

Classification from Positive, Unlabeled and Biased Negative Data

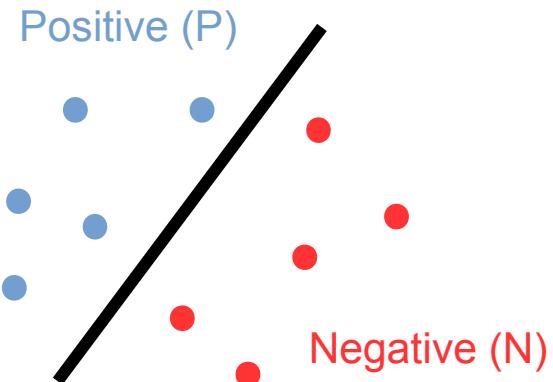
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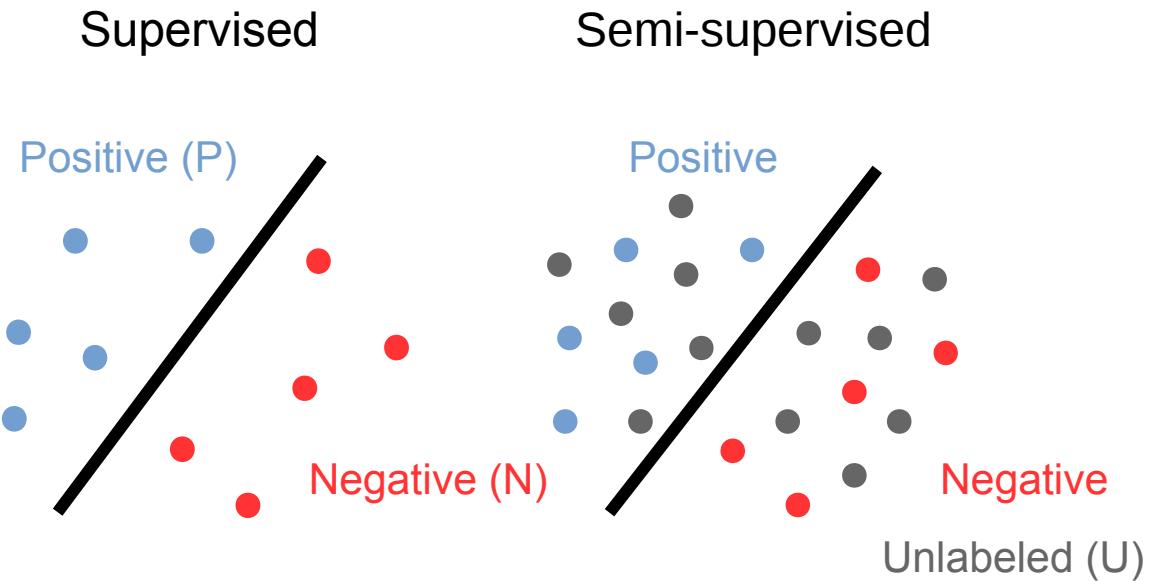
Background and problem setup

Background and problem setup

Supervised



Background and problem setup



Background and problem setup

Supervised

Positive (P)

Negative (N)

Semi-supervised

Positive

Negative
Unlabeled (U)

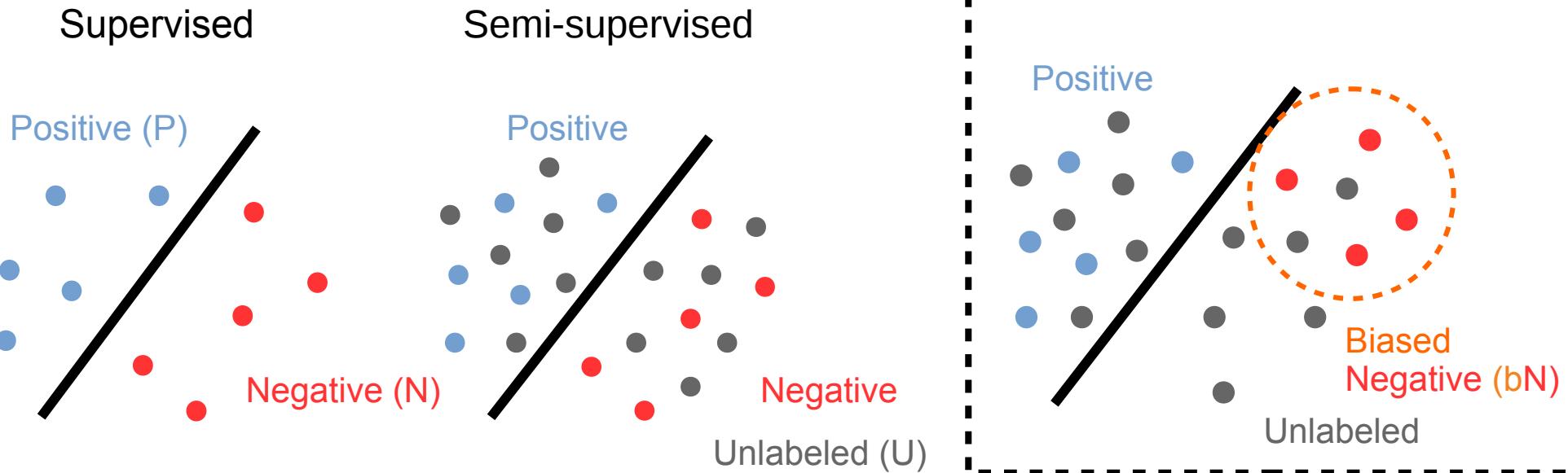
PUbN

Positive

Biased
Negative (bN)

Unlabeled

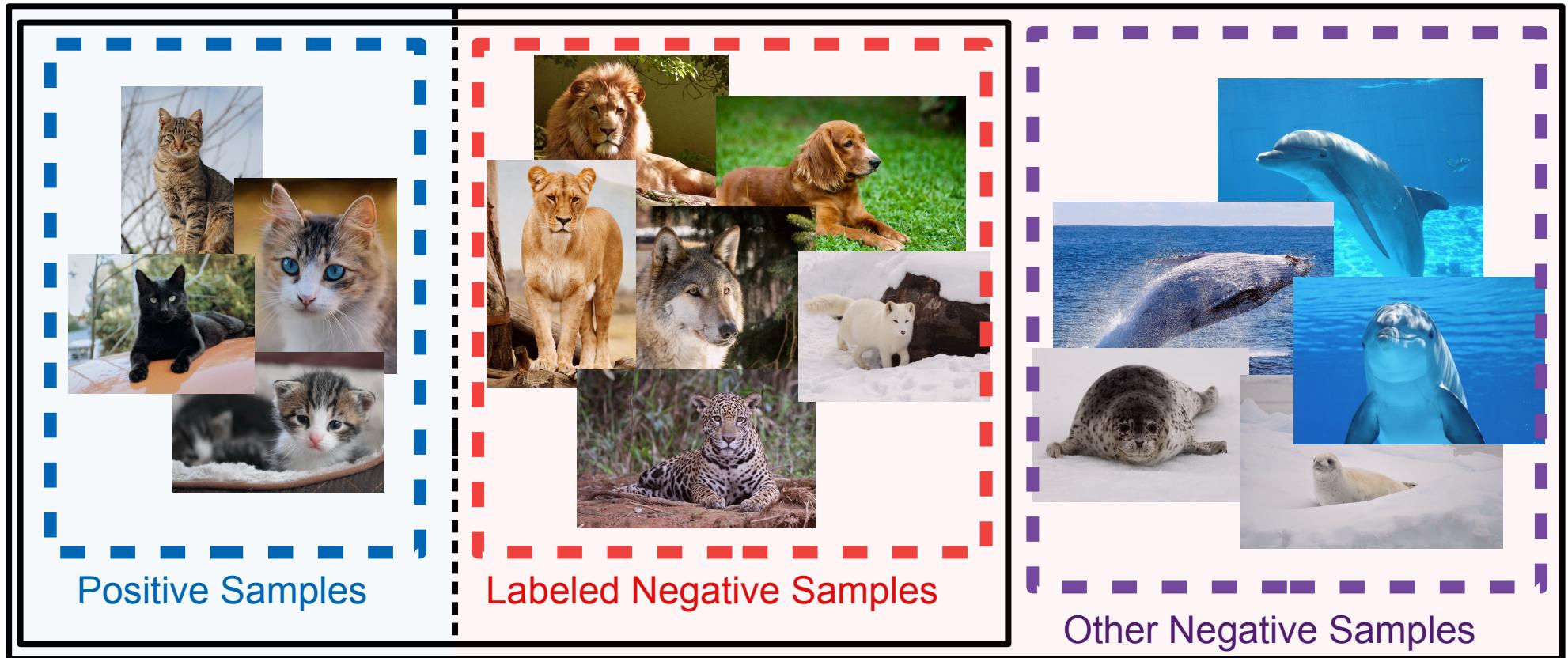
Background and problem setup



- $\sim p_P(x) := p(x \mid y = +1)$
- $\sim p_N(x) := p(x \mid y = -1)$

- $\sim p(x)$
- $\sim p_{bN}(x) := p(x \mid y = -1, s = +1)$

Motivating examples



- Information retrieval, text classification, sentiment analysis
- Medical diagnosis: healthy population requesting physical exams is biased

Method: Empirical risk estimator

$$\min_{g \in \mathcal{G}} \underbrace{\mathbb{E}_{(x,y) \sim p(x,y)} [\ell(yg(x))]}_{R(g)}$$

Risk Minimization

←
Unbiased
Estimator

$$\min_{g \in \mathcal{G}} \frac{1}{n} \sum_{i=1}^n \ell(y_i g(x_i))$$

$\hat{R}(g)$

Unbiased labeled data
Empirical Risk Minimization

Method: Empirical risk estimator

$$\begin{aligned} R(g) = & \pi \mathbb{E}_{x \sim p_{\mathcal{P}}(x)} [\ell(g(x))] + \pi \mathbb{E}_{x \sim p_{\mathcal{P}}(x)} \left[1_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right] \\ & + \rho \mathbb{E}_{x \sim p_{\text{bN}}(x)} [\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{\text{bN}}} \left[1_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right] \\ & + \mathbb{E}_{x \sim p(x)} [1_{\sigma(x) \leq \eta} \ell(-g(x))(1 - \sigma(x))] \end{aligned}$$

$\sigma(x) = p(s=+1|x)$ probability of x being labeled

$\eta > 0$ determining how we rely on the U data to approximate the risk

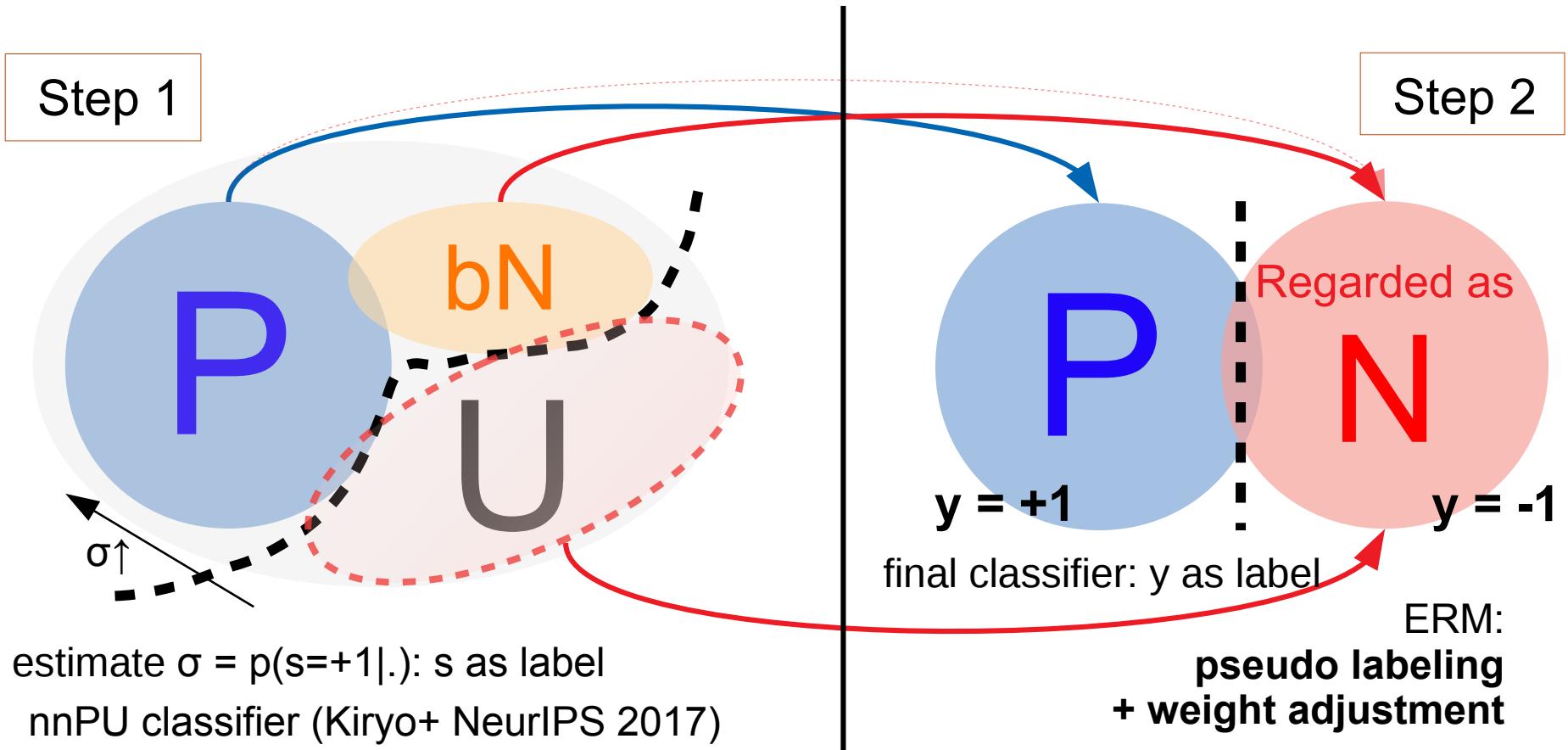
Method: Empirical risk estimator

$$R(g) = \boxed{\pi \mathbb{E}_{x \sim p_{\text{P}}(x)} [\ell(g(x))] + \pi \mathbb{E}_{x \sim p_{\text{P}}(x)} \left[1_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]} \quad \#\text{P data}$$
$$+ \boxed{\rho \mathbb{E}_{x \sim p_{\text{bN}}(x)} [\ell(-g(x))] + \rho \mathbb{E}_{x \sim p_{\text{bN}}} \left[1_{\sigma(x) > \eta} \ell(-g(x)) \frac{1 - \sigma(x)}{\sigma(x)} \right]} \quad \#\text{bN data}$$
$$+ \boxed{\mathbb{E}_{x \sim p(x)} [1_{\sigma(x) \leq \eta} \ell(-g(x))(1 - \sigma(x))]} \quad \#\text{U data}$$

$\sigma(x) = p(s=+1|x)$ probability of x being labeled

$\eta > 0$ determining how we rely on the U data to approximate the risk

Method: Illustration



Estimation error bound

With probability at least $1-\delta$

$$R(\hat{g}) - R(g^*) \leq \frac{4\pi L_\ell}{\eta} \mathfrak{R}_{n_P, p_P}(\mathcal{G}) + \frac{2\pi C_\ell}{\eta} \sqrt{\frac{\ln(6/\delta)}{2n_P}} + \frac{4\rho L_\ell}{\eta} \mathfrak{R}_{n_{bN}, p_{bN}}(\mathcal{G}) + \frac{2\rho C_\ell}{\eta} \sqrt{\frac{\ln(6/\delta)}{2n_{bN}}} \\ \text{\#P data} \quad \quad \quad \text{\#bN data} \\ + 4L_\ell \mathfrak{R}_{n_U, p}(\mathcal{G}) + 2C_\ell \sqrt{\frac{\ln(6/\delta)}{2n_U}} + 2C_\ell \sqrt{\zeta\epsilon} + \frac{2C_\ell}{\eta} \sqrt{(1-\zeta)\epsilon} \\ \text{\#U data} \quad \quad \quad \text{Bias due to inexact approximation of } \sigma$$

Experiments

Models: ConvNet / ResNet / FCN + Training: Amsgrad

Dataset	P	π	bN	ρ	nnPU/nnPNU	PUBN(N)	PU → PN
MNIST	2, 4, 6, 8, 10	0.49	Not given	NA	5.76 ± 1.04	4.64 ± 0.62	NA
			1, 3, 5	0.3	5.33 ± 0.97	<u>4.05 ± 0.27</u>	<u>4.00 ± 0.30</u>
			9 > 5 > others	0.2	4.60 ± 0.65	<u>3.91 ± 0.66</u>	<u>3.77 ± 0.31</u>
CIFAR-10	Airplane, automobile, ship, truck	0.4	Not given	NA	12.02 ± 0.65	10.70 ± 0.57	NA
			Cat, dog, horse	0.3	10.25 ± 0.38	<u>9.71 ± 0.51</u>	10.37 ± 0.65
			Horse > deer = frog > others	0.25	<u>9.98 ± 0.53</u>	<u>9.92 ± 0.42</u>	<u>10.17 ± 0.35</u>
CIFAR-10	Cat, deer, dog, horse	0.4	Not given	NA	23.78 ± 1.04	21.13 ± 0.90	NA
			Bird, frog	0.2	22.00 ± 0.53	<u>18.83 ± 0.71</u>	<u>19.88 ± 0.62</u>
			Car, truck	0.2	22.00 ± 0.74	<u>20.19 ± 1.06</u>	21.83 ± 1.36
20 Newsgroups	alt., comp., misc., rec.	0.56	Not given	NA	14.67 ± 0.87	13.30 ± 0.53	NA
			sci.	0.21	14.69 ± 0.46	13.10 ± 0.90	13.58 ± 0.97
			talk.	0.17	14.38 ± 0.74	<u>12.61 ± 0.75</u>	13.76 ± 0.66
			soc. > talk. > sci.	0.1	14.41 ± 0.76	<u>12.18 ± 0.59</u>	12.92 ± 0.51