10-708 22/05/2020

L2 - Bayes ball / d-sep. (suppl.)

(*) 2 main perspectives; both encoding same set of rules on d-separation
- Jordan (2003): Bayes-ball algorithm / blocking
- Koller (2009): probabilistic influence / active trails

(*) d-separation has to account for naive graph sep. and fails with V-structures.

Jordan (2003): Ch 2

(*) Decide whether a given condit. independence statement $X_A \perp\!\!\!\perp X_B \mid X_C$ is true for a directed graph G.

(*) Formally; this means that the statement holds for every distribution that factors according to G (later).

(*) Use 3 canonical graphs (local structures)

(*) Reachability algorithm:-
- Shade node you are conditioning on i.e. X_C
- Place a ball at one of r.v.s i.e. X_A (source)
- Send it to the other r.v. i.e. X_B (destination)

(*): Does a ball reach the destination X_B from source X_A?

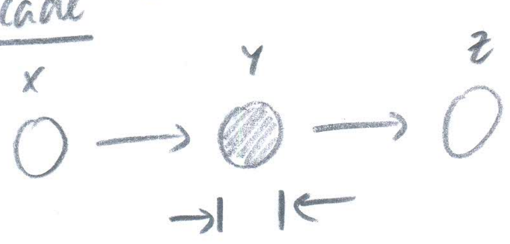
YES $\longrightarrow$ $X_A \perp\!\!\!\perp X_B \mid X_C$ is not true

NO $\longrightarrow$ $X_A \perp\!\!\!\perp X_B \mid X_C$ is true.

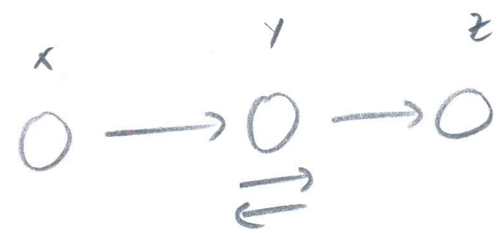
(*) Bayes ball specifies a set of rules on ball movement along directed edges

(*) Balls can travel in any dir

- cascade



(i)



(ii)

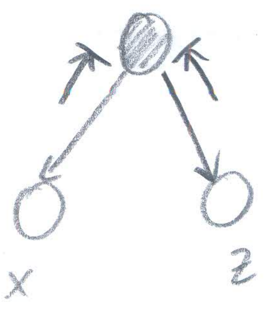
(2.9)

(i) Assert $X \perp\!\!\!\perp Z \mid Y$

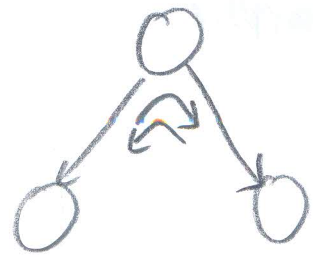
(ii) Do not assert $X \perp\!\!\!\perp Z \mid Y$

• For this structure; ball is blocked when conditioning

common parent



(i)



(ii) (2.10)

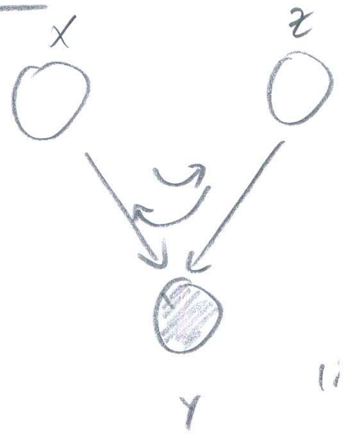
i) Assert $X \perp\!\!\!\perp Z \mid Y$

ii) Do not assert $X \perp\!\!\!\perp Z \mid Y$

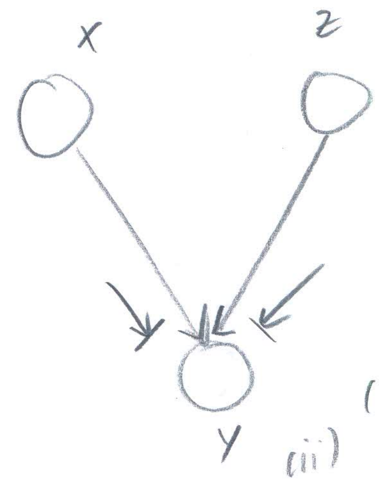
· For this structure, ball is <u>blocked</u> when conditioning.

(*) Note for cascade and common parents; conditioning on a node has the effect of blocking balls. This does not extend to V-structures; as Jordan points out.

V-structure



(i)



(ii) (2.11)

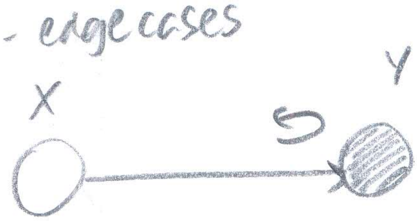
i) Do not assert $X \perp\!\!\!\perp Z \mid Y$

ii) Assert $X \perp\!\!\!\perp Z \mid Y$

- For this structure, ball <u>passes through</u> when conditioning.

(*) source and destination node same - edge cases

(i)



(ii) (2.12)

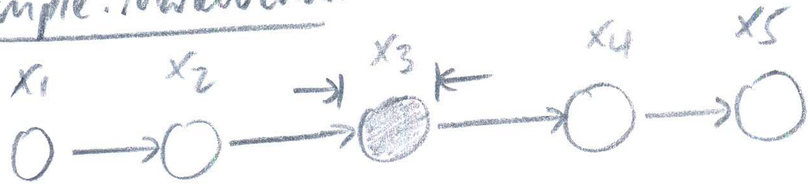
· ~~on~~ For this structure; ball <u>passes through</u> when conditioning

(*) Source and destination node - edge cases

(2.13)

- For this structure; ball is <u>blocked</u> ~~when~~ <u>conditioning</u>

(*) <u>Example: Markov chain</u>



- without d-sep:-

$p(x_1, x_2, \ldots, x_6) = p(x_1) p(x_2|x_1) p(x_3|x_2) p(x_4|x_3) p(x_5|x_4)$

- arbitrarily (for ordering I: 1, 2, 3, 4, 5, 6)

$p(x_1, \ldots, x_6) = p(x_1) p(x_2|x_1) p(x_3|x_2, x_1) p(x_4|x_3, x_2, x_1) p(x_5|x_4, x_3, x_2, x_1)$

- Hence; <u>without d-sep</u>:-

$X_5 \perp\!\!\!\perp \{X_3, X_2, X_1\} \mid X_4$

$X_4 \perp\!\!\!\perp \{X_2, X_1\} \mid X_3$

$X_3 \perp\!\!\!\perp X_1 \mid X_2$

$X_2 \perp\!\!\!\perp \emptyset \mid X_1$

$X_1 \perp\!\!\!\perp \emptyset \mid \emptyset$

(*) with <u>d-sep</u>:

- observe, as an example; that X_3 blocks subsets $\{X_1, X_2\}$ and $\{X_4, X_5\}$.
- Yielding some additional condit. indep:-

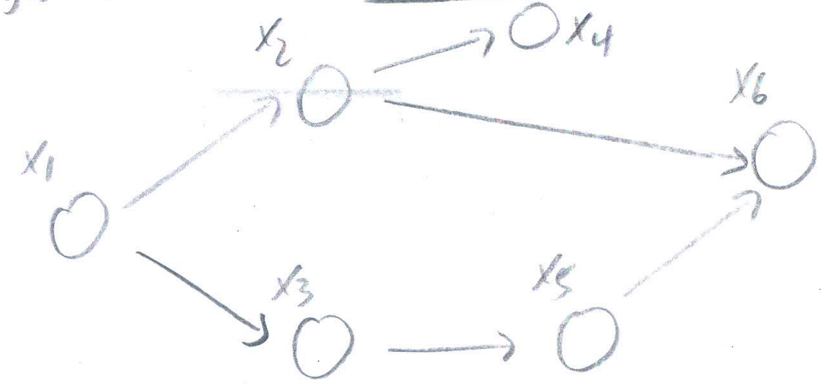
(not exhaustive list) (using cascade + d-sep)

$X_1 \perp\!\!\!\perp X_5 \mid X_4$ $\quad X_1 \perp\!\!\!\perp X_5 \mid X_2$ $\quad X_1 \perp\!\!\!\perp X_5 \mid \{X_2, X_4\}$ etc.

- Bayes ball shortcuts algebraic manipulation

(*) In general, in this cascade, with time as a mental model, ~~we can~~ any subset of 'future' nodes is conditionally indep. of any subs 'past' nodes given the subset that <u>blocks</u> them.

(*) Bayes ball - another composite example



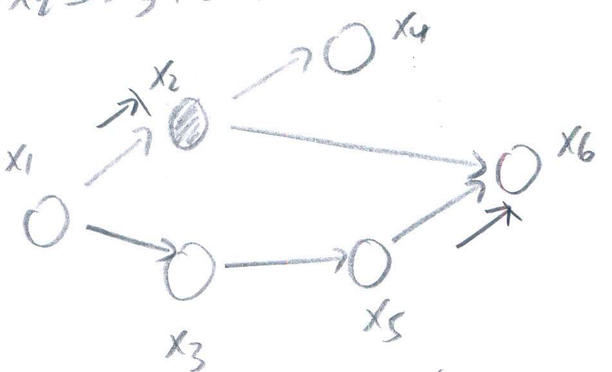
- other than existing (C.I) statements in (2.9)-(2.14), we have further C.I. statements

- i) $X_4 \perp\!\!\!\perp \{X_1, X_3\} \mid X_2$

  ii) $X_1 \perp\!\!\!\perp X_6 \mid \{X_2, X_3\}$

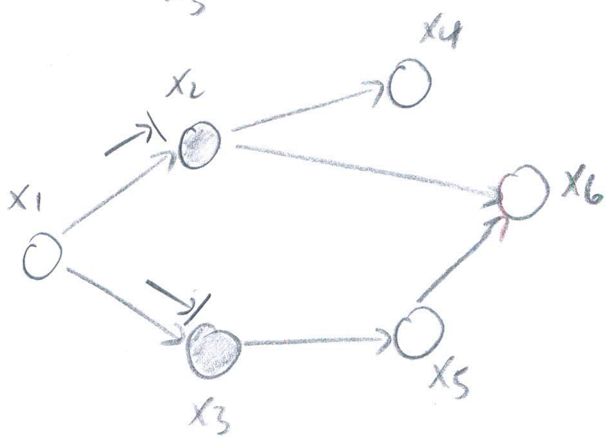
  iii) $X_2 \perp\!\!\!\perp X_3 \mid \{X_1, X_6\}$ not true

i)



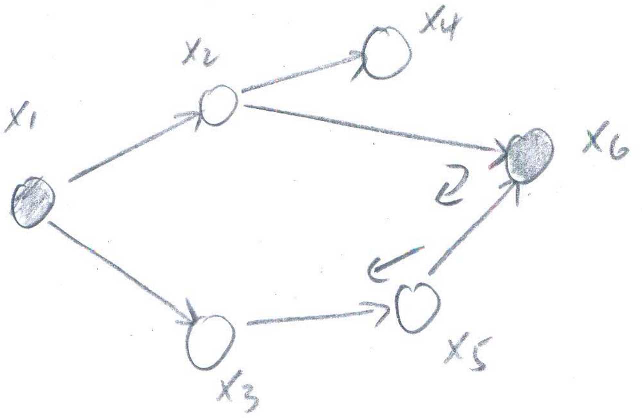
via 2.9 (i)
2.12 (i)

ii)



via 2.9 (i)

iii)



2.9 (i)
via 2.11 (i)

- obs. of X_6 → explaining away dep between X_2, X_5 (v-struc.)

- X_5 and X_3 dep. (?)
- X_2, X_3 dep.