Paper Implementation: Reproduce some basic results of the article "Generating Coherent Patterns of Activity from Chaotic Neural Networks"

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Summary

In this report we'll be working on the article "Generating coherent patterns of activity from chaotic neural networks" published by David Sussillo and L.F. Abbott in 2009. In the work of D. Sussillo and L.F. Abbott, they were interested in neural networks that are capable of generating complex activity patterns either spontaneously or in response to different stimuli.

The network is meant to be chaotic before training, i.e. its activity should be highly sensitive to initial conditions. This is acheieved by the usage of strong recurrent synaptic connections inside it. The so-called FORCE learning algorithm is then employed to modify synaptic weights either external to or within the network. After the training phase, the network will be able to produce a wide variety of complex output patterns and we can switch between different outputs by controlling the input signal.

Now let's come back to this report itself. I only worked on a relatively small part of the article: the generation of periodic patterns in the absence of inputs by using FORCE learning. Moreover, three distinct network structures are considered in the article (this will be detailed later on), but only the first one, which is probably also the simpest, is implemented here.

Introduction

When we're performing some motor action, what is the source of the nerual activity that initiates and carries out this behavior? In the paper of D. Sussillo and L.F. Abbott, they examined the idea that such action may come from the reorgnaization of spontaneous neural activity, which then leads us to another question: how can a such reorgnaization happen?

The objective is thus to find a possible synaptic weight modification rule that allows network activity to be reorgnaized into coherent patterns required to produce complex motor actions after training. However, comparing with feedfoward architectures, the training of a recurrent network is shown to be much more difficult due to several reasons. First, feeding erroneous output back into a network during training may prevent its activity from converging. Secondly, in a recurrent network the credit assignement for output errors becomes quite a hard problem because of the presence of recurrent weights. Finally, while using a network that display chaotic activity prior to training can be beneficial, chaos must be avoided during learning.

FORCE learning solved these problems by keeping the errors always small and forcing the synaptic modifications to be strong and rapid over training. This is quite different from classic learning algorithms which usually reduce the errors little by little. Therefore, the goal of FORCE learning is not significant error reduction, but rather reducing the amount of modification needed to keep the errors small.

According to the article, the contribution of FORCE learning can be viewed from two different angles. From a biological point of view, it can be regarded as a model for learning-induced moficifation of biological networks. Or more simply, it can just be seen as a powerful algorithm that allow us to construct recurrent networks being able to generate complex and controlable output patterns.

Network Arcchitectures

The network that was studied in the paper and will be study here is composed of individual neurons characterized by their firing rates and sparsely interconnected through excitatory and inhibiroty synapses of various strengths (when it comes to the biological reality, we may need to seperate group of excitatory and inhibitory neurons, but when we're modeling neural circuits, such details become less important). Initially, the connectivity and synaptic strengths of the network are chosen randomly.

The next problem is the definition of the output of networks. The technical choice is to define it as a linear combination of neuronal activities. Formally, if at time t, we assemble the firing rates of all the neurons into a column vector $\mathbf{r}(t)$, and the weights connecting these neurons to the output is described by the column vector $\mathbf{w}(t)$, then the network output is

$$z(t) = \mathbf{w}^\top \mathbf{r}(t).$$

Schematically, this output will be stored in a readout unit and its value may be fed back into the network (it depends on the concrete structure). Though thus far the network output is only a scalar, by using several different decoding vectors $\mathbf{w}$ we can easily define a multidimensional readout. This is exactly what D. Sussillo and L.F. Abbott did in their paper when they later asked the network to generate multidimensional outputs; however this point will not covered by the report.

Now that we have defined z as the output of the network, the goal of the algorithm is to set $z = f$ for somme target function $f : t \mapsto f(t)$. Either z is generated in the abscence of any input or it may depend on inputs of the network in a specific way. In the scope of this report, only the first case will be discussed.

Finally, as shown in Figure 1, three different network architectures are proposed in the article. In the first architecture (Figure 1A) feedback to the generator network is provided directly by the readout unit. During training, we modify only the decoding vector $\mathbf{w}$ and the internal recurrent synaptic strengths are left intact. Such a network is simple but far from realistic: every neuron receives the same feedback (though it may be weighted differently) and this feedback itself originates from an abstract output that doesn't exist in real circuit. To address this problem, two other structures more biologically plausible are given. In the second structure (Figure 1B) we use a separated feedback network to supply feedback and in the third one (Figure 1C) modifications on internal synaptic strengths are allowed. Without giving more details here, I just mention that after proper adaptions, FORCE learning can be carried out on the three structures and yield promising results indifferent to the underlying network structure.

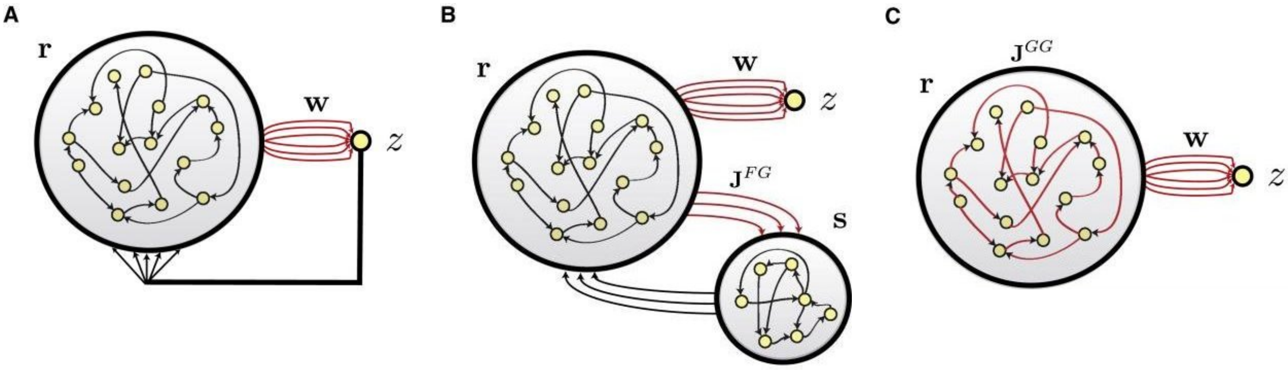


Figure 1: Network Architectures (picture from the paper).
In all three cases, a linear readout unit derives an output z from the firing rates $\mathbf{r}$ of the recurrent generator network. Only connections shown in red are subject to modification.
(A) The output of the readout unit is directly fed back to the generator network.
(B) Feedback to the generator network is provided by a separated feedback network recurrently connected and whose inputs come from the generator network through synapses of strength $\mathbf{J}^{FG}$ (red) that are also modified during training.
(C) A simple recurrent network with no external feedback, but all synaptic strengths $\mathbf{J}^{GG}$ are subject modifications by the learning algorithm.

Now let's put all the elements together and give a formal description of the network that will be used in the framework of this report. First we have a column vector $\mathbf{x}$ containing some abstract values of each neuron (just for modeling convenience, or one may be able to find some biological meaning for this vector that I ignore here). The firing rates are computed by $\mathbf{r} = \tanh(\mathbf{x})$ (application of the function term by term). The dynamic of the network is governed by the equation

$$\tau \frac{d\mathbf{x}}{dt} = -\mathbf{x} + g_{GG}\mathbf{J}^{GG}\mathbf{r} + g_{Gz}\mathbf{J}^{Gz}z,$$

where τ is the caracteristic time, $\mathbf{J}^{GG}$ and $\mathbf{J}^{Gz}$ are respectively the matrix of internal synaptic weights and the vector of connection weights from the readout unit to the generator network (the feadback loop), and z is the output defined earlier. g_{GG} is introduced to scale the strengths of the recurrent connections. When $g_{GG} < 1$ the network is inactive prior to training; on the contrary if $g_{GG} > 1$ the network exhibits chaotic spontaneous activity. g_{Gz} plays a similar role with regard to the connections from the readout unit to the main network.

We use moreover a sparseness parameter ρ to impose sparsity on the network: each pairwise connection is set and held to 0 with probability $1 - \rho$. In the whole report, we'll choose $\tau = 10$ ms, $g_{GG} = 1.5$, $g_{Gz} = 1$, $\rho = 0.1$ and the network contains $N = 1000$ neurons unless otherwise stated. For initialization, nonzero elements of $\mathbf{J}^{GG}$ are drawn independently from a Gaussian distribution with zero mean and variance $1/(\rho N)$ while elements of $\mathbf{J}^{Gz}$ are drawn from a uniform distribution between -1 and 1. We notice that feedback synapses are stronger than internal ones in order to have an appreciable effect on the network activity and this is necessary for suppressing the inherent chaos of the network. Similarly, elements of $\mathbf{w}$ and $\mathbf{x}$ are respectively generated by Gaussian distributions with zero means and standard deviations $\sqrt{1/(\rho N)}$ or $\sigma_x = 0.5$.

FORCE learning

The term FORCE learning stands for first-order reduced and controlled error learning. Training of recurrent networks is in general a hard problem as already mentioned in the introduction (even though here we modify only the readout vector $\mathbf{w}$, the presence of the feedback loop makes any effect of the modification difficult to predict). Since erroneous output shouldn't be fed back to the network, the proposed algorithm must rapidly reduce the magnitude of the difference between the actual and desired output to a small value, and then keep it small while searching for the proper fixed readout vector that can generate the target function without further modification on this vector.

The general schema of such an algorithm consists of updating $\mathbf{w}$ at times separated by an interval Δt based on the readout error at this moment which is defined by:

$$e_-(t) = \mathbf{w}^\top(t-\Delta t)\mathbf{r}(t) - f(t).$$

The minus subscript signifies that this is the error prior to weight update at time t. The update of $\mathbf{w}(t-\Delta t)$ to $\mathbf{w}(t)$ should allow us to decrease the error so that

$$e_+(t) = \mathbf{w}^\top(t)\mathbf{r}(t) - f(t)$$

satisfies $|e_+(t)| < |e_-(t)|$. Since the end of the training takes place only when the decoding vector $\mathbf{w}$ stabilizes and no longer changes, this also requires $e_+(t)/e_-(t) \to 1$. Note that Δt is the interval of time between modifications of readout weights and doesn't necessarily equal to the basic integration time step for the network simulation. In fact, for the simulations carried out throughout this report, I take $\Delta t = 1$ ms whereas the basic integration time step is 0.1 ms.

Knowing all of this, it's still far from obvious to deduce the appropriate modification rule that will work. In the paper, D. Sussillo and L.F. Abbott express their favor to use the recursive least-squares (RLS) algorithm here. I'll not dig into the mathematical details of this algorithm, but briefly in our case it gives

$$\mathbf{w}(t) = \mathbf{w}(t-\Delta t) - e_-(t)\mathbf{P}(t)\mathbf{r}(t),$$

where $\mathbf{P}(t)$ is an $N \times N$ matrix that is updated at the same time as the weights according to the rule

$$\mathbf{P}(t) = \mathbf{P}(t-\Delta t) - \frac{\mathbf{P}(t-\Delta t)\mathbf{r}(t)\mathbf{r}^\top(t)\mathbf{P}(t-\Delta t)}{1+\mathbf{r}^\top\mathbf{P}(t-\Delta t)\mathbf{r}(t)}.$$

We set the initial value for $\mathbf{P}$ to be $\mathbf{P}(0) = \mathbf{I}/\alpha$ where $\mathbf{I}$ is the identity matrix and α is a constant parameter that needs to be chosen carefully according to the target function being learned. In this report I use $\alpha = 1$ as suggested by the article. The modification rule can then be viewed as a starndard delta-type rule, but instead of a scalar quantity, multiple learning rates are specified by $\mathbf{P}$, which is in reality a running estimate of the quantity $(\sum_t \mathbf{r}(t)\mathbf{r}^\top(t) + \alpha\mathbf{I})^{-1}$. The use of $\mathbf{P}$ allows us to have a finer control over learning because it adapts the learning rate to the magnitude of different principal components of the network activity. However, concerning how it works and why it works, it will not be explained here.

One can easily show that if we assume that $\mathbf{w}(0) = \mathbf{0}$ for simplicity, the error after the first weight update at time Δt is

$$e_+(\Delta t) = -\frac{\alpha f(\Delta t)}{\alpha + \mathbf{r}^\top(\Delta t)\mathbf{r}(\Delta t)}$$

which is small as long as $\alpha \ll N$ knowing that $\mathbf{r}^\top \mathbf{r}$ is of order N. Furthermore, every time when the weights are updated, we have

$$e_+(t) = e_-(t)(1 - \mathbf{r}^\top(t)\mathbf{P}(t)\mathbf{r}(t)).$$

The quantity $\mathbf{r}^\top \mathbf{P}\mathbf{r}$ is always positive and decreases to zero over the course of learning; this means the size of error is reduced by the weight update and as required $e_+(t)/e_-(t) \to 1$.

FORCE Learning Examples

In this section we'll apply the FORCE procedure on the network for different target functions. The network is able to learn to generate a variety of patterns after training. The first example presented here is a triangle wave of period 0.6 s (Figure 3). The network activity is chaotic before training (Figure 3A, blue lines), and during the training phase (Figure 3B), as predicted the error is reduced dramatically after a first weight update and the output of the network (red line) matches target function throughout the rest of training. The progression of learning can be observed through the decrease of size of fluctuations of the readout vector (orange trace). $\mathbf{w}$ fluctuates rapidly at the beginning of training to force the network to produce the desired periodic pattern, but gradually it stabilizes and find the static weights that are needed to generate the target function without requiring further modifications. After training (Figure 3C), the network continues to generate the desired pattern indefinitely without need of modifcation on decoding weights.

We also notice that the network activity chaotic prior to training becomes periodic in order to produce the periodic target function, and it continues to be periodic after learning. The learning process is indeed quite rapid here, only five cycles of the traingle wave is sufficient here (and in the paper the convergens takes place only after four cycles). This time depends however a lot on the form of the target function. For example, to learn the complicated periodic functions shown in Figure 5B, the network needs to be trained for a time that corresponds to 10 to 20 cycles of this function ($\sim$ 30 s). And for the sine wave with extremely small period (Figure 5D), hundreds of cycles are even needed since the network converges only after a few tens of seconds.

We show in Figure 5 that the network is able to produce periodic functions of different complexity and form, even when the target function is corrupted by noise (Figure 5E). Nonetheless, the article claims that in these examples, training typically converges in 1000τ, that is, 10 s, while in my simulations it takes much longer time. In most of the cases, at least half a minute is required for the convergence to take place and sometimes even longer time is needed. For example, in Figure 5D the network is trained for a minute (simulation time) but the generated sine wave is still not yet perfect.

According to the paper, with the parameters that are used here, the network is able to generate sine wave with periods ranging from 60 ms to 8 s after training. As just mentioned, though not perfect, I managed to reproduce this result for a sine wave with period 60 ms. This is however not the case for a sine wave with period 8 s (Figure 6). We see that after one and half a minute of training the network is still not able to produce the target funciton. I don't know if more time is needed for the training to converge since the function is of great period or my network is just not able to learn to produce this function using the FORCE procedure.

In the paper, it is also shown that using FORCE learning the network is capable of generating non-periodic patterns as well, such as the dynamic variables of the three-dimensional chaotic Lorenz attractor (even though the match lasts only for a finite amount of time). By modifying slightly the network structure and the training algorithm (adding a second readout unit and feedback loop), it's also possible to produce a segment matching a one-shot target function. However, due to the complexity of these two experiments, they're not reproduced in my report.

Another point worth mentioning is that with the presence of feedback loop and if the parameters are initialized as described above, the network is in fact incapable of displaying spontaneous chaotic activity prior to training in my simulation. Two possible scenarios are present: the whole system can converge to some fixed point and become inactive (Figure 2) or the network activity may become periodic (Figure 4A). As a result, in Figure 3 I in fact set $g_{Gz} = 0$ before training to exhibit the chaotic behavior. Since the initial value of $\mathbf{x}$ doesn't seem to be important for FORCE learning, the fact that the network doesn't demonstrate chaotic activity when there is feedback and when the parameters are initialized in the way described earlier has no influence on the learning procedure that takes place later.

A possible hypothesis is that strong feedback connection weights $\mathbf{J}^{Gz}$ may prevent the network from being chaotic in any situations even when training is not taking place. This explication can however be denied by Figure 4. In this figure two things are shown. Since strong feedbacks are indespensable for suppressing the chaotic behavior of the network, when the target function has a too small magnitude the activity of the network neurons is chaotic during training and learning doesn't converge to a successful solution. According to the paper, this problem wouldn't occur with the networks shown in Figure 1B and 1C. The second obeservation is more interesting. The network activity is in fact not chaotic before training, but it becomes chaotic during learning and remains chaotic even after learning. This suggests that even with the presence of feedback loop the network can be spontaneously chaotic. One possible reason is that after learning the output z becomes smaller in magnitude (this can be seen in the figure) and the feedback into the network is now too small to suppress the chaos. This explanation is still far from satisfying since the change of $|z|$ is not always significant but by traing the network with a target function with small magnitude we are always able to exhibit chaotic activity at the end. I really have difficulty in finding a good reason to explain this.

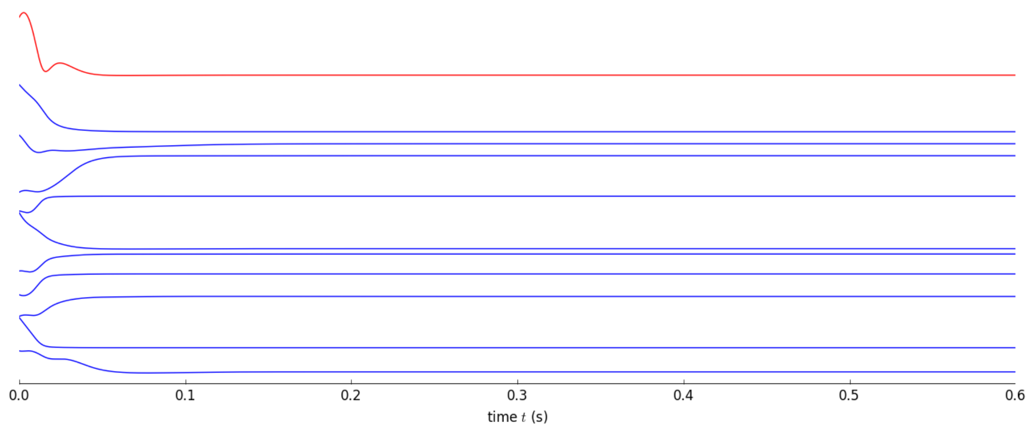


Figure 2: Network Spontaneous Activity with the Presence of Feedback Loop. When the feedback loop is present (so $g_{Gz} = 1$) and the initialization is carried out as decribed in the text, the network doesn't exhibit chaotic activity prior to training. In this particular example the system converges to a fixed point and becomes inactive.

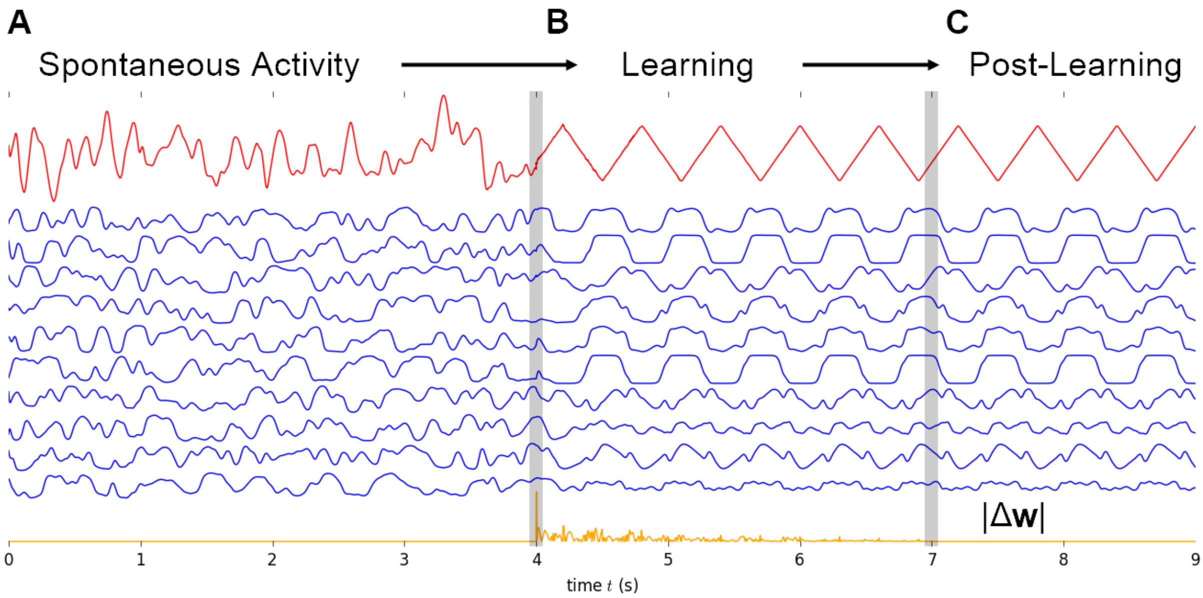


Figure 3: FORCE Learning Applied to a Triangle Wave.
Network output z is in red, the firing rates of 10 sample neurons are in blue and the orange trace represents the magnitude of fluctuations of $\mathbf{w}$ ($\Delta\mathbf{w} = \mathbf{w}(t) - \mathbf{w}(t - \Delta t)$).
(A) The network exhibits chaotic spontaneous activity before training.
(B) During training, the readout error is reduced to small magnitude rapidly and then the output matches always the target function. Readout weights fluctuate greatly at first, gradually the fluctuations diminish and finally almost no more modifications of $\mathbf{w}$ are needed.
(C) At this moment we can turn off the learning procedure and the network persists in generating the desired triangle wave indefinitely without requiring any weight modifcation.

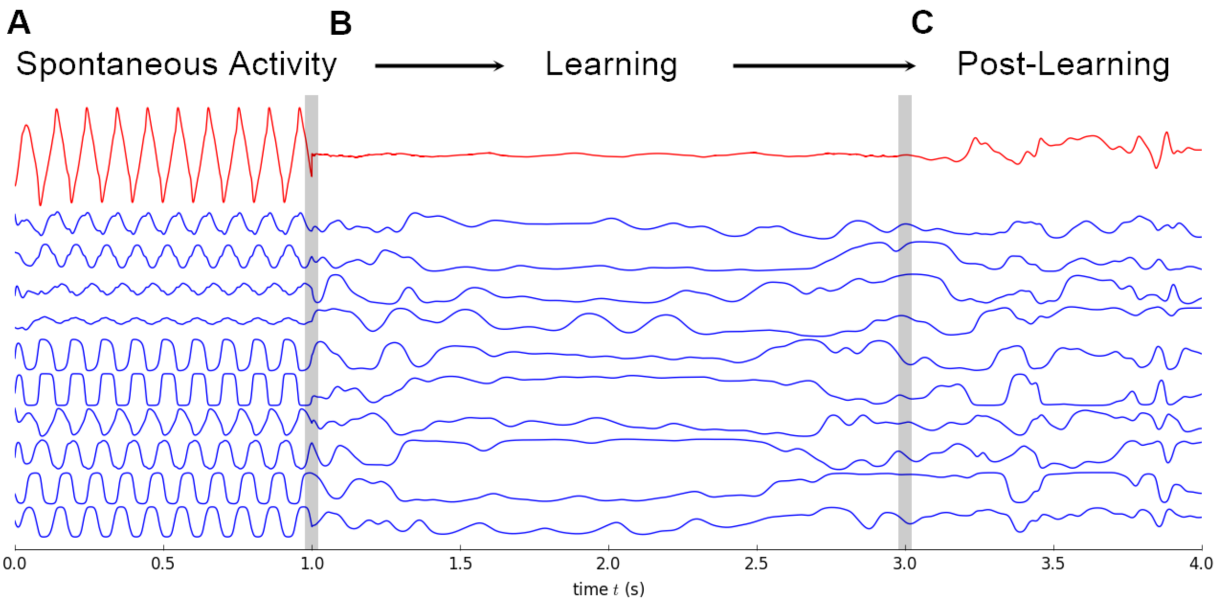
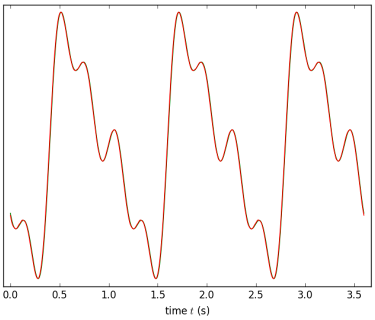
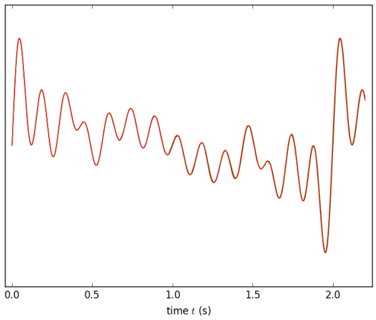


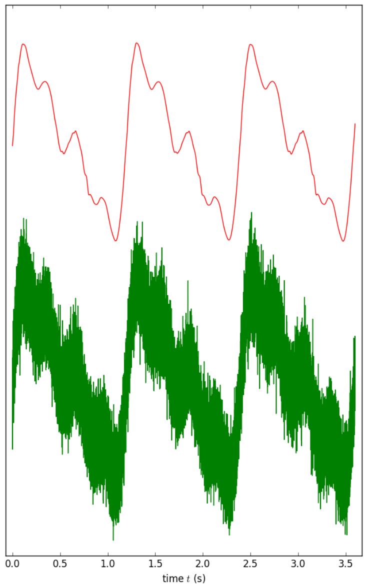
Figure 4: FORCE Learning Applied to Sine wave with Small Magnitude.
FORCE Learning fails for this low amplitude sine wave because feedback is not strong enough. What is interesting is that the network activity isn't chaotic before training (see text and Figure 2) but it becomes chaotic during training and remains chaotic after training.



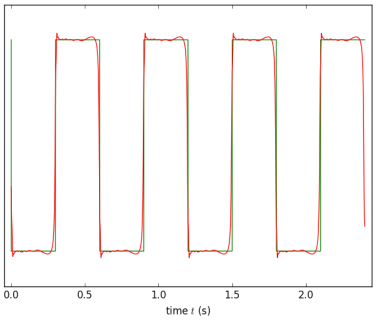
(A) Periodic



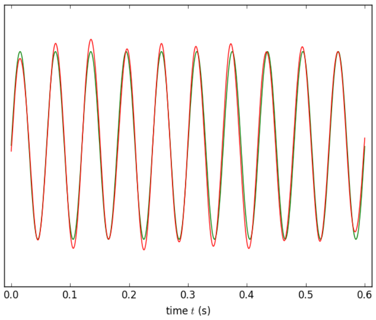
(B) Compicated periodic



(E) Extremely noisy target



(C) Discontinuos target



(D) Sine wave period 6τ

Figure 5: FORCE Learning Applied to Various Patterns.
We show here several examples of successful FORCE learning. Network outputs after training are in red and green traces are target functions (often covered by the red traces, in subfigure (E) the two traces are separated for a better visualization).
(A) Simple periodic function composed of four sinusoids.
(B) Periodic function composed of sixteen sinusoids.
(C) Square-wave.
(D) Sine wave with pariod of 60 ms.
(E) Periodic function of four sinusoids learned from a noisy target functions.

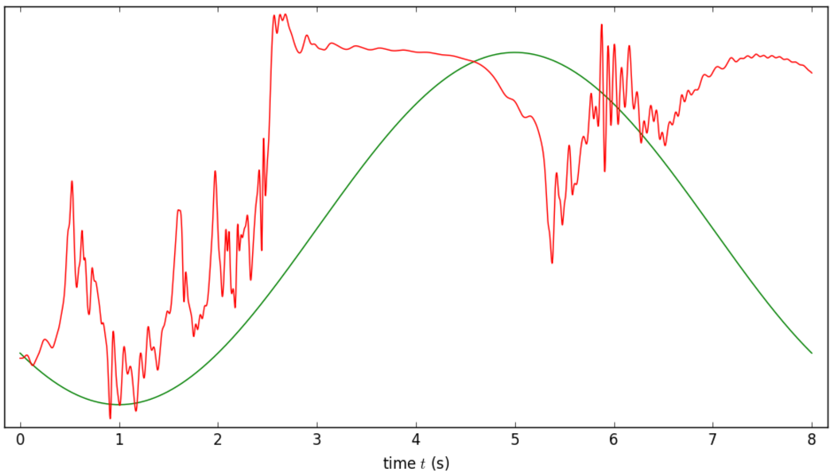


Figure 6: FORCE Learning Applied to a Sine Wave with Period of 8 s.
After one and half of minute of training, the network still fails to produce the desired pattern and the generated trace (red) doesn't resemble at all the target function (green). Either more time is needed for training to converge or this is simply an impossible task for the network.

Principal Component Analysis of FORCE Learning

We examine the activity of a network after training by the FORCE procedure by the usage of principal component analysis (PCA). The network is first trained to produce the periodic pattern shown in Figure 7A (the same target function as Figure 5A). The distribution of PCA eigenvalues (Figure 7C) shows that the trajectory of the network activity concentrates on a subspace of much lower dimension than the number of available neurons in the network. For example, by projecting the neural activity on the eight leading PC vectors we can already construct a very good approximation of the real output (Figure 7A).

In the paper, it is shown that after each training, if the readout vector is projected onto the PC vectors of the network activity, about the top 50 of these projections converge always to the same specific values. In addition, in the RLS algorithm, the matrix $\mathbf{P}$ is adjusted to adapt the learning rates for different PC vectors. Nevertheless, further discussion will be omitted here.

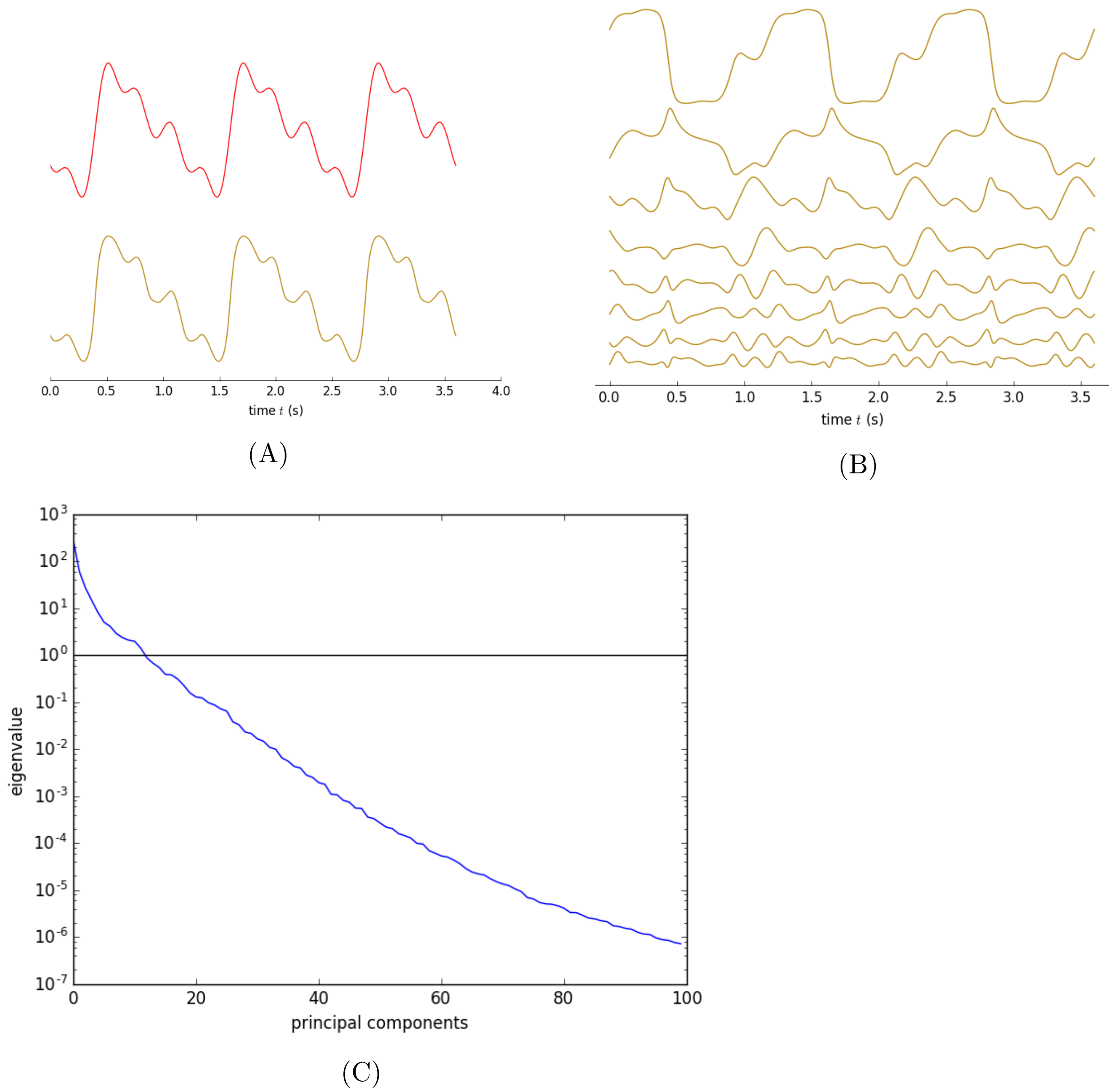


Figure 7: Principal Component Analysis of Netwrok Activity
(A) Output after training a network to produce a sum of four sinusoids (red) and the approximation (brown) obtained using activity projected onto the 8 leading principal components.
(B) Projections of network activity onto the leadaing 8 PC vectors.
(C) PCA eigenvalues for the network activity. Only the largest 100 of 1000 eigenvalues are shown.

Conclusion

Neural networks in biological systems seem to exhibit complex and irregular spontaneous activity that has both stochastic and chaotic sources. In the article "Generating coherent patterns of activity from chaotic neural networks", D. Sussillo and L.F. Abbott propose an algorithm called FORCE learning that can endow such a chaotic network with ability to generate coherent patterns that may be useful for, for example, motor controls.

In this report I implemented this algorithm on one of the three network structures given in the article. The network has an linear readout unit and feedback to the generator network comes directly from this readout unit. Only readout weights $\mathbf{w}$ are modified during learning. I was able to reproduce several results of the article. After learning the network manages to generate a variety of periodic output patterns in the abscence of input, even though all the patterns presented in the article were not successfully learned (sine wave with period 8 s) and the learning time is longer than what is given in the article. Notice that on a laptop one second in the simulation can be much longer in reality, so if the training is meant to be quite long in simulation the result may not be easily reproducible. This is in fact the case for Figure 5 in the original text: it's shown that the performance of the network is the best when it's at the edge of chaos by training networks of differents values of g_{GG}. It's theoretically easy to reproduce this result because the algorithm used is exactly as what was implemented in this report. Nevertheless, too many simulations that are possibly very long are needed to be carried out to produce a similar figure.

I had also a doubt on the source of the chaotic activity in the model, since it's not clear if the feedback is connected to the network when we say that the network is chaotic before training. PCA carried out on the network activity after applying the FORCE procedure is in fact important to explain how the algorithm works and the final dynamic of the system, but details are not presented in my report. The paper went much farther than what has been done until here. The feedback can be distorted and delayed during training. FORCE algorithm was also adapted to be used on the network architectures of Figure 1B and 1C. The network can have multiple and multidimensional outputs controlable by some particular input signals. Finally, in the paper they also succeeded in creating a network that can generate multiple, high-dimensional and nonperiodic patterns that resemble complex human motions.

The FORCE algorithm given here may not be very biologically plausible (even when applying on the second and the third network structure) because updates of readout weights rely on a global error and another global quantity $\mathbf{P}$. A simpler plasticity mechanism is proposed by D. Sussillo and L.F. Abbott in the supplemental data of their article, but on the other hand it's not as powerful as the version presented here.

Another surprising fact is that throughout the report, at no time we're modifying the internal recurrent synaptic strengths of the network, but the network is still able to produce the desired patterns after training. Only looking at the internal synaptic connectivity, we would not be able to tell what the network is doing. Feedback loops passing through distal networks may play a role far more important than one imagines. This also means that the network is able to learn new tasks without need to have great modifications on the orignial circuit whose synaptic connectivity are probably already fixed to be able to carry out some basic but priomordial tasks.

Reference

David Sussillo and L.F. Abbott. Generating coherent patterns of activity from chaotic neural networks. *Neuron* 63:544-47, 2009.