Problem Set #1: Population Growth Model

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March 10, 2017

1 Problem description

Here we want to model the growth of an animal population. Every year, the growth of the population is determined according to its original size. The population value of each year that we get will be stored in an array p so that we can plot its evolution over time.

2 When growth rate α stays constant

As a first approximation, we suppose that the growth rate α is contant. That is, for all n, we can write $p_n = p_{n-1} + \alpha p_{n-1}$, where p_n is the population value of the n^{th} year (counting from zero) and α doesn't depend on n.

At first, we fix $p_0 = 2$ and $\alpha = 0.1$ and simulate the population growth over a hundred years. The result is shown below:

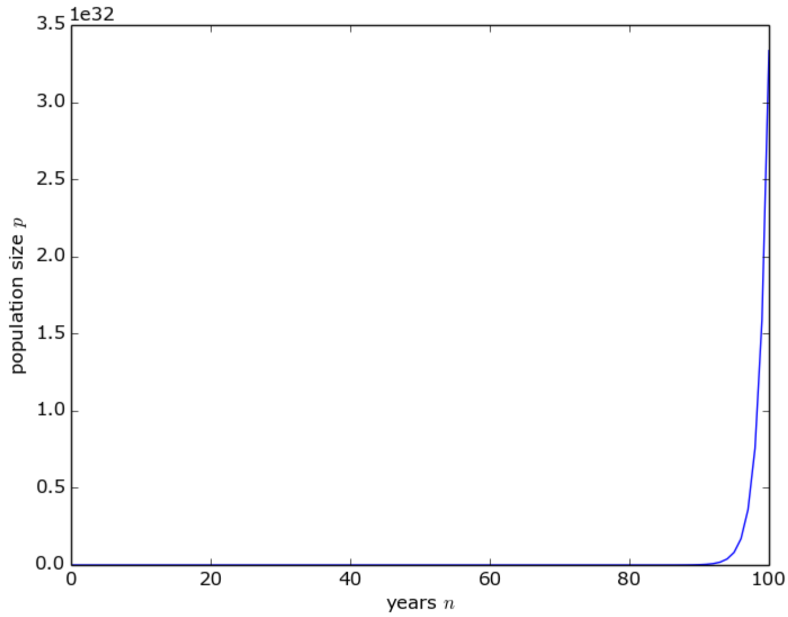

Figure 1: Population growth with $p_0 = 2$ and $\alpha = 0.1$

As we can see, the population grows exponentially and explodes very fast, 10^{32} individuals at the end of a century! But it's somehow not very surprising because it's exactly what our equation indicates as it can also be written in the form $p_n = (1+\alpha)p_{n-1}$. Now let's change the value of α and see what happens.

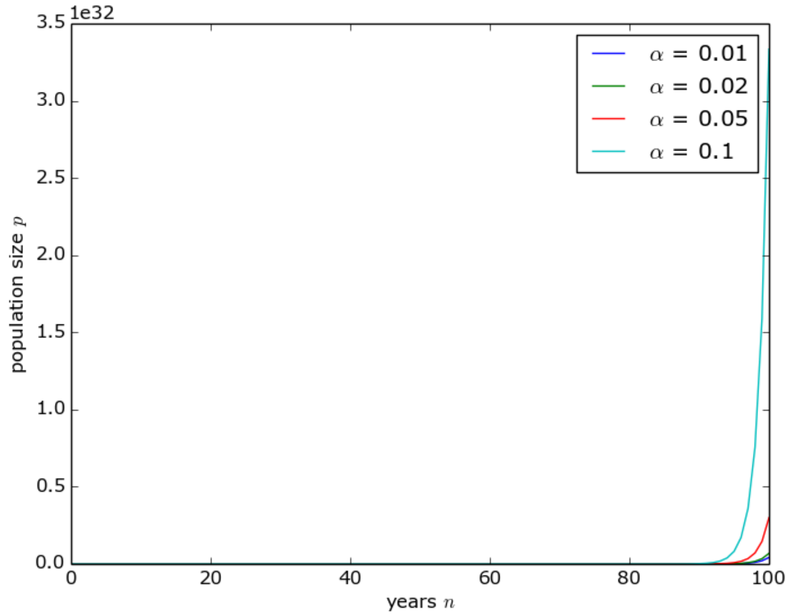

FIGURE 2: Population growth with $p_0 = 2$ and different α

The population grows more or less fast when α varies. I always keep $\alpha \leq 0.1$ in this figure because if α gets even bigger, the population grows really fast and we will not be able to see other curves with smaller α values.

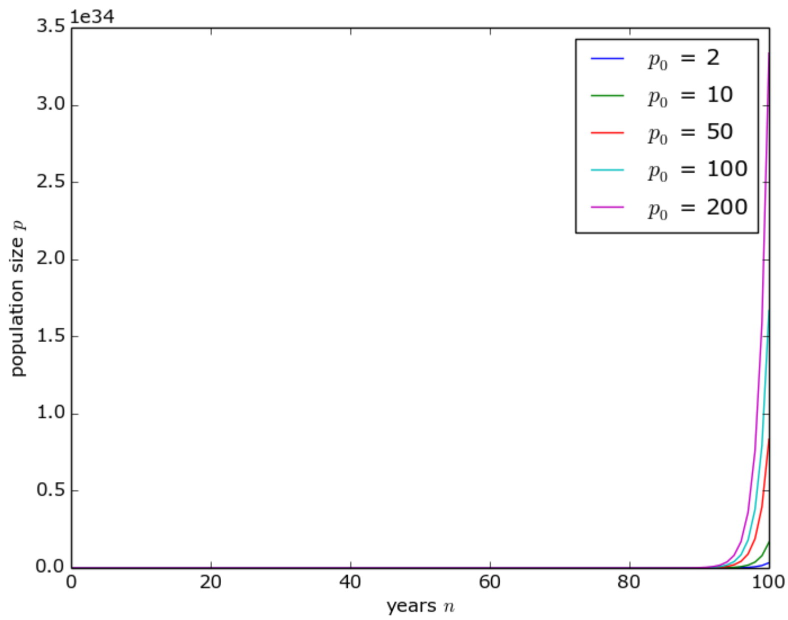

FIGURE 3: Population growth with $\alpha = 0.1$ and different p_0

In constrast, as shown in the figure above, changing initial population has a smaller impact. In fact, the ratio between two different population values stays constant over time.

3 Towards a more realistic model

Nevertheless, the model proposed in the last section is not very satisfying since the animal population is not meant to grow forever without limitation. For instance, the problem of resources should be considered, we need to modify our model such that α becomes a funcion of p, and of course we want α to decrease when p increases. Let's choose $\alpha = 200 - p$.

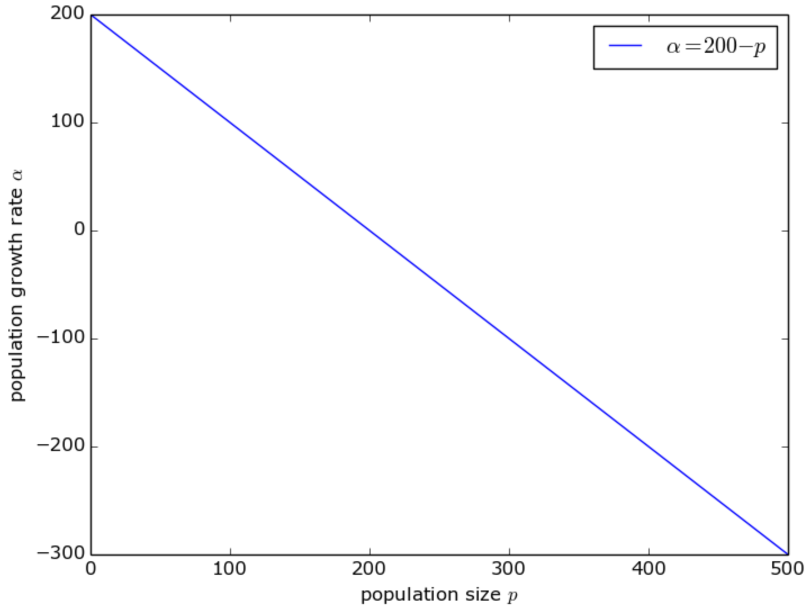

FIGURE 4: Plot of $\alpha = 200 - p$

It's quite nice. α is a decreasing function, which means that the population will grow more and more slowly as time goes by. It gets even negative when p is greater than 200. This prevents the population from growing too large. However, the real value of α here is too big, hence we would rather use $\delta_n = 0.001 p_{n-1}(200 - p_{n-1})$ (in this case $\alpha = 0.001(200 - p_{n-1})$). We can now plot the varaiation of the population.

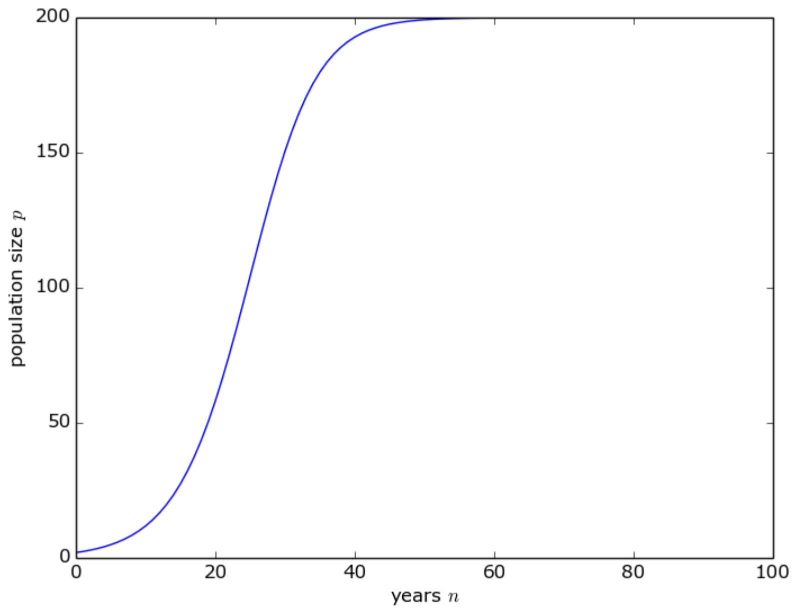

FIGURE 5: Population growth with the law $p_n = p_{n-1} + 0.001 p_{n-1}(200 - p_{n-1})$

It's closer to the reality this time. We obtain a curve with a sigmoid shape. The population grows quite fast at the beginning, but then the growth slows down and becomes almost linear, and finally after about fifty years, the population gets saturated with $p \sim 200$. We can still try to change different parameters in the equation, like the coefficient before $200 - p$ in α.

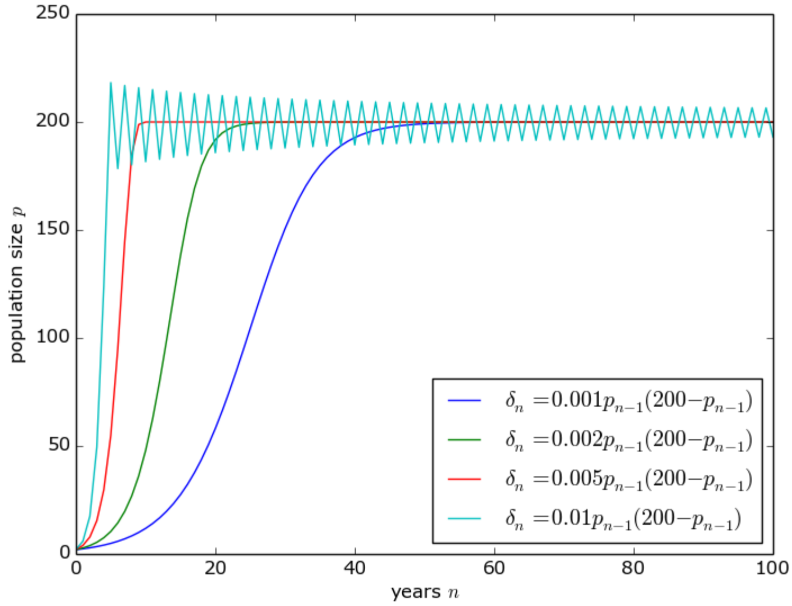

FIGURE 6: Population growth with some different parameters in the law

In most of the cases, we see something that is already obeserved: bigger the α, faster the population grows. Nonetheless, things get more interesting when this coefficient becomes bigger (0.01 here). An oscillation behavior appears since the population size can exceed the environment limit from time to time (but if we keep increasing this parameter, the curve will not make sense anymore). At the end of the report, we'd like to see the influence of p_0 in this model.

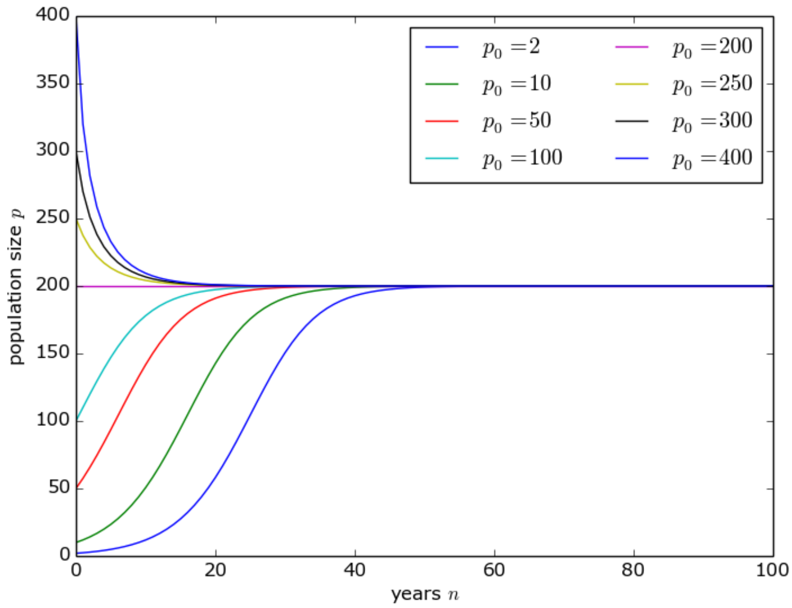

FIGURE 7: Population growth with the same law but different p_0

It turns out that the result is not that different from what we get from the first model, but when p_0 is larger, the population growth could become slower from the very beginning (say, we're already in the linear phase). Furthermore, if p_0 is greater than 200, there aren't enough environmental resources for all the individuals, and the population value will decrease to fit this limitation. A case particular is when p_0 is exactly 200, the population size will be fixed from the beginning at 200.

4 Conclusion

In conclusion, we have tried to simulate the animal population growth with two different models. In the first one, we consider the population growth rate as a constant and see that the population size can explode very quickly. Therefore, we decide to take account of resource issues in the second model. With this extra condition, population saturation is reached after a period of time and too large population gets penalized.