Given: Random variable with uniform distribution between 0 and θ.

Ordered statistics $X_1, X_2, \dots X_k, \dots\dots X_n$

Pdf: $f(x) = \begin{cases} 1/\theta & \text{for } 0 \leq x \leq \theta \\ 0 & \text{otherwise} \end{cases}$

We need to find whether $X_{(n-13)}$ i.e. 14^{th} largest observation converges at some point or not.

We observe endpoints $\rightarrow$ X_1 is the minimum which always converges to 0, X_n converges to θ similarly (at $n \to \infty$)

We expect X_{n-13} to converge to θ as it is almost equal to X_n (as $13 << \infty \Rightarrow n-13 \sim n$ at $n \to \infty$)

$\exists\, x \in \mathbb{R}$ s.t. $x < \theta$.

$P(X_{n-13} \leq x) = P$ (atleast $n-13$ observations are $\leq x$)

For some fixed $x < \theta$, $P(X_{n-13} \leq x)$ approaches 0 for larger n. Thus, $X_{n-13} \longrightarrow \theta$ in probability.

However, $X_{n-13} \not\equiv \theta$ yet.

To prove such, we find $P(|X_{n-13} - \theta| > \varepsilon) = P$

$\forall\, \varepsilon > 0$ as $n \to \infty$.

Let $\varepsilon = |X_{n-k} - \theta| > 0$ $(k \in [0, 12] \cap \mathbb{Z})$

$$\big|\, |X_{n-13} - \theta| - \varepsilon \,\big| = \big|\, |X_{n-13} - \theta| - |X_{n-k} - \theta| \,\big|$$

$$\leq |x_{n-13} - \theta - x_{n-k} + \theta|$$

$$= |x_{n-13} - x_{n-k}|$$

$x_{n-13} < x_{n-k}$ $\forall$ k as defined

$p \neq 0 \Rightarrow x_{n-13}$ does not converge to a finite value.