Problem Set #3: Spikes

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Introduction

Neurons in our brain need to fire signals to communicate with each other. These signals, electrochemical in nature, are refered to as spikes, or action potentials. Here we'll look at several different aspects of this essential element in our nervous system.

1 Spike trains

1.1 Poisson Model

Before discussing how spikes are produced, we'll first work on the statistical description of spike trains (i.e. a seqeunce of spikes and silences from a single neuron). As a first approximation, the generation of a random spike train can be simulated by a Poisson process. We assume that individual spikes are generated mutaually independently with some probability that can be deduced from the instantaneous firing rate.

Since the computer is a discret system, a spike train will just be modeled as an array of 0s and 1s. For example, we create a vector of 1000 elements such that each element of the vector has 25% to be 1.

FIGURE 1: A Bernoulli process of 1000 trials with $p = 0.25$

Next we introduce time units, every 0 or 1 is assocaited with a time bin of lenght Δt ms. Here we choose $\Delta t = 2$ ms and generate a spike train of length 1 sec with the firing rate 25 spikes/sec.

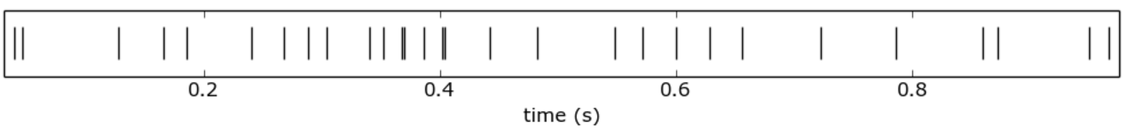

FIGURE 2: A Poisson spike train with an average rate of 25 spikes/sec

In the above figure, there are in effect 29 spikes that are generated. We may be insterested in the distribution of the total number of spikes in each simulation that we refer to as total spike count here. Thus we'll generate 50 spike trains with the same parameters.

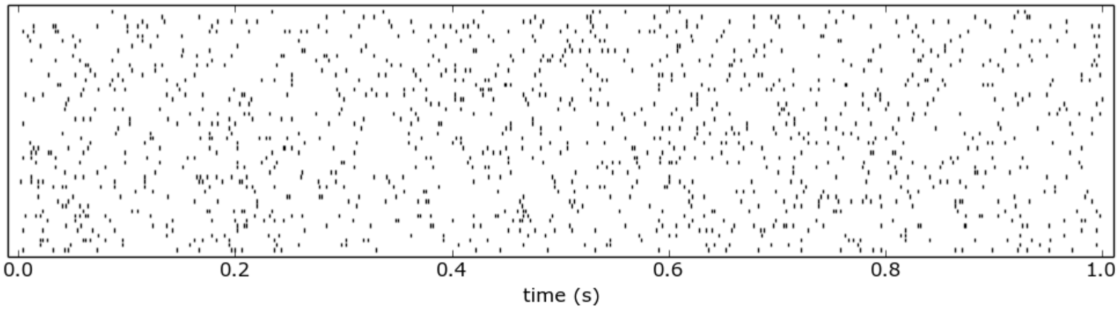

FIGURE 3: 50 Poisson spike trains with firing rate 25 spikes/sec

Then we plot the histogram of total spike counts.

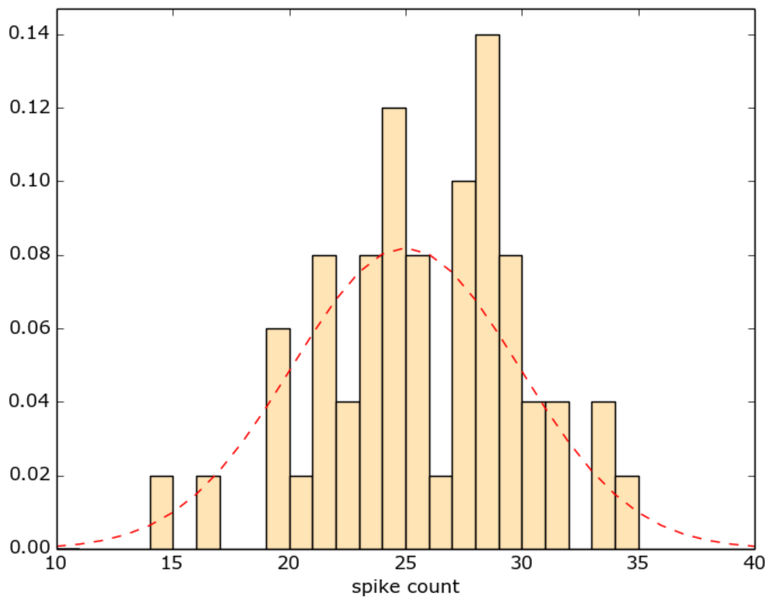

FIGURE 4: Histogram of total spike counts for 50 simulations

According to the central limit theorem, the distribution of spike counts here (which is in fact the binomial distribution $B(500, 0.05)$) can be approximated by the normal distribution $\mathcal{N}(np, np(1-p))$ with $n = 500$ and $p = 0.05$ (the red dashed line in the figure). This can be more or less seen above. However, the theoretical line doesn't fit yet very well the simulation results. It's simply due to the fact that we have too few samples here to describe the distribution, but as we can see later the approximation itself works indeed pretty well.

We also plot the histogram of interspike intervals for the same set of spike trains. This time the histogram follows an exponential distribution, as one might expect (it's a property of the Poisson process).

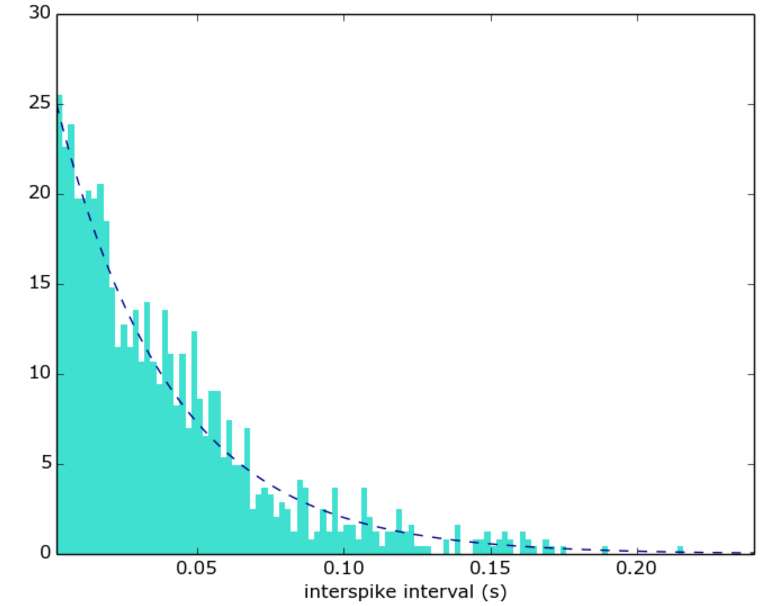

FIGURE 5: Histogram of interspike intervals counts for 50 simulations

We redo the same plots but with now 500 simulated spike trains.

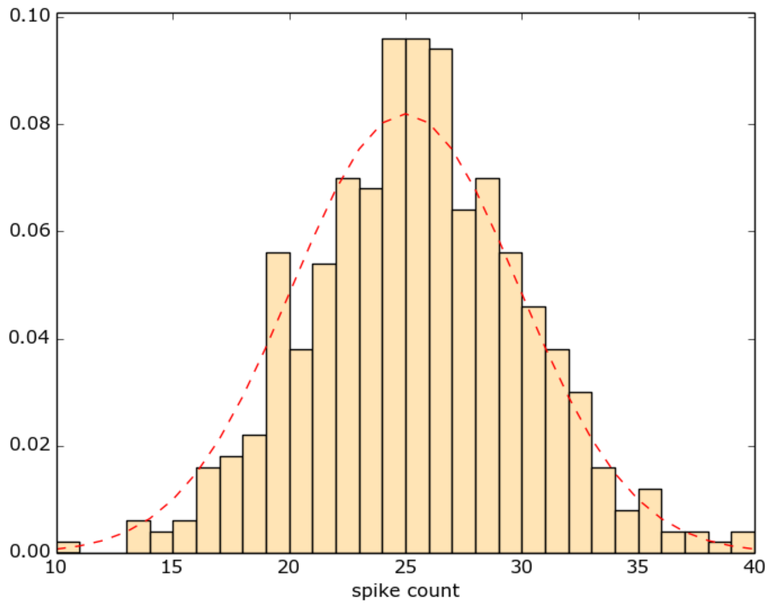

FIGURE 6: Histogram of total spike counts for 500 simulations

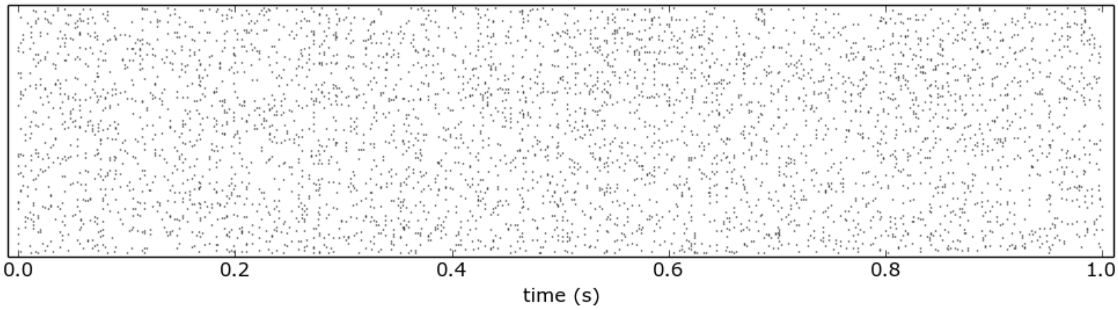

FIGURE 7: 500 Poisson spike trains with firing rate 25 spikes/sec

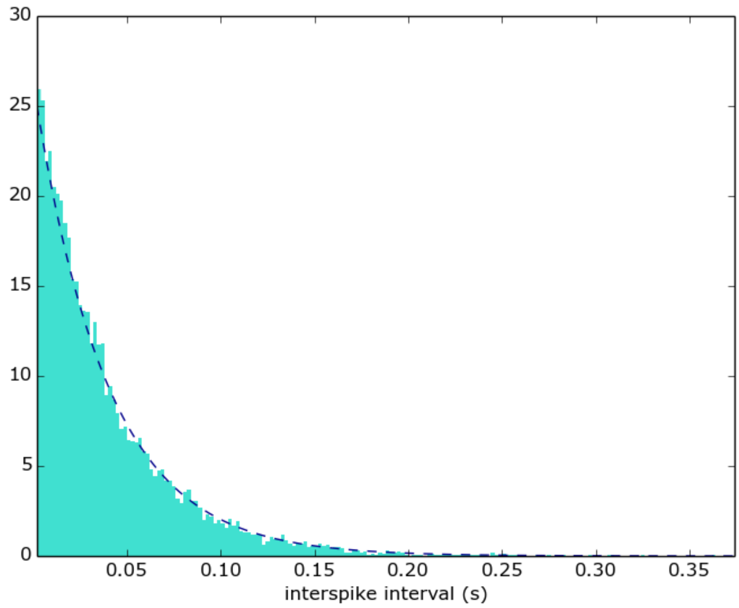

FIGURE 8: Histogram of interspike intervals counts for 500 simulations

We observe that theoretical results fit much better.

1.2 Analysis of spike trains

Besides modeling the spike train generations, we'd also like to do some simple analysis of real spike trains. We use thus the experimental data recorded from a single neuron in the primary somatosensory cortex of a monkey that was experiencing a vibratory stimulus. First we plot the spike trains for the stimulus f = 8.4 Hz into the graph below.

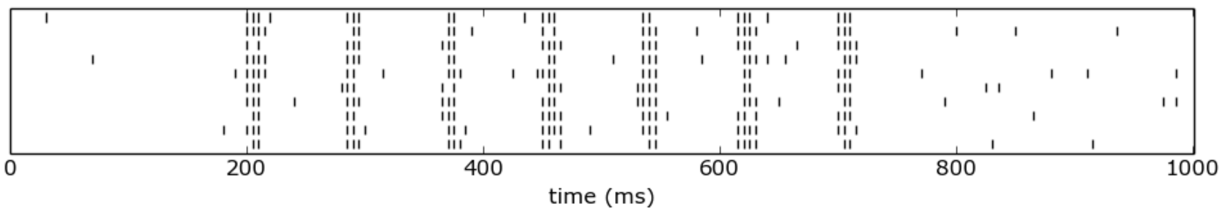

FIGURE 9: Real spike trains recorded from a nueron in the primary somatosensory cortex of a monkey that was experiencing a vibratory stimulus with f = 8.4 z

Here we don't observe anymore the poisson process. Instead, we tend to see more spikes at some specific moments that are separated by some fixed length time intervals. We now plot all the recorded spike trains into the same graph. Alternate backgroud colors are meant to indicate different stimuli.

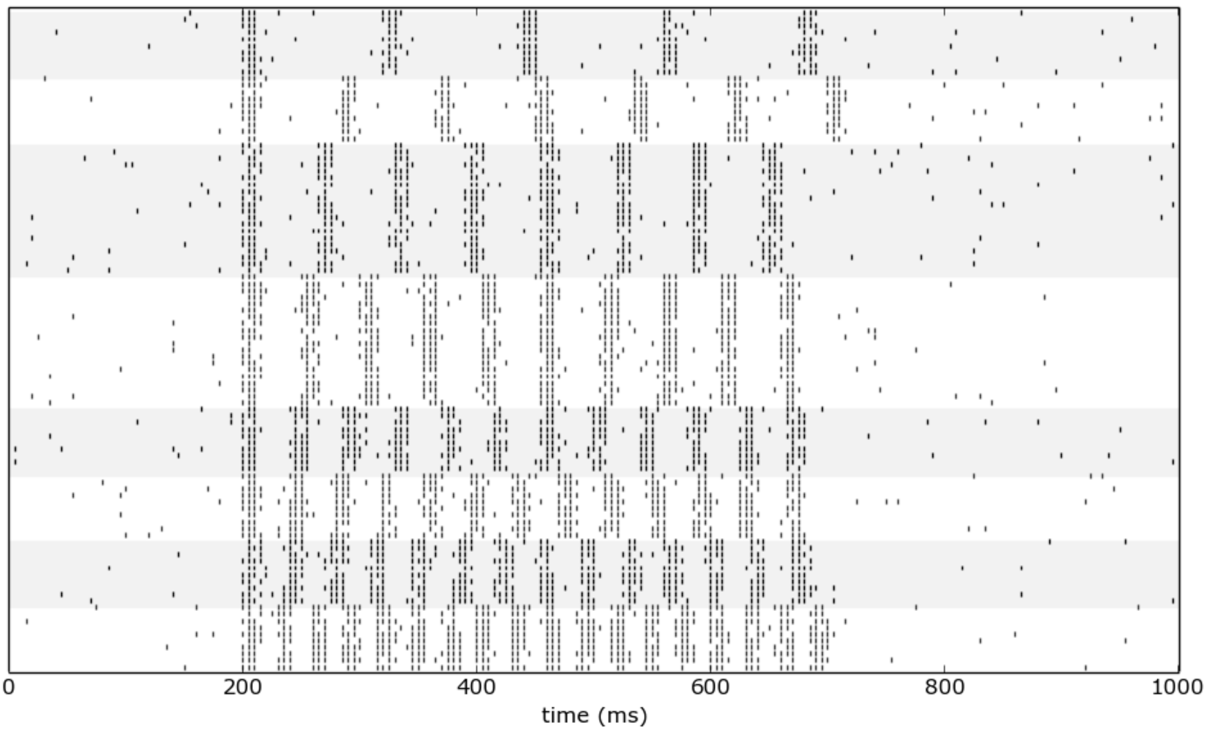

FIGURE 10: Real spike trains recorded from a nueron in the primary somatosensory cortex of a monkey that was experiencing some vibratory stimulus with different frequencies

The corresponding frequency for each dataset is not shown in the graph. In fact from top to bottom the frequency increases and we see that the seperating time intervals also become shorter and the spike count augments. This shall be even clearer if we give the exact numbers. The simulus is only present between $t = 200$ ms and $t = 700$ ms. We compute the average spike count and the standard deviation of spike counts for each stimulus.

TABLE 1: Mean values and standard deviations of spike counts for different stimuli

Frequency (Hz)	8.4	12	15.7	19.6	23.6	25.9	27.7	35
Average spike count m	16.5	19.2	23.6	29.9	35.6	39.5	41.8	52.3
Standard deviation σ	1.80	1.47	1.96	1.58	2.50	5.94	1.89	3.26

Sure when the frequency gets higher, the average spike count increase as well. We plot also the tuing curve of the neuron.

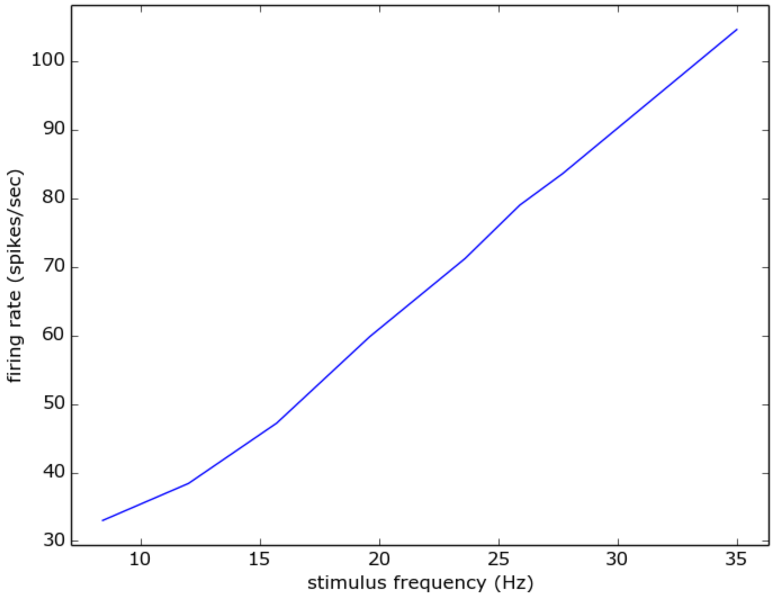

FIGURE 11: Tunning curve of the neuron

The relation between the average firing rate and the stimulus frequency is almost linear. We may also want to show that the mean value and the standad deviation of spike counts are positively correlated and even try to find an explicit relation between these two quantities (for example in the case when the spike count is sampled from some poisson distributions we have $\mu = \sigma^2$). However, with the values given in Table 1, we can not easily draw a conclusion.