four roots are $\mathcal{O}(1), \mathcal{O}(p), \mathcal{O}(p^2), \mathcal{O}(p^3)$ as p-adic numbers.

We may label the eigenvalues so that λ_1 is the eigenvalue that is $\mathcal{O}(1)$. Dwork observes that this eigenvalue may be computed as the limit, as φ tends to a Teichmüller value, of

$$\lambda_1 = \frac{\varpi_0(\varphi)}{\varpi_0(\varphi^p)},$$

where ϖ_0 is the fundamental period defined in (2.2). Again, the series expansion of the period ϖ_0 has radius of convergence $\|\varphi\|_p < 1$ but the ratio of periods has a radius of convergence $1+\delta$ for some $\delta > 0$.

The unit eigenvalue may also be computed, more conveniently, as the limit

$$\lambda_1 = \lim_{s \rightarrow \infty} \frac{(s)\varpi_0(\varphi)}{(s-1)\varpi_0(\varphi)} \quad \text{where} \quad {}^{(r)}\varpi_0(\varphi) = \sum_{n=0}^{p^r-1} a_n \varphi^n.$$

In fact we observe, for the examples that we study here, that

$$\frac{(s)\varpi_0(\varphi)}{(s-1)\varpi_0(\varphi)} = \lambda_1 + \mathcal{O}(p^s) .$$

If R is irreducible over the integers then a calculation of λ_1 for $s = 6$ is likely to give R unambiguously. Given λ_1 , it is straightforward to construct a table of possible values $R(a, b, \lambda_1)$ for values of a and b corresponding to the allowed region of Figure 5 and there are approximately $\frac{32}{3}p^{\frac{7}{2}}$ such values. We then find (a, b) as the pair of integers for which $R(a, b, \lambda_1)$ vanishes to highest p-adic order.

This procedure works well for small values of p , say for $p \leq 19$, but, since the number of a_n coefficients that need to be calculated is p^6 , becomes rapidly impractical for larger values of p . We can however use this method to calculate R for $p = 2, 3, 5$, for which our previous method does not apply. We can also use it as a check, for values of p for which both methods are practical, and to fill in values for apparent singularities, for which our primary method fails.

On the other hand, the unit eigenvalue method has limitations for those cases that R factorises. For example, at conifold points we observe a factorisation of the form

$$R = (1 - p\chi T)(1 - \beta T + p^3 T^2) , \quad (5.1)$$

where $\chi = \pm 1$ is a character. In these cases we are led to a root of the quadratic factor, and so find a value for β , but we have no information about the character χ . Similarly when R factors in to a product of two quadrics

$$R = (1 - p\alpha T + p^3 T^2)(1 - \beta T + p^3 T^2) ,$$

and we are particularly interested in factorisations of this form in relation to attractor points, then the method provides us with a root of the second factor, but no information about the coefficient α of the first factor.

5.4. The form of $R(\varphi, T)$ and bounds on the coefficients a, b

The considerations of this subsection are, in part, adapted from the thesis of Kira Samol [23].

We have already observed that R has the form

$$R = \det(1 - TU) = 1 + aT + bpT + ap^3T^3 + p^6T^4 ,$$

where the third expression holds when the manifold is smooth. The coefficients a and b are most easily calculated as

$$a = -\text{Tr}(U) ; \quad b = \frac{1}{2p} \left((\text{Tr } U)^2 - \text{Tr}(U^2) \right) . \quad (5.2)$$

Denoting the eigenvalues of U by λ_i , $i = 1, \dots, 4$ we have

$$a = -\sum_i \lambda_i \quad \text{and} \quad bp = \sum_{i < j} \lambda_i \lambda_j ,$$

It follows from the Weil conjectures that, as complex numbers, $|\lambda_i| = p^{3/2}$. These eigenvalues are the inverse roots of R , so come in complex conjugate pairs. If we write $p^{3/2}e^{\pm i\theta_1}$ and $p^{3/2}e^{\pm i\theta_2}$ for the four eigenvalues, and also set $a = p^{3/2}\tilde{a}$ and $b = p^2\tilde{b}$, we have

$$\tilde{a} = -2(\cos \theta_1 + \cos \theta_2); \quad \tilde{b} = 2 + 4 \cos \theta_1 \cos \theta_2.$$

From the first relation we have immediately that

$$|\tilde{a}| \leq 4 \quad \text{and so} \quad |a| \leq 4p^{3/2}. \quad (5.3)$$

On eliminating $\cos \theta_2$, the second relation becomes

$$\tilde{b} = 2 - 4 \cos \theta_1 \left(\cos \theta_1 + \frac{\tilde{a}}{2} \right).$$

An upper bound is obtained by maximising the right hand side, while lower bounds are obtained from the limiting values $\cos \theta_1 = \pm 1$. In this way, we find

$$-2 + 2|\tilde{a}| \leq \tilde{b} \leq 2 + \frac{\tilde{a}^2}{4}.$$

This region, which we denote by $\tilde{\mathcal{S}}$, is sketched, on the right, in Figure 5.

Let \tilde{b}_{mid} be the midpoint of the maximum and minimum values of \tilde{b} , for given \tilde{a} :

$$\tilde{b}_{\text{mid}} = 2 \left(\left(1 + \frac{|\tilde{a}|}{4} \right)^2 - 1 \right).$$

Then

$$\left| \tilde{b} - \tilde{b}_{\text{mid}} \right| \leq 2 \left(1 - \frac{|\tilde{a}|}{4} \right)^2 \leq 2$$

and, given a , we have

$$|b - b_{\text{mid}}| \leq 2p^2 + 1, \quad \text{where} \quad b_{\text{mid}} = \left[2p^2 \left(\left(1 + \frac{a}{4p^{3/2}} \right)^2 - 1 \right) \right]$$

and, in this last relation [...] denotes the nearest integer function. In virtue of the relations above, we are able to identify a and b from a knowledge of their values mod p^3 , for $p \geq 5$.

For values of φ for which the manifold is singular, we expect one of the roots of the quartic to go to zero and for the resulting cubic to factor in the form

$$R = (1 - \chi pT)(1 - \beta T + p^3 T^2) \quad (5.4)$$

where $\chi = \pm 1$ is a character. Given this form, the quantities χ and α may be reconstructed from a knowledge of the linear and quadratic terms, as a polynomial in T . The roots of the quadratic factor have complex norm $p^{3/2}$, and $|\beta| \leq 2p^{3/2}$ so a calculation mod p^3 serves in these cases also.

There is a natural way to place coordinates on the region $\tilde{\mathcal{S}}$. Consider the factorisation

$$1 + aT + bpT^2 + ap^3T^3 + p^6T^4 = (1 - \alpha pT + p^3T^2)(1 - \beta T + p^3T^2) .$$

We are most interested in the cases that the coefficients a, b, α, β are integers, but let us temporarily take them to be merely real. Over \mathbb{R} , factorisation, as above, is always possible and we have the relations

$$a = -(p\alpha + \beta) , \quad b = 2p^2 + \alpha\beta , \quad (5.5)$$

which we rewrite as

$$\tilde{a} = -(\tilde{\alpha} + \tilde{\beta}) , \quad \tilde{b} = 2 + \tilde{\alpha}\tilde{\beta} , \quad (5.6)$$

with

$$\tilde{\alpha} = \frac{\alpha}{p^{1/2}} = -2 \cos \theta_1 , \quad \tilde{\beta} = \frac{\beta}{p^{3/2}} = -2 \cos \theta_2 .$$

and we see that $\tilde{\alpha}$ and $\tilde{\beta}$ are simply related to the parameters θ_1 and θ_2 introduced previously. Owing to the symmetry of (5.6) under interchange of $\tilde{\alpha}$ and $\tilde{\beta}$, the map to $\tilde{\mathcal{S}}$ is generically 2–1 with $(\tilde{\alpha}, \tilde{\beta})$ and $(\tilde{\beta}, \tilde{\alpha})$ mapping to the same point. We can divide the square into two triangles by the diagonal $\tilde{\alpha} = \tilde{\beta}$; and choose the lower triangle, say, to be a fundamental region for parametrising the points (\tilde{a}, \tilde{b}) . We denote this fundamental region by $\tilde{\mathcal{S}}_0$, this region is sketched on the left in Figure 5.

In Figure 6 we indicate the distribution of the pairs (a, b) as they map to $\tilde{\mathcal{S}}$ and to $\tilde{\mathcal{S}}_0$. A histogram corresponding to the 1000 primes $p_3 \leq p \leq p_{1002}$, and all values of φ for which the manifold is smooth, is presented for the manifold AESZ34 in Figure 7. The histogram is remarkably close to the probability distribution function $f d\theta_1 d\theta_2$ with f given by

$$f = \frac{16}{\pi^2} \sin^2 \theta_1 \sin^2 \theta_2 (\cos \theta_1 - \cos \theta_2)^2 .$$

This probability distribution function corresponds to the eigenvalue distribution of $\mathrm{USp}(4)$ matrices that are distributed randomly with respect to the Haar measure. We have conjectured [3] that this is the correct distribution function for AESZ34 and we conjecture that this is also the correct distribution function for all one-parameter examples.

5.5. Slopes

The polynomials $\tilde{\mathcal{U}}$ and $\widehat{\mathcal{U}}$ from (1.5) and (1.6) show interesting regularities with respect to degree and which admit of some refinement. Considering first our examples that have no apparent singularities, let us write

$$U(\varphi) = \tilde{\mathcal{U}}_j(\varphi) + \mathcal{O}(p^j) ; \quad j = 1, 2, 3, 4 .$$

We term $\deg \tilde{\mathcal{U}}_4$ the *slope* of the corresponding manifold X and, more generally, refer to the vector $\deg \tilde{\mathcal{U}}_j$, $j = 1, \dots, 4$ as the *slope vector*. We give these slope vectors in Table 1.

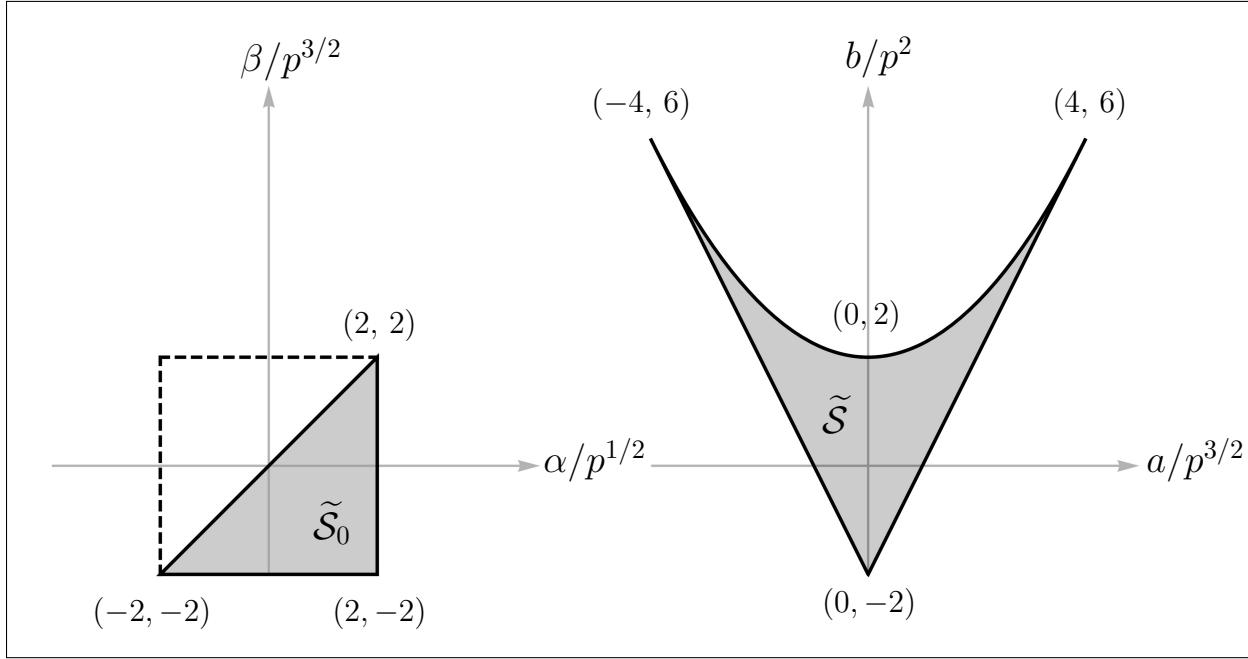


Figure 5: The allowed region $\tilde{\mathcal{S}}$ for the coefficients (a, b) is shown on the right and the fundamental region $\tilde{\mathcal{S}}_0$ for the parameters (α, β) , defined by relations (5.6), is shown on the left.

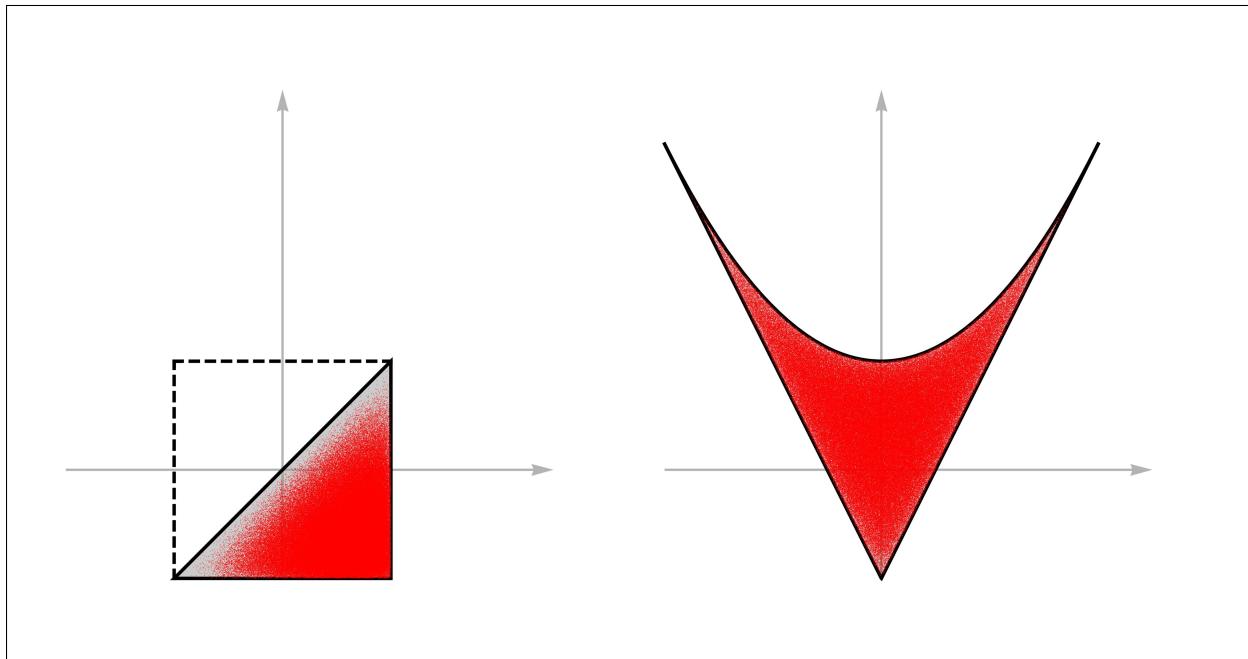


Figure 6: On the right is shown a plot of the pairs (a, b) for the 75 primes $p_{427} \leq p \leq p_{502}$ and all values of φ for which the manifold AESZ34 is smooth. On the left, these points are mapped back onto the fundamental region. Note how the density decreases near the diagonal boundary.

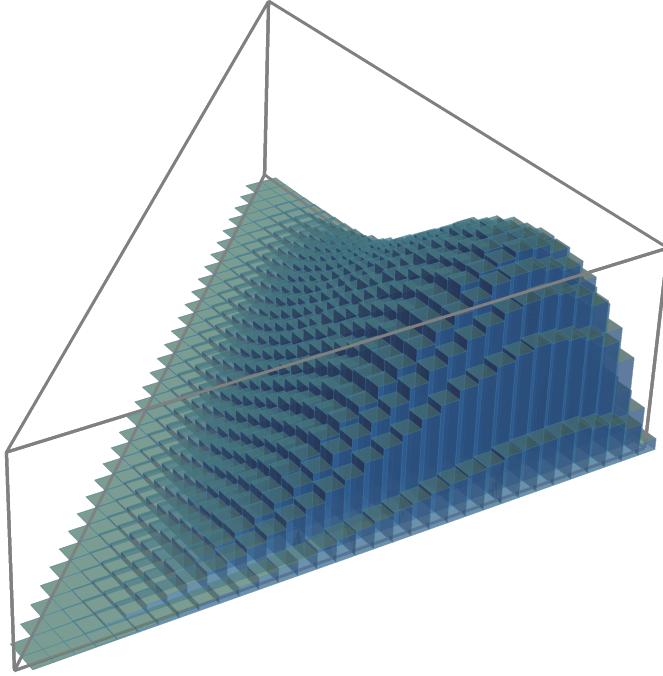


Figure 7: A histogram showing the density of pairs (a, b) , mapped back to the region \tilde{S}_0 . The data shown is for the manifold AESZ34 and all values of φ for which the manifold is smooth, for the 1000 primes $p_3 \leq p \leq p_{1002}$.

Let us take the three examples without apparent singularities first. For these we make the observation that the coefficients of p in the slope vectors coincide, in each case, with the indices of the corresponding differential operators at infinity. For the three examples with apparent singularities the corresponding statement is that the coefficients of p differ from the corresponding index at infinity by 2. The difference of 2 accounts for the denominator that appears in (1.6).

While the slope vectors that appear for the examples without apparent singularities are exact, for the range of p for which we have data. We have to make caveats for the examples with apparent singularities. For the Rødland manifold the stated expressions for the slope vectors are exact (always for the range of primes for which we have data). For the three generation manifold with Hodge numbers $(4, 1)$, the first component of the slope vector is $3p - 1$, apart from for $p = 11$, for which it is $3p - 2$. The second component is $4p - 1$, apart from $p = 139$, for which it is $4p - 2$, while the remaining components are exact. For the manifold with Hodge numbers $(1, 1)$, the first three components of the slope vector are exact, while the fourth component is $5p - 1$, apart from for $p = 11$, for which it is $5p - 2$. These caveats do not seem to change the relation between the slope vectors and the indices at infinity.

Manifold	$j = 1$	$j = 2$	$j = 3$	$j = 4$	Indices at ∞
Quintic	$\left[\frac{p}{5} - \frac{1}{2} \right]$	$\left[\frac{2p}{5} - \frac{1}{2} \right]$	$\left[\frac{3p}{5} - \frac{1}{2} \right]$	$\left[\frac{4p}{5} - \frac{1}{2} \right]$	$\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}$
AESZ25	$\left[\frac{p}{2} - 1 \right]$	$\left[\frac{p}{2} - 1 \right]$	$\left[\frac{3p}{2} - 1 \right]$	$\left[\frac{3p}{2} - 1 \right]$	$\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{3}{2}$
AESZ34	$p - \frac{3}{2} + \frac{1}{2} \left(\frac{p}{15} \right)$	$p - 1$	$2p - \frac{3}{2} - \frac{1}{2} \left(\frac{p}{15} \right)$	$2p - 2$	1, 1, 2, 2
Rødland	$3p - 1$	$3p - 1$	$3p - 1$	$3p - 1$	1, 1, 1, 1
(4,1) Mfld.	$3p - 1$	$4p - 1$	$5p - 1$	$6p - 1$	1, 2, 3, 4
(1,1) Mfld.	$3p - 1$	$4p - 2$	$4p - 1$	$5p - 1$	1, 2, 2, 3

Table 1: *The slopes for our six examples. The first three have no apparent singularities, while the remaining three do have them. Square brackets denote the round function, with the convention that, for $n \in \mathbb{Z}$, $[n + \frac{1}{2}]$ rounds to the nearest even integer. In the third row ($\frac{p}{15}$) denotes the Jacobi symbol. The last column gives the indices at infinity, these are taken from the corresponding Riemann symbols.*

5.6. Evaluating U to higher p -adic order

We have seen that for the purpose of evaluating the numerator of the ζ -function, for $p > 5$, it is sufficient to work mod p^4 . We ask now how the calculation of $U(\varphi)$ changes if we work mod p^n , for $n > 4$. We have looked only at the case of the quintic, in this regard, and then only for $p = 7$ and $p = 3$. We want to examine exponents n that are in the vicinity of powers p^k for a few values of k , and this is only currently practicable for small p . Let us start with $p = 7$, for which we examine powers $n \leq 360$, that is $n \leq p^3 + 17$. The relation that we find, in this case, is

$$U(\varphi) = \frac{\tilde{\mathcal{U}}_n(\varphi)}{\Delta(\varphi)^{p(n-4+\epsilon_n)}} + \mathcal{O}(p^n),$$

where

$$\epsilon_n = \alpha_{n_0} + \beta_{n_0} \delta_{n_1,6} + \gamma_{n_0} \delta_{n_1,6} \delta_{n_2,6}$$

where the n_j are the first three p-adic digits of $n - 5$ and α , β and γ are the following related vectors

$$\begin{aligned}\alpha &= (0, 0, 1, 1, 1, 0) \\ \beta &= (0, 2, 1, 1, 1, 0, 0) \\ \gamma &= (3, 1, 1, 1, 0, 0, 0).\end{aligned}$$

As the degree of the denominator increases, the degree of the numerator $\tilde{\mathcal{U}}_n(\varphi)$ also increases and, for the range of n that we examine, the degree of the numerator differs from the degree of the denominator by a constant independent of n .

We can go to powers n in the vicinity of higher powers p^k for $p = 3$. For this prime the form that U takes is

$$U(\varphi) = \frac{\tilde{\mathcal{U}}_n(\varphi)}{\Delta(\varphi)^{p(n-3+\epsilon_n)}} + \mathcal{O}(p^n).$$

Note the power of Δ in the denominator. One consequence is that there are already inverse powers of Δ if we work mod p^4 .

The quantities ϵ_n naturally form a hypercubical array, as is evident from the following 3^5 values for the range $3 \leq n \leq 247$, which should be read by proceeding in the natural order within each matrix.

$$\left\{ \begin{array}{l} \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right] \\ \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 0\ 1) \\ (0\ 1\ 1) & (0\ 1\ 1) & (2\ 2\ 2) \\ (1\ 1\ 1) & (1\ 1\ 1) & (2\ 2\ 1) \end{array} \right] \\ \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 2\ 3) \\ (0\ 1\ 1) & (0\ 1\ 1) & (3\ 3\ 3) \\ (1\ 1\ 1) & (1\ 1\ 1) & (3\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (0\ 2\ 3) \\ (0\ 1\ 1) & (0\ 1\ 1) & (3\ 3\ 3) \\ (1\ 1\ 1) & (1\ 1\ 1) & (3\ 2\ 1) \end{array} \right], \quad \left[\begin{array}{ccc} (0\ 0\ 0) & (0\ 0\ 0) & (3\ 4\ 4) \\ (0\ 1\ 1) & (0\ 1\ 1) & (4\ 4\ 4) \\ (1\ 1\ 1) & (1\ 1\ 1) & (3\ 2\ 1) \end{array} \right] \end{array} \right\}$$

The lines of a small matrix fit together to form a square, and three squares form a cube. The matrices bounded by square brackets are four-cubes, and three of these form a five-cube. Notice that the first two areas are the same while the third is different, and the first two cubes are the same while the third is different. This structure repeats: the first two four-cubes are the same, while the third is different, and the differences reside in the final cube of each four-cube. We do not however have a good formula to describe this.

6. The Manifolds

We describe here in telegraphic form the manifolds that are our examples and give references to more complete descriptions.

6.1. The mirror of the quintic threefold, AESZ 1

The quintic threefold and its mirror must be the most studied of all Calabi-Yau manifolds and the following analysis can be found in many references, however we take the opportunity to use this case to describe a procedure that can be applied to all our examples. This proceeds as follows:

- Given a presentation of the manifold by projective or toric polynomial, calculate a sufficient number of the coefficients, say 50, of the fundamental period, so as to be able to write

$$\varpi_0(\varphi) = \sum_{n=0}^N a_n \varphi^n + \mathcal{O}(\varphi^{N+1})$$

with known coefficients, for some sufficiently large N .

- Choose a degree k_{\max} for the coefficient polynomials S_j and set

$$S_j = \sum_{k=0}^{k_{\max}} s_{jk} \varphi^k$$

then use the equation $\mathcal{L}\varpi_0 = 0$ to solve for the s_{jk} . If k_{\max} is chosen too small, then the only solution is $s_{jk} = 0$. So we increase k_{\max} until a non-trivial solution appears. In order to have sufficient conditions to solve for the s_{jk} we have to take the “sufficient number” N of known coefficients a_n to satisfy $N > 5k_{\max}$.

- Once the Picard-Fuchs operator \mathcal{L} has been found, it is a simple matter to seek a Frobenius solution

$$\varpi(\varphi, \epsilon) = \sum_{n=0}^{\infty} A_n(\epsilon) \varphi^{n+\epsilon} \quad \text{with} \quad A_0(\epsilon) = 1 ,$$

and find a recurrence relation for the coefficients $A_n(\epsilon)$. On setting

$$A_n(\epsilon) = a_n + b_n \epsilon + \frac{1}{2!} c_n \epsilon^2 + \frac{1}{3!} d_n \epsilon^3 + \mathcal{O}(\epsilon^4)$$

and expanding the recurrence relation we find recurrence relations for the coefficients a_n, b_n, c_n, d_n of the series f_0, f_1, f_2, f_3 . Given these functions, we construct the matrix E and the inverse Frobenius matrix U .

The mirror quintic is realised by taking a quotient of the hypersurface in \mathbb{P}^4

$$\sum_{i=1}^5 X_i^5 - 5\psi \prod_{i=1}^5 X_i = 0$$

by the automorphism $x_i \rightarrow \alpha_i^n X_i$, with α a nontrivial fifth root of unity and $\sum_i n_i = 0 \pmod{5}$.

By a series of standard manoeuvres the fundamental period can be represented by the integral

$$\varpi_0 = \frac{1}{(2\pi i)^5} \int_{\gamma} \frac{d^5 X}{\prod_{i=1}^5 X_i} \left(1 - \frac{\sum_{i=1}^5 X_i^5}{5\psi \prod_{i=1}^5 X_i} \right)^{-1}, \quad (6.1)$$

where now the variables X_i are viewed as taking values in \mathbb{C}^5 , the contour γ is a product of five small circles enclosing the loci $X_i = 0$, and ψ is taken sufficiently large. The integral is evaluated by expanding the bracket in inverse powers of ψ and evaluating term by term by residues. The only terms that contribute are the terms independent of X in the powers

$$\left(\frac{\sum_{i=1}^5 X_i^5}{5\psi \prod_{i=1}^5 X_i} \right)^m.$$

Such a term is zero unless $m = 5n$ and in that case is $(5n)!/(n!)^5$. So, for the mirror quintic,

$$\varpi_0(\varphi) = \sum_{n=0}^{\infty} \frac{(5n)!}{(n!)^5} \varphi^n \quad \text{where we have taken } \varphi = \frac{1}{(5\psi)^5}.$$

In this case, we have been able to evaluate the coefficients of the fundamental period at one blow. In more difficult cases we will often not be able to find a closed form expression for these coefficients. However, it will always be the case that there is a Laurent polynomial $F(X)$ in variables X_i such that

$$a_n = [F(X)^n]_0,$$

where $[\dots]_0$ denotes the operation of taking the part independent of the coordinates X_i , and we will need to evaluate the a_n , for sufficiently many n , in order to get started.

Returning to the mirror quintic, we see from (6.1) that the fundamental period satisfies the differential equation $\mathcal{L}\varpi_0 = 0$, with

$$\mathcal{L} = \vartheta^4 - 5^5 \varphi \left(\vartheta + \frac{1}{5} \right) \left(\vartheta + \frac{2}{5} \right) \left(\vartheta + \frac{3}{5} \right) \left(\vartheta + \frac{4}{5} \right).$$

This operator can, of course, be written in the form (1.3). The coefficient polynomials S_j are then linear in φ .

$$\mathcal{L} = (1 - 3125 \varphi) \vartheta^4 - 6250 \varphi \vartheta^3 - 4375 \varphi \vartheta^2 - 1250 \varphi \vartheta - 120 \varphi.$$

This operator corresponds to the Riemann symbol

$$\mathcal{P} \left\{ \begin{array}{ccc} 0 & 5^{-5} & \infty \\ 0 & 0 & \frac{1}{5} \\ 0 & 1 & \frac{2}{5} \\ 0 & 1 & \frac{3}{5} \\ 0 & 2 & \frac{4}{5} \end{array} \mid \varphi \right\}.$$

If we now seek a Frobenius solution of the form (2.1) we find a recurrence relation for the coefficients $A_n(\epsilon)$ of the form

$$(n + \epsilon)^4 A_n(\epsilon) - 5^5 \left(n + \epsilon - \frac{1}{5}\right) \left(n + \epsilon - \frac{2}{5}\right) \left(n + \epsilon - \frac{3}{5}\right) \left(n + \epsilon - \frac{4}{5}\right) A_{n-1}(\epsilon) = 0 .$$

In this case, we have a two term recurrence relation which we could solve directly in terms of Γ -functions. Instead we will follow the procedure and expand this last relation in powers of ϵ . This yields the following relations

$$\begin{aligned} n^4 a_n &= 5(5n-1)(5n-2)(5n-3)(5n-4)a_{n-1} \\ n^4 b_n &= -4n^3 a_n + 1250(2n-1)(5n^2-5n+1)a_{n-1} \\ &\quad + 5(5n-1)(5n-2)(5n-3)(5n-4)b_{n-1} \\ n^4 c_n &= -12n^2 a_n + 1250(30n^2-30n+7)a_{n-1} \\ &\quad - 8n^3 b_n + 2500(2n-1)(5n^2-5n+1)b_{n-1} \\ &\quad + 5(5n-1)(5n-2)(5n-3)(5n-4)c_{n-1} \\ n^4 d_n &= -24na_n + 37500(2n-1)a_{n-1} \\ &\quad - 36n^2 b_n + 3750(30n^2-30n+7)b_{n-1} \\ &\quad - 12n^3 c_n + 3750(2n-1)(5n^2-5n+1)c_{n-1} \\ &\quad + 5(5n-1)(5n-2)(5n-3)(5n-4)d_{n-1} , \end{aligned}$$

subject to the initial conditions implied by $A_0(\epsilon) = 1$, so $a_0 = 1$ and $b_0 = c_0 = d_0 = 0$.

For uniformity of presentation we give the Hodge numbers

$$h^{pq} = \begin{matrix} & & 1 & & \\ & & 0 & & 0 \\ & 0 & & 101 & 0 \\ 1 & & 1 & & 1 & & 1 \\ & 0 & & 101 & 0 & \\ & & 0 & & 0 & \\ & & & & & 1 \end{matrix} .$$

For the quintic, so the manifold with $h^{11} = 1$, we have the numerical invariants

$$\chi = -200 , \quad c_2 H = 50 , \quad H^3 = 5 .$$

The figure showing the numbers of factorisations into two quadrics appears as Figure 2 of §1.

The discriminant for this manifold is simply $\Delta = v - 5^5 u$ and the hyperdiscriminant is

$$\Delta = 1 .$$

The only bad prime is the notorious prime $p = 5$, for which, as discussed in §2.2, the manifold is singular mod 5, for all values of φ .

6.2. The mirror of a Calabi-Yau hypersurface in $G(2,5)$, AESZ 25

In [33] Batyrev, Ciocan-Fontanine, Kim and one of the present authors studied Calabi-Yau complete intersections in Grassmannians and their mirror manifolds. One of the cases is the manifold X with $h^{11} = 1, h^{21} = 61$, realised as the intersection of hypersurfaces of degree $(1, 2, 2)$ in the six dimensional Grassmannian $G(2, 5)$, in its Plücker embedding. Its characteristic numbers are:

$$H^3 = 20, \quad c_2 H = 68, \quad \chi = -120.$$

The mirror manifold \tilde{X} can be described in terms of a toric Laurent polynomial

$$F_0 = 1 - \varphi f g,$$

with

$$\begin{aligned} f(Y) &= \frac{(1+Y_1)^2(1+Y_2)^2}{Y_1 Y_2}, \\ g(Y) &= \left(1 + \frac{1}{Y_3}\right) \left(1 + \frac{1}{Y_4}\right) (1 + Y_3 + Y_4). \end{aligned}$$

The Newton polyhedron of F_0 is reflexive and has twenty vertices and nine facets. Five of the facets are hexahedra and four are prisms over a pentagonal base. The way in which these facets meet is sketched in Figure 8. The hexahedra are first stacked to form a column, then the top and bottom faces of the column are identified to form a ring. The five pentagonal prisms are also stacked and the top and bottom faces identified to form a second ring. The two rings are linked, as in the sub-figure on the right. The figure is deceptive insofar as it suggests that the Newton Polyhedron has a symmetry generated by rotating each of the two rings. There are, however, no matrices that act on the points of the Newton polyhedron that correspond to these operations.

The facets of the dual polyhedron are sketched in Figure 9. There are twenty facets and nine vertices. The twenty facets are all tetrahedra. One way to think of how these facets are arranged is to first join the tetrahedra in fives so as to form diamonds, as in the subfigure on the left, and then to join the four diamonds into a cluster, as on the right of the figure. The apparently exposed faces are then all identified in pairs.

The variety corresponding to F_0 is singular with 6 nodes. A subdivision of the quadrilateral two-faces corresponds to a conifold resolution of these nodes and yields a manifold X^* with the following Hodge numbers. Thus $\chi(X^*) = 2(h^{11} - h^{21}) = 120$.

$$h^{pq}(X^*) = \begin{matrix} & & 1 & & \\ & & 0 & & 0 \\ & 0 & & 61 & & 0 \\ & 1 & 1 & 1 & 1 & \\ 0 & & 61 & & 0 \\ & & 0 & & 0 \\ & & & & 1 \end{matrix}.$$

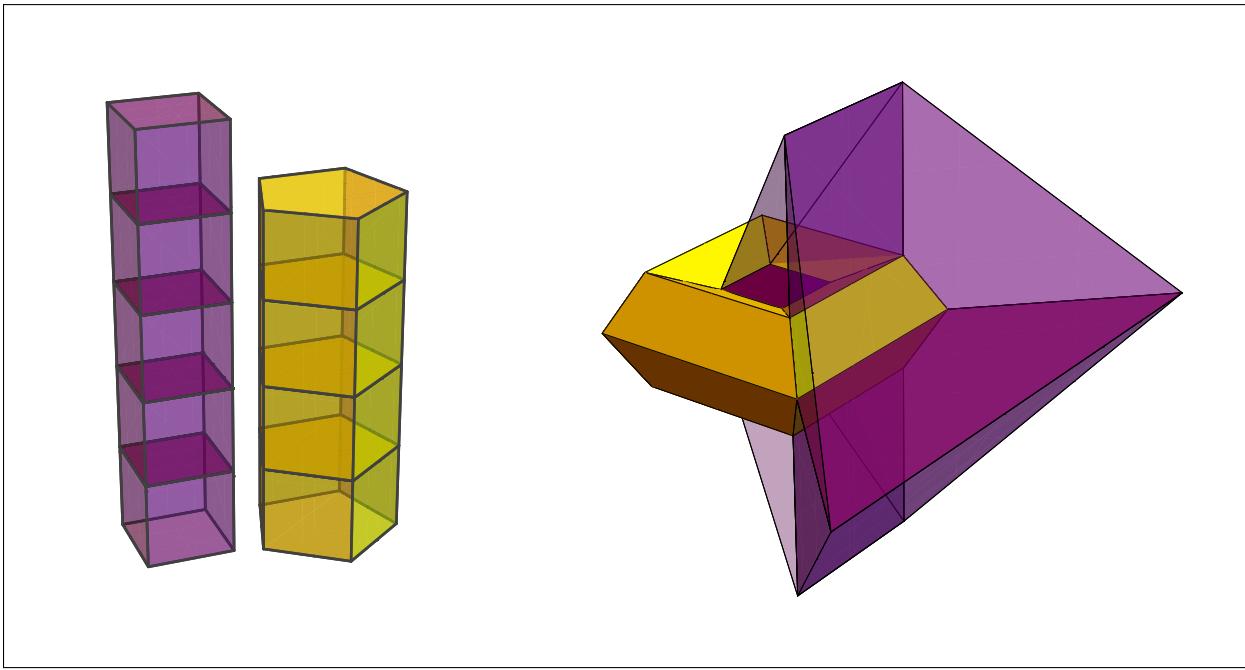


Figure 8: *A sketch indicating how the facets of the toric polyhedron for X fit together. There are five facets that are hexahedra and four that are prisms of pentagonal section.*

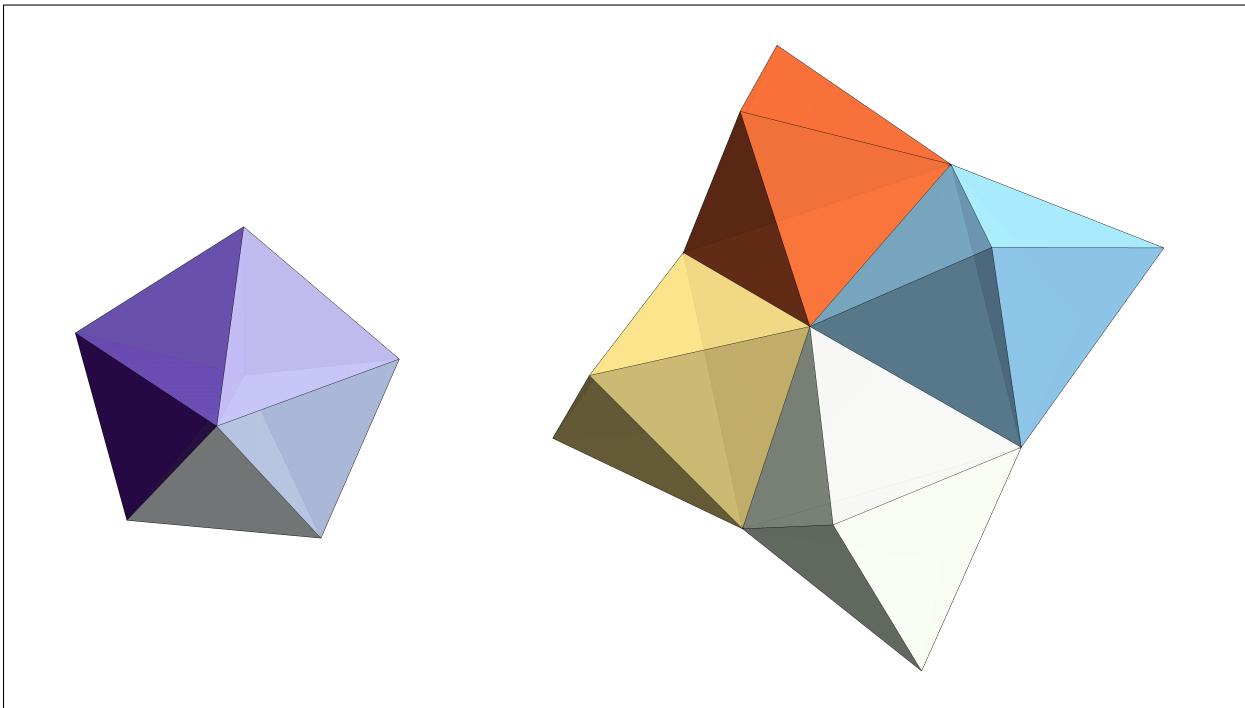


Figure 9: *A sketch indicating how the facets of the dual polyhedron for X fit together. There are now twenty facets that are all tetrahedra. These fit together in fives to form diamonds, as in the figure on the left, and four diamonds fit together as indicated in the figure on the right. Apparently exposed faces are identified, so that, in reality, there is no boundary.*

The coefficients for the fundamental period, for this case, admit an interesting closed form

$$a_n = \binom{2n}{n}^2 \sum_{k=0}^n \binom{n}{k}^2 \binom{n+k}{n} .$$

which equals the famous Apéry numbers (for $\zeta(2)$), multiplied by the square of the central binomial coefficients. They satisfy the following three term recurrence relation

$$n^4 a_n = 4(2n-1)^2(11n^2 - 11n + 3) a_{n-1} + 16(2n-1)^2(2n-3)^2 a_{n-2} .$$

The corresponding Picard-Fuchs operator

$$\mathcal{L} = \vartheta^4 - 4\varphi(2\vartheta+1)^2(11\vartheta^2 + 11\vartheta + 3) - 16\varphi^2(2\vartheta+1)^2(2\vartheta+3)^2 .$$

was listed as number 25 in [34]. Its Riemann symbol is

$$\mathcal{P} \left\{ \begin{array}{cccc} 0 & \varphi_+ & \varphi_- & \infty \\ \hline 0 & 0 & 0 & \frac{1}{2} \\ 0 & 1 & 1 & \frac{1}{2} \\ 0 & 1 & 1 & \frac{3}{2} \\ 0 & 2 & 2 & \frac{3}{2} \end{array} \right| \varphi \right\}$$

The coefficient functions of the Picard-Fuchs operator are quadrics and $\varphi_{\pm} = \frac{1}{32}(-11 \pm 5\sqrt{5})$ are the roots of S_4 . These exist in \mathbb{F}_p only if 5 is a square mod p, so when $p = \pm 1 \pmod{5}$.

The coefficient functions of the Picard-Fuchs operator are

$$\begin{aligned} S_4 &= 256\varphi^2 + 176\varphi - 1 . \\ S_3 &= 32\varphi(32\varphi + 11) \\ S_2 &= 4\varphi(352\varphi + 67) \\ S_1 &= 4\varphi(192\varphi + 23) \\ S_0 &= 12\varphi(12\varphi + 1) . \end{aligned}$$

The discriminant of the manifold is given by the homogenisation of S_4 , above, after setting $\varphi = u/v$. The hyperdiscriminant is

$$\Delta = 2^8 5^3 v^2$$

so the bad primes are 2 and 5. If we reduce $\Delta \pmod{2}$ we find

$$\Delta = v^2 \pmod{2}$$

which has a repeated zero when $\varphi = \infty$. If we reduce $\Delta \pmod{5}$ we have

$$\Delta = (u - 2v)^2 \pmod{5} ,$$

which has a repeated root when $\varphi = 2$.

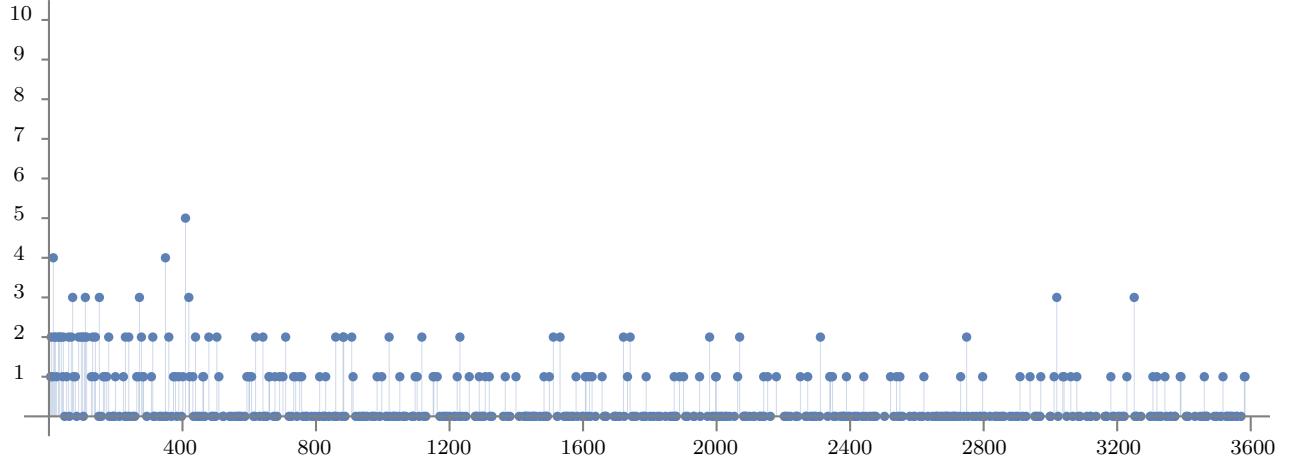


Figure 10: The plot shows the number of factorisations into two quadrics as φ varies over each \mathbb{F}_p , $7 \leq p \leq 3583$, for the mirror of the hypersurface in $G(2, 5)$.

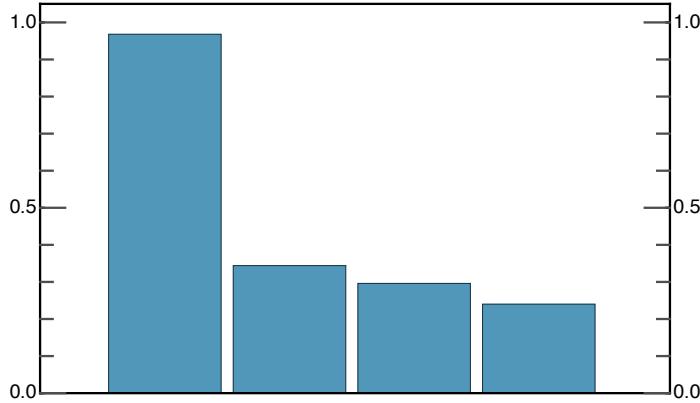


Figure 11: Running averages for data taken from Figure 10. The averages are taken for bins of 125 primes.

The manifold has conifold singularities when the discriminant S_4 vanishes. These roots are

$$\varphi = -\frac{11}{32} \pm \frac{5\sqrt{5}}{32}$$

and exist in \mathbb{F}_p only when 5 is a square mod p , so, by quadratic reciprocity, when p is a square mod 5, which is when $p \equiv 1$ or $4 \pmod{5}$. For these cases, the Frobenius polynomial factorises in the form (5.1) and we record the data corresponding to this factorization in Table 2. The Table records the values of the character χ and the coefficients β_{\pm} , corresponding to the roots φ_{\pm} of the discriminant. While β_+ corresponds to φ_+ and β_- to φ_- , the labels \pm , however, have no intrinsic meaning and are assigned simply on the grounds that $\varphi_- < \varphi_+$, considered as integers. It was established by A. Thorne [2] that the β_{\pm} coefficients correspond to a [4,4] Hilbert modular form of weight [256,16,16].

We are grateful to J. Voight for pointing out to us the following simplification. The coefficients β_{\pm} agree up to sign, and for $p = 1$ or $9 \pmod{20}$ they have the same sign, so $\beta_+ = \beta_-$. Moreover, for $p = 1$ or $4 \pmod{5}$, the β_{\pm} agree up to sign with the corresponding coefficients of the weight 4 and level 20 classical modular form with LMFDB denomination **20.4.c.a**. The coefficients in this expansion involve the quantity $\gamma = 2\sqrt{-19}$.

$$\begin{aligned} f_{\mathbf{20.4.c.a}} = & q - \gamma q^3 + (7 + \gamma)q^5 + \gamma q^7 - 49q^9 + 20q^{11} + 6\gamma q^{13} + (76 - 7\gamma)q^{15} - 8\gamma q^{17} - 84q^{19} + 76q^{21} - \\ & 7\gamma q^{23} - (27 - 14\gamma)q^{25} + 22\gamma q^{27} + \mathbf{6}q^{29} - 224q^{31} - 20\gamma q^{33} - (76 - 7\gamma)q^{35} - 14\gamma q^{37} + \\ & 456q^{39} + \mathbf{266}q^{41} + 35\gamma q^{43} - 49(7 + \gamma)q^{45} - 43\gamma q^{47} + 267q^{49} - 608q^{51} + 42\gamma q^{53} + \\ & 20(7 + \gamma)q^{55} + 84\gamma q^{57} - 28q^{59} + \mathbf{182}q^{61} - 49\gamma q^{63} - 6(76 - 7\gamma)q^{65} - 49\gamma q^{67} - 532q^{69} + \\ & 408q^{71} - 124\gamma q^{73} + (1064 + 27\gamma)q^{75} + 20\gamma q^{77} + 48q^{79} + 349q^{81} + 23\gamma q^{83} + 8(76 - 7\gamma)q^{85} - \\ & 6\gamma q^{87} - \mathbf{1526}q^{89} - 456q^{91} + 224\gamma q^{93} - 84(7 + \gamma)q^{95} - 64\gamma q^{97} - 980q^{99} + \mathbf{1246}q^{101} + \\ & 97\gamma q^{103} + 76(7 + \gamma)q^{105} + 147\gamma q^{107} + \mathbf{902}q^{109} + \mathcal{O}(q^{111}) . \end{aligned}$$

Notice that the coefficients of q^p , for $p = 29, 41, 61, 89, 101, 109$ do indeed agree, up to sign, with the corresponding coefficients of Table 2. Since, for $p = 1$ or $9 \pmod{20}$, we have $\beta_+ = \beta_-$, there is a well defined sign, which we denote by $\chi_{f,g}$, that relates the sign of the coefficient of q^p in the modular form $f_{\mathbf{20.4.c.a}}$ to that of the Hilbert modular form g of the Table. This quantity is recorded in the third column of the Table.

If $p = \pm 1 \pmod{5}$, then, as has already been observed, 5 is a square mod p . Consider, in relation to this, the quantity

$$\tilde{\chi} = -5^{\frac{p-1}{4}} \pmod{p}$$

mod p , we have $5 = z^2$, say. Apart from a sign, the right hand side of the above expression is $z^{\frac{p-1}{2}}$, which for $p > 2$ is a square root of unity so $\pm 1 \pmod{p}$. The values of $\tilde{\chi}$ are listed in the fourth column of the Table. From the Table, we see that $\tilde{\chi}$ repeats modulo 20. The set $\mathbb{Z}/20\mathbb{Z}$ has generators $\langle 11, 17 \rangle$ so $\tilde{\chi}$ can be extended to all p , as a Dirichlet character, by assigning the values

$$\tilde{\chi}(11) = -1, \quad \tilde{\chi}(17) = 1,$$

which are consistent with the Table. We can similarly extend χ , up to complex conjugation by assigning the values

$$\chi(11) = 1, \quad \chi(17) = i.$$

Now for $p = 1$ or $9 \pmod{20}$. The coefficients β_{\pm} have the same sign and we can compare this to the sign of the corresponding coefficient in the modular form $f_{\mathbf{20.4.c.a}}$. Let us define a quantity $\chi_{f,g}$ by taking $-\chi_{f,g}$ to be this ratio. The notation recalls that we are comparing the signs of the coefficients of the modular form f , with those of a Hilbert modular form g of the Table. The quantity $\chi_{f,g}$ is recorded in the third column of the Table. Note that, where we know $\chi_{f,g}$, we have the relation

$$\chi_{f,g} = \chi \tilde{\chi}.$$

p	χ	$\chi_{f,g}$	$\tilde{\chi}$	φ_-	β_-	φ_+	β_+
11	-1		-1	2	20	9	-20
19	1		-1	3	-84	7	84
29	-1	-1	1	5	6	7	6
31	-1		-1	19	224	21	-224
41	1	1	1	13	-266	35	-266
59	1		-1	15	28	47	-28
61	1	1	1	28	-182	59	-182
71	-1		-1	60	-408	68	408
79	1		-1	25	-48	78	48
89	-1	-1	1	36	-1526	69	-1526
101	1	-1	-1	13	1246	81	1246
109	-1	1	-1	10	-902	37	-902
131	-1		-1	36	-2940	37	2940
139	1		-1	58	-364	89	364
149	-1	1	-1	3	-254	136	-254
151	-1		-1	46	-2360	76	2360
179	1		-1	133	-1972	146	1972
181	1	-1	-1	52	-1330	117	-1330
191	-1		-1	87	1728	163	-1728
199	1		-1	167	-1512	193	1512
211	-1		-1	23	1644	95	-1644
229	-1	-1	1	103	3934	111	3934
239	1		-1	124	-3856	189	3856
241	1	1	1	76	-994	89	-994
251	-1		-1	5	6300	10	-6300
269	-1	1	-1	22	5082	95	5082
271	-1		-1	130	2800	225	-2800
281	1	1	1	146	1254	187	1254
311	-1		-1	98	5208	154	-5208
331	-1		-1	46	-7828	305	7828

Table 2: The discriminant S_4 factors in \mathbb{F}_p when $p = \pm 1 \pmod{5}$. These roots are denoted by φ_- and φ_+ . The corresponding β -coefficients are denoted by β_- and β_+ . These are the p 'th coefficients of a [4, 4] Hilbert modular form of level [256, 16, 16]. The table gives also the values for the character χ , and also, for $p = 1$ or $9 \pmod{20}$, the relative sign $-\chi_{f,g}$ of the β coefficients in this table and in the modular form $f_{20.4.c.a}$.

6.3. The Hulek–Verrill manifold, AESZ34

Hulek and Verrill in [35] consider the family of Calabi-Yau manifolds that are birational to the variety defined on $T = \mathbb{P}^4 \setminus \{X_1X_2X_3X_4X_5 = 0\}$ by the equation

$$(X_1 + X_2 + X_3 + X_4 + X_5) \left(\frac{\mu_1}{X_1} + \frac{\mu_2}{X_2} + \frac{\mu_3}{X_3} + \frac{\mu_4}{X_4} + \frac{\mu_5}{X_5} \right) = \mu_6 .$$

A multiplication of the coefficients μ_j , $j = 1, \dots, 6$ by a common scale has no effect, so superficially this equation defines a five parameter family of manifolds. The equation defines a reflexive polyhedron, in the sense of Batyrev. This polyhedron has, as noted by Hulek and Verrill, 10 facets that are tetrahedra and 20 that are prisms with triangular section. The way in which these fit together and a similar description of the dual polyhedron can be found in [35] and in Appendix A of [3]. Analysis of the polyhedron reveals that the superficial count of complex structure parameters is in fact correct and that the Hodge numbers for a generic member of the family are given by

$$h^{pq} = \begin{matrix} & & 1 \\ & 0 & & 0 \\ 0 & & 45 & & 0 \\ 1 & 5 & & 5 & & 1 \\ 0 & & 45 & & 0 \\ & 0 & & 0 \\ & & & & 1 \end{matrix} .$$

Thus $\chi = 2(h^{11} - h^{21}) = 80$.

If we consider now a 1-parameter subfamily where $\mu_j = 1$, $j = 1, \dots, 5$ and $\mu_6 = 1/\varphi$ then the manifold admits symmetries with $\mathbb{Z}/5\mathbb{Z}$ and $\mathbb{Z}/2\mathbb{Z}$ generators

$$X_i \rightarrow X_{i+1} \quad \text{and} \quad X_i \rightarrow 1/X_i ,$$

where the indices are understood mod 5. It is easy to see that these symmetries are fixed point free if $\varphi \neq 1, \frac{1}{9}, \frac{1}{25}$. Taking the quotient by either the $\mathbb{Z}/5\mathbb{Z}$ or $\mathbb{Z}/10\mathbb{Z}$ symmetry yields a family of smooth manifolds with one complex structure parameter and Hodge numbers

$$h^{pq} = \begin{matrix} & & 1 \\ & 0 & & 0 \\ 0 & & 4\kappa+1 & & 0 \\ 1 & 1 & & 1 & & 1 \\ 0 & & 4\kappa+1 & & 0 \\ & 0 & & 0 \\ & & & & 1 \end{matrix} ,$$

where $\kappa = 1, 2$ according as the quotient is taken by a group of order 10 or 5.

The coefficients of the fundamental period have a closed form expression

$$a_n = \sum_{i+j+k+l+m=n} \left(\frac{n!}{i!j!k!l!m!} \right)^2 ,$$

which satisfy the recurrence relation

$$\begin{aligned} n^4 a_n = & \left(35n^4 - 70n^3 + 63n^2 - 28n + 5 \right) a_{n-1} \\ & - (n-1)^2 (259n^2 - 518n + 285) a_{n-2} \\ & + 225(n-1)^2(n-2)^2 a_{n-3}. \end{aligned}$$

corresponding to the Picard-Fuchs operator

$$\mathcal{L} = \vartheta^4 - \varphi(35\vartheta^4 + 70\vartheta^3 + 63\vartheta^2 + 28\vartheta + 5) + \varphi^2(\vartheta+1)^2(259\vartheta^2 + 518\vartheta + 285) - 225\varphi^3(\vartheta+1)^2(\vartheta+2)^2.$$

It was listed as operator 34 in [34]. The Riemann symbol is

$$\mathcal{P} \left\{ \begin{array}{ccccc} 0 & \frac{1}{25} & \frac{1}{9} & 1 & \infty \\ \hline 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & 2 & 2 \end{array} \right\} \varphi$$

and the singular points at $\varphi = \frac{1}{25}, \frac{1}{9}, 1$ correspond to hyperconifold singularities. The coefficient functions of the Picard-Fuchs operator are the cubics

$$\begin{aligned} S_4 &= (\varphi - 1)(9\varphi - 1)(25\varphi - 1) \\ S_3 &= 2\varphi(675\varphi^2 - 518\varphi + 35) \\ S_2 &= \varphi(2925\varphi^2 - 1580\varphi + 63) \\ S_1 &= 4\varphi(675\varphi^2 - 272\varphi + 7) \\ S_0 &= 5\varphi(180\varphi^2 - 57\varphi + 1). \end{aligned}$$

The figure showing the numbers of factorisations into two quadrics has been given in §1.

We observe for this manifold, factorisations of the form (5.4), corresponding to conifold and hyperconifold singularities, and note that the β_p coefficients derive from modular forms of weight 4. These facts were known to Hulek and Verrill [36] and to Meyer [37].

The discriminant of the manifold is given by the homogenisation of S_4 , above. The hyper-discriminant is

$$\Delta = 2^{20} 3^2 v^6,$$

and we see that 2 and 3 are the bad primes. If we reduce Δ modulo these primes, we find

$$\Delta \equiv (u-v)^3 \pmod{2},$$

so Δ has a cubic root mod 2, and

$$\Delta \equiv 2v(u-v)^2 \pmod{3},$$

and we see that Δ has a double root mod 3.

Singularity	$\frac{1}{9}$	$\frac{1}{25}$	1
Meyer label	6/1	30/1	6/1
LMFDB	6.4.a.a	30.4.a.a	6.4.a.a
$p = 5$	6	*	6
$p = 7$	- 16	32	- 16
$p = 11$	12	- 60	12
$p = 13$	38	- 34	38
$p = 17$	- 126	42	- 126
$p = 19$	20	- 76	20
$p = 23$	168	0	168
$p = 29$	30	6	30
$p = 31$	- 88	- 232	- 88
$p = 37$	254	134	254
$p = 41$	42	234	42
$p = 43$	- 52	- 412	- 52
$p = 47$	- 96	- 360	- 96
$p = 53$	198	222	198
$p = 59$	- 660	660	- 660
$p = 61$	- 538	- 490	- 538
$p = 67$	884	812	884
$p = 71$	792	120	792
$p = 73$	218	746	218
$p = 79$	- 520	152	- 520
$p = 83$	- 492	- 804	- 492
$p = 89$	810	- 678	810
$p = 97$	1154	194	1154
$p = 101$	- 618	798	- 618
$p = 103$	128	1088	128
$p = 107$	- 1476	1716	- 1476
$p = 109$	1190	- 970	1190
$p = 113$	- 462	426	- 462
$p = 127$	- 2536	200	- 2536
$p = 131$	2292	60	2292
$p = 137$	- 726	642	- 726

Table 3: For the manifold corresponding to AESZ34, the coefficient β_p for the characteristic polynomial of Frobenius for the cases that the manifold has (hyper-) conifold singularities.

$\varphi = -\frac{1}{7}$			$\varphi = 33 \pm 8\sqrt{17}$		
p	α	β	p	α	β
5	0	-14	13	2	-42
7			17	-6	34
11	0	-28	19	-4	60
13	-4	18	43	-4	508
17	6	74	47	0	-136
19	2	80	53	6	318
23	0	-112	59	12	300
29	-6	190	67	-4	-676
31	-4	72	83	-12	-1132
37	2	-346	89	6	-350
41	6	162	101	-6	-1218
43	8	-412	103	8	8
47	-12	24	127	-16	-1216
53	6	318	137	-18	1954
59	-6	-200	149	6	-1010
61	8	-198	151	8	-968
67	-4	-716	157	14	1654
71	0	392	179	12	-980
73	2	538	191	0	952
79	8	240	223	-16	-712
83	-6	-1072	229	-22	5230
89	-6	810	239	0	2040
97	-10	1354	251	-12	-5868
101	0	-1358	257	6	-4646
103	-4	-832	263	24	-6472
107	12	444	271	-16	8312
109	2	1870	281	18	-518
113	6	1378	293	6	-6402
127	-16	1944	307	20	-3516
131	18	-848	331	-4	2892
137	18	-2966	349	-34	5270

Table 4: The (α, β) -coefficients for the attractor points $\varphi = -\frac{1}{7}$ and $\varphi = 33 \pm 8\sqrt{17}$. For $\varphi = -\frac{1}{7}$ the α_p are the p 'th coefficients of a weight 2 modular form, with LMFDB designation **14.2.a.a**, for $\Gamma_0(14)$. The coefficients β_p are the p 'th coefficients of a weight four modular form, with designation **14.4.a.a**, also for $\Gamma_0(14)$. For $\varphi = 33 \pm 8\sqrt{17}$, with the exception of $p = 17$, the correspondence is for primes such that $\varphi^2 - 66\varphi + 1$ factorises in \mathbb{F}_p . For these primes, the α_p are the p 'th coefficients of a weight two modular form, with designation **34.2.b.a** and the β_p are the p 'th coefficients the weight 4 modular form **34.4.4.a**, both for the group $\Gamma_1(34)$.

6.4. The Rødland manifold, AESZ 27

The Rødland manifold, which is defined by the vanishing of the 7 cubic Pfaffians of a generic anti-symmetric 7×7 matrix of linear forms in 7 variables, is a Calabi-Yau threefold X with $h^{11} = 1$ and $h^{21} = 440$ and characteristic numbers

$$H^3 = 378, \quad c_2 H = -252, \quad \chi = -882.$$

Rødland in ref. [38], which also describes the manifold, constructed a mirror manifold \tilde{X} by an orbifold construction and determined the associated Picard-Fuchs equation. This turned out to be the same operator as obtained in [33] for the mirror of Calabi-Yau obtained intersection of 7 linear hyperplanes in the 10-dimensional Grassmannian $G(2, 7)$. So the very different Pfaffian and Grassmannian Calabi-Yau threefolds turn out to have the same mirror manifold, which led to the suspicion that these manifolds have equivalent categories of coherent sheaves, a fact proved in [39].

The coefficient functions of the differential equation take the following form

$$\begin{aligned} S_4 &= (\varphi - 3)^2 (\varphi^3 - 289\varphi^2 - 57\varphi + 1) \\ S_3 &= 4\varphi(\varphi - 3) (\varphi^3 - 149\varphi^2 + 867\varphi + 85) \\ S_2 &= 2\varphi (3\varphi^4 - 239\varphi^3 + 2353\varphi^2 - 7597\varphi - 408) \\ S_1 &= 2\varphi (2\varphi^4 - 87\varphi^3 + 675\varphi^2 - 4773\varphi - 153) \\ S_0 &= \varphi (\varphi^4 - 26\varphi^3 + 12\varphi^2 - 2166\varphi - 45) \end{aligned}$$

The Riemann symbol is

$$\mathcal{P} \left\{ \begin{array}{cccccc} 0 & 3 & \varphi_1 & \varphi_2 & \varphi_3 & \infty \\ \hline 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 1 & 1 & 1 & 1 \\ 0 & 4 & 2 & 2 & 2 & 1 \end{array} \right\} .$$

where the φ_j , $j = 1, 2, 3$, are the roots of $\varphi^3 - 289\varphi^2 - 57\varphi + 1$. Note that the Picard-Fuchs operator has two points of maximal unipotent monodromy: the degeneration at 0 corresponds to the mirror of the Pfaffian, the point ∞ corresponds to the mirror of the Grassmannian Calabi-Yau manifold.

The coefficients of the fundamental period satisfy the following recurrence relation

$$\begin{aligned} 9n^4 a_n &= 3(173n^4 - 352n^3 + 290n^2 - 114n + 18) a_{n-1} \\ &\quad + 2(1129n^4 - 4000n^3 + 4501n^2 - 1359n - 267) a_{n-2} \\ &\quad - 2(843n^4 - 7488n^3 + 24223n^2 - 33531n + 16485) a_{n-3} \\ &\quad + (295n^4 - 4112n^3 + 21502n^2 - 49986n + 43586) a_{n-4} \\ &\quad - (n-4)^4 a_{n-5}. \end{aligned}$$

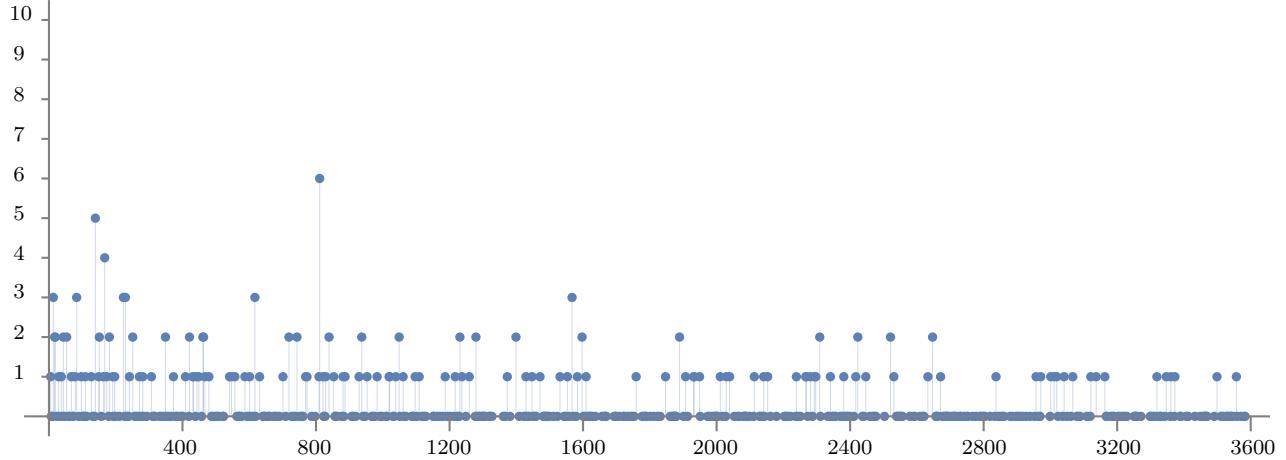


Figure 12: The figure shows the number of factorisations of the numerator of the ζ -function into two factors, for the Rødland manifold.

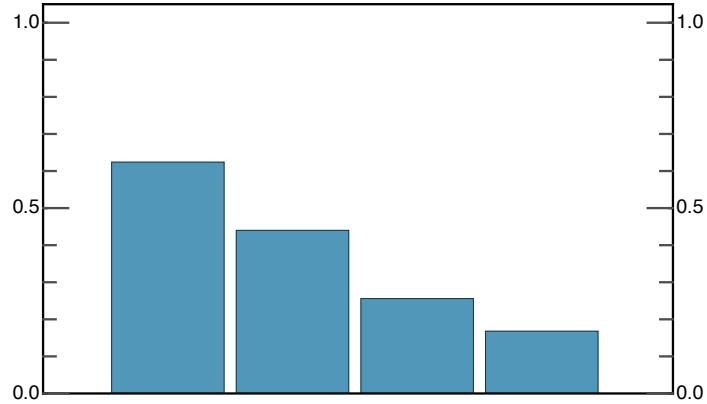


Figure 13: Running averages for the data of the three generation manifold taken from Figure 12. The averages are taken for bins of 125 primes for the 500 primes $5 \leq p \leq 3583$.

p	3	R
5	3	$1 - 2T - 20pT - 2p^3T^3 + p^6T^4$
7	3	no unit root
11	3	$1 + 34T + 238pT + 34p^3T^3 + p^6T^4$
13	3	$1 + 14T + 142pT + 14p^3T^3 + p^6T^4$
17	3	$1 + 58T + 396pT + 58p^3T^3 + p^6T^4$

Table 5: The Frobenius polynomials for the primes 5, 7, 11, 13 at the apparent singularity $\varphi = 3$. These are calculated by the unit root method.

With this manifold we come to an example that has an apparent singularity. The hyperdiscriminant of the manifold is

$$\Delta = 2^6 7^8 v^6 ,$$

while the hyperdiscriminant of the differential equation is

$$\mathbb{S}_4 = 2^{12} 7^{14} v^{12} .$$

Reducing S_4 modulo the bad primes, we find

$$\begin{aligned} S_4 &= (u - v)^5 \pmod{2} , \\ &= (u - 3v)^5 \pmod{7} . \end{aligned}$$

Finally, we wish to make some remarks regarding the splitting field of the discriminant

$$\Delta(\varphi) = \varphi^3 - 289\varphi^2 - 57\varphi + 1$$

since these are relevant to understanding the factorisations of the Frobenius polynomial at the three conifold points.

First note that setting

$$\varphi = \frac{-2\xi + 1}{4\xi + 5}$$

brings the discriminant to the form

$$\Delta = -2744 \frac{g(\xi)}{(4\xi + 5)^3} \quad \text{where} \quad g(\xi) = \xi^3 - \xi^2 - 2\xi + 1 .$$

So the three roots φ_j , $j = 1, 2, 3$, of Δ are given by $\varphi_j = \varphi(\xi_j)$, in terms of the three roots ξ_j of g . It is easy to check that the ξ_j are

$$2 \cos \frac{5\pi}{7}, 2 \cos \frac{3\pi}{7}, 2 \cos \frac{\pi}{7} .$$

Let ζ denote *any* nontrivial seventh root of unity, then it is a quick check that the quantities

$$-(\zeta + \zeta^{-1}), -(\zeta^2 + \zeta^{-2}), -(\zeta^3 + \zeta^{-3})$$

are the three roots above, up to cyclic order. We learn that the splitting field of g is $\mathbb{Q}(\zeta + \zeta^{-1})$ and that the Galois group of g is $\mathbb{Z}/3\mathbb{Z}$. While, for a generic cubic, the Galois group is S_3 , the permutation group on three objects.

Note also that the bad prime 7 factors nontrivially in this field, since if $\nu_j = 2 + \xi_j$ we have

$$7 = \nu_1 \nu_2 \nu_3 .$$

Each ν_j is an integer of the field and, by the above relation, has field-norm 7. Since the norm of an integer of the field is a rational integer, the ν_j cannot themselves factor nontrivially and so are primes of the field. In fact the ν_j are equal, up to units, so the factorisation of 7 is

best expressed in terms of ideals. The ideals (ν_j) are all equal so let \mathfrak{P} denote this common ideal, and let \mathcal{O} denote the ring of integers of the field, then

$$7\mathcal{O} = \mathfrak{P}^3.$$

On the other hand, the prime 2 does not factor nontrivially.

J. Voight has recognised the β -coefficients of Table 6 as corresponding to a Hilbert modular form for the field $\mathbb{Q}(\zeta + \zeta^{-1})$, with weight $[4, 4, 4]$ and level $2\mathfrak{P}$.

p	χ	φ_1	β_1	φ_2	β_2	φ_3	β_3
13	1	2	-42	5	42	9	-14
29	1	12	-306	17	-110	28	282
41	1	23	70	27	-70	34	-434
43	1	1	128	36	-264	37	-68
71	1	5	604	25	-180	46	-376
83	1	10	-1148	14	1372	16	84
97	1	2	854	19	-546	74	-798
113	1	42	-54	65	-1818	69	-642
127	1	19	1556	68	-404	75	2536
139	1	41	-812	119	-756	129	-2100
167	1	60	112	110	-3696	119	-1288
181	1	45	-1498	102	-3850	142	2030
197	1	68	-1762	93	3138	128	2
211	1	111	2396	188	1024	201	-3680
223	1	9	3864	130	-2912	150	392
239	1	107	4832	188	2872	233	-7320
251	1	46	5236	57	-5460	186	1148
281	1	130	-3078	219	2214	221	-138
293	1	25	-8050	59	1638	205	-1722
307	1	112	-8036	180	5796	304	10164
337	1	174	1794	220	11006	232	-6242
349	1	169	10346	173	-9310	296	-5306
379	1	182	-2980	194	-12388	292	-1020
419	1	170	13524	262	6804	276	-4676
421	1	73	14002	293	4398	344	9298
433	1	23	2562	126	10094	140	-11914
449	1	24	-1398	349	366	365	-7866
461	1	126	8106	214	12250	410	-2002
463	1	128	-6844	229	2172	395	9032
491	1	159	9116	186	5980	435	13428
503	1	109	18144	310	-2744	373	-8064
547	1	28	25356	308	-11492	500	7324
587	1	27	-11004	116	-3444	146	140

Table 6: For the Rødland manifold, the character, which is in all cases trivial, together with the coefficients β_p , for the three roots of the discriminant for the cases that the roots exist in \mathbb{F}_p , which is when $p = \pm 1 \pmod{7}$.

6.5. Three-generation manifolds with $(h^{1,1}, h^{2,1}) = (4, 1)$

These manifolds may be realised as the mirror manifolds of quotients of hypersurfaces in $dP_6 \times dP_6$, where dP_6 is the del Pezzo surface of degree 6, by a freely acting group G of order 12. There are, in reality, two variants of this ‘manifold’ since there are two choices for G , which is either $\mathbb{Z}/12\mathbb{Z}$ or the nonabelian group Dyc_3 . Before taking the mirror, this space can be realised as the CICY [40, 41]

$$\begin{aligned} & \mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix}^{(1,4)} \\ & \mathbb{P}^2 \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \end{bmatrix} \\ & \mathbb{P}^2 \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \end{bmatrix} \\ & \mathbb{P}^2 \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \end{bmatrix} / G \end{aligned}$$

or as a reflexive polyhedron, corresponding to the fact that the covering space is a hypersurface in the toric variety $dP_6 \times dP_6$. A detailed description of the manifolds can be found in Refs. [42, 43]. For the basic invariants of X_φ we have

$$(h^{1,1}, h^{2,1}) = (4, 1) ; \quad \chi = 6 ; \quad c_2 H = 12 ; \quad H^3 = 18 .$$

From [42] we have that the fundamental period is given by the following integral representation

$$\varpi_0(\varphi) = \frac{1}{(2\pi i)^4} \int \frac{d^4 t}{t_1 t_2 t_3 t_4 (1 - \varphi F(t))}$$

where

$$F(t) = W(t_1, t_2) + W(t_3, t_4) \quad \text{with} \quad W(t_1, t_2) = t_1 + \frac{1}{t_1} + t_2 + \frac{1}{t_2} + \frac{t_1}{t_2} + \frac{t_2}{t_1} , \quad (6.2)$$

and the contour, for the integral, consists of a product of four loops enclosing the poles $t_j = 0$. From the integral we see that the coefficients of the fundamental period are given, as anticipated in §6.1, by

$$a_n = [F(t)^n]_0 .$$

The question arises as to how to calculate say the first 50 coefficients a_n . Naive computation of $[F(t)^n]_0$ becomes very slow as n increases. In order to compute these coefficients more efficiently, we set $W_k = [W(t_1, t_2)^k]_0$ and note that

$$[F(t)^n]_0 = \sum_{r=0}^n \binom{n}{r} W_{n-r} W_r = \begin{cases} 2 \sum_{r=0}^{\frac{1}{2}(n-1)} \binom{n}{r} W_{n-r} W_r ; & \text{if } n \text{ is odd} \\ 2 \sum_{r=0}^{\frac{n}{2}-1} \binom{n}{r} W_{n-r} W_r + \binom{n}{\frac{n}{2}} W_{\frac{n}{2}}^2 ; & \text{if } n \text{ is even} . \end{cases}$$

We can calculate the quantities W_k explicitly by collecting powers of t_1

$$W(t_1, t_2) = t_1 \left(1 + \frac{1}{t_2} \right) + \left(t_2 + \frac{1}{t_2} \right) + \frac{1}{t_1} (1 + t_2)$$

and picking out the terms in the expansion of $S(t_1, t_2)^k$ that are independent of t_1 and then of t_2 . In this way we find

$$W_k = \sum_{r=0}^{\lfloor \frac{k}{2} \rfloor} \sum_{s=\max(0, \lceil \frac{k-3r}{2} \rceil)}^{\min(k-2r, \lfloor \frac{k-r}{2} \rfloor)} \frac{k! (2r)!}{(r!)^2 s! (3r+2s-k)! (k-r-2s)! (k-2r-s)!}.$$

Availing ourselves of these expressions, there is no difficulty in computing the first few hundred a_n .

If we now seek the differential equation that the fundamental period satisfies, we find the following coefficient functions S_j :

$$\begin{aligned} S_4 &= 69120 \left(\varphi - \frac{3}{2}\right)^2 \left(\varphi + \frac{1}{4}\right) \left(\varphi + \frac{1}{5}\right) \left(\varphi + \frac{1}{6}\right) \left(\varphi - \frac{1}{12}\right) \left(\varphi - \frac{1}{4}\right) \left(\varphi - \frac{1}{3}\right) \\ S_3 &= 8\varphi \left(\varphi - \frac{3}{2}\right) (86400\varphi^6 - 158976\varphi^5 - 1512\varphi^4 + 15964\varphi^3 + 160\varphi^2 - 345\varphi - 5) \\ S_2 &= \varphi (2419200\varphi^7 - 8581248\varphi^6 + 7771104\varphi^5 + 274360\varphi^4 - 552220\varphi^3 - 5250\varphi^2 \\ &\quad + 6917\varphi + 39) \\ S_1 &= \varphi (3456000\varphi^7 - 12745728\varphi^6 + 12372480\varphi^5 + 166288\varphi^4 - 679952\varphi^3 - 1584\varphi^2 \\ &\quad + 5532\varphi + 9) \\ S_0 &= 48\varphi^2 (34560\varphi^6 - 130464\varphi^5 + 132120\varphi^4 + 284\varphi^3 - 6182\varphi^2 + 9\varphi + 36) \end{aligned}$$

The differential operator defined by these coefficient functions appears, after transformation to a new coordinate

$$\tilde{\varphi} = \frac{3\varphi}{3-2\varphi}$$

as operator 6.24, under the revised naming convention, in the AESZ list. In terms of the variable $\tilde{\varphi}$ the differential operator is slightly simplified and the new coefficient functions have degree 6, rather than 8 as above. The designation 6.24 indicates that this is the 24th operator in the list whose coefficient functions have degree 6.

From the ratio S_3/S_4 we may compute the Yukawa coupling

$$\begin{aligned} y_{\varphi\varphi\varphi} &= \frac{1}{\varphi^3} \exp \left(-\frac{1}{2} \int \frac{d\varphi}{\varphi} \frac{S_3(\varphi)}{S_4(\varphi)} \right) \\ &= \frac{\varphi - \frac{3}{2}}{1440\varphi^3 (\varphi + \frac{1}{4}) (\varphi + \frac{1}{5}) (\varphi + \frac{1}{6}) (\varphi - \frac{1}{12}) (\varphi - \frac{1}{4}) (\varphi - \frac{1}{3})}. \end{aligned} \tag{6.3}$$

Notice that the coupling has a zero at $\varphi = 3/2$.

The Riemann symbol for the Picard-Fuchs operator is

$$\mathcal{P} \left\{ \begin{array}{cccccccccc} 0 & \infty & \frac{3}{2} & -\frac{1}{4} & -\frac{1}{5} & -\frac{1}{6} & \frac{1}{12} & \frac{1}{4} & \frac{1}{3} \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 3 & 3 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 4 & 4 & 2 & 2 & 2 & 2 & 2 & 2 \end{array} \varphi \right\}$$

The recurrence relation for the coefficients of the fundamental period follows from the form of the differential equation.

$$\begin{aligned} 9n^4 a_n = & 3(n-1)(16n^3 - 28n^2 + 21n - 6) a_{n-1} \\ & + (983n^4 - 3764n^3 + 5909n^2 - 4392n + 1260) a_{n-2} \\ & - 2(727n^4 - 6384n^3 + 20823n^2 - 30294n + 16740) a_{n-3} \\ & - 4(5849n^4 - 46012n^3 + 128695n^2 - 148340n + 55848) a_{n-4} \\ & + 8(n-4)(3019n^3 - 30072n^2 + 93377n - 90756) a_{n-5} \\ & + 288(n-5)(n-4)(539n^2 - 1503n + 624) a_{n-6} \\ & - 3456(n-6)(n-5)(n-4)(61n - 125) a_{n-7} \\ & + 69120(n-7)(n-6)(n-5)(n-4) a_{n-8} \end{aligned}$$

We do not give the explicit form of the recurrence relations for the coefficients b_n, c_n, d_n of the functions $f_j(\varphi)$ for $j \neq 0$. However these are obtained from the above relation by replacing n by $n + \epsilon$ and a_n by $A_n(\epsilon)$ and expanding in powers of ϵ .

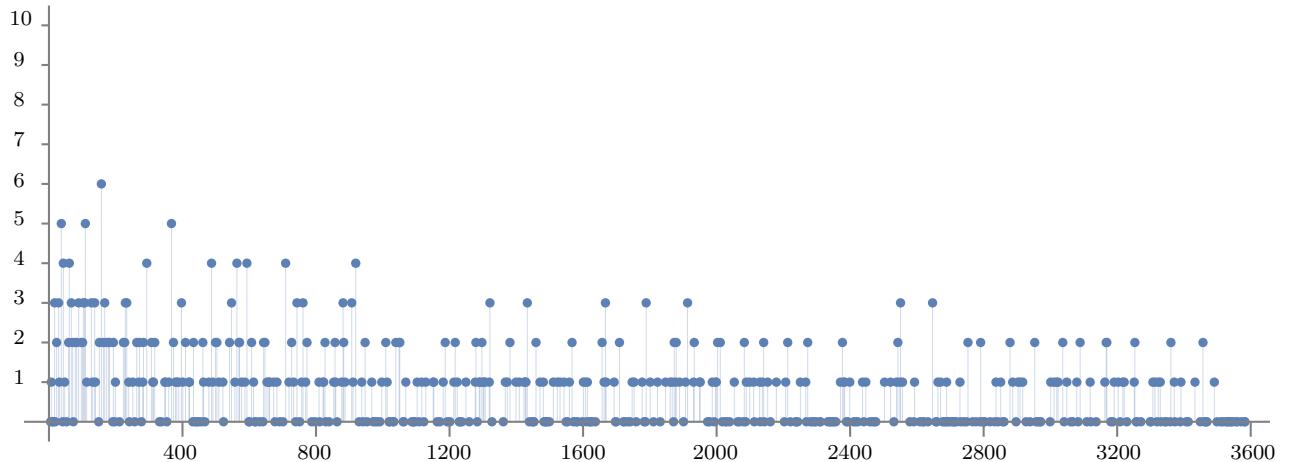


Figure 14: The figure shows the numbers of factorisations into two quadrics for the manifold with Hodge numbers (1,4).

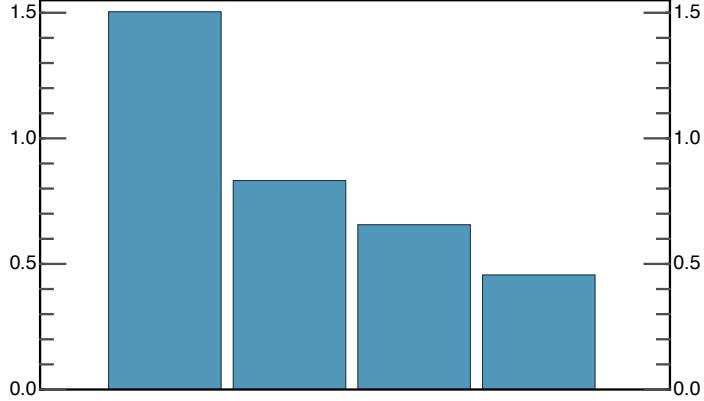


Figure 15: *Running averages for the data of the three generation manifold taken from Figure 14. The averages are taken for bins of 125 primes for the 500 primes $5 \leq p \leq 3583$.*

p	$\frac{3}{2}$	R
2	1	$(1 - \alpha pT + p^3 T^2)(1 + 3T + p^3 T^2)$
3	0	singular
5	4	no unit root
7	5	no unit root
11	7	$(1 - \alpha pT + p^3 T^2)(1 + 48T + p^3 T^2)$
13	8	$1 + 8T - 162pT^2 + 8p^3 T^3 + p^6 T^4$
17	10	no unit root
19	11	$1 + 140T + 690pT^2 + 140p^3 T^3 + p^6 T^4$
23	13	$1 - 6T + 554pT^2 - 6p^3 T^3 + p^6 T^4$
29	16	$1 + 96T + 926pT^2 + 96p^3 T^3 + p^6 T^4$
31	17	$(1 - \alpha pT + p^3 T^2)(1 - 56T + p^3 T^2)$
37	20	$1 + 128T + 2190pT^2 + 128p^3 T^3 + p^6 T^4$

Table 7: *The Frobenius polynomials for the first few primes at the apparent singularity $\varphi = \frac{3}{2}$. These are calculated by the unit root method. For the cases that R factorises, the coefficient α is left undetermined by this process.*

For this manifold, the hyperdiscriminants of the manifold and of the differential equation are

$$\Delta = 2^{32} 3^{16} 5^2 7^2 17^2 v^{30} \quad \text{and} \quad \mathbb{S}_4 = 2^{42} 3^{16} 5^6 7^6 17^6 v^{42} .$$

On reducing S_4 modulo the bad primes, we find

$$\begin{aligned}
 S_4 &= v^6(u-v)^2 \pmod{2}, \\
 &= -u^2v^3(u-2v)(u-v)^2 \pmod{3}, \\
 &= -v(u+v)^4(u+2v)(u+3v)(u+4v) \pmod{5}, \\
 &= 2(u+2v)^4(u+3v)(u+4v)(u+5v)(u+6v) \pmod{7}, \\
 &= -2(u+3v)(u+4v)(u+7v)^4(u+11v)(u+13v) \pmod{17}.
 \end{aligned}$$

Finally, we present Table 8 which presents the β_p -coefficients for the values of φ for which the manifold is singular. For $p \leq 41$ these were calculated using the unit root method (and, for the larger values of p , a supercomputer) and, for $p \geq 7$, also by the method of replacing the power series for $U(\varphi)$ by a rational function. Certain entries to the Table are recorded in gray, such as the entry for $p = 31$ and $\varphi = 1/12$ and those for $p = 17$ and $\varphi = -1/5$ and $\varphi = 1/12$.

Singularity	$-\frac{1}{4}$	$-\frac{1}{5}$	$-\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{4}$	$\frac{1}{3}$
Meyer label	14/2	17/1	60/1	102/3	10/1	21/2
LMFDB	14.4.a.b	17.4.a.a	60.4.a.a	102.4.a.c	10.4.a.a	21.4.a.a
$p = 2$	2	-3	0	2	2	-3
$p = 3$	-2	-8	-3	-3	-8	-3
$p = 5$	-12	6	-5	-12	5	-18
$p = 7$	7	-28	-28	-22	-4	7
$p = 11$	48	-24	-24	-48	12	-36
$p = 13$	56	-58	-70	2	-58	-34
$p = 17$	-114	17	102	-17	66	42
$p = 19$	2	116	20	20	-100	-124
$p = 23$	-120	-60	-72	-54	132	0
$p = 29$	-54	30	306	84	-90	102
$p = 31$	236	-172	-136	62	152	-160
$p = 37$	146	-58	-214	44	-34	398
$p = 41$	126	-342	-150	-138	-438	-318
$p = 43$	-376	-148	-292	428	32	-268
$p = 47$	-12	288	-72	-516	-204	240
$p = 53$	174	318	-414	174	222	-498
$p = 59$	138	252	-744	-852	420	-132
$p = 61$	380	110	-418	908	902	398
$p = 67$	-484	-484	188	-508	-1024	92
$p = 71$	576	-708	480	-426	432	-720
$p = 73$	-1150	362	434	-574	362	-502
$p = 79$	776	-484	1352	110	-160	-1024
$p = 83$	378	756	-612	-1308	72	-204
$p = 89$	-390	-774	-30	798	810	354
$p = 97$	-1330	-382	-286	-1690	1106	-286
$p = 101$	-1500	-210	-1542	-1890	-258	414
$p = 103$	380	-232	1172	-1900	-988	56
$p = 107$	636	432	1956	-480	-24	12
$p = 109$	146	-1186	-1858	1424	950	1478
$p = 113$	198	-366	174	402	-1038	402
$p = 127$	-376	-472	-2068	2336	-124	1280
$p = 131$	2130	2760	312	768	132	1764
$p = 137$	-78	1098	2646	2106	-1254	-2358

Table 8: For the (1,4)-manifold, the coefficient β_p for the characteristic polynomial of Frobenius for the cases that the manifold has (hyper-) conifold singularities.

6.6. A quotient of the 24 cell with $(h^{1,1}, h^{2,1}) = (1, 1)$, AESZ 366

It was known already to Kreuzer and Skarke [44] that the 24-cell is a reflexive polyhedron and occurs in their list of such polyhedra. It was observed by Braun [4] that the threefold that is thereby defined admits free quotients by three groups G , of order 24, where G is homomorphic to $\mathrm{SL}(2, 3)$, $\mathbb{Z}_3 \rtimes \mathbb{Z}_8$ or $\mathbb{Z}_3 \times \mathbb{Q}_8$, and that these quotients are Calabi-Yau manifolds with $(h^{1,1}, h^{2,1}) = (1, 1)$. The Picard-Fuchs equation is common to these three manifolds and was already known, owing to the fact that the fundamental period has an interpretation as the generating function for lattice walks that return to the origin after n steps. It had been observed by Guttman [45, 46] and Broadhurst [47] that, what is to us the fundamental period, but which they viewed as a lattice generating function, satisfies a differential equation of Calabi-Yau type. This was recorded as operator 366 of [34] and has designation 7.3 in the revised numbering. The manifold and the periods, for this case, are studied in somewhat greater detail in [48].

For the basic invariants of the manifold we have

$$(h^{1,1}, h^{2,1}) = (1, 1) ; \quad \chi = 0 ; \quad c_2 H = 4 ; \quad H^3 = 4 .$$

The defining toric equation is

$$1 - \varphi F(t) = 0 ,$$

where, in this case, the Laurent polynomial $F(t)$ takes the form

$$\begin{aligned} F(t) &= t_1 + t_2 + t_3 + t_4 + \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \frac{1}{t_4} \\ &\quad + \frac{t_1}{t_2} + \frac{t_1}{t_3} + \frac{t_1}{t_4} + \frac{t_2}{t_1} + \frac{t_3}{t_1} + \frac{t_4}{t_1} + \frac{t_4}{t_2} + \frac{t_2}{t_4} + \frac{t_3}{t_4} + \frac{t_4}{t_3} \\ &\quad + \frac{t_1}{t_2 t_3} + \frac{t_2 t_3}{t_1} + \frac{t_2 t_3}{t_4} + \frac{t_4}{t_2 t_3} + \frac{t_1 t_4}{t_2 t_3} + \frac{t_2 t_3}{t_1 t_4} \\ &= W(t_1, t_2) + W(t_3, t_4) + W\left(\frac{t_1}{t_3}, \frac{t_2}{t_4}\right) + W\left(\frac{t_1}{t_4}, \frac{t_2 t_3}{t_4}\right) , \end{aligned}$$

where we used the same expression for W as in eq. (6.2). The fundamental period has again an expansion with coefficients $[F(t)^n]_0$ and one can use the last expression above to generate a sufficient number of the coefficients a_n .

Explicit expressions may also be given in several ways and Refs. [46, 47] give, for example, the convenient series

$$\begin{aligned} \varpi_0(\varphi) &= \sum_{i,j,k,l,m=0}^{\infty} \binom{2i}{i} \binom{2j}{j} \binom{2k}{k} \binom{l+m}{m} \binom{2(l+m)}{l+m}^2 \times \\ &\quad \times \binom{i+j+k+l+m}{2(l+m)} \binom{i+j+k-l-m}{-i+j+k} \binom{2i-l-m}{i-k-l} \varphi^{i+j+k+l+m} . \end{aligned}$$

The differential operator that annihilates this function has coefficient functions

$$\begin{aligned}
S_4 &= 8957952 \left(\varphi + \frac{1}{18} \right)^2 \left(\varphi + \frac{1}{3} \right) \left(\varphi + \frac{1}{4} \right) \left(\varphi + \frac{1}{8} \right) \left(\varphi + \frac{1}{12} \right) \left(\varphi - \frac{1}{24} \right) \\
S_3 &= 36\varphi \left(\varphi + \frac{1}{18} \right) (1990656\varphi^5 + 1257984\varphi^4 + 264384\varphi^3 + 22320\varphi^2 + 800\varphi + 15) \\
S_2 &= \varphi (206032896\varphi^6 + 118195200\varphi^5 + 24103872\varphi^4 + 2276640\varphi^3 + 105552\varphi^2 + 2114\varphi + 19) \\
S_1 &= 72\varphi \left(\varphi + \frac{1}{18} \right) (3483648\varphi^5 + 1548288\varphi^4 + 225072\varphi^3 + 13572\varphi^2 + 320\varphi + 1) \\
S_0 &= 96\varphi^2 (1119744\varphi^5 + 508032\varphi^4 + 82512\varphi^3 + 6318\varphi^2 + 237\varphi + 4) .
\end{aligned}$$

The singularities of the Picard-Fuchs equation are evident from the factorisation of S_4 . For the present manifold $\varphi = -1/18$ is an apparent singularity.

The Riemann symbol for this manifold is

$$\mathcal{P} \left\{ \begin{array}{cccccccccc} 0 & \infty & -\frac{1}{3} & -\frac{1}{4} & -\frac{1}{8} & -\frac{1}{12} & -\frac{1}{18} & \frac{1}{24} \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 & 1 & 3 & 1 \\ 0 & 3 & 2 & 2 & 2 & 2 & 4 & 2 \end{array} \mid \varphi \right\}, \quad (6.4)$$

From the differential equation the recurrence relation for the a_n follows

$$\begin{aligned}
n^4 a_n &= -(n-1)(39n^3 - 147n^2 + 158n - 54) a_{n-1} \\
&\quad - 2(16n^4 - 1198n^3 + 5747n^2 - 9800n + 5748) a_{n-2} \\
&\quad + 36(3n-7)(171n^3 - 973n^2 + 1821n - 1007) a_{n-3} \\
&\quad + 864(384n^4 - 4602n^3 + 20995n^2 - 43195n + 33786) a_{n-4} \\
&\quad + 1728(n-4)(1393n^3 - 15324n^2 + 57143n - 72156) a_{n-5} \\
&\quad + 248832(n-4)(n-5)(31n^2 - 267n + 584) a_{n-6} \\
&\quad + 8957952(n-4)(n-5)^2(n-6) a_{n-7} .
\end{aligned}$$

Again, the Frobenius period may be obtained from this recurrence by replacing n by $n + \epsilon$ and a_n by $A_n(\epsilon)$.

If we now return to the Riemann symbol (6.4) and make the change of variable

$$\varphi = \frac{\tilde{\varphi}}{1 - 18\tilde{\varphi}}, \quad \text{so} \quad \tilde{\varphi} = \frac{\varphi}{1 + 18\varphi} .$$

Then, using the properties of the Riemann symbol we see that (6.4) is equivalent to the scheme

$$\frac{1}{1 + 18\varphi} \mathcal{P} \left\{ \begin{array}{cccccccc} 0 & \infty & \frac{1}{42} & \frac{1}{18} & \frac{1}{15} & \frac{1}{14} & \frac{1}{10} & \frac{1}{6} \\ \hline 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 4 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 5 & 2 & 2 & 2 & 2 & 2 & 2 \end{array} \right. \frac{\varphi}{1 + 18\varphi} \right\} .$$

The change of variables has interchanged the roles of the singularities at $\varphi = \infty$ and $\varphi = \frac{1}{18}$ with those at $\tilde{\varphi} = \frac{1}{18}$ and $\tilde{\varphi} = \infty$. The six conifold points now appear on an equal footing in the symbol. Based on this new symbol we can choose new period vector

$$\tilde{\varpi}_j(\tilde{\varphi}) = (1 + 18\varphi) \varpi_j(\varphi)$$

This satisfies a somewhat simpler Picard-Fuchs equation with coefficient functions that are sixth order polynomials rather than seventh order, for detail see [47, §4.3]. The simpler differential operator may also be found in the AESZ list as operator 6.18.

The properly normalized Yukawa coupling is

$$y_{\varphi\varphi\varphi} = \frac{1}{\varphi^3} \exp \left(-\frac{1}{2} \int \frac{d\varphi}{\varphi} \frac{S_3(\varphi)}{S_4(\varphi)} \right) = -\frac{1}{384} \frac{\varphi + \frac{1}{18}}{\varphi^3 (\varphi + \frac{1}{3}) (\varphi + \frac{1}{4}) (\varphi + \frac{1}{8}) (\varphi + \frac{1}{12}) (\varphi - \frac{1}{24})}$$

where we have fixed the integration constant to match the classical intersection number (times φ^{-3}) at the large complex structure limit $\varphi = 0$.

For this manifold, the hyperdiscriminants of the manifold and of the differential equation are

$$\Delta = 2^{32} 3^{14} 5^2 7^2 v^{20} \quad \text{and} \quad \mathbb{S}_4 = 2^{40} 3^{20} 5^6 7^6 v^{30} .$$

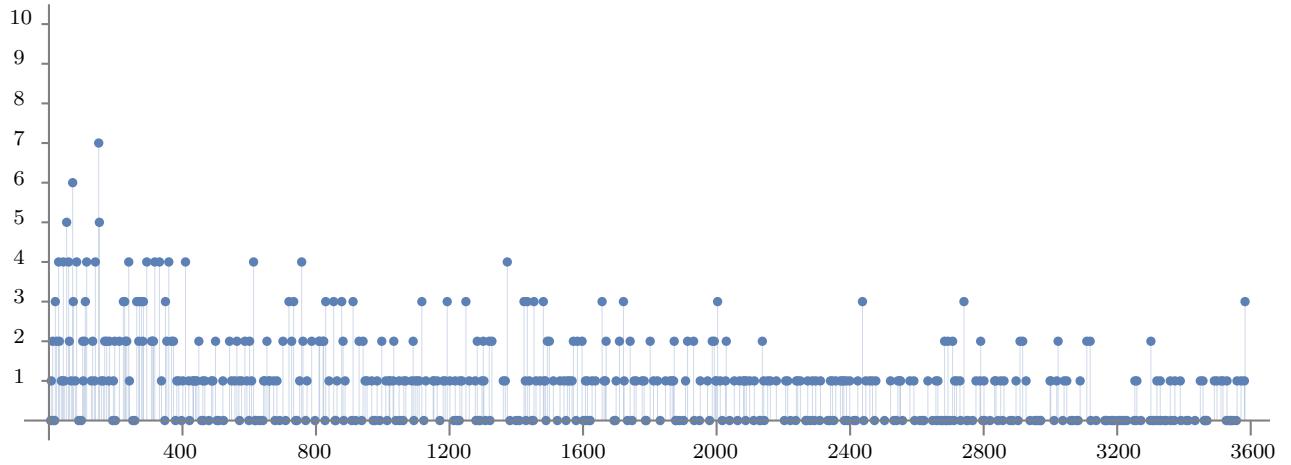


Figure 16: The figure shows the number of factorisations into two quadrics for the manifold with Hodge numbers (1,1).

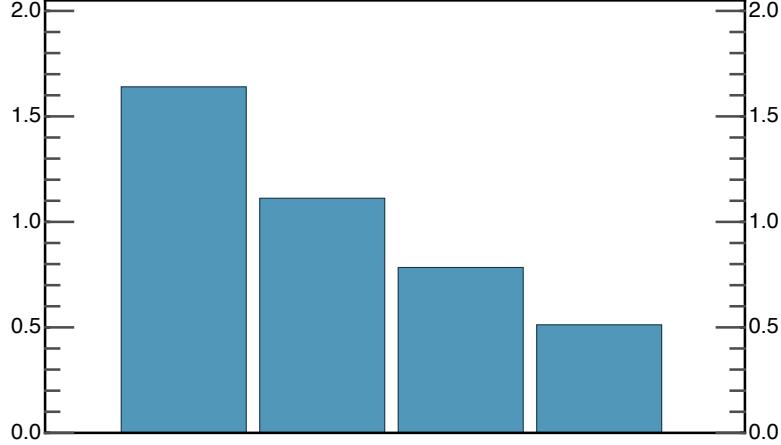


Figure 17: *Running averages for the data from Figure 16. The averages are taken for bins of 125 primes for the 500 primes $5 \leq p \leq 3583$.*

p	$-\frac{1}{18}$	R
2	∞	
3	∞	
5	3	no unit root
7	5	no unit root
11	3	$(1 - \alpha pT + p^3 T^2)(1 - 12T + p^3 T^2)$
13	8	no unit root
17	16	$(1 - \alpha pT + p^3 T^2)(1 + 1187T + p^3 T^2)$
19	1	$(1 - \alpha pT + p^3 T^2)(1 + 52T + p^3 T^2)$
23	14	$1 - 132T + 578pT^2 - 132p^3T^3 + p^6T^4$
29	8	$1 - 16T + 446pT^2 - 16p^3T^3 + p^6T^4$
31	12	$1 + 76T - 766pT^2 + 76p^3T^3 + p^6T^4$
37	2	$1 - 44T - 730pT^2 - 44p^3T^3 + p^6T^4$

Table 9: *The Frobenius polynomials for the first few primes at the apparent singularity $\varphi = -\frac{1}{18}$. These are calculated by the unit root method. For the cases that R factorises, the coefficient α is left undetermined by this process.*

On reducing S_4 modulo the bad primes, we find

$$\begin{aligned}
 S_4 &= v^6(u+v) \pmod{2}, \\
 &= v^5(u+v)(u+2v) \pmod{3}, \\
 &= 2(u+v)(u+2v)^4(u+3v)(u+4v) \pmod{5}, \\
 &= 3(u+v)(u+2v)^4(u+3v)(u+5v) \pmod{7}.
 \end{aligned}$$

Singularity	$-\frac{1}{3}$	$-\frac{1}{4}$	$-\frac{1}{8}$	$-\frac{1}{12}$	$\frac{1}{24}$
Meyer label	15/2	28/2	10/1	12/1	42/2
LMFDB	15.4.a.a	28.4.a.b	10.4.a.a	12.4.a.a	42.4.a.b
$p = 2$	1	0	2	0	2
$p = 3$	3	4	-8	3	3
$p = 5$	5	6	5	-18	2
$p = 7$	-24	7	-4	8	-7
$p = 11$	52	-12	12	36	-8
$p = 13$	22	-82	-58	-10	-42
$p = 17$	-14	-30	66	18	-2
$p = 19$	-20	68	-100	-100	-124
$p = 23$	-168	216	132	72	76
$p = 29$	230	246	-90	-234	254
$p = 31$	-288	-112	152	-16	-72
$p = 37$	-34	110	-34	-226	398
$p = 41$	122	-246	-438	90	462
$p = 43$	-188	-172	32	452	212
$p = 47$	256	192	-204	432	-264
$p = 53$	-338	558	222	414	-162
$p = 59$	100	540	420	-684	-772
$p = 61$	742	110	902	422	30
$p = 67$	-84	140	-1024	332	-764
$p = 71$	-328	-840	432	-360	-236
$p = 73$	-38	-550	362	26	418
$p = 79$	-240	-208	-160	512	552
$p = 83$	1212	516	72	-1188	1036
$p = 89$	330	-1398	810	-630	30
$p = 97$	866	1586	1106	-1054	-1190
$p = 101$	-1218	-1242	-258	558	1370
$p = 103$	-88	680	-988	8	464
$p = 107$	36	996	-24	1764	-2136
$p = 109$	-970	1382	950	1622	-1226
$p = 113$	1042	-750	-1038	-1134	338
$p = 127$	1936	176	-124	-592	2088
$p = 131$	732	-1548	132	-1908	-292
$p = 137$	-2214	378	-1254	954	818

Table 10: For the (1,1)-manifold, the coefficient a_p for the characteristic polynomial of Frobenius for the cases that the manifold has (hyper-) conifold singularities.

Acknowledgements

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A. A Corollary to the Frobenius-Chebotarëv Theorem

It follows from the celebrated Frobenius-Chebotarëv theorem [49] that if f is a polynomial with rational coefficients, that is irreducible over \mathbb{Q} , then the number of roots of f in \mathbb{F}_p , averaged over p , is one.

For the proof of this corollary, which must be quite well known, we first recall the Cauchy-Frobenius-Burnside lemma. This is often referred to as Burnside's lemma, but was earlier known to both Cauchy and Frobenius [50]. This lemma states that if X is a set on which a group G acts, and X^g denotes the set of elements of X that are fixed by $g \in G$. Then

$$\frac{1}{|G|} \sum_{g \in G} |X^g| = |X/G| . \quad (\text{A.1})$$

Thus the average of the size of the fixed point sets, over the group, is given by the number of group orbits.

For the application to the Frobenius-Chebotarëv theorem, take a polynomial f , as above, X to be the set whose elements are the roots of f and G to be the Galois group of f . Each $g \in G$ permutes the roots. Let us suppose that g has cycle type (n_1, n_2, \dots, n_r) , with $n_1 + \dots + n_r = n$, the degree of f . Since the roots of an irreducible polynomial are permuted transitively by the Galois group, there is only one orbit in X/G and the right hand side of (A.1) is one. If we reduce f mod p (and avoid a finite number of bad primes) then f will factor into irreducible factors mod p of degree (m_1, m_2, \dots, m_r) with $m_1 + \dots + m_r = n$. If f mod p has k linear factors, exactly k of the m_j are equal to one, and f has exactly k roots in \mathbb{F}_p . In (A.1) we have $\sum_{g \in G} |X^g| = k$.

The Frobenius-Chebotarëv theorem states that the density of primes p where we have factorization type (m_1, m_2, \dots, m_r) is equal to the proportion of elements $g \in G$ which have cycle type (m_1, m_2, \dots, m_r) . So the average number of roots of f in \mathbb{F}_p is indeed one.

The statement of the corollary has been proved for the case that f is irreducible over \mathbb{Q} . For the case that a polynomial F , with rational coefficients, is reducible, we consider the factorisation of F into irreducible factors. Suppose there are k such factors f_j , so

$$F = \prod_{j=1}^k f_j .$$

Applying the corollary to each factor, we see that the number of roots of F in \mathbb{F}_p , averaged over p , is k .

B. The p-adic Γ and ζ functions

We set out here a brief account of some results that relate to the p-adic Γ and ζ functions. Excellent textbook accounts can be found in [51–54]. There are however inequivalent definitions of $\zeta_p(3)$ in the literature, so, since this quantity is important to us, we recall the salient points of its definition here.

We begin with the p-adic Γ -function. First note that as a p-adic function the factorial function $n!$ tends rapidly to zero as n becomes large, since $n!$ then contains many factors of p . A better behaved function is obtained by omitting the terms divisible by p . Following from this observation, the Morita p-adic Γ -function is defined initially for $n \in \mathbb{Z}_{\geq 0}$ by

$$\Gamma_p(n) = (-1)^n \prod_{\substack{k=1 \\ p \nmid k}}^{n-1} k .$$

The definition can be extended to $\Gamma_p(z)$ for $z \in \mathbb{Z}_p$ by choosing a sequence of integers $\{n_j\}$ that converge to z and noting that Γ_p is p-adically smooth, that is

$$\Gamma_p(n + ap^m) \rightarrow \Gamma_p(n) \text{ as } m \rightarrow \infty .$$

The process of extending the definition from a subset of \mathbb{Z} to \mathbb{Z}_p , as above, is known as p-adic interpolation.

The p-adic function satisfies a recurrence relation that is an analogue of the familiar relation:

$$\Gamma_p(z+1) = \begin{cases} -z\Gamma_p(z) & \text{if } z \in \mathbb{Z}_p^*, \\ -\Gamma_p(z) & \text{if } z \in p\mathbb{Z}_p . \end{cases}$$

While $\Gamma_p(z)$ can be computed from the definition, the following procedure is more convenient. Define coefficients c_n via the expansion

$$\exp\left(x + \frac{x^p}{p}\right) = \sum_{n=0}^{\infty} c_n x^n .$$

By differentiating this expression, it is easy to show that the c_n satisfy the recursion relation

$$nc_n = c_{n-1} + c_{n-p} , \quad c_0 = 1 , \quad c_n = 0 \text{ for } n < 0 .$$

It was shown by Dwork that if $0 \leq a \leq p-1$ then

$$\Gamma_p(-a + pz) = \sum_{k=0}^{\infty} p^k c_{a+kp}(z)_k ,$$

where

$$(z)_k = z(z+1)\dots(z+k-1) ; \quad (z)_0 = 1$$

is the Pochammer symbol.

The function Γ_p is locally analytic, that is it can be expanded as a power series about every $z \in \mathbb{Z}_p$. This being so, it makes sense to define the p-adic digamma function on $p\mathbb{Z}_p$

$$\psi_p(z) = \frac{\Gamma'_p(z)}{\Gamma_p(z)}$$

which satisfies the relation

$$\psi_p(z+1) - \psi_p(z) = \begin{cases} \frac{1}{z} & \text{for } z \in \mathbb{Z}_p^*, \\ 0 & \text{for } z \in p\mathbb{Z}_p. \end{cases}$$

The digamma function is also locally analytic on $p\mathbb{Z}_p$

$$\psi_p(z) = b_0 - \sum_{n=1}^{\infty} \frac{b_n}{n} z^n; \quad \|z\|_p < 1. \quad (\text{B.1})$$

The coefficients b_n that appear in this relation can be expressed as integrals

$$b_0 = \int_{\mathbb{Z}_p^*} \log z \, dz; \quad b_n = \int_{\mathbb{Z}_p^*} z^{-n} \, dz, \quad n \geq 1. \quad (\text{B.2})$$

We note in passing that the coefficient $b_0 = \psi_p(0) = \psi_p(1)$ that appears in the expansion above is the p-adic analogue of Euler's constant $\gamma_E = -\Gamma'(1)$.

The integral that appears here is the Volkenborn integral [52]. It is somewhat analogous to a Riemann sum and has the definition

$$\int_{\mathbb{Z}_p} f(z) \, dz = \lim_{n \rightarrow \infty} \frac{1}{p^n} \sum_{k=0}^{p^n-1} f(k).$$

and for compact subsets $Y \subset \mathbb{Z}_p$ the integral is defined by

$$\int_Y f(z) \, dz = \int_{\mathbb{Z}_p} \Theta_Y(z) f(z) \, dz,$$

where Θ_Y is the characteristic function of Y .

It is also straightforward to show that, for $k \in \mathbb{Z}_{\geq 0}$

$$\int_{\mathbb{Z}_p^*} z^k \, dz = (1 - p^{k-1}) B_k, \quad (\text{B.3})$$

where the B_k are the Bernoulli numbers

$$B_0 = 1, \quad B_1 = -\frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_3 = 0, \quad B_4 = -\frac{1}{30}, \dots$$

and have generating function

$$\frac{t}{e^t - 1} = \sum_{k=0}^{\infty} B_k \frac{t^k}{k!}.$$

The Bernoulli numbers B_{2k+1} vanish for $k \geq 1$, and so the quantities $(1 - p^{k-1})B_k$ vanish for k odd, including $k = 1$. The numbers b_n also vanish for odd n .

Our interest in (B.3) and (B.2) is that $\zeta(s)$ can be related to the Bernoulli numbers and these are, in a sense that we shall now examine, related to the constants b_n . The question arises as to what extent the constants b_n and B_n can be interpolated as functions of n . In order to examine this, we enquire to what extent the function z^s , for $z \in \mathbb{Z}_p^*$ can be defined and is smooth, as a function of p -adic s .

Note first that it is not a priori obvious that this can be done. For suppose $z = n$, a rational integer not divisible by p , then, if n^s were continuous in s , the power n^{1+p^m} would tend to n as $m \rightarrow \infty$. However,

$$n^{1+p^m} = n n^{p^m} = n(n + \mathcal{O}(p)) = n^2 + \mathcal{O}(p),$$

so, if $n \neq 1 \pmod{p}$, this is not close to n .

On the other hand, there are two cases that can be interpolated. If $z = 1 + \mathcal{O}(p)$, that is $z = 1 + pz_1$, with $z_1 \in \mathbb{Z}_p$ then

$$z^s = (1 + pz_1)^s$$

and the expression on the right is defined through the binomial expansion and is continuous as a function of s . If the leading digit of z is not 1, we can still proceed by considering a set of integers that are equal mod $p - 1$

$$s = s_0 + (p - 1)s_1, \quad (\text{B.4})$$

We have

$$z^s = z^{s_0} (z^{p-1})^{s_1}$$

and $z^{p-1} = 1 + \mathcal{O}(p)$. This returns us to the previous case and we see that z^s is p -adically smooth since $z^{s+a(p-1)p^m} \rightarrow z^s$ as $m \rightarrow \infty$.

In virtue of the above we can write

$$b_{2n} = \lim_{m \rightarrow \infty} \int_{\mathbb{Z}_p^*} z^{(p-1)p^m - 2n} dz = \lim_{m \rightarrow \infty} B_{(p-1)p^m - 2n}. \quad (\text{B.5})$$

The numbers $(p - 1)p^m - 2n$ are, for $p > 2$, a sequence of positive even integers that tend p -adically to $-2n$.

We can now apply these considerations to the definition of a p -adic ζ -function.

The Riemann z -function is defined, for $\text{Re}(s) > 1$, by either of the relations

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{q \text{ prime}} \frac{1}{1 - q^{-s}}.$$

Consider first the representation as a sum. The sum contains terms for which $p|n$ and it seems best to remove these terms. So we define an apocopated function

$$\zeta^*(s) = \sum_{\substack{n=1 \\ p \nmid n}}^{\infty} \frac{1}{n^s} = \sum_{n=1}^{\infty} \frac{1}{n^s} - \sum_{n=1}^{\infty} \frac{1}{(pn)^s} = (1 - p^{-s})\zeta(s).$$

Note that

$$\zeta^*(s) = \prod_{\substack{q \text{ prime} \\ q \neq p}} \frac{1}{1 - q^{-s}} .$$

For this reason, the process of passing from $\zeta(s)$ to $\zeta^*(s)$ is known as “removing the Euler factor”.

For s a positive even integer, we have the formula, due to Euler,

$$\zeta(2n) = \frac{2^{2n-1}\pi^{2n}}{(2n)!} |B_{2n}| ,$$

with B_{2n} a Bernoulli number. For $s > 1$, a positive odd number, we have the values $\zeta(3)$, $\zeta(5)$, \dots , which are believed to be algebraically independent transcendentals. In neither case can the $\zeta(n)$, for n a positive integer, be understood as p -adic numbers.

The situation is better for the negative integers. For the negative even integers, we have

$$\zeta(-2k) = \begin{cases} -\frac{1}{2} & \text{for } k = 0 , \\ 0 & \text{for } k \geq 1 . \end{cases}$$

While for the negative odd integers we have

$$\zeta(1 - 2k) = -\frac{B_{2k}}{2k} ,$$

and so

$$\zeta^*(1 - 2k) = -\frac{1}{2k}(1 - p^{2k-1})B_{2k} = -\frac{b_{-2k}}{2k} .$$

We define $\zeta_p(s)$ by interpolating these values. Writing $1 - 2k = s$, we have

$$\zeta_p(s) = \frac{b_{s-1}}{s-1} .$$

and, in particular, $\zeta_p(3) = b_2/2$. The coefficient b_2 may be computed from (B.5) or, more efficiently, by noting that

$$b_1 = 0 \quad \text{and} \quad b_2 = -\psi_p''(0)$$

from which we learn that

$$\Gamma_p''(0) = \Gamma_p'(0)^2 \tag{B.6}$$

and

$$\zeta_p(3) = \frac{1}{2}b_2 = -\frac{1}{2}(\Gamma_p'''(0) - \Gamma_p'(0)^3) .$$

Although the definition that we have used for $\zeta_p(3)$ is widely used, other choices are possible. In a celebrated paper, Kubota and Leopoldt [55] seek a p -adic analogue $L_p(s, \chi)$ of the L -function

$$L(s, \chi) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s} = \prod_{q \text{ prime}} \frac{1}{1 - \chi(q)q^{-s}} ,$$

p	$\zeta_p(3)$
5	5072570838186014721957419926005297
	{2,4,1,2,3,1,4,2,0,2,0,0,2,3,3,0,0,4,2,4,0,1,4,0,2,3,2,2,3,2}
7	117061310239336308334025231434747323634935
	{1,3,5,4,1,0,2,0,2,4,1,2,5,0,4,6,2,4,4,5,6,3,2,2,5,3,2,1,5,2}
11	149134818187057310308096045871911437019561347245302
	{5,4,1,0,5,8,5,2,6,8,3,1,4,3,4,7,0,10,10,3,9,5,10,7,0,9,7,5,7}
13	1959147439192239725032744234064691597958143832554774385
	{6,6,5,2,11,12,5,1,5,2,7,5,10,10,6,7,12,0,10,2,12,0,8,3,1,8,12,7,0,2}
17	1047241053891811275086066324080237679295844245162126884817922
	{7,10,6,1,1,1,12,12,1,15,10,4,1,0,3,1,14,3,7,13,8,12,0,7,10,16,13,16,8,9}
19	366705732565363086376907501556670998446992440018555462542006132
	{5,16,17,8,0,5,13,12,3,14,7,9,8,13,17,18,10,10,2,2,11,9,10,15,12,14,3,11,3}

Table 11: The values of $\zeta_p(3)$ for $5 \leq p \leq 19$. These are given as integers mod p^{50} and also, mod p^{30} , as a list of p -adic digits .

where χ is a Dirichlet character. Clearly $L(s, 1) = \zeta(s)$, however the procedure of p -adic interpolation followed by Kubota and Leopoldt leads to the relation

$$\zeta_p(n) = L_p(n, \omega^{1-n}) ,$$

where ω is the Teichmüller character. In concrete terms, this means

$$\zeta_p(n) = \sum_{a=1}^{p-1} \omega(a)^{1-n} H_p(n, a, p) , \quad (\text{B.7})$$

with

$$H_p(n, a, p) = \frac{1}{(n-1)p} \langle a \rangle^{1-n} \sum_{j=0}^{\infty} \binom{1-n}{j} \left(\frac{p}{a}\right)^j B_j ,$$

where [56, 57]

$$\langle a \rangle = \frac{a}{\omega(a)} .$$

In fact one can replace the Teichmüller character $\omega(a)$ in (B.7) by $\omega(a)^k$ for the $p-1$ values $0 \leq k \leq p-2$ and these have an equal right to be called the p -adic zeta value. The element of choice goes back to the choice made in (B.4) of interpolating the zeta function through a particular sequence of integers. Other choices are possible and this leads to a situation whereby ζ_p is a function with $p-1$ ‘branches’.

It has been shown that for $p = 2, 3$ the number $\zeta_p(3)$ is irrational, [57, 58] (in the case $p = 2$, there is only one branch and, in the case $p = 3$, the two possible definitions coincide!). However, it is currently not even known if $\zeta_p(3)$ is nonzero for all p .

Let us return to the p -adic gamma function. Even the briefest account must make mention of two fundamental identities, presented here without proof, that are very close to the corresponding identities for the classical gamma function. The first is the analogue of Euler's reflection formula. For $z \in \mathbb{Z}_p$ write

$$z = z_0 + pz_1$$

where $z_0 \in \{1, 2, \dots, p\}$, in other words z_0 is the first p -adic digit of z , unless $z \in p\mathbb{Z}_p$, in which case $z_0 = p$. Then we have

$$\Gamma_p(z)\Gamma_p(1-z) = (-1)^{z_0}.$$

The second identity is the analogue of the multiplication formula, which can be written in the form

$$\frac{\prod_{r=0}^{m-1} \Gamma_p\left(\frac{z+r}{m}\right)}{\Gamma_p(z) \prod_{r=1}^{m-1} \Gamma_p\left(\frac{r}{m}\right)} = m^{1-z_0} (m^{-(p-1)})^{z_1},$$

with z_0 and z_1 as above. Remarkably, the classical analogue of this formula can be written by replacing the p -adic gamma functions by the classical functions and the right hand side of the expression by $m^{(1-z)}$.

One use of the reflection formula is the following. If $z \in p\mathbb{Z}_p$, then the reflection formula becomes

$$\Gamma_p(z)\Gamma_p(-z) = 1.$$

On expanding the gamma functions on the left as Taylor series, we discover many identities relating the derivatives $\Gamma_p^{(n)}(0)$; of which (B.6) is but the first.

Finally, let us return to the expansion for the digamma function (B.6), substitute what we know for the coefficients and integrate. We come to the useful expression

$$\Gamma_p(z) = \exp \left(\Gamma'_p(0)z - \sum_{k=1}^{\infty} \frac{\zeta_p(2k+1)}{2k+1} z^{2k+1} \right). \quad (\text{B.8})$$

כִּתְבֵּ קָיוֹן וְבָאָר עַל־הַלְּחֹות לְמַעַן יָרוֹץ קֹרְאָ בָּזָ:

*Write the vision, and make it plain in tables,
so that he that runs past may read it.*

Habakkuk 2.2

C. ζ -Function Tables

C.1. The ζ -function for the mirror of the quintic threefold, AESZ 1

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 5T + 55pT^2 + 5p^3T^3 + p^6T^4$
2	smooth		$1 + 25T + 50pT^2 + 25p^3T^3 + p^6T^4$
3	smooth		$1 + 10T + 60pT^2 + 10p^3T^3 + p^6T^4$
4	smooth		$1 - 5T - 30pT^2 - 5p^3T^3 + p^6T^4$
5	singular	5^{-5}	$(1 + pT)(1 - 6T + p^3T^2)$
6	smooth		$1 - 5pT + 115pT^2 - 5p^4T^3 + p^6T^4$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	5^{-5}	$(1 - pT)(1 + 43T + p^3T^2)$
2	smooth		$1 - 14T - 74pT^2 - 14p^3T^3 + p^6T^4$
3	smooth		$1 - 29T + 6p^2T^2 - 29p^3T^3 + p^6T^4$
4	smooth		$1 + 31T + 131pT^2 + 31p^3T^3 + p^6T^4$
5	smooth		$1 + T - 14pT^2 + p^3T^3 + p^6T^4$
6	smooth		$1 - 9T - 4pT^2 - 9p^3T^3 + p^6T^4$
7	smooth		$1 - 54T + 266pT^2 - 54p^3T^3 + p^6T^4$
8	smooth		$1 + 31T + 206pT^2 + 31p^3T^3 + p^6T^4$
9	smooth		$1 + 26T + 236pT^2 + 26p^3T^3 + p^6T^4$
10	smooth		$1 - 14T + 76pT^2 - 14p^3T^3 + p^6T^4$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 5T + 160pT^2 - 5p^3T^3 + p^6T^4$
2	smooth		$1 + 15T + 120pT^2 + 15p^3T^3 + p^6T^4$

Continued on the following page

$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 + 85T + 405pT^2 + 85p^3T^3 + p^6T^4$
4	smooth		$(1 + 6pT + p^3T^2)(1 - 68T + p^3T^2)$
5	smooth		$1 - 15T - 20pT^2 - 15p^3T^3 + p^6T^4$
6	smooth		$(1 - 6pT + p^3T^2)(1 - 42T + p^3T^2)$
7	smooth		$1 + 20T - 90pT^2 + 20p^3T^3 + p^6T^4$
8	singular	5^{-5}	$(1 + pT)(1 + 28T + p^3T^2)$
9	smooth		$1 - 5T + 20p^2T^2 - 5p^3T^3 + p^6T^4$
10	smooth		$1 + 25T + 100pT^2 + 25p^3T^3 + p^6T^4$
11	smooth		$1 - 25T + 275pT^2 - 25p^3T^3 + p^6T^4$
12	smooth		$1 - 25T + 100pT^2 - 25p^3T^3 + p^6T^4$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 5pT + 660pT^2 + 5p^4T^3 + p^6T^4$
2	smooth		$1 + 70T + 270pT^2 + 70p^3T^3 + p^6T^4$
3	smooth		$1 - 25T - 25pT^2 - 25p^3T^3 + p^6T^4$
4	smooth		$1 + 25T + 225pT^2 + 25p^3T^3 + p^6T^4$
5	smooth		$1 + 20T + 120pT^2 + 20p^3T^3 + p^6T^4$
6	smooth		$1 + 15T + 90pT^2 + 15p^3T^3 + p^6T^4$
7	smooth		$1 - 20T + 130pT^2 - 20p^3T^3 + p^6T^4$
8	smooth		$1 + 55T + 280pT^2 + 55p^3T^3 + p^6T^4$
9	smooth		$1 - 45T + 530pT^2 - 45p^3T^3 + p^6T^4$
10	smooth		$1 + 75T + 100pT^2 + 75p^3T^3 + p^6T^4$
11	singular	5^{-5}	$(1 + pT)(1 - 91T + p^3T^2)$
12	smooth		$1 + 5T + 5pT^2 + 5p^3T^3 + p^6T^4$
13	smooth		$1 - 75T + 150pT^2 - 75p^3T^3 + p^6T^4$
14	smooth		$1 - 140T + 710pT^2 - 140p^3T^3 + p^6T^4$
15	smooth		$1 + 120T + 570pT^2 + 120p^3T^3 + p^6T^4$
16	smooth		$1 - 90T + 610pT^2 - 90p^3T^3 + p^6T^4$

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 60T + 122pT^2 - 60p^3T^3 + p^6T^4$
2	smooth		$1 - 120T + 522pT^2 - 120p^3T^3 + p^6T^4$
3	smooth		$1 - 90T + 372pT^2 - 90p^3T^3 + p^6T^4$
4	smooth		$1 + 45T - 153pT^2 + 45p^3T^3 + p^6T^4$
5	smooth		$1 + 15T - 378pT^2 + 15p^3T^3 + p^6T^4$
6	smooth		$1 + 25T - 12p^2T^2 + 25p^3T^3 + p^6T^4$

Continued on the following page

$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 + 185T + 1097pT^2 + 185p^3T^3 + p^6T^4$
8	smooth		$1 + 130T + 822pT^2 + 130p^3T^3 + p^6T^4$
9	smooth		$1 + 65T + 597pT^2 + 65p^3T^3 + p^6T^4$
10	smooth		$1 + 5T + 197pT^2 + 5p^3T^3 + p^6T^4$
11	smooth		$1 - 130T + 572pT^2 - 130p^3T^3 + p^6T^4$
12	smooth		$1 + 55T + 597pT^2 + 55p^3T^3 + p^6T^4$
13	smooth		$(1 - 5pT + p^3T^2)(1 + 100T + p^3T^2)$
14	smooth		$1 - 75T + 422pT^2 - 75p^3T^3 + p^6T^4$
15	smooth		$1 + 20T + 22pT^2 + 20p^3T^3 + p^6T^4$
16	smooth		$1 + 20T + 222pT^2 + 20p^3T^3 + p^6T^4$
17	singular	5^{-5}	$(1 - pT)(1 + 35T + p^3T^2)$
18	smooth		$1 - 110T + 422pT^2 - 110p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 130T + 240pT^2 + 130p^3T^3 + p^6T^4$
2	smooth		$1 + 40T + 670pT^2 + 40p^3T^3 + p^6T^4$
3	smooth		$1 + 160T + 930pT^2 + 160p^3T^3 + p^6T^4$
4	smooth		$1 + 105T + 90pT^2 + 105p^3T^3 + p^6T^4$
5	smooth		$1 - 15T + 730pT^2 - 15p^3T^3 + p^6T^4$
6	smooth		$1 - 55T - 90pT^2 - 55p^3T^3 + p^6T^4$
7	smooth		$1 + 50T + 450pT^2 + 50p^3T^3 + p^6T^4$
8	smooth		$1 - 60T + 220pT^2 - 60p^3T^3 + p^6T^4$
9	smooth		$1 - 135T + 395pT^2 - 135p^3T^3 + p^6T^4$
10	smooth		$1 - 5T + 210pT^2 - 5p^3T^3 + p^6T^4$
11	smooth		$1 - 40T - 520pT^2 - 40p^3T^3 + p^6T^4$
12	smooth		$1 - 5pT + 605pT^2 - 5p^4T^3 + p^6T^4$
13	smooth		$1 + 45T + 335pT^2 + 45p^3T^3 + p^6T^4$
14	smooth		$1 + 175T + 1100pT^2 + 175p^3T^3 + p^6T^4$
15	singular	5^{-5}	$(1 + pT)(1 - 162T + p^3T^2)$
16	smooth		$1 + 150T + 850pT^2 + 150p^3T^3 + p^6T^4$
17	smooth		$1 - 245T + 1665pT^2 - 245p^3T^3 + p^6T^4$
18	smooth		$1 + 350pT^2 + p^6T^4$
19	smooth		$1 + 105T + 365pT^2 + 105p^3T^3 + p^6T^4$
20	smooth		$1 - 100T + 850pT^2 - 100p^3T^3 + p^6T^4$
21	smooth		$1 + 55T + 865pT^2 + 55p^3T^3 + p^6T^4$
22	smooth		$1 - 105T + 960pT^2 - 105p^3T^3 + p^6T^4$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 240T + 1582pT^2 + 240p^3T^3 + p^6T^4$
2	smooth		$1 - 245T + 1732pT^2 - 245p^3T^3 + p^6T^4$
3	smooth		$1 - 105T + 182pT^2 - 105p^3T^3 + p^6T^4$
4	singular	5^{-5}	$(1 - pT)(1 - 160T + p^3T^2)$
5	smooth		$1 + 80T + 782pT^2 + 80p^3T^3 + p^6T^4$
6	smooth		$1 + 125T + 8p^2T^2 + 125p^3T^3 + p^6T^4$
7	smooth		$1 + 420T + 3032pT^2 + 420p^3T^3 + p^6T^4$
8	smooth		$1 - 90T - 118pT^2 - 90p^3T^3 + p^6T^4$
9	smooth		$1 - 15T - 68pT^2 - 15p^3T^3 + p^6T^4$
10	smooth		$1 + 15T - 318pT^2 + 15p^3T^3 + p^6T^4$
11	smooth		$1 - 10T + 932pT^2 - 10p^3T^3 + p^6T^4$
12	smooth		$1 - 175T + 1607pT^2 - 175p^3T^3 + p^6T^4$
13	smooth		$1 - 175T + 1607pT^2 - 175p^3T^3 + p^6T^4$
14	smooth		$1 + 135T + 557pT^2 + 135p^3T^3 + p^6T^4$
15	smooth		$1 + 45T + 932pT^2 + 45p^3T^3 + p^6T^4$
16	smooth		$1 + 50T + 1132pT^2 + 50p^3T^3 + p^6T^4$
17	smooth		$1 + 140T + 582pT^2 + 140p^3T^3 + p^6T^4$
18	smooth		$1 - 25T + 757pT^2 - 25p^3T^3 + p^6T^4$
19	smooth		$1 + 75T + 1357pT^2 + 75p^3T^3 + p^6T^4$
20	smooth		$1 - 35T + 432pT^2 - 35p^3T^3 + p^6T^4$
21	smooth		$1 + 90T + 682pT^2 + 90p^3T^3 + p^6T^4$
22	smooth		$1 - 185T + 682pT^2 - 185p^3T^3 + p^6T^4$
23	smooth		$1 - 80T - 718pT^2 - 80p^3T^3 + p^6T^4$
24	smooth		$1 - 125T + 107pT^2 - 125p^3T^3 + p^6T^4$
25	smooth		$1 + 45T + 482pT^2 + 45p^3T^3 + p^6T^4$
26	smooth		$1 - 20T - 718pT^2 - 20p^3T^3 + p^6T^4$
27	smooth		$1 - 250T + 1482pT^2 - 250p^3T^3 + p^6T^4$
28	smooth		$1 + 265T + 1932pT^2 + 265p^3T^3 + p^6T^4$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 - 7pT + p^3T^2)(1 + 108T + p^3T^2)$
2	smooth		$1 - 114T + 846pT^2 - 114p^3T^3 + p^6T^4$
3	smooth		$1 + 111T + 946pT^2 + 111p^3T^3 + p^6T^4$
4	smooth		$1 + 366T + 2966pT^2 + 366p^3T^3 + p^6T^4$
5	singular	5^{-5}	$(1 - pT)(1 - 42T + p^3T^2)$
6	smooth		$1 + 66T + 566pT^2 + 66p^3T^3 + p^6T^4$
7	smooth		$1 + 81T + 1401pT^2 + 81p^3T^3 + p^6T^4$
8	smooth		$1 - 144T + 1026pT^2 - 144p^3T^3 + p^6T^4$

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$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
9	smooth		$1 - 39T + 471pT^2 - 39p^3T^3 + p^6T^4$
10	smooth		$1 + 191T + 1091pT^2 + 191p^3T^3 + p^6T^4$
11	smooth		$1 + 16T - 14p^2T^2 + 16p^3T^3 + p^6T^4$
12	smooth		$1 - 174T + 1906pT^2 - 174p^3T^3 + p^6T^4$
13	smooth		$1 + 26T + 156pT^2 + 26p^3T^3 + p^6T^4$
14	smooth		$1 - 4T - 214pT^2 - 4p^3T^3 + p^6T^4$
15	smooth		$1 - 139T + 796pT^2 - 139p^3T^3 + p^6T^4$
16	smooth		$1 - 44T - 774pT^2 - 44p^3T^3 + p^6T^4$
17	smooth		$1 + T + 206pT^2 + p^3T^3 + p^6T^4$
18	smooth		$1 - 154T + 6p^2T^2 - 154p^3T^3 + p^6T^4$
19	smooth		$1 + 81T - 99pT^2 + 81p^3T^3 + p^6T^4$
20	smooth		$1 - 4pT + 1056pT^2 - 4p^4T^3 + p^6T^4$
21	smooth		$1 + 111T + 846pT^2 + 111p^3T^3 + p^6T^4$
22	smooth		$1 + 171T + 1611pT^2 + 171p^3T^3 + p^6T^4$
23	smooth		$1 + pT + 1576pT^2 + p^4T^3 + p^6T^4$
24	smooth		$1 - 119T + 76pT^2 - 119p^3T^3 + p^6T^4$
25	smooth		$1 + 391T + 86p^2T^2 + 391p^3T^3 + p^6T^4$
26	smooth		$1 - 259T + 1591pT^2 - 259p^3T^3 + p^6T^4$
27	smooth		$1 - 339T + 2746pT^2 - 339p^3T^3 + p^6T^4$
28	smooth		$1 + 176T + 1406pT^2 + 176p^3T^3 + p^6T^4$
29	smooth		$1 - 49T + 26p^2T^2 - 49p^3T^3 + p^6T^4$
30	smooth		$1 + 66T - 684pT^2 + 66p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 265T + 1885pT^2 - 265p^3T^3 + p^6T^4$
2	smooth		$1 - 540T + 4210pT^2 - 540p^3T^3 + p^6T^4$
3	smooth		$1 + 180T + 1080pT^2 + 180p^3T^3 + p^6T^4$
4	smooth		$1 + 5pT + 1585pT^2 + 5p^4T^3 + p^6T^4$
5	smooth		$1 + 160T + 510pT^2 + 160p^3T^3 + p^6T^4$
6	smooth		$1 - 205T + 1470pT^2 - 205p^3T^3 + p^6T^4$
7	smooth		$1 - 315T + 1760pT^2 - 315p^3T^3 + p^6T^4$
8	smooth		$1 + 5pT + 960pT^2 + 5p^4T^3 + p^6T^4$
9	smooth		$1 - 15T - 1265pT^2 - 15p^3T^3 + p^6T^4$
10	smooth		$1 + 130T + 2130pT^2 + 130p^3T^3 + p^6T^4$
11	smooth		$1 + 155T - 245pT^2 + 155p^3T^3 + p^6T^4$
12	smooth		$1 + 385T + 2910pT^2 + 385p^3T^3 + p^6T^4$
13	smooth		$1 - 155T + 2645pT^2 - 155p^3T^3 + p^6T^4$
14	smooth		$1 - 55T + 60p^2T^2 - 55p^3T^3 + p^6T^4$

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$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
15	smooth		$1 - 130T + 2170pT^2 - 130p^3T^3 + p^6T^4$
16	smooth		$1 - 55T + 845pT^2 - 55p^3T^3 + p^6T^4$
17	smooth		$1 + 15T + 2040pT^2 + 15p^3T^3 + p^6T^4$
18	smooth		$1 - 325T + 1900pT^2 - 325p^3T^3 + p^6T^4$
19	smooth		$1 + 155T + 1080pT^2 + 155p^3T^3 + p^6T^4$
20	smooth		$1 - 105T - 705pT^2 - 105p^3T^3 + p^6T^4$
21	smooth		$1 + 80T + 1430pT^2 + 80p^3T^3 + p^6T^4$
22	smooth		$1 + 145T + 70pT^2 + 145p^3T^3 + p^6T^4$
23	smooth		$1 + 75T - 800pT^2 + 75p^3T^3 + p^6T^4$
24	singular	5^{-5}	$(1 + pT)(1 + 314T + p^3T^2)$
25	smooth		$1 - 320T + 1430pT^2 - 320p^3T^3 + p^6T^4$
26	smooth		$1 - 140T + 1910pT^2 - 140p^3T^3 + p^6T^4$
27	smooth		$1 + 380T + 3580pT^2 + 380p^3T^3 + p^6T^4$
28	smooth		$1 + 135T + 1310pT^2 + 135p^3T^3 + p^6T^4$
29	smooth		$1 + 30T - 1020pT^2 + 30p^3T^3 + p^6T^4$
30	smooth		$1 + 25T + 500pT^2 + 25p^3T^3 + p^6T^4$
31	smooth		$1 + 120T + 170pT^2 + 120p^3T^3 + p^6T^4$
32	smooth		$1 + 90T + 1590pT^2 + 90p^3T^3 + p^6T^4$
33	smooth		$1 + 180T + 1780pT^2 + 180p^3T^3 + p^6T^4$
34	smooth		$1 - 210T + 1290pT^2 - 210p^3T^3 + p^6T^4$
35	smooth		$1 - 495T + 3855pT^2 - 495p^3T^3 + p^6T^4$
36	smooth		$1 + 170T + 1320pT^2 + 170p^3T^3 + p^6T^4$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 + 8pT + p^3T^2)(1 - 372T + p^3T^2)$
2	smooth		$1 - 154T - 204pT^2 - 154p^3T^3 + p^6T^4$
3	smooth		$1 - 219T - 389pT^2 - 219p^3T^3 + p^6T^4$
4	smooth		$1 + 371T + 2646pT^2 + 371p^3T^3 + p^6T^4$
5	smooth		$1 - 44T - 914pT^2 - 44p^3T^3 + p^6T^4$
6	smooth		$1 - 509T + 4676pT^2 - 509p^3T^3 + p^6T^4$
7	smooth		$1 + 111T + 2906pT^2 + 111p^3T^3 + p^6T^4$
8	smooth		$1 - 4pT + 1206pT^2 - 4p^4T^3 + p^6T^4$
9	smooth		$1 + 581T + 4761pT^2 + 581p^3T^3 + p^6T^4$
10	smooth		$1 + 546T + 4196pT^2 + 546p^3T^3 + p^6T^4$
11	smooth		$1 + 146T - 854pT^2 + 146p^3T^3 + p^6T^4$
12	smooth		$1 - 184T - 124pT^2 - 184p^3T^3 + p^6T^4$
13	smooth		$1 + 201T + 716pT^2 + 201p^3T^3 + p^6T^4$
14	smooth		$1 - 144T + 2086pT^2 - 144p^3T^3 + p^6T^4$

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$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
15	smooth		$1 + 126T - 134pT^2 + 126p^3T^3 + p^6T^4$
16	smooth		$1 + 11T + 1956pT^2 + 11p^3T^3 + p^6T^4$
17	smooth		$1 + 301T + 2291pT^2 + 301p^3T^3 + p^6T^4$
18	smooth		$1 + 381T + 2186pT^2 + 381p^3T^3 + p^6T^4$
19	smooth		$1 + T - 2284pT^2 + p^3T^3 + p^6T^4$
20	smooth		$1 - 154T + 846pT^2 - 154p^3T^3 + p^6T^4$
21	smooth		$1 + 266T + 1576pT^2 + 266p^3T^3 + p^6T^4$
22	smooth		$1 - 434T + 4226pT^2 - 434p^3T^3 + p^6T^4$
23	smooth		$1 - 194T + 1586pT^2 - 194p^3T^3 + p^6T^4$
24	smooth		$1 - 239T + 1906pT^2 - 239p^3T^3 + p^6T^4$
25	smooth		$1 - 14T + 406pT^2 - 14p^3T^3 + p^6T^4$
26	smooth		$1 - 179T + 371pT^2 - 179p^3T^3 + p^6T^4$
27	smooth		$1 - 219T + 2736pT^2 - 219p^3T^3 + p^6T^4$
28	smooth		$1 + 216T + 2276pT^2 + 216p^3T^3 + p^6T^4$
29	smooth		$1 + 31T - 989pT^2 + 31p^3T^3 + p^6T^4$
30	smooth		$1 + 66T + 1976pT^2 + 66p^3T^3 + p^6T^4$
31	smooth		$1 - 469T + 3611pT^2 - 469p^3T^3 + p^6T^4$
32	singular	5^{-5}	$(1 - pT)(1 + 203T + p^3T^2)$
33	smooth		$1 - 169T + 936pT^2 - 169p^3T^3 + p^6T^4$
34	smooth		$1 + 301T + 1966pT^2 + 301p^3T^3 + p^6T^4$
35	smooth		$1 + 56T + 2386pT^2 + 56p^3T^3 + p^6T^4$
36	smooth		$1 - 89T + 2781pT^2 - 89p^3T^3 + p^6T^4$
37	smooth		$1 + 61T + 2256pT^2 + 61p^3T^3 + p^6T^4$
38	smooth		$1 - 269T + 2336pT^2 - 269p^3T^3 + p^6T^4$
39	smooth		$1 + T + 1216pT^2 + p^3T^3 + p^6T^4$
40	smooth		$1 - 44T + 2886pT^2 - 44p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 135T + 670pT^2 - 135p^3T^3 + p^6T^4$
2	smooth		$1 - 540T + 4330pT^2 - 540p^3T^3 + p^6T^4$
3	singular	5^{-5}	$(1 + pT)(1 - 92T + p^3T^2)$
4	smooth		$1 + 180T + 1590pT^2 + 180p^3T^3 + p^6T^4$
5	smooth		$1 - 80T - 390pT^2 - 80p^3T^3 + p^6T^4$
6	smooth		$1 - 140T + 10p^2T^2 - 140p^3T^3 + p^6T^4$
7	smooth		$1 - 875T + 8125pT^2 - 875p^3T^3 + p^6T^4$
8	smooth		$1 + 100T + 2800pT^2 + 100p^3T^3 + p^6T^4$
9	smooth		$1 + 160T - 570pT^2 + 160p^3T^3 + p^6T^4$
10	smooth		$1 - 365T + 4430pT^2 - 365p^3T^3 + p^6T^4$

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$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 - 370T + 3790pT^2 - 370p^3T^3 + p^6T^4$
12	smooth		$1 - 25T + 2250pT^2 - 25p^3T^3 + p^6T^4$
13	smooth		$1 - 55T + 510pT^2 - 55p^3T^3 + p^6T^4$
14	smooth		$1 - 20T + 2290pT^2 - 20p^3T^3 + p^6T^4$
15	smooth		$1 + 375T + 2450pT^2 + 375p^3T^3 + p^6T^4$
16	smooth		$1 - 105T + 1410pT^2 - 105p^3T^3 + p^6T^4$
17	smooth		$1 + 600T + 4450pT^2 + 600p^3T^3 + p^6T^4$
18	smooth		$1 + 115T + 195pT^2 + 115p^3T^3 + p^6T^4$
19	smooth		$1 - 445T + 2890pT^2 - 445p^3T^3 + p^6T^4$
20	smooth		$1 - 265T + 2155pT^2 - 265p^3T^3 + p^6T^4$
21	smooth		$1 - 105T + 3110pT^2 - 105p^3T^3 + p^6T^4$
22	smooth		$1 + 160T + 330pT^2 + 160p^3T^3 + p^6T^4$
23	smooth		$1 - 185T + 670pT^2 - 185p^3T^3 + p^6T^4$
24	smooth		$1 + 565T + 5370pT^2 + 565p^3T^3 + p^6T^4$
25	smooth		$1 + 405T + 3765pT^2 + 405p^3T^3 + p^6T^4$
26	smooth		$1 + 165T + 3445pT^2 + 165p^3T^3 + p^6T^4$
27	smooth		$1 + 180T + 390pT^2 + 180p^3T^3 + p^6T^4$
28	smooth		$1 - 100T - 50pT^2 - 100p^3T^3 + p^6T^4$
29	smooth		$1 + 245T + 960pT^2 + 245p^3T^3 + p^6T^4$
30	smooth		$1 + 230T + 1440pT^2 + 230p^3T^3 + p^6T^4$
31	smooth		$1 - 85T - 1605pT^2 - 85p^3T^3 + p^6T^4$
32	smooth		$1 + 125T + 2500pT^2 + 125p^3T^3 + p^6T^4$
33	smooth		$1 + 25T - 1850pT^2 + 25p^3T^3 + p^6T^4$
34	smooth		$1 - 25T + 2350pT^2 - 25p^3T^3 + p^6T^4$
35	smooth		$1 + 425T + 3975pT^2 + 425p^3T^3 + p^6T^4$
36	smooth		$1 - 30T + 3310pT^2 - 30p^3T^3 + p^6T^4$
37	smooth		$1 + 90T + 40p^2T^2 + 90p^3T^3 + p^6T^4$
38	smooth		$1 - 90T - 40p^2T^2 - 90p^3T^3 + p^6T^4$
39	smooth		$1 - 105T + 1185pT^2 - 105p^3T^3 + p^6T^4$
40	smooth		$1 + 175T + 1350pT^2 + 175p^3T^3 + p^6T^4$
41	smooth		$1 - 275T + 700pT^2 - 275p^3T^3 + p^6T^4$
42	smooth		$1 + 150T + 750pT^2 + 150p^3T^3 + p^6T^4$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 65T + 1790pT^2 + 65p^3T^3 + p^6T^4$
2	smooth		$1 - 190T + 3410pT^2 - 190p^3T^3 + p^6T^4$
3	smooth		$1 + 110T + 1110pT^2 + 110p^3T^3 + p^6T^4$
4	smooth		$1 + 330T + 4530pT^2 + 330p^3T^3 + p^6T^4$

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$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
5	smooth		$1 + 385T + 2610pT^2 + 385p^3T^3 + p^6T^4$
6	smooth		$1 + 390T + 4040pT^2 + 390p^3T^3 + p^6T^4$
7	smooth		$1 + 450T + 4450pT^2 + 450p^3T^3 + p^6T^4$
8	smooth		$1 - 325T + 3825pT^2 - 325p^3T^3 + p^6T^4$
9	smooth		$1 + 555T + 3930pT^2 + 555p^3T^3 + p^6T^4$
10	smooth		$1 - 45T - 1145pT^2 - 45p^3T^3 + p^6T^4$
11	smooth		$1 + 230T - 1070pT^2 + 230p^3T^3 + p^6T^4$
12	smooth		$1 + 40T - 30p^2T^2 + 40p^3T^3 + p^6T^4$
13	smooth		$1 - 450T + 3500pT^2 - 450p^3T^3 + p^6T^4$
14	smooth		$1 + 120T + 2420pT^2 + 120p^3T^3 + p^6T^4$
15	smooth		$1 - 285T + 3065pT^2 - 285p^3T^3 + p^6T^4$
16	smooth		$1 + 80T + 480pT^2 + 80p^3T^3 + p^6T^4$
17	smooth		$1 + 410T + 1910pT^2 + 410p^3T^3 + p^6T^4$
18	smooth		$1 - 205T + 770pT^2 - 205p^3T^3 + p^6T^4$
19	smooth		$1 + 285T + 3410pT^2 + 285p^3T^3 + p^6T^4$
20	smooth		$1 - 15T - 640pT^2 - 15p^3T^3 + p^6T^4$
21	smooth		$1 - 385T + 3165pT^2 - 385p^3T^3 + p^6T^4$
22	smooth		$1 - 105T + 770pT^2 - 105p^3T^3 + p^6T^4$
23	smooth		$1 - 435T + 3065pT^2 - 435p^3T^3 + p^6T^4$
24	smooth		$1 + 145T + 1345pT^2 + 145p^3T^3 + p^6T^4$
25	smooth		$1 + 365T + 2340pT^2 + 365p^3T^3 + p^6T^4$
26	smooth		$1 + 215T + 2065pT^2 + 215p^3T^3 + p^6T^4$
27	smooth		$1 + 415T + 4065pT^2 + 415p^3T^3 + p^6T^4$
28	smooth		$1 - 720T + 5530pT^2 - 720p^3T^3 + p^6T^4$
29	smooth		$1 - 240T + 610pT^2 - 240p^3T^3 + p^6T^4$
30	smooth		$1 - 15T - 1765pT^2 - 15p^3T^3 + p^6T^4$
31	smooth		$1 - 305T + 3170pT^2 - 305p^3T^3 + p^6T^4$
32	smooth		$1 + 280T + 2780pT^2 + 280p^3T^3 + p^6T^4$
33	smooth		$1 - 210T + 2090pT^2 - 210p^3T^3 + p^6T^4$
34	smooth		$1 - 45T + 1780pT^2 - 45p^3T^3 + p^6T^4$
35	smooth		$1 - 75T - 275pT^2 - 75p^3T^3 + p^6T^4$
36	smooth		$1 - 145T + 2755pT^2 - 145p^3T^3 + p^6T^4$
37	smooth		$1 - 335T + 1690pT^2 - 335p^3T^3 + p^6T^4$
38	smooth		$1 - 5pT + 2190pT^2 - 5p^4T^3 + p^6T^4$
39	smooth		$1 - 870T + 7730pT^2 - 870p^3T^3 + p^6T^4$
40	smooth		$1 + 355T + 755pT^2 + 355p^3T^3 + p^6T^4$
41	smooth		$1 + 480T + 4430pT^2 + 480p^3T^3 + p^6T^4$
42	smooth		$(1 - 12pT + p^3T^2)(1 + 264T + p^3T^2)$
43	smooth		$1 - 70T + 2080pT^2 - 70p^3T^3 + p^6T^4$
44	smooth		$1 + 260T + 1010pT^2 + 260p^3T^3 + p^6T^4$

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$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
45	singular	5^{-5}	$(1 + pT)(1 - 196T + p^3T^2)$
46	smooth		$1 + 190T + 3990pT^2 + 190p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 270T + 2890pT^2 - 270p^3T^3 + p^6T^4$
2	smooth		$1 + 10T - 270pT^2 + 10p^3T^3 + p^6T^4$
3	smooth		$1 + 125T - 3200pT^2 + 125p^3T^3 + p^6T^4$
4	smooth		$1 - 405T + 4435pT^2 - 405p^3T^3 + p^6T^4$
5	smooth		$1 - 710T + 5670pT^2 - 710p^3T^3 + p^6T^4$
6	smooth		$1 + 210T + 2030pT^2 + 210p^3T^3 + p^6T^4$
7	smooth		$1 + 465T + 1645pT^2 + 465p^3T^3 + p^6T^4$
8	smooth		$1 + 245T - 590pT^2 + 245p^3T^3 + p^6T^4$
9	smooth		$1 + 310T + 3130pT^2 + 310p^3T^3 + p^6T^4$
10	smooth		$1 + 140T + 3570pT^2 + 140p^3T^3 + p^6T^4$
11	smooth		$1 - 425T + 4100pT^2 - 425p^3T^3 + p^6T^4$
12	smooth		$1 - 180T + 1010pT^2 - 180p^3T^3 + p^6T^4$
13	smooth		$1 + 185T + 4530pT^2 + 185p^3T^3 + p^6T^4$
14	smooth		$1 + 430T + 4590pT^2 + 430p^3T^3 + p^6T^4$
15	smooth		$1 + 90T + 2570pT^2 + 90p^3T^3 + p^6T^4$
16	smooth		$1 + 640T + 4870pT^2 + 640p^3T^3 + p^6T^4$
17	smooth		$1 + 10T + 2530pT^2 + 10p^3T^3 + p^6T^4$
18	smooth		$1 + 10pT + 4890pT^2 + 10p^4T^3 + p^6T^4$
19	smooth		$1 + 15T + 2395pT^2 + 15p^3T^3 + p^6T^4$
20	smooth		$1 - 75T + 5475pT^2 - 75p^3T^3 + p^6T^4$
21	smooth		$1 - 985T + 9970pT^2 - 985p^3T^3 + p^6T^4$
22	smooth		$1 + 340T + 5770pT^2 + 340p^3T^3 + p^6T^4$
23	smooth		$1 + 150T + 3900pT^2 + 150p^3T^3 + p^6T^4$
24	smooth		$1 - 160T + 4470pT^2 - 160p^3T^3 + p^6T^4$
25	smooth		$1 + 30T + 2140pT^2 + 30p^3T^3 + p^6T^4$
26	singular	5^{-5}	$(1 + pT)(1 - 82T + p^3T^2)$
27	smooth		$1 + 5T + 2540pT^2 + 5p^3T^3 + p^6T^4$
28	smooth		$1 - 225T - 475pT^2 - 225p^3T^3 + p^6T^4$
29	smooth		$1 + 70T - 1090pT^2 + 70p^3T^3 + p^6T^4$
30	smooth		$1 - 555T + 4060pT^2 - 555p^3T^3 + p^6T^4$
31	smooth		$1 + 260T - 1170pT^2 + 260p^3T^3 + p^6T^4$
32	smooth		$1 - 235T - 55pT^2 - 235p^3T^3 + p^6T^4$
33	smooth		$1 + 695T + 5435pT^2 + 695p^3T^3 + p^6T^4$
34	smooth		$1 - 90T + 1430pT^2 - 90p^3T^3 + p^6T^4$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 + 35T - 1595pT^2 + 35p^3T^3 + p^6T^4$
36	smooth		$1 + 370T + 5610pT^2 + 370p^3T^3 + p^6T^4$
37	smooth		$1 - 405T + 2360pT^2 - 405p^3T^3 + p^6T^4$
38	smooth		$1 - 305T + 3760pT^2 - 305p^3T^3 + p^6T^4$
39	smooth		$1 - 225T + 5375pT^2 - 225p^3T^3 + p^6T^4$
40	smooth		$1 + 5T + 340pT^2 + 5p^3T^3 + p^6T^4$
41	smooth		$1 - 555T + 4360pT^2 - 555p^3T^3 + p^6T^4$
42	smooth		$1 + 825T + 8000pT^2 + 825p^3T^3 + p^6T^4$
43	smooth		$1 - 100T - 3700pT^2 - 100p^3T^3 + p^6T^4$
44	smooth		$1 + 55T - 3785pT^2 + 55p^3T^3 + p^6T^4$
45	smooth		$1 + 50T + 3050pT^2 + 50p^3T^3 + p^6T^4$
46	smooth		$1 + 575T + 5600pT^2 + 575p^3T^3 + p^6T^4$
47	smooth		$1 - 165T + 1380pT^2 - 165p^3T^3 + p^6T^4$
48	smooth		$1 - 5pT + 4230pT^2 - 5p^4T^3 + p^6T^4$
49	smooth		$1 - 165T + 355pT^2 - 165p^3T^3 + p^6T^4$
50	smooth		$1 - 1005T + 9860pT^2 - 1005p^3T^3 + p^6T^4$
51	smooth		$1 + 615T + 3695pT^2 + 615p^3T^3 + p^6T^4$
52	smooth		$1 + 50T + 1250pT^2 + 50p^3T^3 + p^6T^4$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 70T + 2812pT^2 - 70p^3T^3 + p^6T^4$
2	smooth		$1 + 70T - 4938pT^2 + 70p^3T^3 + p^6T^4$
3	smooth		$1 - 195T + 3212pT^2 - 195p^3T^3 + p^6T^4$
4	smooth		$1 + 1185T + 12712pT^2 + 1185p^3T^3 + p^6T^4$
5	smooth		$1 - 375T + 662pT^2 - 375p^3T^3 + p^6T^4$
6	smooth		$1 + 4162pT^2 + p^6T^4$
7	smooth		$1 + 380T - 538pT^2 + 380p^3T^3 + p^6T^4$
8	smooth		$1 - 385T + 1987pT^2 - 385p^3T^3 + p^6T^4$
9	smooth		$1 - 200T + 4162pT^2 - 200p^3T^3 + p^6T^4$
10	smooth		$1 - 355T + 2837pT^2 - 355p^3T^3 + p^6T^4$
11	smooth		$1 + 1160T + 11962pT^2 + 1160p^3T^3 + p^6T^4$
12	smooth		$1 - 65T + 2362pT^2 - 65p^3T^3 + p^6T^4$
13	smooth		$1 + 145T - 3538pT^2 + 145p^3T^3 + p^6T^4$
14	smooth		$1 - 765T + 8962pT^2 - 765p^3T^3 + p^6T^4$
15	smooth		$1 - 475T + 3387pT^2 - 475p^3T^3 + p^6T^4$
16	smooth		$1 - 455T + 2837pT^2 - 455p^3T^3 + p^6T^4$
17	smooth		$1 + 510T + 4862pT^2 + 510p^3T^3 + p^6T^4$
18	smooth		$1 - 220T + 5812pT^2 - 220p^3T^3 + p^6T^4$

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$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$1 + 225T + 2787pT^2 + 225p^3T^3 + p^6T^4$
20	smooth		$1 + 30T + 3212pT^2 + 30p^3T^3 + p^6T^4$
21	smooth		$1 + 705T + 6087pT^2 + 705p^3T^3 + p^6T^4$
22	smooth		$1 - 370T + 2212pT^2 - 370p^3T^3 + p^6T^4$
23	smooth		$1 - 315T + 1862pT^2 - 315p^3T^3 + p^6T^4$
24	smooth		$1 - 510T + 6362pT^2 - 510p^3T^3 + p^6T^4$
25	smooth		$1 - 130T + 1812pT^2 - 130p^3T^3 + p^6T^4$
26	smooth		$1 - 250T + 5162pT^2 - 250p^3T^3 + p^6T^4$
27	smooth		$(1 + 10pT + p^3T^2)(1 + 150T + p^3T^2)$
28	smooth		$1 + 230T + 6562pT^2 + 230p^3T^3 + p^6T^4$
29	singular	5^{-5}	$(1 - pT)(1 + 280T + p^3T^2)$
30	smooth		$1 - 60T - 4588pT^2 - 60p^3T^3 + p^6T^4$
31	smooth		$1 + 260T - 138pT^2 + 260p^3T^3 + p^6T^4$
32	smooth		$(1 - 5pT + p^3T^2)(1 - 50T + p^3T^2)$
33	smooth		$1 + 450T + 5762pT^2 + 450p^3T^3 + p^6T^4$
34	smooth		$1 + 645T + 5212pT^2 + 645p^3T^3 + p^6T^4$
35	smooth		$1 + 195T + 562pT^2 + 195p^3T^3 + p^6T^4$
36	smooth		$1 + 35T + 587pT^2 + 35p^3T^3 + p^6T^4$
37	smooth		$1 - 165T + 6587pT^2 - 165p^3T^3 + p^6T^4$
38	smooth		$1 + 105T + 18p^2T^2 + 105p^3T^3 + p^6T^4$
39	smooth		$1 - 530T + 3562pT^2 - 530p^3T^3 + p^6T^4$
40	smooth		$1 - 475T + 2262pT^2 - 475p^3T^3 + p^6T^4$
41	smooth		$1 - 195T + 1337pT^2 - 195p^3T^3 + p^6T^4$
42	smooth		$1 - 565T + 4862pT^2 - 565p^3T^3 + p^6T^4$
43	smooth		$1 - 995T + 9962pT^2 - 995p^3T^3 + p^6T^4$
44	smooth		$1 + 185T + 1412pT^2 + 185p^3T^3 + p^6T^4$
45	smooth		$1 - 75T + 6662pT^2 - 75p^3T^3 + p^6T^4$
46	smooth		$(1 + p^3T^2)(1 + 770T + p^3T^2)$
47	smooth		$1 + 175T - 213pT^2 + 175p^3T^3 + p^6T^4$
48	smooth		$1 - 355T - 563pT^2 - 355p^3T^3 + p^6T^4$
49	smooth		$1 - 825T + 7262pT^2 - 825p^3T^3 + p^6T^4$
50	smooth		$1 + 350T + 4262pT^2 + 350p^3T^3 + p^6T^4$
51	smooth		$1 - 415T + 3462pT^2 - 415p^3T^3 + p^6T^4$
52	smooth		$1 + 455T + 2062pT^2 + 455p^3T^3 + p^6T^4$
53	smooth		$1 + 540T + 3212pT^2 + 540p^3T^3 + p^6T^4$
54	smooth		$1 - 200T + 4162pT^2 - 200p^3T^3 + p^6T^4$
55	smooth		$1 - 105T + 312pT^2 - 105p^3T^3 + p^6T^4$
56	smooth		$1 - 120T + 962pT^2 - 120p^3T^3 + p^6T^4$
57	smooth		$1 + 425T + 287pT^2 + 425p^3T^3 + p^6T^4$
58	smooth		$1 + 370T - 988pT^2 + 370p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 11T + 2276pT^2 + 11p^3T^3 + p^6T^4$
2	smooth		$1 - 124T + 4236pT^2 - 124p^3T^3 + p^6T^4$
3	smooth		$1 - 869T + 6356pT^2 - 869p^3T^3 + p^6T^4$
4	smooth		$1 - 384T + 4796pT^2 - 384p^3T^3 + p^6T^4$
5	smooth		$1 - 344T + 4206pT^2 - 344p^3T^3 + p^6T^4$
6	smooth		$1 + 401T - 664pT^2 + 401p^3T^3 + p^6T^4$
7	smooth		$1 + 216T + 3996pT^2 + 216p^3T^3 + p^6T^4$
8	smooth		$1 - 64T + 1926pT^2 - 64p^3T^3 + p^6T^4$
9	smooth		$1 - 599T + 1936pT^2 - 599p^3T^3 + p^6T^4$
10	smooth		$1 - 419T + 96p^2T^2 - 419p^3T^3 + p^6T^4$
11	smooth		$1 + 636T + 8526pT^2 + 636p^3T^3 + p^6T^4$
12	smooth		$1 - 249T - 1489pT^2 - 249p^3T^3 + p^6T^4$
13	smooth		$1 - 214T + 3726pT^2 - 214p^3T^3 + p^6T^4$
14	smooth		$1 + 486T + 326pT^2 + 486p^3T^3 + p^6T^4$
15	smooth		$1 - 429T + 1666pT^2 - 429p^3T^3 + p^6T^4$
16	smooth		$1 + 736T + 7126pT^2 + 736p^3T^3 + p^6T^4$
17	smooth		$1 + 21T + 2916pT^2 + 21p^3T^3 + p^6T^4$
18	smooth		$1 + 351T + 1436pT^2 + 351p^3T^3 + p^6T^4$
19	smooth		$1 + 836T + 9126pT^2 + 836p^3T^3 + p^6T^4$
20	smooth		$1 - 494T + 8156pT^2 - 494p^3T^3 + p^6T^4$
21	smooth		$1 - 364T + 526pT^2 - 364p^3T^3 + p^6T^4$
22	smooth		$1 - 134T - 1554pT^2 - 134p^3T^3 + p^6T^4$
23	smooth		$1 - 529T + 1966pT^2 - 529p^3T^3 + p^6T^4$
24	smooth		$1 - 509T + 5796pT^2 - 509p^3T^3 + p^6T^4$
25	smooth		$1 + 501T + 786pT^2 + 501p^3T^3 + p^6T^4$
26	smooth		$1 + 116T + 3696pT^2 + 116p^3T^3 + p^6T^4$
27	smooth		$1 + 1036T + 11326pT^2 + 1036p^3T^3 + p^6T^4$
28	smooth		$1 - 594T + 3356pT^2 - 594p^3T^3 + p^6T^4$
29	smooth		$1 + 561T + 8176pT^2 + 561p^3T^3 + p^6T^4$
30	smooth		$1 + 346T + 4866pT^2 + 346p^3T^3 + p^6T^4$
31	smooth		$1 - 9pT + 7961pT^2 - 9p^4T^3 + p^6T^4$
32	smooth		$1 - 489T + 3276pT^2 - 489p^3T^3 + p^6T^4$
33	smooth		$1 - 1024T + 9786pT^2 - 1024p^3T^3 + p^6T^4$
34	smooth		$(1 + 13pT + p^3T^2)(1 - 182T + p^3T^2)$
35	smooth		$1 + 486T + 6676pT^2 + 486p^3T^3 + p^6T^4$
36	smooth		$1 - 279T + 1091pT^2 - 279p^3T^3 + p^6T^4$
37	smooth		$1 - 64T + 926pT^2 - 64p^3T^3 + p^6T^4$
38	smooth		$1 + 381T + 3756pT^2 + 381p^3T^3 + p^6T^4$
39	smooth		$1 - 29T - 2309pT^2 - 29p^3T^3 + p^6T^4$
40	smooth		$1 - 489T + 2651pT^2 - 489p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 - 34T + 3996pT^2 - 34p^3T^3 + p^6T^4$
42	smooth		$1 - 79T + 5766pT^2 - 79p^3T^3 + p^6T^4$
43	smooth		$1 + 31T - 494pT^2 + 31p^3T^3 + p^6T^4$
44	smooth		$1 + 541T + 3746pT^2 + 541p^3T^3 + p^6T^4$
45	smooth		$1 + 206T + 2306pT^2 + 206p^3T^3 + p^6T^4$
46	smooth		$1 + 21T - 6684pT^2 + 21p^3T^3 + p^6T^4$
47	smooth		$1 + 11T - 2099pT^2 + 11p^3T^3 + p^6T^4$
48	singular	5^{-5}	$(1 - pT)(1 + 518T + p^3T^2)$
49	smooth		$1 + 111T - 1299pT^2 + 111p^3T^3 + p^6T^4$
50	smooth		$1 - 689T + 8176pT^2 - 689p^3T^3 + p^6T^4$
51	smooth		$1 - 154T + 4266pT^2 - 154p^3T^3 + p^6T^4$
52	smooth		$1 + 191T + 2171pT^2 + 191p^3T^3 + p^6T^4$
53	smooth		$1 - 594T + 5156pT^2 - 594p^3T^3 + p^6T^4$
54	smooth		$1 - 274T + 4786pT^2 - 274p^3T^3 + p^6T^4$
55	smooth		$1 + 271T + 3891pT^2 + 271p^3T^3 + p^6T^4$
56	smooth		$1 - 44T + 7056pT^2 - 44p^3T^3 + p^6T^4$
57	smooth		$1 + 91T + 2771pT^2 + 91p^3T^3 + p^6T^4$
58	smooth		$1 + 876T + 10386pT^2 + 876p^3T^3 + p^6T^4$
59	smooth		$1 + 711T + 4751pT^2 + 711p^3T^3 + p^6T^4$
60	smooth		$1 - 139T - 299pT^2 - 139p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 550T + 4650pT^2 + 550p^3T^3 + p^6T^4$
2	smooth		$1 - 260T + 5490pT^2 - 260p^3T^3 + p^6T^4$
3	smooth		$1 + 435T + 7885pT^2 + 435p^3T^3 + p^6T^4$
4	smooth		$1 + 160T - 5890pT^2 + 160p^3T^3 + p^6T^4$
5	smooth		$1 + 40T + 4490pT^2 + 40p^3T^3 + p^6T^4$
6	smooth		$1 + 315T + 7615pT^2 + 315p^3T^3 + p^6T^4$
7	smooth		$1 - 185T - 1810pT^2 - 185p^3T^3 + p^6T^4$
8	smooth		$1 + 60T + 4610pT^2 + 60p^3T^3 + p^6T^4$
9	smooth		$1 - 190T - 1190pT^2 - 190p^3T^3 + p^6T^4$
10	smooth		$1 + 390T - 30p^2T^2 + 390p^3T^3 + p^6T^4$
11	smooth		$1 + 5450pT^2 + p^6T^4$
12	smooth		$1 - 135T + 915pT^2 - 135p^3T^3 + p^6T^4$
13	smooth		$1 + 145T + 1370pT^2 + 145p^3T^3 + p^6T^4$
14	smooth		$1 - 435T + 115pT^2 - 435p^3T^3 + p^6T^4$
15	smooth		$1 + 610T + 3910pT^2 + 610p^3T^3 + p^6T^4$
16	smooth		$1 - 555T + 2370pT^2 - 555p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 265T - 2015pT^2 - 265p^3T^3 + p^6T^4$
18	smooth		$1 + 280T + 2830pT^2 + 280p^3T^3 + p^6T^4$
19	smooth		$1 - 210T + 4090pT^2 - 210p^3T^3 + p^6T^4$
20	smooth		$1 + 115T + 7390pT^2 + 115p^3T^3 + p^6T^4$
21	smooth		$1 + 470T + 3370pT^2 + 470p^3T^3 + p^6T^4$
22	smooth		$1 - 130T + 5920pT^2 - 130p^3T^3 + p^6T^4$
23	smooth		$1 - 850T + 7800pT^2 - 850p^3T^3 + p^6T^4$
24	smooth		$1 - 185T - 1635pT^2 - 185p^3T^3 + p^6T^4$
25	smooth		$1 - 955T + 9220pT^2 - 955p^3T^3 + p^6T^4$
26	smooth		$1 - 45T - 2270pT^2 - 45p^3T^3 + p^6T^4$
27	smooth		$1 + 580T + 9880pT^2 + 580p^3T^3 + p^6T^4$
28	smooth		$1 + 790T + 5290pT^2 + 790p^3T^3 + p^6T^4$
29	smooth		$1 + 35T + 710pT^2 + 35p^3T^3 + p^6T^4$
30	smooth		$1 - 170T + 1030pT^2 - 170p^3T^3 + p^6T^4$
31	smooth		$1 + 15T + 6140pT^2 + 15p^3T^3 + p^6T^4$
32	smooth		$1 + 6900pT^2 + p^6T^4$
33	smooth		$1 + 365T + 1590pT^2 + 365p^3T^3 + p^6T^4$
34	smooth		$1 - 10T - 3060pT^2 - 10p^3T^3 + p^6T^4$
35	smooth		$1 + 265T + 4440pT^2 + 265p^3T^3 + p^6T^4$
36	smooth		$1 - 45T + 5530pT^2 - 45p^3T^3 + p^6T^4$
37	smooth		$1 - 605T + 1445pT^2 - 605p^3T^3 + p^6T^4$
38	smooth		$1 - 905T + 9320pT^2 - 905p^3T^3 + p^6T^4$
39	smooth		$1 + 310T + 3610pT^2 + 310p^3T^3 + p^6T^4$
40	smooth		$1 + 1660T + 18460pT^2 + 1660p^3T^3 + p^6T^4$
41	smooth		$1 - 325T + 4750pT^2 - 325p^3T^3 + p^6T^4$
42	smooth		$1 - 425T + 7750pT^2 - 425p^3T^3 + p^6T^4$
43	smooth		$1 + 220T - 1580pT^2 + 220p^3T^3 + p^6T^4$
44	smooth		$1 + 30T + 4230pT^2 + 30p^3T^3 + p^6T^4$
45	smooth		$1 - 245T + 4405pT^2 - 245p^3T^3 + p^6T^4$
46	smooth		$1 + 1035T + 8510pT^2 + 1035p^3T^3 + p^6T^4$
47	smooth		$1 + 995T + 9870pT^2 + 995p^3T^3 + p^6T^4$
48	smooth		$1 + 255T + 5530pT^2 + 255p^3T^3 + p^6T^4$
49	smooth		$1 + 645T + 5095pT^2 + 645p^3T^3 + p^6T^4$
50	smooth		$1 + 615T + 7015pT^2 + 615p^3T^3 + p^6T^4$
51	smooth		$1 + 310T + 3910pT^2 + 310p^3T^3 + p^6T^4$
52	smooth		$1 - 850T + 150p^2T^2 - 850p^3T^3 + p^6T^4$
53	singular	5^{-5}	$(1 + pT)(1 - 141T + p^3T^2)$
54	smooth		$1 + 35T + 4810pT^2 + 35p^3T^3 + p^6T^4$
55	smooth		$1 + 390T + 3190pT^2 + 390p^3T^3 + p^6T^4$
56	smooth		$1 + 60T + 2610pT^2 + 60p^3T^3 + p^6T^4$

Continued on the following page

$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
57	smooth		$1 - 35T + 2790pT^2 - 35p^3T^3 + p^6T^4$
58	smooth		$1 - 755T + 7920pT^2 - 755p^3T^3 + p^6T^4$
59	smooth		$1 + 320T + 820pT^2 + 320p^3T^3 + p^6T^4$
60	smooth		$1 + 445T + 2870pT^2 + 445p^3T^3 + p^6T^4$
61	smooth		$1 - 1385T + 14040pT^2 - 1385p^3T^3 + p^6T^4$
62	smooth		$1 + 5T + 2480pT^2 + 5p^3T^3 + p^6T^4$
63	smooth		$1 + 25T + 2450pT^2 + 25p^3T^3 + p^6T^4$
64	smooth		$1 - 1235T + 12190pT^2 - 1235p^3T^3 + p^6T^4$
65	smooth		$1 - 75T + 6125pT^2 - 75p^3T^3 + p^6T^4$
66	smooth		$1 - 1435T + 16140pT^2 - 1435p^3T^3 + p^6T^4$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	5^{-5}	$(1 - pT)(1 - 412T + p^3T^2)$
2	smooth		$1 + 726T + 10846pT^2 + 726p^3T^3 + p^6T^4$
3	smooth		$1 - 569T + 10666pT^2 - 569p^3T^3 + p^6T^4$
4	smooth		$1 - 109T + 2131pT^2 - 109p^3T^3 + p^6T^4$
5	smooth		$1 - 279T - 3374pT^2 - 279p^3T^3 + p^6T^4$
6	smooth		$1 - 379T + 9751pT^2 - 379p^3T^3 + p^6T^4$
7	smooth		$1 - 689T + 7686pT^2 - 689p^3T^3 + p^6T^4$
8	smooth		$1 + 1186T + 14536pT^2 + 1186p^3T^3 + p^6T^4$
9	smooth		$1 - 204T + 8526pT^2 - 204p^3T^3 + p^6T^4$
10	smooth		$1 - 434T - 694pT^2 - 434p^3T^3 + p^6T^4$
11	smooth		$1 + 121T + 6526pT^2 + 121p^3T^3 + p^6T^4$
12	smooth		$1 + 186T + 1586pT^2 + 186p^3T^3 + p^6T^4$
13	smooth		$1 + 86T - 2814pT^2 + 86p^3T^3 + p^6T^4$
14	smooth		$1 - 789T + 3761pT^2 - 789p^3T^3 + p^6T^4$
15	smooth		$1 + 116T - 894pT^2 + 116p^3T^3 + p^6T^4$
16	smooth		$1 - 19T + 3591pT^2 - 19p^3T^3 + p^6T^4$
17	smooth		$1 + 256T + 2866pT^2 + 256p^3T^3 + p^6T^4$
18	smooth		$1 + 1666T + 19406pT^2 + 1666p^3T^3 + p^6T^4$
19	smooth		$1 + 491T + 1081pT^2 + 491p^3T^3 + p^6T^4$
20	smooth		$1 + 601T + 3121pT^2 + 601p^3T^3 + p^6T^4$
21	smooth		$1 - 224T + 5946pT^2 - 224p^3T^3 + p^6T^4$
22	smooth		$1 - 169T + 7741pT^2 - 169p^3T^3 + p^6T^4$
23	smooth		$1 + 176T + 6846pT^2 + 176p^3T^3 + p^6T^4$
24	smooth		$1 + 1076T + 11346pT^2 + 1076p^3T^3 + p^6T^4$
25	smooth		$1 - 379T + 10526pT^2 - 379p^3T^3 + p^6T^4$
26	smooth		$1 - 624T + 1946pT^2 - 624p^3T^3 + p^6T^4$

Continued on the following page

$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
27	smooth		$(1 - 12pT + p^3T^2)(1 + 153T + p^3T^2)$
28	smooth		$1 - 744T + 3966pT^2 - 744p^3T^3 + p^6T^4$
29	smooth		$1 - 594T + 4466pT^2 - 594p^3T^3 + p^6T^4$
30	smooth		$1 - 549T + 1546pT^2 - 549p^3T^3 + p^6T^4$
31	smooth		$1 + 76T + 126p^2T^2 + 76p^3T^3 + p^6T^4$
32	smooth		$1 + 726T + 5996pT^2 + 726p^3T^3 + p^6T^4$
33	smooth		$1 - 699T + 3296pT^2 - 699p^3T^3 + p^6T^4$
34	smooth		$1 + 301T - 279pT^2 + 301p^3T^3 + p^6T^4$
35	smooth		$1 - 1449T + 16121pT^2 - 1449p^3T^3 + p^6T^4$
36	smooth		$1 + 406T + 6266pT^2 + 406p^3T^3 + p^6T^4$
37	smooth		$1 - 299T + 421pT^2 - 299p^3T^3 + p^6T^4$
38	smooth		$1 + 176T + 7446pT^2 + 176p^3T^3 + p^6T^4$
39	smooth		$1 - 524T + 5996pT^2 - 524p^3T^3 + p^6T^4$
40	smooth		$1 - 149T - 3754pT^2 - 149p^3T^3 + p^6T^4$
41	smooth		$1 + 626T + 8196pT^2 + 626p^3T^3 + p^6T^4$
42	smooth		$1 + 151T + 7771pT^2 + 151p^3T^3 + p^6T^4$
43	smooth		$1 + 306T + 8316pT^2 + 306p^3T^3 + p^6T^4$
44	smooth		$1 + 331T + 6266pT^2 + 331p^3T^3 + p^6T^4$
45	smooth		$1 - 1049T + 10046pT^2 - 1049p^3T^3 + p^6T^4$
46	smooth		$1 + 126T - 3604pT^2 + 126p^3T^3 + p^6T^4$
47	smooth		$1 + 321T - 3899pT^2 + 321p^3T^3 + p^6T^4$
48	smooth		$1 - 299T + 7296pT^2 - 299p^3T^3 + p^6T^4$
49	smooth		$1 - 349T - 3154pT^2 - 349p^3T^3 + p^6T^4$
50	smooth		$1 + 731T + 5966pT^2 + 731p^3T^3 + p^6T^4$
51	smooth		$1 + 726T + 4746pT^2 + 726p^3T^3 + p^6T^4$
52	smooth		$1 + 151T + 1321pT^2 + 151p^3T^3 + p^6T^4$
53	smooth		$1 - 189T + 6386pT^2 - 189p^3T^3 + p^6T^4$
54	smooth		$1 - 464T + 2236pT^2 - 464p^3T^3 + p^6T^4$
55	smooth		$1 + 56T + 96p^2T^2 + 56p^3T^3 + p^6T^4$
56	smooth		$1 - 119T - 7309pT^2 - 119p^3T^3 + p^6T^4$
57	smooth		$1 + 561T + 7086pT^2 + 561p^3T^3 + p^6T^4$
58	smooth		$1 + 266T + 1906pT^2 + 266p^3T^3 + p^6T^4$
59	smooth		$1 - 34T - 894pT^2 - 34p^3T^3 + p^6T^4$
60	smooth		$1 + 616T + 3156pT^2 + 616p^3T^3 + p^6T^4$
61	smooth		$1 - 404T + 5076pT^2 - 404p^3T^3 + p^6T^4$
62	smooth		$1 + 1261T + 14161pT^2 + 1261p^3T^3 + p^6T^4$
63	smooth		$1 - 29T + 7151pT^2 - 29p^3T^3 + p^6T^4$
64	smooth		$1 - 9T + 4106pT^2 - 9p^3T^3 + p^6T^4$
65	smooth		$1 - 234T - 2494pT^2 - 234p^3T^3 + p^6T^4$
66	smooth		$1 - 904T + 7826pT^2 - 904p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
67	smooth		$1 + 596T + 7826pT^2 + 596p^3T^3 + p^6T^4$
68	smooth		$1 - 1039T + 8786pT^2 - 1039p^3T^3 + p^6T^4$
69	smooth		$1 + 331T + 2266pT^2 + 331p^3T^3 + p^6T^4$
70	smooth		$1 + 651T + 2646pT^2 + 651p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 395T + 8410pT^2 + 395p^3T^3 + p^6T^4$
2	smooth		$1 + 1005T + 11890pT^2 + 1005p^3T^3 + p^6T^4$
3	smooth		$1 - 560T + 5270pT^2 - 560p^3T^3 + p^6T^4$
4	smooth		$1 + 175T + 2200pT^2 + 175p^3T^3 + p^6T^4$
5	smooth		$1 + 250T + 4050pT^2 + 250p^3T^3 + p^6T^4$
6	smooth		$1 + 555T + 4240pT^2 + 555p^3T^3 + p^6T^4$
7	smooth		$1 + 445T + 6735pT^2 + 445p^3T^3 + p^6T^4$
8	smooth		$1 - 140T + 4730pT^2 - 140p^3T^3 + p^6T^4$
9	smooth		$1 + 945T + 11035pT^2 + 945p^3T^3 + p^6T^4$
10	smooth		$1 + 345T + 4985pT^2 + 345p^3T^3 + p^6T^4$
11	smooth		$1 - 275T + 7700pT^2 - 275p^3T^3 + p^6T^4$
12	smooth		$1 + 115T + 4045pT^2 + 115p^3T^3 + p^6T^4$
13	smooth		$1 + 535T + 5680pT^2 + 535p^3T^3 + p^6T^4$
14	smooth		$1 - 10pT + 70p^2T^2 - 10p^4T^3 + p^6T^4$
15	smooth		$1 - 420T - 1360pT^2 - 420p^3T^3 + p^6T^4$
16	smooth		$1 - 275T + 7275pT^2 - 275p^3T^3 + p^6T^4$
17	smooth		$1 - 65T - 2895pT^2 - 65p^3T^3 + p^6T^4$
18	smooth		$1 - 570T + 1890pT^2 - 570p^3T^3 + p^6T^4$
19	smooth		$1 + 1185T + 13905pT^2 + 1185p^3T^3 + p^6T^4$
20	smooth		$1 + 120T - 3490pT^2 + 120p^3T^3 + p^6T^4$
21	smooth		$1 - 110T + 1320pT^2 - 110p^3T^3 + p^6T^4$
22	smooth		$1 + 555T + 6415pT^2 + 555p^3T^3 + p^6T^4$
23	smooth		$1 + 280T + 390pT^2 + 280p^3T^3 + p^6T^4$
24	smooth		$1 - 620T + 2740pT^2 - 620p^3T^3 + p^6T^4$
25	smooth		$1 + 795T + 6760pT^2 + 795p^3T^3 + p^6T^4$
26	singular	5^{-5}	$(1 + pT)(1 + 763T + p^3T^2)$
27	smooth		$1 + 490T + 9520pT^2 + 490p^3T^3 + p^6T^4$
28	smooth		$1 + 655T + 1665pT^2 + 655p^3T^3 + p^6T^4$
29	smooth		$1 - 1035T + 10120pT^2 - 1035p^3T^3 + p^6T^4$
30	smooth		$1 + 555T - 685pT^2 + 555p^3T^3 + p^6T^4$
31	smooth		$1 - 20T - 3210pT^2 - 20p^3T^3 + p^6T^4$
32	smooth		$1 - 810T + 10770pT^2 - 810p^3T^3 + p^6T^4$

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$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
33	smooth		$1 + 105T + 2740pT^2 + 105p^3T^3 + p^6T^4$
34	smooth		$1 - 1415T + 14280pT^2 - 1415p^3T^3 + p^6T^4$
35	smooth		$1 + 990T + 11470pT^2 + 990p^3T^3 + p^6T^4$
36	smooth		$1 + 190T + 4070pT^2 + 190p^3T^3 + p^6T^4$
37	smooth		$1 + 220T + 6510pT^2 + 220p^3T^3 + p^6T^4$
38	smooth		$(1 - 9pT + p^3T^2)(1 - 508T + p^3T^2)$
39	smooth		$1 + 550T + 10450pT^2 + 550p^3T^3 + p^6T^4$
40	smooth		$1 + 470T - 440pT^2 + 470p^3T^3 + p^6T^4$
41	smooth		$1 + 285T - 4120pT^2 + 285p^3T^3 + p^6T^4$
42	smooth		$1 + 400T + 5350pT^2 + 400p^3T^3 + p^6T^4$
43	smooth		$1 - 850T + 11050pT^2 - 850p^3T^3 + p^6T^4$
44	smooth		$(1 + 14pT + p^3T^2)(1 - 872T + p^3T^2)$
45	smooth		$1 - 75T + 1100pT^2 - 75p^3T^3 + p^6T^4$
46	smooth		$1 + 425T + 1025pT^2 + 425p^3T^3 + p^6T^4$
47	smooth		$1 - 260T - 5230pT^2 - 260p^3T^3 + p^6T^4$
48	smooth		$1 + 595T + 11435pT^2 + 595p^3T^3 + p^6T^4$
49	smooth		$1 - 230T + 860pT^2 - 230p^3T^3 + p^6T^4$
50	smooth		$1 - 90T + 1930pT^2 - 90p^3T^3 + p^6T^4$
51	smooth		$1 - 1010T + 10570pT^2 - 1010p^3T^3 + p^6T^4$
52	smooth		$1 - 1430T + 16410pT^2 - 1430p^3T^3 + p^6T^4$
53	smooth		$1 + 535T + 10055pT^2 + 535p^3T^3 + p^6T^4$
54	smooth		$1 + 495T + 3960pT^2 + 495p^3T^3 + p^6T^4$
55	smooth		$1 - 335T + 3945pT^2 - 335p^3T^3 + p^6T^4$
56	smooth		$1 - 860T + 6270pT^2 - 860p^3T^3 + p^6T^4$
57	smooth		$1 - 790T + 7930pT^2 - 790p^3T^3 + p^6T^4$
58	smooth		$1 - 1315T + 12180pT^2 - 1315p^3T^3 + p^6T^4$
59	smooth		$1 - 10T - 3430pT^2 - 10p^3T^3 + p^6T^4$
60	smooth		$1 + 140T + 3320pT^2 + 140p^3T^3 + p^6T^4$
61	smooth		$1 - 430T + 10860pT^2 - 430p^3T^3 + p^6T^4$
62	smooth		$1 + 470T + 2410pT^2 + 470p^3T^3 + p^6T^4$
63	smooth		$1 + 435T + 8705pT^2 + 435p^3T^3 + p^6T^4$
64	smooth		$1 - 250T - 2550pT^2 - 250p^3T^3 + p^6T^4$
65	smooth		$1 - 215T + 7855pT^2 - 215p^3T^3 + p^6T^4$
66	smooth		$1 + 130T + 6940pT^2 + 130p^3T^3 + p^6T^4$
67	smooth		$1 + 485T + 6030pT^2 + 485p^3T^3 + p^6T^4$
68	smooth		$1 - 465T + 10380pT^2 - 465p^3T^3 + p^6T^4$
69	smooth		$1 - 855T + 7885pT^2 - 855p^3T^3 + p^6T^4$
70	smooth		$1 + 25T - 4700pT^2 + 25p^3T^3 + p^6T^4$
71	smooth		$1 + 1115T + 11020pT^2 + 1115p^3T^3 + p^6T^4$
72	smooth		$1 - 765T + 8180pT^2 - 765p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 845T + 6982pT^2 + 845p^3T^3 + p^6T^4$
2	smooth		$1 + 420T + 4482pT^2 + 420p^3T^3 + p^6T^4$
3	smooth		$1 + 20T + 8982pT^2 + 20p^3T^3 + p^6T^4$
4	smooth		$1 - 425T - 4068pT^2 - 425p^3T^3 + p^6T^4$
5	smooth		$1 + 210T + 6132pT^2 + 210p^3T^3 + p^6T^4$
6	smooth		$1 - 190T + 4632pT^2 - 190p^3T^3 + p^6T^4$
7	smooth		$1 - 460T + 7882pT^2 - 460p^3T^3 + p^6T^4$
8	smooth		$1 + 975T + 8932pT^2 + 975p^3T^3 + p^6T^4$
9	singular	5^{-5}	$(1 - pT)(1 - 510T + p^3T^2)$
10	smooth		$1 + 75T + 182pT^2 + 75p^3T^3 + p^6T^4$
11	smooth		$1 - 415T + 9882pT^2 - 415p^3T^3 + p^6T^4$
12	smooth		$1 - 105T + 5957pT^2 - 105p^3T^3 + p^6T^4$
13	smooth		$1 - 845T + 13482pT^2 - 845p^3T^3 + p^6T^4$
14	smooth		$1 + 200T + 8032pT^2 + 200p^3T^3 + p^6T^4$
15	smooth		$1 + 115T - 5018pT^2 + 115p^3T^3 + p^6T^4$
16	smooth		$1 - 885T + 13382pT^2 - 885p^3T^3 + p^6T^4$
17	smooth		$1 - 5pT + 2082pT^2 - 5p^4T^3 + p^6T^4$
18	smooth		$1 + 2482pT^2 + p^6T^4$
19	smooth		$1 - 715T + 4557pT^2 - 715p^3T^3 + p^6T^4$
20	smooth		$1 + 515T + 5007pT^2 + 515p^3T^3 + p^6T^4$
21	smooth		$1 - 385T + 9382pT^2 - 385p^3T^3 + p^6T^4$
22	smooth		$1 + 435T + 4107pT^2 + 435p^3T^3 + p^6T^4$
23	smooth		$1 + 1260T + 15382pT^2 + 1260p^3T^3 + p^6T^4$
24	smooth		$1 - 745T + 2957pT^2 - 745p^3T^3 + p^6T^4$
25	smooth		$1 - 425T + 8107pT^2 - 425p^3T^3 + p^6T^4$
26	smooth		$1 + 450T + 12532pT^2 + 450p^3T^3 + p^6T^4$
27	smooth		$1 - 955T + 11232pT^2 - 955p^3T^3 + p^6T^4$
28	smooth		$(1 + p^3T^2)(1 + 435T + p^3T^2)$
29	smooth		$1 - 1470T + 14832pT^2 - 1470p^3T^3 + p^6T^4$
30	smooth		$1 + 380T + 2232pT^2 + 380p^3T^3 + p^6T^4$
31	smooth		$1 - 795T + 6482pT^2 - 795p^3T^3 + p^6T^4$
32	smooth		$1 - 495T + 8482pT^2 - 495p^3T^3 + p^6T^4$
33	smooth		$1 - 565T + 2482pT^2 - 565p^3T^3 + p^6T^4$
34	smooth		$1 + 1140T + 11982pT^2 + 1140p^3T^3 + p^6T^4$
35	smooth		$1 + 80T + 6732pT^2 + 80p^3T^3 + p^6T^4$
36	smooth		$1 - 1020T + 6082pT^2 - 1020p^3T^3 + p^6T^4$
37	smooth		$1 + 225T + 8657pT^2 + 225p^3T^3 + p^6T^4$
38	smooth		$1 - 510T + 7182pT^2 - 510p^3T^3 + p^6T^4$
39	smooth		$(1 + p^3T^2)(1 + 1055T + p^3T^2)$
40	smooth		$1 + 190T - 2618pT^2 + 190p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 + 1765T + 19732pT^2 + 1765p^3T^3 + p^6T^4$
42	smooth		$1 + 430T - 2918pT^2 + 430p^3T^3 + p^6T^4$
43	smooth		$1 + 1290T + 14882pT^2 + 1290p^3T^3 + p^6T^4$
44	smooth		$1 - 135T + 1432pT^2 - 135p^3T^3 + p^6T^4$
45	smooth		$1 + 80T + 7482pT^2 + 80p^3T^3 + p^6T^4$
46	smooth		$1 - 350T + 1282pT^2 - 350p^3T^3 + p^6T^4$
47	smooth		$1 - 30T - 10418pT^2 - 30p^3T^3 + p^6T^4$
48	smooth		$1 - 1135T + 13757pT^2 - 1135p^3T^3 + p^6T^4$
49	smooth		$1 + 1525T + 16307pT^2 + 1525p^3T^3 + p^6T^4$
50	smooth		$1 + 180T + 5782pT^2 + 180p^3T^3 + p^6T^4$
51	smooth		$1 + 30T - 5918pT^2 + 30p^3T^3 + p^6T^4$
52	smooth		$1 - 80T + 58p^2T^2 - 80p^3T^3 + p^6T^4$
53	smooth		$1 - 95T + 6732pT^2 - 95p^3T^3 + p^6T^4$
54	smooth		$1 - 400T + 1282pT^2 - 400p^3T^3 + p^6T^4$
55	smooth		$1 + 515T + 11982pT^2 + 515p^3T^3 + p^6T^4$
56	smooth		$1 + 225T - 6818pT^2 + 225p^3T^3 + p^6T^4$
57	smooth		$1 + 1025T + 8682pT^2 + 1025p^3T^3 + p^6T^4$
58	smooth		$1 - 140T - 2118pT^2 - 140p^3T^3 + p^6T^4$
59	smooth		$1 - 645T + 11582pT^2 - 645p^3T^3 + p^6T^4$
60	smooth		$1 + 335T + 6232pT^2 + 335p^3T^3 + p^6T^4$
61	smooth		$1 + 160T + 382pT^2 + 160p^3T^3 + p^6T^4$
62	smooth		$1 + 485T + 4232pT^2 + 485p^3T^3 + p^6T^4$
63	smooth		$1 - 265T + 1557pT^2 - 265p^3T^3 + p^6T^4$
64	smooth		$1 + 1335T + 14982pT^2 + 1335p^3T^3 + p^6T^4$
65	smooth		$1 + 110T - 618pT^2 + 110p^3T^3 + p^6T^4$
66	smooth		$1 - 1105T + 10957pT^2 - 1105p^3T^3 + p^6T^4$
67	smooth		$1 + 610T + 6882pT^2 + 610p^3T^3 + p^6T^4$
68	smooth		$1 - 875T + 6682pT^2 - 875p^3T^3 + p^6T^4$
69	smooth		$1 - 120T + 9282pT^2 - 120p^3T^3 + p^6T^4$
70	smooth		$1 - 105T - 4718pT^2 - 105p^3T^3 + p^6T^4$
71	smooth		$1 - 1015T + 8732pT^2 - 1015p^3T^3 + p^6T^4$
72	smooth		$1 + 280T + 3232pT^2 + 280p^3T^3 + p^6T^4$
73	smooth		$1 + 1315T + 14607pT^2 + 1315p^3T^3 + p^6T^4$
74	smooth		$1 - 1380T + 12832pT^2 - 1380p^3T^3 + p^6T^4$
75	smooth		$1 - 65T - 5993pT^2 - 65p^3T^3 + p^6T^4$
76	smooth		$1 - 1080T + 14082pT^2 - 1080p^3T^3 + p^6T^4$
77	smooth		$1 + 755T + 9857pT^2 + 755p^3T^3 + p^6T^4$
78	smooth		$1 - 65T - 8118pT^2 - 65p^3T^3 + p^6T^4$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 275T + 12350pT^2 - 275p^3T^3 + p^6T^4$
2	smooth		$1 - 1075T + 10225pT^2 - 1075p^3T^3 + p^6T^4$
3	smooth		$1 + 295T - 5815pT^2 + 295p^3T^3 + p^6T^4$
4	smooth		$1 - 525T + 175pT^2 - 525p^3T^3 + p^6T^4$
5	smooth		$1 + 1465T + 15170pT^2 + 1465p^3T^3 + p^6T^4$
6	smooth		$1 - 980T + 15010pT^2 - 980p^3T^3 + p^6T^4$
7	smooth		$1 - 195T - 260pT^2 - 195p^3T^3 + p^6T^4$
8	smooth		$1 - 345T + 11340pT^2 - 345p^3T^3 + p^6T^4$
9	smooth		$1 - 895T + 3515pT^2 - 895p^3T^3 + p^6T^4$
10	smooth		$1 - 825T + 11150pT^2 - 825p^3T^3 + p^6T^4$
11	smooth		$1 + 45T - 6590pT^2 + 45p^3T^3 + p^6T^4$
12	smooth		$1 - 855T + 13910pT^2 - 855p^3T^3 + p^6T^4$
13	smooth		$1 + 35T + 4230pT^2 + 35p^3T^3 + p^6T^4$
14	smooth		$1 - 40T + 13330pT^2 - 40p^3T^3 + p^6T^4$
15	smooth		$1 + 465T - 205pT^2 + 465p^3T^3 + p^6T^4$
16	smooth		$1 + 575T + 3225pT^2 + 575p^3T^3 + p^6T^4$
17	smooth		$1 + 115T + 5470pT^2 + 115p^3T^3 + p^6T^4$
18	smooth		$1 - 270T + 13190pT^2 - 270p^3T^3 + p^6T^4$
19	smooth		$1 + 225T - 3250pT^2 + 225p^3T^3 + p^6T^4$
20	singular	5^{-5}	$(1 + pT)(1 - 777T + p^3T^2)$
21	smooth		$1 + 1070T + 14210pT^2 + 1070p^3T^3 + p^6T^4$
22	smooth		$1 + 1180T + 15340pT^2 + 1180p^3T^3 + p^6T^4$
23	smooth		$1 - 105T + 8060pT^2 - 105p^3T^3 + p^6T^4$
24	smooth		$1 - 15T - 10570pT^2 - 15p^3T^3 + p^6T^4$
25	smooth		$1 - 710T + 4370pT^2 - 710p^3T^3 + p^6T^4$
26	smooth		$1 - 365T + 12230pT^2 - 365p^3T^3 + p^6T^4$
27	smooth		$1 - 120T + 6590pT^2 - 120p^3T^3 + p^6T^4$
28	smooth		$1 + 495T + 4785pT^2 + 495p^3T^3 + p^6T^4$
29	smooth		$1 + 870T + 5010pT^2 + 870p^3T^3 + p^6T^4$
30	smooth		$1 - 570T + 13440pT^2 - 570p^3T^3 + p^6T^4$
31	smooth		$1 + 310T + 530pT^2 + 310p^3T^3 + p^6T^4$
32	smooth		$1 - 435T - 2330pT^2 - 435p^3T^3 + p^6T^4$
33	smooth		$1 + 325T - 5275pT^2 + 325p^3T^3 + p^6T^4$
34	smooth		$1 + 365T - 4080pT^2 + 365p^3T^3 + p^6T^4$
35	smooth		$1 - 750T + 10350pT^2 - 750p^3T^3 + p^6T^4$
36	smooth		$1 + 1390T + 13170pT^2 + 1390p^3T^3 + p^6T^4$
37	smooth		$1 - 1020T + 13040pT^2 - 1020p^3T^3 + p^6T^4$
38	smooth		$1 - 155T + 10085pT^2 - 155p^3T^3 + p^6T^4$
39	smooth		$1 - 1730T + 20760pT^2 - 1730p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 - 635T + 4595pT^2 - 635p^3T^3 + p^6T^4$
41	smooth		$1 + 1210T + 16280pT^2 + 1210p^3T^3 + p^6T^4$
42	smooth		$1 - 1270T + 15940pT^2 - 1270p^3T^3 + p^6T^4$
43	smooth		$1 - 1290T + 15630pT^2 - 1290p^3T^3 + p^6T^4$
44	smooth		$1 - 190T + 5430pT^2 - 190p^3T^3 + p^6T^4$
45	smooth		$1 - 55T + 2985pT^2 - 55p^3T^3 + p^6T^4$
46	smooth		$1 - 925T + 11750pT^2 - 925p^3T^3 + p^6T^4$
47	smooth		$1 - 1160T + 10370pT^2 - 1160p^3T^3 + p^6T^4$
48	smooth		$1 - 600T + 9950pT^2 - 600p^3T^3 + p^6T^4$
49	smooth		$1 - 205T - 6940pT^2 - 205p^3T^3 + p^6T^4$
50	smooth		$1 + 835T + 12005pT^2 + 835p^3T^3 + p^6T^4$
51	smooth		$1 + 1170T + 10410pT^2 + 1170p^3T^3 + p^6T^4$
52	smooth		$1 - 435T + 12970pT^2 - 435p^3T^3 + p^6T^4$
53	smooth		$1 + 310T + 4430pT^2 + 310p^3T^3 + p^6T^4$
54	smooth		$1 + 350T + 400pT^2 + 350p^3T^3 + p^6T^4$
55	smooth		$1 - 1205T + 14160pT^2 - 1205p^3T^3 + p^6T^4$
56	smooth		$1 - 185T + 1645pT^2 - 185p^3T^3 + p^6T^4$
57	smooth		$1 - 1335T + 17870pT^2 - 1335p^3T^3 + p^6T^4$
58	smooth		$1 + 815T + 8970pT^2 + 815p^3T^3 + p^6T^4$
59	smooth		$1 - 555T + 9985pT^2 - 555p^3T^3 + p^6T^4$
60	smooth		$1 + 275T + 5075pT^2 + 275p^3T^3 + p^6T^4$
61	smooth		$1 + 570T + 8810pT^2 + 570p^3T^3 + p^6T^4$
62	smooth		$1 + 590T + 12270pT^2 + 590p^3T^3 + p^6T^4$
63	smooth		$1 - 605T + 1110pT^2 - 605p^3T^3 + p^6T^4$
64	smooth		$1 + 135T - 1020pT^2 + 135p^3T^3 + p^6T^4$
65	smooth		$1 + 1565T + 17070pT^2 + 1565p^3T^3 + p^6T^4$
66	smooth		$1 + 410T - 2170pT^2 + 410p^3T^3 + p^6T^4$
67	smooth		$1 + 550T + 13050pT^2 + 550p^3T^3 + p^6T^4$
68	smooth		$1 - 510T + 7020pT^2 - 510p^3T^3 + p^6T^4$
69	smooth		$1 + 335T - 920pT^2 + 335p^3T^3 + p^6T^4$
70	smooth		$1 - 950T + 13650pT^2 - 950p^3T^3 + p^6T^4$
71	smooth		$1 + 110T - 4870pT^2 + 110p^3T^3 + p^6T^4$
72	smooth		$1 + 630T + 2240pT^2 + 630p^3T^3 + p^6T^4$
73	smooth		$1 + 630T + 1140pT^2 + 630p^3T^3 + p^6T^4$
74	smooth		$1 + 100T + 3300pT^2 + 100p^3T^3 + p^6T^4$
75	smooth		$1 + 5pT + 10870pT^2 + 5p^4T^3 + p^6T^4$
76	smooth		$1 + 580T + 940pT^2 + 580p^3T^3 + p^6T^4$
77	smooth		$1 + 1255T + 17390pT^2 + 1255p^3T^3 + p^6T^4$
78	smooth		$1 + 1820T + 21010pT^2 + 1820p^3T^3 + p^6T^4$
79	smooth		$1 + 805T + 3890pT^2 + 805p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 + 350T + 6950pT^2 + 350p^3T^3 + p^6T^4$
81	smooth		$1 + 345T - 4540pT^2 + 345p^3T^3 + p^6T^4$
82	smooth		$1 - 325T + 1800pT^2 - 325p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 355T - 6033pT^2 + 355p^3T^3 + p^6T^4$
2	smooth		$1 + 200T - 4858pT^2 + 200p^3T^3 + p^6T^4$
3	smooth		$1 - 785T + 5492pT^2 - 785p^3T^3 + p^6T^4$
4	smooth		$1 + 320T + 11642pT^2 + 320p^3T^3 + p^6T^4$
5	smooth		$(1 + 15pT + p^3T^2)(1 - 570T + p^3T^2)$
6	smooth		$1 - 185T - 97p^2T^2 - 185p^3T^3 + p^6T^4$
7	smooth		$1 + 1570T + 17442pT^2 + 1570p^3T^3 + p^6T^4$
8	smooth		$1 + 275T - 3333pT^2 + 275p^3T^3 + p^6T^4$
9	singular	5^{-5}	$(1 - pT)(1 + 945T + p^3T^2)$
10	smooth		$1 - 675T + 10792pT^2 - 675p^3T^3 + p^6T^4$
11	smooth		$1 + 80T + 13642pT^2 + 80p^3T^3 + p^6T^4$
12	smooth		$1 - 1690T + 20342pT^2 - 1690p^3T^3 + p^6T^4$
13	smooth		$1 - 450T + 14142pT^2 - 450p^3T^3 + p^6T^4$
14	smooth		$1 + 285T + 3742pT^2 + 285p^3T^3 + p^6T^4$
15	smooth		$1 + 1295T + 17217pT^2 + 1295p^3T^3 + p^6T^4$
16	smooth		$1 - 870T + 2942pT^2 - 870p^3T^3 + p^6T^4$
17	smooth		$1 + 895T + 13717pT^2 + 895p^3T^3 + p^6T^4$
18	smooth		$1 - 655T + 7267pT^2 - 655p^3T^3 + p^6T^4$
19	smooth		$1 - 865T + 14342pT^2 - 865p^3T^3 + p^6T^4$
20	smooth		$1 - 180T - 9358pT^2 - 180p^3T^3 + p^6T^4$
21	smooth		$1 + 860T + 8742pT^2 + 860p^3T^3 + p^6T^4$
22	smooth		$1 - 600T + 14792pT^2 - 600p^3T^3 + p^6T^4$
23	smooth		$1 + 310T + 1842pT^2 + 310p^3T^3 + p^6T^4$
24	smooth		$1 - 1655T + 22217pT^2 - 1655p^3T^3 + p^6T^4$
25	smooth		$1 + 700T + 8342pT^2 + 700p^3T^3 + p^6T^4$
26	smooth		$1 + 330T - 5558pT^2 + 330p^3T^3 + p^6T^4$
27	smooth		$1 + 130T + 5842pT^2 + 130p^3T^3 + p^6T^4$
28	smooth		$1 - 2070T + 26342pT^2 - 2070p^3T^3 + p^6T^4$
29	smooth		$1 + 1060T + 11592pT^2 + 1060p^3T^3 + p^6T^4$
30	smooth		$1 - 1115T + 12242pT^2 - 1115p^3T^3 + p^6T^4$
31	smooth		$1 - 180T + 10342pT^2 - 180p^3T^3 + p^6T^4$
32	smooth		$1 - 555T + 7842pT^2 - 555p^3T^3 + p^6T^4$
33	smooth		$1 + 655T + 7092pT^2 + 655p^3T^3 + p^6T^4$

Continued on the following page

$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 + 1755T + 23017pT^2 + 1755p^3T^3 + p^6T^4$
35	smooth		$1 + 1645T + 20092pT^2 + 1645p^3T^3 + p^6T^4$
36	smooth		$1 - 1105T + 12192pT^2 - 1105p^3T^3 + p^6T^4$
37	smooth		$1 - 775T + 11517pT^2 - 775p^3T^3 + p^6T^4$
38	smooth		$1 - 815T + 1867pT^2 - 815p^3T^3 + p^6T^4$
39	smooth		$1 + 1140T + 10492pT^2 + 1140p^3T^3 + p^6T^4$
40	smooth		$1 + 850T + 17042pT^2 + 850p^3T^3 + p^6T^4$
41	smooth		$1 - 150T + 7292pT^2 - 150p^3T^3 + p^6T^4$
42	smooth		$1 - 290T - 5508pT^2 - 290p^3T^3 + p^6T^4$
43	smooth		$1 + 250T - 3958pT^2 + 250p^3T^3 + p^6T^4$
44	smooth		$1 - 885T + 2342pT^2 - 885p^3T^3 + p^6T^4$
45	smooth		$1 + 270T + 8642pT^2 + 270p^3T^3 + p^6T^4$
46	smooth		$1 - 755T + 8317pT^2 - 755p^3T^3 + p^6T^4$
47	smooth		$1 + 1080T + 15342pT^2 + 1080p^3T^3 + p^6T^4$
48	smooth		$1 - 360T + 12742pT^2 - 360p^3T^3 + p^6T^4$
49	smooth		$1 - 425T + 4592pT^2 - 425p^3T^3 + p^6T^4$
50	smooth		$1 - 1070T + 4842pT^2 - 1070p^3T^3 + p^6T^4$
51	smooth		$1 + 640T + 8492pT^2 + 640p^3T^3 + p^6T^4$
52	smooth		$1 + 175T - 9108pT^2 + 175p^3T^3 + p^6T^4$
53	smooth		$1 - 325T - 7283pT^2 - 325p^3T^3 + p^6T^4$
54	smooth		$1 - 700T + 13142pT^2 - 700p^3T^3 + p^6T^4$
55	smooth		$1 - 590T + 242pT^2 - 590p^3T^3 + p^6T^4$
56	smooth		$1 - 490T + 3742pT^2 - 490p^3T^3 + p^6T^4$
57	smooth		$1 - 390T + 9242pT^2 - 390p^3T^3 + p^6T^4$
58	smooth		$1 + 1230T + 8342pT^2 + 1230p^3T^3 + p^6T^4$
59	smooth		$1 + 300T + 10292pT^2 + 300p^3T^3 + p^6T^4$
60	smooth		$1 - 555T - 1033pT^2 - 555p^3T^3 + p^6T^4$
61	smooth		$1 - 55T + 2317pT^2 - 55p^3T^3 + p^6T^4$
62	smooth		$1 - 525T + 16142pT^2 - 525p^3T^3 + p^6T^4$
63	smooth		$1 + 285T - 4633pT^2 + 285p^3T^3 + p^6T^4$
64	smooth		$1 + 465T + 5417pT^2 + 465p^3T^3 + p^6T^4$
65	smooth		$1 - 15T + 1867pT^2 - 15p^3T^3 + p^6T^4$
66	smooth		$1 + 1320T + 15342pT^2 + 1320p^3T^3 + p^6T^4$
67	smooth		$1 + 510T + 7492pT^2 + 510p^3T^3 + p^6T^4$
68	smooth		$1 + 265T + 6242pT^2 + 265p^3T^3 + p^6T^4$
69	smooth		$1 + 750T + 8392pT^2 + 750p^3T^3 + p^6T^4$
70	smooth		$1 - 1175T + 16917pT^2 - 1175p^3T^3 + p^6T^4$
71	smooth		$1 - 505T + 3092pT^2 - 505p^3T^3 + p^6T^4$
72	smooth		$1 + 1125T + 15142pT^2 + 1125p^3T^3 + p^6T^4$
73	smooth		$1 - 875T + 8967pT^2 - 875p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	smooth		$1 + 1755T + 23592pT^2 + 1755p^3T^3 + p^6T^4$
75	smooth		$1 + 1265T + 17967pT^2 + 1265p^3T^3 + p^6T^4$
76	smooth		$1 - 110T + 7242pT^2 - 110p^3T^3 + p^6T^4$
77	smooth		$1 + 225T + 292pT^2 + 225p^3T^3 + p^6T^4$
78	smooth		$1 - 705T + 11517pT^2 - 705p^3T^3 + p^6T^4$
79	smooth		$1 - 380T + 2942pT^2 - 380p^3T^3 + p^6T^4$
80	smooth		$1 - 15T + 10292pT^2 - 15p^3T^3 + p^6T^4$
81	smooth		$1 + 1340T + 14242pT^2 + 1340p^3T^3 + p^6T^4$
82	smooth		$1 - 1005T + 17192pT^2 - 1005p^3T^3 + p^6T^4$
83	smooth		$1 - 1625T + 17792pT^2 - 1625p^3T^3 + p^6T^4$
84	smooth		$1 - 340T - 3508pT^2 - 340p^3T^3 + p^6T^4$
85	smooth		$1 - 330T + 1442pT^2 - 330p^3T^3 + p^6T^4$
86	smooth		$1 - 580T - 4558pT^2 - 580p^3T^3 + p^6T^4$
87	smooth		$1 + 875T + 12092pT^2 + 875p^3T^3 + p^6T^4$
88	smooth		$1 - 5T - 3408pT^2 - 5p^3T^3 + p^6T^4$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 15pT + 21730pT^2 + 15p^4T^3 + p^6T^4$
2	smooth		$1 - 370T + 13230pT^2 - 370p^3T^3 + p^6T^4$
3	smooth		$1 - 555T + 14795pT^2 - 555p^3T^3 + p^6T^4$
4	smooth		$1 + 15pT + 22730pT^2 + 15p^4T^3 + p^6T^4$
5	smooth		$1 - 285T - 460pT^2 - 285p^3T^3 + p^6T^4$
6	smooth		$1 - 2440T + 29610pT^2 - 2440p^3T^3 + p^6T^4$
7	smooth		$1 - 1470T + 21580pT^2 - 1470p^3T^3 + p^6T^4$
8	smooth		$1 + 1255T + 13505pT^2 + 1255p^3T^3 + p^6T^4$
9	smooth		$1 + 175T + 7225pT^2 + 175p^3T^3 + p^6T^4$
10	smooth		$1 - 300T + 12950pT^2 - 300p^3T^3 + p^6T^4$
11	smooth		$1 - 385T + 14415pT^2 - 385p^3T^3 + p^6T^4$
12	smooth		$1 - 305T + 17920pT^2 - 305p^3T^3 + p^6T^4$
13	smooth		$1 - 275T - 1000pT^2 - 275p^3T^3 + p^6T^4$
14	smooth		$1 + 890T + 5590pT^2 + 890p^3T^3 + p^6T^4$
15	smooth		$1 + 1495T + 16895pT^2 + 1495p^3T^3 + p^6T^4$
16	smooth		$1 - 775T + 19500pT^2 - 775p^3T^3 + p^6T^4$
17	smooth		$1 + 115T + 12840pT^2 + 115p^3T^3 + p^6T^4$
18	smooth		$1 + 1185T + 17360pT^2 + 1185p^3T^3 + p^6T^4$
19	smooth		$1 - 400T + 350pT^2 - 400p^3T^3 + p^6T^4$
20	smooth		$1 + 120T + 9270pT^2 + 120p^3T^3 + p^6T^4$
21	smooth		$1 + 870T + 1170pT^2 + 870p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 + 1605T + 21105pT^2 + 1605p^3T^3 + p^6T^4$
23	smooth		$1 + 1180T + 20230pT^2 + 1180p^3T^3 + p^6T^4$
24	smooth		$1 - 55T + 3120pT^2 - 55p^3T^3 + p^6T^4$
25	smooth		$1 - 160T + 9490pT^2 - 160p^3T^3 + p^6T^4$
26	smooth		$1 + 475T - 3400pT^2 + 475p^3T^3 + p^6T^4$
27	smooth		$1 - 2410T + 29390pT^2 - 2410p^3T^3 + p^6T^4$
28	smooth		$1 + 1410T + 12910pT^2 + 1410p^3T^3 + p^6T^4$
29	smooth		$1 + 165T + 11415pT^2 + 165p^3T^3 + p^6T^4$
30	smooth		$1 + 1390T + 22590pT^2 + 1390p^3T^3 + p^6T^4$
31	smooth		$1 + 1345T + 12420pT^2 + 1345p^3T^3 + p^6T^4$
32	smooth		$1 - 795T + 14005pT^2 - 795p^3T^3 + p^6T^4$
33	smooth		$1 + 50T + 8850pT^2 + 50p^3T^3 + p^6T^4$
34	smooth		$1 - 55T - 9880pT^2 - 55p^3T^3 + p^6T^4$
35	smooth		$1 + 175T + 900pT^2 + 175p^3T^3 + p^6T^4$
36	smooth		$1 - 450T + 5600pT^2 - 450p^3T^3 + p^6T^4$
37	singular	5^{-5}	$(1 + pT)(1 - 1246T + p^3T^2)$
38	smooth		$1 - 505T + 6020pT^2 - 505p^3T^3 + p^6T^4$
39	smooth		$1 - 105T - 1755pT^2 - 105p^3T^3 + p^6T^4$
40	smooth		$1 - 1250T + 17550pT^2 - 1250p^3T^3 + p^6T^4$
41	smooth		$1 - 320T + 13230pT^2 - 320p^3T^3 + p^6T^4$
42	smooth		$1 + 20T + 17920pT^2 + 20p^3T^3 + p^6T^4$
43	smooth		$1 - 460T + 17740pT^2 - 460p^3T^3 + p^6T^4$
44	smooth		$1 - 175T - 13825pT^2 - 175p^3T^3 + p^6T^4$
45	smooth		$1 - 265T + 4835pT^2 - 265p^3T^3 + p^6T^4$
46	smooth		$1 - 300T + 150p^2T^2 - 300p^3T^3 + p^6T^4$
47	smooth		$1 - 595T + 280pT^2 - 595p^3T^3 + p^6T^4$
48	smooth		$1 + 1340T + 15590pT^2 + 1340p^3T^3 + p^6T^4$
49	smooth		$1 - 685T + 590pT^2 - 685p^3T^3 + p^6T^4$
50	smooth		$1 + 1355T + 14655pT^2 + 1355p^3T^3 + p^6T^4$
51	smooth		$1 + 225T - 3250pT^2 + 225p^3T^3 + p^6T^4$
52	smooth		$1 + 430T + 2880pT^2 + 430p^3T^3 + p^6T^4$
53	smooth		$1 + 1025T + 13900pT^2 + 1025p^3T^3 + p^6T^4$
54	smooth		$1 + 965T + 17340pT^2 + 965p^3T^3 + p^6T^4$
55	smooth		$1 + 575T - 2375pT^2 + 575p^3T^3 + p^6T^4$
56	smooth		$1 - 1540T + 19060pT^2 - 1540p^3T^3 + p^6T^4$
57	smooth		$1 - 1235T + 10840pT^2 - 1235p^3T^3 + p^6T^4$
58	smooth		$1 - 325T - 1100pT^2 - 325p^3T^3 + p^6T^4$
59	smooth		$1 + 675T - 725pT^2 + 675p^3T^3 + p^6T^4$
60	smooth		$1 - 1565T + 21510pT^2 - 1565p^3T^3 + p^6T^4$
61	smooth		$1 - 730T + 2770pT^2 - 730p^3T^3 + p^6T^4$

Continued on the following page

$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 - 1460T + 15690pT^2 - 1460p^3T^3 + p^6T^4$
63	smooth		$1 - 1320T + 11980pT^2 - 1320p^3T^3 + p^6T^4$
64	smooth		$1 + 790T + 7490pT^2 + 790p^3T^3 + p^6T^4$
65	smooth		$1 + 195T + 9795pT^2 + 195p^3T^3 + p^6T^4$
66	smooth		$1 + 1360T + 15410pT^2 + 1360p^3T^3 + p^6T^4$
67	smooth		$1 - 2050T + 27200pT^2 - 2050p^3T^3 + p^6T^4$
68	smooth		$1 + 2045T + 26120pT^2 + 2045p^3T^3 + p^6T^4$
69	smooth		$1 + 95T - 3855pT^2 + 95p^3T^3 + p^6T^4$
70	smooth		$1 - 170T - 6120pT^2 - 170p^3T^3 + p^6T^4$
71	smooth		$1 + 340T - 7710pT^2 + 340p^3T^3 + p^6T^4$
72	smooth		$1 + 90T + 1290pT^2 + 90p^3T^3 + p^6T^4$
73	smooth		$1 + 1420T + 10070pT^2 + 1420p^3T^3 + p^6T^4$
74	smooth		$1 - 1580T + 16470pT^2 - 1580p^3T^3 + p^6T^4$
75	smooth		$1 + 705T + 3805pT^2 + 705p^3T^3 + p^6T^4$
76	smooth		$1 + 355T + 6330pT^2 + 355p^3T^3 + p^6T^4$
77	smooth		$1 - 845T + 11330pT^2 - 845p^3T^3 + p^6T^4$
78	smooth		$1 + 590T + 4090pT^2 + 590p^3T^3 + p^6T^4$
79	smooth		$1 - 455T + 10320pT^2 - 455p^3T^3 + p^6T^4$
80	smooth		$1 + 410T + 6510pT^2 + 410p^3T^3 + p^6T^4$
81	smooth		$1 - 465T + 8460pT^2 - 465p^3T^3 + p^6T^4$
82	smooth		$1 - 780T - 1330pT^2 - 780p^3T^3 + p^6T^4$
83	smooth		$1 - 300T + 550pT^2 - 300p^3T^3 + p^6T^4$
84	smooth		$1 + 205T + 13180pT^2 + 205p^3T^3 + p^6T^4$
85	smooth		$1 + 1060T + 13510pT^2 + 1060p^3T^3 + p^6T^4$
86	smooth		$1 - 1440T + 20410pT^2 - 1440p^3T^3 + p^6T^4$
87	smooth		$1 - 1140T + 9110pT^2 - 1140p^3T^3 + p^6T^4$
88	smooth		$1 + 1585T + 20285pT^2 + 1585p^3T^3 + p^6T^4$
89	smooth		$1 - 130T - 7730pT^2 - 130p^3T^3 + p^6T^4$
90	smooth		$1 + 345T - 180pT^2 + 345p^3T^3 + p^6T^4$
91	smooth		$1 + 450T + 12050pT^2 + 450p^3T^3 + p^6T^4$
92	smooth		$1 + 235T + 7210pT^2 + 235p^3T^3 + p^6T^4$
93	smooth		$1 + 365T + 8315pT^2 + 365p^3T^3 + p^6T^4$
94	smooth		$1 - 1325T + 18500pT^2 - 1325p^3T^3 + p^6T^4$
95	smooth		$1 - 295T - 6620pT^2 - 295p^3T^3 + p^6T^4$
96	smooth		$1 + 385T - 5015pT^2 + 385p^3T^3 + p^6T^4$

C.2. The ζ -function for the mirror of a hypersurface in $G(2,5)$, AESZ 25

$p = 5$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 13T + 16pT^2 + 13p^3T^3 + p^6T^4$
2	singular	2	$1 + 14T + p^3T^2$
3	smooth		$(1 - 2pT + p^3T^2)(1 + 12T + p^3T^2)$
4	smooth		$1 - 3T + 16pT^2 - 3p^3T^3 + p^6T^4$

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 - 2pT + p^3T^2)(1 + 24T + p^3T^2)$
2	smooth		$1 + 25T + 74pT^2 + 25p^3T^3 + p^6T^4$
3	smooth		$(1 + p^3T^2)(1 + 30T + p^3T^2)$
4	smooth		$1 - 10T + 82pT^2 - 10p^3T^3 + p^6T^4$
5	smooth		$1 + 10T - 30pT^2 + 10p^3T^3 + p^6T^4$
6	smooth		$1 - 15T + 26pT^2 - 15p^3T^3 + p^6T^4$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 39T + 262pT^2 + 39p^3T^3 + p^6T^4$
2	singular	2	$(1 + pT)(1 - 20T + p^3T^2)$
3	smooth		$(1 - 4pT + p^3T^2)(1 + 46T + p^3T^2)$
4	smooth		$1 - 26T + 42pT^2 - 26p^3T^3 + p^6T^4$
5	smooth		$1 + 15T + 166pT^2 + 15p^3T^3 + p^6T^4$
6	smooth		$1 + 10T - 134pT^2 + 10p^3T^3 + p^6T^4$
7	smooth		$1 + 39T + 142pT^2 + 39p^3T^3 + p^6T^4$
8	smooth		$1 + 47T + 78pT^2 + 47p^3T^3 + p^6T^4$
9	singular	9	$(1 + pT)(1 + 20T + p^3T^2)$
10	smooth		$1 - 26T + 122pT^2 - 26p^3T^3 + p^6T^4$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 + 2pT + p^3T^2)(1 - 86T + p^3T^2)$
2	smooth		$(1 + p^3T^2)(1 - 50T + p^3T^2)$
3	smooth		$1 - 5pT + 332pT^2 - 5p^4T^3 + p^6T^4$
4	smooth		$(1 + p^3T^2)(1 + 90T + p^3T^2)$
5	smooth		$(1 + p^3T^2)(1 + 70T + p^3T^2)$
6	smooth		$1 - 30T + 90pT^2 - 30p^3T^3 + p^6T^4$

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$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 + 20T + 214pT^2 + 20p^3T^3 + p^6T^4$
8	smooth		$1 + 35T + 120pT^2 + 35p^3T^3 + p^6T^4$
9	smooth		$1 + 50T + 98pT^2 + 50p^3T^3 + p^6T^4$
10	smooth		$1 + 60T + 246pT^2 + 60p^3T^3 + p^6T^4$
11	smooth		$1 + 35T - 40pT^2 + 35p^3T^3 + p^6T^4$
12	smooth		$1 + 15T + 12pT^2 + 15p^3T^3 + p^6T^4$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 60T + 22p^2T^2 - 60p^3T^3 + p^6T^4$
2	smooth		$1 + 115T + 744pT^2 + 115p^3T^3 + p^6T^4$
3	smooth		$(1 + 6pT + p^3T^2)(1 - 82T + p^3T^2)$
4	smooth		$1 - 10T + 50pT^2 - 10p^3T^3 + p^6T^4$
5	smooth		$1 - 15T + 368pT^2 - 15p^3T^3 + p^6T^4$
6	smooth		$1 - 25T + 60pT^2 - 25p^3T^3 + p^6T^4$
7	smooth		$(1 + 2pT + p^3T^2)(1 + 106T + p^3T^2)$
8	smooth		$1 + 35T - 56pT^2 + 35p^3T^3 + p^6T^4$
9	smooth		$1 - 5pT + 632pT^2 - 5p^4T^3 + p^6T^4$
10	smooth		$1 - 15T + 208pT^2 - 15p^3T^3 + p^6T^4$
11	smooth		$1 + 20T - 394pT^2 + 20p^3T^3 + p^6T^4$
12	smooth		$1 - 20T - 330pT^2 - 20p^3T^3 + p^6T^4$
13	smooth		$1 + 446pT^2 + p^6T^4$
14	smooth		$1 + 55T + 540pT^2 + 55p^3T^3 + p^6T^4$
15	smooth		$1 + 75T + 632pT^2 + 75p^3T^3 + p^6T^4$
16	smooth		$1 + 60T + 134pT^2 + 60p^3T^3 + p^6T^4$

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 145T + 838pT^2 + 145p^3T^3 + p^6T^4$
2	smooth		$1 + 33T - 6p^2T^2 + 33p^3T^3 + p^6T^4$
3	singular	3	$(1 - pT)(1 + 84T + p^3T^2)$
4	smooth		$1 - 62pT^2 + p^6T^4$
5	smooth		$1 + 113T + 710pT^2 + 113p^3T^3 + p^6T^4$
6	smooth		$1 + 56T + 322pT^2 + 56p^3T^3 + p^6T^4$
7	singular	7	$(1 - pT)(1 - 84T + p^3T^2)$
8	smooth		$1 - 31T + 654pT^2 - 31p^3T^3 + p^6T^4$
9	smooth		$1 - 17T + 286pT^2 - 17p^3T^3 + p^6T^4$
10	smooth		$(1 + 4pT + p^3T^2)(1 - 124T + p^3T^2)$

Continued on the following page

$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 + 4T + 562pT^2 + 4p^3T^3 + p^6T^4$
12	smooth		$1 + 79T + 662pT^2 + 79p^3T^3 + p^6T^4$
13	smooth		$1 - 24T - 238pT^2 - 24p^3T^3 + p^6T^4$
14	smooth		$(1 + p^3T^2)(1 + 124T + p^3T^2)$
15	smooth		$1 + 15T + 438pT^2 + 15p^3T^3 + p^6T^4$
16	smooth		$1 - T + 382pT^2 - p^3T^3 + p^6T^4$
17	smooth		$1 - 16T + 34pT^2 - 16p^3T^3 + p^6T^4$
18	smooth		$1 - 32T - 350pT^2 - 32p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 35T + 842pT^2 + 35p^3T^3 + p^6T^4$
2	smooth		$1 - 40T + 130pT^2 - 40p^3T^3 + p^6T^4$
3	smooth		$1 - 30T + 626pT^2 - 30p^3T^3 + p^6T^4$
4	smooth		$1 + 60T + 258pT^2 + 60p^3T^3 + p^6T^4$
5	smooth		$1 + 170T + 1074pT^2 + 170p^3T^3 + p^6T^4$
6	smooth		$1 - 100T + 738pT^2 - 100p^3T^3 + p^6T^4$
7	smooth		$1 + 30T + 290pT^2 + 30p^3T^3 + p^6T^4$
8	smooth		$1 - 50T - 334pT^2 - 50p^3T^3 + p^6T^4$
9	smooth		$1 + 100T + 418pT^2 + 100p^3T^3 + p^6T^4$
10	smooth		$1 - 35T - 26p^2T^2 - 35p^3T^3 + p^6T^4$
11	smooth		$1 + 50T + 578pT^2 + 50p^3T^3 + p^6T^4$
12	smooth		$1 + 30T + 370pT^2 + 30p^3T^3 + p^6T^4$
13	smooth		$(1 - 5pT + p^3T^2)(1 + 200T + p^3T^2)$
14	smooth		$1 - 180T + 722pT^2 - 180p^3T^3 + p^6T^4$
15	smooth		$1 - 85T + 218pT^2 - 85p^3T^3 + p^6T^4$
16	smooth		$1 - 70T + 82pT^2 - 70p^3T^3 + p^6T^4$
17	smooth		$1 + 100T + 562pT^2 + 100p^3T^3 + p^6T^4$
18	smooth		$1 + 245T + 1530pT^2 + 245p^3T^3 + p^6T^4$
19	smooth		$1 + 240T + 1570pT^2 + 240p^3T^3 + p^6T^4$
20	smooth		$1 + 205T + 1450pT^2 + 205p^3T^3 + p^6T^4$
21	smooth		$1 - 180T + 802pT^2 - 180p^3T^3 + p^6T^4$
22	smooth		$1 - 50T + 274pT^2 - 50p^3T^3 + p^6T^4$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 49T - 832pT^2 + 49p^3T^3 + p^6T^4$
2	smooth		$1 - 195T + 1748pT^2 - 195p^3T^3 + p^6T^4$
<i>Continued on the following page</i>			

$p = 29$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 - 35T - 492pT^2 - 35p^3T^3 + p^6T^4$
4	smooth		$1 - 214T + 914pT^2 - 214p^3T^3 + p^6T^4$
5	singular	5	$(1 + pT)(1 - 6T + p^3T^2)$
6	smooth		$1 + 209T + 1248pT^2 + 209p^3T^3 + p^6T^4$
7	singular	7	$(1 + pT)(1 - 6T + p^3T^2)$
8	smooth		$(1 + 6pT + p^3T^2)(1 - 214T + p^3T^2)$
9	smooth		$(1 + 8pT + p^3T^2)(1 - 166T + p^3T^2)$
10	smooth		$1 + 71T + 832pT^2 + 71p^3T^3 + p^6T^4$
11	smooth		$1 + 26T + 394pT^2 + 26p^3T^3 + p^6T^4$
12	smooth		$1 - 66T + 898pT^2 - 66p^3T^3 + p^6T^4$
13	smooth		$1 + 94T + 1178pT^2 + 94p^3T^3 + p^6T^4$
14	smooth		$1 - 306T + 2402pT^2 - 306p^3T^3 + p^6T^4$
15	smooth		$1 + 286T + 1538pT^2 + 286p^3T^3 + p^6T^4$
16	smooth		$1 + 266T + 1394pT^2 + 266p^3T^3 + p^6T^4$
17	smooth		$1 + 126T + 322pT^2 + 126p^3T^3 + p^6T^4$
18	smooth		$1 - 41T - 448pT^2 - 41p^3T^3 + p^6T^4$
19	smooth		$1 + 114T + 714pT^2 + 114p^3T^3 + p^6T^4$
20	smooth		$1 - 94T - 286pT^2 - 94p^3T^3 + p^6T^4$
21	smooth		$1 + 260T + 2006pT^2 + 260p^3T^3 + p^6T^4$
22	smooth		$1 - 133T + 372pT^2 - 133p^3T^3 + p^6T^4$
23	smooth		$1 + 16T + 1214pT^2 + 16p^3T^3 + p^6T^4$
24	smooth		$1 + 267T + 1652pT^2 + 267p^3T^3 + p^6T^4$
25	smooth		$1 - 106T + 778pT^2 - 106p^3T^3 + p^6T^4$
26	smooth		$1 + 12T + 1366pT^2 + 12p^3T^3 + p^6T^4$
27	smooth		$1 + 188T + 566pT^2 + 188p^3T^3 + p^6T^4$
28	smooth		$1 - 24T - 146pT^2 - 24p^3T^3 + p^6T^4$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 156T + 962pT^2 + 156p^3T^3 + p^6T^4$
2	smooth		$1 - 19T - 350pT^2 - 19p^3T^3 + p^6T^4$
3	smooth		$1 + 245T + 2018pT^2 + 245p^3T^3 + p^6T^4$
4	smooth		$1 - 22T + 802pT^2 - 22p^3T^3 + p^6T^4$
5	smooth		$1 + 54T - 510pT^2 + 54p^3T^3 + p^6T^4$
6	smooth		$1 + 21T + 1282pT^2 + 21p^3T^3 + p^6T^4$
7	smooth		$1 - 16T + 322pT^2 - 16p^3T^3 + p^6T^4$
8	smooth		$1 + 298T + 2242pT^2 + 298p^3T^3 + p^6T^4$
9	smooth		$1 + 221T + 1506pT^2 + 221p^3T^3 + p^6T^4$
10	smooth		$1 + 157T + 802pT^2 + 157p^3T^3 + p^6T^4$

Continued on the following page

$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$(1 + p^3T^2)(1 - 185T + p^3T^2)$
12	smooth		$1 + 357T + 2242pT^2 + 357p^3T^3 + p^6T^4$
13	smooth		$1 + 86T - 510pT^2 + 86p^3T^3 + p^6T^4$
14	smooth		$1 - 122T + 834pT^2 - 122p^3T^3 + p^6T^4$
15	smooth		$1 + 82T - 78pT^2 + 82p^3T^3 + p^6T^4$
16	smooth		$1 + 127T + 1474pT^2 + 127p^3T^3 + p^6T^4$
17	smooth		$1 + 48T + 2pT^2 + 48p^3T^3 + p^6T^4$
18	smooth		$1 + 112T + 322pT^2 + 112p^3T^3 + p^6T^4$
19	singular	19	$(1 + pT)(1 - 224T + p^3T^2)$
20	smooth		$1 - 211T + 962pT^2 - 211p^3T^3 + p^6T^4$
21	singular	21	$(1 + pT)(1 + 224T + p^3T^2)$
22	smooth		$1 - 340T + 2578pT^2 - 340p^3T^3 + p^6T^4$
23	smooth		$1 - 361T + 2530pT^2 - 361p^3T^3 + p^6T^4$
24	smooth		$1 + 22T - 206pT^2 + 22p^3T^3 + p^6T^4$
25	smooth		$(1 + p^3T^2)(1 - 65T + p^3T^2)$
26	smooth		$1 + 176T + 962pT^2 + 176p^3T^3 + p^6T^4$
27	smooth		$1 + 4T + 882pT^2 + 4p^3T^3 + p^6T^4$
28	smooth		$1 + 4T - 894pT^2 + 4p^3T^3 + p^6T^4$
29	smooth		$1 + 213T + 674pT^2 + 213p^3T^3 + p^6T^4$
30	smooth		$1 - 142T + 322pT^2 - 142p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 50T - 86pT^2 - 50p^3T^3 + p^6T^4$
2	smooth		$1 + 425T + 3332pT^2 + 425p^3T^3 + p^6T^4$
3	smooth		$1 + 80T - 1250pT^2 + 80p^3T^3 + p^6T^4$
4	smooth		$1 + 35T + 1676pT^2 + 35p^3T^3 + p^6T^4$
5	smooth		$1 + 10T + 490pT^2 + 10p^3T^3 + p^6T^4$
6	smooth		$1 + 55T + 1400pT^2 + 55p^3T^3 + p^6T^4$
7	smooth		$1 + 195T + 812pT^2 + 195p^3T^3 + p^6T^4$
8	smooth		$1 + 5pT + 1604pT^2 + 5p^4T^3 + p^6T^4$
9	smooth		$1 + 90T + 2018pT^2 + 90p^3T^3 + p^6T^4$
10	smooth		$1 + 125T + 1872pT^2 + 125p^3T^3 + p^6T^4$
11	smooth		$1 - 150T - 702pT^2 - 150p^3T^3 + p^6T^4$
12	smooth		$1 - 60T + 358pT^2 - 60p^3T^3 + p^6T^4$
13	smooth		$(1 - 10pT + p^3T^2)(1 + 90T + p^3T^2)$
14	smooth		$1 + 360T + 3134pT^2 + 360p^3T^3 + p^6T^4$
15	smooth		$1 + 40T + 894pT^2 + 40p^3T^3 + p^6T^4$
16	smooth		$1 - 220T + 46p^2T^2 - 220p^3T^3 + p^6T^4$

Continued on the following page

$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 55T + 164pT^2 - 55p^3T^3 + p^6T^4$
18	smooth		$1 + 180T + 2534pT^2 + 180p^3T^3 + p^6T^4$
19	smooth		$1 + 280T + 1038pT^2 + 280p^3T^3 + p^6T^4$
20	smooth		$1 - 310T + 2842pT^2 - 310p^3T^3 + p^6T^4$
21	smooth		$1 + 45T + 1232pT^2 + 45p^3T^3 + p^6T^4$
22	smooth		$1 - 160T + 1246pT^2 - 160p^3T^3 + p^6T^4$
23	smooth		$1 - 160T + 2046pT^2 - 160p^3T^3 + p^6T^4$
24	smooth		$1 - 20T - 506pT^2 - 20p^3T^3 + p^6T^4$
25	smooth		$1 + 230T + 2234pT^2 + 230p^3T^3 + p^6T^4$
26	smooth		$1 + 210T + 2514pT^2 + 210p^3T^3 + p^6T^4$
27	smooth		$1 - 140T + 2182pT^2 - 140p^3T^3 + p^6T^4$
28	smooth		$1 - 190T + 914pT^2 - 190p^3T^3 + p^6T^4$
29	smooth		$1 - 30T + 762pT^2 - 30p^3T^3 + p^6T^4$
30	smooth		$1 + 35T + 972pT^2 + 35p^3T^3 + p^6T^4$
31	smooth		$1 - 135T - 348pT^2 - 135p^3T^3 + p^6T^4$
32	smooth		$1 + 200T + 1262pT^2 + 200p^3T^3 + p^6T^4$
33	smooth		$1 - 40T + 1710pT^2 - 40p^3T^3 + p^6T^4$
34	smooth		$1 - 45T + 396pT^2 - 45p^3T^3 + p^6T^4$
35	smooth		$1 + 135T + 760pT^2 + 135p^3T^3 + p^6T^4$
36	smooth		$(1 + 10pT + p^3T^2)(1 + 130T + p^3T^2)$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 284T + 1030pT^2 + 284p^3T^3 + p^6T^4$
2	smooth		$1 + 30T - 334pT^2 + 30p^3T^3 + p^6T^4$
3	smooth		$1 - 232T - 178pT^2 - 232p^3T^3 + p^6T^4$
4	smooth		$1 + 76T - 730pT^2 + 76p^3T^3 + p^6T^4$
5	smooth		$1 + 242T + 1514pT^2 + 242p^3T^3 + p^6T^4$
6	smooth		$1 + 206T + 1242pT^2 + 206p^3T^3 + p^6T^4$
7	smooth		$1 + 142T + 2538pT^2 + 142p^3T^3 + p^6T^4$
8	smooth		$1 + 534T + 4482pT^2 + 534p^3T^3 + p^6T^4$
9	smooth		$1 - 182T + 474pT^2 - 182p^3T^3 + p^6T^4$
10	smooth		$1 - 60T + 3062pT^2 - 60p^3T^3 + p^6T^4$
11	smooth		$1 + 157T + 3328pT^2 + 157p^3T^3 + p^6T^4$
12	smooth		$1 - 166T + 1890pT^2 - 166p^3T^3 + p^6T^4$
13	singular	13	$(1 - pT)(1 + 266T + p^3T^2)$
14	smooth		$1 - 46T + 50p^2T^2 - 46p^3T^3 + p^6T^4$
15	smooth		$1 - 472T + 3502pT^2 - 472p^3T^3 + p^6T^4$
16	smooth		$1 + 7pT + 3208pT^2 + 7p^4T^3 + p^6T^4$

Continued on the following page

$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 116T + 1750pT^2 - 116p^3T^3 + p^6T^4$
18	smooth		$1 - 342T + 2234pT^2 - 342p^3T^3 + p^6T^4$
19	smooth		$1 + 237T + 608pT^2 + 237p^3T^3 + p^6T^4$
20	smooth		$1 + 401T + 3092pT^2 + 401p^3T^3 + p^6T^4$
21	smooth		$1 + 126T + 2962pT^2 + 126p^3T^3 + p^6T^4$
22	smooth		$1 + 304T + 2110pT^2 + 304p^3T^3 + p^6T^4$
23	smooth		$1 - 78T - 1366pT^2 - 78p^3T^3 + p^6T^4$
24	smooth		$1 + 102T - 502pT^2 + 102p^3T^3 + p^6T^4$
25	smooth		$1 + 420T + 3062pT^2 + 420p^3T^3 + p^6T^4$
26	smooth		$1 - 59T - 1168pT^2 - 59p^3T^3 + p^6T^4$
27	smooth		$(1 + 10pT + p^3T^2)(1 + 225T + p^3T^2)$
28	smooth		$1 + 326T + 1562pT^2 + 326p^3T^3 + p^6T^4$
29	smooth		$1 - 165T + 2412pT^2 - 165p^3T^3 + p^6T^4$
30	smooth		$1 - 256T + 190pT^2 - 256p^3T^3 + p^6T^4$
31	smooth		$1 + 63T + 1608pT^2 + 63p^3T^3 + p^6T^4$
32	smooth		$1 - 359T + 2116pT^2 - 359p^3T^3 + p^6T^4$
33	smooth		$1 + 129T + 1332pT^2 + 129p^3T^3 + p^6T^4$
34	smooth		$1 + 4T + 70p^2T^2 + 4p^3T^3 + p^6T^4$
35	singular	35	$(1 - pT)(1 + 266T + p^3T^2)$
36	smooth		$1 - 151T - 604pT^2 - 151p^3T^3 + p^6T^4$
37	smooth		$1 - 120T + 2638pT^2 - 120p^3T^3 + p^6T^4$
38	smooth		$1 - 219T + 592pT^2 - 219p^3T^3 + p^6T^4$
39	smooth		$1 - 210T + 1298pT^2 - 210p^3T^3 + p^6T^4$
40	smooth		$1 - 240T + 2558pT^2 - 240p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 70T + 1706pT^2 + 70p^3T^3 + p^6T^4$
2	smooth		$1 - 10T - 1750pT^2 - 10p^3T^3 + p^6T^4$
3	smooth		$1 - 240T + 3650pT^2 - 240p^3T^3 + p^6T^4$
4	smooth		$1 + 80T + 898pT^2 + 80p^3T^3 + p^6T^4$
5	smooth		$1 - 110T - 1878pT^2 - 110p^3T^3 + p^6T^4$
6	smooth		$1 - 355T + 1454pT^2 - 355p^3T^3 + p^6T^4$
7	smooth		$1 - 125T - 98pT^2 - 125p^3T^3 + p^6T^4$
8	smooth		$1 - 150T + 3210pT^2 - 150p^3T^3 + p^6T^4$
9	smooth		$1 - 75T + 2286pT^2 - 75p^3T^3 + p^6T^4$
10	smooth		$1 + 100T + 2898pT^2 + 100p^3T^3 + p^6T^4$
11	smooth		$(1 + 6pT + p^3T^2)(1 - 268T + p^3T^2)$
12	smooth		$1 - 235T + 2374pT^2 - 235p^3T^3 + p^6T^4$

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$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
13	smooth		$1 + 250T - 102pT^2 + 250p^3T^3 + p^6T^4$
14	smooth		$1 + 75T - 522pT^2 + 75p^3T^3 + p^6T^4$
15	smooth		$1 + 160T + 1922pT^2 + 160p^3T^3 + p^6T^4$
16	smooth		$1 - 160T + 1122pT^2 - 160p^3T^3 + p^6T^4$
17	smooth		$1 + 205T + 10p^2T^2 + 205p^3T^3 + p^6T^4$
18	smooth		$1 + 125T + 2886pT^2 + 125p^3T^3 + p^6T^4$
19	smooth		$1 + 205T + 710pT^2 + 205p^3T^3 + p^6T^4$
20	smooth		$1 - 325T + 1662pT^2 - 325p^3T^3 + p^6T^4$
21	smooth		$1 + 360T + 3042pT^2 + 360p^3T^3 + p^6T^4$
22	smooth		$1 + 70T + 2154pT^2 + 70p^3T^3 + p^6T^4$
23	smooth		$1 - 435T + 3694pT^2 - 435p^3T^3 + p^6T^4$
24	smooth		$1 + 565T + 4398pT^2 + 565p^3T^3 + p^6T^4$
25	smooth		$1 + 200T - 30pT^2 + 200p^3T^3 + p^6T^4$
26	smooth		$1 + 165T + 2822pT^2 + 165p^3T^3 + p^6T^4$
27	smooth		$1 + 40T - 318pT^2 + 40p^3T^3 + p^6T^4$
28	smooth		$1 + 340T + 1682pT^2 + 340p^3T^3 + p^6T^4$
29	smooth		$1 - 275T + 1126pT^2 - 275p^3T^3 + p^6T^4$
30	smooth		$1 - 20T + 1986pT^2 - 20p^3T^3 + p^6T^4$
31	smooth		$1 - 265T + 3334pT^2 - 265p^3T^3 + p^6T^4$
32	smooth		$1 - 25T + 2510pT^2 - 25p^3T^3 + p^6T^4$
33	smooth		$1 + 440T + 4578pT^2 + 440p^3T^3 + p^6T^4$
34	smooth		$1 + 615T + 4846pT^2 + 615p^3T^3 + p^6T^4$
35	smooth		$1 - 350T + 3450pT^2 - 350p^3T^3 + p^6T^4$
36	smooth		$1 + 360T + 3170pT^2 + 360p^3T^3 + p^6T^4$
37	smooth		$(1 + 4pT + p^3T^2)(1 - 4pT + p^3T^2)$
38	smooth		$1 + 140T - 1006pT^2 + 140p^3T^3 + p^6T^4$
39	smooth		$1 + 120T + 3218pT^2 + 120p^3T^3 + p^6T^4$
40	smooth		$1 + 115T + 438pT^2 + 115p^3T^3 + p^6T^4$
41	smooth		$1 - 105T + 1510pT^2 - 105p^3T^3 + p^6T^4$
42	smooth		$1 + 320T + 3138pT^2 + 320p^3T^3 + p^6T^4$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 30T + 2594pT^2 + 30p^3T^3 + p^6T^4$
2	smooth		$1 - 480T + 3394pT^2 - 480p^3T^3 + p^6T^4$
3	smooth		$1 - 160T - 318pT^2 - 160p^3T^3 + p^6T^4$
4	smooth		$1 + 395T + 2930pT^2 + 395p^3T^3 + p^6T^4$
5	smooth		$1 - 10pT + 2578pT^2 - 10p^4T^3 + p^6T^4$
6	smooth		$1 - 360T + 3522pT^2 - 360p^3T^3 + p^6T^4$

Continued on the following page

$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 - 420T + 3330pT^2 - 420p^3T^3 + p^6T^4$
8	smooth		$1 + 260T + 898pT^2 + 260p^3T^3 + p^6T^4$
9	smooth		$1 + 305T + 178pT^2 + 305p^3T^3 + p^6T^4$
10	smooth		$1 + 530T + 5538pT^2 + 530p^3T^3 + p^6T^4$
11	smooth		$1 + 110T + 1058pT^2 + 110p^3T^3 + p^6T^4$
12	smooth		$1 + 230T + 2242pT^2 + 230p^3T^3 + p^6T^4$
13	smooth		$1 + 625T + 4818pT^2 + 625p^3T^3 + p^6T^4$
14	smooth		$1 + 320T + 1730pT^2 + 320p^3T^3 + p^6T^4$
15	smooth		$1 - 540T + 5026pT^2 - 540p^3T^3 + p^6T^4$
16	smooth		$1 + 585T + 4594pT^2 + 585p^3T^3 + p^6T^4$
17	smooth		$1 - 310T + 4642pT^2 - 310p^3T^3 + p^6T^4$
18	smooth		$1 + 220T + 2178pT^2 + 220p^3T^3 + p^6T^4$
19	smooth		$1 - 320T + 1986pT^2 - 320p^3T^3 + p^6T^4$
20	smooth		$1 - 15T + 146pT^2 - 15p^3T^3 + p^6T^4$
21	smooth		$1 - 615T + 5682pT^2 - 615p^3T^3 + p^6T^4$
22	smooth		$1 - 160T - 462pT^2 - 160p^3T^3 + p^6T^4$
23	smooth		$1 - 580T + 4482pT^2 - 580p^3T^3 + p^6T^4$
24	smooth		$1 + 75T - 2606pT^2 + 75p^3T^3 + p^6T^4$
25	smooth		$1 + 280T + 2498pT^2 + 280p^3T^3 + p^6T^4$
26	smooth		$1 + 635T + 4530pT^2 + 635p^3T^3 + p^6T^4$
27	smooth		$1 + 145T - 1614pT^2 + 145p^3T^3 + p^6T^4$
28	smooth		$1 - 300T + 2850pT^2 - 300p^3T^3 + p^6T^4$
29	smooth		$1 + 355T + 1106pT^2 + 355p^3T^3 + p^6T^4$
30	smooth		$1 - 175T + 722pT^2 - 175p^3T^3 + p^6T^4$
31	smooth		$1 - 135T + 1874pT^2 - 135p^3T^3 + p^6T^4$
32	smooth		$1 + 50T - 478pT^2 + 50p^3T^3 + p^6T^4$
33	smooth		$1 - 45T - 1870pT^2 - 45p^3T^3 + p^6T^4$
34	smooth		$1 - 205T + 690pT^2 - 205p^3T^3 + p^6T^4$
35	smooth		$1 + 575T + 5522pT^2 + 575p^3T^3 + p^6T^4$
36	smooth		$1 + 135T - 142pT^2 + 135p^3T^3 + p^6T^4$
37	smooth		$1 + 100T - 1406pT^2 + 100p^3T^3 + p^6T^4$
38	smooth		$1 + 310T + 3346pT^2 + 310p^3T^3 + p^6T^4$
39	smooth		$1 - 60T - 190pT^2 - 60p^3T^3 + p^6T^4$
40	smooth		$1 - 20T + 2562pT^2 - 20p^3T^3 + p^6T^4$
41	smooth		$1 - 440T + 4322pT^2 - 440p^3T^3 + p^6T^4$
42	smooth		$1 + 825T + 7378pT^2 + 825p^3T^3 + p^6T^4$
43	smooth		$1 + 480T + 4450pT^2 + 480p^3T^3 + p^6T^4$
44	smooth		$1 + 160T + 1058pT^2 + 160p^3T^3 + p^6T^4$
45	smooth		$1 + 180T + 690pT^2 + 180p^3T^3 + p^6T^4$
46	smooth		$1 + 105T + 3602pT^2 + 105p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 25T - 924pT^2 - 25p^3T^3 + p^6T^4$
2	smooth		$1 + 665T + 5012pT^2 + 665p^3T^3 + p^6T^4$
3	smooth		$1 - 615T + 4532pT^2 - 615p^3T^3 + p^6T^4$
4	smooth		$1 + 185T + 632pT^2 + 185p^3T^3 + p^6T^4$
5	smooth		$1 + 35T + 1488pT^2 + 35p^3T^3 + p^6T^4$
6	smooth		$1 + 455T + 2116pT^2 + 455p^3T^3 + p^6T^4$
7	smooth		$1 - 120T - 2674pT^2 - 120p^3T^3 + p^6T^4$
8	smooth		$1 - 510T + 6330pT^2 - 510p^3T^3 + p^6T^4$
9	smooth		$1 - 955T + 8432pT^2 - 955p^3T^3 + p^6T^4$
10	smooth		$1 - 280T + 1998pT^2 - 280p^3T^3 + p^6T^4$
11	smooth		$1 - 20T - 1770pT^2 - 20p^3T^3 + p^6T^4$
12	smooth		$1 - 135T - 844pT^2 - 135p^3T^3 + p^6T^4$
13	smooth		$1 - 15T + 3336pT^2 - 15p^3T^3 + p^6T^4$
14	smooth		$1 + 10pT + 3610pT^2 + 10p^4T^3 + p^6T^4$
15	smooth		$1 + 100T + 4390pT^2 + 100p^3T^3 + p^6T^4$
16	smooth		$1 + 520T + 5646pT^2 + 520p^3T^3 + p^6T^4$
17	smooth		$1 + 900T + 8006pT^2 + 900p^3T^3 + p^6T^4$
18	smooth		$1 - 360T + 4334pT^2 - 360p^3T^3 + p^6T^4$
19	smooth		$1 + 35T + 80p^2T^2 + 35p^3T^3 + p^6T^4$
20	smooth		$1 + 120T - 866pT^2 + 120p^3T^3 + p^6T^4$
21	smooth		$1 - 170T + 3122pT^2 - 170p^3T^3 + p^6T^4$
22	smooth		$1 + 140T + 966pT^2 + 140p^3T^3 + p^6T^4$
23	smooth		$1 - 210T + 2962pT^2 - 210p^3T^3 + p^6T^4$
24	smooth		$1 + 565T + 6256pT^2 + 565p^3T^3 + p^6T^4$
25	smooth		$1 - 280T + 526pT^2 - 280p^3T^3 + p^6T^4$
26	smooth		$1 + 40T - 1122pT^2 + 40p^3T^3 + p^6T^4$
27	smooth		$1 - 135T + 916pT^2 - 135p^3T^3 + p^6T^4$
28	smooth		$1 - 215T - 8pT^2 - 215p^3T^3 + p^6T^4$
29	smooth		$1 + 625T + 6376pT^2 + 625p^3T^3 + p^6T^4$
30	smooth		$1 - 260T + 5222pT^2 - 260p^3T^3 + p^6T^4$
31	smooth		$1 - 365T + 2000pT^2 - 365p^3T^3 + p^6T^4$
32	smooth		$1 + 460T + 3782pT^2 + 460p^3T^3 + p^6T^4$
33	smooth		$1 - 125T + 2768pT^2 - 125p^3T^3 + p^6T^4$
34	smooth		$(1 - 2pT + p^3T^2)(1 + 486T + p^3T^2)$
35	smooth		$1 + 490T + 3066pT^2 + 490p^3T^3 + p^6T^4$
36	smooth		$1 - 75T + 2352pT^2 - 75p^3T^3 + p^6T^4$
37	smooth		$1 + 360T + 2414pT^2 + 360p^3T^3 + p^6T^4$
38	smooth		$1 - 280T + 5614pT^2 - 280p^3T^3 + p^6T^4$
39	smooth		$1 + 370T + 2042pT^2 + 370p^3T^3 + p^6T^4$
40	smooth		$1 + 5T - 464pT^2 + 5p^3T^3 + p^6T^4$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 + 240T + 158pT^2 + 240p^3T^3 + p^6T^4$
42	smooth		$1 + 120T + 4654pT^2 + 120p^3T^3 + p^6T^4$
43	smooth		$1 - 360T + 5998pT^2 - 360p^3T^3 + p^6T^4$
44	smooth		$1 + 620T + 4726pT^2 + 620p^3T^3 + p^6T^4$
45	smooth		$1 + 610T + 5082pT^2 + 610p^3T^3 + p^6T^4$
46	smooth		$1 + 80T - 1186pT^2 + 80p^3T^3 + p^6T^4$
47	smooth		$1 + 50T + 2610pT^2 + 50p^3T^3 + p^6T^4$
48	smooth		$1 + 240T + 5534pT^2 + 240p^3T^3 + p^6T^4$
49	smooth		$1 + 120T + 5486pT^2 + 120p^3T^3 + p^6T^4$
50	smooth		$1 + 65T - 2780pT^2 + 65p^3T^3 + p^6T^4$
51	smooth		$1 - 175T - 1340pT^2 - 175p^3T^3 + p^6T^4$
52	smooth		$1 - 630T + 4130pT^2 - 630p^3T^3 + p^6T^4$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 557T + 7910pT^2 - 557p^3T^3 + p^6T^4$
2	smooth		$1 + 352T + 5042pT^2 + 352p^3T^3 + p^6T^4$
3	smooth		$1 + 647T + 6806pT^2 + 647p^3T^3 + p^6T^4$
4	smooth		$1 + 140T + 4818pT^2 + 140p^3T^3 + p^6T^4$
5	smooth		$1 + 132T + 3730pT^2 + 132p^3T^3 + p^6T^4$
6	smooth		$1 + 244T - 686pT^2 + 244p^3T^3 + p^6T^4$
7	smooth		$1 + 26T - 422pT^2 + 26p^3T^3 + p^6T^4$
8	smooth		$1 - 30T + 1146pT^2 - 30p^3T^3 + p^6T^4$
9	smooth		$1 - 82T + 3530pT^2 - 82p^3T^3 + p^6T^4$
10	smooth		$1 - 210T - 1222pT^2 - 210p^3T^3 + p^6T^4$
11	smooth		$1 + 7T - 1954pT^2 + 7p^3T^3 + p^6T^4$
12	smooth		$1 + 164T + 882pT^2 + 164p^3T^3 + p^6T^4$
13	smooth		$1 + 337T + 4582pT^2 + 337p^3T^3 + p^6T^4$
14	smooth		$1 - 1052T + 11026pT^2 - 1052p^3T^3 + p^6T^4$
15	singular	15	$(1 - pT)(1 - 28T + p^3T^2)$
16	smooth		$1 - 430T + 4058pT^2 - 430p^3T^3 + p^6T^4$
17	smooth		$1 - 599T + 5134pT^2 - 599p^3T^3 + p^6T^4$
18	smooth		$1 + 40T - 1774pT^2 + 40p^3T^3 + p^6T^4$
19	smooth		$1 + 223T + 3542pT^2 + 223p^3T^3 + p^6T^4$
20	smooth		$1 - 356T + 5522pT^2 - 356p^3T^3 + p^6T^4$
21	smooth		$1 + 274T + 378pT^2 + 274p^3T^3 + p^6T^4$
22	smooth		$1 + 61T - 1378pT^2 + 61p^3T^3 + p^6T^4$
23	smooth		$1 - 4T + 5634pT^2 - 4p^3T^3 + p^6T^4$
24	smooth		$1 + 381T + 1142pT^2 + 381p^3T^3 + p^6T^4$

Continued on the following page

$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
25	smooth		$1 + 432T + 38p^2T^2 + 432p^3T^3 + p^6T^4$
26	smooth		$1 - 8T + 5250pT^2 - 8p^3T^3 + p^6T^4$
27	smooth		$1 - 63T - 2578pT^2 - 63p^3T^3 + p^6T^4$
28	smooth		$1 + 592T + 4802pT^2 + 592p^3T^3 + p^6T^4$
29	smooth		$1 - 452T + 6610pT^2 - 452p^3T^3 + p^6T^4$
30	smooth		$1 - 702T + 7562pT^2 - 702p^3T^3 + p^6T^4$
31	smooth		$1 - 276T + 2434pT^2 - 276p^3T^3 + p^6T^4$
32	smooth		$1 + 680T + 4626pT^2 + 680p^3T^3 + p^6T^4$
33	smooth		$1 - 78T + 2906pT^2 - 78p^3T^3 + p^6T^4$
34	smooth		$1 - 497T + 2142pT^2 - 497p^3T^3 + p^6T^4$
35	smooth		$1 - 804T + 7762pT^2 - 804p^3T^3 + p^6T^4$
36	smooth		$1 + 846T + 126p^2T^2 + 846p^3T^3 + p^6T^4$
37	smooth		$1 + 116T + 2642pT^2 + 116p^3T^3 + p^6T^4$
38	smooth		$1 - 263T + 2630pT^2 - 263p^3T^3 + p^6T^4$
39	smooth		$1 + 86T + 5194pT^2 + 86p^3T^3 + p^6T^4$
40	smooth		$1 + 192T + 4402pT^2 + 192p^3T^3 + p^6T^4$
41	smooth		$1 - 82T + 1546pT^2 - 82p^3T^3 + p^6T^4$
42	smooth		$1 + 10T - 3014pT^2 + 10p^3T^3 + p^6T^4$
43	smooth		$1 + 936T + 8034pT^2 + 936p^3T^3 + p^6T^4$
44	smooth		$1 + 124T + 642pT^2 + 124p^3T^3 + p^6T^4$
45	smooth		$1 + 777T + 7310pT^2 + 777p^3T^3 + p^6T^4$
46	smooth		$1 - 264T + 1698pT^2 - 264p^3T^3 + p^6T^4$
47	singular	47	$(1 - pT)(1 + 28T + p^3T^2)$
48	smooth		$1 + 375T + 4278pT^2 + 375p^3T^3 + p^6T^4$
49	smooth		$(1 + 12pT + p^3T^2)(1 - 92T + p^3T^2)$
50	smooth		$(1 - 4pT + p^3T^2)(1 + 560T + p^3T^2)$
51	smooth		$1 + 54T + 1034pT^2 + 54p^3T^3 + p^6T^4$
52	smooth		$1 - 258T + 1850pT^2 - 258p^3T^3 + p^6T^4$
53	smooth		$1 + 18T + 2522pT^2 + 18p^3T^3 + p^6T^4$
54	smooth		$1 + 718T + 7610pT^2 + 718p^3T^3 + p^6T^4$
55	smooth		$1 - 39T + 5894pT^2 - 39p^3T^3 + p^6T^4$
56	smooth		$1 + 659T + 4014pT^2 + 659p^3T^3 + p^6T^4$
57	smooth		$1 + 148T + 370pT^2 + 148p^3T^3 + p^6T^4$
58	smooth		$1 - 25T + 158pT^2 - 25p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 416T + 6750pT^2 - 416p^3T^3 + p^6T^4$
2	smooth		$1 - 89T - 3968pT^2 - 89p^3T^3 + p^6T^4$

Continued on the following page

$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 + 1067T + 11444pT^2 + 1067p^3T^3 + p^6T^4$
4	smooth		$1 - 151T - 1808pT^2 - 151p^3T^3 + p^6T^4$
5	smooth		$1 - 64T - 1890pT^2 - 64p^3T^3 + p^6T^4$
6	smooth		$1 - 772T + 7014pT^2 - 772p^3T^3 + p^6T^4$
7	smooth		$1 + 147T + 2952pT^2 + 147p^3T^3 + p^6T^4$
8	smooth		$1 + 1269T + 13636pT^2 + 1269p^3T^3 + p^6T^4$
9	smooth		$1 + 58T - 3502pT^2 + 58p^3T^3 + p^6T^4$
10	smooth		$1 - 41T + 1216pT^2 - 41p^3T^3 + p^6T^4$
11	smooth		$1 - 292T + 6694pT^2 - 292p^3T^3 + p^6T^4$
12	smooth		$1 + 240T + 2942pT^2 + 240p^3T^3 + p^6T^4$
13	smooth		$1 + 523T + 5972pT^2 + 523p^3T^3 + p^6T^4$
14	smooth		$1 + 111T - 1988pT^2 + 111p^3T^3 + p^6T^4$
15	smooth		$1 + 834T + 8482pT^2 + 834p^3T^3 + p^6T^4$
16	smooth		$1 - 58T - 1558pT^2 - 58p^3T^3 + p^6T^4$
17	smooth		$1 - 277T + 5144pT^2 - 277p^3T^3 + p^6T^4$
18	smooth		$1 - 708T + 8902pT^2 - 708p^3T^3 + p^6T^4$
19	smooth		$1 - 602T + 7658pT^2 - 602p^3T^3 + p^6T^4$
20	smooth		$1 + 550T + 1578pT^2 + 550p^3T^3 + p^6T^4$
21	smooth		$1 + 84T + 3766pT^2 + 84p^3T^3 + p^6T^4$
22	smooth		$1 - 597T + 5844pT^2 - 597p^3T^3 + p^6T^4$
23	smooth		$1 + 444T + 3222pT^2 + 444p^3T^3 + p^6T^4$
24	smooth		$1 + 92T - 3578pT^2 + 92p^3T^3 + p^6T^4$
25	smooth		$1 - 137T - 2068pT^2 - 137p^3T^3 + p^6T^4$
26	smooth		$1 + 442T + 18p^2T^2 + 442p^3T^3 + p^6T^4$
27	smooth		$1 - 682T + 4202pT^2 - 682p^3T^3 + p^6T^4$
28	singular	28	$(1 - pT)(1 + 182T + p^3T^2)$
29	smooth		$1 + 763T + 8184pT^2 + 763p^3T^3 + p^6T^4$
30	smooth		$1 + 144T - 1714pT^2 + 144p^3T^3 + p^6T^4$
31	smooth		$1 + 119T + 5536pT^2 + 119p^3T^3 + p^6T^4$
32	smooth		$1 - 886T + 7802pT^2 - 886p^3T^3 + p^6T^4$
33	smooth		$1 + 151T + 5472pT^2 + 151p^3T^3 + p^6T^4$
34	smooth		$1 + 54T - 118pT^2 + 54p^3T^3 + p^6T^4$
35	smooth		$1 - 326T + 2746pT^2 - 326p^3T^3 + p^6T^4$
36	smooth		$1 - 80T - 258pT^2 - 80p^3T^3 + p^6T^4$
37	smooth		$1 - 96T - 594pT^2 - 96p^3T^3 + p^6T^4$
38	smooth		$1 + 404T + 6006pT^2 + 404p^3T^3 + p^6T^4$
39	smooth		$1 + 942T + 9434pT^2 + 942p^3T^3 + p^6T^4$
40	smooth		$1 + 764T + 7062pT^2 + 764p^3T^3 + p^6T^4$
41	smooth		$1 - 254T + 4162pT^2 - 254p^3T^3 + p^6T^4$
42	smooth		$1 - 778T + 8778pT^2 - 778p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
43	smooth		$1 + 504T + 4206pT^2 + 504p^3T^3 + p^6T^4$
44	smooth		$1 + 834T + 8442pT^2 + 834p^3T^3 + p^6T^4$
45	smooth		$1 - 194T + 2842pT^2 - 194p^3T^3 + p^6T^4$
46	smooth		$1 - 130T + 378pT^2 - 130p^3T^3 + p^6T^4$
47	smooth		$1 + 761T + 8112pT^2 + 761p^3T^3 + p^6T^4$
48	smooth		$1 - 38T + 1618pT^2 - 38p^3T^3 + p^6T^4$
49	smooth		$1 - 393T + 1292pT^2 - 393p^3T^3 + p^6T^4$
50	smooth		$1 + 578T + 3978pT^2 + 578p^3T^3 + p^6T^4$
51	smooth		$1 - 296T + 5806pT^2 - 296p^3T^3 + p^6T^4$
52	smooth		$1 + 747T + 6132pT^2 + 747p^3T^3 + p^6T^4$
53	smooth		$1 - 13T - 4728pT^2 - 13p^3T^3 + p^6T^4$
54	smooth		$1 - 491T + 6116pT^2 - 491p^3T^3 + p^6T^4$
55	smooth		$1 - 822T + 7978pT^2 - 822p^3T^3 + p^6T^4$
56	smooth		$1 + 94T + 4122pT^2 + 94p^3T^3 + p^6T^4$
57	smooth		$1 - 81T - 3108pT^2 - 81p^3T^3 + p^6T^4$
58	smooth		$1 + 718T + 6394pT^2 + 718p^3T^3 + p^6T^4$
59	singular	59	$(1 - pT)(1 + 182T + p^3T^2)$
60	smooth		$1 - 194T + 5402pT^2 - 194p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 1290T + 13242pT^2 - 1290p^3T^3 + p^6T^4$
2	smooth		$1 + 880T + 7650pT^2 + 880p^3T^3 + p^6T^4$
3	smooth		$1 - 935T + 9470pT^2 - 935p^3T^3 + p^6T^4$
4	smooth		$1 - 215T + 5942pT^2 - 215p^3T^3 + p^6T^4$
5	smooth		$1 + 40T + 2pT^2 + 40p^3T^3 + p^6T^4$
6	smooth		$1 + 600T + 6210pT^2 + 600p^3T^3 + p^6T^4$
7	smooth		$1 + 505T + 3678pT^2 + 505p^3T^3 + p^6T^4$
8	smooth		$1 + 675T + 5622pT^2 + 675p^3T^3 + p^6T^4$
9	smooth		$1 + 155T + 2846pT^2 + 155p^3T^3 + p^6T^4$
10	smooth		$1 - 400T + 4482pT^2 - 400p^3T^3 + p^6T^4$
11	smooth		$1 - 65T - 7706pT^2 - 65p^3T^3 + p^6T^4$
12	smooth		$1 - 120T + 5906pT^2 - 120p^3T^3 + p^6T^4$
13	smooth		$1 + 130T + 3370pT^2 + 130p^3T^3 + p^6T^4$
14	smooth		$1 + 200T + 1410pT^2 + 200p^3T^3 + p^6T^4$
15	smooth		$1 + 400T + 1698pT^2 + 400p^3T^3 + p^6T^4$
16	smooth		$1 + 520T + 7810pT^2 + 520p^3T^3 + p^6T^4$
17	smooth		$1 - 610T + 2682pT^2 - 610p^3T^3 + p^6T^4$
18	smooth		$1 - 725T + 7734pT^2 - 725p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$1 - 600T + 5378pT^2 - 600p^3T^3 + p^6T^4$
20	smooth		$1 - 295T + 6526pT^2 - 295p^3T^3 + p^6T^4$
21	smooth		$1 + 115T - 930pT^2 + 115p^3T^3 + p^6T^4$
22	smooth		$1 - 880T + 8482pT^2 - 880p^3T^3 + p^6T^4$
23	smooth		$1 + 255T + 4270pT^2 + 255p^3T^3 + p^6T^4$
24	smooth		$1 + 1740T + 20178pT^2 + 1740p^3T^3 + p^6T^4$
25	smooth		$1 - 175T + 2870pT^2 - 175p^3T^3 + p^6T^4$
26	smooth		$1 - 40T + 482pT^2 - 40p^3T^3 + p^6T^4$
27	smooth		$1 + 320T + 4290pT^2 + 320p^3T^3 + p^6T^4$
28	smooth		$1 + 30T + 8218pT^2 + 30p^3T^3 + p^6T^4$
29	smooth		$1 + 400T + 9122pT^2 + 400p^3T^3 + p^6T^4$
30	smooth		$1 + 495T + 3430pT^2 + 495p^3T^3 + p^6T^4$
31	smooth		$1 + 160T + 8178pT^2 + 160p^3T^3 + p^6T^4$
32	smooth		$1 + 480T + 4338pT^2 + 480p^3T^3 + p^6T^4$
33	smooth		$1 + 5pT - 594pT^2 + 5p^4T^3 + p^6T^4$
34	smooth		$1 - 565T + 18p^2T^2 - 565p^3T^3 + p^6T^4$
35	smooth		$1 + 610T + 2058pT^2 + 610p^3T^3 + p^6T^4$
36	smooth		$1 - 190T + 4682pT^2 - 190p^3T^3 + p^6T^4$
37	smooth		$1 + 355T + 3678pT^2 + 355p^3T^3 + p^6T^4$
38	smooth		$1 - 510T + 94p^2T^2 - 510p^3T^3 + p^6T^4$
39	smooth		$(1 - 8pT + p^3T^2)(1 + 636T + p^3T^2)$
40	smooth		$1 + 190T + 826pT^2 + 190p^3T^3 + p^6T^4$
41	smooth		$1 + 930T + 10202pT^2 + 930p^3T^3 + p^6T^4$
42	smooth		$1 - 385T + 7270pT^2 - 385p^3T^3 + p^6T^4$
43	smooth		$1 - 160T - 4702pT^2 - 160p^3T^3 + p^6T^4$
44	smooth		$1 + 465T + 4350pT^2 + 465p^3T^3 + p^6T^4$
45	smooth		$1 + 150T - 1862pT^2 + 150p^3T^3 + p^6T^4$
46	smooth		$1 + 435T + 310pT^2 + 435p^3T^3 + p^6T^4$
47	smooth		$1 - 280T - 3646pT^2 - 280p^3T^3 + p^6T^4$
48	smooth		$1 - 270T + 5786pT^2 - 270p^3T^3 + p^6T^4$
49	smooth		$1 - 100T + 7890pT^2 - 100p^3T^3 + p^6T^4$
50	smooth		$1 + 440T + 7938pT^2 + 440p^3T^3 + p^6T^4$
51	smooth		$1 + 360T + 722pT^2 + 360p^3T^3 + p^6T^4$
52	smooth		$1 + 160T + 4450pT^2 + 160p^3T^3 + p^6T^4$
53	smooth		$1 - 1100T + 9490pT^2 - 1100p^3T^3 + p^6T^4$
54	smooth		$1 - 375T + 4150pT^2 - 375p^3T^3 + p^6T^4$
55	smooth		$1 - 615T + 4854pT^2 - 615p^3T^3 + p^6T^4$
56	smooth		$1 - 485T + 2494pT^2 - 485p^3T^3 + p^6T^4$
57	smooth		$1 + 100T - 2318pT^2 + 100p^3T^3 + p^6T^4$
58	smooth		$1 + 660T + 3570pT^2 + 660p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
59	smooth		$1 + 815T + 9870pT^2 + 815p^3T^3 + p^6T^4$
60	smooth		$1 + 110T + 634pT^2 + 110p^3T^3 + p^6T^4$
61	smooth		$1 - 10T + 2874pT^2 - 10p^3T^3 + p^6T^4$
62	smooth		$1 + 450T + 5034pT^2 + 450p^3T^3 + p^6T^4$
63	smooth		$1 + 390T + 3146pT^2 + 390p^3T^3 + p^6T^4$
64	smooth		$1 + 1250T + 14730pT^2 + 1250p^3T^3 + p^6T^4$
65	smooth		$1 - 100T + 2130pT^2 - 100p^3T^3 + p^6T^4$
66	smooth		$(1 - 8pT + p^3T^2)(1 + 116T + p^3T^2)$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 54T - 8494pT^2 + 54p^3T^3 + p^6T^4$
2	smooth		$1 - 1185T + 13082pT^2 - 1185p^3T^3 + p^6T^4$
3	smooth		$1 - 1353T + 13722pT^2 - 1353p^3T^3 + p^6T^4$
4	smooth		$1 + 309T + 2602pT^2 + 309p^3T^3 + p^6T^4$
5	smooth		$1 + 348T - 1918pT^2 + 348p^3T^3 + p^6T^4$
6	smooth		$1 + 356T - 510pT^2 + 356p^3T^3 + p^6T^4$
7	smooth		$1 + 63T + 7082pT^2 + 63p^3T^3 + p^6T^4$
8	smooth		$1 - 713T + 5242pT^2 - 713p^3T^3 + p^6T^4$
9	smooth		$(1 + 8pT + p^3T^2)(1 - 672T + p^3T^2)$
10	smooth		$1 - 394T + 7186pT^2 - 394p^3T^3 + p^6T^4$
11	smooth		$1 - 513T + 3242pT^2 - 513p^3T^3 + p^6T^4$
12	smooth		$1 + 505T + 10282pT^2 + 505p^3T^3 + p^6T^4$
13	smooth		$1 + 223T + 1002pT^2 + 223p^3T^3 + p^6T^4$
14	smooth		$1 - 844T + 11602pT^2 - 844p^3T^3 + p^6T^4$
15	smooth		$1 + 184T + 8866pT^2 + 184p^3T^3 + p^6T^4$
16	smooth		$1 - 240T + 2082pT^2 - 240p^3T^3 + p^6T^4$
17	smooth		$1 + 234T + 8466pT^2 + 234p^3T^3 + p^6T^4$
18	smooth		$1 - 1052T + 9602pT^2 - 1052p^3T^3 + p^6T^4$
19	smooth		$1 + 303T - 3942pT^2 + 303p^3T^3 + p^6T^4$
20	smooth		$1 + 402T + 8338pT^2 + 402p^3T^3 + p^6T^4$
21	smooth		$1 + 544T + 9986pT^2 + 544p^3T^3 + p^6T^4$
22	smooth		$1 + 972T + 11714pT^2 + 972p^3T^3 + p^6T^4$
23	smooth		$1 - 705T + 8298pT^2 - 705p^3T^3 + p^6T^4$
24	smooth		$1 + 117T + 4202pT^2 + 117p^3T^3 + p^6T^4$
25	smooth		$1 + 647T + 3802pT^2 + 647p^3T^3 + p^6T^4$
26	smooth		$1 - 339T + 4442pT^2 - 339p^3T^3 + p^6T^4$
27	smooth		$1 - 350T + 3858pT^2 - 350p^3T^3 + p^6T^4$
28	smooth		$1 - 67T + 2842pT^2 - 67p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
29	smooth		$1 + 988T + 7842pT^2 + 988p^3T^3 + p^6T^4$
30	smooth		$1 - 12T + 6722pT^2 - 12p^3T^3 + p^6T^4$
31	smooth		$(1 - 15pT + p^3T^2)(1 + 744T + p^3T^2)$
32	smooth		$1 + 480T + 8738pT^2 + 480p^3T^3 + p^6T^4$
33	smooth		$1 + 310T - 2702pT^2 + 310p^3T^3 + p^6T^4$
34	smooth		$1 + 359T + 4362pT^2 + 359p^3T^3 + p^6T^4$
35	smooth		$1 + 428T - 2238pT^2 + 428p^3T^3 + p^6T^4$
36	smooth		$1 + 1032T + 11362pT^2 + 1032p^3T^3 + p^6T^4$
37	smooth		$1 - 956T + 11490pT^2 - 956p^3T^3 + p^6T^4$
38	smooth		$1 + 233T + 1514pT^2 + 233p^3T^3 + p^6T^4$
39	smooth		$1 + 700T + 7218pT^2 + 700p^3T^3 + p^6T^4$
40	smooth		$1 + 439T + 6522pT^2 + 439p^3T^3 + p^6T^4$
41	smooth		$1 - 12T + 2274pT^2 - 12p^3T^3 + p^6T^4$
42	smooth		$(1 + p^3T^2)(1 + 880T + p^3T^2)$
43	smooth		$1 + 1896T + 22306pT^2 + 1896p^3T^3 + p^6T^4$
44	smooth		$1 + 23T + 2602pT^2 + 23p^3T^3 + p^6T^4$
45	smooth		$1 + 400T + 6818pT^2 + 400p^3T^3 + p^6T^4$
46	smooth		$1 + 111T + 8810pT^2 + 111p^3T^3 + p^6T^4$
47	smooth		$1 + 760T + 8738pT^2 + 760p^3T^3 + p^6T^4$
48	smooth		$1 - 156T + 1442pT^2 - 156p^3T^3 + p^6T^4$
49	smooth		$1 - 129T + 6682pT^2 - 129p^3T^3 + p^6T^4$
50	smooth		$1 + 884T + 6466pT^2 + 884p^3T^3 + p^6T^4$
51	smooth		$1 + 560T + 5938pT^2 + 560p^3T^3 + p^6T^4$
52	smooth		$1 + 455T - 118pT^2 + 455p^3T^3 + p^6T^4$
53	smooth		$1 - 490T + 498pT^2 - 490p^3T^3 + p^6T^4$
54	smooth		$1 - 657T + 9722pT^2 - 657p^3T^3 + p^6T^4$
55	smooth		$1 - 548T + 2802pT^2 - 548p^3T^3 + p^6T^4$
56	smooth		$1 - 175T - 1318pT^2 - 175p^3T^3 + p^6T^4$
57	smooth		$1 - 489T + 7226pT^2 - 489p^3T^3 + p^6T^4$
58	smooth		$1 + 152T + 4002pT^2 + 152p^3T^3 + p^6T^4$
59	smooth		$1 - 120T + 2178pT^2 - 120p^3T^3 + p^6T^4$
60	singular	60	$(1 + pT)(1 + 408T + p^3T^2)$
61	smooth		$1 + 52T + 8162pT^2 + 52p^3T^3 + p^6T^4$
62	smooth		$1 + 170T - 814pT^2 + 170p^3T^3 + p^6T^4$
63	smooth		$1 - 312T + 6962pT^2 - 312p^3T^3 + p^6T^4$
64	smooth		$1 + 103T - 1286pT^2 + 103p^3T^3 + p^6T^4$
65	smooth		$1 - 188T - 318pT^2 - 188p^3T^3 + p^6T^4$
66	smooth		$1 - 32T + 2322pT^2 - 32p^3T^3 + p^6T^4$
67	smooth		$1 + 656T + 2786pT^2 + 656p^3T^3 + p^6T^4$
68	singular	68	$(1 + pT)(1 - 408T + p^3T^2)$

Continued on the following page

$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
69	smooth		$1 + 135T + 1066pT^2 + 135p^3T^3 + p^6T^4$
70	smooth		$1 - 111T - 4390pT^2 - 111p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 960T + 11038pT^2 + 960p^3T^3 + p^6T^4$
2	smooth		$1 + 420T + 1718pT^2 + 420p^3T^3 + p^6T^4$
3	smooth		$1 - 1475T + 16252pT^2 - 1475p^3T^3 + p^6T^4$
4	smooth		$1 - 655T + 8820pT^2 - 655p^3T^3 + p^6T^4$
5	smooth		$1 + 480T + 3678pT^2 + 480p^3T^3 + p^6T^4$
6	smooth		$1 + 170T + 7930pT^2 + 170p^3T^3 + p^6T^4$
7	smooth		$1 + 355T + 5084pT^2 + 355p^3T^3 + p^6T^4$
8	smooth		$1 + 90T - 7142pT^2 + 90p^3T^3 + p^6T^4$
9	smooth		$1 + 1085T + 156p^2T^2 + 1085p^3T^3 + p^6T^4$
10	smooth		$1 - 400T + 30p^2T^2 - 400p^3T^3 + p^6T^4$
11	smooth		$1 + 690T + 114p^2T^2 + 690p^3T^3 + p^6T^4$
12	smooth		$1 + 710T + 7874pT^2 + 710p^3T^3 + p^6T^4$
13	smooth		$1 - 390T + 3490pT^2 - 390p^3T^3 + p^6T^4$
14	smooth		$1 + 320T + 7326pT^2 + 320p^3T^3 + p^6T^4$
15	smooth		$1 + 675T + 1212pT^2 + 675p^3T^3 + p^6T^4$
16	smooth		$1 - 1155T + 14588pT^2 - 1155p^3T^3 + p^6T^4$
17	smooth		$1 - 1155T + 10176pT^2 - 1155p^3T^3 + p^6T^4$
18	smooth		$1 + 330T - 3142pT^2 + 330p^3T^3 + p^6T^4$
19	smooth		$1 - 390T + 58pT^2 - 390p^3T^3 + p^6T^4$
20	smooth		$1 + 40T - 7250pT^2 + 40p^3T^3 + p^6T^4$
21	smooth		$1 - 605T + 8988pT^2 - 605p^3T^3 + p^6T^4$
22	smooth		$1 - 420T + 3702pT^2 - 420p^3T^3 + p^6T^4$
23	smooth		$(1 - 10pT + p^3T^2)(1 + 995T + p^3T^2)$
24	smooth		$1 - 70T + 8410pT^2 - 70p^3T^3 + p^6T^4$
25	smooth		$1 + 55T + 5624pT^2 + 55p^3T^3 + p^6T^4$
26	smooth		$1 - 415T - 2584pT^2 - 415p^3T^3 + p^6T^4$
27	smooth		$1 - 540T + 1078pT^2 - 540p^3T^3 + p^6T^4$
28	smooth		$1 - 575T + 4776pT^2 - 575p^3T^3 + p^6T^4$
29	smooth		$1 - 80T - 7250pT^2 - 80p^3T^3 + p^6T^4$
30	smooth		$1 - 100T + 4918pT^2 - 100p^3T^3 + p^6T^4$
31	smooth		$1 - 125T - 1316pT^2 - 125p^3T^3 + p^6T^4$
32	smooth		$1 + 270T + 10770pT^2 + 270p^3T^3 + p^6T^4$
33	smooth		$1 + 610T + 11842pT^2 + 610p^3T^3 + p^6T^4$
34	smooth		$1 + 620T + 10838pT^2 + 620p^3T^3 + p^6T^4$

Continued on the following page

$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 - 3234pT^2 + p^6T^4$
36	smooth		$1 + 705T + 2260pT^2 + 705p^3T^3 + p^6T^4$
37	smooth		$1 - 230T - 1254pT^2 - 230p^3T^3 + p^6T^4$
38	smooth		$1 - 1015T + 8708pT^2 - 1015p^3T^3 + p^6T^4$
39	smooth		$1 + 80T + 1566pT^2 + 80p^3T^3 + p^6T^4$
40	smooth		$1 + 645T + 5904pT^2 + 645p^3T^3 + p^6T^4$
41	smooth		$1 + 790T + 10370pT^2 + 790p^3T^3 + p^6T^4$
42	smooth		$1 + 1210T + 15394pT^2 + 1210p^3T^3 + p^6T^4$
43	smooth		$1 + 240T - 5474pT^2 + 240p^3T^3 + p^6T^4$
44	smooth		$1 + 130T - 2638pT^2 + 130p^3T^3 + p^6T^4$
45	smooth		$1 + 20T + 2438pT^2 + 20p^3T^3 + p^6T^4$
46	smooth		$1 + 1130T + 14298pT^2 + 1130p^3T^3 + p^6T^4$
47	smooth		$1 - 1660T + 19398pT^2 - 1660p^3T^3 + p^6T^4$
48	smooth		$1 + 45T + 5852pT^2 + 45p^3T^3 + p^6T^4$
49	smooth		$1 + 100T + 6070pT^2 + 100p^3T^3 + p^6T^4$
50	smooth		$1 + 1040T + 11006pT^2 + 1040p^3T^3 + p^6T^4$
51	smooth		$1 + 120T + 590pT^2 + 120p^3T^3 + p^6T^4$
52	smooth		$1 + 1080T + 10846pT^2 + 1080p^3T^3 + p^6T^4$
53	smooth		$1 + 900T + 8022pT^2 + 900p^3T^3 + p^6T^4$
54	smooth		$1 + 660T + 10198pT^2 + 660p^3T^3 + p^6T^4$
55	smooth		$1 + 870T + 9122pT^2 + 870p^3T^3 + p^6T^4$
56	smooth		$1 - 30T + 1890pT^2 - 30p^3T^3 + p^6T^4$
57	smooth		$1 - 735T + 5652pT^2 - 735p^3T^3 + p^6T^4$
58	smooth		$1 - 80T + 9598pT^2 - 80p^3T^3 + p^6T^4$
59	smooth		$1 + 595T + 3068pT^2 + 595p^3T^3 + p^6T^4$
60	smooth		$1 + 45T + 6336pT^2 + 45p^3T^3 + p^6T^4$
61	smooth		$1 - 1300T + 13670pT^2 - 1300p^3T^3 + p^6T^4$
62	smooth		$1 + 805T + 10224pT^2 + 805p^3T^3 + p^6T^4$
63	smooth		$1 + 250T + 10162pT^2 + 250p^3T^3 + p^6T^4$
64	smooth		$1 - 175T - 2508pT^2 - 175p^3T^3 + p^6T^4$
65	smooth		$1 - 370T + 722pT^2 - 370p^3T^3 + p^6T^4$
66	smooth		$1 - 790T + 5954pT^2 - 790p^3T^3 + p^6T^4$
67	smooth		$1 + 1340T + 13670pT^2 + 1340p^3T^3 + p^6T^4$
68	smooth		$1 - 5pT - 3908pT^2 - 5p^4T^3 + p^6T^4$
69	smooth		$1 - 630T + 3706pT^2 - 630p^3T^3 + p^6T^4$
70	smooth		$1 + 855T + 4984pT^2 + 855p^3T^3 + p^6T^4$
71	smooth		$1 - 80T - 2pT^2 - 80p^3T^3 + p^6T^4$
72	smooth		$1 - 460T + 5398pT^2 - 460p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 792T + 13602pT^2 + 792p^3T^3 + p^6T^4$
2	smooth		$1 - 404T + 3714pT^2 - 404p^3T^3 + p^6T^4$
3	smooth		$1 - 326T + 18pT^2 - 326p^3T^3 + p^6T^4$
4	smooth		$1 + 1097T + 13042pT^2 + 1097p^3T^3 + p^6T^4$
5	smooth		$1 - 198T - 2718pT^2 - 198p^3T^3 + p^6T^4$
6	smooth		$1 - 21T + 12402pT^2 - 21p^3T^3 + p^6T^4$
7	smooth		$1 - 226T + 9874pT^2 - 226p^3T^3 + p^6T^4$
8	smooth		$1 - 582T + 6242pT^2 - 582p^3T^3 + p^6T^4$
9	smooth		$1 - 349T + 10642pT^2 - 349p^3T^3 + p^6T^4$
10	smooth		$1 + 1290T + 13986pT^2 + 1290p^3T^3 + p^6T^4$
11	smooth		$1 + 1167T + 14386pT^2 + 1167p^3T^3 + p^6T^4$
12	smooth		$1 - 184T + 5202pT^2 - 184p^3T^3 + p^6T^4$
13	smooth		$1 - 504T + 7234pT^2 - 504p^3T^3 + p^6T^4$
14	smooth		$1 + 821T + 5714pT^2 + 821p^3T^3 + p^6T^4$
15	smooth		$1 - 858T + 11042pT^2 - 858p^3T^3 + p^6T^4$
16	smooth		$1 + 258T + 5922pT^2 + 258p^3T^3 + p^6T^4$
17	smooth		$1 - 198T + 2642pT^2 - 198p^3T^3 + p^6T^4$
18	smooth		$1 - 950T + 9026pT^2 - 950p^3T^3 + p^6T^4$
19	smooth		$1 + 478T + 10530pT^2 + 478p^3T^3 + p^6T^4$
20	smooth		$1 + 455T - 718pT^2 + 455p^3T^3 + p^6T^4$
21	smooth		$1 - 355T + 1586pT^2 - 355p^3T^3 + p^6T^4$
22	smooth		$1 + 1350T + 12706pT^2 + 1350p^3T^3 + p^6T^4$
23	smooth		$1 - 206T + 4994pT^2 - 206p^3T^3 + p^6T^4$
24	smooth		$1 + 1886T + 22738pT^2 + 1886p^3T^3 + p^6T^4$
25	singular	25	$(1 - pT)(1 + 48T + p^3T^2)$
26	smooth		$1 - 1143T + 12370pT^2 - 1143p^3T^3 + p^6T^4$
27	smooth		$1 + 708T + 11970pT^2 + 708p^3T^3 + p^6T^4$
28	smooth		$1 + 1088T + 11330pT^2 + 1088p^3T^3 + p^6T^4$
29	smooth		$1 - 162T + 2466pT^2 - 162p^3T^3 + p^6T^4$
30	smooth		$1 - 809T + 10994pT^2 - 809p^3T^3 + p^6T^4$
31	smooth		$1 + 240T - 1214pT^2 + 240p^3T^3 + p^6T^4$
32	smooth		$1 - 558T + 9890pT^2 - 558p^3T^3 + p^6T^4$
33	smooth		$1 + 482T + 722pT^2 + 482p^3T^3 + p^6T^4$
34	smooth		$1 + 204T + 3954pT^2 + 204p^3T^3 + p^6T^4$
35	smooth		$1 - 7T + 6130pT^2 - 7p^3T^3 + p^6T^4$
36	smooth		$1 - 304T - 6846pT^2 - 304p^3T^3 + p^6T^4$
37	smooth		$1 + 506T - 622pT^2 + 506p^3T^3 + p^6T^4$
38	smooth		$1 + 727T + 3250pT^2 + 727p^3T^3 + p^6T^4$
39	smooth		$1 - 354T + 3442pT^2 - 354p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 110T - 4702pT^2 + 110p^3T^3 + p^6T^4$
41	smooth		$1 + 448T + 770pT^2 + 448p^3T^3 + p^6T^4$
42	smooth		$1 + 1110T + 11778pT^2 + 1110p^3T^3 + p^6T^4$
43	smooth		$1 + 319T - 3086pT^2 + 319p^3T^3 + p^6T^4$
44	smooth		$1 + 264T + 4802pT^2 + 264p^3T^3 + p^6T^4$
45	smooth		$1 + 402T + 3362pT^2 + 402p^3T^3 + p^6T^4$
46	smooth		$1 - 664T + 3202pT^2 - 664p^3T^3 + p^6T^4$
47	smooth		$1 + 122T + 10530pT^2 + 122p^3T^3 + p^6T^4$
48	smooth		$1 - 1385T + 16178pT^2 - 1385p^3T^3 + p^6T^4$
49	smooth		$1 + 173T + 2610pT^2 + 173p^3T^3 + p^6T^4$
50	smooth		$1 - 346T - 4318pT^2 - 346p^3T^3 + p^6T^4$
51	smooth		$1 + 278T + 7522pT^2 + 278p^3T^3 + p^6T^4$
52	smooth		$1 - 64T - 3678pT^2 - 64p^3T^3 + p^6T^4$
53	smooth		$1 - 143T + 8082pT^2 - 143p^3T^3 + p^6T^4$
54	smooth		$1 - 488T + 2962pT^2 - 488p^3T^3 + p^6T^4$
55	smooth		$1 - 338T + 6370pT^2 - 338p^3T^3 + p^6T^4$
56	smooth		$1 + 394T - 286pT^2 + 394p^3T^3 + p^6T^4$
57	smooth		$1 + 294T + 2994pT^2 + 294p^3T^3 + p^6T^4$
58	smooth		$1 + 735T + 10482pT^2 + 735p^3T^3 + p^6T^4$
59	smooth		$1 - 379T + 3058pT^2 - 379p^3T^3 + p^6T^4$
60	smooth		$1 - 373T + 7602pT^2 - 373p^3T^3 + p^6T^4$
61	smooth		$1 + 137T + 690pT^2 + 137p^3T^3 + p^6T^4$
62	smooth		$1 + 228T + 3010pT^2 + 228p^3T^3 + p^6T^4$
63	smooth		$1 + 744T + 4482pT^2 + 744p^3T^3 + p^6T^4$
64	smooth		$1 + 464T + 6594pT^2 + 464p^3T^3 + p^6T^4$
65	smooth		$1 - 193T + 11890pT^2 - 193p^3T^3 + p^6T^4$
66	smooth		$1 - 1118T + 12962pT^2 - 1118p^3T^3 + p^6T^4$
67	smooth		$1 - 879T + 12914pT^2 - 879p^3T^3 + p^6T^4$
68	smooth		$(1 - 8pT + p^3T^2)(1 + 1368T + p^3T^2)$
69	smooth		$1 + 238T - 2334pT^2 + 238p^3T^3 + p^6T^4$
70	smooth		$1 - 466T + 9698pT^2 - 466p^3T^3 + p^6T^4$
71	smooth		$1 - 968T + 10050pT^2 - 968p^3T^3 + p^6T^4$
72	smooth		$1 + 467T + 6322pT^2 + 467p^3T^3 + p^6T^4$
73	smooth		$1 + 30T + 6626pT^2 + 30p^3T^3 + p^6T^4$
74	smooth		$1 + 1578T + 17122pT^2 + 1578p^3T^3 + p^6T^4$
75	smooth		$1 + 929T + 11154pT^2 + 929p^3T^3 + p^6T^4$
76	smooth		$1 - 191T - 3566pT^2 - 191p^3T^3 + p^6T^4$
77	smooth		$1 - 656T + 4882pT^2 - 656p^3T^3 + p^6T^4$
78	singular	78	$(1 - pT)(1 - 48T + p^3T^2)$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 960T + 4514pT^2 + 960p^3T^3 + p^6T^4$
2	smooth		$1 + 300T - 4030pT^2 + 300p^3T^3 + p^6T^4$
3	smooth		$1 - 570T + 3706pT^2 - 570p^3T^3 + p^6T^4$
4	smooth		$1 - 320T - 2238pT^2 - 320p^3T^3 + p^6T^4$
5	smooth		$1 - 610T + 3642pT^2 - 610p^3T^3 + p^6T^4$
6	smooth		$1 + 170T + 8010pT^2 + 170p^3T^3 + p^6T^4$
7	smooth		$1 + 180T + 1586pT^2 + 180p^3T^3 + p^6T^4$
8	smooth		$1 + 1515T + 17638pT^2 + 1515p^3T^3 + p^6T^4$
9	smooth		$1 + 610T + 3690pT^2 + 610p^3T^3 + p^6T^4$
10	smooth		$1 - 235T - 7050pT^2 - 235p^3T^3 + p^6T^4$
11	smooth		$1 + 1575T + 16574pT^2 + 1575p^3T^3 + p^6T^4$
12	smooth		$1 - 110T + 3498pT^2 - 110p^3T^3 + p^6T^4$
13	smooth		$1 + 50T + 12970pT^2 + 50p^3T^3 + p^6T^4$
14	smooth		$1 + 295T + 4566pT^2 + 295p^3T^3 + p^6T^4$
15	smooth		$1 + 1720T + 18978pT^2 + 1720p^3T^3 + p^6T^4$
16	smooth		$1 + 520T + 9922pT^2 + 520p^3T^3 + p^6T^4$
17	smooth		$1 - 605T + 14062pT^2 - 605p^3T^3 + p^6T^4$
18	smooth		$1 + 275T + 8326pT^2 + 275p^3T^3 + p^6T^4$
19	smooth		$1 + 340T + 13410pT^2 + 340p^3T^3 + p^6T^4$
20	smooth		$1 + 1058pT^2 + p^6T^4$
21	smooth		$1 - 590T + 10666pT^2 - 590p^3T^3 + p^6T^4$
22	smooth		$1 + 540T + 7154pT^2 + 540p^3T^3 + p^6T^4$
23	smooth		$1 + 860T + 6930pT^2 + 860p^3T^3 + p^6T^4$
24	smooth		$1 + 675T + 8166pT^2 + 675p^3T^3 + p^6T^4$
25	smooth		$1 - 10T - 1286pT^2 - 10p^3T^3 + p^6T^4$
26	smooth		$1 - 590T + 5162pT^2 - 590p^3T^3 + p^6T^4$
27	smooth		$1 + 35T + 11054pT^2 + 35p^3T^3 + p^6T^4$
28	smooth		$1 + 600T + 1794pT^2 + 600p^3T^3 + p^6T^4$
29	smooth		$1 - 1525T + 16334pT^2 - 1525p^3T^3 + p^6T^4$
30	smooth		$1 - 1360T + 14562pT^2 - 1360p^3T^3 + p^6T^4$
31	smooth		$1 - 310T + 2250pT^2 - 310p^3T^3 + p^6T^4$
32	smooth		$1 - 555T + 3070pT^2 - 555p^3T^3 + p^6T^4$
33	smooth		$1 + 235T + 3374pT^2 + 235p^3T^3 + p^6T^4$
34	smooth		$1 - 475T + 11902pT^2 - 475p^3T^3 + p^6T^4$
35	smooth		$1 + 930T + 14714pT^2 + 930p^3T^3 + p^6T^4$
36	smooth		$1 + 175T + 4542pT^2 + 175p^3T^3 + p^6T^4$
37	smooth		$1 + 1430T + 13946pT^2 + 1430p^3T^3 + p^6T^4$
38	smooth		$1 + 770T + 6954pT^2 + 770p^3T^3 + p^6T^4$
39	smooth		$1 + 730T + 8026pT^2 + 730p^3T^3 + p^6T^4$

Continued on the following page

$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 955T + 14542pT^2 + 955p^3T^3 + p^6T^4$
41	smooth		$1 - 45T + 11054pT^2 - 45p^3T^3 + p^6T^4$
42	smooth		$1 + 15T + 9846pT^2 + 15p^3T^3 + p^6T^4$
43	smooth		$1 + 160T + 5282pT^2 + 160p^3T^3 + p^6T^4$
44	smooth		$1 - 355T - 5482pT^2 - 355p^3T^3 + p^6T^4$
45	smooth		$1 + 1405T + 13342pT^2 + 1405p^3T^3 + p^6T^4$
46	smooth		$1 - 1290T + 16346pT^2 - 1290p^3T^3 + p^6T^4$
47	smooth		$1 - 160T + 226pT^2 - 160p^3T^3 + p^6T^4$
48	smooth		$1 + 840T + 13442pT^2 + 840p^3T^3 + p^6T^4$
49	smooth		$1 + 4642pT^2 + p^6T^4$
50	smooth		$1 - 180T + 1250pT^2 - 180p^3T^3 + p^6T^4$
51	smooth		$1 + 510T + 10330pT^2 + 510p^3T^3 + p^6T^4$
52	smooth		$1 - 245T - 3866pT^2 - 245p^3T^3 + p^6T^4$
53	smooth		$1 + 1395T + 18278pT^2 + 1395p^3T^3 + p^6T^4$
54	smooth		$1 - 1190T + 9450pT^2 - 1190p^3T^3 + p^6T^4$
55	smooth		$1 - 225T + 598pT^2 - 225p^3T^3 + p^6T^4$
56	smooth		$1 - 500T + 2466pT^2 - 500p^3T^3 + p^6T^4$
57	smooth		$1 - 1990T + 24218pT^2 - 1990p^3T^3 + p^6T^4$
58	smooth		$1 - 530T + 9034pT^2 - 530p^3T^3 + p^6T^4$
59	smooth		$1 - 65T - 12066pT^2 - 65p^3T^3 + p^6T^4$
60	smooth		$1 + 1035T + 11206pT^2 + 1035p^3T^3 + p^6T^4$
61	smooth		$1 + 200T + 5378pT^2 + 200p^3T^3 + p^6T^4$
62	smooth		$1 - 80T + 7378pT^2 - 80p^3T^3 + p^6T^4$
63	smooth		$1 - 330T + 8090pT^2 - 330p^3T^3 + p^6T^4$
64	smooth		$1 + 110T + 2458pT^2 + 110p^3T^3 + p^6T^4$
65	smooth		$1 - 140T + 9554pT^2 - 140p^3T^3 + p^6T^4$
66	smooth		$1 - 1400T + 17378pT^2 - 1400p^3T^3 + p^6T^4$
67	smooth		$1 + 1310T + 13690pT^2 + 1310p^3T^3 + p^6T^4$
68	smooth		$1 - 75T + 10518pT^2 - 75p^3T^3 + p^6T^4$
69	smooth		$1 + 350T + 7162pT^2 + 350p^3T^3 + p^6T^4$
70	smooth		$1 - 965T + 9806pT^2 - 965p^3T^3 + p^6T^4$
71	smooth		$1 + 160T + 4146pT^2 + 160p^3T^3 + p^6T^4$
72	smooth		$1 - 1330T + 12298pT^2 - 1330p^3T^3 + p^6T^4$
73	smooth		$1 + 510T + 10010pT^2 + 510p^3T^3 + p^6T^4$
74	smooth		$1 - 5T - 474pT^2 - 5p^3T^3 + p^6T^4$
75	smooth		$1 - 120T - 4862pT^2 - 120p^3T^3 + p^6T^4$
76	smooth		$1 + 400T - 798pT^2 + 400p^3T^3 + p^6T^4$
77	smooth		$1 - 340T - 3310pT^2 - 340p^3T^3 + p^6T^4$
78	smooth		$1 + 780T + 5330pT^2 + 780p^3T^3 + p^6T^4$
79	smooth		$1 - 250T + 8234pT^2 - 250p^3T^3 + p^6T^4$

Continued on the following page

$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 + 440T + 4178pT^2 + 440p^3T^3 + p^6T^4$
81	smooth		$1 + 410T - 2870pT^2 + 410p^3T^3 + p^6T^4$
82	smooth		$1 + 90T + 5402pT^2 + 90p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 352T + 3742pT^2 - 352p^3T^3 + p^6T^4$
2	smooth		$1 + 37T + 1116pT^2 + 37p^3T^3 + p^6T^4$
3	smooth		$1 + 1668T + 19446pT^2 + 1668p^3T^3 + p^6T^4$
4	smooth		$1 + 290T + 5738pT^2 + 290p^3T^3 + p^6T^4$
5	smooth		$1 - 724T + 10982pT^2 - 724p^3T^3 + p^6T^4$
6	smooth		$1 + 278T + 10890pT^2 + 278p^3T^3 + p^6T^4$
7	smooth		$1 + 40T - 34pT^2 + 40p^3T^3 + p^6T^4$
8	smooth		$1 + 541T + 12204pT^2 + 541p^3T^3 + p^6T^4$
9	smooth		$1 + 172T + 5606pT^2 + 172p^3T^3 + p^6T^4$
10	smooth		$1 - 360T + 12142pT^2 - 360p^3T^3 + p^6T^4$
11	smooth		$1 + 30T + 8978pT^2 + 30p^3T^3 + p^6T^4$
12	smooth		$1 + 2813T + 37712pT^2 + 2813p^3T^3 + p^6T^4$
13	smooth		$1 - 6T + 2114pT^2 - 6p^3T^3 + p^6T^4$
14	smooth		$(1 + 2pT + p^3T^2)(1 - 1054T + p^3T^2)$
15	smooth		$1 + 455T - 172pT^2 + 455p^3T^3 + p^6T^4$
16	smooth		$1 - 1145T + 18792pT^2 - 1145p^3T^3 + p^6T^4$
17	smooth		$(1 - 10pT + p^3T^2)(1 + 1410T + p^3T^2)$
18	smooth		$1 + 1566T + 21362pT^2 + 1566p^3T^3 + p^6T^4$
19	smooth		$1 + 562T + 5282pT^2 + 562p^3T^3 + p^6T^4$
20	smooth		$1 + 335T - 2408pT^2 + 335p^3T^3 + p^6T^4$
21	smooth		$1 - 523T + 2876pT^2 - 523p^3T^3 + p^6T^4$
22	smooth		$1 + 664T + 15534pT^2 + 664p^3T^3 + p^6T^4$
23	smooth		$1 - 1022T + 17762pT^2 - 1022p^3T^3 + p^6T^4$
24	smooth		$1 + 304T - 6658pT^2 + 304p^3T^3 + p^6T^4$
25	smooth		$1 - 289T + 1272pT^2 - 289p^3T^3 + p^6T^4$
26	smooth		$1 - 123T - 10528pT^2 - 123p^3T^3 + p^6T^4$
27	smooth		$1 - 188T + 7766pT^2 - 188p^3T^3 + p^6T^4$
28	smooth		$1 + 68T - 10554pT^2 + 68p^3T^3 + p^6T^4$
29	smooth		$1 - 1443T + 18192pT^2 - 1443p^3T^3 + p^6T^4$
30	smooth		$1 - 1601T + 18884pT^2 - 1601p^3T^3 + p^6T^4$
31	smooth		$1 - 421T + 11452pT^2 - 421p^3T^3 + p^6T^4$
32	smooth		$1 + 904T + 12814pT^2 + 904p^3T^3 + p^6T^4$
33	smooth		$1 + 640T + 2078pT^2 + 640p^3T^3 + p^6T^4$

Continued on the following page

$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 + 156T + 262pT^2 + 156p^3T^3 + p^6T^4$
35	smooth		$1 - 402T - 518pT^2 - 402p^3T^3 + p^6T^4$
36	singular	36	$(1 + pT)(1 + 1526T + p^3T^2)$
37	smooth		$1 + 9886pT^2 + p^6T^4$
38	smooth		$1 + 1493T + 20096pT^2 + 1493p^3T^3 + p^6T^4$
39	smooth		$1 + 1415T + 17896pT^2 + 1415p^3T^3 + p^6T^4$
40	smooth		$1 + 66T + 2762pT^2 + 66p^3T^3 + p^6T^4$
41	smooth		$1 - 505T - 3692pT^2 - 505p^3T^3 + p^6T^4$
42	smooth		$1 + 997T + 11356pT^2 + 997p^3T^3 + p^6T^4$
43	smooth		$1 + 342T + 11162pT^2 + 342p^3T^3 + p^6T^4$
44	smooth		$1 - 265T + 3592pT^2 - 265p^3T^3 + p^6T^4$
45	smooth		$1 + 1010T + 14538pT^2 + 1010p^3T^3 + p^6T^4$
46	smooth		$1 - 44T + 4422pT^2 - 44p^3T^3 + p^6T^4$
47	smooth		$1 - 924T + 3062pT^2 - 924p^3T^3 + p^6T^4$
48	smooth		$1 + 1135T + 17508pT^2 + 1135p^3T^3 + p^6T^4$
49	smooth		$1 - 114T + 5522pT^2 - 114p^3T^3 + p^6T^4$
50	smooth		$1 + 135T - 664pT^2 + 135p^3T^3 + p^6T^4$
51	smooth		$1 + 1280T + 142p^2T^2 + 1280p^3T^3 + p^6T^4$
52	smooth		$1 - 426T + 4762pT^2 - 426p^3T^3 + p^6T^4$
53	smooth		$1 - 945T + 13592pT^2 - 945p^3T^3 + p^6T^4$
54	smooth		$1 + 84T + 9222pT^2 + 84p^3T^3 + p^6T^4$
55	smooth		$1 - 1683T + 17036pT^2 - 1683p^3T^3 + p^6T^4$
56	smooth		$1 + 197T + 9056pT^2 + 197p^3T^3 + p^6T^4$
57	smooth		$1 - 126T + 3114pT^2 - 126p^3T^3 + p^6T^4$
58	smooth		$1 + 142T + 12170pT^2 + 142p^3T^3 + p^6T^4$
59	smooth		$1 + 576T + 5342pT^2 + 576p^3T^3 + p^6T^4$
60	smooth		$1 + 953T + 8360pT^2 + 953p^3T^3 + p^6T^4$
61	smooth		$1 + 826T + 10114pT^2 + 826p^3T^3 + p^6T^4$
62	smooth		$1 + 62T - 2710pT^2 + 62p^3T^3 + p^6T^4$
63	smooth		$1 - 124T + 5254pT^2 - 124p^3T^3 + p^6T^4$
64	smooth		$1 + 590T + 7538pT^2 + 590p^3T^3 + p^6T^4$
65	smooth		$1 - 369T + 14084pT^2 - 369p^3T^3 + p^6T^4$
66	smooth		$1 + 460T + 358pT^2 + 460p^3T^3 + p^6T^4$
67	smooth		$1 - 886T + 11994pT^2 - 886p^3T^3 + p^6T^4$
68	smooth		$1 - 364T + 4822pT^2 - 364p^3T^3 + p^6T^4$
69	singular	69	$(1 + pT)(1 + 1526T + p^3T^2)$
70	smooth		$1 + 260T - 442pT^2 + 260p^3T^3 + p^6T^4$
71	smooth		$1 + 1306T + 12602pT^2 + 1306p^3T^3 + p^6T^4$
72	smooth		$1 - 344T - 4018pT^2 - 344p^3T^3 + p^6T^4$
73	smooth		$1 + 330T + 42p^2T^2 + 330p^3T^3 + p^6T^4$

Continued on the following page

$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	smooth		$1 + 95T + 11588pT^2 + 95p^3T^3 + p^6T^4$
75	smooth		$1 - 3pT - 448pT^2 - 3p^4T^3 + p^6T^4$
76	smooth		$1 - 108T + 11686pT^2 - 108p^3T^3 + p^6T^4$
77	smooth		$1 - 1383T + 11720pT^2 - 1383p^3T^3 + p^6T^4$
78	smooth		$1 + 591T - 488pT^2 + 591p^3T^3 + p^6T^4$
79	smooth		$1 + 336T + 13342pT^2 + 336p^3T^3 + p^6T^4$
80	smooth		$1 - 928T + 11806pT^2 - 928p^3T^3 + p^6T^4$
81	smooth		$1 - 72T + 14702pT^2 - 72p^3T^3 + p^6T^4$
82	smooth		$1 - 114T - 13158pT^2 - 114p^3T^3 + p^6T^4$
83	smooth		$1 - 229T + 2172pT^2 - 229p^3T^3 + p^6T^4$
84	smooth		$1 - 243T - 1204pT^2 - 243p^3T^3 + p^6T^4$
85	smooth		$1 - 2203T + 27356pT^2 - 2203p^3T^3 + p^6T^4$
86	smooth		$1 - 1082T + 12650pT^2 - 1082p^3T^3 + p^6T^4$
87	smooth		$1 - 70T + 13178pT^2 - 70p^3T^3 + p^6T^4$
88	smooth		$1 + 1261T + 11404pT^2 + 1261p^3T^3 + p^6T^4$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 285T + 15000pT^2 - 285p^3T^3 + p^6T^4$
2	smooth		$1 - 770T + 8546pT^2 - 770p^3T^3 + p^6T^4$
3	smooth		$1 + 120T + 7630pT^2 + 120p^3T^3 + p^6T^4$
4	smooth		$1 + 120T + 3918pT^2 + 120p^3T^3 + p^6T^4$
5	smooth		$1 - 450T + 9978pT^2 - 450p^3T^3 + p^6T^4$
6	smooth		$1 + 1285T + 20372pT^2 + 1285p^3T^3 + p^6T^4$
7	smooth		$(1 + 4pT + p^3T^2)(1 + 1002T + p^3T^2)$
8	smooth		$1 - 570T + 11314pT^2 - 570p^3T^3 + p^6T^4$
9	smooth		$1 + 175T + 14208pT^2 + 175p^3T^3 + p^6T^4$
10	smooth		$1 + 420T - 2906pT^2 + 420p^3T^3 + p^6T^4$
11	smooth		$1 - 250T + 6450pT^2 - 250p^3T^3 + p^6T^4$
12	smooth		$1 - 380T + 7158pT^2 - 380p^3T^3 + p^6T^4$
13	smooth		$1 - 905T + 10988pT^2 - 905p^3T^3 + p^6T^4$
14	smooth		$1 + 1650T + 15522pT^2 + 1650p^3T^3 + p^6T^4$
15	smooth		$1 + 1260T + 22742pT^2 + 1260p^3T^3 + p^6T^4$
16	smooth		$1 + 2490T + 33610pT^2 + 2490p^3T^3 + p^6T^4$
17	smooth		$1 + 140T + 11862pT^2 + 140p^3T^3 + p^6T^4$
18	smooth		$1 - 865T + 7968pT^2 - 865p^3T^3 + p^6T^4$
19	smooth		$1 + 500T + 1318pT^2 + 500p^3T^3 + p^6T^4$
20	smooth		$1 - 760T + 11630pT^2 - 760p^3T^3 + p^6T^4$
21	smooth		$1 - 390T + 9378pT^2 - 390p^3T^3 + p^6T^4$

Continued on the following page

$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 + 50T + 7770pT^2 + 50p^3T^3 + p^6T^4$
23	smooth		$1 - 1860T + 19638pT^2 - 1860p^3T^3 + p^6T^4$
24	smooth		$1 + 600T + 1422pT^2 + 600p^3T^3 + p^6T^4$
25	smooth		$1 + 1085T + 15076pT^2 + 1085p^3T^3 + p^6T^4$
26	smooth		$1 + 1050T + 9858pT^2 + 1050p^3T^3 + p^6T^4$
27	smooth		$1 + 1280T + 12670pT^2 + 1280p^3T^3 + p^6T^4$
28	smooth		$1 + 1840T + 19694pT^2 + 1840p^3T^3 + p^6T^4$
29	smooth		$1 + 1575T + 23244pT^2 + 1575p^3T^3 + p^6T^4$
30	smooth		$1 + 1200T + 8030pT^2 + 1200p^3T^3 + p^6T^4$
31	smooth		$1 + 1600T + 15870pT^2 + 1600p^3T^3 + p^6T^4$
32	smooth		$1 + 230T - 4110pT^2 + 230p^3T^3 + p^6T^4$
33	smooth		$(1 - 2pT + p^3T^2)(1 - 796T + p^3T^2)$
34	smooth		$1 + 1365T + 11816pT^2 + 1365p^3T^3 + p^6T^4$
35	smooth		$1 - 740T + 5830pT^2 - 740p^3T^3 + p^6T^4$
36	smooth		$1 - 775T + 17964pT^2 - 775p^3T^3 + p^6T^4$
37	smooth		$1 + 600T + 2990pT^2 + 600p^3T^3 + p^6T^4$
38	smooth		$1 - 10T - 2822pT^2 - 10p^3T^3 + p^6T^4$
39	smooth		$1 - 190T + 3522pT^2 - 190p^3T^3 + p^6T^4$
40	smooth		$1 + 1310T + 20810pT^2 + 1310p^3T^3 + p^6T^4$
41	smooth		$1 + 240T + 2270pT^2 + 240p^3T^3 + p^6T^4$
42	smooth		$1 + 55T + 2668pT^2 + 55p^3T^3 + p^6T^4$
43	smooth		$1 - 750T + 2650pT^2 - 750p^3T^3 + p^6T^4$
44	smooth		$1 + 950T + 10834pT^2 + 950p^3T^3 + p^6T^4$
45	smooth		$1 - 1530T + 23418pT^2 - 1530p^3T^3 + p^6T^4$
46	smooth		$1 - 2245T + 26548pT^2 - 2245p^3T^3 + p^6T^4$
47	smooth		$1 - 1240T + 22382pT^2 - 1240p^3T^3 + p^6T^4$
48	smooth		$1 - 80T - 8546pT^2 - 80p^3T^3 + p^6T^4$
49	smooth		$1 + 605T + 16356pT^2 + 605p^3T^3 + p^6T^4$
50	smooth		$1 - 270T + 6554pT^2 - 270p^3T^3 + p^6T^4$
51	smooth		$1 + 30T + 2298pT^2 + 30p^3T^3 + p^6T^4$
52	smooth		$1 - 920T + 5310pT^2 - 920p^3T^3 + p^6T^4$
53	smooth		$1 - 270T - 4806pT^2 - 270p^3T^3 + p^6T^4$
54	smooth		$1 - 1140T + 19494pT^2 - 1140p^3T^3 + p^6T^4$
55	smooth		$1 + 665T + 3696pT^2 + 665p^3T^3 + p^6T^4$
56	smooth		$1 + 325T - 10904pT^2 + 325p^3T^3 + p^6T^4$
57	smooth		$1 - 240T + 30p^2T^2 - 240p^3T^3 + p^6T^4$
58	smooth		$1 + 925T + 11608pT^2 + 925p^3T^3 + p^6T^4$
59	smooth		$1 - 480T + 6270pT^2 - 480p^3T^3 + p^6T^4$
60	smooth		$1 - 270T + 82pT^2 - 270p^3T^3 + p^6T^4$
61	smooth		$1 + 595T + 18360pT^2 + 595p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 + 550T + 7282pT^2 + 550p^3T^3 + p^6T^4$
63	smooth		$1 + 680T - 3330pT^2 + 680p^3T^3 + p^6T^4$
64	smooth		$1 - 220T + 14774pT^2 - 220p^3T^3 + p^6T^4$
65	smooth		$1 - 355T + 7780pT^2 - 355p^3T^3 + p^6T^4$
66	smooth		$1 + 805T + 16660pT^2 + 805p^3T^3 + p^6T^4$
67	smooth		$1 + 470T + 6890pT^2 + 470p^3T^3 + p^6T^4$
68	smooth		$1 + 520T + 5134pT^2 + 520p^3T^3 + p^6T^4$
69	smooth		$1 + 455T - 756pT^2 + 455p^3T^3 + p^6T^4$
70	smooth		$1 + 85T + 6260pT^2 + 85p^3T^3 + p^6T^4$
71	smooth		$1 - 1015T + 19088pT^2 - 1015p^3T^3 + p^6T^4$
72	smooth		$1 + 1200T + 13854pT^2 + 1200p^3T^3 + p^6T^4$
73	smooth		$1 + 1130T + 42p^2T^2 + 1130p^3T^3 + p^6T^4$
74	smooth		$1 + 420T + 11494pT^2 + 420p^3T^3 + p^6T^4$
75	smooth		$1 + 540T - 58pT^2 + 540p^3T^3 + p^6T^4$
76	smooth		$1 - 1335T + 12848pT^2 - 1335p^3T^3 + p^6T^4$
77	smooth		$1 - 630T - 1198pT^2 - 630p^3T^3 + p^6T^4$
78	smooth		$1 - 1290T + 10986pT^2 - 1290p^3T^3 + p^6T^4$
79	smooth		$1 + 1990T + 25266pT^2 + 1990p^3T^3 + p^6T^4$
80	smooth		$1 + 205T + 9624pT^2 + 205p^3T^3 + p^6T^4$
81	smooth		$1 - 140T + 10262pT^2 - 140p^3T^3 + p^6T^4$
82	smooth		$1 + 205T + 15864pT^2 + 205p^3T^3 + p^6T^4$
83	smooth		$1 - 595T + 4888pT^2 - 595p^3T^3 + p^6T^4$
84	smooth		$1 + 825T + 9296pT^2 + 825p^3T^3 + p^6T^4$
85	smooth		$1 + 1640T + 15790pT^2 + 1640p^3T^3 + p^6T^4$
86	smooth		$1 - 675T + 6180pT^2 - 675p^3T^3 + p^6T^4$
87	smooth		$1 + 620T + 9718pT^2 + 620p^3T^3 + p^6T^4$
88	smooth		$1 - 320T + 1278pT^2 - 320p^3T^3 + p^6T^4$
89	smooth		$1 - 350T - 10086pT^2 - 350p^3T^3 + p^6T^4$
90	smooth		$1 - 700T + 3718pT^2 - 700p^3T^3 + p^6T^4$
91	smooth		$1 - 1275T + 16532pT^2 - 1275p^3T^3 + p^6T^4$
92	smooth		$1 - 5pT - 2732pT^2 - 5p^4T^3 + p^6T^4$
93	smooth		$1 + 15T + 1088pT^2 + 15p^3T^3 + p^6T^4$
94	smooth		$1 - 500T - 5562pT^2 - 500p^3T^3 + p^6T^4$
95	smooth		$1 + 735T + 16928pT^2 + 735p^3T^3 + p^6T^4$
96	smooth		$1 - 1895T + 26604pT^2 - 1895p^3T^3 + p^6T^4$

C.3. The ζ -function for a Hulek–Verrill manifold, AESZ34

$p = 5$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 6T + p^3T^2)$
2	smooth		$(1 + p^3T^2)(1 + 14T + p^3T^2)$
3	smooth		$1 + 4T - 2p^2T^2 + 4p^3T^3 + p^6T^4$
4	singular	$\frac{1}{9}$	$(1 + pT)(1 - 6T + p^3T^2)$

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 16T + p^3T^2)$
2	singular	$\frac{1}{25}$	$(1 + pT)(1 - 32T + p^3T^2)$
3	smooth		$1 + 2T - 54pT^2 + 2p^3T^3 + p^6T^4$
4	singular	$\frac{1}{9}$	$(1 - pT)(1 + 16T + p^3T^2)$
5	smooth		$(1 + 4pT + p^3T^2)(1 - 34T + p^3T^2)$
6	smooth		$1 + 12T - 2p^2T^2 + 12p^3T^3 + p^6T^4$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 12T + p^3T^2)$
2	smooth		$1 + 42T + 194pT^2 + 42p^3T^3 + p^6T^4$
3	smooth		$(1 + p^3T^2)(1 + 28T + p^3T^2)$
4	singular	$\frac{1}{25}$	$(1 - pT)(1 + 60T + p^3T^2)$
5	singular	$\frac{1}{9}$	$(1 + pT)(1 - 12T + p^3T^2)$
6	smooth		$1 - 2T + 2pT^2 - 2p^3T^3 + p^6T^4$
7	smooth		$1 - 2pT + 122pT^2 - 2p^4T^3 + p^6T^4$
8	smooth		$1 - 16T + 50pT^2 - 16p^3T^3 + p^6T^4$
9	smooth		$1 - 32T + 2pT^2 - 32p^3T^3 + p^6T^4$
10	smooth		$1 - 2pT + 2pT^2 - 2p^4T^3 + p^6T^4$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 38T + p^3T^2)$
2	smooth		$(1 - 6pT + p^3T^2)(1 + 34T + p^3T^2)$
3	singular	$\frac{1}{9}$	$(1 - pT)(1 - 38T + p^3T^2)$
4	smooth		$(1 - 2pT + p^3T^2)(1 + 42T + p^3T^2)$
5	smooth		$1 - 36T + 166pT^2 - 36p^3T^3 + p^6T^4$
6	smooth		$1 - 2pT + 266pT^2 - 2p^4T^3 + p^6T^4$

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$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 + 14T - 54pT^2 + 14p^3T^3 + p^6T^4$
8	smooth		$(1 + 4pT + p^3T^2)(1 - 6T + p^3T^2)$
9	smooth		$1 + 36T - 2p^2T^2 + 36p^3T^3 + p^6T^4$
10	smooth		$(1 - 2pT + p^3T^2)(1 + 42T + p^3T^2)$
11	smooth		$(1 + 4pT + p^3T^2)(1 - 18T + p^3T^2)$
12	singular	$\frac{1}{25}$	$(1 + pT)(1 + 34T + p^3T^2)$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 126T + p^3T^2)$
2	singular	$\frac{1}{9}$	$(1 + pT)(1 + 126T + p^3T^2)$
3	smooth		$1 + 98T + 674pT^2 + 98p^3T^3 + p^6T^4$
4	smooth		$(1 + 6pT + p^3T^2)(1 - 114T + p^3T^2)$
5	smooth		$1 - 4T - 202pT^2 - 4p^3T^3 + p^6T^4$
6	smooth		$1 - 44T + 38pT^2 - 44p^3T^3 + p^6T^4$
7	smooth		$1 - 2T + 194pT^2 - 2p^3T^3 + p^6T^4$
8	smooth		$(1 + 6pT + p^3T^2)(1 - 94T + p^3T^2)$
9	smooth		$1 + 4T + 182pT^2 + 4p^3T^3 + p^6T^4$
10	smooth		$1 + 2pT + 2pT^2 + 2p^4T^3 + p^6T^4$
11	smooth		$1 - 124T + 518pT^2 - 124p^3T^3 + p^6T^4$
12	smooth		$(1 - 6pT + p^3T^2)(1 - 74T + p^3T^2)$
13	smooth		$(1 - 2pT + p^3T^2)(1 + 78T + p^3T^2)$
14	smooth		$(1 + p^3T^2)(1 - 134T + p^3T^2)$
15	singular	$\frac{1}{25}$	$(1 + pT)(1 - 42T + p^3T^2)$
16	smooth		$(1 + 6pT + p^3T^2)(1 - 2pT + p^3T^2)$

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 20T + p^3T^2)$
2	smooth		$1 + 4pT + 2pT^2 + 4p^4T^3 + p^6T^4$
3	smooth		$1 - 8T + 242pT^2 - 8p^3T^3 + p^6T^4$
4	smooth		$(1 + 4pT + p^3T^2)(1 - 60T + p^3T^2)$
5	smooth		$(1 + 4pT + p^3T^2)(1 - 60T + p^3T^2)$
6	smooth		$1 + 8T - 318pT^2 + 8p^3T^3 + p^6T^4$
7	smooth		$1 - 44T - 238pT^2 - 44p^3T^3 + p^6T^4$
8	smooth		$(1 - 2pT + p^3T^2)(1 - 80T + p^3T^2)$
9	smooth		$(1 + 4pT + p^3T^2)(1 - 160T + p^3T^2)$
10	smooth		$1 + 12T + 562pT^2 + 12p^3T^3 + p^6T^4$

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$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$(1 + 4pT + p^3T^2)(1 - 140T + p^3T^2)$
12	smooth		$1 + 12T + 82pT^2 + 12p^3T^3 + p^6T^4$
13	smooth		$1 + 178T + 1082pT^2 + 178p^3T^3 + p^6T^4$
14	smooth		$1 + 12T - 158pT^2 + 12p^3T^3 + p^6T^4$
15	smooth		$1 + 42T - 2p^2T^2 + 42p^3T^3 + p^6T^4$
16	singular	$\frac{1}{25}$	$(1 - pT)(1 + 76T + p^3T^2)$
17	singular	$\frac{1}{9}$	$(1 - pT)(1 - 20T + p^3T^2)$
18	smooth		$1 - 54T + 322pT^2 - 54p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 168T + p^3T^2)$
2	smooth		$1 - 28T + 98pT^2 - 28p^3T^3 + p^6T^4$
3	smooth		$1 - 68T + 98pT^2 - 68p^3T^3 + p^6T^4$
4	smooth		$(1 + 4pT + p^3T^2)(1 - 120T + p^3T^2)$
5	smooth		$1 - 90T + 890pT^2 - 90p^3T^3 + p^6T^4$
6	smooth		$1 + 72T + 98pT^2 + 72p^3T^3 + p^6T^4$
7	smooth		$1 + 52T + 338pT^2 + 52p^3T^3 + p^6T^4$
8	smooth		$1 + 12T + 98pT^2 + 12p^3T^3 + p^6T^4$
9	smooth		$1 + 32T + 98pT^2 + 32p^3T^3 + p^6T^4$
10	smooth		$(1 + 4pT + p^3T^2)(1 - 132T + p^3T^2)$
11	smooth		$1 + 94T - 94pT^2 + 94p^3T^3 + p^6T^4$
12	singular	$\frac{1}{25}$	$(1 + pT)(1 + p^3T^2)$
13	smooth		$(1 + p^3T^2)(1 + 112T + p^3T^2)$
14	smooth		$1 + 94T + 266pT^2 + 94p^3T^3 + p^6T^4$
15	smooth		$1 + 2T - 142pT^2 + 2p^3T^3 + p^6T^4$
16	smooth		$1 - 28T + 98pT^2 - 28p^3T^3 + p^6T^4$
17	smooth		$1 + 144T + 626pT^2 + 144p^3T^3 + p^6T^4$
18	singular	$\frac{1}{9}$	$(1 + pT)(1 - 168T + p^3T^2)$
19	smooth		$1 + 162T + 1178pT^2 + 162p^3T^3 + p^6T^4$
20	smooth		$1 - 120T + 530pT^2 - 120p^3T^3 + p^6T^4$
21	smooth		$1 - 120T + 530pT^2 - 120p^3T^3 + p^6T^4$
22	smooth		$1 + 94T + 866pT^2 + 94p^3T^3 + p^6T^4$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 30T + p^3T^2)$
2	smooth		$1 - 150T + 362pT^2 - 150p^3T^3 + p^6T^4$

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$p = 29$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 - 6T + 722pT^2 - 6p^3T^3 + p^6T^4$
4	smooth		$(1 + 6pT + p^3T^2)(1 - 190T + p^3T^2)$
5	smooth		$1 + 44T + 182pT^2 + 44p^3T^3 + p^6T^4$
6	smooth		$1 + 96T - 418pT^2 + 96p^3T^3 + p^6T^4$
7	singular	$\frac{1}{25}$	$(1 - pT)(1 - 6T + p^3T^2)$
8	smooth		$1 + 24T + 542pT^2 + 24p^3T^3 + p^6T^4$
9	smooth		$1 - 16T - 418pT^2 - 16p^3T^3 + p^6T^4$
10	smooth		$1 - 80T + 542pT^2 - 80p^3T^3 + p^6T^4$
11	smooth		$1 - 126T + 242pT^2 - 126p^3T^3 + p^6T^4$
12	smooth		$1 + 100T + 182pT^2 + 100p^3T^3 + p^6T^4$
13	singular	$\frac{1}{9}$	$(1 + pT)(1 - 30T + p^3T^2)$
14	smooth		$1 - 24T + 542pT^2 - 24p^3T^3 + p^6T^4$
15	smooth		$1 - 156T + 1382pT^2 - 156p^3T^3 + p^6T^4$
16	smooth		$1 - 396T + 2822pT^2 - 396p^3T^3 + p^6T^4$
17	smooth		$1 + 6T - 238pT^2 + 6p^3T^3 + p^6T^4$
18	smooth		$1 + 10T - 1438pT^2 + 10p^3T^3 + p^6T^4$
19	smooth		$1 + 130T + 722pT^2 + 130p^3T^3 + p^6T^4$
20	smooth		$1 + 276T + 1862pT^2 + 276p^3T^3 + p^6T^4$
21	smooth		$1 + 1502pT^2 + p^6T^4$
22	smooth		$1 + 24T + 1262pT^2 + 24p^3T^3 + p^6T^4$
23	smooth		$(1 + 10pT + p^3T^2)(1 - 294T + p^3T^2)$
24	smooth		$1 + 56T + 782pT^2 + 56p^3T^3 + p^6T^4$
25	smooth		$1 - 44T - 2p^2T^2 - 44p^3T^3 + p^6T^4$
26	smooth		$1 + 206T + 1922pT^2 + 206p^3T^3 + p^6T^4$
27	smooth		$1 + 6T - 118pT^2 + 6p^3T^3 + p^6T^4$
28	smooth		$1 + 136T + 782pT^2 + 136p^3T^3 + p^6T^4$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 88T + p^3T^2)$
2	smooth		$1 - 288T + 2434pT^2 - 288p^3T^3 + p^6T^4$
3	smooth		$1 + 256T + 1106pT^2 + 256p^3T^3 + p^6T^4$
4	smooth		$1 + 352T + 2114pT^2 + 352p^3T^3 + p^6T^4$
5	singular	$\frac{1}{25}$	$(1 - pT)(1 + 232T + p^3T^2)$
6	smooth		$1 + 258T + 2122pT^2 + 258p^3T^3 + p^6T^4$
7	singular	$\frac{1}{9}$	$(1 - pT)(1 + 88T + p^3T^2)$
8	smooth		$(1 - 8pT + p^3T^2)(1 + 268T + p^3T^2)$
9	smooth		$(1 - 8pT + p^3T^2)(1 + 168T + p^3T^2)$
10	smooth		$1 + 52T - 286pT^2 + 52p^3T^3 + p^6T^4$

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$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 - 98T + 1514pT^2 - 98p^3T^3 + p^6T^4$
12	smooth		$1 - 264T + 1906pT^2 - 264p^3T^3 + p^6T^4$
13	smooth		$1 - 98T + 674pT^2 - 98p^3T^3 + p^6T^4$
14	smooth		$1 - 8T + 1154pT^2 - 8p^3T^3 + p^6T^4$
15	smooth		$1 - 200T + 1058pT^2 - 200p^3T^3 + p^6T^4$
16	smooth		$(1 + 4pT + p^3T^2)(1 - 192T + p^3T^2)$
17	smooth		$1 - 90T - 422pT^2 - 90p^3T^3 + p^6T^4$
18	smooth		$(1 - 8pT + p^3T^2)(1 + 128T + p^3T^2)$
19	smooth		$1 - 148T + 1794pT^2 - 148p^3T^3 + p^6T^4$
20	smooth		$(1 + 8pT + p^3T^2)(1 - 8pT + p^3T^2)$
21	smooth		$1 + 12T + 754pT^2 + 12p^3T^3 + p^6T^4$
22	smooth		$(1 + 4pT + p^3T^2)(1 - 72T + p^3T^2)$
23	smooth		$1 - 4T + 1266pT^2 - 4p^3T^3 + p^6T^4$
24	smooth		$1 + 82T + 1394pT^2 + 82p^3T^3 + p^6T^4$
25	smooth		$1 + 72T - 446pT^2 + 72p^3T^3 + p^6T^4$
26	smooth		$(1 + 4pT + p^3T^2)(1 - 202T + p^3T^2)$
27	smooth		$1 + 198T + 562pT^2 + 198p^3T^3 + p^6T^4$
28	smooth		$1 + 1858pT^2 + p^6T^4$
29	smooth		$1 + 196T + 1346pT^2 + 196p^3T^3 + p^6T^4$
30	smooth		$1 - 318T + 1954pT^2 - 318p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 254T + p^3T^2)$
2	smooth		$1 + 54T + 322pT^2 + 54p^3T^3 + p^6T^4$
3	singular	$\frac{1}{25}$	$(1 + pT)(1 - 134T + p^3T^2)$
4	smooth		$1 - 48T + 2206pT^2 - 48p^3T^3 + p^6T^4$
5	smooth		$(1 - 2pT + p^3T^2)(1 - 404T + p^3T^2)$
6	smooth		$1 + 64T + 302pT^2 + 64p^3T^3 + p^6T^4$
7	smooth		$1 - 204T + 838pT^2 - 204p^3T^3 + p^6T^4$
8	smooth		$1 - 274T + 2418pT^2 - 274p^3T^3 + p^6T^4$
9	smooth		$1 + 232T + 1166pT^2 + 232p^3T^3 + p^6T^4$
10	smooth		$1 - 244T + 2358pT^2 - 244p^3T^3 + p^6T^4$
11	smooth		$1 + 252T + 1366pT^2 + 252p^3T^3 + p^6T^4$
12	smooth		$1 + 176T + 1758pT^2 + 176p^3T^3 + p^6T^4$
13	smooth		$1 - 10T + 930pT^2 - 10p^3T^3 + p^6T^4$
14	smooth		$1 - 18T + 826pT^2 - 18p^3T^3 + p^6T^4$
15	smooth		$1 + 182T + 186pT^2 + 182p^3T^3 + p^6T^4$
16	smooth		$1 - 28T - 474pT^2 - 28p^3T^3 + p^6T^4$

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$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 18T - 1454pT^2 - 18p^3T^3 + p^6T^4$
18	smooth		$1 + 316T + 3158pT^2 + 316p^3T^3 + p^6T^4$
19	smooth		$1 + 162T + 1186pT^2 + 162p^3T^3 + p^6T^4$
20	smooth		$1 - 188T + 1766pT^2 - 188p^3T^3 + p^6T^4$
21	smooth		$(1 - 2pT + p^3T^2)(1 + 346T + p^3T^2)$
22	smooth		$1 - 258T + 58p^2T^2 - 258p^3T^3 + p^6T^4$
23	smooth		$1 + 170T + 1890pT^2 + 170p^3T^3 + p^6T^4$
24	smooth		$(1 - 2pT + p^3T^2)(1 + 358T + p^3T^2)$
25	smooth		$1 - 24T + 238pT^2 - 24p^3T^3 + p^6T^4$
26	smooth		$1 + 12T + 1846pT^2 + 12p^3T^3 + p^6T^4$
27	smooth		$1 + 432T + 3166pT^2 + 432p^3T^3 + p^6T^4$
28	smooth		$1 - 88T - 1074pT^2 - 88p^3T^3 + p^6T^4$
29	smooth		$1 + 432T + 3886pT^2 + 432p^3T^3 + p^6T^4$
30	smooth		$1 - 8T + 2126pT^2 - 8p^3T^3 + p^6T^4$
31	smooth		$1 + 30T + 1090pT^2 + 30p^3T^3 + p^6T^4$
32	smooth		$1 + 30T + 2410pT^2 + 30p^3T^3 + p^6T^4$
33	singular	$\frac{1}{9}$	$(1 - pT)(1 - 254T + p^3T^2)$
34	smooth		$1 - 28T + 1446pT^2 - 28p^3T^3 + p^6T^4$
35	smooth		$1 - 196T + 1062pT^2 - 196p^3T^3 + p^6T^4$
36	smooth		$1 - 308T + 2486pT^2 - 308p^3T^3 + p^6T^4$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 42T + p^3T^2)$
2	smooth		$1 - 48T + 1214pT^2 - 48p^3T^3 + p^6T^4$
3	smooth		$1 - 230T + 2018pT^2 - 230p^3T^3 + p^6T^4$
4	smooth		$1 + 112T + 2174pT^2 + 112p^3T^3 + p^6T^4$
5	smooth		$1 + 192T - 226pT^2 + 192p^3T^3 + p^6T^4$
6	smooth		$1 + 396T + 2966pT^2 + 396p^3T^3 + p^6T^4$
7	smooth		$1 - 12T + 2342pT^2 - 12p^3T^3 + p^6T^4$
8	smooth		$1 - 396T + 2390pT^2 - 396p^3T^3 + p^6T^4$
9	smooth		$1 + 24T - 850pT^2 + 24p^3T^3 + p^6T^4$
10	smooth		$(1 + 6pT + p^3T^2)(1 - 422T + p^3T^2)$
11	smooth		$1 - 6pT + 1970pT^2 - 6p^4T^3 + p^6T^4$
12	smooth		$1 - 282T + 482pT^2 - 282p^3T^3 + p^6T^4$
13	smooth		$1 - 106T + 290pT^2 - 106p^3T^3 + p^6T^4$
14	smooth		$1 + 8T - 2338pT^2 + 8p^3T^3 + p^6T^4$
15	smooth		$1 + 182T + 1154pT^2 + 182p^3T^3 + p^6T^4$
16	smooth		$1 - 36T - 2170pT^2 - 36p^3T^3 + p^6T^4$

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$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 + 148T + 422pT^2 + 148p^3T^3 + p^6T^4$
18	smooth		$1 - 76T - 490pT^2 - 76p^3T^3 + p^6T^4$
19	smooth		$1 - 30T + 578pT^2 - 30p^3T^3 + p^6T^4$
20	smooth		$1 - 108T - 106pT^2 - 108p^3T^3 + p^6T^4$
21	smooth		$1 - 128T - 1186pT^2 - 128p^3T^3 + p^6T^4$
22	smooth		$1 - 336T + 2030pT^2 - 336p^3T^3 + p^6T^4$
23	singular	$\frac{1}{25}$	$(1 - pT)(1 - 234T + p^3T^2)$
24	smooth		$1 + 4T + 470pT^2 + 4p^3T^3 + p^6T^4$
25	smooth		$1 - 36T - 250pT^2 - 36p^3T^3 + p^6T^4$
26	smooth		$1 - 312T + 1742pT^2 - 312p^3T^3 + p^6T^4$
27	smooth		$1 + 168T - 898pT^2 + 168p^3T^3 + p^6T^4$
28	smooth		$1 + 48T - 1138pT^2 + 48p^3T^3 + p^6T^4$
29	smooth		$1 + 172T + 614pT^2 + 172p^3T^3 + p^6T^4$
30	smooth		$1 - 2pT + 2642pT^2 - 2p^4T^3 + p^6T^4$
31	smooth		$1 + 104T + 590pT^2 + 104p^3T^3 + p^6T^4$
32	singular	$\frac{1}{9}$	$(1 + pT)(1 - 42T + p^3T^2)$
33	smooth		$1 + 292T + 1334pT^2 + 292p^3T^3 + p^6T^4$
34	smooth		$1 + 282T + 3554pT^2 + 282p^3T^3 + p^6T^4$
35	smooth		$(1 - 6pT + p^3T^2)(1 - 162T + p^3T^2)$
36	smooth		$1 + 392T + 3374pT^2 + 392p^3T^3 + p^6T^4$
37	smooth		$1 + 284T + 1670pT^2 + 284p^3T^3 + p^6T^4$
38	smooth		$1 - 44T - 154pT^2 - 44p^3T^3 + p^6T^4$
39	smooth		$1 + 552T + 4334pT^2 + 552p^3T^3 + p^6T^4$
40	smooth		$1 + 92T + 2054pT^2 + 92p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 52T + p^3T^2)$
2	smooth		$1 - 212T + 482pT^2 - 212p^3T^3 + p^6T^4$
3	smooth		$1 + 94T + 2186pT^2 + 94p^3T^3 + p^6T^4$
4	smooth		$1 + 48T + 1282pT^2 + 48p^3T^3 + p^6T^4$
5	smooth		$1 - 72T + 1762pT^2 - 72p^3T^3 + p^6T^4$
6	smooth		$(1 - 8pT + p^3T^2)(1 + 412T + p^3T^2)$
7	smooth		$1 + 356T + 2994pT^2 + 356p^3T^3 + p^6T^4$
8	smooth		$1 - 336T + 2626pT^2 - 336p^3T^3 + p^6T^4$
9	smooth		$1 - 196T + 3666pT^2 - 196p^3T^3 + p^6T^4$
10	smooth		$(1 + 4pT + p^3T^2)(1 - 508T + p^3T^2)$
11	smooth		$1 + 84T - 1454pT^2 + 84p^3T^3 + p^6T^4$
12	smooth		$1 + 228T + 322pT^2 + 228p^3T^3 + p^6T^4$

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$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
13	smooth		$(1 + 4pT + p^3T^2)(1 - 508T + p^3T^2)$
14	smooth		$1 + 368T + 2562pT^2 + 368p^3T^3 + p^6T^4$
15	smooth		$1 + 48T + 1282pT^2 + 48p^3T^3 + p^6T^4$
16	smooth		$1 - 196T - 654pT^2 - 196p^3T^3 + p^6T^4$
17	smooth		$1 + 224T + 1986pT^2 + 224p^3T^3 + p^6T^4$
18	smooth		$1 + 152T + 2418pT^2 + 152p^3T^3 + p^6T^4$
19	smooth		$1 - 128T + 818pT^2 - 128p^3T^3 + p^6T^4$
20	smooth		$1 - 12T - 1358pT^2 - 12p^3T^3 + p^6T^4$
21	smooth		$1 + 168T - 26p^2T^2 + 168p^3T^3 + p^6T^4$
22	smooth		$1 + 38T - 2238pT^2 + 38p^3T^3 + p^6T^4$
23	smooth		$1 + 44T + 2226pT^2 + 44p^3T^3 + p^6T^4$
24	singular	$\frac{1}{9}$	$(1 - pT)(1 + 52T + p^3T^2)$
25	smooth		$1 + 124T + 1586pT^2 + 124p^3T^3 + p^6T^4$
26	smooth		$1 - 178T + 1938pT^2 - 178p^3T^3 + p^6T^4$
27	smooth		$(1 - 8pT + p^3T^2)(1 + 292T + p^3T^2)$
28	smooth		$1 + 96T - 1166pT^2 + 96p^3T^3 + p^6T^4$
29	smooth		$1 - 162T + 1882pT^2 - 162p^3T^3 + p^6T^4$
30	smooth		$1 - 138T + 1738pT^2 - 138p^3T^3 + p^6T^4$
31	singular	$\frac{1}{25}$	$(1 + pT)(1 + 412T + p^3T^2)$
32	smooth		$1 - 214T - 366pT^2 - 214p^3T^3 + p^6T^4$
33	smooth		$1 + 326T + 2394pT^2 + 326p^3T^3 + p^6T^4$
34	smooth		$1 - 498T + 4618pT^2 - 498p^3T^3 + p^6T^4$
35	smooth		$1 - 176T + 1346pT^2 - 176p^3T^3 + p^6T^4$
36	smooth		$(1 + 4pT + p^3T^2)(1 - 244T + p^3T^2)$
37	smooth		$1 - 136T + 3426pT^2 - 136p^3T^3 + p^6T^4$
38	smooth		$1 + 188T + 3282pT^2 + 188p^3T^3 + p^6T^4$
39	smooth		$1 + 274T + 3506pT^2 + 274p^3T^3 + p^6T^4$
40	smooth		$1 + 264T + 1186pT^2 + 264p^3T^3 + p^6T^4$
41	smooth		$(1 + 4pT + p^3T^2)(1 - 428T + p^3T^2)$
42	smooth		$1 + 42T - 1982pT^2 + 42p^3T^3 + p^6T^4$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 96T + p^3T^2)$
2	smooth		$(1 + p^3T^2)(1 - 264T + p^3T^2)$
3	smooth		$(1 + p^3T^2)(1 + 136T + p^3T^2)$
4	smooth		$1 - 240T + 1730pT^2 - 240p^3T^3 + p^6T^4$
5	smooth		$1 - 216T + 1202pT^2 - 216p^3T^3 + p^6T^4$
6	smooth		$1 - 480T + 3650pT^2 - 480p^3T^3 + p^6T^4$

Continued on the following page

$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 + 8pT + 3458pT^2 + 8p^4T^3 + p^6T^4$
8	smooth		$1 - 144T + 1538pT^2 - 144p^3T^3 + p^6T^4$
9	smooth		$(1 + p^3T^2)(1 - 444T + p^3T^2)$
10	smooth		$1 + 72T + 866pT^2 + 72p^3T^3 + p^6T^4$
11	smooth		$(1 + 8pT + p^3T^2)(1 - 402T + p^3T^2)$
12	smooth		$1 - 124T + 578pT^2 - 124p^3T^3 + p^6T^4$
13	smooth		$1 - 2T + 74pT^2 - 2p^3T^3 + p^6T^4$
14	smooth		$1 - 264T + 578pT^2 - 264p^3T^3 + p^6T^4$
15	smooth		$1 - 276T + 3842pT^2 - 276p^3T^3 + p^6T^4$
16	smooth		$(1 + p^3T^2)(1 + 136T + p^3T^2)$
17	smooth		$(1 - 12pT + p^3T^2)(1 + 416T + p^3T^2)$
18	smooth		$1 - 64T + 3458pT^2 - 64p^3T^3 + p^6T^4$
19	smooth		$1 + 98T + 1754pT^2 + 98p^3T^3 + p^6T^4$
20	smooth		$(1 + 12pT + p^3T^2)(1 - 24T + p^3T^2)$
21	singular	$\frac{1}{9}$	$(1 + pT)(1 + 96T + p^3T^2)$
22	smooth		$1 + 436T + 3218pT^2 + 436p^3T^3 + p^6T^4$
23	smooth		$1 + 18T + 434pT^2 + 18p^3T^3 + p^6T^4$
24	smooth		$1 + 440T + 3650pT^2 + 440p^3T^3 + p^6T^4$
25	smooth		$1 - 244T + 2018pT^2 - 244p^3T^3 + p^6T^4$
26	smooth		$(1 - 12pT + p^3T^2)(1 - 32T + p^3T^2)$
27	smooth		$1 - 288T + 1346pT^2 - 288p^3T^3 + p^6T^4$
28	smooth		$1 - 304T + 3458pT^2 - 304p^3T^3 + p^6T^4$
29	smooth		$1 + 166T + 218pT^2 + 166p^3T^3 + p^6T^4$
30	smooth		$1 - 106T + 3602pT^2 - 106p^3T^3 + p^6T^4$
31	smooth		$1 + 156T - 382pT^2 + 156p^3T^3 + p^6T^4$
32	singular	$\frac{1}{25}$	$(1 + pT)(1 + 360T + p^3T^2)$
33	smooth		$1 + 536T + 4898pT^2 + 536p^3T^3 + p^6T^4$
34	smooth		$1 + 676T + 4898pT^2 + 676p^3T^3 + p^6T^4$
35	smooth		$1 - 560T + 5570pT^2 - 560p^3T^3 + p^6T^4$
36	smooth		$1 - 304T + 578pT^2 - 304p^3T^3 + p^6T^4$
37	smooth		$1 + 16T - 2302pT^2 + 16p^3T^3 + p^6T^4$
38	smooth		$1 + 296T + 2978pT^2 + 296p^3T^3 + p^6T^4$
39	smooth		$1 + 708T + 5714pT^2 + 708p^3T^3 + p^6T^4$
40	smooth		$1 + 270T + 2930pT^2 + 270p^3T^3 + p^6T^4$
41	smooth		$1 + 222T + 1826pT^2 + 222p^3T^3 + p^6T^4$
42	smooth		$1 - 168T + 3266pT^2 - 168p^3T^3 + p^6T^4$
43	smooth		$1 + 488T + 3314pT^2 + 488p^3T^3 + p^6T^4$
44	smooth		$1 - 158T + 1826pT^2 - 158p^3T^3 + p^6T^4$
45	smooth		$1 - 366T + 3122pT^2 - 366p^3T^3 + p^6T^4$
46	smooth		$1 - 612T + 5714pT^2 - 612p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 198T + p^3T^2)$
2	smooth		$1 - 560T + 6350pT^2 - 560p^3T^3 + p^6T^4$
3	smooth		$1 + 744T + 7886pT^2 + 744p^3T^3 + p^6T^4$
4	smooth		$1 + 984T + 8846pT^2 + 984p^3T^3 + p^6T^4$
5	smooth		$1 + 330T + 5450pT^2 + 330p^3T^3 + p^6T^4$
6	singular	$\frac{1}{9}$	$(1 + pT)(1 - 198T + p^3T^2)$
7	smooth		$1 + 444T + 4886pT^2 + 444p^3T^3 + p^6T^4$
8	smooth		$1 + 198T + 5042pT^2 + 198p^3T^3 + p^6T^4$
9	smooth		$1 + 280T + 4430pT^2 + 280p^3T^3 + p^6T^4$
10	smooth		$(1 - 6pT + p^3T^2)(1 + 498T + p^3T^2)$
11	smooth		$1 - 200T + 2030pT^2 - 200p^3T^3 + p^6T^4$
12	smooth		$1 - 30T + 3290pT^2 - 30p^3T^3 + p^6T^4$
13	smooth		$1 + 64T - 4354pT^2 + 64p^3T^3 + p^6T^4$
14	smooth		$1 + 336T + 4574pT^2 + 336p^3T^3 + p^6T^4$
15	smooth		$(1 - 6pT + p^3T^2)2$
16	smooth		$1 - 240T + 4190pT^2 - 240p^3T^3 + p^6T^4$
17	singular	$\frac{1}{25}$	$(1 + pT)(1 - 222T + p^3T^2)$
18	smooth		$1 + 442T + 4298pT^2 + 442p^3T^3 + p^6T^4$
19	smooth		$1 + 6pT + 1082pT^2 + 6p^4T^3 + p^6T^4$
20	smooth		$1 - 336T + 4766pT^2 - 336p^3T^3 + p^6T^4$
21	smooth		$1 + 108T - 58pT^2 + 108p^3T^3 + p^6T^4$
22	smooth		$1 + 182T + 698pT^2 + 182p^3T^3 + p^6T^4$
23	smooth		$1 - 8pT + 3614pT^2 - 8p^4T^3 + p^6T^4$
24	smooth		$(1 - 6pT + p^3T^2)2$
25	smooth		$1 - 460T + 3590pT^2 - 460p^3T^3 + p^6T^4$
26	smooth		$1 + 182T + 338pT^2 + 182p^3T^3 + p^6T^4$
27	smooth		$1 - 66T + 2906pT^2 - 66p^3T^3 + p^6T^4$
28	smooth		$1 + 580T + 4070pT^2 + 580p^3T^3 + p^6T^4$
29	smooth		$1 - 480T + 1790pT^2 - 480p^3T^3 + p^6T^4$
30	smooth		$1 - 316T - 154pT^2 - 316p^3T^3 + p^6T^4$
31	smooth		$1 - 194T + 4394pT^2 - 194p^3T^3 + p^6T^4$
32	smooth		$1 + 270T + 1130pT^2 + 270p^3T^3 + p^6T^4$
33	smooth		$1 + 14T + 26pT^2 + 14p^3T^3 + p^6T^4$
34	smooth		$1 - 148T + 1478pT^2 - 148p^3T^3 + p^6T^4$
35	smooth		$1 + 702T + 5018pT^2 + 702p^3T^3 + p^6T^4$
36	smooth		$1 - 256T + 4286pT^2 - 256p^3T^3 + p^6T^4$
37	smooth		$1 + 300T + 2870pT^2 + 300p^3T^3 + p^6T^4$
38	smooth		$1 + 420T + 5990pT^2 + 420p^3T^3 + p^6T^4$
39	smooth		$1 + 16T + 254pT^2 + 16p^3T^3 + p^6T^4$
40	smooth		$1 + 100T + 1190pT^2 + 100p^3T^3 + p^6T^4$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 - 798T + 6938pT^2 - 798p^3T^3 + p^6T^4$
42	smooth		$(1 - 6pT + p^3T^2)2$
43	smooth		$1 - 296T + 686pT^2 - 296p^3T^3 + p^6T^4$
44	smooth		$1 + 4T - 154pT^2 + 4p^3T^3 + p^6T^4$
45	smooth		$1 + 218T - 358pT^2 + 218p^3T^3 + p^6T^4$
46	smooth		$(1 + 14pT + p^3T^2)(1 - 6pT + p^3T^2)$
47	smooth		$1 + 384T + 2366pT^2 + 384p^3T^3 + p^6T^4$
48	smooth		$1 - 550T + 5690pT^2 - 550p^3T^3 + p^6T^4$
49	smooth		$(1 - 6pT + p^3T^2)(1 + 42T + p^3T^2)$
50	smooth		$1 - 310T + 4250pT^2 - 310p^3T^3 + p^6T^4$
51	smooth		$1 + 224T + 4286pT^2 + 224p^3T^3 + p^6T^4$
52	smooth		$1 - 120T - 2770pT^2 - 120p^3T^3 + p^6T^4$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 660T + p^3T^2)$
2	smooth		$1 + 200T + 482pT^2 + 200p^3T^3 + p^6T^4$
3	smooth		$1 + 72T - 1438pT^2 + 72p^3T^3 + p^6T^4$
4	smooth		$1 + 268T + 242pT^2 + 268p^3T^3 + p^6T^4$
5	smooth		$1 - 392T - 478pT^2 - 392p^3T^3 + p^6T^4$
6	smooth		$1 + 850T + 7442pT^2 + 850p^3T^3 + p^6T^4$
7	smooth		$1 + 300T - 2158pT^2 + 300p^3T^3 + p^6T^4$
8	smooth		$1 + 648T + 4322pT^2 + 648p^3T^3 + p^6T^4$
9	smooth		$1 - 840T + 9122pT^2 - 840p^3T^3 + p^6T^4$
10	smooth		$1 + 96T - 4558pT^2 + 96p^3T^3 + p^6T^4$
11	smooth		$1 + 240T - 958pT^2 + 240p^3T^3 + p^6T^4$
12	smooth		$1 + 512T + 3842pT^2 + 512p^3T^3 + p^6T^4$
13	smooth		$1 + 108T + 2pT^2 + 108p^3T^3 + p^6T^4$
14	smooth		$1 + 86T - 2p^2T^2 + 86p^3T^3 + p^6T^4$
15	smooth		$1 + 160T + 1922pT^2 + 160p^3T^3 + p^6T^4$
16	smooth		$1 - 448T + 962pT^2 - 448p^3T^3 + p^6T^4$
17	smooth		$1 - 80T + 962pT^2 - 80p^3T^3 + p^6T^4$
18	smooth		$1 - 54T - 958pT^2 - 54p^3T^3 + p^6T^4$
19	smooth		$1 + 288T + 2pT^2 + 288p^3T^3 + p^6T^4$
20	smooth		$1 - 480T + 5762pT^2 - 480p^3T^3 + p^6T^4$
21	smooth		$(1 - 12pT + p^3T^2)(1 - 300T + p^3T^2)$
22	smooth		$1 - 292T + 1202pT^2 - 292p^3T^3 + p^6T^4$
23	smooth		$1 - 10T - 958pT^2 - 10p^3T^3 + p^6T^4$
24	smooth		$1 - 470T + 6122pT^2 - 470p^3T^3 + p^6T^4$

Continued on the following page

$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
25	smooth		$1 - 72T - 478pT^2 - 72p^3T^3 + p^6T^4$
26	singular	$\frac{1}{25}$	$(1 - pT)(1 - 660T + p^3T^2)$
27	smooth		$1 + 100T + 1202pT^2 + 100p^3T^3 + p^6T^4$
28	smooth		$1 + 112T + 6722pT^2 + 112p^3T^3 + p^6T^4$
29	smooth		$1 - 180T + 242pT^2 - 180p^3T^3 + p^6T^4$
30	smooth		$1 - 786T + 7682pT^2 - 786p^3T^3 + p^6T^4$
31	smooth		$1 - 210T + 602pT^2 - 210p^3T^3 + p^6T^4$
32	smooth		$1 - 54T + 3002pT^2 - 54p^3T^3 + p^6T^4$
33	smooth		$1 - 464T + 5042pT^2 - 464p^3T^3 + p^6T^4$
34	smooth		$1 + 192T + 2pT^2 + 192p^3T^3 + p^6T^4$
35	smooth		$1 - 232T + 3362pT^2 - 232p^3T^3 + p^6T^4$
36	smooth		$1 - 188T - 238pT^2 - 188p^3T^3 + p^6T^4$
37	smooth		$1 - 56T + 242pT^2 - 56p^3T^3 + p^6T^4$
38	smooth		$1 + 776T + 6482pT^2 + 776p^3T^3 + p^6T^4$
39	smooth		$1 - 680T + 4322pT^2 - 680p^3T^3 + p^6T^4$
40	smooth		$1 - 1006T + 10202pT^2 - 1006p^3T^3 + p^6T^4$
41	smooth		$1 + 912T + 8642pT^2 + 912p^3T^3 + p^6T^4$
42	smooth		$(1 + 6pT + p^3T^2)(1 + 200T + p^3T^2)$
43	smooth		$1 + 978T + 8282pT^2 + 978p^3T^3 + p^6T^4$
44	smooth		$1 + 726T + 7562pT^2 + 726p^3T^3 + p^6T^4$
45	smooth		$(1 - 12pT + p^3T^2)(1 - 300T + p^3T^2)$
46	singular	$\frac{1}{9}$	$(1 + pT)(1 + 660T + p^3T^2)$
47	smooth		$1 + 380T + 1922pT^2 + 380p^3T^3 + p^6T^4$
48	smooth		$1 - 632T + 8162pT^2 - 632p^3T^3 + p^6T^4$
49	smooth		$1 - 148T + 1202pT^2 - 148p^3T^3 + p^6T^4$
50	smooth		$1 + 2T + 1202pT^2 + 2p^3T^3 + p^6T^4$
51	smooth		$1 - 280T - 478pT^2 - 280p^3T^3 + p^6T^4$
52	smooth		$1 + 54T - 1918pT^2 + 54p^3T^3 + p^6T^4$
53	smooth		$1 + 208T - 1918pT^2 + 208p^3T^3 + p^6T^4$
54	smooth		$1 + 6pT - 238pT^2 + 6p^4T^3 + p^6T^4$
55	smooth		$1 + 544T + 2162pT^2 + 544p^3T^3 + p^6T^4$
56	smooth		$1 - 240T + 5762pT^2 - 240p^3T^3 + p^6T^4$
57	smooth		$1 - 352T + 1922pT^2 - 352p^3T^3 + p^6T^4$
58	smooth		$1 + 342T + 3362pT^2 + 342p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 538T + p^3T^2)$
2	smooth		$1 - 278T + 3914pT^2 - 278p^3T^3 + p^6T^4$
<i>Continued on the following page</i>			

$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 + 376T + 1646pT^2 + 376p^3T^3 + p^6T^4$
4	smooth		$1 - 1264T + 13566pT^2 - 1264p^3T^3 + p^6T^4$
5	smooth		$1 - 564T + 46p^2T^2 - 564p^3T^3 + p^6T^4$
6	smooth		$1 + 330T + 7258pT^2 + 330p^3T^3 + p^6T^4$
7	smooth		$1 - 60T + 118pT^2 - 60p^3T^3 + p^6T^4$
8	smooth		$1 - 274T - 534pT^2 - 274p^3T^3 + p^6T^4$
9	smooth		$1 + 8pT + 1902pT^2 + 8p^4T^3 + p^6T^4$
10	smooth		$1 - 1410T + 15298pT^2 - 1410p^3T^3 + p^6T^4$
11	smooth		$1 + 168T + 4462pT^2 + 168p^3T^3 + p^6T^4$
12	smooth		$(1 - 14pT + p^3T^2)(1 - 102T + p^3T^2)$
13	smooth		$1 - 276T + 5110pT^2 - 276p^3T^3 + p^6T^4$
14	smooth		$1 - 272T + 1982pT^2 - 272p^3T^3 + p^6T^4$
15	smooth		$1 + 556T + 1526pT^2 + 556p^3T^3 + p^6T^4$
16	smooth		$1 + 628T + 2822pT^2 + 628p^3T^3 + p^6T^4$
17	smooth		$(1 - 2pT + p^3T^2)(1 + 824T + p^3T^2)$
18	smooth		$1 - 388T + 2214pT^2 - 388p^3T^3 + p^6T^4$
19	smooth		$1 + 116T + 966pT^2 + 116p^3T^3 + p^6T^4$
20	smooth		$1 + 844T + 8630pT^2 + 844p^3T^3 + p^6T^4$
21	smooth		$1 + 276T + 5446pT^2 + 276p^3T^3 + p^6T^4$
22	singular	$\frac{1}{25}$	$(1 - pT)(1 + 490T + p^3T^2)$
23	smooth		$1 - 470T + 6218pT^2 - 470p^3T^3 + p^6T^4$
24	smooth		$1 - 532T + 7542pT^2 - 532p^3T^3 + p^6T^4$
25	smooth		$1 + 648T + 6382pT^2 + 648p^3T^3 + p^6T^4$
26	smooth		$(1 - 8pT + p^3T^2)(1 + 198T + p^3T^2)$
27	smooth		$1 - 744T + 4846pT^2 - 744p^3T^3 + p^6T^4$
28	smooth		$1 - 60T + 3478pT^2 - 60p^3T^3 + p^6T^4$
29	smooth		$1 + 42T + 2194pT^2 + 42p^3T^3 + p^6T^4$
30	smooth		$1 - 30T - 3182pT^2 - 30p^3T^3 + p^6T^4$
31	smooth		$1 - 1000T + 9438pT^2 - 1000p^3T^3 + p^6T^4$
32	smooth		$1 + 188T + 2262pT^2 + 188p^3T^3 + p^6T^4$
33	smooth		$1 + 110T + 858pT^2 + 110p^3T^3 + p^6T^4$
34	singular	$\frac{1}{9}$	$(1 - pT)(1 + 538T + p^3T^2)$
35	smooth		$1 - 332T + 62p^2T^2 - 332p^3T^3 + p^6T^4$
36	smooth		$(1 - 2pT + p^3T^2)(1 - 590T + p^3T^2)$
37	smooth		$1 + 408T + 382pT^2 + 408p^3T^3 + p^6T^4$
38	smooth		$1 - 180T + 838pT^2 - 180p^3T^3 + p^6T^4$
39	smooth		$1 - 424T + 2286pT^2 - 424p^3T^3 + p^6T^4$
40	smooth		$1 - 700T + 5718pT^2 - 700p^3T^3 + p^6T^4$
41	smooth		$1 - 96T + 2110pT^2 - 96p^3T^3 + p^6T^4$
42	smooth		$1 - 252T + 1702pT^2 - 252p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
43	smooth		$1 + 498T + 2362pT^2 + 498p^3T^3 + p^6T^4$
44	smooth		$1 - 246T - 1790pT^2 - 246p^3T^3 + p^6T^4$
45	smooth		$1 + 248T - 18pT^2 + 248p^3T^3 + p^6T^4$
46	smooth		$1 - 192T + 862pT^2 - 192p^3T^3 + p^6T^4$
47	smooth		$1 - 264T + 526pT^2 - 264p^3T^3 + p^6T^4$
48	smooth		$1 - 264T + 5326pT^2 - 264p^3T^3 + p^6T^4$
49	smooth		$1 + 476T + 4566pT^2 + 476p^3T^3 + p^6T^4$
50	smooth		$(1 + 8pT + p^3T^2)(1 + 718T + p^3T^2)$
51	smooth		$1 + 636T + 9046pT^2 + 636p^3T^3 + p^6T^4$
52	smooth		$1 + 996T + 10726pT^2 + 996p^3T^3 + p^6T^4$
53	smooth		$1 + 378T + 4762pT^2 + 378p^3T^3 + p^6T^4$
54	smooth		$1 + 570T + 6658pT^2 + 570p^3T^3 + p^6T^4$
55	smooth		$1 + 128T - 2178pT^2 + 128p^3T^3 + p^6T^4$
56	smooth		$1 + 168T - 1298pT^2 + 168p^3T^3 + p^6T^4$
57	smooth		$1 + 168T + 2542pT^2 + 168p^3T^3 + p^6T^4$
58	smooth		$1 + 268T - 1738pT^2 + 268p^3T^3 + p^6T^4$
59	smooth		$1 - 326T + 7370pT^2 - 326p^3T^3 + p^6T^4$
60	smooth		$1 - 264T + 1006pT^2 - 264p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 884T + p^3T^2)$
2	smooth		$1 + 460T + 1730pT^2 + 460p^3T^3 + p^6T^4$
3	smooth		$1 - 594T + 9418pT^2 - 594p^3T^3 + p^6T^4$
4	smooth		$(1 + 12pT + p^3T^2)(1 - 1004T + p^3T^2)$
5	smooth		$1 - 364T + 3858pT^2 - 364p^3T^3 + p^6T^4$
6	smooth		$1 - 476T + 6482pT^2 - 476p^3T^3 + p^6T^4$
7	smooth		$1 - 156T - 1118pT^2 - 156p^3T^3 + p^6T^4$
8	smooth		$1 - 388T + 8226pT^2 - 388p^3T^3 + p^6T^4$
9	smooth		$1 + 364T + 3602pT^2 + 364p^3T^3 + p^6T^4$
10	smooth		$1 - 556T + 8562pT^2 - 556p^3T^3 + p^6T^4$
11	smooth		$1 + 202T + 6386pT^2 + 202p^3T^3 + p^6T^4$
12	smooth		$1 - 102T + 2074pT^2 - 102p^3T^3 + p^6T^4$
13	smooth		$1 + 112T + 4946pT^2 + 112p^3T^3 + p^6T^4$
14	smooth		$1 - 16T + 2082pT^2 - 16p^3T^3 + p^6T^4$
15	singular	$\frac{1}{9}$	$(1 - pT)(1 - 884T + p^3T^2)$
16	smooth		$1 + 304T + 2402pT^2 + 304p^3T^3 + p^6T^4$
17	smooth		$1 - 836T + 10802pT^2 - 836p^3T^3 + p^6T^4$
18	smooth		$1 + 278T + 6354pT^2 + 278p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$(1 + 4pT + p^3T^2)(1 + 716T + p^3T^2)$
20	smooth		$1 + 848T + 8274pT^2 + 848p^3T^3 + p^6T^4$
21	smooth		$1 + 64T + 7202pT^2 + 64p^3T^3 + p^6T^4$
22	smooth		$1 - 1032T + 10114pT^2 - 1032p^3T^3 + p^6T^4$
23	smooth		$1 + 144T - 1118pT^2 + 144p^3T^3 + p^6T^4$
24	smooth		$1 - 56T + 1922pT^2 - 56p^3T^3 + p^6T^4$
25	smooth		$1 - 136T - 1278pT^2 - 136p^3T^3 + p^6T^4$
26	smooth		$(1 + 12pT + p^3T^2)(1 - 524T + p^3T^2)$
27	smooth		$1 - 1158T + 13426pT^2 - 1158p^3T^3 + p^6T^4$
28	smooth		$1 - 630T + 1930pT^2 - 630p^3T^3 + p^6T^4$
29	smooth		$(1 + 4pT + p^3T^2)(1 + 676T + p^3T^2)$
30	smooth		$1 + 294T - 158pT^2 + 294p^3T^3 + p^6T^4$
31	smooth		$1 - 84T + 8578pT^2 - 84p^3T^3 + p^6T^4$
32	smooth		$1 + 248T + 102p^2T^2 + 248p^3T^3 + p^6T^4$
33	smooth		$1 - 456T + 5122pT^2 - 456p^3T^3 + p^6T^4$
34	smooth		$1 + 614T + 1602pT^2 + 614p^3T^3 + p^6T^4$
35	smooth		$(1 + 4pT + p^3T^2)(1 + 36T + p^3T^2)$
36	smooth		$1 - 216T + 4162pT^2 - 216p^3T^3 + p^6T^4$
37	smooth		$(1 + 4pT + p^3T^2)(1 + 676T + p^3T^2)$
38	smooth		$1 - 210T + 2050pT^2 - 210p^3T^3 + p^6T^4$
39	smooth		$1 - 936T + 8002pT^2 - 936p^3T^3 + p^6T^4$
40	smooth		$1 + 164T - 78pT^2 + 164p^3T^3 + p^6T^4$
41	smooth		$1 + 920T + 7170pT^2 + 920p^3T^3 + p^6T^4$
42	smooth		$1 + 124T + 1202pT^2 + 124p^3T^3 + p^6T^4$
43	smooth		$1 + 344T - 2238pT^2 + 344p^3T^3 + p^6T^4$
44	smooth		$1 - 298T - 294pT^2 - 298p^3T^3 + p^6T^4$
45	smooth		$1 - 92T + 2354pT^2 - 92p^3T^3 + p^6T^4$
46	smooth		$1 - 90T - 3110pT^2 - 90p^3T^3 + p^6T^4$
47	smooth		$1 + 44T + 1362pT^2 + 44p^3T^3 + p^6T^4$
48	smooth		$1 + 840T + 10690pT^2 + 840p^3T^3 + p^6T^4$
49	smooth		$1 + 664T + 8642pT^2 + 664p^3T^3 + p^6T^4$
50	smooth		$1 - 6T + 6802pT^2 - 6p^3T^3 + p^6T^4$
51	smooth		$1 + 188T + 5634pT^2 + 188p^3T^3 + p^6T^4$
52	smooth		$1 + 132T + 5986pT^2 + 132p^3T^3 + p^6T^4$
53	smooth		$1 - 76T + 4722pT^2 - 76p^3T^3 + p^6T^4$
54	smooth		$1 + 960T + 10210pT^2 + 960p^3T^3 + p^6T^4$
55	smooth		$1 + 4T - 3598pT^2 + 4p^3T^3 + p^6T^4$
56	smooth		$1 - 356T + 2162pT^2 - 356p^3T^3 + p^6T^4$
57	smooth		$1 - 122T + 1394pT^2 - 122p^3T^3 + p^6T^4$
58	smooth		$1 + 164T - 2958pT^2 + 164p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
59	singular	$\frac{1}{25}$	$(1 + pT)(1 - 812T + p^3T^2)$
60	smooth		$1 - 156T + 5362pT^2 - 156p^3T^3 + p^6T^4$
61	smooth		$1 + 416T + 7698pT^2 + 416p^3T^3 + p^6T^4$
62	smooth		$1 + 80T + 4770pT^2 + 80p^3T^3 + p^6T^4$
63	smooth		$1 - 498T + 8626pT^2 - 498p^3T^3 + p^6T^4$
64	smooth		$1 + 20T - 1230pT^2 + 20p^3T^3 + p^6T^4$
65	smooth		$1 - 180T + 2290pT^2 - 180p^3T^3 + p^6T^4$
66	smooth		$1 + 672T + 5266pT^2 + 672p^3T^3 + p^6T^4$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 792T + p^3T^2)$
2	smooth		$1 + 1068T + 9122pT^2 + 1068p^3T^3 + p^6T^4$
3	smooth		$1 + 436T + 6626pT^2 + 436p^3T^3 + p^6T^4$
4	smooth		$1 + 248T - 478pT^2 + 248p^3T^3 + p^6T^4$
5	smooth		$1 + 240T - 1822pT^2 + 240p^3T^3 + p^6T^4$
6	smooth		$1 - 800T + 6818pT^2 - 800p^3T^3 + p^6T^4$
7	smooth		$1 + 654T + 3530pT^2 + 654p^3T^3 + p^6T^4$
8	singular	$\frac{1}{9}$	$(1 + pT)(1 - 792T + p^3T^2)$
9	smooth		$1 - 532T + 2882pT^2 - 532p^3T^3 + p^6T^4$
10	smooth		$(1 + p^3T^2)(1 - 392T + p^3T^2)$
11	smooth		$1 + 526T + 6506pT^2 + 526p^3T^3 + p^6T^4$
12	smooth		$1 + 160T - 862pT^2 + 160p^3T^3 + p^6T^4$
13	smooth		$1 + 260T - 2p^2T^2 + 260p^3T^3 + p^6T^4$
14	smooth		$1 - 1218T + 11474pT^2 - 1218p^3T^3 + p^6T^4$
15	smooth		$(1 + p^3T^2)(1 - 912T + p^3T^2)$
16	smooth		$1 - 132T - 2878pT^2 - 132p^3T^3 + p^6T^4$
17	smooth		$1 - 654T + 6266pT^2 - 654p^3T^3 + p^6T^4$
18	smooth		$1 - 112T - 4318pT^2 - 112p^3T^3 + p^6T^4$
19	smooth		$1 - 320T + 5858pT^2 - 320p^3T^3 + p^6T^4$
20	smooth		$1 + 168T + 4322pT^2 + 168p^3T^3 + p^6T^4$
21	smooth		$(1 - 12pT + p^3T^2)(1 + 126T + p^3T^2)$
22	smooth		$1 - 420T + 7778pT^2 - 420p^3T^3 + p^6T^4$
23	smooth		$1 + 644T + 7730pT^2 + 644p^3T^3 + p^6T^4$
24	smooth		$1 - 232T - 1438pT^2 - 232p^3T^3 + p^6T^4$
25	smooth		$1 + 1176T + 12386pT^2 + 1176p^3T^3 + p^6T^4$
26	smooth		$1 - 140T + 5138pT^2 - 140p^3T^3 + p^6T^4$
27	smooth		$1 + 28T + 3362pT^2 + 28p^3T^3 + p^6T^4$
28	smooth		$1 + 936T + 6146pT^2 + 936p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
29	smooth		$1 - 112T - 6238pT^2 - 112p^3T^3 + p^6T^4$
30	smooth		$(1 + 8pT + p^3T^2)(1 + 768T + p^3T^2)$
31	smooth		$1 - 706T + 9770pT^2 - 706p^3T^3 + p^6T^4$
32	smooth		$1 - 512T + 6242pT^2 - 512p^3T^3 + p^6T^4$
33	smooth		$1 - 660T + 4178pT^2 - 660p^3T^3 + p^6T^4$
34	smooth		$1 + 808T + 7202pT^2 + 808p^3T^3 + p^6T^4$
35	smooth		$1 + 102T + 1154pT^2 + 102p^3T^3 + p^6T^4$
36	smooth		$1 - 152T - 1438pT^2 - 152p^3T^3 + p^6T^4$
37	smooth		$1 - 672T + 2402pT^2 - 672p^3T^3 + p^6T^4$
38	smooth		$1 + 1368T + 13922pT^2 + 1368p^3T^3 + p^6T^4$
39	smooth		$1 - 248T + 94p^2T^2 - 248p^3T^3 + p^6T^4$
40	smooth		$1 - 184T + 8546pT^2 - 184p^3T^3 + p^6T^4$
41	smooth		$1 - 1318T + 13514pT^2 - 1318p^3T^3 + p^6T^4$
42	smooth		$1 - 252T - 1438pT^2 - 252p^3T^3 + p^6T^4$
43	smooth		$1 + 368T + 2402pT^2 + 368p^3T^3 + p^6T^4$
44	smooth		$1 + 46T + 7706pT^2 + 46p^3T^3 + p^6T^4$
45	smooth		$1 + 8pT + 4322pT^2 + 8p^4T^3 + p^6T^4$
46	smooth		$1 - 282T + 3002pT^2 - 282p^3T^3 + p^6T^4$
47	smooth		$1 - 442T + 4082pT^2 - 442p^3T^3 + p^6T^4$
48	smooth		$1 + 796T + 6626pT^2 + 796p^3T^3 + p^6T^4$
49	smooth		$1 + 216T + 2786pT^2 + 216p^3T^3 + p^6T^4$
50	smooth		$1 + 800T + 6818pT^2 + 800p^3T^3 + p^6T^4$
51	smooth		$1 + 210T - 4822pT^2 + 210p^3T^3 + p^6T^4$
52	smooth		$1 - 144T - 2014pT^2 - 144p^3T^3 + p^6T^4$
53	smooth		$1 + 14T + 7490pT^2 + 14p^3T^3 + p^6T^4$
54	singular	$\frac{1}{25}$	$(1 - pT)(1 - 120T + p^3T^2)$
55	smooth		$1 - 66T + 4250pT^2 - 66p^3T^3 + p^6T^4$
56	smooth		$1 + 564T + 4370pT^2 + 564p^3T^3 + p^6T^4$
57	smooth		$1 - 840T + 10658pT^2 - 840p^3T^3 + p^6T^4$
58	smooth		$1 - 472T + 7202pT^2 - 472p^3T^3 + p^6T^4$
59	smooth		$1 - 318T + 7034pT^2 - 318p^3T^3 + p^6T^4$
60	smooth		$1 + 28T + 3842pT^2 + 28p^3T^3 + p^6T^4$
61	smooth		$1 + 1112T + 11474pT^2 + 1112p^3T^3 + p^6T^4$
62	smooth		$1 + 492T + 4754pT^2 + 492p^3T^3 + p^6T^4$
63	smooth		$1 + 366T + 5906pT^2 + 366p^3T^3 + p^6T^4$
64	smooth		$1 + 228T - 4798pT^2 + 228p^3T^3 + p^6T^4$
65	smooth		$1 - 82T + 3962pT^2 - 82p^3T^3 + p^6T^4$
66	smooth		$1 - 216T + 1490pT^2 - 216p^3T^3 + p^6T^4$
67	smooth		$(1 + p^3T^2)(1 + 488T + p^3T^2)$
68	smooth		$1 + 162T + 4754pT^2 + 162p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
69	smooth		$1 - 182T + 1922pT^2 - 182p^3T^3 + p^6T^4$
70	smooth		$1 - 390T - 2302pT^2 - 390p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 218T + p^3T^2)$
2	smooth		$1 - 44T - 4906pT^2 - 44p^3T^3 + p^6T^4$
3	smooth		$1 - 640T + 6750pT^2 - 640p^3T^3 + p^6T^4$
4	smooth		$(1 + 6pT + p^3T^2)(1 + 22T + p^3T^2)$
5	smooth		$1 + 560T + 5790pT^2 + 560p^3T^3 + p^6T^4$
6	smooth		$1 - 448T + 7998pT^2 - 448p^3T^3 + p^6T^4$
7	smooth		$1 - 1236T + 13846pT^2 - 1236p^3T^3 + p^6T^4$
8	smooth		$1 + 76T + 7334pT^2 + 76p^3T^3 + p^6T^4$
9	smooth		$1 + 156T + 7174pT^2 + 156p^3T^3 + p^6T^4$
10	smooth		$1 - 274T + 4794pT^2 - 274p^3T^3 + p^6T^4$
11	smooth		$1 - 212T + 4982pT^2 - 212p^3T^3 + p^6T^4$
12	smooth		$1 + 536T + 7374pT^2 + 536p^3T^3 + p^6T^4$
13	smooth		$1 - 270T + 7570pT^2 - 270p^3T^3 + p^6T^4$
14	smooth		$1 - 54T + 1354pT^2 - 54p^3T^3 + p^6T^4$
15	smooth		$1 + 348T + 2902pT^2 + 348p^3T^3 + p^6T^4$
16	smooth		$1 - 284T - 2506pT^2 - 284p^3T^3 + p^6T^4$
17	smooth		$1 + 462T + 1018pT^2 + 462p^3T^3 + p^6T^4$
18	smooth		$1 - 68T - 6202pT^2 - 68p^3T^3 + p^6T^4$
19	smooth		$1 + 256T - 226pT^2 + 256p^3T^3 + p^6T^4$
20	smooth		$1 + 1110T + 14170pT^2 + 1110p^3T^3 + p^6T^4$
21	smooth		$1 + 492T + 9958pT^2 + 492p^3T^3 + p^6T^4$
22	smooth		$1 + 306T - 3926pT^2 + 306p^3T^3 + p^6T^4$
23	smooth		$1 - 648T + 9358pT^2 - 648p^3T^3 + p^6T^4$
24	smooth		$1 + 552T + 2638pT^2 + 552p^3T^3 + p^6T^4$
25	smooth		$1 - 348T + 2038pT^2 - 348p^3T^3 + p^6T^4$
26	smooth		$1 - 26T + 7706pT^2 - 26p^3T^3 + p^6T^4$
27	smooth		$1 - 488T + 9038pT^2 - 488p^3T^3 + p^6T^4$
28	smooth		$1 + 230T + 5850pT^2 + 230p^3T^3 + p^6T^4$
29	smooth		$1 + 164T + 8646pT^2 + 164p^3T^3 + p^6T^4$
30	smooth		$1 - 296T - 3874pT^2 - 296p^3T^3 + p^6T^4$
31	smooth		$1 + 520T + 6350pT^2 + 520p^3T^3 + p^6T^4$
32	smooth		$1 + 252T + 8518pT^2 + 252p^3T^3 + p^6T^4$
33	smooth		$1 + 166T + 7754pT^2 + 166p^3T^3 + p^6T^4$
34	smooth		$1 - 618T + 6178pT^2 - 618p^3T^3 + p^6T^4$

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$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 + 776T + 5934pT^2 + 776p^3T^3 + p^6T^4$
36	smooth		$1 + 916T + 10454pT^2 + 916p^3T^3 + p^6T^4$
37	smooth		$1 + 636T + 6214pT^2 + 636p^3T^3 + p^6T^4$
38	singular	$\frac{1}{25}$	$(1 + pT)(1 - 746T + p^3T^2)$
39	smooth		$1 - 406T - 1854pT^2 - 406p^3T^3 + p^6T^4$
40	smooth		$1 - 936T + 7006pT^2 - 936p^3T^3 + p^6T^4$
41	smooth		$1 - 224T + 5534pT^2 - 224p^3T^3 + p^6T^4$
42	smooth		$1 - 540T + 3670pT^2 - 540p^3T^3 + p^6T^4$
43	smooth		$1 + 116T + 7254pT^2 + 116p^3T^3 + p^6T^4$
44	smooth		$1 - 994T + 10194pT^2 - 994p^3T^3 + p^6T^4$
45	smooth		$1 - 760T + 9390pT^2 - 760p^3T^3 + p^6T^4$
46	smooth		$1 + 260T + 6390pT^2 + 260p^3T^3 + p^6T^4$
47	smooth		$1 + 272T + 5358pT^2 + 272p^3T^3 + p^6T^4$
48	smooth		$1 - 1304T + 14894pT^2 - 1304p^3T^3 + p^6T^4$
49	smooth		$1 - 368T + 1598pT^2 - 368p^3T^3 + p^6T^4$
50	smooth		$1 + 200T + 30p^2T^2 + 200p^3T^3 + p^6T^4$
51	smooth		$1 + 224T + 4446pT^2 + 224p^3T^3 + p^6T^4$
52	smooth		$(1 - 2pT + p^3T^2)(1 - 538T + p^3T^2)$
53	smooth		$1 + 1218T + 11482pT^2 + 1218p^3T^3 + p^6T^4$
54	smooth		$1 - 304T + 4254pT^2 - 304p^3T^3 + p^6T^4$
55	smooth		$1 - 184T + 1614pT^2 - 184p^3T^3 + p^6T^4$
56	smooth		$1 + 16T - 4786pT^2 + 16p^3T^3 + p^6T^4$
57	smooth		$1 + 636T + 10054pT^2 + 636p^3T^3 + p^6T^4$
58	smooth		$1 + 340T - 1690pT^2 + 340p^3T^3 + p^6T^4$
59	smooth		$(1 + 14pT + p^3T^2)(1 - 764T + p^3T^2)$
60	smooth		$1 - 284T + 9014pT^2 - 284p^3T^3 + p^6T^4$
61	smooth		$(1 + 10pT + p^3T^2)(1 + 142T + p^3T^2)$
62	smooth		$1 - 504T + 2014pT^2 - 504p^3T^3 + p^6T^4$
63	smooth		$1 - 776T + 7166pT^2 - 776p^3T^3 + p^6T^4$
64	smooth		$(1 - 2pT + p^3T^2)(1 + 1062T + p^3T^2)$
65	singular	$\frac{1}{9}$	$(1 - pT)(1 - 218T + p^3T^2)$
66	smooth		$1 + 406T + 7754pT^2 + 406p^3T^3 + p^6T^4$
67	smooth		$1 - 160T + 9150pT^2 - 160p^3T^3 + p^6T^4$
68	smooth		$1 + 566T + 7194pT^2 + 566p^3T^3 + p^6T^4$
69	smooth		$1 - 4T + 3654pT^2 - 4p^3T^3 + p^6T^4$
70	smooth		$1 + 112T + 9278pT^2 + 112p^3T^3 + p^6T^4$
71	smooth		$1 + 660T + 1750pT^2 + 660p^3T^3 + p^6T^4$
72	smooth		$1 - 404T + 2534pT^2 - 404p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 520T + p^3T^2)$
2	smooth		$1 + 1308T + 13762pT^2 + 1308p^3T^3 + p^6T^4$
3	smooth		$1 - 82T - 7158pT^2 - 82p^3T^3 + p^6T^4$
4	smooth		$1 + 404T + 2082pT^2 + 404p^3T^3 + p^6T^4$
5	smooth		$1 - 752T + 4802pT^2 - 752p^3T^3 + p^6T^4$
6	smooth		$1 - 620T + 7682pT^2 - 620p^3T^3 + p^6T^4$
7	smooth		$1 + 440T + 4722pT^2 + 440p^3T^3 + p^6T^4$
8	smooth		$1 + 1048T + 10562pT^2 + 1048p^3T^3 + p^6T^4$
9	smooth		$1 + 288T - 2558pT^2 + 288p^3T^3 + p^6T^4$
10	smooth		$1 - 92T - 7678pT^2 - 92p^3T^3 + p^6T^4$
11	smooth		$1 - 296T + 2882pT^2 - 296p^3T^3 + p^6T^4$
12	smooth		$1 + 1202T + 14442pT^2 + 1202p^3T^3 + p^6T^4$
13	smooth		$(1 + p^3T^2)(1 - 776T + p^3T^2)$
14	smooth		$1 - 342T + 6922pT^2 - 342p^3T^3 + p^6T^4$
15	smooth		$1 - 710T + 5042pT^2 - 710p^3T^3 + p^6T^4$
16	smooth		$1 - 272T - 2878pT^2 - 272p^3T^3 + p^6T^4$
17	smooth		$1 - 706T + 8682pT^2 - 706p^3T^3 + p^6T^4$
18	smooth		$1 + 528T + 322pT^2 + 528p^3T^3 + p^6T^4$
19	singular	$\frac{1}{25}$	$(1 - pT)(1 - 152T + p^3T^2)$
20	smooth		$1 - 912T + 13762pT^2 - 912p^3T^3 + p^6T^4$
21	smooth		$1 - 104T - 6718pT^2 - 104p^3T^3 + p^6T^4$
22	smooth		$(1 - 8pT + p^3T^2)(1 + 880T + p^3T^2)$
23	smooth		$1 - 156T + 2722pT^2 - 156p^3T^3 + p^6T^4$
24	smooth		$1 + 190T - 2278pT^2 + 190p^3T^3 + p^6T^4$
25	smooth		$1 - 1512T + 15682pT^2 - 1512p^3T^3 + p^6T^4$
26	smooth		$1 + 736T + 962pT^2 + 736p^3T^3 + p^6T^4$
27	smooth		$1 - 64T - 1278pT^2 - 64p^3T^3 + p^6T^4$
28	smooth		$1 + 398T - 3918pT^2 + 398p^3T^3 + p^6T^4$
29	smooth		$1 + 1164T + 15442pT^2 + 1164p^3T^3 + p^6T^4$
30	smooth		$1 + 972T + 9682pT^2 + 972p^3T^3 + p^6T^4$
31	smooth		$1 - 32T - 6718pT^2 - 32p^3T^3 + p^6T^4$
32	smooth		$1 + 1368T + 13762pT^2 + 1368p^3T^3 + p^6T^4$
33	smooth		$1 - 1008T + 8722pT^2 - 1008p^3T^3 + p^6T^4$
34	smooth		$1 - 224T + 1922pT^2 - 224p^3T^3 + p^6T^4$
35	smooth		$1 - 362T + 11042pT^2 - 362p^3T^3 + p^6T^4$
36	smooth		$1 - 232T - 318pT^2 - 232p^3T^3 + p^6T^4$
37	smooth		$1 - 1312T + 13122pT^2 - 1312p^3T^3 + p^6T^4$
38	smooth		$1 - 904T + 5442pT^2 - 904p^3T^3 + p^6T^4$
39	smooth		$1 + 254T + 2922pT^2 + 254p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 888T + 9922pT^2 + 888p^3T^3 + p^6T^4$
41	smooth		$1 - 952T + 7842pT^2 - 952p^3T^3 + p^6T^4$
42	smooth		$(1 - 8pT + p^3T^2)(1 + 320T + p^3T^2)$
43	smooth		$1 - 420T + 6562pT^2 - 420p^3T^3 + p^6T^4$
44	singular	$\frac{1}{9}$	$(1 - pT)(1 + 520T + p^3T^2)$
45	smooth		$(1 - 8pT + p^3T^2)(1 - 240T + p^3T^2)$
46	smooth		$1 + 808T + 6722pT^2 + 808p^3T^3 + p^6T^4$
47	smooth		$1 - 20T + 1442pT^2 - 20p^3T^3 + p^6T^4$
48	smooth		$1 + 830T + 9042pT^2 + 830p^3T^3 + p^6T^4$
49	smooth		$1 + 776T + 9282pT^2 + 776p^3T^3 + p^6T^4$
50	smooth		$1 - 576T + 4162pT^2 - 576p^3T^3 + p^6T^4$
51	smooth		$1 - 964T + 10242pT^2 - 964p^3T^3 + p^6T^4$
52	smooth		$1 - 756T + 6562pT^2 - 756p^3T^3 + p^6T^4$
53	smooth		$1 - 294T - 5558pT^2 - 294p^3T^3 + p^6T^4$
54	smooth		$1 + 582T - 878pT^2 + 582p^3T^3 + p^6T^4$
55	smooth		$1 + 88T + 4802pT^2 + 88p^3T^3 + p^6T^4$
56	smooth		$1 + 104T - 7758pT^2 + 104p^3T^3 + p^6T^4$
57	smooth		$1 + 500T + 1842pT^2 + 500p^3T^3 + p^6T^4$
58	smooth		$1 + 408T + 1282pT^2 + 408p^3T^3 + p^6T^4$
59	smooth		$1 - 214T + 8802pT^2 - 214p^3T^3 + p^6T^4$
60	smooth		$1 + 1330T + 15122pT^2 + 1330p^3T^3 + p^6T^4$
61	smooth		$1 + 406T + 3602pT^2 + 406p^3T^3 + p^6T^4$
62	smooth		$(1 - 8pT + p^3T^2)(1 - 200T + p^3T^2)$
63	smooth		$1 - 196T + 3762pT^2 - 196p^3T^3 + p^6T^4$
64	smooth		$1 - 552T + 3202pT^2 - 552p^3T^3 + p^6T^4$
65	smooth		$(1 - 8pT + p^3T^2)(1 + 1080T + p^3T^2)$
66	smooth		$1 + 266T + 7962pT^2 + 266p^3T^3 + p^6T^4$
67	smooth		$1 + 56T - 7998pT^2 + 56p^3T^3 + p^6T^4$
68	smooth		$1 + 10T + 3482pT^2 + 10p^3T^3 + p^6T^4$
69	smooth		$1 - 620T + 7922pT^2 - 620p^3T^3 + p^6T^4$
70	smooth		$1 + 564T + 2962pT^2 + 564p^3T^3 + p^6T^4$
71	smooth		$1 - 1294T + 14322pT^2 - 1294p^3T^3 + p^6T^4$
72	smooth		$1 + 848T + 13122pT^2 + 848p^3T^3 + p^6T^4$
73	smooth		$1 - 232T + 7362pT^2 - 232p^3T^3 + p^6T^4$
74	smooth		$1 + 750T + 5962pT^2 + 750p^3T^3 + p^6T^4$
75	smooth		$1 + 90T - 8798pT^2 + 90p^3T^3 + p^6T^4$
76	smooth		$1 + 616T + 2882pT^2 + 616p^3T^3 + p^6T^4$
77	smooth		$1 - 40T + 882pT^2 - 40p^3T^3 + p^6T^4$
78	smooth		$1 - 852T + 12802pT^2 - 852p^3T^3 + p^6T^4$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 + 492T + p^3T^2)$
2	smooth		$1 - 688T + 14018pT^2 - 688p^3T^3 + p^6T^4$
3	smooth		$1 - 1964T + 22994pT^2 - 1964p^3T^3 + p^6T^4$
4	smooth		$1 + 36T + 11474pT^2 + 36p^3T^3 + p^6T^4$
5	smooth		$1 - 1728T + 19778pT^2 - 1728p^3T^3 + p^6T^4$
6	smooth		$1 - 1048T + 15938pT^2 - 1048p^3T^3 + p^6T^4$
7	smooth		$1 + 296T - 2686pT^2 + 296p^3T^3 + p^6T^4$
8	smooth		$1 - 446T - 2734pT^2 - 446p^3T^3 + p^6T^4$
9	smooth		$1 + 216T + 7874pT^2 + 216p^3T^3 + p^6T^4$
10	singular	$\frac{1}{25}$	$(1 + pT)(1 + 804T + p^3T^2)$
11	smooth		$1 + 868T + 9842pT^2 + 868p^3T^3 + p^6T^4$
12	smooth		$1 - 1072T + 6242pT^2 - 1072p^3T^3 + p^6T^4$
13	smooth		$1 + 76T - 5086pT^2 + 76p^3T^3 + p^6T^4$
14	smooth		$1 - 514T + 1754pT^2 - 514p^3T^3 + p^6T^4$
15	smooth		$1 - 10pT + 14810pT^2 - 10p^4T^3 + p^6T^4$
16	smooth		$1 + 176T + 7394pT^2 + 176p^3T^3 + p^6T^4$
17	smooth		$1 + 48T - 1438pT^2 + 48p^3T^3 + p^6T^4$
18	smooth		$1 + 302T + 4898pT^2 + 302p^3T^3 + p^6T^4$
19	smooth		$1 + 420T - 2110pT^2 + 420p^3T^3 + p^6T^4$
20	smooth		$1 + 382T + 9098pT^2 + 382p^3T^3 + p^6T^4$
21	smooth		$1 - 8pT + 6914pT^2 - 8p^4T^3 + p^6T^4$
22	smooth		$1 + 408T + 1922pT^2 + 408p^3T^3 + p^6T^4$
23	smooth		$1 + 776T + 7874pT^2 + 776p^3T^3 + p^6T^4$
24	smooth		$1 - 1038T + 8738pT^2 - 1038p^3T^3 + p^6T^4$
25	smooth		$1 + 376T + 6914pT^2 + 376p^3T^3 + p^6T^4$
26	smooth		$1 + 96T - 8926pT^2 + 96p^3T^3 + p^6T^4$
27	smooth		$1 + 32T - 862pT^2 + 32p^3T^3 + p^6T^4$
28	smooth		$1 - 8T + 8258pT^2 - 8p^3T^3 + p^6T^4$
29	smooth		$1 - 552T + 3842pT^2 - 552p^3T^3 + p^6T^4$
30	smooth		$(1 + 12pT + p^3T^2)(1 + 1132T + p^3T^2)$
31	smooth		$1 + 536T + 118p^2T^2 + 536p^3T^3 + p^6T^4$
32	smooth		$1 + 126T - 7726pT^2 + 126p^3T^3 + p^6T^4$
33	smooth		$1 - 1452T + 18962pT^2 - 1452p^3T^3 + p^6T^4$
34	smooth		$1 - 476T + 1586pT^2 - 476p^3T^3 + p^6T^4$
35	smooth		$1 - 244T + 1154pT^2 - 244p^3T^3 + p^6T^4$
36	smooth		$(1 + 12pT + p^3T^2)(1 + 1132T + p^3T^2)$
37	singular	$\frac{1}{9}$	$(1 + pT)(1 + 492T + p^3T^2)$
38	smooth		$(1 + 4pT + p^3T^2)(1 + 1116T + p^3T^2)$
39	smooth		$1 + 98T + 962pT^2 + 98p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 168T + 6722pT^2 + 168p^3T^3 + p^6T^4$
41	smooth		$1 - 1112T + 10562pT^2 - 1112p^3T^3 + p^6T^4$
42	smooth		$1 + 780T + 10610pT^2 + 780p^3T^3 + p^6T^4$
43	smooth		$(1 - 12pT + p^3T^2)(1 + 1362T + p^3T^2)$
44	smooth		$1 - 828T + 6578pT^2 - 828p^3T^3 + p^6T^4$
45	smooth		$1 + 740T + 70p^2T^2 + 740p^3T^3 + p^6T^4$
46	smooth		$1 + 240T - 2350pT^2 + 240p^3T^3 + p^6T^4$
47	smooth		$1 + 124T + 9506pT^2 + 124p^3T^3 + p^6T^4$
48	smooth		$1 - 888T + 11138pT^2 - 888p^3T^3 + p^6T^4$
49	smooth		$1 - 32T - 478pT^2 - 32p^3T^3 + p^6T^4$
50	smooth		$1 - 1586T + 19106pT^2 - 1586p^3T^3 + p^6T^4$
51	smooth		$1 + 876T + 15314pT^2 + 876p^3T^3 + p^6T^4$
52	smooth		$1 + 240T - 9070pT^2 + 240p^3T^3 + p^6T^4$
53	smooth		$1 - 206T + 26pT^2 - 206p^3T^3 + p^6T^4$
54	smooth		$1 + 864T + 6866pT^2 + 864p^3T^3 + p^6T^4$
55	smooth		$1 - 388T - 1582pT^2 - 388p^3T^3 + p^6T^4$
56	smooth		$1 - 812T + 12242pT^2 - 812p^3T^3 + p^6T^4$
57	smooth		$1 + 196T - 2926pT^2 + 196p^3T^3 + p^6T^4$
58	smooth		$1 - 12pT + 12866pT^2 - 12p^4T^3 + p^6T^4$
59	smooth		$1 - 1044T + 8594pT^2 - 1044p^3T^3 + p^6T^4$
60	smooth		$1 + 222T + 2978pT^2 + 222p^3T^3 + p^6T^4$
61	smooth		$1 + 776T + 8834pT^2 + 776p^3T^3 + p^6T^4$
62	smooth		$1 + 342T + 10178pT^2 + 342p^3T^3 + p^6T^4$
63	smooth		$1 - 408T + 2498pT^2 - 408p^3T^3 + p^6T^4$
64	smooth		$1 - 304T + 7394pT^2 - 304p^3T^3 + p^6T^4$
65	smooth		$1 + 948T + 8882pT^2 + 948p^3T^3 + p^6T^4$
66	smooth		$1 + 198T + 1922pT^2 + 198p^3T^3 + p^6T^4$
67	smooth		$1 - 70T - 3670pT^2 - 70p^3T^3 + p^6T^4$
68	smooth		$1 + 252T + 338pT^2 + 252p^3T^3 + p^6T^4$
69	smooth		$1 - 1048T + 13058pT^2 - 1048p^3T^3 + p^6T^4$
70	smooth		$1 - 632T + 4802pT^2 - 632p^3T^3 + p^6T^4$
71	smooth		$(1 + 6pT + p^3T^2)(1 + 1072T + p^3T^2)$
72	smooth		$1 + 28T + 6482pT^2 + 28p^3T^3 + p^6T^4$
73	smooth		$1 - 56T + 2546pT^2 - 56p^3T^3 + p^6T^4$
74	smooth		$1 - 196T - 3454pT^2 - 196p^3T^3 + p^6T^4$
75	smooth		$1 + 1392T + 11618pT^2 + 1392p^3T^3 + p^6T^4$
76	smooth		$1 + 140T - 1390pT^2 + 140p^3T^3 + p^6T^4$
77	smooth		$1 - 1168T + 9698pT^2 - 1168p^3T^3 + p^6T^4$
78	smooth		$1 + 208T - 3358pT^2 + 208p^3T^3 + p^6T^4$
79	smooth		$1 - 550T + 3170pT^2 - 550p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 + 1078T + 10442pT^2 + 1078p^3T^3 + p^6T^4$
81	smooth		$1 - 552T + 11522pT^2 - 552p^3T^3 + p^6T^4$
82	smooth		$1 + 1164T + 13106pT^2 + 1164p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 810T + p^3T^2)$
2	smooth		$1 + 124T + 9542pT^2 + 124p^3T^3 + p^6T^4$
3	smooth		$1 - 600T + 12302pT^2 - 600p^3T^3 + p^6T^4$
4	smooth		$1 + 1236T + 9302pT^2 + 1236p^3T^3 + p^6T^4$
5	smooth		$1 - 264T - 3538pT^2 - 264p^3T^3 + p^6T^4$
6	smooth		$1 + 1114T + 6242pT^2 + 1114p^3T^3 + p^6T^4$
7	smooth		$1 - 294T + 15002pT^2 - 294p^3T^3 + p^6T^4$
8	smooth		$1 - 1256T + 16622pT^2 - 1256p^3T^3 + p^6T^4$
9	smooth		$1 + 968T + 5582pT^2 + 968p^3T^3 + p^6T^4$
10	singular	$\frac{1}{9}$	$(1 + pT)(1 - 810T + p^3T^2)$
11	smooth		$1 - 68T + 13382pT^2 - 68p^3T^3 + p^6T^4$
12	smooth		$1 - 1384T + 17822pT^2 - 1384p^3T^3 + p^6T^4$
13	smooth		$1 - 538T - 3598pT^2 - 538p^3T^3 + p^6T^4$
14	smooth		$1 - 1990T + 21962pT^2 - 1990p^3T^3 + p^6T^4$
15	smooth		$1 - 348T + 4742pT^2 - 348p^3T^3 + p^6T^4$
16	smooth		$1 + 764T + 9542pT^2 + 764p^3T^3 + p^6T^4$
17	smooth		$1 + 376T + 6062pT^2 + 376p^3T^3 + p^6T^4$
18	smooth		$1 + 64T - 6178pT^2 + 64p^3T^3 + p^6T^4$
19	smooth		$1 + 666T + 16322pT^2 + 666p^3T^3 + p^6T^4$
20	smooth		$1 + 856T + 5582pT^2 + 856p^3T^3 + p^6T^4$
21	smooth		$(1 + 10pT + p^3T^2)(1 - 1146T + p^3T^2)$
22	smooth		$1 - 656T - 2818pT^2 - 656p^3T^3 + p^6T^4$
23	smooth		$1 + 782T + 5402pT^2 + 782p^3T^3 + p^6T^4$
24	smooth		$1 + 576T + 7262pT^2 + 576p^3T^3 + p^6T^4$
25	smooth		$1 + 1364T + 13142pT^2 + 1364p^3T^3 + p^6T^4$
26	smooth		$1 - 434T + 10682pT^2 - 434p^3T^3 + p^6T^4$
27	smooth		$1 - 410T - 478pT^2 - 410p^3T^3 + p^6T^4$
28	smooth		$1 + 1002T + 11882pT^2 + 1002p^3T^3 + p^6T^4$
29	smooth		$1 - 1116T + 12662pT^2 - 1116p^3T^3 + p^6T^4$
30	smooth		$1 + 310T - 5758pT^2 + 310p^3T^3 + p^6T^4$
31	smooth		$1 - 330T + 13922pT^2 - 330p^3T^3 + p^6T^4$
32	smooth		$1 + 64T + 9182pT^2 + 64p^3T^3 + p^6T^4$
33	smooth		$1 - 20T - 6058pT^2 - 20p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 + 1236T + 16022pT^2 + 1236p^3T^3 + p^6T^4$
35	smooth		$1 - 992T + 14222pT^2 - 992p^3T^3 + p^6T^4$
36	smooth		$1 - 504T + 302pT^2 - 504p^3T^3 + p^6T^4$
37	smooth		$1 + 258T + 9362pT^2 + 258p^3T^3 + p^6T^4$
38	smooth		$(1 + 6pT + p^3T^2)(1 - 810T + p^3T^2)$
39	smooth		$(1 + 6pT + p^3T^2)(1 - 610T + p^3T^2)$
40	smooth		$1 - 88T + 3662pT^2 - 88p^3T^3 + p^6T^4$
41	smooth		$1 - 790T + 8642pT^2 - 790p^3T^3 + p^6T^4$
42	smooth		$1 + 896T + 5822pT^2 + 896p^3T^3 + p^6T^4$
43	smooth		$1 - 1772T + 21542pT^2 - 1772p^3T^3 + p^6T^4$
44	smooth		$(1 - 10pT + p^3T^2)(1 + 654T + p^3T^2)$
45	smooth		$1 + 144T + 10622pT^2 + 144p^3T^3 + p^6T^4$
46	smooth		$1 - 292T + 13382pT^2 - 292p^3T^3 + p^6T^4$
47	smooth		$1 - 136T + 11342pT^2 - 136p^3T^3 + p^6T^4$
48	smooth		$1 - 330T + 2042pT^2 - 330p^3T^3 + p^6T^4$
49	smooth		$1 - 884T + 12902pT^2 - 884p^3T^3 + p^6T^4$
50	smooth		$1 - 84T - 2458pT^2 - 84p^3T^3 + p^6T^4$
51	smooth		$1 + 12T + 662pT^2 + 12p^3T^3 + p^6T^4$
52	smooth		$1 - 300T + 3302pT^2 - 300p^3T^3 + p^6T^4$
53	smooth		$1 - 136T + 1742pT^2 - 136p^3T^3 + p^6T^4$
54	smooth		$1 + 634T + 2882pT^2 + 634p^3T^3 + p^6T^4$
55	smooth		$(1 + 14pT + p^3T^2)(1 + 510T + p^3T^2)$
56	smooth		$1 - 322T + 9482pT^2 - 322p^3T^3 + p^6T^4$
57	singular	$\frac{1}{25}$	$(1 - pT)(1 + 678T + p^3T^2)$
58	smooth		$1 - 318T - 2878pT^2 - 318p^3T^3 + p^6T^4$
59	smooth		$1 - 480T + 13262pT^2 - 480p^3T^3 + p^6T^4$
60	smooth		$1 + 518T - 958pT^2 + 518p^3T^3 + p^6T^4$
61	smooth		$1 + 1206T + 9842pT^2 + 1206p^3T^3 + p^6T^4$
62	smooth		$1 - 66T + 6722pT^2 - 66p^3T^3 + p^6T^4$
63	smooth		$1 - 110T + 3722pT^2 - 110p^3T^3 + p^6T^4$
64	smooth		$1 + 76T + 2342pT^2 + 76p^3T^3 + p^6T^4$
65	smooth		$1 + 44T + 5222pT^2 + 44p^3T^3 + p^6T^4$
66	smooth		$1 + 662T - 1918pT^2 + 662p^3T^3 + p^6T^4$
67	smooth		$1 + 88T + 7022pT^2 + 88p^3T^3 + p^6T^4$
68	smooth		$1 + 816T + 13502pT^2 + 816p^3T^3 + p^6T^4$
69	smooth		$1 - 464T + 8702pT^2 - 464p^3T^3 + p^6T^4$
70	smooth		$1 + 148T + 10022pT^2 + 148p^3T^3 + p^6T^4$
71	smooth		$(1 - 6pT + p^3T^2)(1 + 350T + p^3T^2)$
72	smooth		$1 - 708T + 16262pT^2 - 708p^3T^3 + p^6T^4$
73	smooth		$1 + 1484T + 17702pT^2 + 1484p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	smooth		$1 + 76T + 7142pT^2 + 76p^3T^3 + p^6T^4$
75	smooth		$1 + 774T + 13202pT^2 + 774p^3T^3 + p^6T^4$
76	smooth		$1 - 244T + 422pT^2 - 244p^3T^3 + p^6T^4$
77	smooth		$1 - 40T - 7618pT^2 - 40p^3T^3 + p^6T^4$
78	smooth		$(1 + 6pT + p^3T^2)(1 - 1530T + p^3T^2)$
79	smooth		$1 + 396T + 422pT^2 + 396p^3T^3 + p^6T^4$
80	smooth		$1 + 276T - 1258pT^2 + 276p^3T^3 + p^6T^4$
81	smooth		$1 - 1644T + 16982pT^2 - 1644p^3T^3 + p^6T^4$
82	smooth		$1 + 1244T + 18182pT^2 + 1244p^3T^3 + p^6T^4$
83	smooth		$1 - 202T + 9602pT^2 - 202p^3T^3 + p^6T^4$
84	smooth		$(1 - 6pT + p^3T^2)(1 + 350T + p^3T^2)$
85	smooth		$1 - 696T + 4622pT^2 - 696p^3T^3 + p^6T^4$
86	smooth		$1 + 172T + 6422pT^2 + 172p^3T^3 + p^6T^4$
87	smooth		$1 - 676T + 16262pT^2 - 676p^3T^3 + p^6T^4$
88	smooth		$(1 + 6pT + p^3T^2)(1 + 1510T + p^3T^2)$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 1154T + p^3T^2)$
2	smooth		$1 - 232T + 654pT^2 - 232p^3T^3 + p^6T^4$
3	smooth		$(1 - 10pT + p^3T^2)(1 + 406T + p^3T^2)$
4	smooth		$1 + 848T + 9054pT^2 + 848p^3T^3 + p^6T^4$
5	smooth		$1 - 172T - 1866pT^2 - 172p^3T^3 + p^6T^4$
6	smooth		$1 + 1172T + 8406pT^2 + 1172p^3T^3 + p^6T^4$
7	smooth		$1 + 182T - 4614pT^2 + 182p^3T^3 + p^6T^4$
8	smooth		$1 + 1352T + 22926pT^2 + 1352p^3T^3 + p^6T^4$
9	smooth		$1 + 92T - 4794pT^2 + 92p^3T^3 + p^6T^4$
10	smooth		$1 + 46T + 11498pT^2 + 46p^3T^3 + p^6T^4$
11	smooth		$1 + 216T + 3118pT^2 + 216p^3T^3 + p^6T^4$
12	smooth		$1 + 248T + 9294pT^2 + 248p^3T^3 + p^6T^4$
13	smooth		$1 - 150T + 4210pT^2 - 150p^3T^3 + p^6T^4$
14	smooth		$1 + 156T - 1082pT^2 + 156p^3T^3 + p^6T^4$
15	smooth		$1 - 1864T + 24558pT^2 - 1864p^3T^3 + p^6T^4$
16	smooth		$1 - 124T + 54p^2T^2 - 124p^3T^3 + p^6T^4$
17	smooth		$1 + 430T - 6070pT^2 + 430p^3T^3 + p^6T^4$
18	smooth		$1 - 684T + 10198pT^2 - 684p^3T^3 + p^6T^4$
19	smooth		$1 - 784T + 7758pT^2 - 784p^3T^3 + p^6T^4$
20	smooth		$1 + 216T - 8882pT^2 + 216p^3T^3 + p^6T^4$
21	smooth		$1 - 1462T + 14154pT^2 - 1462p^3T^3 + p^6T^4$

Continued on the following page

$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 + 432T + 13726pT^2 + 432p^3T^3 + p^6T^4$
23	smooth		$1 - 604T + 16278pT^2 - 604p^3T^3 + p^6T^4$
24	smooth		$(1 + 18pT + p^3T^2)(1 + 46T + p^3T^2)$
25	smooth		$1 - 484T + 6918pT^2 - 484p^3T^3 + p^6T^4$
26	smooth		$1 + 1220T + 11910pT^2 + 1220p^3T^3 + p^6T^4$
27	smooth		$1 + 888T + 17134pT^2 + 888p^3T^3 + p^6T^4$
28	smooth		$1 - 1220T + 12470pT^2 - 1220p^3T^3 + p^6T^4$
29	smooth		$1 - 1382T + 14234pT^2 - 1382p^3T^3 + p^6T^4$
30	smooth		$1 + 260T + 1350pT^2 + 260p^3T^3 + p^6T^4$
31	smooth		$1 - 808T + 17646pT^2 - 808p^3T^3 + p^6T^4$
32	smooth		$1 - 192T - 10946pT^2 - 192p^3T^3 + p^6T^4$
33	smooth		$1 + 972T + 1126pT^2 + 972p^3T^3 + p^6T^4$
34	smooth		$1 - 1970T + 22250pT^2 - 1970p^3T^3 + p^6T^4$
35	smooth		$1 + 972T + 12646pT^2 + 972p^3T^3 + p^6T^4$
36	smooth		$(1 - 2pT + p^3T^2)(1 + 1206T + p^3T^2)$
37	smooth		$1 - 100T - 10890pT^2 - 100p^3T^3 + p^6T^4$
38	smooth		$1 + 1530T + 20650pT^2 + 1530p^3T^3 + p^6T^4$
39	smooth		$1 - 258T - 1694pT^2 - 258p^3T^3 + p^6T^4$
40	smooth		$1 + 508T + 8774pT^2 + 508p^3T^3 + p^6T^4$
41	smooth		$1 - 1394T + 10418pT^2 - 1394p^3T^3 + p^6T^4$
42	smooth		$1 - 32T + 14pT^2 - 32p^3T^3 + p^6T^4$
43	smooth		$1 - 468T + 13606pT^2 - 468p^3T^3 + p^6T^4$
44	smooth		$1 - 584T + 2318pT^2 - 584p^3T^3 + p^6T^4$
45	smooth		$1 - 82T + 9594pT^2 - 82p^3T^3 + p^6T^4$
46	smooth		$1 + 810T + 6490pT^2 + 810p^3T^3 + p^6T^4$
47	smooth		$1 + 316T - 1402pT^2 + 316p^3T^3 + p^6T^4$
48	smooth		$1 + 1488T + 14494pT^2 + 1488p^3T^3 + p^6T^4$
49	smooth		$1 - 624T + 15838pT^2 - 624p^3T^3 + p^6T^4$
50	smooth		$1 + 12T + 14566pT^2 + 12p^3T^3 + p^6T^4$
51	smooth		$1 - 894T + 16018pT^2 - 894p^3T^3 + p^6T^4$
52	smooth		$1 - 210T - 110pT^2 - 210p^3T^3 + p^6T^4$
53	smooth		$1 + 848T + 9534pT^2 + 848p^3T^3 + p^6T^4$
54	singular	$\frac{1}{9}$	$(1 - pT)(1 - 1154T + p^3T^2)$
55	smooth		$1 - 708T + 18886pT^2 - 708p^3T^3 + p^6T^4$
56	smooth		$1 + 170T + 90pT^2 + 170p^3T^3 + p^6T^4$
57	smooth		$1 + 40T - 13570pT^2 + 40p^3T^3 + p^6T^4$
58	smooth		$1 - 632T + 5534pT^2 - 632p^3T^3 + p^6T^4$
59	smooth		$1 - 524T + 10838pT^2 - 524p^3T^3 + p^6T^4$
60	smooth		$1 + 1500T + 19990pT^2 + 1500p^3T^3 + p^6T^4$
61	smooth		$1 - 1024T + 12798pT^2 - 1024p^3T^3 + p^6T^4$

Continued on the following page

$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 + 348T - 1466pT^2 + 348p^3T^3 + p^6T^4$
63	smooth		$1 + 572T + 6pT^2 + 572p^3T^3 + p^6T^4$
64	smooth		$1 - 324T + 3718pT^2 - 324p^3T^3 + p^6T^4$
65	smooth		$1 + 1308T + 19654pT^2 + 1308p^3T^3 + p^6T^4$
66	singular	$\frac{1}{25}$	$(1 + pT)(1 - 194T + p^3T^2)$
67	smooth		$1 + 710T + 10890pT^2 + 710p^3T^3 + p^6T^4$
68	smooth		$1 + 182T + 13746pT^2 + 182p^3T^3 + p^6T^4$
69	smooth		$1 + 678T + 12514pT^2 + 678p^3T^3 + p^6T^4$
70	smooth		$1 + 56T + 8238pT^2 + 56p^3T^3 + p^6T^4$
71	smooth		$1 + 1324T + 18902pT^2 + 1324p^3T^3 + p^6T^4$
72	smooth		$1 + 1256T + 13518pT^2 + 1256p^3T^3 + p^6T^4$
73	smooth		$1 - 208T - 8034pT^2 - 208p^3T^3 + p^6T^4$
74	smooth		$1 + 834T + 6802pT^2 + 834p^3T^3 + p^6T^4$
75	smooth		$1 - 68T - 634pT^2 - 68p^3T^3 + p^6T^4$
76	smooth		$1 + 322T - 6574pT^2 + 322p^3T^3 + p^6T^4$
77	smooth		$1 - 778T + 8466pT^2 - 778p^3T^3 + p^6T^4$
78	smooth		$1 + 280T - 7810pT^2 + 280p^3T^3 + p^6T^4$
79	smooth		$1 - 104T - 4882pT^2 - 104p^3T^3 + p^6T^4$
80	smooth		$1 + 264T - 1778pT^2 + 264p^3T^3 + p^6T^4$
81	smooth		$1 + 692T - 2154pT^2 + 692p^3T^3 + p^6T^4$
82	smooth		$1 - 1346T + 10442pT^2 - 1346p^3T^3 + p^6T^4$
83	smooth		$(1 + 10pT + p^3T^2)(1 - 1354T + p^3T^2)$
84	smooth		$1 + 1382T + 12786pT^2 + 1382p^3T^3 + p^6T^4$
85	smooth		$1 - 1144T + 13518pT^2 - 1144p^3T^3 + p^6T^4$
86	smooth		$1 - 592T + 1374pT^2 - 592p^3T^3 + p^6T^4$
87	smooth		$1 + 186T + 7858pT^2 + 186p^3T^3 + p^6T^4$
88	smooth		$1 - 44T - 6442pT^2 - 44p^3T^3 + p^6T^4$
89	smooth		$1 - 1448T + 22286pT^2 - 1448p^3T^3 + p^6T^4$
90	smooth		$1 - 2424T + 32638pT^2 - 2424p^3T^3 + p^6T^4$
91	smooth		$1 + 876T + 8998pT^2 + 876p^3T^3 + p^6T^4$
92	smooth		$1 + 1380T + 9670pT^2 + 1380p^3T^3 + p^6T^4$
93	smooth		$1 - 68T - 634pT^2 - 68p^3T^3 + p^6T^4$
94	smooth		$1 - 804T + 7558pT^2 - 804p^3T^3 + p^6T^4$
95	smooth		$1 + 476T - 4602pT^2 + 476p^3T^3 + p^6T^4$
96	smooth		$1 - 288T + 766pT^2 - 288p^3T^3 + p^6T^4$

C.4. The ζ function for the Rødland manifold, AESZ 27

$p = 5$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 + p^3T^2)(1 + 12T + p^3T^2)$
2	smooth		$1 + 26T + 78pT^2 + 26p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 9T + 8pT^2 - 9p^3T^3 + p^6T^4$

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 8T + 8pT^2 - 8p^3T^3 + p^6T^4$
2	smooth		$1 + 20T + 64pT^2 + 20p^3T^3 + p^6T^4$
3	singular	{3, 3, 3, 3}	
4	smooth		$1 + 6T - 6pT^2 + 6p^3T^3 + p^6T^4$
5	smooth		$1 + 20T + 50pT^2 + 20p^3T^3 + p^6T^4$
6	smooth		$1 + 6T + 22pT^2 + 6p^3T^3 + p^6T^4$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - T + 182pT^2 - p^3T^3 + p^6T^4$
2	smooth		$1 + 48T + 182pT^2 + 48p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 6T + 14p^2T^2 + 6p^3T^3 + p^6T^4$
5	smooth		$1 + 6T - 42pT^2 + 6p^3T^3 + p^6T^4$
6	smooth		$1 - 64T + 238pT^2 - 64p^3T^3 + p^6T^4$
7	smooth		$1 + 41T + 210pT^2 + 41p^3T^3 + p^6T^4$
8	smooth		$1 + 13T - 70pT^2 + 13p^3T^3 + p^6T^4$
9	smooth		$1 + 69T + 294pT^2 + 69p^3T^3 + p^6T^4$
10	smooth		$1 - 29T + 98pT^2 - 29p^3T^3 + p^6T^4$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 35T + 44pT^2 - 35p^3T^3 + p^6T^4$
2	singular	2	$(1 - pT)(1 + 42T + p^3T^2)$
3	smooth*	3	
4	smooth		$(1 + p^3T^2)(1 + 84T + p^3T^2)$
5	singular	5	$(1 - pT)(1 - 42T + p^3T^2)$
6	smooth		$1 + 28T - 54pT^2 + 28p^3T^3 + p^6T^4$

Continued on the following page

$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$(1 + p^3T^2)(1 + 63T + p^3T^2)$
8	smooth		$1 + 49T + 142pT^2 + 49p^3T^3 + p^6T^4$
9	singular	9	$(1 - pT)(1 + 14T + p^3T^2)$
10	smooth		$1 - 14T + 142pT^2 - 14p^3T^3 + p^6T^4$
11	smooth		$1 - 42T + 240pT^2 - 42p^3T^3 + p^6T^4$
12	smooth		$(1 + 7pT + p^3T^2)(1 - 42T + p^3T^2)$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 3pT + 452pT^2 + 3p^4T^3 + p^6T^4$
2	smooth		$(1 + p^3T^2)(1 + 58T + p^3T^2)$
3	smooth*	3	
4	smooth		$1 - 96T + 438pT^2 - 96p^3T^3 + p^6T^4$
5	smooth		$1 + 100T + 620pT^2 + 100p^3T^3 + p^6T^4$
6	smooth		$1 + 44T - 66pT^2 + 44p^3T^3 + p^6T^4$
7	smooth		$1 - 33T - 10pT^2 - 33p^3T^3 + p^6T^4$
8	smooth		$1 - 54T + 466pT^2 - 54p^3T^3 + p^6T^4$
9	smooth		$1 + 72T + 270pT^2 + 72p^3T^3 + p^6T^4$
10	smooth		$1 + 2T + 466pT^2 + 2p^3T^3 + p^6T^4$
11	smooth		$(1 - 4pT + p^3T^2)(1 + 63T + p^3T^2)$
12	smooth		$1 - 5T - 178pT^2 - 5p^3T^3 + p^6T^4$
13	smooth		$1 + 142T + 634pT^2 + 142p^3T^3 + p^6T^4$
14	smooth		$1 - 33T + 382pT^2 - 33p^3T^3 + p^6T^4$
15	smooth		$1 + 16T + 158pT^2 + 16p^3T^3 + p^6T^4$
16	smooth		$1 - 26T + 242pT^2 - 26p^3T^3 + p^6T^4$

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 72T + 610pT^2 - 72p^3T^3 + p^6T^4$
2	smooth		$1 + 12T - 258pT^2 + 12p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 54T + 610pT^2 + 54p^3T^3 + p^6T^4$
5	smooth		$1 + 12T - 20pT^2 + 12p^3T^3 + p^6T^4$
6	smooth		$1 + 166T + 876pT^2 + 166p^3T^3 + p^6T^4$
7	smooth		$1 + 33T - 62pT^2 + 33p^3T^3 + p^6T^4$
8	smooth		$1 + 5T + 330pT^2 + 5p^3T^3 + p^6T^4$
9	smooth		$1 + 110T + 484pT^2 + 110p^3T^3 + p^6T^4$
10	smooth		$(1 + 8pT + p^3T^2)(1 - 35T + p^3T^2)$

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$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 + pT - 426pT^2 + p^4T^3 + p^6T^4$
12	smooth		$1 - 9T - 230pT^2 - 9p^3T^3 + p^6T^4$
13	smooth		$1 + 40T - 118pT^2 + 40p^3T^3 + p^6T^4$
14	smooth		$1 + 26T + 358pT^2 + 26p^3T^3 + p^6T^4$
15	smooth		$(1 - 6pT + p^3T^2)(1 + 98T + p^3T^2)$
16	smooth		$1 - 163T + 1030pT^2 - 163p^3T^3 + p^6T^4$
17	smooth		$1 + 131T + 610pT^2 + 131p^3T^3 + p^6T^4$
18	smooth		$1 + 12T + 106pT^2 + 12p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 71T + 14pT^2 - 71p^3T^3 + p^6T^4$
2	smooth		$1 - 99T + 630pT^2 - 99p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 20T - 616pT^2 + 20p^3T^3 + p^6T^4$
5	smooth		$1 + 132T + 350pT^2 + 132p^3T^3 + p^6T^4$
6	smooth		$1 - 4pT + 84pT^2 - 4p^4T^3 + p^6T^4$
7	smooth		$1 + 48T - 350pT^2 + 48p^3T^3 + p^6T^4$
8	smooth		$1 - 176T + 1050pT^2 - 176p^3T^3 + p^6T^4$
9	smooth		$1 + 90T + 980pT^2 + 90p^3T^3 + p^6T^4$
10	smooth		$1 + 216T + 1442pT^2 + 216p^3T^3 + p^6T^4$
11	smooth		$1 + 20T + 70pT^2 + 20p^3T^3 + p^6T^4$
12	smooth		$1 - 43T - 210pT^2 - 43p^3T^3 + p^6T^4$
13	smooth		$1 + 48T + 728pT^2 + 48p^3T^3 + p^6T^4$
14	smooth		$1 - 8T - 686pT^2 - 8p^3T^3 + p^6T^4$
15	smooth		$1 - 78T + 658pT^2 - 78p^3T^3 + p^6T^4$
16	smooth		$1 + 3pT + 854pT^2 + 3p^4T^3 + p^6T^4$
17	smooth		$1 + 41T + 490pT^2 + 41p^3T^3 + p^6T^4$
18	smooth		$1 - 78T + 658pT^2 - 78p^3T^3 + p^6T^4$
19	smooth		$1 + 125T + 602pT^2 + 125p^3T^3 + p^6T^4$
20	smooth		$1 + 160T + 714pT^2 + 160p^3T^3 + p^6T^4$
21	smooth		$1 + 97T + 826pT^2 + 97p^3T^3 + p^6T^4$
22	smooth		$1 + 48T + 1022pT^2 + 48p^3T^3 + p^6T^4$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 122T + 734pT^2 + 122p^3T^3 + p^6T^4$
2	smooth		$1 - 123T - 190pT^2 - 123p^3T^3 + p^6T^4$

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$p = 29$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth*	3	
4	smooth		$1 + 31T + 1056pT^2 + 31p^3T^3 + p^6T^4$
5	smooth		$1 - 95T + 524pT^2 - 95p^3T^3 + p^6T^4$
6	smooth		$1 - 95T + 720pT^2 - 95p^3T^3 + p^6T^4$
7	smooth		$1 + 38T + 1658pT^2 + 38p^3T^3 + p^6T^4$
8	smooth		$1 - 88T - 568pT^2 - 88p^3T^3 + p^6T^4$
9	smooth		$1 - 11T + 1280pT^2 - 11p^3T^3 + p^6T^4$
10	smooth		$1 + 80T + 1462pT^2 + 80p^3T^3 + p^6T^4$
11	smooth		$1 + 437T + 3058pT^2 + 437p^3T^3 + p^6T^4$
12	singular	12	$(1 - pT)(1 + 306T + p^3T^2)$
13	smooth		$1 + 45T - 876pT^2 + 45p^3T^3 + p^6T^4$
14	smooth		$(1 - 2pT + p^3T^2)(1 + 194T + p^3T^2)$
15	smooth		$1 + 38T + 1210pT^2 + 38p^3T^3 + p^6T^4$
16	smooth		$1 + 192T + 1658pT^2 + 192p^3T^3 + p^6T^4$
17	singular	17	$(1 - pT)(1 + 110T + p^3T^2)$
18	smooth		$1 - 137T + 958pT^2 - 137p^3T^3 + p^6T^4$
19	smooth		$1 - 123T - 134pT^2 - 123p^3T^3 + p^6T^4$
20	smooth		$1 + 17T + 384pT^2 + 17p^3T^3 + p^6T^4$
21	smooth		$1 + 276T + 1644pT^2 + 276p^3T^3 + p^6T^4$
22	smooth		$1 + 115T + 356pT^2 + 115p^3T^3 + p^6T^4$
23	smooth		$1 + 360T + 2554pT^2 + 360p^3T^3 + p^6T^4$
24	smooth		$1 - 25T + 804pT^2 - 25p^3T^3 + p^6T^4$
25	smooth		$1 - 284T + 1434pT^2 - 284p^3T^3 + p^6T^4$
26	smooth		$1 + 24T + 944pT^2 + 24p^3T^3 + p^6T^4$
27	smooth		$1 - 4T + 524pT^2 - 4p^3T^3 + p^6T^4$
28	singular	28	$(1 - pT)(1 - 282T + p^3T^2)$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 5T + 578pT^2 - 5p^3T^3 + p^6T^4$
2	smooth		$1 + 219T + 2202pT^2 + 219p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 135T + 1138pT^2 + 135p^3T^3 + p^6T^4$
5	smooth		$1 + 247T + 1894pT^2 + 247p^3T^3 + p^6T^4$
6	smooth		$1 + 44T + 522pT^2 + 44p^3T^3 + p^6T^4$
7	smooth		$1 - 110T + 1124pT^2 - 110p^3T^3 + p^6T^4$
8	smooth		$1 + 16T + 18pT^2 + 16p^3T^3 + p^6T^4$
9	smooth		$1 + 170T + 368pT^2 + 170p^3T^3 + p^6T^4$
10	smooth		$1 - 61T + 914pT^2 - 61p^3T^3 + p^6T^4$

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$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 + 114T + 74pT^2 + 114p^3T^3 + p^6T^4$
12	smooth		$1 + 324T + 1810pT^2 + 324p^3T^3 + p^6T^4$
13	smooth		$1 - 96T + 130pT^2 - 96p^3T^3 + p^6T^4$
14	smooth		$1 + 142T + 634pT^2 + 142p^3T^3 + p^6T^4$
15	smooth		$1 - 166T + 438pT^2 - 166p^3T^3 + p^6T^4$
16	smooth		$1 + 331T + 2398pT^2 + 331p^3T^3 + p^6T^4$
17	smooth		$1 + 296T + 1978pT^2 + 296p^3T^3 + p^6T^4$
18	smooth		$1 - 12T - 724pT^2 - 12p^3T^3 + p^6T^4$
19	smooth		$1 - 243T + 1446pT^2 - 243p^3T^3 + p^6T^4$
20	smooth		$1 - 131T + 2034pT^2 - 131p^3T^3 + p^6T^4$
21	smooth		$1 - 40T - 542pT^2 - 40p^3T^3 + p^6T^4$
22	smooth		$1 + 128T - 430pT^2 + 128p^3T^3 + p^6T^4$
23	smooth		$1 + 9T + 1754pT^2 + 9p^3T^3 + p^6T^4$
24	smooth		$1 - 180T + 494pT^2 - 180p^3T^3 + p^6T^4$
25	smooth		$1 - 208T + 1348pT^2 - 208p^3T^3 + p^6T^4$
26	smooth		$1 - 131T + 130pT^2 - 131p^3T^3 + p^6T^4$
27	smooth		$1 + 233T + 2090pT^2 + 233p^3T^3 + p^6T^4$
28	smooth		$1 - 40T - 598pT^2 - 40p^3T^3 + p^6T^4$
29	smooth		$1 - 33T - 38pT^2 - 33p^3T^3 + p^6T^4$
30	smooth		$1 + 37T - 766pT^2 + 37p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 76T + 1694pT^2 + 76p^3T^3 + p^6T^4$
2	smooth		$1 + 62T + 1904pT^2 + 62p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 244T + 742pT^2 + 244p^3T^3 + p^6T^4$
5	smooth		$1 - 92T + 2156pT^2 - 92p^3T^3 + p^6T^4$
6	smooth		$1 + 202T + 1470pT^2 + 202p^3T^3 + p^6T^4$
7	smooth		$1 + 272T + 3066pT^2 + 272p^3T^3 + p^6T^4$
8	smooth		$1 + 447T + 2842pT^2 + 447p^3T^3 + p^6T^4$
9	smooth		$1 - 414T + 2478pT^2 - 414p^3T^3 + p^6T^4$
10	smooth		$1 - 36T + 2198pT^2 - 36p^3T^3 + p^6T^4$
11	smooth		$1 + 181T + 1344pT^2 + 181p^3T^3 + p^6T^4$
12	smooth		$1 - 99T + 2016pT^2 - 99p^3T^3 + p^6T^4$
13	smooth		$1 + 13T - 1134pT^2 + 13p^3T^3 + p^6T^4$
14	smooth		$1 - 120T + 714pT^2 - 120p^3T^3 + p^6T^4$
15	smooth		$1 - 22T - 168pT^2 - 22p^3T^3 + p^6T^4$
16	smooth		$1 + 545T + 4508pT^2 + 545p^3T^3 + p^6T^4$

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$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 8T - 1064pT^2 - 8p^3T^3 + p^6T^4$
18	smooth		$1 - 120T + 910pT^2 - 120p^3T^3 + p^6T^4$
19	smooth		$1 - 50T + 938pT^2 - 50p^3T^3 + p^6T^4$
20	smooth		$1 + 328T + 84p^2T^2 + 328p^3T^3 + p^6T^4$
21	smooth		$1 - 57T + 1876pT^2 - 57p^3T^3 + p^6T^4$
22	smooth		$1 + 230T + 1442pT^2 + 230p^3T^3 + p^6T^4$
23	smooth		$1 + 83T - 714pT^2 + 83p^3T^3 + p^6T^4$
24	smooth		$1 - 57T + 2170pT^2 - 57p^3T^3 + p^6T^4$
25	smooth		$1 + 104T + 686pT^2 + 104p^3T^3 + p^6T^4$
26	smooth		$1 - 106T + 798pT^2 - 106p^3T^3 + p^6T^4$
27	smooth		$1 - 484T + 4018pT^2 - 484p^3T^3 + p^6T^4$
28	smooth		$1 - 29T + 1848pT^2 - 29p^3T^3 + p^6T^4$
29	smooth		$1 + 55T + 1666pT^2 + 55p^3T^3 + p^6T^4$
30	smooth		$1 + 230T + 462pT^2 + 230p^3T^3 + p^6T^4$
31	smooth		$1 - 316T + 2478pT^2 - 316p^3T^3 + p^6T^4$
32	smooth		$1 - 8T + 1092pT^2 - 8p^3T^3 + p^6T^4$
33	smooth		$1 + 181T + 952pT^2 + 181p^3T^3 + p^6T^4$
34	smooth		$(1 - pT + p^3T^2)(1 - 202T + p^3T^2)$
35	smooth		$1 + 293T + 3094pT^2 + 293p^3T^3 + p^6T^4$
36	smooth		$1 + 90T - 182pT^2 + 90p^3T^3 + p^6T^4$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 21T - 1636pT^2 - 21p^3T^3 + p^6T^4$
2	smooth		$1 + 175T + 2088pT^2 + 175p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 448T + 3166pT^2 - 448p^3T^3 + p^6T^4$
5	smooth		$1 + 469T + 4048pT^2 + 469p^3T^3 + p^6T^4$
6	smooth		$1 + 217T + 1402pT^2 + 217p^3T^3 + p^6T^4$
7	smooth		$1 + 476T + 3558pT^2 + 476p^3T^3 + p^6T^4$
8	smooth		$1 + 266T + 2578pT^2 + 266p^3T^3 + p^6T^4$
9	smooth		$1 + 28T + 30pT^2 + 28p^3T^3 + p^6T^4$
10	smooth		$1 - 70T + 1794pT^2 - 70p^3T^3 + p^6T^4$
11	smooth		$1 - 336T + 2480pT^2 - 336p^3T^3 + p^6T^4$
12	smooth		$1 + 406T + 1892pT^2 + 406p^3T^3 + p^6T^4$
13	smooth		$1 + 70T - 950pT^2 + 70p^3T^3 + p^6T^4$
14	smooth		$1 - 70T + 422pT^2 - 70p^3T^3 + p^6T^4$
15	smooth		$1 + 35T - 754pT^2 + 35p^3T^3 + p^6T^4$
16	smooth		$1 - 623T + 5224pT^2 - 623p^3T^3 + p^6T^4$

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$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 133T + 1402pT^2 - 133p^3T^3 + p^6T^4$
18	smooth		$1 - 21T - 68pT^2 - 21p^3T^3 + p^6T^4$
19	smooth		$1 - 518T + 4146pT^2 - 518p^3T^3 + p^6T^4$
20	smooth		$1 - 210T + 226pT^2 - 210p^3T^3 + p^6T^4$
21	smooth		$1 + 203T + 3068pT^2 + 203p^3T^3 + p^6T^4$
22	smooth		$1 + 182T + 2480pT^2 + 182p^3T^3 + p^6T^4$
23	singular	23	$(1 - pT)(1 - 70T + p^3T^2)$
24	smooth		$1 - 140T - 460pT^2 - 140p^3T^3 + p^6T^4$
25	smooth		$1 + 140T - 166pT^2 + 140p^3T^3 + p^6T^4$
26	smooth		$1 + 252T + 1304pT^2 + 252p^3T^3 + p^6T^4$
27	singular	27	$(1 - pT)(1 + 70T + p^3T^2)$
28	smooth		$1 + 182T + 1598pT^2 + 182p^3T^3 + p^6T^4$
29	smooth		$1 + 182T + 3068pT^2 + 182p^3T^3 + p^6T^4$
30	smooth		$1 + 266T + 618pT^2 + 266p^3T^3 + p^6T^4$
31	smooth		$1 + 14T - 2910pT^2 + 14p^3T^3 + p^6T^4$
32	smooth		$1 - 539T + 3852pT^2 - 539p^3T^3 + p^6T^4$
33	smooth		$1 + 196T + 2382pT^2 + 196p^3T^3 + p^6T^4$
34	singular	34	$(1 - pT)(1 + 434T + p^3T^2)$
35	smooth		$1 - 70T + 128pT^2 - 70p^3T^3 + p^6T^4$
36	smooth		$1 + 532T + 4734pT^2 + 532p^3T^3 + p^6T^4$
37	smooth		$1 + 35T + 2284pT^2 + 35p^3T^3 + p^6T^4$
38	smooth		$1 - 301T + 2578pT^2 - 301p^3T^3 + p^6T^4$
39	smooth		$1 - 224T + 1598pT^2 - 224p^3T^3 + p^6T^4$
40	smooth		$1 + 308T + 2774pT^2 + 308p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	1	$(1 - pT)(1 - 128T + p^3T^2)$
2	smooth		$1 - 67T - 638pT^2 - 67p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 283T + 2330pT^2 + 283p^3T^3 + p^6T^4$
5	smooth		$1 - 11T + 1602pT^2 - 11p^3T^3 + p^6T^4$
6	smooth		$1 + 234T + 1392pT^2 + 234p^3T^3 + p^6T^4$
7	smooth		$1 - 354T + 3254pT^2 - 354p^3T^3 + p^6T^4$
8	smooth		$1 + 185T + 2834pT^2 + 185p^3T^3 + p^6T^4$
9	smooth		$1 + 787T + 7062pT^2 + 787p^3T^3 + p^6T^4$
10	smooth		$1 + 24T - 2360pT^2 + 24p^3T^3 + p^6T^4$
11	smooth		$1 - 74T - 974pT^2 - 74p^3T^3 + p^6T^4$
12	smooth		$1 + 150T + 2414pT^2 + 150p^3T^3 + p^6T^4$

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$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
13	smooth		$1 - 81T + 1266pT^2 - 81p^3T^3 + p^6T^4$
14	smooth		$1 + 395T + 4290pT^2 + 395p^3T^3 + p^6T^4$
15	smooth		$1 - 459T + 3842pT^2 - 459p^3T^3 + p^6T^4$
16	smooth		$1 - 529T + 4542pT^2 - 529p^3T^3 + p^6T^4$
17	smooth		$1 - 494T + 4752pT^2 - 494p^3T^3 + p^6T^4$
18	smooth		$1 - 60T + 1910pT^2 - 60p^3T^3 + p^6T^4$
19	smooth		$1 - 564T + 4514pT^2 - 564p^3T^3 + p^6T^4$
20	smooth		$1 + 31T - 78pT^2 + 31p^3T^3 + p^6T^4$
21	smooth		$1 + 10T - 1786pT^2 + 10p^3T^3 + p^6T^4$
22	smooth		$1 + 122T + 1714pT^2 + 122p^3T^3 + p^6T^4$
23	smooth		$1 + 311T + 3142pT^2 + 311p^3T^3 + p^6T^4$
24	smooth		$1 - 130T - 1170pT^2 - 130p^3T^3 + p^6T^4$
25	smooth		$1 + 101T + 1042pT^2 + 101p^3T^3 + p^6T^4$
26	smooth		$1 + 136T + 34p^2T^2 + 136p^3T^3 + p^6T^4$
27	smooth		$1 + 661T + 5410pT^2 + 661p^3T^3 + p^6T^4$
28	smooth		$1 - 242T - 386pT^2 - 242p^3T^3 + p^6T^4$
29	smooth		$1 + 192T + 1714pT^2 + 192p^3T^3 + p^6T^4$
30	smooth		$1 - 144T - 1226pT^2 - 144p^3T^3 + p^6T^4$
31	smooth		$1 + 10T - 2038pT^2 + 10p^3T^3 + p^6T^4$
32	smooth		$1 + 185T + 762pT^2 + 185p^3T^3 + p^6T^4$
33	smooth		$1 - 158T - 554pT^2 - 158p^3T^3 + p^6T^4$
34	smooth		$1 - 375T + 3730pT^2 - 375p^3T^3 + p^6T^4$
35	smooth		$(1 + 12pT + p^3T^2)(1 - 44T + p^3T^2)$
36	singular	36	$(1 - pT)(1 + 264T + p^3T^2)$
37	singular	37	$(1 - pT)(1 + 68T + p^3T^2)$
38	smooth		$1 + 248T + 1490pT^2 + 248p^3T^3 + p^6T^4$
39	smooth		$1 + 262T + 398pT^2 + 262p^3T^3 + p^6T^4$
40	smooth		$1 + 66T - 36p^2T^2 + 66p^3T^3 + p^6T^4$
41	smooth		$1 + 234T + 2218pT^2 + 234p^3T^3 + p^6T^4$
42	smooth		$(1 + 5pT + p^3T^2)(1 - 128T + p^3T^2)$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 30T - 2890pT^2 - 30p^3T^3 + p^6T^4$
2	smooth		$1 + 222T + 3802pT^2 + 222p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 23T + 3214pT^2 - 23p^3T^3 + p^6T^4$
5	smooth		$1 - 13pT + 4754pT^2 - 13p^4T^3 + p^6T^4$
6	smooth		$1 + 166T + 1520pT^2 + 166p^3T^3 + p^6T^4$

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$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 - 205T + 2234pT^2 - 205p^3T^3 + p^6T^4$
8	smooth		$1 - 23T + 2430pT^2 - 23p^3T^3 + p^6T^4$
9	smooth		$1 + 68T + 1338pT^2 + 68p^3T^3 + p^6T^4$
10	smooth		$1 - 58T + 750pT^2 - 58p^3T^3 + p^6T^4$
11	smooth		$1 - 296T + 1730pT^2 - 296p^3T^3 + p^6T^4$
12	smooth		$1 + 446T + 4782pT^2 + 446p^3T^3 + p^6T^4$
13	smooth		$1 + 166T + 3074pT^2 + 166p^3T^3 + p^6T^4$
14	smooth		$1 - 58T - 3198pT^2 - 58p^3T^3 + p^6T^4$
15	smooth		$1 + 5T - 1238pT^2 + 5p^3T^3 + p^6T^4$
16	smooth		$1 + 866T + 8114pT^2 + 866p^3T^3 + p^6T^4$
17	smooth		$1 - 65T - 482pT^2 - 65p^3T^3 + p^6T^4$
18	smooth		$1 + pT + 862pT^2 + p^4T^3 + p^6T^4$
19	smooth		$1 + 159T - 678pT^2 + 159p^3T^3 + p^6T^4$
20	smooth		$1 + 82T - 622pT^2 + 82p^3T^3 + p^6T^4$
21	smooth		$1 - 58T - 1168pT^2 - 58p^3T^3 + p^6T^4$
22	smooth		$1 + 404T + 3466pT^2 + 404p^3T^3 + p^6T^4$
23	smooth		$1 - 170T + 2710pT^2 - 170p^3T^3 + p^6T^4$
24	smooth		$1 + 467T + 2934pT^2 + 467p^3T^3 + p^6T^4$
25	smooth		$1 + 383T + 4978pT^2 + 383p^3T^3 + p^6T^4$
26	smooth		$1 - 44T + 2654pT^2 - 44p^3T^3 + p^6T^4$
27	smooth		$1 - 114T + 1268pT^2 - 114p^3T^3 + p^6T^4$
28	smooth		$1 + 166T + 2346pT^2 + 166p^3T^3 + p^6T^4$
29	smooth		$1 - 716T + 6322pT^2 - 716p^3T^3 + p^6T^4$
30	smooth		$1 + 68T + 3410pT^2 + 68p^3T^3 + p^6T^4$
31	smooth		$1 - 9T + 3522pT^2 - 9p^3T^3 + p^6T^4$
32	smooth		$1 + 138T + 4222pT^2 + 138p^3T^3 + p^6T^4$
33	smooth		$1 - 100T + 274pT^2 - 100p^3T^3 + p^6T^4$
34	smooth		$1 + 12T + 1450pT^2 + 12p^3T^3 + p^6T^4$
35	smooth		$1 + 677T + 6210pT^2 + 677p^3T^3 + p^6T^4$
36	smooth		$1 + 467T + 2934pT^2 + 467p^3T^3 + p^6T^4$
37	smooth		$1 + 96T + 3536pT^2 + 96p^3T^3 + p^6T^4$
38	smooth		$1 + 117T + 3914pT^2 + 117p^3T^3 + p^6T^4$
39	smooth		$1 + 271T + 2234pT^2 + 271p^3T^3 + p^6T^4$
40	smooth		$1 + 222T + 2570pT^2 + 222p^3T^3 + p^6T^4$
41	smooth		$1 + 103T - 1294pT^2 + 103p^3T^3 + p^6T^4$
42	smooth		$1 - 492T + 4810pT^2 - 492p^3T^3 + p^6T^4$
43	smooth		$1 + 285T + 50pT^2 + 285p^3T^3 + p^6T^4$
44	smooth		$1 + 12T - 1574pT^2 + 12p^3T^3 + p^6T^4$
45	smooth		$1 - 247T + 1842pT^2 - 247p^3T^3 + p^6T^4$
46	smooth		$1 - 191T + 890pT^2 - 191p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 267T + 4172pT^2 - 267p^3T^3 + p^6T^4$
2	smooth		$1 - 22T - 532pT^2 - 22p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 552T + 4130pT^2 + 552p^3T^3 + p^6T^4$
5	smooth		$1 - 232T + 5012pT^2 - 232p^3T^3 + p^6T^4$
6	smooth		$1 + 545T + 4256pT^2 + 545p^3T^3 + p^6T^4$
7	smooth		$1 - 533T + 4844pT^2 - 533p^3T^3 + p^6T^4$
8	smooth		$1 - 57T + 3038pT^2 - 57p^3T^3 + p^6T^4$
9	smooth		$1 + 412T + 5082pT^2 + 412p^3T^3 + p^6T^4$
10	smooth		$1 + 27T + 3976pT^2 + 27p^3T^3 + p^6T^4$
11	smooth		$1 + 111T - 280pT^2 + 111p^3T^3 + p^6T^4$
12	smooth		$1 + 1105T + 10542pT^2 + 1105p^3T^3 + p^6T^4$
13	smooth		$1 - 36T - 2338pT^2 - 36p^3T^3 + p^6T^4$
14	smooth		$1 - 148T + 4088pT^2 - 148p^3T^3 + p^6T^4$
15	smooth		$1 - 50T + 1050pT^2 - 50p^3T^3 + p^6T^4$
16	smooth		$1 + 48T + 4382pT^2 + 48p^3T^3 + p^6T^4$
17	smooth		$1 + 5pT + 2828pT^2 + 5p^4T^3 + p^6T^4$
18	smooth		$1 - 204T - 1666pT^2 - 204p^3T^3 + p^6T^4$
19	smooth		$1 + 76T + 2506pT^2 + 76p^3T^3 + p^6T^4$
20	smooth		$1 + 468T + 3290pT^2 + 468p^3T^3 + p^6T^4$
21	smooth		$(1 - 4pT + p^3T^2)(1 + 498T + p^3T^2)$
22	smooth		$1 - 743T + 5586pT^2 - 743p^3T^3 + p^6T^4$
23	smooth		$1 + 230T + 4732pT^2 + 230p^3T^3 + p^6T^4$
24	smooth		$1 + 314T + 5866pT^2 + 314p^3T^3 + p^6T^4$
25	smooth		$(1 - 4pT + p^3T^2)(1 + 624T + p^3T^2)$
26	smooth		$1 + 244T + 1246pT^2 + 244p^3T^3 + p^6T^4$
27	smooth		$1 + 118T + 2828pT^2 + 118p^3T^3 + p^6T^4$
28	smooth		$1 - 2pT + 4998pT^2 - 2p^4T^3 + p^6T^4$
29	smooth		$1 + 202T - 1330pT^2 + 202p^3T^3 + p^6T^4$
30	smooth		$1 + 195T - 1106pT^2 + 195p^3T^3 + p^6T^4$
31	smooth		$1 + 251T + 2982pT^2 + 251p^3T^3 + p^6T^4$
32	smooth		$1 + 384T + 3724pT^2 + 384p^3T^3 + p^6T^4$
33	smooth		$1 + 272T + 4270pT^2 + 272p^3T^3 + p^6T^4$
34	smooth		$1 - 813T + 7630pT^2 - 813p^3T^3 + p^6T^4$
35	smooth		$1 + 202T + 3570pT^2 + 202p^3T^3 + p^6T^4$
36	smooth		$1 - 176T - 2366pT^2 - 176p^3T^3 + p^6T^4$
37	smooth		$1 - 8T + 882pT^2 - 8p^3T^3 + p^6T^4$
38	smooth		$1 - 407T + 812pT^2 - 407p^3T^3 + p^6T^4$
39	smooth		$1 + 48T + 3500pT^2 + 48p^3T^3 + p^6T^4$
40	smooth		$1 + 608T + 4886pT^2 + 608p^3T^3 + p^6T^4$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 - 596T + 5978pT^2 - 596p^3T^3 + p^6T^4$
42	smooth		$1 + 13T - 1652pT^2 + 13p^3T^3 + p^6T^4$
43	smooth		$1 + 321T + 2016pT^2 + 321p^3T^3 + p^6T^4$
44	smooth		$1 + 321T + 4172pT^2 + 321p^3T^3 + p^6T^4$
45	smooth		$1 - 190T + 2002pT^2 - 190p^3T^3 + p^6T^4$
46	smooth		$1 - 64T + 2478pT^2 - 64p^3T^3 + p^6T^4$
47	smooth		$1 - 316T + 3878pT^2 - 316p^3T^3 + p^6T^4$
48	smooth		$1 - 218T + 252pT^2 - 218p^3T^3 + p^6T^4$
49	smooth		$1 - 344T + 658pT^2 - 344p^3T^3 + p^6T^4$
50	smooth		$1 - 64T - 1834pT^2 - 64p^3T^3 + p^6T^4$
51	smooth		$1 + 496T + 3374pT^2 + 496p^3T^3 + p^6T^4$
52	smooth		$1 + 335T + 5096pT^2 + 335p^3T^3 + p^6T^4$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 446T + 3434pT^2 - 446p^3T^3 + p^6T^4$
2	smooth		$1 + 569T + 2762pT^2 + 569p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 334T + 200pT^2 - 334p^3T^3 + p^6T^4$
5	smooth		$1 - 579T + 3406pT^2 - 579p^3T^3 + p^6T^4$
6	smooth		$1 + 3pT + 1306pT^2 + 3p^4T^3 + p^6T^4$
7	smooth		$1 + 394T + 2804pT^2 + 394p^3T^3 + p^6T^4$
8	smooth		$1 - 96T - 2698pT^2 - 96p^3T^3 + p^6T^4$
9	smooth		$1 + 373T + 5450pT^2 + 373p^3T^3 + p^6T^4$
10	smooth		$1 + 2T + 2370pT^2 + 2p^3T^3 + p^6T^4$
11	smooth		$1 + 534T + 3042pT^2 + 534p^3T^3 + p^6T^4$
12	smooth		$1 + 800T + 6612pT^2 + 800p^3T^3 + p^6T^4$
13	smooth		$1 + 44T + 3518pT^2 + 44p^3T^3 + p^6T^4$
14	smooth		$1 + 639T + 4442pT^2 + 639p^3T^3 + p^6T^4$
15	smooth		$1 + 324T + 6066pT^2 + 324p^3T^3 + p^6T^4$
16	smooth		$1 + 44T + 4386pT^2 + 44p^3T^3 + p^6T^4$
17	smooth		$1 + 436T + 5170pT^2 + 436p^3T^3 + p^6T^4$
18	smooth		$1 - 1118T + 10854pT^2 - 1118p^3T^3 + p^6T^4$
19	smooth		$1 + 247T + 3378pT^2 + 247p^3T^3 + p^6T^4$
20	smooth		$1 - 642T + 4330pT^2 - 642p^3T^3 + p^6T^4$
21	smooth		$1 + 310T - 402pT^2 + 310p^3T^3 + p^6T^4$
22	smooth		$1 + 128T - 4294pT^2 + 128p^3T^3 + p^6T^4$
23	smooth		$1 + 30T - 850pT^2 + 30p^3T^3 + p^6T^4$
24	smooth		$1 - 320T + 1026pT^2 - 320p^3T^3 + p^6T^4$

Continued on the following page

$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
25	smooth		$1 + 478T + 4204pT^2 + 478p^3T^3 + p^6T^4$
26	smooth		$1 - 523T + 3126pT^2 - 523p^3T^3 + p^6T^4$
27	smooth		$1 + 366T + 2398pT^2 + 366p^3T^3 + p^6T^4$
28	smooth		$1 - 250T + 4890pT^2 - 250p^3T^3 + p^6T^4$
29	smooth		$1 + 674T + 5100pT^2 + 674p^3T^3 + p^6T^4$
30	smooth		$1 + 891T + 9818pT^2 + 891p^3T^3 + p^6T^4$
31	smooth		$1 - 803T + 7914pT^2 - 803p^3T^3 + p^6T^4$
32	smooth		$1 + 723T + 6458pT^2 + 723p^3T^3 + p^6T^4$
33	smooth		$1 + 93T - 1942pT^2 + 93p^3T^3 + p^6T^4$
34	smooth		$1 + 646T + 6206pT^2 + 646p^3T^3 + p^6T^4$
35	smooth		$1 - 12T + 410pT^2 - 12p^3T^3 + p^6T^4$
36	smooth		$1 + 506T + 2762pT^2 + 506p^3T^3 + p^6T^4$
37	smooth		$1 - 824T + 7214pT^2 - 824p^3T^3 + p^6T^4$
38	smooth		$1 - 320T + 1810pT^2 - 320p^3T^3 + p^6T^4$
39	smooth		$1 + 366T + 1222pT^2 + 366p^3T^3 + p^6T^4$
40	smooth		$1 + 338T + 6318pT^2 + 338p^3T^3 + p^6T^4$
41	smooth		$1 - 159T + 158pT^2 - 159p^3T^3 + p^6T^4$
42	smooth		$1 + 548T + 6094pT^2 + 548p^3T^3 + p^6T^4$
43	smooth		$1 - 572T + 7270pT^2 - 572p^3T^3 + p^6T^4$
44	smooth		$1 + 247T + 186pT^2 + 247p^3T^3 + p^6T^4$
45	smooth		$1 + 254T + 2272pT^2 + 254p^3T^3 + p^6T^4$
46	smooth		$1 - 47T - 318pT^2 - 47p^3T^3 + p^6T^4$
47	smooth		$1 - 124T - 962pT^2 - 124p^3T^3 + p^6T^4$
48	smooth		$1 - 250T - 962pT^2 - 250p^3T^3 + p^6T^4$
49	smooth		$1 + 478T + 6108pT^2 + 478p^3T^3 + p^6T^4$
50	smooth		$1 - 320T + 1726pT^2 - 320p^3T^3 + p^6T^4$
51	smooth		$1 + 555T + 5702pT^2 + 555p^3T^3 + p^6T^4$
52	smooth		$1 - 47T + 4106pT^2 - 47p^3T^3 + p^6T^4$
53	smooth		$1 - 950T + 9538pT^2 - 950p^3T^3 + p^6T^4$
54	smooth		$1 + 324T + 2622pT^2 + 324p^3T^3 + p^6T^4$
55	smooth		$1 + 436T + 1110pT^2 + 436p^3T^3 + p^6T^4$
56	smooth		$1 - 138T + 3014pT^2 - 138p^3T^3 + p^6T^4$
57	smooth		$1 + 121T + 5086pT^2 + 121p^3T^3 + p^6T^4$
58	smooth		$1 - 572T + 7046pT^2 - 572p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 422T + 134pT^2 - 422p^3T^3 + p^6T^4$
2	smooth		$1 + 8pT + 3914pT^2 + 8p^4T^3 + p^6T^4$

Continued on the following page

$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth*	3	
4	smooth		$1 + 439T + 3060pT^2 + 439p^3T^3 + p^6T^4$
5	smooth		$1 + 516T + 6070pT^2 + 516p^3T^3 + p^6T^4$
6	smooth		$1 + 1181T + 12090pT^2 + 1181p^3T^3 + p^6T^4$
7	smooth		$1 + 194T - 160pT^2 + 194p^3T^3 + p^6T^4$
8	smooth		$1 + 537T + 3074pT^2 + 537p^3T^3 + p^6T^4$
9	smooth		$1 - 170T - 678pT^2 - 170p^3T^3 + p^6T^4$
10	smooth		$1 - 121T + 2990pT^2 - 121p^3T^3 + p^6T^4$
11	smooth		$1 + 397T + 4670pT^2 + 397p^3T^3 + p^6T^4$
12	smooth		$1 + 656T + 4558pT^2 + 656p^3T^3 + p^6T^4$
13	smooth		$1 + 250T + 4194pT^2 + 250p^3T^3 + p^6T^4$
14	smooth		$1 + 390T + 1674pT^2 + 390p^3T^3 + p^6T^4$
15	smooth		$1 + 47T + 1828pT^2 + 47p^3T^3 + p^6T^4$
16	smooth		$1 + 334T + 1702pT^2 + 334p^3T^3 + p^6T^4$
17	smooth		$1 - 828T + 8142pT^2 - 828p^3T^3 + p^6T^4$
18	smooth		$1 + 110T + 4292pT^2 + 110p^3T^3 + p^6T^4$
19	smooth		$1 + 565T + 8100pT^2 + 565p^3T^3 + p^6T^4$
20	smooth		$1 + 663T + 7288pT^2 + 663p^3T^3 + p^6T^4$
21	smooth		$1 + 439T + 5202pT^2 + 439p^3T^3 + p^6T^4$
22	smooth		$1 + 775T + 8884pT^2 + 775p^3T^3 + p^6T^4$
23	smooth		$1 + 362T + 2108pT^2 + 362p^3T^3 + p^6T^4$
24	smooth		$1 - 324T + 3354pT^2 - 324p^3T^3 + p^6T^4$
25	smooth		$1 - 296T + 5202pT^2 - 296p^3T^3 + p^6T^4$
26	smooth		$1 + 159T + 2654pT^2 + 159p^3T^3 + p^6T^4$
27	smooth		$1 + 334T + 2822pT^2 + 334p^3T^3 + p^6T^4$
28	smooth		$1 - 576T + 4726pT^2 - 576p^3T^3 + p^6T^4$
29	smooth		$1 - 548T + 6364pT^2 - 548p^3T^3 + p^6T^4$
30	smooth		$1 - 6pT + 1758pT^2 - 6p^4T^3 + p^6T^4$
31	smooth		$1 - 205T + 7190pT^2 - 205p^3T^3 + p^6T^4$
32	smooth		$1 + 236T + 526pT^2 + 236p^3T^3 + p^6T^4$
33	smooth		$1 - 282T + 358pT^2 - 282p^3T^3 + p^6T^4$
34	smooth		$1 - 688T + 7106pT^2 - 688p^3T^3 + p^6T^4$
35	smooth		$1 + 33T + 3690pT^2 + 33p^3T^3 + p^6T^4$
36	smooth		$1 - 877T + 7232pT^2 - 877p^3T^3 + p^6T^4$
37	smooth		$1 - 359T + 3718pT^2 - 359p^3T^3 + p^6T^4$
38	smooth		$1 - 380T + 2934pT^2 - 380p^3T^3 + p^6T^4$
39	smooth		$1 - 352T + 778pT^2 - 352p^3T^3 + p^6T^4$
40	smooth		$1 + 1013T + 9570pT^2 + 1013p^3T^3 + p^6T^4$
41	smooth		$1 + 320T + 4894pT^2 + 320p^3T^3 + p^6T^4$
42	smooth		$1 + 68T - 650pT^2 + 68p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
43	smooth		$1 - 170T + 890pT^2 - 170p^3T^3 + p^6T^4$
44	smooth		$1 + 712T + 4278pT^2 + 712p^3T^3 + p^6T^4$
45	smooth		$1 - 149T + 3088pT^2 - 149p^3T^3 + p^6T^4$
46	smooth		$1 + 537T + 4292pT^2 + 537p^3T^3 + p^6T^4$
47	smooth		$1 - 604T + 2346pT^2 - 604p^3T^3 + p^6T^4$
48	smooth		$1 + 208T + 4726pT^2 + 208p^3T^3 + p^6T^4$
49	smooth		$1 + 40T - 426pT^2 + 40p^3T^3 + p^6T^4$
50	smooth		$1 - 86T + 7120pT^2 - 86p^3T^3 + p^6T^4$
51	smooth		$1 - 352T + 1310pT^2 - 352p^3T^3 + p^6T^4$
52	smooth		$1 - 184T + 946pT^2 - 184p^3T^3 + p^6T^4$
53	smooth		$1 + 117T + 1170pT^2 + 117p^3T^3 + p^6T^4$
54	smooth		$1 + 82T + 4614pT^2 + 82p^3T^3 + p^6T^4$
55	smooth		$1 - 275T + 1030pT^2 - 275p^3T^3 + p^6T^4$
56	smooth		$1 + 68T - 2050pT^2 + 68p^3T^3 + p^6T^4$
57	smooth		$1 + 831T + 8800pT^2 + 831p^3T^3 + p^6T^4$
58	smooth		$1 - 380T - 1070pT^2 - 380p^3T^3 + p^6T^4$
59	smooth		$1 - 828T + 9010pT^2 - 828p^3T^3 + p^6T^4$
60	smooth		$1 + 222T + 5986pT^2 + 222p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 435T + 70pT^2 - 435p^3T^3 + p^6T^4$
2	smooth		$1 + 55T - 7182pT^2 + 55p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 148T - 148p^3T^3 + p^6T^4$
5	smooth		$1 + 825T + 7378pT^2 + 825p^3T^3 + p^6T^4$
6	smooth		$1 + 286T + 6986pT^2 + 286p^3T^3 + p^6T^4$
7	smooth		$1 - 50T + 7742pT^2 - 50p^3T^3 + p^6T^4$
8	smooth		$1 + 811T + 7042pT^2 + 811p^3T^3 + p^6T^4$
9	smooth		$1 + 118T - 5474pT^2 + 118p^3T^3 + p^6T^4$
10	smooth		$1 - 92T + 6636pT^2 - 92p^3T^3 + p^6T^4$
11	smooth		$1 + 132T + 1918pT^2 + 132p^3T^3 + p^6T^4$
12	smooth		$1 + 223T + 6258pT^2 + 223p^3T^3 + p^6T^4$
13	smooth		$1 - 533T - 126pT^2 - 533p^3T^3 + p^6T^4$
14	smooth		$1 + 321T + 2926pT^2 + 321p^3T^3 + p^6T^4$
15	smooth		$1 + 587T + 3234pT^2 + 587p^3T^3 + p^6T^4$
16	smooth		$1 + 636T + 3234pT^2 + 636p^3T^3 + p^6T^4$
17	smooth		$1 - 169T + 966pT^2 - 169p^3T^3 + p^6T^4$
18	smooth		$1 + 377T + 3290pT^2 + 377p^3T^3 + p^6T^4$

Continued on the following page

$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$1 + 433T + 7378pT^2 + 433p^3T^3 + p^6T^4$
20	smooth		$1 + 552T + 4746pT^2 + 552p^3T^3 + p^6T^4$
21	smooth		$1 - 162T + 1232pT^2 - 162p^3T^3 + p^6T^4$
22	smooth		$1 - 400T + 5418pT^2 - 400p^3T^3 + p^6T^4$
23	smooth		$1 + 258T - 2016pT^2 + 258p^3T^3 + p^6T^4$
24	smooth		$1 + 538T + 5978pT^2 + 538p^3T^3 + p^6T^4$
25	smooth		$1 - 596T + 6202pT^2 - 596p^3T^3 + p^6T^4$
26	smooth		$1 - 204T - 854pT^2 - 204p^3T^3 + p^6T^4$
27	smooth		$1 - 106T + 1694pT^2 - 106p^3T^3 + p^6T^4$
28	smooth		$1 + 97T + 882pT^2 + 97p^3T^3 + p^6T^4$
29	smooth		$1 + 69T + 6286pT^2 + 69p^3T^3 + p^6T^4$
30	smooth		$1 - 106T - 3598pT^2 - 106p^3T^3 + p^6T^4$
31	smooth		$1 + 433T + 8162pT^2 + 433p^3T^3 + p^6T^4$
32	smooth		$1 - 519T + 5306pT^2 - 519p^3T^3 + p^6T^4$
33	smooth		$1 - 484T + 658pT^2 - 484p^3T^3 + p^6T^4$
34	smooth		$1 - 218T + 8414pT^2 - 218p^3T^3 + p^6T^4$
35	smooth		$1 + 678T + 4928pT^2 + 678p^3T^3 + p^6T^4$
36	smooth		$1 + 622T + 6426pT^2 + 622p^3T^3 + p^6T^4$
37	smooth		$1 - 421T + 3542pT^2 - 421p^3T^3 + p^6T^4$
38	smooth		$1 + 727T + 8946pT^2 + 727p^3T^3 + p^6T^4$
39	smooth		$1 - 974T + 10850pT^2 - 974p^3T^3 + p^6T^4$
40	smooth		$1 + 62T - 6818pT^2 + 62p^3T^3 + p^6T^4$
41	smooth		$1 - 708T + 9198pT^2 - 708p^3T^3 + p^6T^4$
42	smooth		$1 - 624T + 4158pT^2 - 624p^3T^3 + p^6T^4$
43	smooth		$1 + 377T + 2506pT^2 + 377p^3T^3 + p^6T^4$
44	smooth		$1 + 419T + 6258pT^2 + 419p^3T^3 + p^6T^4$
45	smooth		$1 - 36T - 6230pT^2 - 36p^3T^3 + p^6T^4$
46	smooth		$(1 + 2pT + p^3T^2)(1 - 688T + p^3T^2)$
47	smooth		$1 + 1532T + 17388pT^2 + 1532p^3T^3 + p^6T^4$
48	smooth		$1 + 356T + 1806pT^2 + 356p^3T^3 + p^6T^4$
49	smooth		$1 + 1042T + 10234pT^2 + 1042p^3T^3 + p^6T^4$
50	smooth		$1 - 484T + 8498pT^2 - 484p^3T^3 + p^6T^4$
51	smooth		$1 - 316T + 2338pT^2 - 316p^3T^3 + p^6T^4$
52	smooth		$1 + 188T + 4242pT^2 + 188p^3T^3 + p^6T^4$
53	smooth		$1 - 554T + 2702pT^2 - 554p^3T^3 + p^6T^4$
54	smooth		$1 - 358T + 4172pT^2 - 358p^3T^3 + p^6T^4$
55	smooth		$1 + 13T + 7686pT^2 + 13p^3T^3 + p^6T^4$
56	smooth		$1 - 330T + 2688pT^2 - 330p^3T^3 + p^6T^4$
57	smooth		$1 - 204T + 8750pT^2 - 204p^3T^3 + p^6T^4$
58	smooth		$1 - 183T - 2702pT^2 - 183p^3T^3 + p^6T^4$

Continued on the following page

$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
59	smooth		$1 + 503T + 7294pT^2 + 503p^3T^3 + p^6T^4$
60	smooth		$1 + 258T - 2310pT^2 + 258p^3T^3 + p^6T^4$
61	smooth		$1 + 223T + 7042pT^2 + 223p^3T^3 + p^6T^4$
62	smooth		$1 + 83T + 3486pT^2 + 83p^3T^3 + p^6T^4$
63	smooth		$1 + 69T - 966pT^2 + 69p^3T^3 + p^6T^4$
64	smooth		$1 + 944T + 9352pT^2 + 944p^3T^3 + p^6T^4$
65	smooth		$1 + 272T + 1652pT^2 + 272p^3T^3 + p^6T^4$
66	smooth		$1 - 652T + 9954pT^2 - 652p^3T^3 + p^6T^4$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 556T + 5354pT^2 + 556p^3T^3 + p^6T^4$
2	smooth		$1 - 578T + 1868pT^2 - 578p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 150T + 4920pT^2 + 150p^3T^3 + p^6T^4$
5	singular	5	$(1 - pT)(1 - 604T + p^3T^2)$
6	smooth		$1 + 654T + 8938pT^2 + 654p^3T^3 + p^6T^4$
7	smooth		$1 + 220T + 706pT^2 + 220p^3T^3 + p^6T^4$
8	smooth		$1 - 606T + 4136pT^2 - 606p^3T^3 + p^6T^4$
9	smooth		$1 + 479T + 4822pT^2 + 479p^3T^3 + p^6T^4$
10	smooth		$1 + 353T + 4486pT^2 + 353p^3T^3 + p^6T^4$
11	smooth		$1 + 871T + 8994pT^2 + 871p^3T^3 + p^6T^4$
12	smooth		$1 + 52T + 7426pT^2 + 52p^3T^3 + p^6T^4$
13	smooth		$1 + 570T + 4738pT^2 + 570p^3T^3 + p^6T^4$
14	smooth		$1 + 752T + 10450pT^2 + 752p^3T^3 + p^6T^4$
15	smooth		$1 + 472T - 2080pT^2 + 472p^3T^3 + p^6T^4$
16	smooth		$1 - 704T + 10352pT^2 - 704p^3T^3 + p^6T^4$
17	smooth		$1 - 1159T + 13586pT^2 - 1159p^3T^3 + p^6T^4$
18	smooth		$1 - 326T + 1028pT^2 - 326p^3T^3 + p^6T^4$
19	smooth		$1 - 88T + 5186pT^2 - 88p^3T^3 + p^6T^4$
20	smooth		$1 + 332T + 580pT^2 + 332p^3T^3 + p^6T^4$
21	smooth		$1 + 108T - 3214pT^2 + 108p^3T^3 + p^6T^4$
22	smooth		$1 + 136T - 1478pT^2 + 136p^3T^3 + p^6T^4$
23	smooth		$1 - 4T + 3898pT^2 - 4p^3T^3 + p^6T^4$
24	smooth		$1 + 318T - 4334pT^2 + 318p^3T^3 + p^6T^4$
25	singular	25	$(1 - pT)(1 + 180T + p^3T^2)$
26	smooth		$1 + 857T + 5130pT^2 + 857p^3T^3 + p^6T^4$
27	smooth		$1 + 577T + 10058pT^2 + 577p^3T^3 + p^6T^4$
28	smooth		$1 - 389T + 7426pT^2 - 389p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
29	smooth		$1 + 311T - 3774pT^2 + 311p^3T^3 + p^6T^4$
30	smooth		$1 + 318T + 9274pT^2 + 318p^3T^3 + p^6T^4$
31	smooth		$1 - 179T + 2442pT^2 - 179p^3T^3 + p^6T^4$
32	smooth		$1 + 500T + 3044pT^2 + 500p^3T^3 + p^6T^4$
33	smooth		$1 - 984T + 7930pT^2 - 984p^3T^3 + p^6T^4$
34	smooth		$1 - 452T + 9638pT^2 - 452p^3T^3 + p^6T^4$
35	smooth		$1 + 500T + 2134pT^2 + 500p^3T^3 + p^6T^4$
36	smooth		$1 + 696T + 8756pT^2 + 696p^3T^3 + p^6T^4$
37	smooth		$1 + 1060T + 9778pT^2 + 1060p^3T^3 + p^6T^4$
38	smooth		$1 - 1054T + 13250pT^2 - 1054p^3T^3 + p^6T^4$
39	smooth		$1 - 389T + 6194pT^2 - 389p^3T^3 + p^6T^4$
40	smooth		$1 + 381T + 8210pT^2 + 381p^3T^3 + p^6T^4$
41	smooth		$1 + 332T + 3058pT^2 + 332p^3T^3 + p^6T^4$
42	smooth		$1 + 395T + 4290pT^2 + 395p^3T^3 + p^6T^4$
43	smooth		$1 + 248T + 1098pT^2 + 248p^3T^3 + p^6T^4$
44	smooth		$1 + 528T + 1378pT^2 + 528p^3T^3 + p^6T^4$
45	smooth		$1 - 606T + 4444pT^2 - 606p^3T^3 + p^6T^4$
46	singular	46	$(1 - pT)(1 + 376T + p^3T^2)$
47	smooth		$1 + 17T - 2206pT^2 + 17p^3T^3 + p^6T^4$
48	smooth		$1 - 536T + 9568pT^2 - 536p^3T^3 + p^6T^4$
49	smooth		$1 - 228T + 6404pT^2 - 228p^3T^3 + p^6T^4$
50	smooth		$1 - 725T + 9890pT^2 - 725p^3T^3 + p^6T^4$
51	smooth		$1 + 199T - 1534pT^2 + 199p^3T^3 + p^6T^4$
52	smooth		$1 - 186T + 7874pT^2 - 186p^3T^3 + p^6T^4$
53	smooth		$1 + 367T + 4682pT^2 + 367p^3T^3 + p^6T^4$
54	smooth		$1 + 66T + 2218pT^2 + 66p^3T^3 + p^6T^4$
55	smooth		$1 + 325T + 5802pT^2 + 325p^3T^3 + p^6T^4$
56	smooth		$1 - 508T + 4066pT^2 - 508p^3T^3 + p^6T^4$
57	smooth		$1 + 262T + 7314pT^2 + 262p^3T^3 + p^6T^4$
58	smooth		$1 + 171T + 4346pT^2 + 171p^3T^3 + p^6T^4$
59	smooth		$1 - 340T + 2834pT^2 - 340p^3T^3 + p^6T^4$
60	smooth		$1 + 80T + 5018pT^2 + 80p^3T^3 + p^6T^4$
61	smooth		$1 + 332T + 1042pT^2 + 332p^3T^3 + p^6T^4$
62	smooth		$1 - 599T + 2442pT^2 - 599p^3T^3 + p^6T^4$
63	smooth		$1 + 920T + 6838pT^2 + 920p^3T^3 + p^6T^4$
64	smooth		$1 - 67T - 4530pT^2 - 67p^3T^3 + p^6T^4$
65	smooth		$1 + 941T + 8714pT^2 + 941p^3T^3 + p^6T^4$
66	smooth		$1 + 17T + 5746pT^2 + 17p^3T^3 + p^6T^4$
67	smooth		$1 - 438T + 7650pT^2 - 438p^3T^3 + p^6T^4$
68	smooth		$1 - 599T + 4794pT^2 - 599p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
69	smooth		$1 - 767T + 4962pT^2 - 767p^3T^3 + p^6T^4$
70	smooth		$1 - 130T - 2878pT^2 - 130p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 2T - 990pT^2 + 2p^3T^3 + p^6T^4$
2	smooth		$1 + 576T + 7214pT^2 + 576p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 1206T + 10042pT^2 + 1206p^3T^3 + p^6T^4$
5	smooth		$1 + 72T - 5232pT^2 + 72p^3T^3 + p^6T^4$
6	smooth		$1 - 726T + 6178pT^2 - 726p^3T^3 + p^6T^4$
7	smooth		$1 + 121T + 7130pT^2 + 121p^3T^3 + p^6T^4$
8	smooth		$1 - 1118T + 13738pT^2 - 1118p^3T^3 + p^6T^4$
9	smooth		$1 + 450T - 1046pT^2 + 450p^3T^3 + p^6T^4$
10	smooth		$1 + 163T + 6906pT^2 + 163p^3T^3 + p^6T^4$
11	smooth		$1 + 149T - 2586pT^2 + 149p^3T^3 + p^6T^4$
12	smooth		$1 + 58T + 7298pT^2 + 58p^3T^3 + p^6T^4$
13	smooth		$1 + 324T + 4176pT^2 + 324p^3T^3 + p^6T^4$
14	smooth		$1 + 156T + 4036pT^2 + 156p^3T^3 + p^6T^4$
15	smooth		$1 - 40T - 2838pT^2 - 40p^3T^3 + p^6T^4$
16	smooth		$1 - 152T + 270pT^2 - 152p^3T^3 + p^6T^4$
17	smooth		$1 + 317T + 8418pT^2 + 317p^3T^3 + p^6T^4$
18	smooth		$1 + 737T + 8180pT^2 + 737p^3T^3 + p^6T^4$
19	smooth		$1 - 201T - 3916pT^2 - 201p^3T^3 + p^6T^4$
20	smooth		$1 + 1010T + 12954pT^2 + 1010p^3T^3 + p^6T^4$
21	smooth		$1 + 30T - 2348pT^2 + 30p^3T^3 + p^6T^4$
22	smooth		$1 + 1164T + 12044pT^2 + 1164p^3T^3 + p^6T^4$
23	smooth		$1 - 726T + 8698pT^2 - 726p^3T^3 + p^6T^4$
24	smooth		$1 - 621T + 10560pT^2 - 621p^3T^3 + p^6T^4$
25	smooth		$1 - 621T + 2076pT^2 - 621p^3T^3 + p^6T^4$
26	smooth		$1 - 873T + 11022pT^2 - 873p^3T^3 + p^6T^4$
27	smooth		$1 + 79T + 2776pT^2 + 79p^3T^3 + p^6T^4$
28	smooth		$1 + 289T - 22p^2T^2 + 289p^3T^3 + p^6T^4$
29	smooth		$1 - 334T + 4274pT^2 - 334p^3T^3 + p^6T^4$
30	smooth		$1 - 684T + 8194pT^2 - 684p^3T^3 + p^6T^4$
31	smooth		$1 - 278T + 7620pT^2 - 278p^3T^3 + p^6T^4$
32	smooth		$1 - 313T - 1256pT^2 - 313p^3T^3 + p^6T^4$
33	smooth		$1 - 96T + 2048pT^2 - 96p^3T^3 + p^6T^4$
34	smooth		$1 + 114T + 8852pT^2 + 114p^3T^3 + p^6T^4$

Continued on the following page

$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 - 530T + 5058pT^2 - 530p^3T^3 + p^6T^4$
36	smooth		$1 + 674T + 3266pT^2 + 674p^3T^3 + p^6T^4$
37	smooth		$1 + 324T + 3910pT^2 + 324p^3T^3 + p^6T^4$
38	smooth		$1 - 348T + 6038pT^2 - 348p^3T^3 + p^6T^4$
39	smooth		$1 + 135T + 3938pT^2 + 135p^3T^3 + p^6T^4$
40	smooth		$1 + 163T - 7374pT^2 + 163p^3T^3 + p^6T^4$
41	smooth		$1 + 478T + 90p^2T^2 + 478p^3T^3 + p^6T^4$
42	smooth		$1 - 264T - 2040pT^2 - 264p^3T^3 + p^6T^4$
43	smooth		$1 + 338T - 3286pT^2 + 338p^3T^3 + p^6T^4$
44	smooth		$1 + 10pT + 5730pT^2 + 10p^4T^3 + p^6T^4$
45	smooth		$1 - 320T + 3196pT^2 - 320p^3T^3 + p^6T^4$
46	smooth		$1 + 1066T + 13850pT^2 + 1066p^3T^3 + p^6T^4$
47	smooth		$1 - 649T + 90p^2T^2 - 649p^3T^3 + p^6T^4$
48	smooth		$1 + 1262T + 13458pT^2 + 1262p^3T^3 + p^6T^4$
49	smooth		$1 - 180T - 6618pT^2 - 180p^3T^3 + p^6T^4$
50	smooth		$1 + 513T + 3336pT^2 + 513p^3T^3 + p^6T^4$
51	smooth		$1 + 282T + 5674pT^2 + 282p^3T^3 + p^6T^4$
52	smooth		$1 + 184T - 4854pT^2 + 184p^3T^3 + p^6T^4$
53	smooth		$1 + 1248T + 13038pT^2 + 1248p^3T^3 + p^6T^4$
54	smooth		$1 - 26T + 410pT^2 - 26p^3T^3 + p^6T^4$
55	smooth		$1 - 397T + 6864pT^2 - 397p^3T^3 + p^6T^4$
56	smooth		$1 - 488T + 7788pT^2 - 488p^3T^3 + p^6T^4$
57	smooth		$1 - 11pT + 7452pT^2 - 11p^4T^3 + p^6T^4$
58	smooth		$1 - 313T + 10630pT^2 - 313p^3T^3 + p^6T^4$
59	smooth		$1 + 10pT + 5534pT^2 + 10p^4T^3 + p^6T^4$
60	smooth		$1 + 51T - 4098pT^2 + 51p^3T^3 + p^6T^4$
61	smooth		$1 - 376T - 3986pT^2 - 376p^3T^3 + p^6T^4$
62	smooth		$1 + 79T + 2594pT^2 + 79p^3T^3 + p^6T^4$
63	smooth		$1 + 394T + 4890pT^2 + 394p^3T^3 + p^6T^4$
64	smooth		$1 + 828T + 4750pT^2 + 828p^3T^3 + p^6T^4$
65	smooth		$1 - 705T + 7480pT^2 - 705p^3T^3 + p^6T^4$
66	smooth		$1 - 4pT + 536pT^2 - 4p^4T^3 + p^6T^4$
67	smooth		$1 + 1045T + 7900pT^2 + 1045p^3T^3 + p^6T^4$
68	smooth		$1 - 460T + 11218pT^2 - 460p^3T^3 + p^6T^4$
69	smooth		$1 + 79T + 5156pT^2 + 79p^3T^3 + p^6T^4$
70	smooth		$1 - 110T + 3322pT^2 - 110p^3T^3 + p^6T^4$
71	smooth		$1 - 173T - 2292pT^2 - 173p^3T^3 + p^6T^4$
72	smooth		$1 + 226T + 1418pT^2 + 226p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 246T + 490pT^2 - 246p^3T^3 + p^6T^4$
2	smooth		$1 + 468T + 1442pT^2 + 468p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 - 687T + 5586pT^2 - 687p^3T^3 + p^6T^4$
5	smooth		$1 + 769T + 9814pT^2 + 769p^3T^3 + p^6T^4$
6	smooth		$1 + 34T + 1386pT^2 + 34p^3T^3 + p^6T^4$
7	smooth		$1 - 197T + 5978pT^2 - 197p^3T^3 + p^6T^4$
8	smooth		$1 + 90T - 2590pT^2 + 90p^3T^3 + p^6T^4$
9	smooth		$1 - 288T + 6216pT^2 - 288p^3T^3 + p^6T^4$
10	smooth		$1 + 125T + 12222pT^2 + 125p^3T^3 + p^6T^4$
11	smooth		$1 + 713T + 2422pT^2 + 713p^3T^3 + p^6T^4$
12	smooth		$1 + 594T + 2002pT^2 + 594p^3T^3 + p^6T^4$
13	smooth		$1 + 433T + 3486pT^2 + 433p^3T^3 + p^6T^4$
14	smooth		$1 + 335T + 2898pT^2 + 335p^3T^3 + p^6T^4$
15	smooth		$1 + 1000T + 9730pT^2 + 1000p^3T^3 + p^6T^4$
16	smooth		$1 + 69T + 2282pT^2 + 69p^3T^3 + p^6T^4$
17	smooth		$1 - 498T + 8974pT^2 - 498p^3T^3 + p^6T^4$
18	smooth		$1 + 517T + 4186pT^2 + 517p^3T^3 + p^6T^4$
19	smooth		$1 - 379T + 9394pT^2 - 379p^3T^3 + p^6T^4$
20	smooth		$1 - 1513T + 17878pT^2 - 1513p^3T^3 + p^6T^4$
21	smooth		$1 - 267T - 1302pT^2 - 267p^3T^3 + p^6T^4$
22	smooth		$1 - 568T + 8162pT^2 - 568p^3T^3 + p^6T^4$
23	smooth		$1 + 1819T + 21602pT^2 + 1819p^3T^3 + p^6T^4$
24	smooth		$(1 - 10pT + p^3T^2)(1 + 1272T + p^3T^2)$
25	smooth		$1 + 314T + 10220pT^2 + 314p^3T^3 + p^6T^4$
26	smooth		$1 + 286T + 2506pT^2 + 286p^3T^3 + p^6T^4$
27	smooth		$1 + 76T - 126pT^2 + 76p^3T^3 + p^6T^4$
28	smooth		$1 - 197T - 6958pT^2 - 197p^3T^3 + p^6T^4$
29	smooth		$1 + 97T - 3822pT^2 + 97p^3T^3 + p^6T^4$
30	smooth		$1 - 232T - 210pT^2 - 232p^3T^3 + p^6T^4$
31	smooth		$1 + 692T + 6706pT^2 + 692p^3T^3 + p^6T^4$
32	smooth		$1 - 78T + 1204pT^2 - 78p^3T^3 + p^6T^4$
33	smooth		$1 + 216T + 5810pT^2 + 216p^3T^3 + p^6T^4$
34	smooth		$1 - 1548T + 16786pT^2 - 1548p^3T^3 + p^6T^4$
35	smooth		$1 - 1156T + 9730pT^2 - 1156p^3T^3 + p^6T^4$
36	smooth		$1 + 762T + 11242pT^2 + 762p^3T^3 + p^6T^4$
37	smooth		$1 - 1646T + 19922pT^2 - 1646p^3T^3 + p^6T^4$
38	smooth		$1 - 1506T + 15862pT^2 - 1506p^3T^3 + p^6T^4$
39	smooth		$1 + 76T + 6342pT^2 + 76p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 - 85T + 2142pT^2 - 85p^3T^3 + p^6T^4$
41	smooth		$1 - 694T + 7014pT^2 - 694p^3T^3 + p^6T^4$
42	smooth		$1 - 302T - 1806pT^2 - 302p^3T^3 + p^6T^4$
43	smooth		$1 + 622T + 9030pT^2 + 622p^3T^3 + p^6T^4$
44	smooth		$1 + 664T + 10164pT^2 + 664p^3T^3 + p^6T^4$
45	smooth		$1 + 762T + 6734pT^2 + 762p^3T^3 + p^6T^4$
46	smooth		$1 - 393T - 98pT^2 - 393p^3T^3 + p^6T^4$
47	smooth		$1 + 979T + 12642pT^2 + 979p^3T^3 + p^6T^4$
48	smooth		$1 - 932T + 8330pT^2 - 932p^3T^3 + p^6T^4$
49	smooth		$1 + 1049T + 7966pT^2 + 1049p^3T^3 + p^6T^4$
50	smooth		$1 - 22T + 3206pT^2 - 22p^3T^3 + p^6T^4$
51	smooth		$1 + 1665T + 21070pT^2 + 1665p^3T^3 + p^6T^4$
52	smooth		$1 + 132T - 6062pT^2 + 132p^3T^3 + p^6T^4$
53	smooth		$1 + 272T + 8498pT^2 + 272p^3T^3 + p^6T^4$
54	smooth		$1 + 146T - 6762pT^2 + 146p^3T^3 + p^6T^4$
55	smooth		$1 - 631T + 6510pT^2 - 631p^3T^3 + p^6T^4$
56	smooth		$1 + 104T - 5054pT^2 + 104p^3T^3 + p^6T^4$
57	smooth		$1 - 1044T + 8442pT^2 - 1044p^3T^3 + p^6T^4$
58	smooth		$1 + 692T + 8274pT^2 + 692p^3T^3 + p^6T^4$
59	smooth		$1 + 20T + 6202pT^2 + 20p^3T^3 + p^6T^4$
60	smooth		$1 - 1485T + 14322pT^2 - 1485p^3T^3 + p^6T^4$
61	smooth		$1 + 1112T + 11578pT^2 + 1112p^3T^3 + p^6T^4$
62	smooth		$1 - 414T + 5852pT^2 - 414p^3T^3 + p^6T^4$
63	smooth		$1 + 48T + 5390pT^2 + 48p^3T^3 + p^6T^4$
64	smooth		$1 - T + 4214pT^2 - p^3T^3 + p^6T^4$
65	smooth		$1 - 64T + 5698pT^2 - 64p^3T^3 + p^6T^4$
66	smooth		$1 + 1098T + 13454pT^2 + 1098p^3T^3 + p^6T^4$
67	smooth		$1 + 1203T + 9086pT^2 + 1203p^3T^3 + p^6T^4$
68	smooth		$1 + 867T + 10794pT^2 + 867p^3T^3 + p^6T^4$
69	smooth		$1 + 1224T + 15974pT^2 + 1224p^3T^3 + p^6T^4$
70	smooth		$1 - 631T + 11018pT^2 - 631p^3T^3 + p^6T^4$
71	smooth		$1 - 806T + 7714pT^2 - 806p^3T^3 + p^6T^4$
72	smooth		$1 + 804T + 5712pT^2 + 804p^3T^3 + p^6T^4$
73	smooth		$1 - 477T + 3906pT^2 - 477p^3T^3 + p^6T^4$
74	smooth		$1 + 433T + 546pT^2 + 433p^3T^3 + p^6T^4$
75	smooth		$1 + 167T + 3066pT^2 + 167p^3T^3 + p^6T^4$
76	smooth		$1 + 699T + 9982pT^2 + 699p^3T^3 + p^6T^4$
77	smooth		$1 + 146T - 686pT^2 + 146p^3T^3 + p^6T^4$
78	smooth		$1 - 85T + 3906pT^2 - 85p^3T^3 + p^6T^4$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 231T + 3194pT^2 - 231p^3T^3 + p^6T^4$
2	smooth		$1 + 259T + 58pT^2 + 259p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 406T - 4940pT^2 + 406p^3T^3 + p^6T^4$
5	smooth		$1 + 1092T + 15542pT^2 + 1092p^3T^3 + p^6T^4$
6	smooth		$1 - 364T + 10446pT^2 - 364p^3T^3 + p^6T^4$
7	smooth		$1 + 189T + 5938pT^2 + 189p^3T^3 + p^6T^4$
8	smooth		$1 + 182T - 334pT^2 + 182p^3T^3 + p^6T^4$
9	smooth		$1 - 98T + 2410pT^2 - 98p^3T^3 + p^6T^4$
10	singular	10	$(1 - pT)(1 + 1148T + p^3T^2)$
11	smooth		$1 + 7pT + 3978pT^2 + 7p^4T^3 + p^6T^4$
12	smooth		$(1 + 14pT + p^3T^2)(1 - 56T + p^3T^2)$
13	smooth		$1 + 196T + 2606pT^2 + 196p^3T^3 + p^6T^4$
14	singular	14	$(1 - pT)(1 - 1372T + p^3T^2)$
15	smooth		$1 - 168T + 8486pT^2 - 168p^3T^3 + p^6T^4$
16	singular	16	$(1 - pT)(1 - 84T + p^3T^2)$
17	smooth		$1 + 182T + 10348pT^2 + 182p^3T^3 + p^6T^4$
18	smooth		$1 + 952T + 4174pT^2 + 952p^3T^3 + p^6T^4$
19	smooth		$1 - 91T + 10642pT^2 - 91p^3T^3 + p^6T^4$
20	smooth		$(1 + 7pT + p^3T^2)(1 - 448T + p^3T^2)$
21	smooth		$1 - 637T + 9858pT^2 - 637p^3T^3 + p^6T^4$
22	smooth		$1 - 420T + 12014pT^2 - 420p^3T^3 + p^6T^4$
23	smooth		$1 - 350T + 5350pT^2 - 350p^3T^3 + p^6T^4$
24	smooth		$1 + 1274T + 18286pT^2 + 1274p^3T^3 + p^6T^4$
25	smooth		$1 + 21T - 7390pT^2 + 21p^3T^3 + p^6T^4$
26	smooth		$1 - 770T + 9956pT^2 - 770p^3T^3 + p^6T^4$
27	smooth		$1 + 189T + 842pT^2 + 189p^3T^3 + p^6T^4$
28	smooth		$1 - 798T + 11034pT^2 - 798p^3T^3 + p^6T^4$
29	smooth		$1 - 91T - 9154pT^2 - 91p^3T^3 + p^6T^4$
30	smooth		$1 + 140T + 11818pT^2 + 140p^3T^3 + p^6T^4$
31	smooth		$1 - 350T + 10250pT^2 - 350p^3T^3 + p^6T^4$
32	smooth		$1 - 266T + 12014pT^2 - 266p^3T^3 + p^6T^4$
33	smooth		$1 - 602T + 9858pT^2 - 602p^3T^3 + p^6T^4$
34	smooth		$1 + 259T + 12210pT^2 + 259p^3T^3 + p^6T^4$
35	smooth		$1 + 231T + 5546pT^2 + 231p^3T^3 + p^6T^4$
36	smooth		$1 - 1134T + 10250pT^2 - 1134p^3T^3 + p^6T^4$
37	smooth		$1 + 210T - 3960pT^2 + 210p^3T^3 + p^6T^4$
38	smooth		$1 - 1554T + 14954pT^2 - 1554p^3T^3 + p^6T^4$
39	smooth		$1 - 112T + 7310pT^2 - 112p^3T^3 + p^6T^4$

Continued on the following page

$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 707T + 3782pT^2 + 707p^3T^3 + p^6T^4$
41	smooth		$1 + 70T + 10348pT^2 + 70p^3T^3 + p^6T^4$
42	smooth		$1 - 308T + 4958pT^2 - 308p^3T^3 + p^6T^4$
43	smooth		$1 - 980T + 5742pT^2 - 980p^3T^3 + p^6T^4$
44	smooth		$1 + 343T + 1430pT^2 + 343p^3T^3 + p^6T^4$
45	smooth		$1 + 441T + 11818pT^2 + 441p^3T^3 + p^6T^4$
46	smooth		$1 - 784T + 4370pT^2 - 784p^3T^3 + p^6T^4$
47	smooth		$1 + 714T + 13582pT^2 + 714p^3T^3 + p^6T^4$
48	smooth		$1 + 1274T + 12504pT^2 + 1274p^3T^3 + p^6T^4$
49	smooth		$1 - 98T + 7800pT^2 - 98p^3T^3 + p^6T^4$
50	smooth		$1 + 1036T + 7310pT^2 + 1036p^3T^3 + p^6T^4$
51	smooth		$1 - 847T + 9074pT^2 - 847p^3T^3 + p^6T^4$
52	smooth		$1 + 245T + 3586pT^2 + 245p^3T^3 + p^6T^4$
53	smooth		$1 + 1526T + 20050pT^2 + 1526p^3T^3 + p^6T^4$
54	smooth		$1 + 266T - 1314pT^2 + 266p^3T^3 + p^6T^4$
55	smooth		$1 - 252T + 12014pT^2 - 252p^3T^3 + p^6T^4$
56	smooth		$(1 + 14pT + p^3T^2)(1 + 546T + p^3T^2)$
57	smooth		$1 + 1960T + 23774pT^2 + 1960p^3T^3 + p^6T^4$
58	smooth		$1 + 217T - 2686pT^2 + 217p^3T^3 + p^6T^4$
59	smooth		$1 - 1820T + 21618pT^2 - 1820p^3T^3 + p^6T^4$
60	smooth		$1 + 14pT + 14758pT^2 + 14p^4T^3 + p^6T^4$
61	smooth		$1 - 14T + 12504pT^2 - 14p^3T^3 + p^6T^4$
62	smooth		$1 - 882T + 10446pT^2 - 882p^3T^3 + p^6T^4$
63	smooth		$1 + 168T + 1920pT^2 + 168p^3T^3 + p^6T^4$
64	smooth		$1 - 224T + 2802pT^2 - 224p^3T^3 + p^6T^4$
65	smooth		$1 + 805T + 5742pT^2 + 805p^3T^3 + p^6T^4$
66	smooth		$1 - 1092T + 14954pT^2 - 1092p^3T^3 + p^6T^4$
67	smooth		$1 + 1764T + 19070pT^2 + 1764p^3T^3 + p^6T^4$
68	smooth		$1 - 175T + 2998pT^2 - 175p^3T^3 + p^6T^4$
69	smooth		$1 + 770T + 7408pT^2 + 770p^3T^3 + p^6T^4$
70	smooth		$1 + 1225T + 11818pT^2 + 1225p^3T^3 + p^6T^4$
71	smooth		$1 - 70T - 4450pT^2 - 70p^3T^3 + p^6T^4$
72	smooth		$1 + 1001T + 13386pT^2 + 1001p^3T^3 + p^6T^4$
73	smooth		$1 - 1064T + 14170pT^2 - 1064p^3T^3 + p^6T^4$
74	smooth		$1 + 644T + 9858pT^2 + 644p^3T^3 + p^6T^4$
75	smooth		$1 - 378T - 4254pT^2 - 378p^3T^3 + p^6T^4$
76	smooth		$1 - 252T - 3862pT^2 - 252p^3T^3 + p^6T^4$
77	smooth		$1 - 966T + 11426pT^2 - 966p^3T^3 + p^6T^4$
78	smooth		$1 - 1015T + 12406pT^2 - 1015p^3T^3 + p^6T^4$
79	smooth		$1 + 798T + 1430pT^2 + 798p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 - 35T + 7898pT^2 - 35p^3T^3 + p^6T^4$
81	smooth		$1 + 238T + 9760pT^2 + 238p^3T^3 + p^6T^4$
82	smooth		$1 + 336T + 4958pT^2 + 336p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 796T + 8198pT^2 + 796p^3T^3 + p^6T^4$
2	smooth		$1 - 1262T + 11810pT^2 - 1262p^3T^3 + p^6T^4$
3	smooth*	3	
4	smooth		$1 + 1720T + 19398pT^2 + 1720p^3T^3 + p^6T^4$
5	smooth		$1 - 1850T + 22842pT^2 - 1850p^3T^3 + p^6T^4$
6	smooth		$1 - 464T + 7946pT^2 - 464p^3T^3 + p^6T^4$
7	smooth		$1 + 460T + 12692pT^2 + 460p^3T^3 + p^6T^4$
8	smooth		$1 + 313T + 10536pT^2 + 313p^3T^3 + p^6T^4$
9	smooth		$1 - 170T + 8002pT^2 - 170p^3T^3 + p^6T^4$
10	smooth		$1 - 975T + 18404pT^2 - 975p^3T^3 + p^6T^4$
11	smooth		$1 + 404T + 5370pT^2 + 404p^3T^3 + p^6T^4$
12	smooth		$1 + 376T + 10074pT^2 + 376p^3T^3 + p^6T^4$
13	smooth		$1 + 173T + 9038pT^2 + 173p^3T^3 + p^6T^4$
14	smooth		$1 + 306T + 1870pT^2 + 306p^3T^3 + p^6T^4$
15	smooth		$1 + 544T + 11628pT^2 + 544p^3T^3 + p^6T^4$
16	smooth		$1 + 320T + 13406pT^2 + 320p^3T^3 + p^6T^4$
17	smooth		$1 + 1545T + 14568pT^2 + 1545p^3T^3 + p^6T^4$
18	smooth		$1 - 1654T + 20994pT^2 - 1654p^3T^3 + p^6T^4$
19	smooth		$1 - 1402T + 18684pT^2 - 1402p^3T^3 + p^6T^4$
20	smooth		$1 - 912T + 10942pT^2 - 912p^3T^3 + p^6T^4$
21	smooth		$1 - 380T + 13574pT^2 - 380p^3T^3 + p^6T^4$
22	smooth		$1 + 922T + 3914pT^2 + 922p^3T^3 + p^6T^4$
23	smooth		$1 + 439T + 6966pT^2 + 439p^3T^3 + p^6T^4$
24	smooth		$1 + 68T + 6322pT^2 + 68p^3T^3 + p^6T^4$
25	smooth		$1 - 478T + 4306pT^2 - 478p^3T^3 + p^6T^4$
26	smooth		$1 + 362T + 12062pT^2 + 362p^3T^3 + p^6T^4$
27	smooth		$1 + 110T - 76pT^2 + 110p^3T^3 + p^6T^4$
28	smooth		$1 + 432T - 6236pT^2 + 432p^3T^3 + p^6T^4$
29	smooth		$1 + 530T - 3646pT^2 + 530p^3T^3 + p^6T^4$
30	smooth		$1 + 1230T + 9542pT^2 + 1230p^3T^3 + p^6T^4$
31	smooth		$1 + 208T + 4950pT^2 + 208p^3T^3 + p^6T^4$
32	smooth		$1 - 268T - 4738pT^2 - 268p^3T^3 + p^6T^4$
33	smooth		$1 + 327T + 3998pT^2 + 327p^3T^3 + p^6T^4$

Continued on the following page

$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 - 485T + 5552pT^2 - 485p^3T^3 + p^6T^4$
35	smooth		$1 - 58T - 9190pT^2 - 58p^3T^3 + p^6T^4$
36	smooth		$1 + 12T - 3674pT^2 + 12p^3T^3 + p^6T^4$
37	smooth		$1 + 1412T + 15170pT^2 + 1412p^3T^3 + p^6T^4$
38	smooth		$1 - 800T + 1912pT^2 - 800p^3T^3 + p^6T^4$
39	smooth		$1 - 212T - 7510pT^2 - 212p^3T^3 + p^6T^4$
40	smooth		$1 + 572T - 2106pT^2 + 572p^3T^3 + p^6T^4$
41	smooth		$1 - 1549T + 16318pT^2 - 1549p^3T^3 + p^6T^4$
42	smooth		$1 - 86T - 4542pT^2 - 86p^3T^3 + p^6T^4$
43	smooth		$1 - 569T + 246pT^2 - 569p^3T^3 + p^6T^4$
44	smooth		$1 + 922T + 13602pT^2 + 922p^3T^3 + p^6T^4$
45	smooth		$1 - 1283T + 11824pT^2 - 1283p^3T^3 + p^6T^4$
46	smooth		$1 - 142T + 4908pT^2 - 142p^3T^3 + p^6T^4$
47	smooth		$1 + 866T + 3522pT^2 + 866p^3T^3 + p^6T^4$
48	smooth		$1 + 1006T + 6504pT^2 + 1006p^3T^3 + p^6T^4$
49	smooth		$1 + 47T + 12244pT^2 + 47p^3T^3 + p^6T^4$
50	smooth		$1 + 229T + 2416pT^2 + 229p^3T^3 + p^6T^4$
51	smooth		$1 - 674T + 6266pT^2 - 674p^3T^3 + p^6T^4$
52	smooth		$1 + 1244T + 18698pT^2 + 1244p^3T^3 + p^6T^4$
53	smooth		$1 - 44T + 3158pT^2 - 44p^3T^3 + p^6T^4$
54	smooth		$1 + 1342T + 18586pT^2 + 1342p^3T^3 + p^6T^4$
55	smooth		$1 - 1542T + 20154pT^2 - 1542p^3T^3 + p^6T^4$
56	smooth		$1 + 1272T + 14288pT^2 + 1272p^3T^3 + p^6T^4$
57	smooth		$1 + 348T + 2738pT^2 + 348p^3T^3 + p^6T^4$
58	smooth		$1 + 1020T + 12818pT^2 + 1020p^3T^3 + p^6T^4$
59	smooth		$1 - 1584T + 18782pT^2 - 1584p^3T^3 + p^6T^4$
60	smooth		$1 - 142T + 9892pT^2 - 142p^3T^3 + p^6T^4$
61	smooth		$1 + 551T + 11334pT^2 + 551p^3T^3 + p^6T^4$
62	smooth		$1 + 1405T + 15590pT^2 + 1405p^3T^3 + p^6T^4$
63	smooth		$1 - 982T + 15632pT^2 - 982p^3T^3 + p^6T^4$
64	smooth		$1 - 44T + 12230pT^2 - 44p^3T^3 + p^6T^4$
65	smooth		$1 - 408T + 13938pT^2 - 408p^3T^3 + p^6T^4$
66	smooth		$1 - 1024T + 15044pT^2 - 1024p^3T^3 + p^6T^4$
67	smooth		$1 - 135T - 4024pT^2 - 135p^3T^3 + p^6T^4$
68	smooth		$1 - 450T + 9402pT^2 - 450p^3T^3 + p^6T^4$
69	smooth		$1 + 607T + 9080pT^2 + 607p^3T^3 + p^6T^4$
70	smooth		$1 + 705T + 7722pT^2 + 705p^3T^3 + p^6T^4$
71	smooth		$1 - 604T + 694pT^2 - 604p^3T^3 + p^6T^4$
72	smooth		$1 - 198T + 6658pT^2 - 198p^3T^3 + p^6T^4$
73	smooth		$1 - 1311T + 15240pT^2 - 1311p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	smooth		$1 + 1076T + 14680pT^2 + 1076p^3T^3 + p^6T^4$
75	smooth		$1 + 26T + 136p^2T^2 + 26p^3T^3 + p^6T^4$
76	smooth		$1 - 527T - 174pT^2 - 527p^3T^3 + p^6T^4$
77	smooth		$1 - 170T + 12356pT^2 - 170p^3T^3 + p^6T^4$
78	smooth		$1 + 439T + 7092pT^2 + 439p^3T^3 + p^6T^4$
79	smooth		$1 + 572T + 7974pT^2 + 572p^3T^3 + p^6T^4$
80	smooth		$1 + 719T + 7204pT^2 + 719p^3T^3 + p^6T^4$
81	smooth		$1 - 331T + 7792pT^2 - 331p^3T^3 + p^6T^4$
82	smooth		$1 + 5T - 7034pT^2 + 5p^3T^3 + p^6T^4$
83	smooth		$1 - 247T + 3270pT^2 - 247p^3T^3 + p^6T^4$
84	smooth		$1 - 114T - 10198pT^2 - 114p^3T^3 + p^6T^4$
85	smooth		$1 + 1132T + 13686pT^2 + 1132p^3T^3 + p^6T^4$
86	smooth		$1 + 1307T + 9066pT^2 + 1307p^3T^3 + p^6T^4$
87	smooth		$1 + 726T + 1338pT^2 + 726p^3T^3 + p^6T^4$
88	smooth		$1 + 2028T + 25670pT^2 + 2028p^3T^3 + p^6T^4$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 21T - 9896pT^2 + 21p^3T^3 + p^6T^4$
2	singular	2	$(1 - pT)(1 - 854T + p^3T^2)$
3	smooth*	3	
4	smooth		$1 - 1232T + 9214pT^2 - 1232p^3T^3 + p^6T^4$
5	smooth		$1 + 1218T + 10978pT^2 + 1218p^3T^3 + p^6T^4$
6	smooth		$1 + 413T + 9704pT^2 + 413p^3T^3 + p^6T^4$
7	smooth		$1 + 273T + 13526pT^2 + 273p^3T^3 + p^6T^4$
8	smooth		$1 - 945T + 11664pT^2 - 945p^3T^3 + p^6T^4$
9	smooth		$1 + 308T + 14114pT^2 + 308p^3T^3 + p^6T^4$
10	smooth		$1 + 1218T + 21170pT^2 + 1218p^3T^3 + p^6T^4$
11	smooth		$1 + 98T + 14506pT^2 + 98p^3T^3 + p^6T^4$
12	smooth		$1 + 1029T + 11860pT^2 + 1029p^3T^3 + p^6T^4$
13	smooth		$1 + 1365T + 11370pT^2 + 1365p^3T^3 + p^6T^4$
14	smooth		$1 - 189T - 390pT^2 - 189p^3T^3 + p^6T^4$
15	smooth		$1 + 301T + 3334pT^2 + 301p^3T^3 + p^6T^4$
16	smooth		$1 - 539T + 5784pT^2 - 539p^3T^3 + p^6T^4$
17	smooth		$(1 + 7pT + p^3T^2)(1 - 1232T + p^3T^2)$
18	smooth		$1 - 1064T + 10390pT^2 - 1064p^3T^3 + p^6T^4$
19	singular	19	$(1 - pT)(1 + 546T + p^3T^2)$
20	smooth		$1 + 112T + 14506pT^2 + 112p^3T^3 + p^6T^4$
21	smooth		$1 + 434T - 2350pT^2 + 434p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 - 1820T + 23522pT^2 - 1820p^3T^3 + p^6T^4$
23	smooth		$1 + 301T + 10194pT^2 + 301p^3T^3 + p^6T^4$
24	smooth		$1 - 672T - 2154pT^2 - 672p^3T^3 + p^6T^4$
25	smooth		$1 - 182T + 6274pT^2 - 182p^3T^3 + p^6T^4$
26	smooth		$1 - 266T + 14310pT^2 - 266p^3T^3 + p^6T^4$
27	smooth		$1 + 455T - 4800pT^2 + 455p^3T^3 + p^6T^4$
28	smooth		$1 + 392T + 1178pT^2 + 392p^3T^3 + p^6T^4$
29	smooth		$1 + 1085T + 7842pT^2 + 1085p^3T^3 + p^6T^4$
30	smooth		$1 + 1400T + 14212pT^2 + 1400p^3T^3 + p^6T^4$
31	smooth		$1 + 28T - 1370pT^2 + 28p^3T^3 + p^6T^4$
32	smooth		$1 - 322T - 3134pT^2 - 322p^3T^3 + p^6T^4$
33	smooth		$1 - 196T + 2354pT^2 - 196p^3T^3 + p^6T^4$
34	smooth		$1 - 665T + 14506pT^2 - 665p^3T^3 + p^6T^4$
35	smooth		$1 + 322T + 14898pT^2 + 322p^3T^3 + p^6T^4$
36	smooth		$1 + 1302T + 18426pT^2 + 1302p^3T^3 + p^6T^4$
37	smooth		$1 - 1582T + 18916pT^2 - 1582p^3T^3 + p^6T^4$
38	smooth		$1 + 686T + 7940pT^2 + 686p^3T^3 + p^6T^4$
39	smooth		$1 - 1617T + 16270pT^2 - 1617p^3T^3 + p^6T^4$
40	smooth		$1 - 7152pT^2 + p^6T^4$
41	smooth		$1 - 294T + 14506pT^2 - 294p^3T^3 + p^6T^4$
42	smooth		$1 - 1449T + 21562pT^2 - 1449p^3T^3 + p^6T^4$
43	smooth		$1 + 42T - 3134pT^2 + 42p^3T^3 + p^6T^4$
44	smooth		$1 - 1001T + 7548pT^2 - 1001p^3T^3 + p^6T^4$
45	smooth		$1 - 413T + 6470pT^2 - 413p^3T^3 + p^6T^4$
46	smooth		$1 - 770T + 3530pT^2 - 770p^3T^3 + p^6T^4$
47	smooth		$1 + 413T + 13624pT^2 + 413p^3T^3 + p^6T^4$
48	smooth		$1 - 854T + 5098pT^2 - 854p^3T^3 + p^6T^4$
49	smooth		$1 - 1330T + 13722pT^2 - 1330p^3T^3 + p^6T^4$
50	smooth		$1 + 847T + 10684pT^2 + 847p^3T^3 + p^6T^4$
51	smooth		$1 + 1918T + 26364pT^2 + 1918p^3T^3 + p^6T^4$
52	smooth		$1 - 917T + 7058pT^2 - 917p^3T^3 + p^6T^4$
53	smooth		$1 + 1974T + 22346pT^2 + 1974p^3T^3 + p^6T^4$
54	smooth		$1 + 2072T + 24894pT^2 + 2072p^3T^3 + p^6T^4$
55	smooth		$1 + 1260T + 19504pT^2 + 1260p^3T^3 + p^6T^4$
56	smooth		$1 + 126T - 9896pT^2 + 126p^3T^3 + p^6T^4$
57	smooth		$1 + 658T + 3040pT^2 + 658p^3T^3 + p^6T^4$
58	smooth		$1 + 812T + 17446pT^2 + 812p^3T^3 + p^6T^4$
59	smooth		$1 + 294T + 1962pT^2 + 294p^3T^3 + p^6T^4$
60	smooth		$1 - 532T + 4020pT^2 - 532p^3T^3 + p^6T^4$
61	smooth		$1 - 238T + 10194pT^2 - 238p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 - 658T + 12546pT^2 - 658p^3T^3 + p^6T^4$
63	smooth		$1 + 574T + 14506pT^2 + 574p^3T^3 + p^6T^4$
64	smooth		$1 + 252T + 786pT^2 + 252p^3T^3 + p^6T^4$
65	smooth		$1 - 700T + 7254pT^2 - 700p^3T^3 + p^6T^4$
66	smooth		$1 - 196T + 9606pT^2 - 196p^3T^3 + p^6T^4$
67	smooth		$1 - 182T + 1276pT^2 - 182p^3T^3 + p^6T^4$
68	smooth		$1 - 511T + 5294pT^2 - 511p^3T^3 + p^6T^4$
69	smooth		$1 + 2016T + 28226pT^2 + 2016p^3T^3 + p^6T^4$
70	smooth		$1 + 532T + 16074pT^2 + 532p^3T^3 + p^6T^4$
71	smooth		$1 - 371T + 9018pT^2 - 371p^3T^3 + p^6T^4$
72	smooth		$1 + 77T + 11860pT^2 + 77p^3T^3 + p^6T^4$
73	smooth		$1 + 1190T + 19602pT^2 + 1190p^3T^3 + p^6T^4$
74	singular	74	$(1 - pT)(1 + 798T + p^3T^2)$
75	smooth		$1 + 1442T + 17250pT^2 + 1442p^3T^3 + p^6T^4$
76	smooth		$1 + 441T + 4118pT^2 + 441p^3T^3 + p^6T^4$
77	smooth		$1 - 364T + 7254pT^2 - 364p^3T^3 + p^6T^4$
78	smooth		$1 - 945T + 19210pT^2 - 945p^3T^3 + p^6T^4$
79	smooth		$1 + 714T + 3922pT^2 + 714p^3T^3 + p^6T^4$
80	smooth		$1 - 644T + 10880pT^2 - 644p^3T^3 + p^6T^4$
81	smooth		$1 + 28T - 2154pT^2 + 28p^3T^3 + p^6T^4$
82	smooth		$1 + 231T - 2546pT^2 + 231p^3T^3 + p^6T^4$
83	smooth		$1 - 231T + 12350pT^2 - 231p^3T^3 + p^6T^4$
84	smooth		$1 + 399T - 3330pT^2 + 399p^3T^3 + p^6T^4$
85	smooth		$1 - 406T + 1570pT^2 - 406p^3T^3 + p^6T^4$
86	smooth		$1 + 329T - 1664pT^2 + 329p^3T^3 + p^6T^4$
87	smooth		$1 + 1323T + 15094pT^2 + 1323p^3T^3 + p^6T^4$
88	smooth		$1 - 203T - 8328pT^2 - 203p^3T^3 + p^6T^4$
89	smooth		$1 - 2247T + 28128pT^2 - 2247p^3T^3 + p^6T^4$
90	smooth		$1 + 1288T + 22640pT^2 + 1288p^3T^3 + p^6T^4$
91	smooth		$1 + 1624T + 17838pT^2 + 1624p^3T^3 + p^6T^4$
92	smooth		$1 + 532T + 2942pT^2 + 532p^3T^3 + p^6T^4$
93	smooth		$1 - 651T - 4408pT^2 - 651p^3T^3 + p^6T^4$
94	smooth		$1 + 1491T + 21268pT^2 + 1491p^3T^3 + p^6T^4$
95	smooth		$1 + 217T - 1076pT^2 + 217p^3T^3 + p^6T^4$
96	smooth		$1 - 476T - 586pT^2 - 476p^3T^3 + p^6T^4$

C.5. The ζ -function for the manifold with $(h^{1,1}, h^{2,1}) = (4, 1)$

$p = 5$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 12T + p^3T^2)$
2	singular	$\frac{1}{3}$	$(1 - pT)(1 + 18T + p^3T^2)$
3	singular	$\frac{1}{12}$	$(1 - pT)(1 + 12T + p^3T^2)$
4	singular	$\{-\frac{1}{6}, \frac{1}{4}, \frac{3}{2}\}$	

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 28T + p^3T^2)$
2	singular	$\frac{1}{4}$	$(1 - pT)(1 + 4T + p^3T^2)$
3	singular	$\frac{1}{12}$	$(1 - pT)(1 + 22T + p^3T^2)$
4	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 28T + p^3T^2)$
5	singular	$\{-\frac{1}{4}, \frac{1}{3}, \frac{3}{2}\}$	
6	smooth		$(1 + 4pT + p^3T^2)(1 - 2pT + p^3T^2)$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$\frac{1}{12}$	$(1 - pT)(1 + 48T + p^3T^2)$
2	singular	$-\frac{1}{5}$	$(1 + pT)(1 + 24T + p^3T^2)$
3	singular	$\frac{1}{4}$	$(1 + pT)(1 - 12T + p^3T^2)$
4	singular	$\frac{1}{3}$	$(1 - pT)(1 + 36T + p^3T^2)$
5	smooth		$1 + 6pT + 314pT^2 + 6p^4T^3 + p^6T^4$
6	smooth		$1 + 170pT^2 + p^6T^4$
7	smooth*	$\frac{3}{2}$	
8	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 48T + p^3T^2)$
9	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 24T + p^3T^2)$
10	smooth		$1 + 24T - 46pT^2 + 24p^3T^3 + p^6T^4$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 8T + 126pT^2 + 8p^3T^3 + p^6T^4$
2	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 70T + p^3T^2)$
3	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 56T + p^3T^2)$
4	smooth		$1 + 32T + 6pT^2 + 32p^3T^3 + p^6T^4$
5	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 58T + p^3T^2)$
6	smooth		$1 + 2pT + 90pT^2 + 2p^4T^3 + p^6T^4$

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$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 - 10T - 54pT^2 - 10p^3T^3 + p^6T^4$
8	smooth*	$\frac{3}{2}$	
9	singular	$\frac{1}{3}$	$(1 - pT)(1 + 34T + p^3T^2)$
10	singular	$\frac{1}{4}$	$(1 - pT)(1 + 58T + p^3T^2)$
11	smooth		$1 - 46T + 90pT^2 - 46p^3T^3 + p^6T^4$
12	singular	$\frac{1}{12}$	$(1 - pT)(1 - 2T + p^3T^2)$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 90T + 290pT^2 - 90p^3T^3 + p^6T^4$
2	smooth		$1 + 36T + 398pT^2 + 36p^3T^3 + p^6T^4$
3	smooth		$1 + 156T + 758pT^2 + 156p^3T^3 + p^6T^4$
4	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 114T + p^3T^2)$
5	smooth		$1 - 6T - 214pT^2 - 6p^3T^3 + p^6T^4$
6	singular	$\frac{1}{3}$	$(1 - pT)(1 - 42T + p^3T^2)$
7	smooth		$(1 - 6pT + p^3T^2)(1 + 90T + p^3T^2)$
8	smooth		$1 - 250pT^2 + p^6T^4$
9	smooth		$1 - 60T + 182pT^2 - 60p^3T^3 + p^6T^4$
10	singular	$\{-\frac{1}{5}, \frac{1}{12}, \frac{3}{2}\}$	
11	smooth		$(1 + 6pT + p^3T^2)(1 + 36T + p^3T^2)$
12	smooth		$1 + 54T + 146pT^2 + 54p^3T^3 + p^6T^4$
13	singular	$\frac{1}{4}$	$(1 + pT)(1 - 66T + p^3T^2)$
14	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 102T + p^3T^2)$
15	smooth		$1 + 60T + 182pT^2 + 60p^3T^3 + p^6T^4$
16	smooth		$(1 + 6pT + p^3T^2)(1 - 6T + p^3T^2)$

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 16T + 498pT^2 - 16p^3T^3 + p^6T^4$
2	smooth		$1 + 194T + 1122pT^2 + 194p^3T^3 + p^6T^4$
3	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 20T + p^3T^2)$
4	smooth		$1 + 32T + 474pT^2 + 32p^3T^3 + p^6T^4$
5	singular	$\frac{1}{4}$	$(1 - pT)(1 + 100T + p^3T^2)$
6	smooth		$1 - 40T + 402pT^2 - 40p^3T^3 + p^6T^4$
7	smooth		$1 + 158T + 762pT^2 + 158p^3T^3 + p^6T^4$
8	singular	$\frac{1}{12}$	$(1 - pT)(1 - 20T + p^3T^2)$
9	smooth		$1 - 64T + 450pT^2 - 64p^3T^3 + p^6T^4$
10	smooth		$1 - 34T - 150pT^2 - 34p^3T^3 + p^6T^4$

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$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth*	$\frac{3}{2}$	
12	smooth		$1 - 46T + 666pT^2 - 46p^3T^3 + p^6T^4$
13	singular	$\frac{1}{3}$	$(1 - pT)(1 + 124T + p^3T^2)$
14	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 2T + p^3T^2)$
15	singular	$-\frac{1}{5}$	$(1 - pT)(1 - 116T + p^3T^2)$
16	smooth		$1 - 28T + 594pT^2 - 28p^3T^3 + p^6T^4$
17	smooth		$1 + 98T + 666pT^2 + 98p^3T^3 + p^6T^4$
18	smooth		$1 + 32T - 30pT^2 + 32p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 12T + 626pT^2 - 12p^3T^3 + p^6T^4$
2	singular	$\frac{1}{12}$	$(1 - pT)(1 + 54T + p^3T^2)$
3	smooth		$1 + 126T + 698pT^2 + 126p^3T^3 + p^6T^4$
4	smooth		$1 + 6T - 382pT^2 + 6p^3T^3 + p^6T^4$
5	smooth		$1 + 48T + 770pT^2 + 48p^3T^3 + p^6T^4$
6	singular	$\frac{1}{4}$	$(1 + pT)(1 - 132T + p^3T^2)$
7	smooth		$1 - 48T + 914pT^2 - 48p^3T^3 + p^6T^4$
8	singular	$\frac{1}{3}$	$(1 - pT)(1 + p^3T^2)$
9	singular	$-\frac{1}{5}$	$(1 + pT)(1 + 60T + p^3T^2)$
10	smooth		$1 - 120T + 482pT^2 - 120p^3T^3 + p^6T^4$
11	smooth		$1 + 96T + 482pT^2 + 96p^3T^3 + p^6T^4$
12	smooth		$1 + 914pT^2 + p^6T^4$
13	smooth*	$\frac{3}{2}$	
14	smooth		$1 + 102T + 626pT^2 + 102p^3T^3 + p^6T^4$
15	smooth		$1 - 42T + 194pT^2 - 42p^3T^3 + p^6T^4$
16	smooth		$1 - 162T + 842pT^2 - 162p^3T^3 + p^6T^4$
17	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 120T + p^3T^2)$
18	smooth		$1 + 60T + 50pT^2 + 60p^3T^3 + p^6T^4$
19	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 72T + p^3T^2)$
20	smooth		$(1 + p^3T^2)(1 + 120T + p^3T^2)$
21	smooth		$1 + 96T + 482pT^2 + 96p^3T^3 + p^6T^4$
22	smooth		$(1 + 6pT + p^3T^2)(1 - 36T + p^3T^2)$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 222T + 1034pT^2 - 222p^3T^3 + p^6T^4$
2	smooth		$1 + 258T + 1250pT^2 + 258p^3T^3 + p^6T^4$

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$p = 29$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 + 72T - 226pT^2 + 72p^3T^3 + p^6T^4$
4	smooth		$(1 + 6pT + p^3T^2)(1 - 78T + p^3T^2)$
5	smooth		$1 + 288T + 2006pT^2 + 288p^3T^3 + p^6T^4$
6	smooth		$1 + 180T + 710pT^2 + 180p^3T^3 + p^6T^4$
7	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 54T + p^3T^2)$
8	smooth		$1 + 60T + 422pT^2 + 60p^3T^3 + p^6T^4$
9	smooth		$1 + 36T + 278pT^2 + 36p^3T^3 + p^6T^4$
10	singular	$\frac{1}{3}$	$(1 - pT)(1 - 102T + p^3T^2)$
11	smooth		$(1 + 6pT + p^3T^2)(1 - 6pT + p^3T^2)$
12	smooth		$1 + 168T + 1214pT^2 + 168p^3T^3 + p^6T^4$
13	smooth		$1 + 192T + 494pT^2 + 192p^3T^3 + p^6T^4$
14	smooth		$1 + 108T + 998pT^2 + 108p^3T^3 + p^6T^4$
15	smooth		$1 - 168T + 782pT^2 - 168p^3T^3 + p^6T^4$
16	smooth*	$\frac{3}{2}$	
17	singular	$\frac{1}{12}$	$(1 - pT)(1 - 84T + p^3T^2)$
18	smooth		$1 - 132T + 998pT^2 - 132p^3T^3 + p^6T^4$
19	smooth		$1 + 926pT^2 + p^6T^4$
20	smooth		$1 + 90T + 386pT^2 + 90p^3T^3 + p^6T^4$
21	smooth		$1 + 138T + 530pT^2 + 138p^3T^3 + p^6T^4$
22	singular	$\frac{1}{4}$	$(1 + pT)(1 + 90T + p^3T^2)$
23	singular	$-\frac{1}{5}$	$(1 + pT)(1 - 30T + p^3T^2)$
24	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 306T + p^3T^2)$
25	smooth		$1 + 210T + 1466pT^2 + 210p^3T^3 + p^6T^4$
26	smooth		$1 - 108T - 154pT^2 - 108p^3T^3 + p^6T^4$
27	smooth		$1 - 156T + 278pT^2 - 156p^3T^3 + p^6T^4$
28	smooth		$(1 - 6pT + p^3T^2)(1 + 78T + p^3T^2)$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 20T - 942pT^2 + 20p^3T^3 + p^6T^4$
2	smooth		$1 + 128T + 642pT^2 + 128p^3T^3 + p^6T^4$
3	smooth		$1 - 64T + 594pT^2 - 64p^3T^3 + p^6T^4$
4	smooth		$1 + 74T + 1074pT^2 + 74p^3T^3 + p^6T^4$
5	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 136T + p^3T^2)$
6	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 172T + p^3T^2)$
7	smooth		$1 + 26T - 1206pT^2 + 26p^3T^3 + p^6T^4$
8	singular	$\frac{1}{4}$	$(1 - pT)(1 - 152T + p^3T^2)$
9	smooth		$1 - 40T + 834pT^2 - 40p^3T^3 + p^6T^4$
10	smooth		$1 + 128T - 222pT^2 + 128p^3T^3 + p^6T^4$

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$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 + 236T + 2082pT^2 + 236p^3T^3 + p^6T^4$
12	smooth		$1 + 20T - 1374pT^2 + 20p^3T^3 + p^6T^4$
13	singular	$\frac{1}{12}$	$(1 - pT)(1 - 62T + p^3T^2)$
14	smooth		$1 + 134T + 234pT^2 + 134p^3T^3 + p^6T^4$
15	smooth		$1 - 88T + 138pT^2 - 88p^3T^3 + p^6T^4$
16	smooth		$1 + 188T + 450pT^2 + 188p^3T^3 + p^6T^4$
17	smooth*	$\frac{3}{2}$	
18	smooth		$1 + 260T + 1170pT^2 + 260p^3T^3 + p^6T^4$
19	smooth		$1 - 292T + 1554pT^2 - 292p^3T^3 + p^6T^4$
20	smooth		$1 - 10T - 18p^2T^2 - 10p^3T^3 + p^6T^4$
21	singular	$\frac{1}{3}$	$(1 - pT)(1 + 160T + p^3T^2)$
22	smooth		$1 + 8T + 810pT^2 + 8p^3T^3 + p^6T^4$
23	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 236T + p^3T^2)$
24	smooth		$1 + 146T + 498pT^2 + 146p^3T^3 + p^6T^4$
25	smooth		$1 + 56T + 786pT^2 + 56p^3T^3 + p^6T^4$
26	smooth		$1 + 200T + 1362pT^2 + 200p^3T^3 + p^6T^4$
27	smooth		$1 + 260T + 1890pT^2 + 260p^3T^3 + p^6T^4$
28	smooth		$(1 + 4pT + p^3T^2)(1 - 224T + p^3T^2)$
29	smooth		$1 + 92T + 66pT^2 + 92p^3T^3 + p^6T^4$
30	smooth		$1 + 32T + 258pT^2 + 32p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 - 2pT + p^3T^2)(1 - 218T + p^3T^2)$
2	smooth		$1 + 26T - 1062pT^2 + 26p^3T^3 + p^6T^4$
3	smooth		$1 - 196T + 246pT^2 - 196p^3T^3 + p^6T^4$
4	smooth		$1 + 164T + 822pT^2 + 164p^3T^3 + p^6T^4$
5	smooth		$1 - 316T + 2358pT^2 - 316p^3T^3 + p^6T^4$
6	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 214T + p^3T^2)$
7	smooth		$1 - 52T - 1338pT^2 - 52p^3T^3 + p^6T^4$
8	smooth		$1 - 40T + 78pT^2 - 40p^3T^3 + p^6T^4$
9	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 146T + p^3T^2)$
10	smooth		$1 + 260T + 1854pT^2 + 260p^3T^3 + p^6T^4$
11	smooth		$1 - 52T + 2262pT^2 - 52p^3T^3 + p^6T^4$
12	smooth		$1 + 104T + 510pT^2 + 104p^3T^3 + p^6T^4$
13	smooth		$1 - 76T - 1002pT^2 - 76p^3T^3 + p^6T^4$
14	smooth		$(1 + 10pT + p^3T^2)(1 + 46T + p^3T^2)$
15	smooth		$(1 - 2pT + p^3T^2)(1 + 178T + p^3T^2)$
16	smooth		$1 - 124T - 186pT^2 - 124p^3T^3 + p^6T^4$

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$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 + 14T - 678pT^2 + 14p^3T^3 + p^6T^4$
18	smooth		$1 - 10T + 1962pT^2 - 10p^3T^3 + p^6T^4$
19	smooth		$1 + 104T - 210pT^2 + 104p^3T^3 + p^6T^4$
20	smooth*	$\frac{3}{2}$	
21	smooth		$1 - 196T + 1254pT^2 - 196p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 58T + p^3T^2)$
23	smooth		$1 + 320T + 1374pT^2 + 320p^3T^3 + p^6T^4$
24	smooth		$1 - 28T + 558pT^2 - 28p^3T^3 + p^6T^4$
25	singular	$\frac{1}{3}$	$(1 - pT)(1 - 398T + p^3T^2)$
26	smooth		$1 - 22T - 1902pT^2 - 22p^3T^3 + p^6T^4$
27	smooth		$1 + 284T + 1734pT^2 + 284p^3T^3 + p^6T^4$
28	singular	$\frac{1}{4}$	$(1 - pT)(1 + 34T + p^3T^2)$
29	smooth		$1 + 146T + 786pT^2 + 146p^3T^3 + p^6T^4$
30	smooth		$1 + 2pT + 1362pT^2 + 2p^4T^3 + p^6T^4$
31	smooth		$(1 + 10pT + p^3T^2)(1 - 362T + p^3T^2)$
32	smooth		$1 + 110T + 1290pT^2 + 110p^3T^3 + p^6T^4$
33	smooth		$1 + 404T + 3294pT^2 + 404p^3T^3 + p^6T^4$
34	singular	$\frac{1}{12}$	$(1 - pT)(1 - 44T + p^3T^2)$
35	smooth		$1 + 176T - 354pT^2 + 176p^3T^3 + p^6T^4$
36	smooth		$(1 - 2pT + p^3T^2)(1 + 394T + p^3T^2)$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 78T + 842pT^2 + 78p^3T^3 + p^6T^4$
2	smooth		$1 + 354T + 2138pT^2 + 354p^3T^3 + p^6T^4$
3	smooth		$1 + 72T + 14pT^2 + 72p^3T^3 + p^6T^4$
4	smooth		$1 - 330T + 2786pT^2 - 330p^3T^3 + p^6T^4$
5	smooth		$1 - 468T + 2966pT^2 - 468p^3T^3 + p^6T^4$
6	smooth		$1 + 138T - 94pT^2 + 138p^3T^3 + p^6T^4$
7	smooth		$1 - 48T + 1166pT^2 - 48p^3T^3 + p^6T^4$
8	singular	$-\frac{1}{5}$	$(1 + pT)(1 + 342T + p^3T^2)$
9	smooth		$1 + 150T + 194pT^2 + 150p^3T^3 + p^6T^4$
10	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 126T + p^3T^2)$
11	smooth		$1 - 558T + 4298pT^2 - 558p^3T^3 + p^6T^4$
12	smooth		$1 - 192T + 1166pT^2 - 192p^3T^3 + p^6T^4$
13	smooth		$1 + 174T + 1922pT^2 + 174p^3T^3 + p^6T^4$
14	singular	$\frac{1}{3}$	$(1 - pT)(1 + 318T + p^3T^2)$
15	smooth		$1 - 108T - 634pT^2 - 108p^3T^3 + p^6T^4$
16	smooth		$1 + 498T + 3722pT^2 + 498p^3T^3 + p^6T^4$

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$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	smooth		$1 - 420T + 3830pT^2 - 420p^3T^3 + p^6T^4$
18	smooth		$1 - 6pT + 3146pT^2 - 6p^4T^3 + p^6T^4$
19	smooth		$1 + 96T - 418pT^2 + 96p^3T^3 + p^6T^4$
20	smooth		$1 - 258T + 626pT^2 - 258p^3T^3 + p^6T^4$
21	smooth		$1 - 36T + 1094pT^2 - 36p^3T^3 + p^6T^4$
22	smooth*	$\frac{3}{2}$	
23	smooth		$1 + 168T + 2750pT^2 + 168p^3T^3 + p^6T^4$
24	singular	$\frac{1}{12}$	$(1 - pT)(1 + 138T + p^3T^2)$
25	smooth		$1 + 486T + 3506pT^2 + 486p^3T^3 + p^6T^4$
26	smooth		$1 - 240T + 1814pT^2 - 240p^3T^3 + p^6T^4$
27	smooth		$1 + 462T + 4442pT^2 + 462p^3T^3 + p^6T^4$
28	smooth		$1 + 564T + 4550pT^2 + 564p^3T^3 + p^6T^4$
29	smooth		$1 - 276T + 2822pT^2 - 276p^3T^3 + p^6T^4$
30	smooth		$1 + 372T + 3542pT^2 + 372p^3T^3 + p^6T^4$
31	singular	$\frac{1}{4}$	$(1 + pT)(1 + 438T + p^3T^2)$
32	smooth		$1 - 396T + 2246pT^2 - 396p^3T^3 + p^6T^4$
33	smooth		$1 - 180T + 2390pT^2 - 180p^3T^3 + p^6T^4$
34	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 150T + p^3T^2)$
35	smooth		$1 + 240T + 2174pT^2 + 240p^3T^3 + p^6T^4$
36	smooth		$1 + 1742pT^2 + p^6T^4$
37	smooth		$1 + 156T + 2894pT^2 + 156p^3T^3 + p^6T^4$
38	smooth		$1 - 120T + 734pT^2 - 120p^3T^3 + p^6T^4$
39	smooth		$1 - 84T + 806pT^2 - 84p^3T^3 + p^6T^4$
40	smooth		$1 + 312T + 878pT^2 + 312p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 + 4pT + p^3T^2)(1 - 152T + p^3T^2)$
2	smooth		$1 + 188T - 990pT^2 + 188p^3T^3 + p^6T^4$
3	smooth		$1 - 268T + 3018pT^2 - 268p^3T^3 + p^6T^4$
4	smooth		$1 + 200T + 2370pT^2 + 200p^3T^3 + p^6T^4$
5	smooth		$1 - 256T + 834pT^2 - 256p^3T^3 + p^6T^4$
6	smooth		$(1 - 8pT + p^3T^2)(1 + 484T + p^3T^2)$
7	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 292T + p^3T^2)$
8	smooth		$1 + 110T + 1146pT^2 + 110p^3T^3 + p^6T^4$
9	smooth		$1 - 16T - 1662pT^2 - 16p^3T^3 + p^6T^4$
10	smooth		$1 + 74T + 714pT^2 + 74p^3T^3 + p^6T^4$
11	singular	$\frac{1}{4}$	$(1 - pT)(1 - 32T + p^3T^2)$
12	smooth		$1 + 572T + 4434pT^2 + 572p^3T^3 + p^6T^4$

Continued on the following page

$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
13	smooth		$1 - 190T - 198pT^2 - 190p^3T^3 + p^6T^4$
14	smooth		$1 + 284T + 186pT^2 + 284p^3T^3 + p^6T^4$
15	smooth		$1 + 92T - 798pT^2 + 92p^3T^3 + p^6T^4$
16	smooth		$1 - 646T + 5682pT^2 - 646p^3T^3 + p^6T^4$
17	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 148T + p^3T^2)$
18	singular	$\frac{1}{12}$	$(1 - pT)(1 - 428T + p^3T^2)$
19	smooth		$1 - 304T + 3810pT^2 - 304p^3T^3 + p^6T^4$
20	smooth		$1 + 110T + 1434pT^2 + 110p^3T^3 + p^6T^4$
21	smooth		$1 + 764T + 6642pT^2 + 764p^3T^3 + p^6T^4$
22	smooth		$1 + 128T + 1002pT^2 + 128p^3T^3 + p^6T^4$
23	smooth*	$\frac{3}{2}$	
24	smooth		$1 + 104T + 6p^2T^2 + 104p^3T^3 + p^6T^4$
25	smooth		$1 - 76T - 606pT^2 - 76p^3T^3 + p^6T^4$
26	smooth		$1 + 38T - 2166pT^2 + 38p^3T^3 + p^6T^4$
27	smooth		$1 + 182T + 138pT^2 + 182p^3T^3 + p^6T^4$
28	smooth		$1 + 368T + 2754pT^2 + 368p^3T^3 + p^6T^4$
29	singular	$\frac{1}{3}$	$(1 - pT)(1 + 268T + p^3T^2)$
30	smooth		$1 + 176T - 606pT^2 + 176p^3T^3 + p^6T^4$
31	smooth		$1 + 284T + 3210pT^2 + 284p^3T^3 + p^6T^4$
32	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 376T + p^3T^2)$
33	smooth		$(1 - 8pT + p^3T^2)(1 + 388T + p^3T^2)$
34	smooth		$1 + 536T + 3858pT^2 + 536p^3T^3 + p^6T^4$
35	smooth		$1 - 544T + 4866pT^2 - 544p^3T^3 + p^6T^4$
36	smooth		$1 - 112T - 750pT^2 - 112p^3T^3 + p^6T^4$
37	smooth		$1 + 8T - 1134pT^2 + 8p^3T^3 + p^6T^4$
38	smooth		$1 + 302T + 3282pT^2 + 302p^3T^3 + p^6T^4$
39	smooth		$1 + 8T + 1026pT^2 + 8p^3T^3 + p^6T^4$
40	smooth		$(1 - 2pT + p^3T^2)(1 + 220T + p^3T^2)$
41	smooth		$1 - 556T + 4530pT^2 - 556p^3T^3 + p^6T^4$
42	smooth		$1 - 58T + 2922pT^2 - 58p^3T^3 + p^6T^4$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 252T - 766pT^2 + 252p^3T^3 + p^6T^4$
2	smooth		$1 - 336T + 3122pT^2 - 336p^3T^3 + p^6T^4$
3	smooth		$1 + 582T + 4346pT^2 + 582p^3T^3 + p^6T^4$
4	singular	$\frac{1}{12}$	$(1 - pT)(1 + 516T + p^3T^2)$
5	smooth		$1 + 480T + 3842pT^2 + 480p^3T^3 + p^6T^4$
6	smooth		$1 + 720T + 6650pT^2 + 720p^3T^3 + p^6T^4$

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$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 - 264T + 3842pT^2 - 264p^3T^3 + p^6T^4$
8	smooth		$1 - 414T + 3554pT^2 - 414p^3T^3 + p^6T^4$
9	smooth		$1 - 678T + 5714pT^2 - 678p^3T^3 + p^6T^4$
10	smooth		$1 + 96T + 1826pT^2 + 96p^3T^3 + p^6T^4$
11	smooth		$1 - 348T + 1970pT^2 - 348p^3T^3 + p^6T^4$
12	singular	$\frac{1}{4}$	$(1 + pT)(1 + 204T + p^3T^2)$
13	smooth		$1 - 270T + 2618pT^2 - 270p^3T^3 + p^6T^4$
14	smooth		$1 + 180T + 4130pT^2 + 180p^3T^3 + p^6T^4$
15	smooth		$1 + 378T + 2258pT^2 + 378p^3T^3 + p^6T^4$
16	singular	$\frac{1}{3}$	$(1 - pT)(1 - 240T + p^3T^2)$
17	smooth		$1 + 228T + 1322pT^2 + 228p^3T^3 + p^6T^4$
18	smooth		$1 + 144T + 1538pT^2 + 144p^3T^3 + p^6T^4$
19	smooth		$1 + 174T + 2474pT^2 + 174p^3T^3 + p^6T^4$
20	smooth		$1 + 510T + 2762pT^2 + 510p^3T^3 + p^6T^4$
21	smooth		$1 - 384T + 4706pT^2 - 384p^3T^3 + p^6T^4$
22	smooth		$1 - 180T + 3698pT^2 - 180p^3T^3 + p^6T^4$
23	smooth		$1 - 96T + 1250pT^2 - 96p^3T^3 + p^6T^4$
24	smooth		$1 + 264T + 3410pT^2 + 264p^3T^3 + p^6T^4$
25	smooth*	$\frac{3}{2}$	
26	smooth		$1 - 144T - 262pT^2 - 144p^3T^3 + p^6T^4$
27	smooth		$1 + 204T + 3554pT^2 + 204p^3T^3 + p^6T^4$
28	singular	$-\frac{1}{5}$	$(1 + pT)(1 - 288T + p^3T^2)$
29	smooth		$1 + 174T + 386pT^2 + 174p^3T^3 + p^6T^4$
30	smooth		$1 + 156T + 1826pT^2 + 156p^3T^3 + p^6T^4$
31	smooth		$1 + 528T + 3410pT^2 + 528p^3T^3 + p^6T^4$
32	smooth		$1 - 264T + 1682pT^2 - 264p^3T^3 + p^6T^4$
33	smooth		$1 - 114T + 2402pT^2 - 114p^3T^3 + p^6T^4$
34	smooth		$1 - 96T + 1466pT^2 - 96p^3T^3 + p^6T^4$
35	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 12T + p^3T^2)$
36	smooth		$1 + 156T + 2258pT^2 + 156p^3T^3 + p^6T^4$
37	smooth		$(1 + 12pT + p^3T^2)(1 + 228T + p^3T^2)$
38	smooth		$1 - 588T + 5426pT^2 - 588p^3T^3 + p^6T^4$
39	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 72T + p^3T^2)$
40	smooth		$1 + 108T + 2402pT^2 + 108p^3T^3 + p^6T^4$
41	smooth		$1 - 132T - 2494pT^2 - 132p^3T^3 + p^6T^4$
42	smooth		$1 + 6pT - 46pT^2 + 6p^4T^3 + p^6T^4$
43	smooth		$1 - 252T + 1826pT^2 - 252p^3T^3 + p^6T^4$
44	smooth		$1 - 156T + 98pT^2 - 156p^3T^3 + p^6T^4$
45	smooth		$1 + 132T - 50p^2T^2 + 132p^3T^3 + p^6T^4$
46	smooth		$1 + 18T + 2546pT^2 + 18p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 18T + 3890pT^2 - 18p^3T^3 + p^6T^4$
2	smooth		$1 + 6T + 4970pT^2 + 6p^3T^3 + p^6T^4$
3	smooth		$1 + 186T + 1946pT^2 + 186p^3T^3 + p^6T^4$
4	smooth		$1 - 168T - 466pT^2 - 168p^3T^3 + p^6T^4$
5	smooth		$1 + 396T + 4646pT^2 + 396p^3T^3 + p^6T^4$
6	smooth		$1 + 762T + 6986pT^2 + 762p^3T^3 + p^6T^4$
7	smooth		$1 - 618T + 5330pT^2 - 618p^3T^3 + p^6T^4$
8	smooth		$1 + 684T + 5366pT^2 + 684p^3T^3 + p^6T^4$
9	smooth		$1 - 90T + 1082pT^2 - 90p^3T^3 + p^6T^4$
10	smooth		$1 - 222T + 2882pT^2 - 222p^3T^3 + p^6T^4$
11	smooth		$1 + 5294pT^2 + p^6T^4$
12	smooth		$1 + 300T - 682pT^2 + 300p^3T^3 + p^6T^4$
13	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 174T + p^3T^2)$
14	smooth		$1 + 120T + 3566pT^2 + 120p^3T^3 + p^6T^4$
15	smooth		$1 + 84T - 2698pT^2 + 84p^3T^3 + p^6T^4$
16	smooth		$1 + 84T - 538pT^2 + 84p^3T^3 + p^6T^4$
17	smooth		$1 + 816T + 7022pT^2 + 816p^3T^3 + p^6T^4$
18	singular	$\frac{1}{3}$	$(1 - pT)(1 + 498T + p^3T^2)$
19	smooth		$1 - 96T - 4066pT^2 - 96p^3T^3 + p^6T^4$
20	smooth		$1 - 120T + 4718pT^2 - 120p^3T^3 + p^6T^4$
21	singular	$-\frac{1}{5}$	$(1 + pT)(1 - 318T + p^3T^2)$
22	smooth		$1 + 228T + 2774pT^2 + 228p^3T^3 + p^6T^4$
23	smooth		$1 + 288T + 70p^2T^2 + 288p^3T^3 + p^6T^4$
24	smooth		$1 - 348T + 2198pT^2 - 348p^3T^3 + p^6T^4$
25	smooth		$1 - 12T + 2486pT^2 - 12p^3T^3 + p^6T^4$
26	smooth		$1 + 24T + 1694pT^2 + 24p^3T^3 + p^6T^4$
27	smooth		$1 - 108T + 2054pT^2 - 108p^3T^3 + p^6T^4$
28	smooth*	$\frac{3}{2}$	
29	smooth		$1 + 324T + 4646pT^2 + 324p^3T^3 + p^6T^4$
30	smooth		$1 - 558T + 5258pT^2 - 558p^3T^3 + p^6T^4$
31	singular	$\frac{1}{12}$	$(1 - pT)(1 - 174T + p^3T^2)$
32	smooth		$1 + 552T + 2990pT^2 + 552p^3T^3 + p^6T^4$
33	smooth		$1 - 222T - 286pT^2 - 222p^3T^3 + p^6T^4$
34	smooth		$1 + 204T - 1834pT^2 + 204p^3T^3 + p^6T^4$
35	smooth		$1 - 156T + 2630pT^2 - 156p^3T^3 + p^6T^4$
36	smooth		$1 + 24T - 34pT^2 + 24p^3T^3 + p^6T^4$
37	smooth		$1 + 90T + 5114pT^2 + 90p^3T^3 + p^6T^4$
38	smooth		$1 - 138T + 1874pT^2 - 138p^3T^3 + p^6T^4$
39	smooth		$1 + 156T + 2414pT^2 + 156p^3T^3 + p^6T^4$
40	singular	$\frac{1}{4}$	$(1 + pT)(1 - 222T + p^3T^2)$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 - 96T + 3782pT^2 - 96p^3T^3 + p^6T^4$
42	smooth		$1 - 66T + 1370pT^2 - 66p^3T^3 + p^6T^4$
43	smooth		$1 - 222T + 1946pT^2 - 222p^3T^3 + p^6T^4$
44	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 414T + p^3T^2)$
45	smooth		$1 + 36T - 1834pT^2 + 36p^3T^3 + p^6T^4$
46	smooth		$1 + 804T + 6806pT^2 + 804p^3T^3 + p^6T^4$
47	smooth		$1 - 12T + 2054pT^2 - 12p^3T^3 + p^6T^4$
48	smooth		$1 + 4286pT^2 + p^6T^4$
49	smooth		$1 + 408T + 5294pT^2 + 408p^3T^3 + p^6T^4$
50	smooth		$1 + 42T + 5186pT^2 + 42p^3T^3 + p^6T^4$
51	smooth		$1 - 312T + 2846pT^2 - 312p^3T^3 + p^6T^4$
52	smooth		$1 + 48T - 898pT^2 + 48p^3T^3 + p^6T^4$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 792T + 6674pT^2 + 792p^3T^3 + p^6T^4$
2	smooth		$1 - 600T + 5954pT^2 - 600p^3T^3 + p^6T^4$
3	smooth		$1 - 636T + 5810pT^2 - 636p^3T^3 + p^6T^4$
4	smooth		$1 + 2498pT^2 + p^6T^4$
5	singular	$\frac{1}{12}$	$(1 - pT)(1 + 852T + p^3T^2)$
6	smooth		$1 + 444T + 50pT^2 + 444p^3T^3 + p^6T^4$
7	smooth		$1 - 180T + 4802pT^2 - 180p^3T^3 + p^6T^4$
8	smooth		$1 - 168T + 2210pT^2 - 168p^3T^3 + p^6T^4$
9	smooth		$1 + 192T - 1822pT^2 + 192p^3T^3 + p^6T^4$
10	smooth		$1 - 408T + 4802pT^2 - 408p^3T^3 + p^6T^4$
11	smooth		$1 - 264T + 4514pT^2 - 264p^3T^3 + p^6T^4$
12	smooth		$1 - 564T + 6602pT^2 - 564p^3T^3 + p^6T^4$
13	smooth		$1 + 288T + 3938pT^2 + 288p^3T^3 + p^6T^4$
14	smooth		$1 + 792T + 8546pT^2 + 792p^3T^3 + p^6T^4$
15	singular	$\frac{1}{4}$	$(1 + pT)(1 - 420T + p^3T^2)$
16	smooth		$1 + 504T + 6242pT^2 + 504p^3T^3 + p^6T^4$
17	smooth		$1 - 60T - 3982pT^2 - 60p^3T^3 + p^6T^4$
18	smooth		$1 - 144T - 238pT^2 - 144p^3T^3 + p^6T^4$
19	smooth		$1 + 288T - 1102pT^2 + 288p^3T^3 + p^6T^4$
20	singular	$\frac{1}{3}$	$(1 - pT)(1 + 132T + p^3T^2)$
21	smooth		$1 - 468T + 4154pT^2 - 468p^3T^3 + p^6T^4$
22	smooth		$1 - 528T + 4226pT^2 - 528p^3T^3 + p^6T^4$
23	smooth		$1 + 432T + 482pT^2 + 432p^3T^3 + p^6T^4$
24	smooth		$1 - 18T + 1994pT^2 - 18p^3T^3 + p^6T^4$

Continued on the following page

$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
25	smooth		$1 - 816T + 7682pT^2 - 816p^3T^3 + p^6T^4$
26	smooth		$1 - 726T + 5882pT^2 - 726p^3T^3 + p^6T^4$
27	smooth		$1 + 48T + 2498pT^2 + 48p^3T^3 + p^6T^4$
28	smooth		$1 - 870T + 9410pT^2 - 870p^3T^3 + p^6T^4$
29	smooth		$1 + 90T - 3262pT^2 + 90p^3T^3 + p^6T^4$
30	smooth		$1 - 306T + 6458pT^2 - 306p^3T^3 + p^6T^4$
31	smooth*	$\frac{3}{2}$	
32	smooth		$1 + 372T - 526pT^2 + 372p^3T^3 + p^6T^4$
33	smooth		$1 + 84T + 3506pT^2 + 84p^3T^3 + p^6T^4$
34	smooth		$1 + 1152T + 11642pT^2 + 1152p^3T^3 + p^6T^4$
35	smooth		$1 + 84T - 1390pT^2 + 84p^3T^3 + p^6T^4$
36	smooth		$1 - 444T + 5810pT^2 - 444p^3T^3 + p^6T^4$
37	smooth		$1 - 174T + 1850pT^2 - 174p^3T^3 + p^6T^4$
38	smooth		$(1 + 12pT + p^3T^2)(1 - 324T + p^3T^2)$
39	smooth		$1 + 132T + 5378pT^2 + 132p^3T^3 + p^6T^4$
40	smooth		$(1 + p^3T^2)(1 + 780T + p^3T^2)$
41	smooth		$1 + 444T + 2930pT^2 + 444p^3T^3 + p^6T^4$
42	smooth		$1 + 432T + 1346pT^2 + 432p^3T^3 + p^6T^4$
43	smooth		$1 - 126T - 2974pT^2 - 126p^3T^3 + p^6T^4$
44	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 138T + p^3T^2)$
45	smooth		$1 + 786T + 6314pT^2 + 786p^3T^3 + p^6T^4$
46	smooth		$1 - 60T - 2542pT^2 - 60p^3T^3 + p^6T^4$
47	singular	$-\frac{1}{5}$	$(1 + pT)(1 - 252T + p^3T^2)$
48	smooth		$1 + 138T + 3578pT^2 + 138p^3T^3 + p^6T^4$
49	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 744T + p^3T^2)$
50	smooth		$1 + 984T + 9122pT^2 + 984p^3T^3 + p^6T^4$
51	smooth		$1 + 492T + 4946pT^2 + 492p^3T^3 + p^6T^4$
52	smooth		$1 - 744T + 5522pT^2 - 744p^3T^3 + p^6T^4$
53	smooth		$1 + 978T + 10058pT^2 + 978p^3T^3 + p^6T^4$
54	smooth		$1 + 120T - 526pT^2 + 120p^3T^3 + p^6T^4$
55	smooth		$1 - 552T + 2210pT^2 - 552p^3T^3 + p^6T^4$
56	smooth		$1 - 258T + 1202pT^2 - 258p^3T^3 + p^6T^4$
57	smooth		$1 + 126T + 6818pT^2 + 126p^3T^3 + p^6T^4$
58	smooth		$1 + 300T + 4514pT^2 + 300p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 1192T + 11886pT^2 - 1192p^3T^3 + p^6T^4$
2	smooth		$1 + 80T - 2610pT^2 + 80p^3T^3 + p^6T^4$
<i>Continued on the following page</i>			

$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 - 382T + 3786pT^2 - 382p^3T^3 + p^6T^4$
4	smooth		$1 - 220T + 438pT^2 - 220p^3T^3 + p^6T^4$
5	smooth		$1 + 44T - 1386pT^2 + 44p^3T^3 + p^6T^4$
6	smooth		$1 + 188T + 4518pT^2 + 188p^3T^3 + p^6T^4$
7	smooth		$1 + 788T + 9726pT^2 + 788p^3T^3 + p^6T^4$
8	smooth		$(1 + 10pT + p^3T^2)(1 - 2T + p^3T^2)$
9	smooth		$1 + 626T + 6162pT^2 + 626p^3T^3 + p^6T^4$
10	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 418T + p^3T^2)$
11	smooth		$1 + 512T + 3150pT^2 + 512p^3T^3 + p^6T^4$
12	singular	$-\frac{1}{5}$	$(1 - pT)(1 - 110T + p^3T^2)$
13	smooth		$1 + 590T + 3354pT^2 + 590p^3T^3 + p^6T^4$
14	smooth		$1 + 1202T + 12930pT^2 + 1202p^3T^3 + p^6T^4$
15	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 380T + p^3T^2)$
16	smooth		$1 + 932T + 8502pT^2 + 932p^3T^3 + p^6T^4$
17	smooth		$1 - 40T + 78pT^2 - 40p^3T^3 + p^6T^4$
18	smooth		$1 - 340T + 6294pT^2 - 340p^3T^3 + p^6T^4$
19	smooth		$1 - 544T + 4974pT^2 - 544p^3T^3 + p^6T^4$
20	smooth		$1 - 412T + 6582pT^2 - 412p^3T^3 + p^6T^4$
21	smooth		$1 + 716T + 4614pT^2 + 716p^3T^3 + p^6T^4$
22	smooth		$1 + 14T + 330pT^2 + 14p^3T^3 + p^6T^4$
23	smooth		$(1 - 14pT + p^3T^2)(1 + 358T + p^3T^2)$
24	smooth		$1 - 166T - 2262pT^2 - 166p^3T^3 + p^6T^4$
25	smooth		$1 + 428T + 2166pT^2 + 428p^3T^3 + p^6T^4$
26	smooth		$1 + 992T + 10398pT^2 + 992p^3T^3 + p^6T^4$
27	smooth		$1 + 452T + 2262pT^2 + 452p^3T^3 + p^6T^4$
28	smooth		$1 - 796T + 5478pT^2 - 796p^3T^3 + p^6T^4$
29	smooth		$1 + 56T + 894pT^2 + 56p^3T^3 + p^6T^4$
30	smooth		$1 - 196T - 186pT^2 - 196p^3T^3 + p^6T^4$
31	smooth		$1 + 314T - 558pT^2 + 314p^3T^3 + p^6T^4$
32	smooth*	$\frac{3}{2}$	
33	smooth		$(1 + 10pT + p^3T^2)(1 - 590T + p^3T^2)$
34	smooth		$1 - 388T + 2646pT^2 - 388p^3T^3 + p^6T^4$
35	smooth		$1 - 1264T + 13758pT^2 - 1264p^3T^3 + p^6T^4$
36	smooth		$1 + 74T + 3306pT^2 + 74p^3T^3 + p^6T^4$
37	smooth		$1 - 184T + 6p^2T^2 - 184p^3T^3 + p^6T^4$
38	smooth		$1 + 344T - 1266pT^2 + 344p^3T^3 + p^6T^4$
39	smooth		$(1 - 14pT + p^3T^2)(1 + 772T + p^3T^2)$
40	smooth		$1 + 944T + 9054pT^2 + 944p^3T^3 + p^6T^4$
41	singular	$\frac{1}{3}$	$(1 - pT)(1 - 398T + p^3T^2)$
42	smooth		$1 + 416T + 4926pT^2 + 416p^3T^3 + p^6T^4$

Continued on the following page

$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
43	smooth		$1 + 164T + 5862pT^2 + 164p^3T^3 + p^6T^4$
44	smooth		$1 + 428T + 2598pT^2 + 428p^3T^3 + p^6T^4$
45	smooth		$1 + 620T + 7110pT^2 + 620p^3T^3 + p^6T^4$
46	singular	$\frac{1}{4}$	$(1 - pT)(1 - 902T + p^3T^2)$
47	smooth		$1 - 184T + 2094pT^2 - 184p^3T^3 + p^6T^4$
48	smooth		$1 + 62T + 954pT^2 + 62p^3T^3 + p^6T^4$
49	smooth		$1 + 308T + 6150pT^2 + 308p^3T^3 + p^6T^4$
50	smooth		$1 + 254T + 66p^2T^2 + 254p^3T^3 + p^6T^4$
51	smooth		$1 + 86T - 1254pT^2 + 86p^3T^3 + p^6T^4$
52	smooth		$1 - 280T + 3942pT^2 - 280p^3T^3 + p^6T^4$
53	smooth		$1 + 644T + 5190pT^2 + 644p^3T^3 + p^6T^4$
54	smooth		$1 + 368T + 1566pT^2 + 368p^3T^3 + p^6T^4$
55	smooth		$1 + 266T - 1398pT^2 + 266p^3T^3 + p^6T^4$
56	singular	$\frac{1}{12}$	$(1 - pT)(1 - 908T + p^3T^2)$
57	smooth		$1 - 148T + 5046pT^2 - 148p^3T^3 + p^6T^4$
58	smooth		$1 + 104T + 6p^2T^2 + 104p^3T^3 + p^6T^4$
59	smooth		$1 - 826T + 8130pT^2 - 826p^3T^3 + p^6T^4$
60	smooth		$1 + 176T - 3522pT^2 + 176p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 284T + 8178pT^2 + 284p^3T^3 + p^6T^4$
2	smooth		$1 + 1112T + 12642pT^2 + 1112p^3T^3 + p^6T^4$
3	smooth		$1 + 488T - 222pT^2 + 488p^3T^3 + p^6T^4$
4	smooth		$1 + 188T + 7794pT^2 + 188p^3T^3 + p^6T^4$
5	smooth		$1 - 58T + 258pT^2 - 58p^3T^3 + p^6T^4$
6	smooth		$1 + 272T - 942pT^2 + 272p^3T^3 + p^6T^4$
7	smooth		$1 - 16pT + 10098pT^2 - 16p^4T^3 + p^6T^4$
8	smooth		$1 + 752T + 4002pT^2 + 752p^3T^3 + p^6T^4$
9	smooth		$1 - 280T - 3870pT^2 - 280p^3T^3 + p^6T^4$
10	smooth		$1 + 284T + 3858pT^2 + 284p^3T^3 + p^6T^4$
11	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 188T + p^3T^2)$
12	smooth		$1 + 56T + 2946pT^2 + 56p^3T^3 + p^6T^4$
13	smooth		$1 + 272T - 4398pT^2 + 272p^3T^3 + p^6T^4$
14	smooth		$1 - 64T + 4482pT^2 - 64p^3T^3 + p^6T^4$
15	smooth		$1 - 436T + 3570pT^2 - 436p^3T^3 + p^6T^4$
16	smooth		$1 + 62T + 2106pT^2 + 62p^3T^3 + p^6T^4$
17	singular	$\frac{1}{4}$	$(1 - pT)(1 + 1024T + p^3T^2)$
18	smooth		$1 - 388T + 4410pT^2 - 388p^3T^3 + p^6T^4$

Continued on the following page

$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$1 + 590T + 5658pT^2 + 590p^3T^3 + p^6T^4$
20	smooth		$1 - 184T + 4578pT^2 - 184p^3T^3 + p^6T^4$
21	smooth		$1 - 448T + 1506pT^2 - 448p^3T^3 + p^6T^4$
22	smooth		$1 + 416T + 30p^2T^2 + 416p^3T^3 + p^6T^4$
23	smooth		$1 + 116T - 270pT^2 + 116p^3T^3 + p^6T^4$
24	smooth		$1 + 926T + 9306pT^2 + 926p^3T^3 + p^6T^4$
25	smooth		$1 - 328T + 1842pT^2 - 328p^3T^3 + p^6T^4$
26	smooth		$1 - 532T + 7938pT^2 - 532p^3T^3 + p^6T^4$
27	smooth		$1 + 302T - 3414pT^2 + 302p^3T^3 + p^6T^4$
28	singular	$\frac{1}{12}$	$(1 - pT)(1 + 508T + p^3T^2)$
29	smooth		$1 - 424T + 8154pT^2 - 424p^3T^3 + p^6T^4$
30	smooth		$1 - 784T + 8226pT^2 - 784p^3T^3 + p^6T^4$
31	smooth		$1 + 266T + 1194pT^2 + 266p^3T^3 + p^6T^4$
32	smooth		$1 + 62T + 810pT^2 + 62p^3T^3 + p^6T^4$
33	smooth		$1 - 592T + 6402pT^2 - 592p^3T^3 + p^6T^4$
34	smooth		$1 + 140T + 3642pT^2 + 140p^3T^3 + p^6T^4$
35	smooth*	$\frac{3}{2}$	
36	smooth		$1 - 52T + 1650pT^2 - 52p^3T^3 + p^6T^4$
37	smooth		$1 + 338T + 3642pT^2 + 338p^3T^3 + p^6T^4$
38	smooth		$1 + 578T + 6906pT^2 + 578p^3T^3 + p^6T^4$
39	smooth		$1 + 524T + 9570pT^2 + 524p^3T^3 + p^6T^4$
40	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 484T + p^3T^2)$
41	smooth		$(1 + 4pT + p^3T^2)2$
42	smooth		$1 - 256T + 2274pT^2 - 256p^3T^3 + p^6T^4$
43	smooth		$1 + 164T + 5970pT^2 + 164p^3T^3 + p^6T^4$
44	smooth		$(1 + 4pT + p^3T^2)(1 - 956T + p^3T^2)$
45	singular	$\frac{1}{3}$	$(1 - pT)(1 - 92T + p^3T^2)$
46	smooth		$1 - 94T + 3210pT^2 - 94p^3T^3 + p^6T^4$
47	smooth		$1 + 410T + 8250pT^2 + 410p^3T^3 + p^6T^4$
48	smooth		$1 - 568T + 5202pT^2 - 568p^3T^3 + p^6T^4$
49	smooth		$(1 + 4pT + p^3T^2)(1 - 734T + p^3T^2)$
50	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 484T + p^3T^2)$
51	smooth		$1 - 640T + 7074pT^2 - 640p^3T^3 + p^6T^4$
52	smooth		$1 + 482T + 7602pT^2 + 482p^3T^3 + p^6T^4$
53	smooth		$1 + 218T + 8706pT^2 + 218p^3T^3 + p^6T^4$
54	smooth		$1 + 32T + 8466pT^2 + 32p^3T^3 + p^6T^4$
55	smooth		$1 + 98T + 3978pT^2 + 98p^3T^3 + p^6T^4$
56	smooth		$1 + 332T + 7794pT^2 + 332p^3T^3 + p^6T^4$
57	smooth		$1 + 704T + 8562pT^2 + 704p^3T^3 + p^6T^4$
58	smooth		$1 + 1112T + 12498pT^2 + 1112p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
59	smooth		$1 + 170T + 5850pT^2 + 170p^3T^3 + p^6T^4$
60	smooth		$1 - 580T + 2994pT^2 - 580p^3T^3 + p^6T^4$
61	smooth		$1 - 604T + 4194pT^2 - 604p^3T^3 + p^6T^4$
62	smooth		$1 - 238T + 3930pT^2 - 238p^3T^3 + p^6T^4$
63	smooth		$1 + 932T + 9042pT^2 + 932p^3T^3 + p^6T^4$
64	smooth		$1 + 308T - 366pT^2 + 308p^3T^3 + p^6T^4$
65	smooth		$1 - 856T + 6642pT^2 - 856p^3T^3 + p^6T^4$
66	smooth		$1 + 638T + 9594pT^2 + 638p^3T^3 + p^6T^4$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 240T - 5902pT^2 + 240p^3T^3 + p^6T^4$
2	smooth		$1 - 156T + 1154pT^2 - 156p^3T^3 + p^6T^4$
3	smooth		$1 + 570T + 6194pT^2 + 570p^3T^3 + p^6T^4$
4	smooth		$1 - 12pT + 8642pT^2 - 12p^4T^3 + p^6T^4$
5	smooth		$(1 - 12pT + p^3T^2)(1 + 1020T + p^3T^2)$
6	singular	$\frac{1}{12}$	$(1 - pT)(1 + 426T + p^3T^2)$
7	smooth		$1 + 282T + 1802pT^2 + 282p^3T^3 + p^6T^4$
8	smooth		$1 + 246T + 9506pT^2 + 246p^3T^3 + p^6T^4$
9	smooth		$1 + 102T + 3242pT^2 + 102p^3T^3 + p^6T^4$
10	smooth		$1 - 156T + 5330pT^2 - 156p^3T^3 + p^6T^4$
11	smooth		$1 - 228T + 1442pT^2 - 228p^3T^3 + p^6T^4$
12	smooth		$1 + 1074T + 10946pT^2 + 1074p^3T^3 + p^6T^4$
13	smooth		$1 + 420T + 578pT^2 + 420p^3T^3 + p^6T^4$
14	singular	$-\frac{1}{5}$	$(1 + pT)(1 + 708T + p^3T^2)$
15	smooth		$1 + 636T + 3746pT^2 + 636p^3T^3 + p^6T^4$
16	smooth		$(1 + p^3T^2)(1 + 480T + p^3T^2)$
17	smooth		$1 - 312T + 3170pT^2 - 312p^3T^3 + p^6T^4$
18	singular	$\frac{1}{4}$	$(1 + pT)(1 - 432T + p^3T^2)$
19	smooth		$1 + 144T + 4322pT^2 + 144p^3T^3 + p^6T^4$
20	smooth		$1 + 360T + 6626pT^2 + 360p^3T^3 + p^6T^4$
21	smooth		$1 + 102T + 1874pT^2 + 102p^3T^3 + p^6T^4$
22	smooth		$1 + 528T + 2234pT^2 + 528p^3T^3 + p^6T^4$
23	smooth		$1 - 504T + 9290pT^2 - 504p^3T^3 + p^6T^4$
24	singular	$\frac{1}{3}$	$(1 - pT)(1 + 720T + p^3T^2)$
25	smooth		$1 - 96T - 7270pT^2 - 96p^3T^3 + p^6T^4$
26	smooth		$1 + 162T + 4898pT^2 + 162p^3T^3 + p^6T^4$
27	smooth		$1 + 384T - 1582pT^2 + 384p^3T^3 + p^6T^4$
28	smooth		$1 - 144T + 7058pT^2 - 144p^3T^3 + p^6T^4$

Continued on the following page

$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
29	smooth		$1 - 102T + 70p^2T^2 - 102p^3T^3 + p^6T^4$
30	smooth		$1 - 456T + 6482pT^2 - 456p^3T^3 + p^6T^4$
31	smooth		$1 - 84T - 2158pT^2 - 84p^3T^3 + p^6T^4$
32	smooth		$1 - 222T + 3458pT^2 - 222p^3T^3 + p^6T^4$
33	smooth		$1 + 606T + 7346pT^2 + 606p^3T^3 + p^6T^4$
34	smooth		$1 - 258T - 2446pT^2 - 258p^3T^3 + p^6T^4$
35	smooth		$1 + 72T - 3598pT^2 + 72p^3T^3 + p^6T^4$
36	smooth		$1 + 180T - 3022pT^2 + 180p^3T^3 + p^6T^4$
37	smooth*	$\frac{3}{2}$	
38	smooth		$1 - 72T + 2306pT^2 - 72p^3T^3 + p^6T^4$
39	smooth		$1 + 468T + 7778pT^2 + 468p^3T^3 + p^6T^4$
40	smooth		$1 - 432T + 6914pT^2 - 432p^3T^3 + p^6T^4$
41	smooth		$1 - 516T + 4466pT^2 - 516p^3T^3 + p^6T^4$
42	smooth		$1 + 324T + 2882pT^2 + 324p^3T^3 + p^6T^4$
43	smooth		$1 - 7198pT^2 + p^6T^4$
44	smooth		$1 + 816T + 11378pT^2 + 816p^3T^3 + p^6T^4$
45	smooth		$1 + 372T + 2594pT^2 + 372p^3T^3 + p^6T^4$
46	smooth		$1 + 192T + 7058pT^2 + 192p^3T^3 + p^6T^4$
47	smooth		$1 - 594T + 9938pT^2 - 594p^3T^3 + p^6T^4$
48	smooth		$1 + 60T - 4606pT^2 + 60p^3T^3 + p^6T^4$
49	smooth		$1 - 324T - 4174pT^2 - 324p^3T^3 + p^6T^4$
50	smooth		$1 - 300T + 6050pT^2 - 300p^3T^3 + p^6T^4$
51	smooth		$1 + 1044T + 13250pT^2 + 1044p^3T^3 + p^6T^4$
52	smooth		$1 - 72T + 2738pT^2 - 72p^3T^3 + p^6T^4$
53	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 576T + p^3T^2)$
54	smooth		$1 + 48T + 3890pT^2 + 48p^3T^3 + p^6T^4$
55	smooth		$1 + 294T + 9218pT^2 + 294p^3T^3 + p^6T^4$
56	smooth		$1 + 264T + 5762pT^2 + 264p^3T^3 + p^6T^4$
57	smooth		$1 + 468T + 5762pT^2 + 468p^3T^3 + p^6T^4$
58	smooth		$1 + 36T + 6338pT^2 + 36p^3T^3 + p^6T^4$
59	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 480T + p^3T^2)$
60	smooth		$1 - 264T - 3742pT^2 - 264p^3T^3 + p^6T^4$
61	smooth		$1 + 24T - 862pT^2 + 24p^3T^3 + p^6T^4$
62	smooth		$1 + 576T + 1442pT^2 + 576p^3T^3 + p^6T^4$
63	smooth		$1 - 420T + 4682pT^2 - 420p^3T^3 + p^6T^4$
64	smooth		$1 + 510T + 2882pT^2 + 510p^3T^3 + p^6T^4$
65	smooth		$1 - 624T + 1442pT^2 - 624p^3T^3 + p^6T^4$
66	smooth		$1 + 822T + 9218pT^2 + 822p^3T^3 + p^6T^4$
67	smooth		$1 + 36T - 2878pT^2 + 36p^3T^3 + p^6T^4$
68	smooth		$1 + 228T + 2306pT^2 + 228p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
69	smooth		$1 + 24T + 6626pT^2 + 24p^3T^3 + p^6T^4$
70	smooth		$1 - 816T + 4610pT^2 - 816p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 400T - 66pT^2 - 400p^3T^3 + p^6T^4$
2	smooth		$1 - 424T + 8478pT^2 - 424p^3T^3 + p^6T^4$
3	smooth		$1 + 380T + 5574pT^2 + 380p^3T^3 + p^6T^4$
4	smooth		$1 + 356T + 5766pT^2 + 356p^3T^3 + p^6T^4$
5	smooth		$1 + 848T + 11262pT^2 + 848p^3T^3 + p^6T^4$
6	smooth		$1 - 136T + 8046pT^2 - 136p^3T^3 + p^6T^4$
7	smooth		$1 - 190T - 3510pT^2 - 190p^3T^3 + p^6T^4$
8	smooth		$1 + 296T + 2574pT^2 + 296p^3T^3 + p^6T^4$
9	smooth		$1 - 1132T + 9606pT^2 - 1132p^3T^3 + p^6T^4$
10	smooth		$1 + 680T + 2094pT^2 + 680p^3T^3 + p^6T^4$
11	smooth		$1 - 160T + 8094pT^2 - 160p^3T^3 + p^6T^4$
12	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 434T + p^3T^2)$
13	smooth		$1 + 56T + 30pT^2 + 56p^3T^3 + p^6T^4$
14	smooth		$1 + 932T + 11742pT^2 + 932p^3T^3 + p^6T^4$
15	smooth		$1 - 148T + 870pT^2 - 148p^3T^3 + p^6T^4$
16	smooth		$1 - 1078T + 11082pT^2 - 1078p^3T^3 + p^6T^4$
17	smooth		$1 + 434T + 3594pT^2 + 434p^3T^3 + p^6T^4$
18	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 1150T + p^3T^2)$
19	smooth		$1 + 428T + 2598pT^2 + 428p^3T^3 + p^6T^4$
20	smooth		$1 + 1136T + 10686pT^2 + 1136p^3T^3 + p^6T^4$
21	smooth		$1 - 496T + 5310pT^2 - 496p^3T^3 + p^6T^4$
22	smooth		$1 + 200T + 7806pT^2 + 200p^3T^3 + p^6T^4$
23	smooth		$1 + 716T + 5334pT^2 + 716p^3T^3 + p^6T^4$
24	smooth		$1 - 352T + 1566pT^2 - 352p^3T^3 + p^6T^4$
25	smooth		$1 + 566T + 7938pT^2 + 566p^3T^3 + p^6T^4$
26	smooth		$1 + 296T + 1134pT^2 + 296p^3T^3 + p^6T^4$
27	smooth		$1 + 932T + 8718pT^2 + 932p^3T^3 + p^6T^4$
28	smooth		$1 + 212T - 5610pT^2 + 212p^3T^3 + p^6T^4$
29	singular	$-\frac{1}{5}$	$(1 - pT)(1 - 362T + p^3T^2)$
30	smooth		$1 + 362T + 3162pT^2 + 362p^3T^3 + p^6T^4$
31	smooth		$1 - 262T + 8802pT^2 - 262p^3T^3 + p^6T^4$
32	smooth		$1 - 208T - 882pT^2 - 208p^3T^3 + p^6T^4$
33	smooth		$1 - 1048T + 13326pT^2 - 1048p^3T^3 + p^6T^4$
34	smooth		$1 + 284T - 4026pT^2 + 284p^3T^3 + p^6T^4$

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$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 + 488T + 8022pT^2 + 488p^3T^3 + p^6T^4$
36	smooth		$1 - 700T + 102p^2T^2 - 700p^3T^3 + p^6T^4$
37	smooth		$1 - 556T - 186pT^2 - 556p^3T^3 + p^6T^4$
38	smooth*	$\frac{3}{2}$	
39	smooth		$1 - 310T - 2118pT^2 - 310p^3T^3 + p^6T^4$
40	smooth		$1 + 992T + 9102pT^2 + 992p^3T^3 + p^6T^4$
41	smooth		$1 - 514T + 4986pT^2 - 514p^3T^3 + p^6T^4$
42	smooth		$1 + 56T + 1326pT^2 + 56p^3T^3 + p^6T^4$
43	smooth		$1 + 536T + 3678pT^2 + 536p^3T^3 + p^6T^4$
44	smooth		$1 - 4pT - 714pT^2 - 4p^4T^3 + p^6T^4$
45	smooth		$1 + 44T + 5526pT^2 + 44p^3T^3 + p^6T^4$
46	smooth		$1 - 1012T + 12102pT^2 - 1012p^3T^3 + p^6T^4$
47	smooth		$1 - 514T + 7506pT^2 - 514p^3T^3 + p^6T^4$
48	smooth		$1 + 542T + 9714pT^2 + 542p^3T^3 + p^6T^4$
49	singular	$\frac{1}{3}$	$(1 - pT)(1 + 502T + p^3T^2)$
50	smooth		$1 - 616T + 8142pT^2 - 616p^3T^3 + p^6T^4$
51	smooth		$1 - 220T + 5262pT^2 - 220p^3T^3 + p^6T^4$
52	smooth		$1 + 134T + 8802pT^2 + 134p^3T^3 + p^6T^4$
53	smooth		$1 + 1058T + 13506pT^2 + 1058p^3T^3 + p^6T^4$
54	smooth		$1 - 640T + 8046pT^2 - 640p^3T^3 + p^6T^4$
55	singular	$\frac{1}{4}$	$(1 - pT)(1 - 362T + p^3T^2)$
56	smooth		$1 - 916T + 5862pT^2 - 916p^3T^3 + p^6T^4$
57	smooth		$1 + 200T + 3054pT^2 + 200p^3T^3 + p^6T^4$
58	smooth		$1 + 200T + 6294pT^2 + 200p^3T^3 + p^6T^4$
59	smooth		$1 - 82T + 4050pT^2 - 82p^3T^3 + p^6T^4$
60	smooth		$1 + 380T + 4422pT^2 + 380p^3T^3 + p^6T^4$
61	smooth		$1 + 536T + 6558pT^2 + 536p^3T^3 + p^6T^4$
62	smooth		$1 + 524T + 7590pT^2 + 524p^3T^3 + p^6T^4$
63	smooth		$1 - 58T - 5070pT^2 - 58p^3T^3 + p^6T^4$
64	smooth		$1 - 304T + 6654pT^2 - 304p^3T^3 + p^6T^4$
65	smooth		$1 + 446T + 6090pT^2 + 446p^3T^3 + p^6T^4$
66	smooth		$1 + 428T + 6342pT^2 + 428p^3T^3 + p^6T^4$
67	singular	$\frac{1}{12}$	$(1 - pT)(1 + 574T + p^3T^2)$
68	smooth		$1 + 236T + 5142pT^2 + 236p^3T^3 + p^6T^4$
69	smooth		$1 - 694T + 4698pT^2 - 694p^3T^3 + p^6T^4$
70	smooth		$1 + 44T + 2574pT^2 + 44p^3T^3 + p^6T^4$
71	smooth		$1 + 596T + 11046pT^2 + 596p^3T^3 + p^6T^4$
72	smooth		$1 - 448T + 1470pT^2 - 448p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 412T + 10506pT^2 - 412p^3T^3 + p^6T^4$
2	smooth		$1 + 326T + 6114pT^2 + 326p^3T^3 + p^6T^4$
3	smooth		$1 - 118T - 1134pT^2 - 118p^3T^3 + p^6T^4$
4	smooth		$1 + 290T - 6846pT^2 + 290p^3T^3 + p^6T^4$
5	smooth		$1 - 112T + 5730pT^2 - 112p^3T^3 + p^6T^4$
6	smooth		$1 + 608T + 3858pT^2 + 608p^3T^3 + p^6T^4$
7	smooth		$1 - 76T - 4782pT^2 - 76p^3T^3 + p^6T^4$
8	smooth		$1 - 184T - 4926pT^2 - 184p^3T^3 + p^6T^4$
9	smooth		$1 + 722T + 2082pT^2 + 722p^3T^3 + p^6T^4$
10	smooth		$1 + 356T + 3426pT^2 + 356p^3T^3 + p^6T^4$
11	smooth		$1 + 572T + 6090pT^2 + 572p^3T^3 + p^6T^4$
12	smooth		$1 - 88T + 6402pT^2 - 88p^3T^3 + p^6T^4$
13	singular	$-\frac{1}{6}$	$(1 - pT)(1 - 1352T + p^3T^2)$
14	smooth		$1 + 1682T + 18162pT^2 + 1682p^3T^3 + p^6T^4$
15	smooth		$1 + 356T - 2334pT^2 + 356p^3T^3 + p^6T^4$
16	smooth		$1 - 616T + 7170pT^2 - 616p^3T^3 + p^6T^4$
17	smooth		$1 + 644T + 5730pT^2 + 644p^3T^3 + p^6T^4$
18	smooth		$1 + 1670T + 19986pT^2 + 1670p^3T^3 + p^6T^4$
19	smooth		$1 + 710T + 8658pT^2 + 710p^3T^3 + p^6T^4$
20	singular	$\frac{1}{4}$	$(1 - pT)(1 + 160T + p^3T^2)$
21	smooth		$1 + 1136T + 13026pT^2 + 1136p^3T^3 + p^6T^4$
22	smooth		$1 - 1456T + 18642pT^2 - 1456p^3T^3 + p^6T^4$
23	smooth		$1 - 1894T + 22290pT^2 - 1894p^3T^3 + p^6T^4$
24	smooth		$1 - 220T + 9834pT^2 - 220p^3T^3 + p^6T^4$
25	smooth		$1 + 512T + 10242pT^2 + 512p^3T^3 + p^6T^4$
26	smooth		$1 - 988T + 13890pT^2 - 988p^3T^3 + p^6T^4$
27	smooth		$1 - 436T + 3138pT^2 - 436p^3T^3 + p^6T^4$
28	smooth		$1 - 322T + 8706pT^2 - 322p^3T^3 + p^6T^4$
29	smooth		$1 + 284T - 1830pT^2 + 284p^3T^3 + p^6T^4$
30	smooth		$1 - 262T + 2754pT^2 - 262p^3T^3 + p^6T^4$
31	smooth		$1 + 488T + 1074pT^2 + 488p^3T^3 + p^6T^4$
32	smooth		$1 + 1100T + 11370pT^2 + 1100p^3T^3 + p^6T^4$
33	singular	$\frac{1}{12}$	$(1 - pT)(1 - 110T + p^3T^2)$
34	smooth		$1 + 680T + 5442pT^2 + 680p^3T^3 + p^6T^4$
35	smooth		$1 - 856T + 5922pT^2 - 856p^3T^3 + p^6T^4$
36	smooth		$1 + 320T + 10050pT^2 + 320p^3T^3 + p^6T^4$
37	smooth		$1 - 292T + 4578pT^2 - 292p^3T^3 + p^6T^4$
38	smooth		$1 + 92T + 10146pT^2 + 92p^3T^3 + p^6T^4$
39	smooth		$1 - 124T + 498pT^2 - 124p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 416T - 222pT^2 + 416p^3T^3 + p^6T^4$
41	smooth*	$\frac{3}{2}$	
42	smooth		$1 + 848T + 11586pT^2 + 848p^3T^3 + p^6T^4$
43	smooth		$1 + 308T + 11010pT^2 + 308p^3T^3 + p^6T^4$
44	smooth		$1 + 152T - 270pT^2 + 152p^3T^3 + p^6T^4$
45	smooth		$1 - 568T + 5778pT^2 - 568p^3T^3 + p^6T^4$
46	smooth		$1 - 1624T + 19842pT^2 - 1624p^3T^3 + p^6T^4$
47	smooth		$1 + 776T + 9858pT^2 + 776p^3T^3 + p^6T^4$
48	smooth		$1 + 920T + 11298pT^2 + 920p^3T^3 + p^6T^4$
49	smooth		$1 + 38T - 1734pT^2 + 38p^3T^3 + p^6T^4$
50	smooth		$1 - 490T + 5442pT^2 - 490p^3T^3 + p^6T^4$
51	smooth		$1 - 400T + 5442pT^2 - 400p^3T^3 + p^6T^4$
52	smooth		$1 + 920T + 5682pT^2 + 920p^3T^3 + p^6T^4$
53	singular	$\frac{1}{3}$	$(1 - pT)(1 + 1024T + p^3T^2)$
54	smooth		$1 + 56T + 2226pT^2 + 56p^3T^3 + p^6T^4$
55	smooth		$1 - 166T - 4134pT^2 - 166p^3T^3 + p^6T^4$
56	smooth		$1 - 238T - 3054pT^2 - 238p^3T^3 + p^6T^4$
57	smooth		$1 - 256T - 1326pT^2 - 256p^3T^3 + p^6T^4$
58	smooth		$1 + 110T + 9858pT^2 + 110p^3T^3 + p^6T^4$
59	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 776T + p^3T^2)$
60	smooth		$1 - 646T + 5754pT^2 - 646p^3T^3 + p^6T^4$
61	smooth		$1 + 416T - 222pT^2 + 416p^3T^3 + p^6T^4$
62	smooth		$(1 - 8pT + p^3T^2)(1 - 380T + p^3T^2)$
63	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 484T + p^3T^2)$
64	smooth		$(1 + 16pT + p^3T^2)(1 - 1352T + p^3T^2)$
65	smooth		$1 - 148T + 4722pT^2 - 148p^3T^3 + p^6T^4$
66	smooth		$1 + 614T + 642pT^2 + 614p^3T^3 + p^6T^4$
67	smooth		$1 - 322T + 1074pT^2 - 322p^3T^3 + p^6T^4$
68	smooth		$1 + 440T + 1746pT^2 + 440p^3T^3 + p^6T^4$
69	smooth		$1 + 1190T + 10002pT^2 + 1190p^3T^3 + p^6T^4$
70	smooth		$1 + 596T + 12162pT^2 + 596p^3T^3 + p^6T^4$
71	smooth		$1 + 368T - 918pT^2 + 368p^3T^3 + p^6T^4$
72	smooth		$1 - 472T + 4290pT^2 - 472p^3T^3 + p^6T^4$
73	smooth		$1 - 760T + 10914pT^2 - 760p^3T^3 + p^6T^4$
74	smooth		$1 - 208T - 2286pT^2 - 208p^3T^3 + p^6T^4$
75	smooth		$1 + 710T + 8658pT^2 + 710p^3T^3 + p^6T^4$
76	smooth		$1 + 620T + 738pT^2 + 620p^3T^3 + p^6T^4$
77	smooth		$1 + 1448T + 16578pT^2 + 1448p^3T^3 + p^6T^4$
78	smooth		$1 - 1528T + 17346pT^2 - 1528p^3T^3 + p^6T^4$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 420T + 10322pT^2 - 420p^3T^3 + p^6T^4$
2	smooth		$1 + 1212T + 12698pT^2 + 1212p^3T^3 + p^6T^4$
3	smooth		$1 - 360T + 4418pT^2 - 360p^3T^3 + p^6T^4$
4	smooth		$1 + 618T + 314pT^2 + 618p^3T^3 + p^6T^4$
5	smooth		$1 - 132T + 9602pT^2 - 132p^3T^3 + p^6T^4$
6	smooth		$1 + 606T + 890pT^2 + 606p^3T^3 + p^6T^4$
7	singular	$\frac{1}{12}$	$(1 - pT)(1 + 1308T + p^3T^2)$
8	smooth		$1 + 1338T + 16730pT^2 + 1338p^3T^3 + p^6T^4$
9	smooth		$1 - 1020T + 10178pT^2 - 1020p^3T^3 + p^6T^4$
10	smooth		$1 + 810T + 8162pT^2 + 810p^3T^3 + p^6T^4$
11	smooth		$1 + 534T + 7514pT^2 + 534p^3T^3 + p^6T^4$
12	smooth		$1 + 348T - 6094pT^2 + 348p^3T^3 + p^6T^4$
13	smooth		$1 + 492T + 6578pT^2 + 492p^3T^3 + p^6T^4$
14	smooth		$1 + 576T + 2690pT^2 + 576p^3T^3 + p^6T^4$
15	smooth		$1 + 1056T + 16802pT^2 + 1056p^3T^3 + p^6T^4$
16	smooth		$1 - 120T + 8450pT^2 - 120p^3T^3 + p^6T^4$
17	smooth		$1 + 288T - 5086pT^2 + 288p^3T^3 + p^6T^4$
18	smooth		$1 + 528T + 3698pT^2 + 528p^3T^3 + p^6T^4$
19	smooth		$1 + 1194T + 11474pT^2 + 1194p^3T^3 + p^6T^4$
20	smooth		$1 - 984T + 6722pT^2 - 984p^3T^3 + p^6T^4$
21	singular	$\frac{1}{4}$	$(1 + pT)(1 - 72T + p^3T^2)$
22	smooth		$1 - 414T + 10034pT^2 - 414p^3T^3 + p^6T^4$
23	smooth		$1 - 378T + 1898pT^2 - 378p^3T^3 + p^6T^4$
24	smooth		$1 - 1044T + 8306pT^2 - 1044p^3T^3 + p^6T^4$
25	smooth		$1 - 480T + 8450pT^2 - 480p^3T^3 + p^6T^4$
26	smooth		$1 + 798T + 8954pT^2 + 798p^3T^3 + p^6T^4$
27	smooth		$1 + 186T + 8378pT^2 + 186p^3T^3 + p^6T^4$
28	singular	$\frac{1}{3}$	$(1 - pT)(1 + 204T + p^3T^2)$
29	smooth		$1 + 678T + 4994pT^2 + 678p^3T^3 + p^6T^4$
30	smooth		$1 + 456T + 2690pT^2 + 456p^3T^3 + p^6T^4$
31	smooth		$1 + 360T + 4850pT^2 + 360p^3T^3 + p^6T^4$
32	smooth		$1 + 102T - 4006pT^2 + 102p^3T^3 + p^6T^4$
33	singular	$-\frac{1}{5}$	$(1 + pT)(1 - 756T + p^3T^2)$
34	smooth		$1 - 12T - 1486pT^2 - 12p^3T^3 + p^6T^4$
35	smooth		$1 - 1188T + 16802pT^2 - 1188p^3T^3 + p^6T^4$
36	smooth		$1 - 234T - 5302pT^2 - 234p^3T^3 + p^6T^4$
37	smooth		$1 + 600T + 6722pT^2 + 600p^3T^3 + p^6T^4$
38	smooth		$1 - 228T - 1774pT^2 - 228p^3T^3 + p^6T^4$
39	smooth		$1 + 312T + 4418pT^2 + 312p^3T^3 + p^6T^4$

Continued on the following page

$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 - 684T + 2762pT^2 - 684p^3T^3 + p^6T^4$
41	smooth		$1 + 1692T + 19538pT^2 + 1692p^3T^3 + p^6T^4$
42	smooth		$1 + 138T + 10250pT^2 + 138p^3T^3 + p^6T^4$
43	smooth*	$\frac{3}{2}$	
44	smooth		$1 - 432T + 5426pT^2 - 432p^3T^3 + p^6T^4$
45	smooth		$1 + 96T + 1538pT^2 + 96p^3T^3 + p^6T^4$
46	smooth		$1 + 1092T + 9962pT^2 + 1092p^3T^3 + p^6T^4$
47	smooth		$1 - 192T + 7082pT^2 - 192p^3T^3 + p^6T^4$
48	smooth		$1 + 1212T + 9170pT^2 + 1212p^3T^3 + p^6T^4$
49	smooth		$1 - 720T + 13202pT^2 - 720p^3T^3 + p^6T^4$
50	smooth		$1 + 366T + 3842pT^2 + 366p^3T^3 + p^6T^4$
51	smooth		$1 + 528T - 3070pT^2 + 528p^3T^3 + p^6T^4$
52	smooth		$1 - 336T - 5158pT^2 - 336p^3T^3 + p^6T^4$
53	smooth		$1 + 1188T + 10034pT^2 + 1188p^3T^3 + p^6T^4$
54	smooth		$1 + 216T + 9890pT^2 + 216p^3T^3 + p^6T^4$
55	smooth		$1 + 228T - 1918pT^2 + 228p^3T^3 + p^6T^4$
56	smooth		$1 - 1020T + 14642pT^2 - 1020p^3T^3 + p^6T^4$
57	smooth		$1 - 810T + 5786pT^2 - 810p^3T^3 + p^6T^4$
58	smooth		$1 + 540T + 3410pT^2 + 540p^3T^3 + p^6T^4$
59	smooth		$1 - 24T - 3502pT^2 - 24p^3T^3 + p^6T^4$
60	smooth		$1 + 24T + 13058pT^2 + 24p^3T^3 + p^6T^4$
61	smooth		$1 - 1284T + 16370pT^2 - 1284p^3T^3 + p^6T^4$
62	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 378T + p^3T^2)$
63	smooth		$1 + 42T - 1126pT^2 + 42p^3T^3 + p^6T^4$
64	smooth		$1 - 324T - 2278pT^2 - 324p^3T^3 + p^6T^4$
65	smooth		$1 + 960T + 13058pT^2 + 960p^3T^3 + p^6T^4$
66	smooth		$1 - 1080T + 9602pT^2 - 1080p^3T^3 + p^6T^4$
67	smooth		$1 - 432T + 11186pT^2 - 432p^3T^3 + p^6T^4$
68	smooth		$(1 - 12pT + p^3T^2)(1 - 390T + p^3T^2)$
69	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 612T + p^3T^2)$
70	smooth		$1 - 36T + 1682pT^2 - 36p^3T^3 + p^6T^4$
71	smooth		$(1 - 12pT + p^3T^2)(1 + 600T + p^3T^2)$
72	smooth		$1 + 450T + 6434pT^2 + 450p^3T^3 + p^6T^4$
73	smooth		$1 - 708T + 3410pT^2 - 708p^3T^3 + p^6T^4$
74	smooth		$1 - 420T - 910pT^2 - 420p^3T^3 + p^6T^4$
75	smooth		$1 + 1584T + 18530pT^2 + 1584p^3T^3 + p^6T^4$
76	smooth		$1 + 1236T + 16370pT^2 + 1236p^3T^3 + p^6T^4$
77	smooth		$1 - 204T + 5714pT^2 - 204p^3T^3 + p^6T^4$
78	smooth		$1 + 156T - 6094pT^2 + 156p^3T^3 + p^6T^4$
79	smooth		$1 + 288T + 4418pT^2 + 288p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 - 384T + 7082pT^2 - 384p^3T^3 + p^6T^4$
81	smooth		$1 - 1026T + 16010pT^2 - 1026p^3T^3 + p^6T^4$
82	smooth		$1 + 162T + 13562pT^2 + 162p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 138T + 938pT^2 + 138p^3T^3 + p^6T^4$
2	smooth		$1 - 1668T + 17174pT^2 - 1668p^3T^3 + p^6T^4$
3	smooth		$1 + 780T + 16454pT^2 + 780p^3T^3 + p^6T^4$
4	smooth		$1 - 48T + 254pT^2 - 48p^3T^3 + p^6T^4$
5	smooth		$1 + 264T + 6086pT^2 + 264p^3T^3 + p^6T^4$
6	smooth		$1 - 552T + 6158pT^2 - 552p^3T^3 + p^6T^4$
7	smooth		$1 + 276T + 3638pT^2 + 276p^3T^3 + p^6T^4$
8	smooth		$1 + 228T - 4714pT^2 + 228p^3T^3 + p^6T^4$
9	smooth		$1 + 276T + 13646pT^2 + 276p^3T^3 + p^6T^4$
10	smooth		$1 + 1200T + 16814pT^2 + 1200p^3T^3 + p^6T^4$
11	smooth		$1 + 222T + 6194pT^2 + 222p^3T^3 + p^6T^4$
12	smooth		$1 + 492T + 10262pT^2 + 492p^3T^3 + p^6T^4$
13	smooth		$1 - 720T + 10910pT^2 - 720p^3T^3 + p^6T^4$
14	smooth		$1 - 522T - 2590pT^2 - 522p^3T^3 + p^6T^4$
15	smooth		$1 - 360T + 3422pT^2 - 360p^3T^3 + p^6T^4$
16	smooth		$(1 + 6pT + p^3T^2)(1 - 510T + p^3T^2)$
17	smooth		$1 + 12T + 2342pT^2 + 12p^3T^3 + p^6T^4$
18	smooth		$1 + 564T + 11702pT^2 + 564p^3T^3 + p^6T^4$
19	smooth		$1 - 96T - 898pT^2 - 96p^3T^3 + p^6T^4$
20	smooth		$1 - 600T + 11054pT^2 - 600p^3T^3 + p^6T^4$
21	smooth		$1 - 798T + 6698pT^2 - 798p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 390T + p^3T^2)$
23	smooth		$1 + 1098T + 12602pT^2 + 1098p^3T^3 + p^6T^4$
24	smooth		$1 + 672T + 13646pT^2 + 672p^3T^3 + p^6T^4$
25	smooth		$1 - 252T - 682pT^2 - 252p^3T^3 + p^6T^4$
26	smooth		$1 - 360T + 8318pT^2 - 360p^3T^3 + p^6T^4$
27	smooth		$1 - 648T - 250pT^2 - 648p^3T^3 + p^6T^4$
28	smooth		$1 - 300T - 250pT^2 - 300p^3T^3 + p^6T^4$
29	smooth		$1 - 540T - 2266pT^2 - 540p^3T^3 + p^6T^4$
30	singular	$\frac{1}{3}$	$(1 - pT)(1 - 354T + p^3T^2)$
31	smooth		$1 - 6T + 3602pT^2 - 6p^3T^3 + p^6T^4$
32	smooth		$1 + 6T + 4826pT^2 + 6p^3T^3 + p^6T^4$
33	smooth		$1 - 462T + 4682pT^2 - 462p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 + 996T + 5222pT^2 + 996p^3T^3 + p^6T^4$
35	smooth		$1 - 210T + 434pT^2 - 210p^3T^3 + p^6T^4$
36	smooth		$1 - 138T + 2306pT^2 - 138p^3T^3 + p^6T^4$
37	smooth		$(1 - 6pT + p^3T^2)(1 + 162T + p^3T^2)$
38	smooth		$1 - 288T + 6302pT^2 - 288p^3T^3 + p^6T^4$
39	smooth		$1 - 414T + 7274pT^2 - 414p^3T^3 + p^6T^4$
40	smooth		$1 + 240T + 3710pT^2 + 240p^3T^3 + p^6T^4$
41	smooth		$1 - 588T + 2198pT^2 - 588p^3T^3 + p^6T^4$
42	smooth		$1 + 1152T + 16094pT^2 + 1152p^3T^3 + p^6T^4$
43	smooth		$1 + 972T + 5222pT^2 + 972p^3T^3 + p^6T^4$
44	smooth		$1 + 426T + 15338pT^2 + 426p^3T^3 + p^6T^4$
45	smooth		$1 - 1080T + 10046pT^2 - 1080p^3T^3 + p^6T^4$
46	smooth*	$\frac{3}{2}$	
47	smooth		$1 + 48T + 4574pT^2 + 48p^3T^3 + p^6T^4$
48	smooth		$1 + 288T + 830pT^2 + 288p^3T^3 + p^6T^4$
49	smooth		$1 - 276T - 4138pT^2 - 276p^3T^3 + p^6T^4$
50	smooth		$1 + 636T + 6950pT^2 + 636p^3T^3 + p^6T^4$
51	smooth		$1 - 990T + 7418pT^2 - 990p^3T^3 + p^6T^4$
52	singular	$\frac{1}{12}$	$(1 - pT)(1 - 798T + p^3T^2)$
53	smooth		$1 + 432T + 10622pT^2 + 432p^3T^3 + p^6T^4$
54	smooth		$1 + 564T + 2198pT^2 + 564p^3T^3 + p^6T^4$
55	smooth		$1 - 726T + 6410pT^2 - 726p^3T^3 + p^6T^4$
56	smooth		$1 - 1230T + 16634pT^2 - 1230p^3T^3 + p^6T^4$
57	smooth		$1 + 672T + 7598pT^2 + 672p^3T^3 + p^6T^4$
58	smooth		$1 - 510T + 5258pT^2 - 510p^3T^3 + p^6T^4$
59	smooth		$1 + 630T + 2954pT^2 + 630p^3T^3 + p^6T^4$
60	smooth		$1 - 1488T + 14366pT^2 - 1488p^3T^3 + p^6T^4$
61	smooth		$1 + 756T + 9830pT^2 + 756p^3T^3 + p^6T^4$
62	smooth		$1 + 666T + 9938pT^2 + 666p^3T^3 + p^6T^4$
63	smooth		$1 + 876T + 902pT^2 + 876p^3T^3 + p^6T^4$
64	smooth		$1 + 1644T + 19334pT^2 + 1644p^3T^3 + p^6T^4$
65	smooth		$1 - 630T + 10010pT^2 - 630p^3T^3 + p^6T^4$
66	smooth		$1 + 390T - 3742pT^2 + 390p^3T^3 + p^6T^4$
67	singular	$\frac{1}{4}$	$(1 + pT)(1 - 810T + p^3T^2)$
68	smooth		$1 - 36T + 13862pT^2 - 36p^3T^3 + p^6T^4$
69	smooth		$1 + 708T + 5366pT^2 + 708p^3T^3 + p^6T^4$
70	smooth		$1 - 1392T + 19262pT^2 - 1392p^3T^3 + p^6T^4$
71	singular	$-\frac{1}{5}$	$(1 + pT)(1 + 774T + p^3T^2)$
72	smooth		$1 + 576T - 2914pT^2 + 576p^3T^3 + p^6T^4$
73	smooth		$(1 + 6pT + p^3T^2)(1 + 222T + p^3T^2)$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 30T + p^3T^2)$
75	smooth		$1 - 306T + 13682pT^2 - 306p^3T^3 + p^6T^4$
76	smooth		$1 - 12T + 398pT^2 - 12p^3T^3 + p^6T^4$
77	smooth		$1 + 120T + 2990pT^2 + 120p^3T^3 + p^6T^4$
78	smooth		$1 - 120T - 9106pT^2 - 120p^3T^3 + p^6T^4$
79	smooth		$1 + 354T - 4822pT^2 + 354p^3T^3 + p^6T^4$
80	smooth		$1 - 558T + 506pT^2 - 558p^3T^3 + p^6T^4$
81	smooth		$1 + 900T + 15446pT^2 + 900p^3T^3 + p^6T^4$
82	smooth		$1 + 1212T + 13430pT^2 + 1212p^3T^3 + p^6T^4$
83	smooth		$1 - 276T + 14726pT^2 - 276p^3T^3 + p^6T^4$
84	smooth		$1 + 1308T + 12854pT^2 + 1308p^3T^3 + p^6T^4$
85	smooth		$1 + 456T - 6946pT^2 + 456p^3T^3 + p^6T^4$
86	smooth		$1 + 1056T + 10190pT^2 + 1056p^3T^3 + p^6T^4$
87	smooth		$1 + 1464T + 14798pT^2 + 1464p^3T^3 + p^6T^4$
88	smooth		$1 - 132T + 4934pT^2 - 132p^3T^3 + p^6T^4$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 164T - 4794pT^2 + 164p^3T^3 + p^6T^4$
2	smooth		$1 + 746T + 2322pT^2 + 746p^3T^3 + p^6T^4$
3	smooth		$(1 + 10pT + p^3T^2)(1 - 1550T + p^3T^2)$
4	smooth		$1 + 554T + 8826pT^2 + 554p^3T^3 + p^6T^4$
5	smooth		$1 + 620T + 8550pT^2 + 620p^3T^3 + p^6T^4$
6	smooth		$1 + 26T - 126p^2T^2 + 26p^3T^3 + p^6T^4$
7	smooth		$1 - 382T + 10266pT^2 - 382p^3T^3 + p^6T^4$
8	smooth		$1 - 64T + 4734pT^2 - 64p^3T^3 + p^6T^4$
9	smooth		$1 + 824T + 1374pT^2 + 824p^3T^3 + p^6T^4$
10	smooth		$1 + 248T + 3822pT^2 + 248p^3T^3 + p^6T^4$
11	smooth		$1 - 112T - 3666pT^2 - 112p^3T^3 + p^6T^4$
12	smooth		$1 + 380T + 3846pT^2 + 380p^3T^3 + p^6T^4$
13	smooth		$1 + 404T + 4086pT^2 + 404p^3T^3 + p^6T^4$
14	smooth		$1 - 604T + 13446pT^2 - 604p^3T^3 + p^6T^4$
15	smooth		$1 - 820T + 4374pT^2 - 820p^3T^3 + p^6T^4$
16	singular	$-\frac{1}{6}$	$(1 - pT)(1 + 286T + p^3T^2)$
17	smooth		$1 - 1540T + 24822pT^2 - 1540p^3T^3 + p^6T^4$
18	smooth		$1 - 952T + 14862pT^2 - 952p^3T^3 + p^6T^4$
19	smooth		$1 - 58T - 12198pT^2 - 58p^3T^3 + p^6T^4$
20	smooth		$1 - 1678T + 23010pT^2 - 1678p^3T^3 + p^6T^4$
21	smooth		$1 - 292T + 7494pT^2 - 292p^3T^3 + p^6T^4$

Continued on the following page

$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 + 1262T + 19074pT^2 + 1262p^3T^3 + p^6T^4$
23	smooth		$1 - 1228T + 12822pT^2 - 1228p^3T^3 + p^6T^4$
24	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 1330T + p^3T^2)$
25	smooth		$1 + 1712T + 17310pT^2 + 1712p^3T^3 + p^6T^4$
26	smooth		$1 + 1016T + 11502pT^2 + 1016p^3T^3 + p^6T^4$
27	smooth		$1 + 440T + 7182pT^2 + 440p^3T^3 + p^6T^4$
28	smooth		$1 + 392T + 7134pT^2 + 392p^3T^3 + p^6T^4$
29	smooth		$1 - 52T + 8454pT^2 - 52p^3T^3 + p^6T^4$
30	smooth		$1 + 1244T + 7518pT^2 + 1244p^3T^3 + p^6T^4$
31	smooth		$1 + 32T - 498pT^2 + 32p^3T^3 + p^6T^4$
32	smooth		$1 + 704T + 1182pT^2 + 704p^3T^3 + p^6T^4$
33	smooth		$1 + 1814T + 19482pT^2 + 1814p^3T^3 + p^6T^4$
34	smooth		$1 - 1360T + 19998pT^2 - 1360p^3T^3 + p^6T^4$
35	smooth		$1 + 1178T + 7722pT^2 + 1178p^3T^3 + p^6T^4$
36	smooth		$1 + 1136T + 16590pT^2 + 1136p^3T^3 + p^6T^4$
37	smooth		$1 + 1694T + 19650pT^2 + 1694p^3T^3 + p^6T^4$
38	smooth		$1 - 1696T + 15486pT^2 - 1696p^3T^3 + p^6T^4$
39	smooth		$1 - 28T + 18342pT^2 - 28p^3T^3 + p^6T^4$
40	smooth		$1 + 26T + 6570pT^2 + 26p^3T^3 + p^6T^4$
41	smooth		$1 + 164T + 606pT^2 + 164p^3T^3 + p^6T^4$
42	smooth		$1 + 608T + 5694pT^2 + 608p^3T^3 + p^6T^4$
43	smooth		$1 + 1160T + 6318pT^2 + 1160p^3T^3 + p^6T^4$
44	smooth		$1 + 494T - 1422pT^2 + 494p^3T^3 + p^6T^4$
45	smooth		$1 - 64T - 2322pT^2 - 64p^3T^3 + p^6T^4$
46	smooth		$1 - 796T + 9078pT^2 - 796p^3T^3 + p^6T^4$
47	smooth		$1 - 1702T + 20394pT^2 - 1702p^3T^3 + p^6T^4$
48	smooth		$1 + 200T + 6942pT^2 + 200p^3T^3 + p^6T^4$
49	smooth		$1 + 8T + 10782pT^2 + 8p^3T^3 + p^6T^4$
50	smooth*	$\frac{3}{2}$	
51	smooth		$1 + 554T + 9186pT^2 + 554p^3T^3 + p^6T^4$
52	smooth		$1 + 1736T + 23886pT^2 + 1736p^3T^3 + p^6T^4$
53	smooth		$1 - 196T + 5718pT^2 - 196p^3T^3 + p^6T^4$
54	smooth		$1 - 22T - 4350pT^2 - 22p^3T^3 + p^6T^4$
55	smooth		$1 + 704T - 1986pT^2 + 704p^3T^3 + p^6T^4$
56	smooth		$1 + 512T - 6498pT^2 + 512p^3T^3 + p^6T^4$
57	smooth		$1 + 506T + 4458pT^2 + 506p^3T^3 + p^6T^4$
58	singular	$-\frac{1}{5}$	$(1 - pT)(1 + 382T + p^3T^2)$
59	smooth		$1 + 608T + 16494pT^2 + 608p^3T^3 + p^6T^4$
60	smooth		$1 - 1744T + 21342pT^2 - 1744p^3T^3 + p^6T^4$
61	smooth		$1 + 488T + 4062pT^2 + 488p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 + 68T + 2166pT^2 + 68p^3T^3 + p^6T^4$
63	smooth		$1 - 1132T + 12918pT^2 - 1132p^3T^3 + p^6T^4$
64	smooth		$1 + 680T - 3090pT^2 + 680p^3T^3 + p^6T^4$
65	singular	$\frac{1}{3}$	$(1 - pT)(1 + 286T + p^3T^2)$
66	smooth		$1 + 218T - 8790pT^2 + 218p^3T^3 + p^6T^4$
67	smooth		$1 + 1070T + 6210pT^2 + 1070p^3T^3 + p^6T^4$
68	smooth		$1 - 4T + 12102pT^2 - 4p^3T^3 + p^6T^4$
69	smooth		$1 + 26T + 11322pT^2 + 26p^3T^3 + p^6T^4$
70	smooth		$1 - 964T + 18486pT^2 - 964p^3T^3 + p^6T^4$
71	smooth		$1 + 836T + 10134pT^2 + 836p^3T^3 + p^6T^4$
72	smooth		$1 + 788T + 8358pT^2 + 788p^3T^3 + p^6T^4$
73	singular	$\frac{1}{4}$	$(1 - pT)(1 - 1106T + p^3T^2)$
74	smooth		$(1 - 8pT + p^3T^2)(1 + 1294T + p^3T^2)$
75	smooth		$1 + 32T + 366pT^2 + 32p^3T^3 + p^6T^4$
76	smooth		$1 + 170T + 11754pT^2 + 170p^3T^3 + p^6T^4$
77	smooth		$1 + 248T + 2526pT^2 + 248p^3T^3 + p^6T^4$
78	smooth		$1 - 148T + 12822pT^2 - 148p^3T^3 + p^6T^4$
79	smooth		$1 - 1888T + 20046pT^2 - 1888p^3T^3 + p^6T^4$
80	smooth		$1 + 524T + 3702pT^2 + 524p^3T^3 + p^6T^4$
81	smooth		$1 + 512T + 10206pT^2 + 512p^3T^3 + p^6T^4$
82	smooth		$1 + 272T + 10110pT^2 + 272p^3T^3 + p^6T^4$
83	smooth		$1 + 1076T + 2886pT^2 + 1076p^3T^3 + p^6T^4$
84	smooth		$1 - 322T + 14754pT^2 - 322p^3T^3 + p^6T^4$
85	smooth		$1 + 140T - 12090pT^2 + 140p^3T^3 + p^6T^4$
86	smooth		$1 - 982T + 14058pT^2 - 982p^3T^3 + p^6T^4$
87	smooth		$1 - 496T + 17550pT^2 - 496p^3T^3 + p^6T^4$
88	smooth		$1 - 178T - 2238pT^2 - 178p^3T^3 + p^6T^4$
89	singular	$\frac{1}{12}$	$(1 - pT)(1 + 1690T + p^3T^2)$
90	smooth		$1 - 1060T + 8454pT^2 - 1060p^3T^3 + p^6T^4$
91	smooth		$1 - 16T + 16302pT^2 - 16p^3T^3 + p^6T^4$
92	smooth		$1 - 748T + 10710pT^2 - 748p^3T^3 + p^6T^4$
93	smooth		$1 - 484T + 2334pT^2 - 484p^3T^3 + p^6T^4$
94	smooth		$1 + 1184T + 18150pT^2 + 1184p^3T^3 + p^6T^4$
95	smooth		$1 + 248T - 2802pT^2 + 248p^3T^3 + p^6T^4$
96	smooth		$1 - 928T + 7614pT^2 - 928p^3T^3 + p^6T^4$

C.6. The ζ -function for a quotient of the 24 cell, AESZ 366

$p = 5$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 6T + p^3T^2)$
2	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 18T + p^3T^2)$
3	singular	$\left\{-\frac{1}{3}, -\frac{1}{8}, -\frac{1}{18}\right\}$	
4	singular	$\frac{1}{24}$	$(1 - pT)(1 - 2T + p^3T^2)$

$p = 7$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 16T + 2pT^2 + 16p^3T^3 + p^6T^4$
2	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 24T + p^3T^2)$
3	smooth		$(1 + 4pT + p^3T^2)(1 - 24T + p^3T^2)$
4	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 8T + p^3T^2)$
5	singular	$\left\{-\frac{1}{4}, \frac{1}{24}, -\frac{1}{18}\right\}$	
6	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 4T + p^3T^2)$

$p = 11$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 20T + 50pT^2 - 20p^3T^3 + p^6T^4$
2	smooth		$(1 - 4pT + p^3T^2)(1 + 12T + p^3T^2)$
3	smooth*	$-\frac{1}{18}$	
4	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 12T + p^3T^2)$
5	smooth		$(1 + 4pT + p^3T^2)(1 - 60T + p^3T^2)$
6	singular	$\frac{1}{24}$	$(1 - pT)(1 + 8T + p^3T^2)$
7	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 52T + p^3T^2)$
8	singular	$-\frac{1}{4}$	$(1 + pT)(1 + 12T + p^3T^2)$
9	smooth		$1 - 32T + 2pT^2 - 32p^3T^3 + p^6T^4$
10	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 36T + p^3T^2)$

$p = 13$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 10T + p^3T^2)$
2	smooth		$1 - 36T + 22pT^2 - 36p^3T^3 + p^6T^4$
3	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 82T + p^3T^2)$
4	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 22T + p^3T^2)$
5	smooth*	$-\frac{1}{18}$	
6	singular	$\frac{1}{24}$	$(1 - pT)(1 + 42T + p^3T^2)$

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$p = 13$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 + 24T + 142pT^2 + 24p^3T^3 + p^6T^4$
8	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 58T + p^3T^2)$
9	smooth		$1 - 12T + 70pT^2 - 12p^3T^3 + p^6T^4$
10	smooth		$1 + 72T + 238pT^2 + 72p^3T^3 + p^6T^4$
11	smooth		$1 + 12T - 74pT^2 + 12p^3T^3 + p^6T^4$
12	smooth		$1 - 56T + 110pT^2 - 56p^3T^3 + p^6T^4$

$p = 17$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 52T + 230pT^2 - 52p^3T^3 + p^6T^4$
2	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 66T + p^3T^2)$
3	smooth		$1 + 12T - 154pT^2 + 12p^3T^3 + p^6T^4$
4	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 30T + p^3T^2)$
5	singular	$\frac{1}{24}$	$(1 - pT)(1 + 2T + p^3T^2)$
6	smooth		$1 + 48T - 2p^2T^2 + 48p^3T^3 + p^6T^4$
7	singular	$-\frac{1}{12}$	$(1 - pT)(1 - 18T + p^3T^2)$
8	smooth		$1 - 60T + 22p^2T^2 - 60p^3T^3 + p^6T^4$
9	smooth		$1 - 4T - 26p^2T^2 - 4p^3T^3 + p^6T^4$
10	smooth		$1 - 36T + 326pT^2 - 36p^3T^3 + p^6T^4$
11	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 14T + p^3T^2)$
12	smooth		$1 + 20T + 278pT^2 + 20p^3T^3 + p^6T^4$
13	smooth		$1 - 28T + 182pT^2 - 28p^3T^3 + p^6T^4$
14	smooth		$1 - 40T + 14pT^2 - 40p^3T^3 + p^6T^4$
15	smooth		$1 - 28T + 182pT^2 - 28p^3T^3 + p^6T^4$
16	smooth*	$-\frac{1}{18}$	

$p = 19$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth*	$-\frac{1}{18}$	
2	smooth		$1 + 72T + 514pT^2 + 72p^3T^3 + p^6T^4$
3	smooth		$1 + 80T + 354pT^2 + 80p^3T^3 + p^6T^4$
4	singular	$\frac{1}{24}$	$(1 - pT)(1 + 124T + p^3T^2)$
5	smooth		$(1 - 4pT + p^3T^2)(1 + 16T + p^3T^2)$
6	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 20T + p^3T^2)$
7	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 100T + p^3T^2)$
8	smooth		$1 - 48T - 158pT^2 - 48p^3T^3 + p^6T^4$
9	smooth		$1 - 64T + 450pT^2 - 64p^3T^3 + p^6T^4$
10	smooth		$1 + 68T + 402pT^2 + 68p^3T^3 + p^6T^4$

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$p = 19$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	singular	$-\frac{1}{12}$	$(1 + pT)(1 + 100T + p^3T^2)$
12	smooth		$(1 + 8pT + p^3T^2)(1 + 4T + p^3T^2)$
13	smooth		$1 - 52T + 6p^2T^2 - 52p^3T^3 + p^6T^4$
14	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 68T + p^3T^2)$
15	smooth		$(1 - 4pT + p^3T^2)(1 + 88T + p^3T^2)$
16	smooth		$1 + 514pT^2 + p^6T^4$
17	smooth		$1 + 32T + 162pT^2 + 32p^3T^3 + p^6T^4$
18	smooth		$1 + 120T + 706pT^2 + 120p^3T^3 + p^6T^4$

$p = 23$			
φ	smooth/sing.	singularity	$R(T)$
1	singular	$\frac{1}{24}$	$(1 - pT)(1 - 76T + p^3T^2)$
2	smooth		$1 - 48T + 98pT^2 - 48p^3T^3 + p^6T^4$
3	smooth		$1 - 96T - 94pT^2 - 96p^3T^3 + p^6T^4$
4	smooth		$1 + 116T + 578pT^2 + 116p^3T^3 + p^6T^4$
5	smooth		$(1 + p^3T^2)(1 - 120T + p^3T^2)$
6	smooth		$1 + 248T + 1442pT^2 + 248p^3T^3 + p^6T^4$
7	smooth		$(1 + p^3T^2)(1 - 192T + p^3T^2)$
8	smooth		$1 + 104T + 386pT^2 + 104p^3T^3 + p^6T^4$
9	smooth		$1 - 144T + 1250pT^2 - 144p^3T^3 + p^6T^4$
10	smooth		$1 - 80T + 290pT^2 - 80p^3T^3 + p^6T^4$
11	smooth		$1 + 12T + 386pT^2 + 12p^3T^3 + p^6T^4$
12	smooth		$1 + 144T + 1250pT^2 + 144p^3T^3 + p^6T^4$
13	smooth		$1 + 72T + 482pT^2 + 72p^3T^3 + p^6T^4$
14	smooth*	$-\frac{1}{18}$	
15	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 168T + p^3T^2)$
16	smooth		$1 + 100T + 770pT^2 + 100p^3T^3 + p^6T^4$
17	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 216T + p^3T^2)$
18	smooth		$1 - 36T - 190pT^2 - 36p^3T^3 + p^6T^4$
19	smooth		$1 - 128T + 866pT^2 - 128p^3T^3 + p^6T^4$
20	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 132T + p^3T^2)$
21	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 72T + p^3T^2)$
22	smooth		$1 - 12T + 194pT^2 - 12p^3T^3 + p^6T^4$

$p = 29$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$(1 + 2pT + p^3T^2)(1 - 198T + p^3T^2)$
2	smooth		$1 - 260T + 1430pT^2 - 260p^3T^3 + p^6T^4$

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$p = 29$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 + 108T + 758pT^2 + 108p^3T^3 + p^6T^4$
4	smooth		$1 - 20T + 1142pT^2 - 20p^3T^3 + p^6T^4$
5	smooth		$1 - 44T + 710pT^2 - 44p^3T^3 + p^6T^4$
6	smooth		$1 + 112T + 62pT^2 + 112p^3T^3 + p^6T^4$
7	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 246T + p^3T^2)$
8	smooth*	$-\frac{1}{18}$	
9	smooth		$1 + 12T - 970pT^2 + 12p^3T^3 + p^6T^4$
10	smooth		$1 + 204T + 1718pT^2 + 204p^3T^3 + p^6T^4$
11	smooth		$1 + 64T + 926pT^2 + 64p^3T^3 + p^6T^4$
12	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 234T + p^3T^2)$
13	smooth		$1 - 72T + 14pT^2 - 72p^3T^3 + p^6T^4$
14	smooth		$1 - 260T + 1814pT^2 - 260p^3T^3 + p^6T^4$
15	smooth		$1 - 76T + 518pT^2 - 76p^3T^3 + p^6T^4$
16	smooth		$(1 - 6pT + p^3T^2)(1 + 250T + p^3T^2)$
17	smooth		$(1 - 6pT + p^3T^2)(1 + 154T + p^3T^2)$
18	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 90T + p^3T^2)$
19	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 230T + p^3T^2)$
20	smooth		$1 + 268T + 46p^2T^2 + 268p^3T^3 + p^6T^4$
21	smooth		$1 + 4T + 1190pT^2 + 4p^3T^3 + p^6T^4$
22	smooth		$1 - 36T - 682pT^2 - 36p^3T^3 + p^6T^4$
23	singular	$\frac{1}{24}$	$(1 - pT)(1 - 254T + p^3T^2)$
24	smooth		$1 - 192T + 1502pT^2 - 192p^3T^3 + p^6T^4$
25	smooth		$(1 - 6pT + p^3T^2)(1 - 86T + p^3T^2)$
26	smooth		$1 + 84T - 2p^2T^2 + 84p^3T^3 + p^6T^4$
27	smooth		$1 - 260T + 2102pT^2 - 260p^3T^3 + p^6T^4$
28	smooth		$1 + 268T + 1526pT^2 + 268p^3T^3 + p^6T^4$

$p = 31$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 12T + 1666pT^2 - 12p^3T^3 + p^6T^4$
2	smooth		$1 + 56T + 258pT^2 + 56p^3T^3 + p^6T^4$
3	smooth		$1 + 80T - 126pT^2 + 80p^3T^3 + p^6T^4$
4	smooth		$1 + 88T + 2pT^2 + 88p^3T^3 + p^6T^4$
5	smooth		$1 + 152T + 834pT^2 + 152p^3T^3 + p^6T^4$
6	smooth		$1 + 280T + 1538pT^2 + 280p^3T^3 + p^6T^4$
7	smooth		$1 + 88T + 578pT^2 + 88p^3T^3 + p^6T^4$
8	smooth		$1 - 120T + 130pT^2 - 120p^3T^3 + p^6T^4$
9	smooth		$1 - 44T + 1346pT^2 - 44p^3T^3 + p^6T^4$
10	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 288T + p^3T^2)$

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$p = 31$, continued			
φ	smooth/sing.	singularity	$R(T)$
11	smooth		$1 - 360T + 2338pT^2 - 360p^3T^3 + p^6T^4$
12	smooth*	$-\frac{1}{18}$	
13	smooth		$1 + 176T + 1410pT^2 + 176p^3T^3 + p^6T^4$
14	smooth		$1 + 40T - 1150pT^2 + 40p^3T^3 + p^6T^4$
15	smooth		$1 + 120T + 1282pT^2 + 120p^3T^3 + p^6T^4$
16	smooth		$(1 + 4pT + p^3T^2)(1 - 48T + p^3T^2)$
17	smooth		$1 + 24T - 1214pT^2 + 24p^3T^3 + p^6T^4$
18	singular	$-\frac{1}{12}$	$(1 + pT)(1 + 16T + p^3T^2)$
19	smooth		$1 + 16T + 386pT^2 + 16p^3T^3 + p^6T^4$
20	smooth		$1 + 120T + 514pT^2 + 120p^3T^3 + p^6T^4$
21	smooth		$(1 - 8pT + p^3T^2)(1 - 8T + p^3T^2)$
22	singular	$\frac{1}{24}$	$(1 - pT)(1 + 72T + p^3T^2)$
23	singular	$-\frac{1}{4}$	$(1 + pT)(1 + 112T + p^3T^2)$
24	smooth		$1 + 232T + 962pT^2 + 232p^3T^3 + p^6T^4$
25	smooth		$1 - 36T + 322pT^2 - 36p^3T^3 + p^6T^4$
26	smooth		$1 - 8T - 766pT^2 - 8p^3T^3 + p^6T^4$
27	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 152T + p^3T^2)$
28	smooth		$1 - 24T + 1282pT^2 - 24p^3T^3 + p^6T^4$
29	smooth		$1 - 100T + 1602pT^2 - 100p^3T^3 + p^6T^4$
30	smooth		$1 + 104T + 30p^2T^2 + 104p^3T^3 + p^6T^4$

$p = 37$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 172T + 1910pT^2 + 172p^3T^3 + p^6T^4$
2	smooth*	$-\frac{1}{18}$	
3	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 226T + p^3T^2)$
4	smooth		$1 - 64T - 642pT^2 - 64p^3T^3 + p^6T^4$
5	smooth		$1 + 360T + 1678pT^2 + 360p^3T^3 + p^6T^4$
6	smooth		$1 - 20T + 758pT^2 - 20p^3T^3 + p^6T^4$
7	smooth		$1 + 408T + 3694pT^2 + 408p^3T^3 + p^6T^4$
8	smooth		$1 + 96T + 1150pT^2 + 96p^3T^3 + p^6T^4$
9	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 110T + p^3T^2)$
10	smooth		$1 + 28T + 2006pT^2 + 28p^3T^3 + p^6T^4$
11	smooth		$1 + 460T + 3254pT^2 + 460p^3T^3 + p^6T^4$
12	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 34T + p^3T^2)$
13	smooth		$1 + 4pT + 1094pT^2 + 4p^4T^3 + p^6T^4$
14	smooth		$1 - 292T + 2646pT^2 - 292p^3T^3 + p^6T^4$
15	smooth		$1 + 120T + 22p^2T^2 + 120p^3T^3 + p^6T^4$
16	smooth		$1 - 36T - 170pT^2 - 36p^3T^3 + p^6T^4$

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$p = 37$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	singular	$\frac{1}{24}$	$(1 - pT)(1 - 398T + p^3T^2)$
18	smooth		$1 - 60T - 1178pT^2 - 60p^3T^3 + p^6T^4$
19	smooth		$1 - 92T + 14p^2T^2 - 92p^3T^3 + p^6T^4$
20	smooth		$1 + 260T + 870pT^2 + 260p^3T^3 + p^6T^4$
21	smooth		$1 + 20T + 2310pT^2 + 20p^3T^3 + p^6T^4$
22	smooth		$1 + 196T + 1382pT^2 + 196p^3T^3 + p^6T^4$
23	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 34T + p^3T^2)$
24	smooth		$1 - 272T + 2270pT^2 - 272p^3T^3 + p^6T^4$
25	smooth		$1 + 16pT + 4766pT^2 + 16p^4T^3 + p^6T^4$
26	smooth		$1 - 268T + 1926pT^2 - 268p^3T^3 + p^6T^4$
27	smooth		$1 - 124T + 294pT^2 - 124p^3T^3 + p^6T^4$
28	smooth		$1 + 220T + 470pT^2 + 220p^3T^3 + p^6T^4$
29	smooth		$1 - 36T - 938pT^2 - 36p^3T^3 + p^6T^4$
30	smooth		$1 + 316T + 1814pT^2 + 316p^3T^3 + p^6T^4$
31	smooth		$1 - 156T + 1318pT^2 - 156p^3T^3 + p^6T^4$
32	smooth		$1 + 292T + 2726pT^2 + 292p^3T^3 + p^6T^4$
33	smooth		$1 - 36T - 938pT^2 - 36p^3T^3 + p^6T^4$
34	smooth		$(1 - 10pT + p^3T^2)(1 + 322T + p^3T^2)$
35	smooth		$1 - 116T + 182pT^2 - 116p^3T^3 + p^6T^4$
36	smooth		$1 - 188T + 230pT^2 - 188p^3T^3 + p^6T^4$

$p = 41$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 60T - 58pT^2 + 60p^3T^3 + p^6T^4$
2	smooth		$1 + 132T - 778pT^2 + 132p^3T^3 + p^6T^4$
3	smooth		$1 - 304T + 2174pT^2 - 304p^3T^3 + p^6T^4$
4	smooth		$1 - 120T + 1550pT^2 - 120p^3T^3 + p^6T^4$
5	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 438T + p^3T^2)$
6	smooth		$1 - 276T + 2534pT^2 - 276p^3T^3 + p^6T^4$
7	smooth		$1 + 76T - 730pT^2 + 76p^3T^3 + p^6T^4$
8	smooth		$1 + 4pT + 950pT^2 + 4p^4T^3 + p^6T^4$
9	smooth		$1 - 152T - 754pT^2 - 152p^3T^3 + p^6T^4$
10	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 246T + p^3T^2)$
11	smooth		$1 + 156T + 134pT^2 + 156p^3T^3 + p^6T^4$
12	singular	$\frac{1}{24}$	$(1 - pT)(1 - 462T + p^3T^2)$
13	smooth		$(1 - 10pT + p^3T^2)(1 + 78T + p^3T^2)$
14	smooth		$1 + 60T + 1382pT^2 + 60p^3T^3 + p^6T^4$
15	smooth		$1 + 152T + 1070pT^2 + 152p^3T^3 + p^6T^4$
16	smooth		$1 + 204T + 38pT^2 + 204p^3T^3 + p^6T^4$

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$p = 41$, continued			
φ	smooth/sing.	singularity	$R(T)$
17	singular	$-\frac{1}{12}$	$(1 - pT)(1 - 90T + p^3T^2)$
18	smooth		$1 + 128T - 418pT^2 + 128p^3T^3 + p^6T^4$
19	smooth		$1 - 252T + 1526pT^2 - 252p^3T^3 + p^6T^4$
20	smooth		$1 - 324T + 3014pT^2 - 324p^3T^3 + p^6T^4$
21	smooth		$1 + 132T + 950pT^2 + 132p^3T^3 + p^6T^4$
22	smooth		$1 + 28T + 518pT^2 + 28p^3T^3 + p^6T^4$
23	smooth		$1 + 504T + 3566pT^2 + 504p^3T^3 + p^6T^4$
24	smooth		$1 + 1694pT^2 + p^6T^4$
25	smooth*	$-\frac{1}{18}$	
26	smooth		$1 + 92T + 134pT^2 + 92p^3T^3 + p^6T^4$
27	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 122T + p^3T^2)$
28	smooth		$1 - 348T + 2678pT^2 - 348p^3T^3 + p^6T^4$
29	smooth		$1 - 44T + 2390pT^2 - 44p^3T^3 + p^6T^4$
30	smooth		$1 - 44T + 1910pT^2 - 44p^3T^3 + p^6T^4$
31	smooth		$1 + 228T - 202pT^2 + 228p^3T^3 + p^6T^4$
32	smooth		$1 + 80T - 898pT^2 + 80p^3T^3 + p^6T^4$
33	smooth		$1 + 20T - 1450pT^2 + 20p^3T^3 + p^6T^4$
34	smooth		$1 + 240T + 2942pT^2 + 240p^3T^3 + p^6T^4$
35	smooth		$1 - 44T + 470pT^2 - 44p^3T^3 + p^6T^4$
36	smooth		$1 - 80T - 1858pT^2 - 80p^3T^3 + p^6T^4$
37	smooth		$1 - 420T + 4070pT^2 - 420p^3T^3 + p^6T^4$
38	smooth		$1 + 12T + 1190pT^2 + 12p^3T^3 + p^6T^4$
39	smooth		$1 + 140T + 1574pT^2 + 140p^3T^3 + p^6T^4$
40	smooth		$1 - 476T + 3446pT^2 - 476p^3T^3 + p^6T^4$

$p = 43$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 20T - 174pT^2 + 20p^3T^3 + p^6T^4$
2	smooth		$1 + 248T + 1218pT^2 + 248p^3T^3 + p^6T^4$
3	smooth		$(1 + 4pT + p^3T^2)(1 - 348T + p^3T^2)$
4	smooth		$1 - 264T + 1186pT^2 - 264p^3T^3 + p^6T^4$
5	smooth		$1 + 392T + 3042pT^2 + 392p^3T^3 + p^6T^4$
6	smooth		$1 + 696T + 5986pT^2 + 696p^3T^3 + p^6T^4$
7	smooth		$1 + 144T - 1790pT^2 + 144p^3T^3 + p^6T^4$
8	smooth		$1 - 504T + 4834pT^2 - 504p^3T^3 + p^6T^4$
9	singular	$\frac{1}{24}$	$(1 - pT)(1 - 212T + p^3T^2)$
10	smooth		$(1 + 4pT + p^3T^2)(1 + 364T + p^3T^2)$
11	smooth		$1 + 272T + 2562pT^2 + 272p^3T^3 + p^6T^4$
12	smooth		$(1 - 12pT + p^3T^2)(1 + 292T + p^3T^2)$

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$p = 43$, continued			
φ	smooth/sing.	singularity	$R(T)$
13	smooth		$1 - 44T + 722pT^2 - 44p^3T^3 + p^6T^4$
14	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 188T + p^3T^2)$
15	smooth		$1 + 128T + 2754pT^2 + 128p^3T^3 + p^6T^4$
16	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 32T + p^3T^2)$
17	smooth		$1 - 232T + 642pT^2 - 232p^3T^3 + p^6T^4$
18	smooth		$1 + 16T - 382pT^2 + 16p^3T^3 + p^6T^4$
19	smooth		$1 - 192T + 3202pT^2 - 192p^3T^3 + p^6T^4$
20	smooth		$1 + 212T + 1170pT^2 + 212p^3T^3 + p^6T^4$
21	smooth		$1 - 212T + 3314pT^2 - 212p^3T^3 + p^6T^4$
22	smooth		$1 + 176T - 126pT^2 + 176p^3T^3 + p^6T^4$
23	smooth		$1 + 16T + 386pT^2 + 16p^3T^3 + p^6T^4$
24	smooth		$1 + 256T + 2114pT^2 + 256p^3T^3 + p^6T^4$
25	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 452T + p^3T^2)$
26	smooth		$1 + 752T + 6402pT^2 + 752p^3T^3 + p^6T^4$
27	smooth		$1 + 96T - 542pT^2 + 96p^3T^3 + p^6T^4$
28	smooth		$1 - 36T - 14pT^2 - 36p^3T^3 + p^6T^4$
29	smooth		$1 + 236T + 1650pT^2 + 236p^3T^3 + p^6T^4$
30	smooth		$1 - 24T + 3298pT^2 - 24p^3T^3 + p^6T^4$
31	smooth*	$-\frac{1}{18}$	
32	singular	$-\frac{1}{4}$	$(1 + pT)(1 + 172T + p^3T^2)$
33	smooth		$1 - 240T + 3586pT^2 - 240p^3T^3 + p^6T^4$
34	smooth		$1 + 24T + 2338pT^2 + 24p^3T^3 + p^6T^4$
35	smooth		$1 + 480T + 70p^2T^2 + 480p^3T^3 + p^6T^4$
36	smooth		$1 - 372T + 1906pT^2 - 372p^3T^3 + p^6T^4$
37	smooth		$1 - 180T + 3826pT^2 - 180p^3T^3 + p^6T^4$
38	smooth		$1 + 136T - 286pT^2 + 136p^3T^3 + p^6T^4$
39	smooth		$(1 + 4pT + p^3T^2)(1 - 92T + p^3T^2)$
40	smooth		$1 - 16T - 1278pT^2 - 16p^3T^3 + p^6T^4$
41	smooth		$1 + 276T + 1618pT^2 + 276p^3T^3 + p^6T^4$
42	smooth		$1 + 128T - 318pT^2 + 128p^3T^3 + p^6T^4$

$p = 47$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 200T - 574pT^2 - 200p^3T^3 + p^6T^4$
2	singular	$\frac{1}{24}$	$(1 - pT)(1 + 264T + p^3T^2)$
3	smooth		$1 + 840T + 7106pT^2 + 840p^3T^3 + p^6T^4$
4	smooth		$1 + 664T + 5186pT^2 + 664p^3T^3 + p^6T^4$
5	smooth		$1 - 128T + 3650pT^2 - 128p^3T^3 + p^6T^4$
6	smooth		$1 - 232T + 3650pT^2 - 232p^3T^3 + p^6T^4$

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$p = 47$, continued			
φ	smooth/sing.	singularity	$R(T)$
7	smooth		$1 - 216T + 1538pT^2 - 216p^3T^3 + p^6T^4$
8	smooth		$1 - 36T - 190pT^2 - 36p^3T^3 + p^6T^4$
9	smooth		$1 - 164T + 3650pT^2 - 164p^3T^3 + p^6T^4$
10	smooth		$1 - 100T + 386pT^2 - 100p^3T^3 + p^6T^4$
11	smooth		$1 + 280T + 1730pT^2 + 280p^3T^3 + p^6T^4$
12	smooth		$1 - 168T - 190pT^2 - 168p^3T^3 + p^6T^4$
13	smooth*	$-\frac{1}{18}$	
14	smooth		$1 - 264T + 194pT^2 - 264p^3T^3 + p^6T^4$
15	smooth		$1 - 60T + 3074pT^2 - 60p^3T^3 + p^6T^4$
16	smooth		$1 - 428T + 3266pT^2 - 428p^3T^3 + p^6T^4$
17	smooth		$1 + 552T + 4034pT^2 + 552p^3T^3 + p^6T^4$
18	smooth		$1 - 856T + 7586pT^2 - 856p^3T^3 + p^6T^4$
19	smooth		$1 + 104T - 958pT^2 + 104p^3T^3 + p^6T^4$
20	smooth		$1 - 136T + 1730pT^2 - 136p^3T^3 + p^6T^4$
21	smooth		$1 + 372T + 3266pT^2 + 372p^3T^3 + p^6T^4$
22	smooth		$1 - 36T + 3650pT^2 - 36p^3T^3 + p^6T^4$
23	smooth		$1 - 4T + 1538pT^2 - 4p^3T^3 + p^6T^4$
24	smooth		$1 - 80T + 1730pT^2 - 80p^3T^3 + p^6T^4$
25	smooth		$1 - 248T + 3266pT^2 - 248p^3T^3 + p^6T^4$
26	smooth		$1 + 128T + 2114pT^2 + 128p^3T^3 + p^6T^4$
27	smooth		$1 - 216T - 190pT^2 - 216p^3T^3 + p^6T^4$
28	smooth		$1 - 72T + 1730pT^2 - 72p^3T^3 + p^6T^4$
29	smooth		$1 + 792T + 7106pT^2 + 792p^3T^3 + p^6T^4$
30	smooth		$1 - 384T + 1634pT^2 - 384p^3T^3 + p^6T^4$
31	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 256T + p^3T^2)$
32	smooth		$1 - 36T + 3842pT^2 - 36p^3T^3 + p^6T^4$
33	smooth		$1 + 160T - 1150pT^2 + 160p^3T^3 + p^6T^4$
34	smooth		$1 + 528T + 3650pT^2 + 528p^3T^3 + p^6T^4$
35	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 192T + p^3T^2)$
36	smooth		$1 - 592T + 5186pT^2 - 592p^3T^3 + p^6T^4$
37	smooth		$1 - 460T + 4994pT^2 - 460p^3T^3 + p^6T^4$
38	smooth		$(1 - 8pT + p^3T^2)(1 - 48T + p^3T^2)$
39	smooth		$1 + 576T + 4034pT^2 + 576p^3T^3 + p^6T^4$
40	smooth		$1 + 216T + 962pT^2 + 216p^3T^3 + p^6T^4$
41	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 204T + p^3T^2)$
42	smooth		$1 - 584T + 5186pT^2 - 584p^3T^3 + p^6T^4$
43	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 432T + p^3T^2)$
44	smooth		$1 - 120T + 2210pT^2 - 120p^3T^3 + p^6T^4$
45	smooth		$1 + 8T - 190pT^2 + 8p^3T^3 + p^6T^4$
46	smooth		$1 + 52T - 2878pT^2 + 52p^3T^3 + p^6T^4$

$p = 53$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 396T + 4550pT^2 - 396p^3T^3 + p^6T^4$
2	smooth		$1 - 556T + 4742pT^2 - 556p^3T^3 + p^6T^4$
3	smooth		$1 - 444T + 5510pT^2 - 444p^3T^3 + p^6T^4$
4	smooth		$1 - 76T + 1094pT^2 - 76p^3T^3 + p^6T^4$
5	smooth		$1 + 92T + 2774pT^2 + 92p^3T^3 + p^6T^4$
6	smooth		$1 + 200T - 1906pT^2 + 200p^3T^3 + p^6T^4$
7	smooth		$1 - 744T + 7598pT^2 - 744p^3T^3 + p^6T^4$
8	smooth		$1 + 672T + 5630pT^2 + 672p^3T^3 + p^6T^4$
9	smooth		$(1 + 10pT + p^3T^2)(1 + 306T + p^3T^2)$
10	smooth		$1 - 760T + 6158pT^2 - 760p^3T^3 + p^6T^4$
11	smooth		$1 - 340T + 1814pT^2 - 340p^3T^3 + p^6T^4$
12	smooth		$1 + 296T + 2126pT^2 + 296p^3T^3 + p^6T^4$
13	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 558T + p^3T^2)$
14	smooth		$1 + 156T + 2966pT^2 + 156p^3T^3 + p^6T^4$
15	smooth		$1 + 44T + 1718pT^2 + 44p^3T^3 + p^6T^4$
16	smooth		$1 - 796T + 7334pT^2 - 796p^3T^3 + p^6T^4$
17	smooth		$(1 - 6pT + p^3T^2)(1 + 594T + p^3T^2)$
18	smooth		$(1 - 6pT + p^3T^2)(1 + 146T + p^3T^2)$
19	smooth		$1 - 180T + 1910pT^2 - 180p^3T^3 + p^6T^4$
20	smooth		$1 + 452T + 2150pT^2 + 452p^3T^3 + p^6T^4$
21	smooth		$1 + 492T + 94p^2T^2 + 492p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{12}$	$(1 - pT)(1 - 414T + p^3T^2)$
23	smooth		$1 - 696T + 6158pT^2 - 696p^3T^3 + p^6T^4$
24	smooth		$1 - 12T + 2246pT^2 - 12p^3T^3 + p^6T^4$
25	smooth		$1 - 216T + 974pT^2 - 216p^3T^3 + p^6T^4$
26	smooth		$(1 - 6pT + p^3T^2)(1 + 594T + p^3T^2)$
27	smooth		$1 - 168T - 1042pT^2 - 168p^3T^3 + p^6T^4$
28	smooth		$1 - 120T - 1330pT^2 - 120p^3T^3 + p^6T^4$
29	smooth		$1 - 272T + 3230pT^2 - 272p^3T^3 + p^6T^4$
30	smooth		$1 + 84T - 1114pT^2 + 84p^3T^3 + p^6T^4$
31	smooth		$1 + 132T - 2458pT^2 + 132p^3T^3 + p^6T^4$
32	smooth		$1 + 428T + 5366pT^2 + 428p^3T^3 + p^6T^4$
33	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 222T + p^3T^2)$
34	smooth		$1 - 444T + 4070pT^2 - 444p^3T^3 + p^6T^4$
35	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 338T + p^3T^2)$
36	smooth		$1 + 80T + 2270pT^2 + 80p^3T^3 + p^6T^4$
37	smooth		$1 + 172T + 2294pT^2 + 172p^3T^3 + p^6T^4$
38	smooth		$1 - 180T + 4406pT^2 - 180p^3T^3 + p^6T^4$
39	smooth		$1 - 28T - 3034pT^2 - 28p^3T^3 + p^6T^4$
40	smooth		$1 + 148T - 1018pT^2 + 148p^3T^3 + p^6T^4$

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$p = 53$, continued			
φ	smooth/sing.	singularity	$R(T)$
41	smooth		$1 - 444T + 3686pT^2 - 444p^3T^3 + p^6T^4$
42	singular	$\frac{1}{24}$	$(1 - pT)(1 + 162T + p^3T^2)$
43	smooth		$1 - 428T + 5894pT^2 - 428p^3T^3 + p^6T^4$
44	smooth		$1 + 208T + 2654pT^2 + 208p^3T^3 + p^6T^4$
45	smooth		$1 - 20T + 1910pT^2 - 20p^3T^3 + p^6T^4$
46	smooth		$1 - 132T + 3446pT^2 - 132p^3T^3 + p^6T^4$
47	smooth		$1 + 380T + 2390pT^2 + 380p^3T^3 + p^6T^4$
48	smooth		$1 + 92T - 298pT^2 + 92p^3T^3 + p^6T^4$
49	smooth		$1 - 108T + 2054pT^2 - 108p^3T^3 + p^6T^4$
50	smooth*	$-\frac{1}{18}$	
51	smooth		$1 + 364T + 4214pT^2 + 364p^3T^3 + p^6T^4$
52	smooth		$(1 + 6pT + p^3T^2)(1 - 654T + p^3T^2)$

$p = 59$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 84T - 2062pT^2 - 84p^3T^3 + p^6T^4$
2	smooth		$1 - 880T + 8642pT^2 - 880p^3T^3 + p^6T^4$
3	smooth		$1 - 168T + 866pT^2 - 168p^3T^3 + p^6T^4$
4	smooth		$1 - 464T + 7106pT^2 - 464p^3T^3 + p^6T^4$
5	smooth		$1 - 900T + 8690pT^2 - 900p^3T^3 + p^6T^4$
6	smooth		$1 + 96T + 770pT^2 + 96p^3T^3 + p^6T^4$
7	smooth		$1 + 96T + 5762pT^2 + 96p^3T^3 + p^6T^4$
8	smooth		$1 + 312T + 3938pT^2 + 312p^3T^3 + p^6T^4$
9	smooth		$1 - 328T - 958pT^2 - 328p^3T^3 + p^6T^4$
10	smooth		$1 - 48T + 2594pT^2 - 48p^3T^3 + p^6T^4$
11	smooth		$1 - 104T - 3934pT^2 - 104p^3T^3 + p^6T^4$
12	smooth		$1 + 40T + 1826pT^2 + 40p^3T^3 + p^6T^4$
13	smooth		$(1 - 8pT + p^3T^2)(1 + 588T + p^3T^2)$
14	smooth		$1 - 8T + 3842pT^2 - 8p^3T^3 + p^6T^4$
15	smooth		$1 - 364T + 6674pT^2 - 364p^3T^3 + p^6T^4$
16	smooth		$1 - 60T + 2066pT^2 - 60p^3T^3 + p^6T^4$
17	smooth		$(1 + 4pT + p^3T^2)(1 - 132T + p^3T^2)$
18	smooth		$1 - 264T + 2402pT^2 - 264p^3T^3 + p^6T^4$
19	smooth		$1 - 56T + 1826pT^2 - 56p^3T^3 + p^6T^4$
20	smooth		$1 - 24T - 3166pT^2 - 24p^3T^3 + p^6T^4$
21	smooth		$1 + 456T + 5282pT^2 + 456p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 420T + p^3T^2)$
23	smooth		$1 - 188T - 1966pT^2 - 188p^3T^3 + p^6T^4$
24	smooth		$1 - 444T + 2450pT^2 - 444p^3T^3 + p^6T^4$

Continued on the following page

$p = 59$, continued			
φ	smooth/sing.	singularity	$R(T)$
25	smooth		$1 + 192T + 2pT^2 + 192p^3T^3 + p^6T^4$
26	smooth		$1 - 260T + 6002pT^2 - 260p^3T^3 + p^6T^4$
27	smooth		$1 + 440T + 98pT^2 + 440p^3T^3 + p^6T^4$
28	smooth		$1 + 208T - 2782pT^2 + 208p^3T^3 + p^6T^4$
29	smooth		$1 - 312T + 7202pT^2 - 312p^3T^3 + p^6T^4$
30	smooth		$1 - 1216T + 11906pT^2 - 1216p^3T^3 + p^6T^4$
31	smooth		$1 - 216T + 2594pT^2 - 216p^3T^3 + p^6T^4$
32	singular	$\frac{1}{24}$	$(1 - pT)(1 + 772T + p^3T^2)$
33	smooth		$(1 + 12pT + p^3T^2)(1 + 252T + p^3T^2)$
34	smooth		$1 + 420T + 3794pT^2 + 420p^3T^3 + p^6T^4$
35	smooth		$(1 + 12pT + p^3T^2)(1 - 852T + p^3T^2)$
36	smooth*	$-\frac{1}{18}$	
37	smooth		$1 - 112T + 1346pT^2 - 112p^3T^3 + p^6T^4$
38	smooth		$1 + 620T + 4658pT^2 + 620p^3T^3 + p^6T^4$
39	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 100T + p^3T^2)$
40	smooth		$1 + 768T + 9410pT^2 + 768p^3T^3 + p^6T^4$
41	smooth		$1 - 136T + 5858pT^2 - 136p^3T^3 + p^6T^4$
42	smooth		$1 - 400T + 5954pT^2 - 400p^3T^3 + p^6T^4$
43	smooth		$1 + 288T + 3458pT^2 + 288p^3T^3 + p^6T^4$
44	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 540T + p^3T^2)$
45	smooth		$1 - 32T + 4610pT^2 - 32p^3T^3 + p^6T^4$
46	smooth		$1 + 612T + 5522pT^2 + 612p^3T^3 + p^6T^4$
47	smooth		$1 - 372T + 1778pT^2 - 372p^3T^3 + p^6T^4$
48	smooth		$1 - 144T + 4802pT^2 - 144p^3T^3 + p^6T^4$
49	smooth		$1 + 400T + 2498pT^2 + 400p^3T^3 + p^6T^4$
50	smooth		$1 - 304T + 3650pT^2 - 304p^3T^3 + p^6T^4$
51	smooth		$1 - 476T + 7058pT^2 - 476p^3T^3 + p^6T^4$
52	smooth		$1 + 180T + 3410pT^2 + 180p^3T^3 + p^6T^4$
53	smooth		$1 - 296T + 5090pT^2 - 296p^3T^3 + p^6T^4$
54	singular	$-\frac{1}{12}$	$(1 + pT)(1 + 684T + p^3T^2)$
55	smooth		$1 - 920T + 8162pT^2 - 920p^3T^3 + p^6T^4$
56	smooth		$1 - 64T + 6530pT^2 - 64p^3T^3 + p^6T^4$
57	smooth		$1 + 296T - 478pT^2 + 296p^3T^3 + p^6T^4$
58	smooth		$1 + 16T + 2690pT^2 + 16p^3T^3 + p^6T^4$

$p = 61$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 756T + 5638pT^2 + 756p^3T^3 + p^6T^4$
2	smooth		$1 + 60T - 2282pT^2 + 60p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
3	smooth		$1 - 792T + 6958pT^2 - 792p^3T^3 + p^6T^4$
4	smooth		$1 + 260T + 4134pT^2 + 260p^3T^3 + p^6T^4$
5	singular	$-\frac{1}{12}$	$(1 - pT)(1 - 422T + p^3T^2)$
6	smooth		$1 + 684T + 142p^2T^2 + 684p^3T^3 + p^6T^4$
7	smooth		$1 - 1020T + 8998pT^2 - 1020p^3T^3 + p^6T^4$
8	smooth		$1 + 1024T + 10718pT^2 + 1024p^3T^3 + p^6T^4$
9	smooth		$1 + 624T + 4798pT^2 + 624p^3T^3 + p^6T^4$
10	smooth		$1 + 420T + 4198pT^2 + 420p^3T^3 + p^6T^4$
11	smooth		$1 + 960T + 9694pT^2 + 960p^3T^3 + p^6T^4$
12	smooth		$1 + 336T + 2686pT^2 + 336p^3T^3 + p^6T^4$
13	smooth		$1 + 740T + 9318pT^2 + 740p^3T^3 + p^6T^4$
14	smooth		$1 + 804T + 6118pT^2 + 804p^3T^3 + p^6T^4$
15	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 110T + p^3T^2)$
16	smooth		$1 + 356T + 4326pT^2 + 356p^3T^3 + p^6T^4$
17	smooth		$1 + 1472T + 15774pT^2 + 1472p^3T^3 + p^6T^4$
18	smooth		$(1 + 10pT + p^3T^2)(1 - 934T + p^3T^2)$
19	smooth		$1 + 468T + 1990pT^2 + 468p^3T^3 + p^6T^4$
20	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 742T + p^3T^2)$
21	smooth		$1 - 56T + 3374pT^2 - 56p^3T^3 + p^6T^4$
22	smooth		$1 - 108T - 4058pT^2 - 108p^3T^3 + p^6T^4$
23	smooth		$1 + 972T + 8374pT^2 + 972p^3T^3 + p^6T^4$
24	smooth		$1 + 84T - 4154pT^2 + 84p^3T^3 + p^6T^4$
25	smooth		$1 + 28T - 2218pT^2 + 28p^3T^3 + p^6T^4$
26	smooth		$1 - 12T + 4102pT^2 - 12p^3T^3 + p^6T^4$
27	smooth		$1 + 60T + 1174pT^2 + 60p^3T^3 + p^6T^4$
28	singular	$\frac{1}{24}$	$(1 - pT)(1 - 30T + p^3T^2)$
29	smooth		$1 + 104T - 1938pT^2 + 104p^3T^3 + p^6T^4$
30	smooth		$1 - 1036T + 10758pT^2 - 1036p^3T^3 + p^6T^4$
31	smooth		$1 - 436T + 6198pT^2 - 436p^3T^3 + p^6T^4$
32	smooth		$1 - 944T + 8894pT^2 - 944p^3T^3 + p^6T^4$
33	smooth		$1 + 252T + 1558pT^2 + 252p^3T^3 + p^6T^4$
34	smooth		$1 + 68T + 1446pT^2 + 68p^3T^3 + p^6T^4$
35	smooth		$1 + 288T + 3934pT^2 + 288p^3T^3 + p^6T^4$
36	smooth		$1 - 12T + 1798pT^2 - 12p^3T^3 + p^6T^4$
37	smooth		$1 + 428T + 6774pT^2 + 428p^3T^3 + p^6T^4$
38	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 902T + p^3T^2)$
39	smooth		$(1 - 14pT + p^3T^2)(1 + 730T + p^3T^2)$
40	smooth		$1 - 512T + 6686pT^2 - 512p^3T^3 + p^6T^4$
41	smooth		$1 + 560T + 6654pT^2 + 560p^3T^3 + p^6T^4$
42	smooth		$1 - 332T + 7046pT^2 - 332p^3T^3 + p^6T^4$

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$p = 61$, continued			
φ	smooth/sing.	singularity	$R(T)$
43	smooth		$1 - 12T + 4102pT^2 - 12p^3T^3 + p^6T^4$
44	smooth*	$-\frac{1}{18}$	
45	smooth		$1 - 540T + 3430pT^2 - 540p^3T^3 + p^6T^4$
46	smooth		$1 - 144T + 4222pT^2 - 144p^3T^3 + p^6T^4$
47	smooth		$1 - 20T + 6134pT^2 - 20p^3T^3 + p^6T^4$
48	smooth		$1 - 12T + 4390pT^2 - 12p^3T^3 + p^6T^4$
49	smooth		$1 + 248T - 114pT^2 + 248p^3T^3 + p^6T^4$
50	smooth		$1 - 276T + 5878pT^2 - 276p^3T^3 + p^6T^4$
51	smooth		$1 - 36T - 2474pT^2 - 36p^3T^3 + p^6T^4$
52	smooth		$1 + 104T - 4050pT^2 + 104p^3T^3 + p^6T^4$
53	smooth		$1 - 36T - 5162pT^2 - 36p^3T^3 + p^6T^4$
54	smooth		$1 - 156T + 1606pT^2 - 156p^3T^3 + p^6T^4$
55	smooth		$1 + 132T - 4826pT^2 + 132p^3T^3 + p^6T^4$
56	smooth		$1 + 140T + 6198pT^2 + 140p^3T^3 + p^6T^4$
57	smooth		$1 + 452T + 7206pT^2 + 452p^3T^3 + p^6T^4$
58	smooth		$1 + 36T + 1606pT^2 + 36p^3T^3 + p^6T^4$
59	smooth		$1 + 668T + 4950pT^2 + 668p^3T^3 + p^6T^4$
60	smooth		$1 - 400T + 6078pT^2 - 400p^3T^3 + p^6T^4$

$p = 67$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 608T + 9858pT^2 + 608p^3T^3 + p^6T^4$
2	smooth		$1 - 72T - 4766pT^2 - 72p^3T^3 + p^6T^4$
3	smooth		$1 - 280T - 3774pT^2 - 280p^3T^3 + p^6T^4$
4	smooth		$1 - 696T + 9154pT^2 - 696p^3T^3 + p^6T^4$
5	smooth		$1 - 504T + 7618pT^2 - 504p^3T^3 + p^6T^4$
6	smooth		$1 - 600T + 4930pT^2 - 600p^3T^3 + p^6T^4$
7	smooth		$1 + 40T + 5570pT^2 + 40p^3T^3 + p^6T^4$
8	smooth		$1 - 960T + 9058pT^2 - 960p^3T^3 + p^6T^4$
9	smooth		$1 + 208T - 2590pT^2 + 208p^3T^3 + p^6T^4$
10	smooth		$1 - 432T + 7330pT^2 - 432p^3T^3 + p^6T^4$
11	smooth		$1 + 80T + 4770pT^2 + 80p^3T^3 + p^6T^4$
12	smooth		$1 + 640T + 2402pT^2 + 640p^3T^3 + p^6T^4$
13	smooth		$1 - 496T + 6498pT^2 - 496p^3T^3 + p^6T^4$
14	singular	$\frac{1}{24}$	$(1 - pT)(1 + 764T + p^3T^2)$
15	smooth		$1 + 264T - 3902pT^2 + 264p^3T^3 + p^6T^4$
16	smooth		$1 - 392T + 6914pT^2 - 392p^3T^3 + p^6T^4$
17	smooth		$1 + 504T + 4738pT^2 + 504p^3T^3 + p^6T^4$
18	smooth		$1 + 104T - 1470pT^2 + 104p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
19	smooth		$1 - 416T + 7394pT^2 - 416p^3T^3 + p^6T^4$
20	smooth		$1 + 800T + 7650pT^2 + 800p^3T^3 + p^6T^4$
21	smooth		$1 + 184T - 4798pT^2 + 184p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 84T + p^3T^2)$
23	smooth		$1 + 328T - 574pT^2 + 328p^3T^3 + p^6T^4$
24	smooth		$1 + 532T + 6866pT^2 + 532p^3T^3 + p^6T^4$
25	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 1024T + p^3T^2)$
26	smooth*	$-\frac{1}{18}$	
27	smooth		$1 + 152T - 1278pT^2 + 152p^3T^3 + p^6T^4$
28	smooth		$1 + 392T + 4290pT^2 + 392p^3T^3 + p^6T^4$
29	smooth		$1 - 164T + 8882pT^2 - 164p^3T^3 + p^6T^4$
30	smooth		$1 + 456T - 1022pT^2 + 456p^3T^3 + p^6T^4$
31	smooth		$1 - 336T + 1954pT^2 - 336p^3T^3 + p^6T^4$
32	smooth		$1 - 108T + 274pT^2 - 108p^3T^3 + p^6T^4$
33	smooth		$1 - 252T + 2962pT^2 - 252p^3T^3 + p^6T^4$
34	smooth		$1 + 216T + 4738pT^2 + 216p^3T^3 + p^6T^4$
35	smooth		$1 - 164T + 38p^2T^2 - 164p^3T^3 + p^6T^4$
36	smooth		$1 - 416T + 5666pT^2 - 416p^3T^3 + p^6T^4$
37	smooth		$1 + 184T + 386pT^2 + 184p^3T^3 + p^6T^4$
38	smooth		$1 + 376T - 1438pT^2 + 376p^3T^3 + p^6T^4$
39	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 332T + p^3T^2)$
40	smooth		$1 - 168T + 6850pT^2 - 168p^3T^3 + p^6T^4$
41	smooth		$1 - 148T + 3762pT^2 - 148p^3T^3 + p^6T^4$
42	smooth		$1 - 512T + 6818pT^2 - 512p^3T^3 + p^6T^4$
43	smooth		$1 + 900T + 9490pT^2 + 900p^3T^3 + p^6T^4$
44	smooth		$1 + 68T - 750pT^2 + 68p^3T^3 + p^6T^4$
45	smooth		$1 + 4pT + 1586pT^2 + 4p^4T^3 + p^6T^4$
46	smooth		$1 + 956T + 10866pT^2 + 956p^3T^3 + p^6T^4$
47	smooth		$1 + 56T - 2046pT^2 + 56p^3T^3 + p^6T^4$
48	smooth		$1 + 468T + 1426pT^2 + 468p^3T^3 + p^6T^4$
49	smooth		$1 + 312T + 5314pT^2 + 312p^3T^3 + p^6T^4$
50	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 140T + p^3T^2)$
51	smooth		$1 + 272T + 1506pT^2 + 272p^3T^3 + p^6T^4$
52	smooth		$1 - 464T + 3362pT^2 - 464p^3T^3 + p^6T^4$
53	smooth		$1 + 136T - 958pT^2 + 136p^3T^3 + p^6T^4$
54	smooth		$1 + 4pT + 3506pT^2 + 4p^4T^3 + p^6T^4$
55	smooth		$1 - 40T - 3198pT^2 - 40p^3T^3 + p^6T^4$
56	smooth		$1 + 116T + 4434pT^2 + 116p^3T^3 + p^6T^4$
57	smooth		$1 + 836T + 4818pT^2 + 836p^3T^3 + p^6T^4$
58	smooth		$1 + 304T + 674pT^2 + 304p^3T^3 + p^6T^4$

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$p = 67$, continued			
φ	smooth/sing.	singularity	$R(T)$
59	smooth		$1 - 144T + 5026pT^2 - 144p^3T^3 + p^6T^4$
60	smooth		$1 + 788T + 6738pT^2 + 788p^3T^3 + p^6T^4$
61	smooth		$1 - 152T + 8642pT^2 - 152p^3T^3 + p^6T^4$
62	smooth		$1 - 128T - 94pT^2 - 128p^3T^3 + p^6T^4$
63	smooth		$1 - 640T + 4578pT^2 - 640p^3T^3 + p^6T^4$
64	smooth		$(1 - 8pT + p^3T^2)(1 + 468T + p^3T^2)$
65	smooth		$1 + 860T + 7410pT^2 + 860p^3T^3 + p^6T^4$
66	smooth		$1 - 776T + 8450pT^2 - 776p^3T^3 + p^6T^4$

$p = 71$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 60T - 4990pT^2 - 60p^3T^3 + p^6T^4$
2	smooth		$1 - 480T - 862pT^2 - 480p^3T^3 + p^6T^4$
3	singular	$\frac{1}{24}$	$(1 - pT)(1 + 236T + p^3T^2)$
4	smooth		$1 + 320T + 674pT^2 + 320p^3T^3 + p^6T^4$
5	smooth		$1 + 536T + 7202pT^2 + 536p^3T^3 + p^6T^4$
6	smooth		$1 + 136T + 3938pT^2 + 136p^3T^3 + p^6T^4$
7	smooth		$1 - 364T + 9986pT^2 - 364p^3T^3 + p^6T^4$
8	smooth		$1 - 352T + 4322pT^2 - 352p^3T^3 + p^6T^4$
9	smooth		$(1 + p^3T^2)(1 + 136T + p^3T^2)$
10	smooth		$(1 + 8pT + p^3T^2)(1 - 936T + p^3T^2)$
11	smooth		$1 + 332T + 6530pT^2 + 332p^3T^3 + p^6T^4$
12	smooth		$1 + 504T + 1634pT^2 + 504p^3T^3 + p^6T^4$
13	smooth		$1 - 912T + 8354pT^2 - 912p^3T^3 + p^6T^4$
14	smooth		$1 - 288T + 2018pT^2 - 288p^3T^3 + p^6T^4$
15	smooth		$1 - 816T + 6434pT^2 - 816p^3T^3 + p^6T^4$
16	smooth		$1 - 456T + 3746pT^2 - 456p^3T^3 + p^6T^4$
17	smooth		$(1 - 8pT + p^3T^2)(1 - 216T + p^3T^2)$
18	smooth		$1 + 688T + 4130pT^2 + 688p^3T^3 + p^6T^4$
19	smooth		$1 - 192T - 3166pT^2 - 192p^3T^3 + p^6T^4$
20	smooth		$1 - 408T + 866pT^2 - 408p^3T^3 + p^6T^4$
21	smooth		$1 - 168T - 4318pT^2 - 168p^3T^3 + p^6T^4$
22	smooth		$1 + 164T - 574pT^2 + 164p^3T^3 + p^6T^4$
23	smooth		$1 + 36T + 9986pT^2 + 36p^3T^3 + p^6T^4$
24	smooth		$1 - 452T + 7298pT^2 - 452p^3T^3 + p^6T^4$
25	smooth		$1 - 804T + 6146pT^2 - 804p^3T^3 + p^6T^4$
26	smooth		$1 - 8T + 5666pT^2 - 8p^3T^3 + p^6T^4$
27	smooth		$1 + 624T + 9506pT^2 + 624p^3T^3 + p^6T^4$
28	smooth		$1 + 800T + 4514pT^2 + 800p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
29	smooth		$1 + 1296T + 14882pT^2 + 1296p^3T^3 + p^6T^4$
30	smooth		$1 + 912T + 8354pT^2 + 912p^3T^3 + p^6T^4$
31	smooth		$1 + 296T + 4514pT^2 + 296p^3T^3 + p^6T^4$
32	smooth		$1 + 360T + 6626pT^2 + 360p^3T^3 + p^6T^4$
33	smooth		$1 + 56T + 2786pT^2 + 56p^3T^3 + p^6T^4$
34	smooth		$1 - 192T + 6050pT^2 - 192p^3T^3 + p^6T^4$
35	smooth		$1 + 56T + 3554pT^2 + 56p^3T^3 + p^6T^4$
36	smooth		$1 - 1012T + 8642pT^2 - 1012p^3T^3 + p^6T^4$
37	smooth		$1 - 1232T + 13730pT^2 - 1232p^3T^3 + p^6T^4$
38	smooth		$1 - 16pT + 9698pT^2 - 16p^4T^3 + p^6T^4$
39	smooth		$1 - 872T + 9026pT^2 - 872p^3T^3 + p^6T^4$
40	smooth		$1 + 12T - 4798pT^2 + 12p^3T^3 + p^6T^4$
41	smooth		$1 - 120T + 8162pT^2 - 120p^3T^3 + p^6T^4$
42	smooth		$1 + 224T - 1726pT^2 + 224p^3T^3 + p^6T^4$
43	smooth		$1 + 112T + 3362pT^2 + 112p^3T^3 + p^6T^4$
44	smooth		$1 + 108T - 1342pT^2 + 108p^3T^3 + p^6T^4$
45	smooth		$1 - 176T + 290pT^2 - 176p^3T^3 + p^6T^4$
46	smooth		$1 - 4pT + 6722pT^2 - 4p^4T^3 + p^6T^4$
47	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 328T + p^3T^2)$
48	smooth		$1 - 124T + 6146pT^2 - 124p^3T^3 + p^6T^4$
49	smooth		$1 - 1128T + 14114pT^2 - 1128p^3T^3 + p^6T^4$
50	smooth		$1 + 800T + 7298pT^2 + 800p^3T^3 + p^6T^4$
51	smooth		$1 + 264T + 674pT^2 + 264p^3T^3 + p^6T^4$
52	smooth		$1 + 576T + 10658pT^2 + 576p^3T^3 + p^6T^4$
53	singular	$-\frac{1}{4}$	$(1 + pT)(1 + 840T + p^3T^2)$
54	smooth		$1 - 608T + 8354pT^2 - 608p^3T^3 + p^6T^4$
55	smooth		$1 + 584T + 5090pT^2 + 584p^3T^3 + p^6T^4$
56	smooth		$(1 + 8pT + p^3T^2)(1 - 912T + p^3T^2)$
57	smooth		$(1 + 12pT + p^3T^2)(1 - 648T + p^3T^2)$
58	smooth		$1 + 360T + 8930pT^2 + 360p^3T^3 + p^6T^4$
59	smooth		$(1 - 8pT + p^3T^2)(1 + 648T + p^3T^2)$
60	smooth		$1 - 392T - 286pT^2 - 392p^3T^3 + p^6T^4$
61	smooth		$1 - 116T + 7490pT^2 - 116p^3T^3 + p^6T^4$
62	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 432T + p^3T^2)$
63	smooth		$1 - 536T + 6626pT^2 - 536p^3T^3 + p^6T^4$
64	smooth		$1 + 336T - 862pT^2 + 336p^3T^3 + p^6T^4$
65	singular	$-\frac{1}{12}$	$(1 + pT)(1 + 360T + p^3T^2)$
66	smooth		$1 - 80T + 1826pT^2 - 80p^3T^3 + p^6T^4$
67	smooth*	$-\frac{1}{18}$	
68	smooth		$1 - 488T + 3938pT^2 - 488p^3T^3 + p^6T^4$

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$p = 71$, continued			
φ	smooth/sing.	singularity	$R(T)$
69	smooth		$1 - 384T - 2302pT^2 - 384p^3T^3 + p^6T^4$
70	smooth		$1 + 324T + 4802pT^2 + 324p^3T^3 + p^6T^4$

$p = 73$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 68T - 8506pT^2 - 68p^3T^3 + p^6T^4$
2	smooth		$1 - 44T + 5846pT^2 - 44p^3T^3 + p^6T^4$
3	smooth		$1 + 408T + 6574pT^2 + 408p^3T^3 + p^6T^4$
4	smooth*	$-\frac{1}{18}$	
5	smooth		$1 + 200T - 2610pT^2 + 200p^3T^3 + p^6T^4$
6	singular	$-\frac{1}{12}$	$(1 - pT)(1 - 26T + p^3T^2)$
7	smooth		$1 - 796T + 10806pT^2 - 796p^3T^3 + p^6T^4$
8	smooth		$1 - 204T - 5738pT^2 - 204p^3T^3 + p^6T^4$
9	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 362T + p^3T^2)$
10	smooth		$1 + 244T + 8726pT^2 + 244p^3T^3 + p^6T^4$
11	smooth		$(1 + 6pT + p^3T^2)(1 + 214T + p^3T^2)$
12	smooth		$1 - 708T + 6598pT^2 - 708p^3T^3 + p^6T^4$
13	smooth		$1 - 68T + 1478pT^2 - 68p^3T^3 + p^6T^4$
14	smooth		$1 - 356T + 1286pT^2 - 356p^3T^3 + p^6T^4$
15	smooth		$1 + 1404T + 16966pT^2 + 1404p^3T^3 + p^6T^4$
16	smooth		$1 + 540T + 4486pT^2 + 540p^3T^3 + p^6T^4$
17	smooth		$1 + 92T - 4218pT^2 + 92p^3T^3 + p^6T^4$
18	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 550T + p^3T^2)$
19	smooth		$1 + 1224T + 12622pT^2 + 1224p^3T^3 + p^6T^4$
20	smooth		$1 + 328T + 10574pT^2 + 328p^3T^3 + p^6T^4$
21	smooth		$1 + 1416T + 15310pT^2 + 1416p^3T^3 + p^6T^4$
22	smooth		$1 + 172T + 86p^2T^2 + 172p^3T^3 + p^6T^4$
23	smooth		$(1 + 6pT + p^3T^2)(1 - 650T + p^3T^2)$
24	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 38T + p^3T^2)$
25	smooth		$1 + 364T - 1690pT^2 + 364p^3T^3 + p^6T^4$
26	smooth		$1 + 256T - 226pT^2 + 256p^3T^3 + p^6T^4$
27	smooth		$1 - 500T + 7334pT^2 - 500p^3T^3 + p^6T^4$
28	smooth		$1 + 756T + 3478pT^2 + 756p^3T^3 + p^6T^4$
29	smooth		$1 - 68T + 8774pT^2 - 68p^3T^3 + p^6T^4$
30	smooth		$1 - 524T + 9494pT^2 - 524p^3T^3 + p^6T^4$
31	smooth		$1 + 84T + 7126pT^2 + 84p^3T^3 + p^6T^4$
32	smooth		$1 - 884T + 10022pT^2 - 884p^3T^3 + p^6T^4$
33	smooth		$1 - 328T + 8430pT^2 - 328p^3T^3 + p^6T^4$
34	smooth		$1 + 572T + 4806pT^2 + 572p^3T^3 + p^6T^4$

Continued on the following page

$p = 73$, continued			
φ	smooth/sing.	singularity	$R(T)$
35	smooth		$1 - 372T + 10246pT^2 - 372p^3T^3 + p^6T^4$
36	smooth		$1 - 44T - 6058pT^2 - 44p^3T^3 + p^6T^4$
37	smooth		$1 - 576T + 7966pT^2 - 576p^3T^3 + p^6T^4$
38	smooth		$1 + 9118pT^2 + p^6T^4$
39	smooth		$1 - 996T + 7174pT^2 - 996p^3T^3 + p^6T^4$
40	smooth		$1 + 1044T + 11350pT^2 + 1044p^3T^3 + p^6T^4$
41	smooth		$1 + 420T + 6070pT^2 + 420p^3T^3 + p^6T^4$
42	smooth		$1 + 700T + 5126pT^2 + 700p^3T^3 + p^6T^4$
43	smooth		$1 - 44T + 5558pT^2 - 44p^3T^3 + p^6T^4$
44	smooth		$1 + 348T + 9862pT^2 + 348p^3T^3 + p^6T^4$
45	smooth		$1 - 796T + 9270pT^2 - 796p^3T^3 + p^6T^4$
46	smooth		$1 - 596T + 1766pT^2 - 596p^3T^3 + p^6T^4$
47	smooth		$1 + 176T + 78p^2T^2 + 176p^3T^3 + p^6T^4$
48	smooth		$1 - 68T + 5318pT^2 - 68p^3T^3 + p^6T^4$
49	smooth		$1 + 316T + 10694pT^2 + 316p^3T^3 + p^6T^4$
50	smooth		$1 - 56T + 9614pT^2 - 56p^3T^3 + p^6T^4$
51	smooth		$1 + 540T + 6790pT^2 + 540p^3T^3 + p^6T^4$
52	smooth		$1 - 132T + 5830pT^2 - 132p^3T^3 + p^6T^4$
53	smooth		$1 - 476T + 2486pT^2 - 476p^3T^3 + p^6T^4$
54	smooth		$1 - 132T + 4294pT^2 - 132p^3T^3 + p^6T^4$
55	smooth		$1 - 84T + 5734pT^2 - 84p^3T^3 + p^6T^4$
56	smooth		$1 + 196T - 5386pT^2 + 196p^3T^3 + p^6T^4$
57	smooth		$1 - 644T + 10598pT^2 - 644p^3T^3 + p^6T^4$
58	smooth		$1 - 228T + 7174pT^2 - 228p^3T^3 + p^6T^4$
59	smooth		$1 + 464T + 7422pT^2 + 464p^3T^3 + p^6T^4$
60	smooth		$1 - 192T - 4706pT^2 - 192p^3T^3 + p^6T^4$
61	smooth		$1 + 860T + 6918pT^2 + 860p^3T^3 + p^6T^4$
62	smooth		$1 + 1044T + 9046pT^2 + 1044p^3T^3 + p^6T^4$
63	smooth		$1 + 836T + 9846pT^2 + 836p^3T^3 + p^6T^4$
64	smooth		$(1 - 10pT + p^3T^2)(1 + 706T + p^3T^2)$
65	smooth		$1 + 492T + 6502pT^2 + 492p^3T^3 + p^6T^4$
66	smooth		$1 + 564T + 1558pT^2 + 564p^3T^3 + p^6T^4$
67	smooth		$1 - 188T + 2678pT^2 - 188p^3T^3 + p^6T^4$
68	smooth		$1 - 444T + 2038pT^2 - 444p^3T^3 + p^6T^4$
69	smooth		$1 - 324T + 4582pT^2 - 324p^3T^3 + p^6T^4$
70	singular	$\frac{1}{24}$	$(1 - pT)(1 - 418T + p^3T^2)$
71	smooth		$1 - 736T + 3870pT^2 - 736p^3T^3 + p^6T^4$
72	smooth		$1 - 220T + 1590pT^2 - 220p^3T^3 + p^6T^4$

$p = 79$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 360T + 3394pT^2 - 360p^3T^3 + p^6T^4$
2	smooth		$1 - 400T + 12354pT^2 - 400p^3T^3 + p^6T^4$
3	smooth		$1 - 224T + 2882pT^2 - 224p^3T^3 + p^6T^4$
4	smooth		$1 + 540T + 6850pT^2 + 540p^3T^3 + p^6T^4$
5	smooth		$1 - 252T + 3394pT^2 - 252p^3T^3 + p^6T^4$
6	smooth		$1 + 232T + 962pT^2 + 232p^3T^3 + p^6T^4$
7	smooth		$1 + 272T + 8898pT^2 + 272p^3T^3 + p^6T^4$
8	smooth		$1 - 788T + 7682pT^2 - 788p^3T^3 + p^6T^4$
9	smooth		$1 - 456T + 9922pT^2 - 456p^3T^3 + p^6T^4$
10	smooth		$1 - 836T + 9986pT^2 - 836p^3T^3 + p^6T^4$
11	smooth		$1 - 192T + 9922pT^2 - 192p^3T^3 + p^6T^4$
12	smooth		$1 + 164T - 6846pT^2 + 164p^3T^3 + p^6T^4$
13	smooth		$1 - 456T - 1022pT^2 - 456p^3T^3 + p^6T^4$
14	smooth		$1 + 692T + 7938pT^2 + 692p^3T^3 + p^6T^4$
15	smooth		$1 - 600T + 5314pT^2 - 600p^3T^3 + p^6T^4$
16	smooth		$1 - 60T + 7234pT^2 - 60p^3T^3 + p^6T^4$
17	smooth		$1 + 504T + 4546pT^2 + 504p^3T^3 + p^6T^4$
18	smooth		$1 + 668T + 10434pT^2 + 668p^3T^3 + p^6T^4$
19	smooth		$1 + 616T + 4610pT^2 + 616p^3T^3 + p^6T^4$
20	smooth		$1 - 640T + 4290pT^2 - 640p^3T^3 + p^6T^4$
21	smooth		$1 + 372T + 2242pT^2 + 372p^3T^3 + p^6T^4$
22	smooth		$1 - 580T + 1410pT^2 - 580p^3T^3 + p^6T^4$
23	smooth		$1 - 272T + 10562pT^2 - 272p^3T^3 + p^6T^4$
24	smooth		$1 + 848T + 3138pT^2 + 848p^3T^3 + p^6T^4$
25	smooth		$1 - 792T + 8386pT^2 - 792p^3T^3 + p^6T^4$
26	singular	$-\frac{1}{3}$	$(1 - pT)(1 + 240T + p^3T^2)$
27	smooth		$1 + 380T + 1410pT^2 + 380p^3T^3 + p^6T^4$
28	smooth		$1 + 932T + 7746pT^2 + 932p^3T^3 + p^6T^4$
29	smooth		$1 - 184T + 11202pT^2 - 184p^3T^3 + p^6T^4$
30	smooth		$1 + 152T - 4446pT^2 + 152p^3T^3 + p^6T^4$
31	smooth		$1 + 248T + 11202pT^2 + 248p^3T^3 + p^6T^4$
32	smooth		$1 - 36T + 130pT^2 - 36p^3T^3 + p^6T^4$
33	smooth		$1 + 464T + 4290pT^2 + 464p^3T^3 + p^6T^4$
34	smooth		$1 + 536T + 11202pT^2 + 536p^3T^3 + p^6T^4$
35	smooth		$1 - 512T + 8162pT^2 - 512p^3T^3 + p^6T^4$
36	smooth		$1 - 512T + 4802pT^2 - 512p^3T^3 + p^6T^4$
37	smooth		$1 - 1668T + 19330pT^2 - 1668p^3T^3 + p^6T^4$
38	smooth		$1 + 700T + 7298pT^2 + 700p^3T^3 + p^6T^4$
39	smooth		$1 + 1392T + 13762pT^2 + 1392p^3T^3 + p^6T^4$

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$p = 79$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 - 136T - 1182pT^2 - 136p^3T^3 + p^6T^4$
41	smooth		$1 + 216T - 62pT^2 + 216p^3T^3 + p^6T^4$
42	smooth		$1 + 40T + 8642pT^2 + 40p^3T^3 + p^6T^4$
43	smooth		$1 + 1664T + 19650pT^2 + 1664p^3T^3 + p^6T^4$
44	smooth		$1 + 72T - 4670pT^2 + 72p^3T^3 + p^6T^4$
45	smooth		$1 - 272T + 7874pT^2 - 272p^3T^3 + p^6T^4$
46	singular	$-\frac{1}{12}$	$(1 + pT)(1 - 512T + p^3T^2)$
47	smooth		$1 + 340T - 5566pT^2 + 340p^3T^3 + p^6T^4$
48	smooth		$1 - 272T - 4798pT^2 - 272p^3T^3 + p^6T^4$
49	smooth		$1 - 1000T + 13122pT^2 - 1000p^3T^3 + p^6T^4$
50	smooth		$1 - 24T - 9950pT^2 - 24p^3T^3 + p^6T^4$
51	smooth		$1 - 680T + 9026pT^2 - 680p^3T^3 + p^6T^4$
52	smooth		$1 - 536T + 2498pT^2 - 536p^3T^3 + p^6T^4$
53	smooth		$1 + 896T + 8898pT^2 + 896p^3T^3 + p^6T^4$
54	smooth		$1 + 1256T + 11202pT^2 + 1256p^3T^3 + p^6T^4$
55	smooth		$1 + 584T + 8610pT^2 + 584p^3T^3 + p^6T^4$
56	singular	$\frac{1}{24}$	$(1 - pT)(1 - 552T + p^3T^2)$
57	smooth*	$-\frac{1}{18}$	
58	smooth		$1 - 52T + 4674pT^2 - 52p^3T^3 + p^6T^4$
59	singular	$-\frac{1}{4}$	$(1 + pT)(1 + 208T + p^3T^2)$
60	smooth		$1 + 216T - 638pT^2 + 216p^3T^3 + p^6T^4$
61	smooth		$1 + 336T - 254pT^2 + 336p^3T^3 + p^6T^4$
62	smooth		$1 + 408T + 1474pT^2 + 408p^3T^3 + p^6T^4$
63	smooth		$1 + 896T + 8514pT^2 + 896p^3T^3 + p^6T^4$
64	smooth		$1 + 96T + 5506pT^2 + 96p^3T^3 + p^6T^4$
65	smooth		$1 - 168T + 2146pT^2 - 168p^3T^3 + p^6T^4$
66	smooth		$1 - 720T + 11458pT^2 - 720p^3T^3 + p^6T^4$
67	smooth		$1 - 796T + 4290pT^2 - 796p^3T^3 + p^6T^4$
68	smooth		$1 + 444T + 9346pT^2 + 444p^3T^3 + p^6T^4$
69	singular	$-\frac{1}{8}$	$(1 - pT)(1 + 160T + p^3T^2)$
70	smooth		$1 + 1552T + 19490pT^2 + 1552p^3T^3 + p^6T^4$
71	smooth		$1 + 552T + 12994pT^2 + 552p^3T^3 + p^6T^4$
72	smooth		$1 + 152T - 3006pT^2 + 152p^3T^3 + p^6T^4$
73	smooth		$1 + 284T + 3906pT^2 + 284p^3T^3 + p^6T^4$
74	smooth		$1 - 24T - 10046pT^2 - 24p^3T^3 + p^6T^4$
75	smooth		$1 + 440T + 2082pT^2 + 440p^3T^3 + p^6T^4$
76	smooth		$1 + 1192T + 12866pT^2 + 1192p^3T^3 + p^6T^4$
77	smooth		$(1 + p^3T^2)(1 + 1012T + p^3T^2)$
78	smooth		$1 - 300T + 6082pT^2 - 300p^3T^3 + p^6T^4$

$p = 83$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 - 424T + 10466pT^2 - 424p^3T^3 + p^6T^4$
2	smooth		$1 + 480T + 6434pT^2 + 480p^3T^3 + p^6T^4$
3	smooth		$1 - 660T + 3890pT^2 - 660p^3T^3 + p^6T^4$
4	smooth		$1 + 8pT + 8258pT^2 + 8p^4T^3 + p^6T^4$
5	smooth		$1 - 600T + 3842pT^2 - 600p^3T^3 + p^6T^4$
6	smooth		$1 + 460T + 1010pT^2 + 460p^3T^3 + p^6T^4$
7	smooth		$1 + 288T - 4702pT^2 + 288p^3T^3 + p^6T^4$
8	smooth		$1 - 936T + 9026pT^2 - 936p^3T^3 + p^6T^4$
9	smooth		$1 - 840T + 12098pT^2 - 840p^3T^3 + p^6T^4$
10	smooth		$1 - 456T + 9026pT^2 - 456p^3T^3 + p^6T^4$
11	smooth		$1 - 488T + 6146pT^2 - 488p^3T^3 + p^6T^4$
12	smooth		$1 - 100T - 7438pT^2 - 100p^3T^3 + p^6T^4$
13	smooth		$(1 - 4pT + p^3T^2)(1 + 852T + p^3T^2)$
14	smooth		$1 - 604T + 4178pT^2 - 604p^3T^3 + p^6T^4$
15	smooth		$1 + 540T + 2546pT^2 + 540p^3T^3 + p^6T^4$
16	smooth		$1 + 672T + 4898pT^2 + 672p^3T^3 + p^6T^4$
17	smooth		$1 + 240T - 670pT^2 + 240p^3T^3 + p^6T^4$
18	smooth		$1 - 508T + 12050pT^2 - 508p^3T^3 + p^6T^4$
19	smooth		$1 - 728T + 674pT^2 - 728p^3T^3 + p^6T^4$
20	smooth		$1 - 288T + 7394pT^2 - 288p^3T^3 + p^6T^4$
21	smooth		$1 - 140T - 4078pT^2 - 140p^3T^3 + p^6T^4$
22	smooth		$1 - 148T - 2446pT^2 - 148p^3T^3 + p^6T^4$
23	smooth*	$-\frac{1}{18}$	
24	smooth		$1 - 12T + 2834pT^2 - 12p^3T^3 + p^6T^4$
25	smooth		$1 - 504T + 5570pT^2 - 504p^3T^3 + p^6T^4$
26	smooth		$1 + 208T - 7582pT^2 + 208p^3T^3 + p^6T^4$
27	smooth		$1 + 648T - 766pT^2 + 648p^3T^3 + p^6T^4$
28	smooth		$1 + 80T + 11042pT^2 + 80p^3T^3 + p^6T^4$
29	smooth		$1 - 1136T + 13538pT^2 - 1136p^3T^3 + p^6T^4$
30	smooth		$1 + 1728T + 19106pT^2 + 1728p^3T^3 + p^6T^4$
31	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 72T + p^3T^2)$
32	smooth		$1 + 24T - 4414pT^2 + 24p^3T^3 + p^6T^4$
33	smooth		$1 - 108T - 8494pT^2 - 108p^3T^3 + p^6T^4$
34	smooth		$1 + 36T + 530pT^2 + 36p^3T^3 + p^6T^4$
35	smooth		$1 - 920T + 12674pT^2 - 920p^3T^3 + p^6T^4$
36	smooth		$1 + 312T - 1246pT^2 + 312p^3T^3 + p^6T^4$
37	smooth		$1 - 1148T + 11282pT^2 - 1148p^3T^3 + p^6T^4$
38	smooth		$1 + 240T + 7010pT^2 + 240p^3T^3 + p^6T^4$
39	smooth		$1 + 416T + 10274pT^2 + 416p^3T^3 + p^6T^4$

Continued on the following page

$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
40	smooth		$1 + 564T + 7442pT^2 + 564p^3T^3 + p^6T^4$
41	smooth		$1 + 732T + 5234pT^2 + 732p^3T^3 + p^6T^4$
42	smooth		$1 - 328T - 3646pT^2 - 328p^3T^3 + p^6T^4$
43	smooth		$1 + 252T - 3022pT^2 + 252p^3T^3 + p^6T^4$
44	smooth		$1 - 384T - 94pT^2 - 384p^3T^3 + p^6T^4$
45	singular	$\frac{1}{24}$	$(1 - pT)(1 - 1036T + p^3T^2)$
46	smooth		$1 + 1080T + 14018pT^2 + 1080p^3T^3 + p^6T^4$
47	smooth		$1 - 104T + 3266pT^2 - 104p^3T^3 + p^6T^4$
48	smooth		$1 - 328T + 1538pT^2 - 328p^3T^3 + p^6T^4$
49	smooth		$1 - 724T + 12146pT^2 - 724p^3T^3 + p^6T^4$
50	smooth		$1 - 544T + 4514pT^2 - 544p^3T^3 + p^6T^4$
51	smooth		$(1 - 12pT + p^3T^2)(1 - 396T + p^3T^2)$
52	smooth		$1 + 84T - 3310pT^2 + 84p^3T^3 + p^6T^4$
53	smooth		$1 - 288T + 11042pT^2 - 288p^3T^3 + p^6T^4$
54	smooth		$1 + 216T - 3550pT^2 + 216p^3T^3 + p^6T^4$
55	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 1212T + p^3T^2)$
56	smooth		$1 - 448T + 9506pT^2 - 448p^3T^3 + p^6T^4$
57	smooth		$1 - 304T - 7102pT^2 - 304p^3T^3 + p^6T^4$
58	smooth		$1 + 320T + 8354pT^2 + 320p^3T^3 + p^6T^4$
59	smooth		$(1 - 4pT + p^3T^2)(1 + 108T + p^3T^2)$
60	smooth		$1 + 1132T + 16370pT^2 + 1132p^3T^3 + p^6T^4$
61	smooth		$1 + 256T - 3934pT^2 + 256p^3T^3 + p^6T^4$
62	singular	$-\frac{1}{4}$	$(1 + pT)(1 - 516T + p^3T^2)$
63	smooth		$1 + 40T - 4606pT^2 + 40p^3T^3 + p^6T^4$
64	smooth		$1 + 1288T + 10178pT^2 + 1288p^3T^3 + p^6T^4$
65	smooth		$1 - 84T - 910pT^2 - 84p^3T^3 + p^6T^4$
66	smooth		$(1 + 12pT + p^3T^2)(1 - 444T + p^3T^2)$
67	smooth		$1 - 1056T + 15650pT^2 - 1056p^3T^3 + p^6T^4$
68	smooth		$1 - 528T + 2018pT^2 - 528p^3T^3 + p^6T^4$
69	smooth		$1 - 472T + 1922pT^2 - 472p^3T^3 + p^6T^4$
70	smooth		$1 - 992T + 11042pT^2 - 992p^3T^3 + p^6T^4$
71	smooth		$1 - 696T + 9026pT^2 - 696p^3T^3 + p^6T^4$
72	smooth		$1 - 840T + 10178pT^2 - 840p^3T^3 + p^6T^4$
73	smooth		$1 - 4pT + 7442pT^2 - 4p^4T^3 + p^6T^4$
74	smooth		$1 + 88T + 8642pT^2 + 88p^3T^3 + p^6T^4$
75	smooth		$1 + 48T - 11422pT^2 + 48p^3T^3 + p^6T^4$
76	singular	$-\frac{1}{12}$	$(1 + pT)(1 + 1188T + p^3T^2)$
77	smooth		$1 - 1344T + 18722pT^2 - 1344p^3T^3 + p^6T^4$
78	smooth		$1 + 1388T + 13298pT^2 + 1388p^3T^3 + p^6T^4$
79	smooth		$1 + 176T + 2786pT^2 + 176p^3T^3 + p^6T^4$

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$p = 83$, continued			
φ	smooth/sing.	singularity	$R(T)$
80	smooth		$1 + 576T + 3746pT^2 + 576p^3T^3 + p^6T^4$
81	smooth		$1 - 552T + 13634pT^2 - 552p^3T^3 + p^6T^4$
82	smooth		$1 - 392T + 4802pT^2 - 392p^3T^3 + p^6T^4$

$p = 89$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 724T + 7766pT^2 + 724p^3T^3 + p^6T^4$
2	smooth		$1 + 852T + 7766pT^2 + 852p^3T^3 + p^6T^4$
3	smooth		$1 - 972T + 13718pT^2 - 972p^3T^3 + p^6T^4$
4	smooth		$1 + 684T + 15206pT^2 + 684p^3T^3 + p^6T^4$
5	smooth		$1 - 1244T + 10550pT^2 - 1244p^3T^3 + p^6T^4$
6	smooth		$1 - 1452T + 18134pT^2 - 1452p^3T^3 + p^6T^4$
7	smooth		$1 + 944T + 14846pT^2 + 944p^3T^3 + p^6T^4$
8	smooth		$1 - 404T - 5146pT^2 - 404p^3T^3 + p^6T^4$
9	smooth		$1 + 160T + 10142pT^2 + 160p^3T^3 + p^6T^4$
10	smooth		$1 - 776T + 15854pT^2 - 776p^3T^3 + p^6T^4$
11	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 810T + p^3T^2)$
12	smooth		$1 + 476T + 7046pT^2 + 476p^3T^3 + p^6T^4$
13	smooth		$1 + 348T + 7046pT^2 + 348p^3T^3 + p^6T^4$
14	smooth		$1 + 676T + 758pT^2 + 676p^3T^3 + p^6T^4$
15	smooth		$1 + 204T + 8486pT^2 + 204p^3T^3 + p^6T^4$
16	smooth		$1 + 244T - 682pT^2 + 244p^3T^3 + p^6T^4$
17	smooth		$1 - 424T + 12782pT^2 - 424p^3T^3 + p^6T^4$
18	smooth		$1 - 516T + 8006pT^2 - 516p^3T^3 + p^6T^4$
19	smooth		$1 + 556T + 8294pT^2 + 556p^3T^3 + p^6T^4$
20	smooth		$1 - 1416T + 16814pT^2 - 1416p^3T^3 + p^6T^4$
21	smooth		$1 + 60T + 4934pT^2 + 60p^3T^3 + p^6T^4$
22	singular	$-\frac{1}{4}$	$(1 - pT)(1 + 1398T + p^3T^2)$
23	smooth		$1 - 1340T + 16118pT^2 - 1340p^3T^3 + p^6T^4$
24	smooth		$1 - 692T + 9830pT^2 - 692p^3T^3 + p^6T^4$
25	smooth		$1 + 296T + 1742pT^2 + 296p^3T^3 + p^6T^4$
26	singular	$\frac{1}{24}$	$(1 - pT)(1 - 30T + p^3T^2)$
27	smooth		$1 - 292T + 134pT^2 - 292p^3T^3 + p^6T^4$
28	smooth		$1 - 212T + 8294pT^2 - 212p^3T^3 + p^6T^4$
29	smooth		$1 - 60T - 11530pT^2 - 60p^3T^3 + p^6T^4$
30	smooth		$1 - 1676T + 19286pT^2 - 1676p^3T^3 + p^6T^4$
31	smooth		$1 - 156T - 4714pT^2 - 156p^3T^3 + p^6T^4$
32	smooth		$1 - 1412T + 14918pT^2 - 1412p^3T^3 + p^6T^4$
33	smooth		$1 + 88T - 2962pT^2 + 88p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
34	smooth		$1 + 808T + 5582pT^2 + 808p^3T^3 + p^6T^4$
35	smooth		$1 + 252T + 9542pT^2 + 252p^3T^3 + p^6T^4$
36	smooth		$1 - 1388T + 14678pT^2 - 1388p^3T^3 + p^6T^4$
37	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 630T + p^3T^2)$
38	smooth		$1 + 1476T + 13430pT^2 + 1476p^3T^3 + p^6T^4$
39	smooth		$1 - 1216T + 16670pT^2 - 1216p^3T^3 + p^6T^4$
40	smooth		$1 - 468T - 3994pT^2 - 468p^3T^3 + p^6T^4$
41	smooth		$1 + 12T + 5798pT^2 + 12p^3T^3 + p^6T^4$
42	smooth		$1 - 200T - 11986pT^2 - 200p^3T^3 + p^6T^4$
43	smooth		$1 + 900T + 5558pT^2 + 900p^3T^3 + p^6T^4$
44	smooth		$1 + 172T + 9062pT^2 + 172p^3T^3 + p^6T^4$
45	smooth		$1 + 652T + 6950pT^2 + 652p^3T^3 + p^6T^4$
46	smooth		$1 - 1188T + 16262pT^2 - 1188p^3T^3 + p^6T^4$
47	smooth		$1 + 132T + 10742pT^2 + 132p^3T^3 + p^6T^4$
48	smooth		$1 - 276T - 8986pT^2 - 276p^3T^3 + p^6T^4$
49	smooth		$1 + 8pT + 2318pT^2 + 8p^4T^3 + p^6T^4$
50	smooth		$1 - 672T + 12062pT^2 - 672p^3T^3 + p^6T^4$
51	smooth		$1 + 132T - 11914pT^2 + 132p^3T^3 + p^6T^4$
52	smooth		$1 + 520T + 7694pT^2 + 520p^3T^3 + p^6T^4$
53	smooth		$1 + 556T + 3686pT^2 + 556p^3T^3 + p^6T^4$
54	smooth		$1 - 140T + 278pT^2 - 140p^3T^3 + p^6T^4$
55	smooth		$1 + 556T + 10982pT^2 + 556p^3T^3 + p^6T^4$
56	smooth		$1 + 628T - 1642pT^2 + 628p^3T^3 + p^6T^4$
57	smooth		$1 - 1524T + 14342pT^2 - 1524p^3T^3 + p^6T^4$
58	smooth		$1 - 708T + 14534pT^2 - 708p^3T^3 + p^6T^4$
59	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 330T + p^3T^2)$
60	smooth		$1 + 676T + 2486pT^2 + 676p^3T^3 + p^6T^4$
61	smooth		$1 + 460T - 1498pT^2 + 460p^3T^3 + p^6T^4$
62	smooth		$1 - 144T - 5506pT^2 - 144p^3T^3 + p^6T^4$
63	smooth		$1 - 1308T + 17078pT^2 - 1308p^3T^3 + p^6T^4$
64	smooth		$1 + 36T + 5942pT^2 + 36p^3T^3 + p^6T^4$
65	smooth		$1 - 1208T + 18062pT^2 - 1208p^3T^3 + p^6T^4$
66	smooth		$1 - 1116T + 12374pT^2 - 1116p^3T^3 + p^6T^4$
67	smooth		$1 + 492T - 1306pT^2 + 492p^3T^3 + p^6T^4$
68	smooth		$1 + 568T - 4114pT^2 + 568p^3T^3 + p^6T^4$
69	smooth		$1 - 88T + 590pT^2 - 88p^3T^3 + p^6T^4$
70	smooth		$1 + 204T - 6874pT^2 + 204p^3T^3 + p^6T^4$
71	smooth		$1 - 20T + 8678pT^2 - 20p^3T^3 + p^6T^4$
72	smooth		$1 + 300T + 6758pT^2 + 300p^3T^3 + p^6T^4$
73	smooth		$1 + 12T - 9562pT^2 + 12p^3T^3 + p^6T^4$

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$p = 89$, continued			
φ	smooth/sing.	singularity	$R(T)$
74	smooth		$1 + 336T + 3902pT^2 + 336p^3T^3 + p^6T^4$
75	smooth		$1 - 524T + 13910pT^2 - 524p^3T^3 + p^6T^4$
76	smooth		$1 + 60T + 8390pT^2 + 60p^3T^3 + p^6T^4$
77	smooth		$1 + 972T + 15782pT^2 + 972p^3T^3 + p^6T^4$
78	smooth		$1 + 12T - 7258pT^2 + 12p^3T^3 + p^6T^4$
79	smooth		$1 - 600T + 5390pT^2 - 600p^3T^3 + p^6T^4$
80	smooth		$1 - 1176T + 19022pT^2 - 1176p^3T^3 + p^6T^4$
81	smooth		$1 + 1612T + 18854pT^2 + 1612p^3T^3 + p^6T^4$
82	smooth		$1 + 844T + 11558pT^2 + 844p^3T^3 + p^6T^4$
83	smooth		$1 - 528T + 8318pT^2 - 528p^3T^3 + p^6T^4$
84	smooth*	$-\frac{1}{18}$	
85	smooth		$1 - 388T + 8774pT^2 - 388p^3T^3 + p^6T^4$
86	smooth		$1 - 1008T + 17726pT^2 - 1008p^3T^3 + p^6T^4$
87	smooth		$1 - 492T + 5270pT^2 - 492p^3T^3 + p^6T^4$
88	smooth		$1 - 900T + 13766pT^2 - 900p^3T^3 + p^6T^4$

$p = 97$			
φ	smooth/sing.	singularity	$R(T)$
1	smooth		$1 + 580T - 5386pT^2 + 580p^3T^3 + p^6T^4$
2	smooth		$1 - 1116T + 17590pT^2 - 1116p^3T^3 + p^6T^4$
3	smooth		$1 + 612T + 9622pT^2 + 612p^3T^3 + p^6T^4$
4	smooth		$1 + 704T + 5502pT^2 + 704p^3T^3 + p^6T^4$
5	smooth		$1 - 496T + 11742pT^2 - 496p^3T^3 + p^6T^4$
6	smooth		$1 + 356T + 13110pT^2 + 356p^3T^3 + p^6T^4$
7	smooth		$1 + 1088T + 126p^2T^2 + 1088p^3T^3 + p^6T^4$
8	singular	$-\frac{1}{12}$	$(1 - pT)(1 + 1054T + p^3T^2)$
9	smooth		$1 + 1808T + 19230pT^2 + 1808p^3T^3 + p^6T^4$
10	smooth		$1 + 3646pT^2 + p^6T^4$
11	smooth		$1 - 1240T + 15726pT^2 - 1240p^3T^3 + p^6T^4$
12	singular	$-\frac{1}{8}$	$(1 - pT)(1 - 1106T + p^3T^2)$
13	smooth		$1 + 684T + 17446pT^2 + 684p^3T^3 + p^6T^4$
14	smooth		$1 - 488T + 4814pT^2 - 488p^3T^3 + p^6T^4$
15	smooth		$1 + 1236T + 20182pT^2 + 1236p^3T^3 + p^6T^4$
16	smooth		$1 + 1244T + 9030pT^2 + 1244p^3T^3 + p^6T^4$
17	smooth		$1 + 516T + 11254pT^2 + 516p^3T^3 + p^6T^4$
18	smooth		$1 + 584T - 5010pT^2 + 584p^3T^3 + p^6T^4$
19	smooth		$1 - 1108T + 19110pT^2 - 1108p^3T^3 + p^6T^4$
20	smooth		$1 + 1004T + 8358pT^2 + 1004p^3T^3 + p^6T^4$
21	smooth		$1 - 216T + 13870pT^2 - 216p^3T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
22	smooth		$1 - 680T + 14414pT^2 - 680p^3T^3 + p^6T^4$
23	smooth		$1 + 496T + 13598pT^2 + 496p^3T^3 + p^6T^4$
24	singular	$-\frac{1}{4}$	$(1 - pT)(1 - 1586T + p^3T^2)$
25	smooth		$1 + 1272T + 12046pT^2 + 1272p^3T^3 + p^6T^4$
26	smooth		$1 + 84T - 12650pT^2 + 84p^3T^3 + p^6T^4$
27	smooth		$1 - 1104T + 20254pT^2 - 1104p^3T^3 + p^6T^4$
28	smooth		$1 - 720T + 5662pT^2 - 720p^3T^3 + p^6T^4$
29	smooth		$1 - 820T + 10854pT^2 - 820p^3T^3 + p^6T^4$
30	smooth		$1 - 1860T + 25702pT^2 - 1860p^3T^3 + p^6T^4$
31	smooth		$1 + 260T + 630pT^2 + 260p^3T^3 + p^6T^4$
32	singular	$-\frac{1}{3}$	$(1 - pT)(1 - 866T + p^3T^2)$
33	smooth		$1 + 1580T + 22950pT^2 + 1580p^3T^3 + p^6T^4$
34	smooth		$1 + 172T - 12634pT^2 + 172p^3T^3 + p^6T^4$
35	smooth		$1 - 164T - 2746pT^2 - 164p^3T^3 + p^6T^4$
36	smooth		$1 - 68T + 4742pT^2 - 68p^3T^3 + p^6T^4$
37	smooth		$1 + 12T + 5350pT^2 + 12p^3T^3 + p^6T^4$
38	smooth		$1 + 1052T + 7878pT^2 + 1052p^3T^3 + p^6T^4$
39	smooth		$1 - 452T + 16262pT^2 - 452p^3T^3 + p^6T^4$
40	smooth		$1 - 60T + 8854pT^2 - 60p^3T^3 + p^6T^4$
41	smooth		$1 + 1176T + 14926pT^2 + 1176p^3T^3 + p^6T^4$
42	smooth		$1 + 140T - 4890pT^2 + 140p^3T^3 + p^6T^4$
43	smooth		$1 - 292T + 11334pT^2 - 292p^3T^3 + p^6T^4$
44	smooth		$1 - 1244T + 17078pT^2 - 1244p^3T^3 + p^6T^4$
45	smooth		$1 - 52T - 7578pT^2 - 52p^3T^3 + p^6T^4$
46	smooth		$1 + 1108T + 17366pT^2 + 1108p^3T^3 + p^6T^4$
47	smooth		$1 - 1140T + 17542pT^2 - 1140p^3T^3 + p^6T^4$
48	smooth		$1 + 164T + 4662pT^2 + 164p^3T^3 + p^6T^4$
49	smooth		$1 - 480T + 13630pT^2 - 480p^3T^3 + p^6T^4$
50	smooth		$1 - 716T + 9878pT^2 - 716p^3T^3 + p^6T^4$
51	smooth		$1 - 708T + 14854pT^2 - 708p^3T^3 + p^6T^4$
52	smooth		$1 - 244T - 11802pT^2 - 244p^3T^3 + p^6T^4$
53	smooth		$1 + 2460T + 33862pT^2 + 2460p^3T^3 + p^6T^4$
54	smooth		$1 + 524T + 7014pT^2 + 524p^3T^3 + p^6T^4$
55	smooth		$1 - 988T + 5622pT^2 - 988p^3T^3 + p^6T^4$
56	smooth		$1 - 1764T + 25318pT^2 - 1764p^3T^3 + p^6T^4$
57	smooth		$1 + 964T + 16022pT^2 + 964p^3T^3 + p^6T^4$
58	smooth		$1 + 2164T + 26006pT^2 + 2164p^3T^3 + p^6T^4$
59	smooth		$1 + 256T - 1282pT^2 + 256p^3T^3 + p^6T^4$
60	smooth		$1 + 220T + 5702pT^2 + 220p^3T^3 + p^6T^4$
61	smooth		$1 + 8pT + 366pT^2 + 8p^4T^3 + p^6T^4$

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$p = 97$, continued			
φ	smooth/sing.	singularity	$R(T)$
62	smooth		$1 + 1344T + 16894pT^2 + 1344p^3T^3 + p^6T^4$
63	smooth		$1 + 428T + 102p^2T^2 + 428p^3T^3 + p^6T^4$
64	smooth		$1 - 1396T + 12006pT^2 - 1396p^3T^3 + p^6T^4$
65	smooth		$1 + 852T + 9430pT^2 + 852p^3T^3 + p^6T^4$
66	smooth		$1 - 180T - 6938pT^2 - 180p^3T^3 + p^6T^4$
67	smooth		$1 + 1904T + 23838pT^2 + 1904p^3T^3 + p^6T^4$
68	smooth		$1 + 840T + 13870pT^2 + 840p^3T^3 + p^6T^4$
69	smooth		$1 + 36T + 9910pT^2 + 36p^3T^3 + p^6T^4$
70	smooth*	$-\frac{1}{18}$	
71	smooth		$1 - 564T - 3098pT^2 - 564p^3T^3 + p^6T^4$
72	smooth		$1 + 668T + 10950pT^2 + 668p^3T^3 + p^6T^4$
73	smooth		$1 + 980T + 11094pT^2 + 980p^3T^3 + p^6T^4$
74	smooth		$1 + 1748T + 23574pT^2 + 1748p^3T^3 + p^6T^4$
75	smooth		$1 + 644T + 15414pT^2 + 644p^3T^3 + p^6T^4$
76	smooth		$1 - 420T + 8518pT^2 - 420p^3T^3 + p^6T^4$
77	smooth		$1 + 588T + 11110pT^2 + 588p^3T^3 + p^6T^4$
78	smooth		$1 + 500T - 6570pT^2 + 500p^3T^3 + p^6T^4$
79	smooth		$1 - 228T - 6458pT^2 - 228p^3T^3 + p^6T^4$
80	smooth		$1 + 360T + 1198pT^2 + 360p^3T^3 + p^6T^4$
81	smooth		$1 + 1372T + 14534pT^2 + 1372p^3T^3 + p^6T^4$
82	smooth		$1 - 1036T + 18198pT^2 - 1036p^3T^3 + p^6T^4$
83	smooth		$1 + 356T + 11574pT^2 + 356p^3T^3 + p^6T^4$
84	smooth		$1 - 1588T + 20070pT^2 - 1588p^3T^3 + p^6T^4$
85	smooth		$1 + 740T + 3894pT^2 + 740p^3T^3 + p^6T^4$
86	smooth		$1 + 540T + 1990pT^2 + 540p^3T^3 + p^6T^4$
87	smooth		$1 - 420T - 9146pT^2 - 420p^3T^3 + p^6T^4$
88	smooth		$1 - 248T + 10094pT^2 - 248p^3T^3 + p^6T^4$
89	smooth		$1 - 1060T + 8262pT^2 - 1060p^3T^3 + p^6T^4$
90	smooth		$1 - 100T + 10950pT^2 - 100p^3T^3 + p^6T^4$
91	smooth		$1 - 276T + 5350pT^2 - 276p^3T^3 + p^6T^4$
92	smooth		$1 + 284T - 8250pT^2 + 284p^3T^3 + p^6T^4$
93	singular	$\frac{1}{24}$	$(1 - pT)(1 + 1190T + p^3T^2)$
94	smooth		$1 + 108T + 6694pT^2 + 108p^3T^3 + p^6T^4$
95	smooth		$1 - 1244T + 14870pT^2 - 1244p^3T^3 + p^6T^4$
96	smooth		$1 - 4T + 6150pT^2 - 4p^3T^3 + p^6T^4$

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