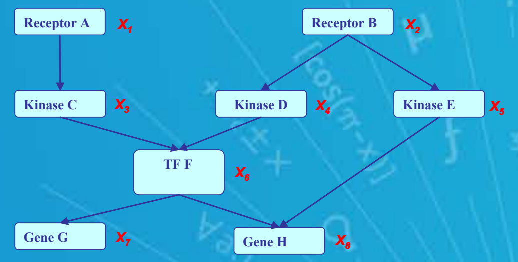
# Probabilistic Graphical Models

## Introduction to GM

Eric Xing

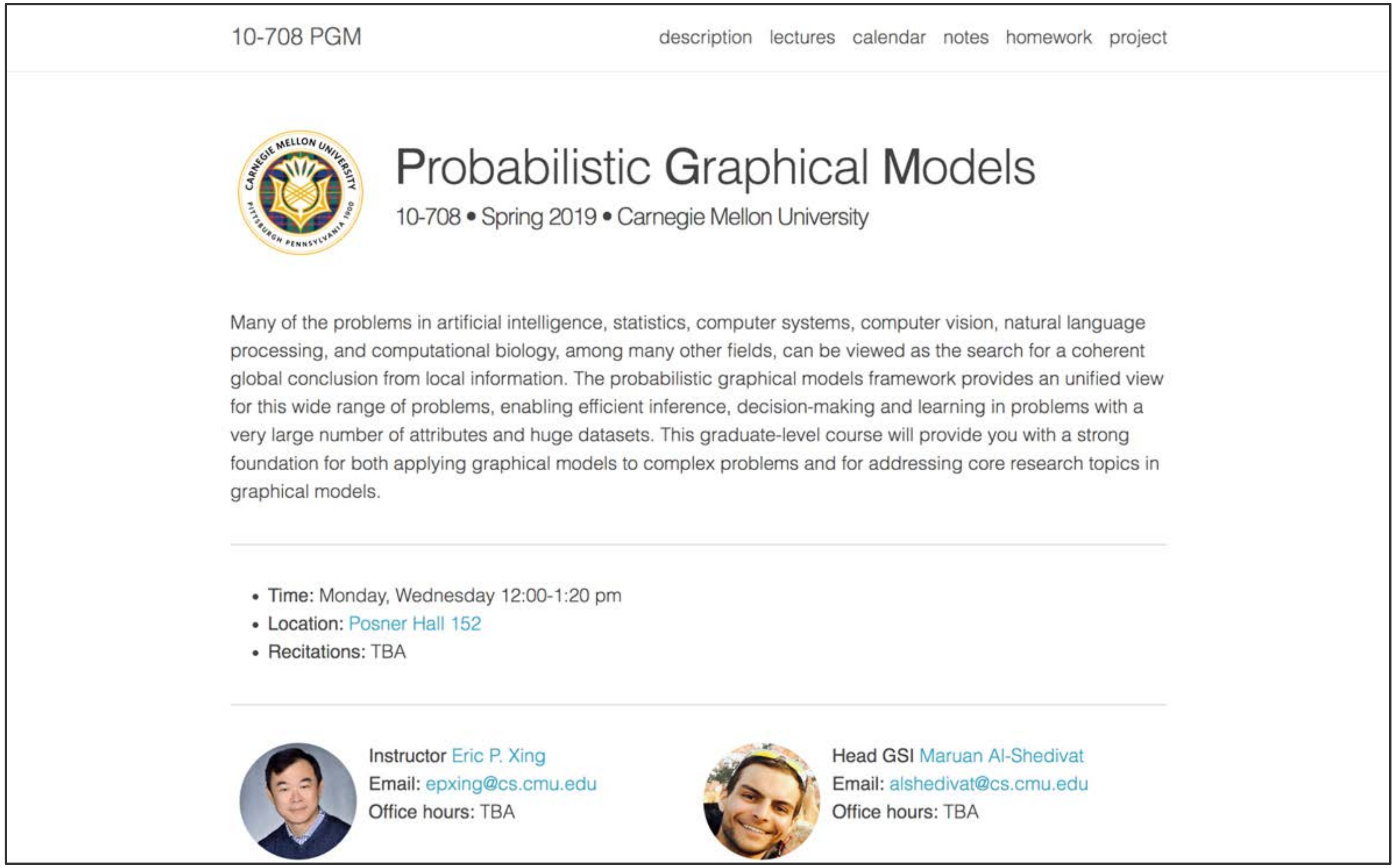
Lecture 1, January 14, 2019




**Reading: see class homepage**

# Logistics

- Class webpage: https://sailinglab.github.io/pgm-spring-2019/



10-708 PGM    description  lectures  calendar  notes  homework  project

## Probabilistic Graphical Models

10-708 • Spring 2019 • Carnegie Mellon University

Many of the problems in artificial intelligence, statistics, computer systems, computer vision, natural language processing, and computational biology, among many other fields, can be viewed as the search for a coherent global conclusion from local information. The probabilistic graphical models framework provides an unified view for this wide range of problems, enabling efficient inference, decision-making and learning in problems with a very large number of attributes and huge datasets. This graduate-level course will provide you with a strong foundation for both applying graphical models to complex problems and for addressing core research topics in graphical models.

- **Time:** Monday, Wednesday 12:00-1:20 pm
- **Location:** Posner Hall 152
- **Recitations:** TBA

Instructor Eric P. Xing
Email: epxing@cs.cmu.edu
Office hours: TBA

Head GSI Maruan Al-Shedivat
Email: alshedivat@cs.cmu.edu
Office hours: TBA

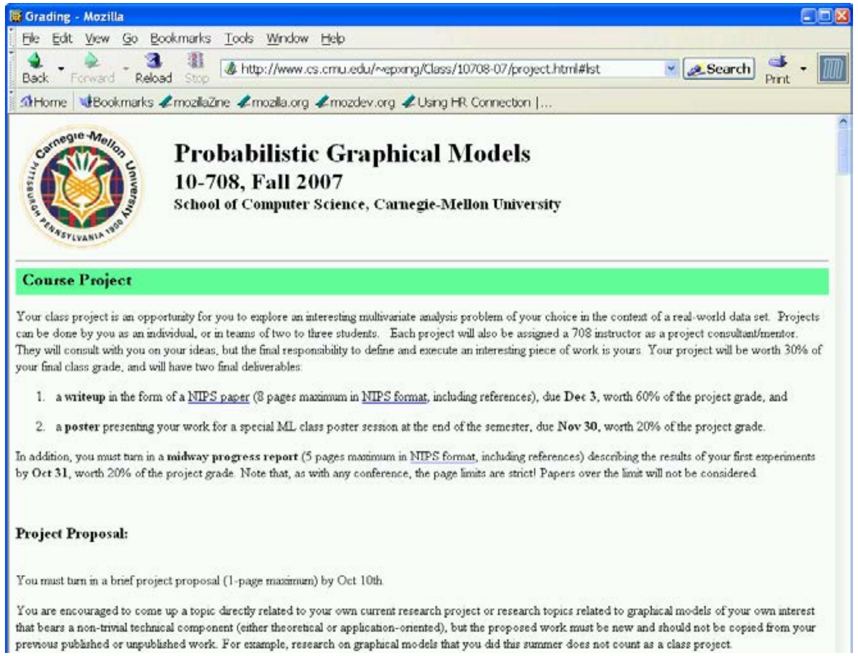
# Logistics

- Text books:
  - Daphne Koller and Nir Friedman, Probabilistic Graphical Models
  - M. I. Jordan, An Introduction to Probabilistic Graphical Models
- Mailing Lists:
  - To contact the instructors: 10708-instructor@cs.cmu.edu
  - Class announcements: Piazza
- TA:
  - Maruan Al-Shedivat, GHC 8229, Office hours: Wednesday 1:30-2:30 pm
  - Lisa Lee, GHC 8011, Office hours: Monday 9-10 am
  - Xun Zheng, GHC 8013, Office hours: Friday 4-5 pm
  - Hao Zhang, NSH 4225, Office hours: Friday 12-1 pm
  - Paul Liang, GHC 8011, Office hours: Thursday 5-6 pm
- Lecturer: Eric Xing, Office hours: Monday 1:30-2:30 pm
- Class Assistant:
  - Amy Protos, GHC 8001
- Instruction aids: Piazza

# Logistics

- 4 homework assignments: 40% of grade
  - Theory exercises, Implementation exercises
- Scribe duties: 10% (~once to twice for the whole semester)
- Class participation: 4% (more details on the course webpage)
  - Contribution to discussion on Piazza (up to 3.5%)
  - Complete mid-semester evaluation (0.5%)
- Final project: 46% of grade
  - Applying PGM to the development of a real, substantial ML system
    - Design and Implement a (record-breaking) distributed Logistic Regression, Gradient Boosted Tree, Deep Network, or Topic model on Petuum and apply to ImageNet, Wikipedia, and/or other data
    - Build a web-scale topic or story line tracking system for news media, or a paper recommendation system for conference review matching
    - An online car or people or event detector for web-images and webcam
    - An automatic "what's up here?" or "photo album" service on iPhone
  - Theoretical and/or algorithmic work
    - a more efficient approximate inference or optimization algorithm, e.g., based on stochastic approximation, proximal average, or other new techniques
    - a distributed sampling scheme with convergence guarantee
  - 3-member team to be formed in the first three weeks, proposal, mid-way report, oral presentation & demo, final report, peer review → possibly conference submission !

# Past projects:


Grading - Mozilla

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Back Forward Reload Stop http://www.cs.cmu.edu/~epxing/Class/10708-07/project.html#lst Search Print

Home Bookmarks mozillaZine mozilla.org mozdev.org Using HR Connection |...

**Probabilistic Graphical Models**
**10-708, Fall 2007**
**School of Computer Science, Carnegie-Mellon University**

**Course Project**

Your class project is an opportunity for you to explore an interesting multivariate analysis problem of your choice in the context of a real-world data set. Projects can be done by you as an individual, or in teams of two to three students. Each project will also be assigned a 708 instructor as a project consultant/mentor. They will consult with you on your ideas, but the final responsibility to define and execute an interesting piece of work is yours. Your project will be worth 30% of your final class grade, and will have two final deliverables:

1. a **writeup** in the form of a NIPS paper (8 pages maximum in NIPS format, including references), due **Dec 3**, worth 60% of the project grade, and
2. a **poster** presenting your work for a special ML class poster session at the end of the semester, due **Nov 30**, worth 20% of the project grade.

In addition, you must turn in a **midway progress report** (5 pages maximum in NIPS format, including references) describing the results of your first experiments by **Oct 31**, worth 20% of the project grade. Note that, as with any conference, the page limits are strict! Papers over the limit will not be considered.

**Project Proposal:**

You must turn in a brief project proposal (1-page maximum) by Oct 10th.

You are encouraged to come up a topic directly related to your own current research project or research topics related to graphical models of your own interest that bears a non-trivial technical component (either theoretical or application-oriented), but the proposed work must be new and should not be copied from your previous published or unpublished work. For example, research on graphical models that you did this summer does not count as a class project.

- We will have a prize for the best project(s) …

- Award Winning Projects:

  J. Yang, Y. Liu, E. P. Xing and A. Hauptmann, Harmonium-Based Models for Semantic Video Representation and Classification , *Proceedings of The Seventh SIAM International Conference on Data Mining* (SDM 2007 best paper)

  Manaal Faruqui, Jesse Dodge, Sujay Kumar Jauhar, Chris Dyer, Eduard Hovy, Noah A. Smith, Retrofitting Word Vectors to Semantic Lexicons, NAACL 2015 best paper

  Others … such as KDD 2014 best paper

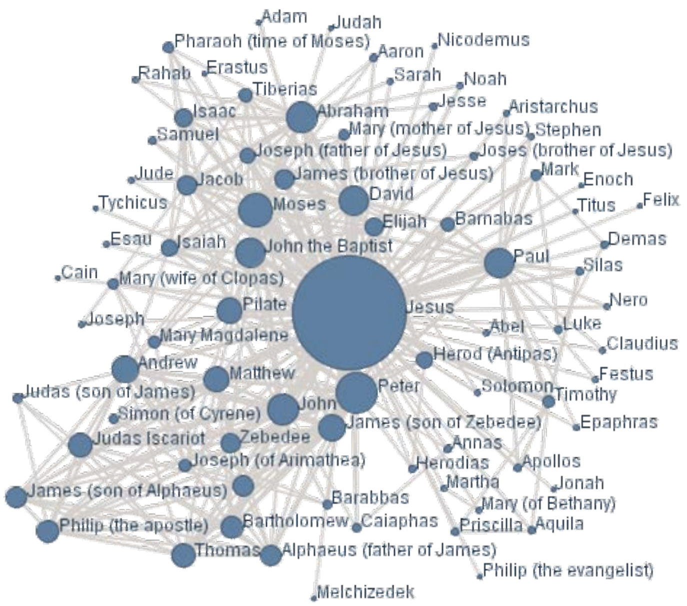
- Other projects:

  Andreas Krause, Jure Leskovec and Carlos Guestrin, Data Association for Topic Intensity Tracking, *23rd International Conference on Machine Learning* (ICML 2006).

  M. Sachan, A. Dubey, S. Srivastava, E. P. Xing and Eduard Hovy, Spatial Compactness meets Topical Consistency: Jointly modeling Links and Content for Community Detection , *Proceedings of The 7th ACM International Conference on Web Search and Data Mining* (WSDM 2014).

# What Are Graphical Models?

**Graph**



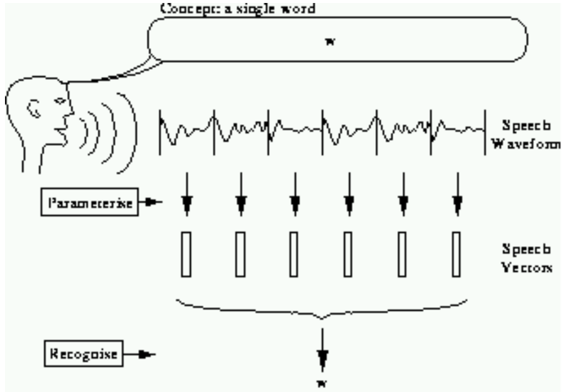

**Model**

$$\mathrm{M_G}$$

**Data**

$$\mathrm{D} = \{X_1^{(i)}, X_2^{(i)}, \ldots, X_m^{(i)}\}_{i=1}^{N}$$

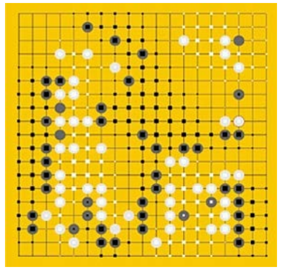
# Reasoning under uncertainty!

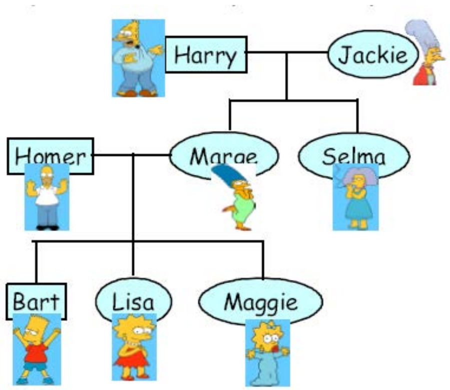


Speech recognition

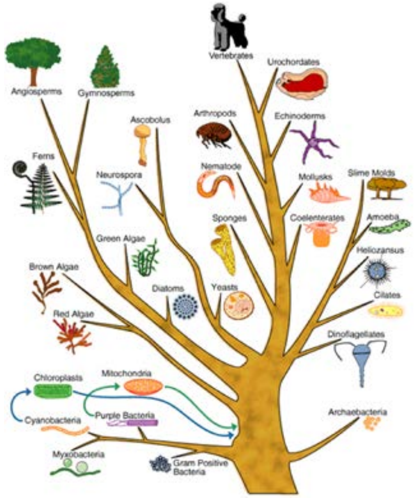
Information retrieval


Computer vision

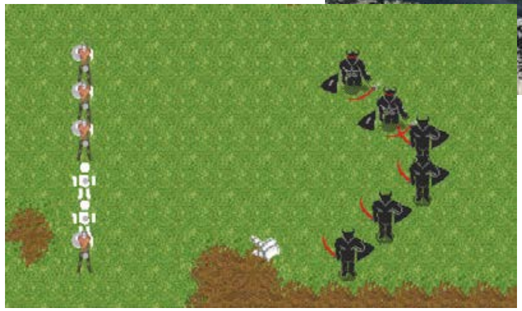

Games



Pedigree


Evolution


Robotic control


Planning

# The Fundamental Questions

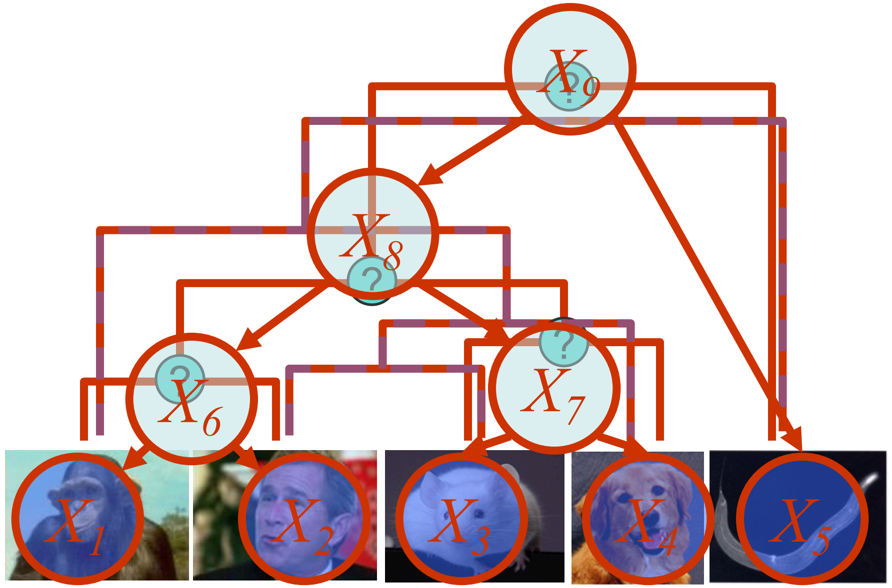
- Representation
  - How to capture/model uncertainties in possible worlds?
  - How to encode our domain knowledge/assumptions/constraints?

- Inference
  - How do I answers questions/queries according to my model and/or based given data?

    e.g.: $P(X_i \mid \mathcal{D})$

- Learning
  - What model is "right" for my data?

    e.g.: $\mathcal{M} = \arg\max_{\mathcal{M} \in M} F(\mathcal{D}; \mathcal{M})$

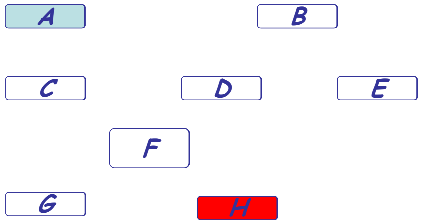


# Recap of Basic Prob. Concepts

- Representation: what is the joint probability dist. on multiple variables?

$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$

  - How many state configurations in total? --- $2^8$
  - Are they all needed to be represented?
  - Do we get any scientific/medical insight?



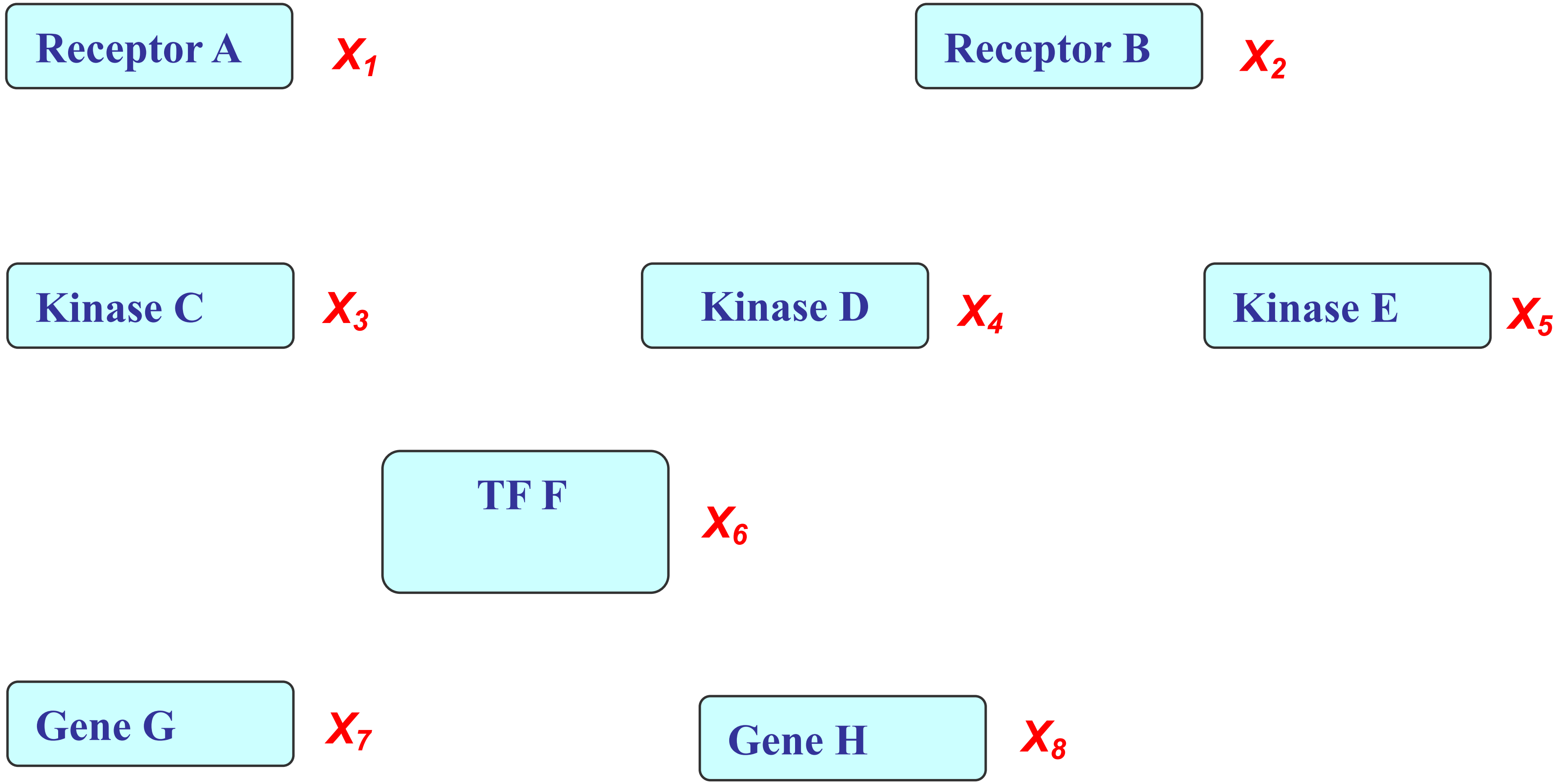

- Learning: where do we get all this probabilities?
  - Maximal-likelihood estimation? but how many data do we need?
  - Are there other est. principles?
  - Where do we put domain knowledge in terms of plausible relationships between variables, and plausible values of the probabilities?

- Inference: If not all variables are observable, how to compute the conditional distribution of latent variables given evidence?
  - Computing $p(H|A)$ would require summing over all $2^6$ configurations of the unobserved variables

# What is a Graphical Model?

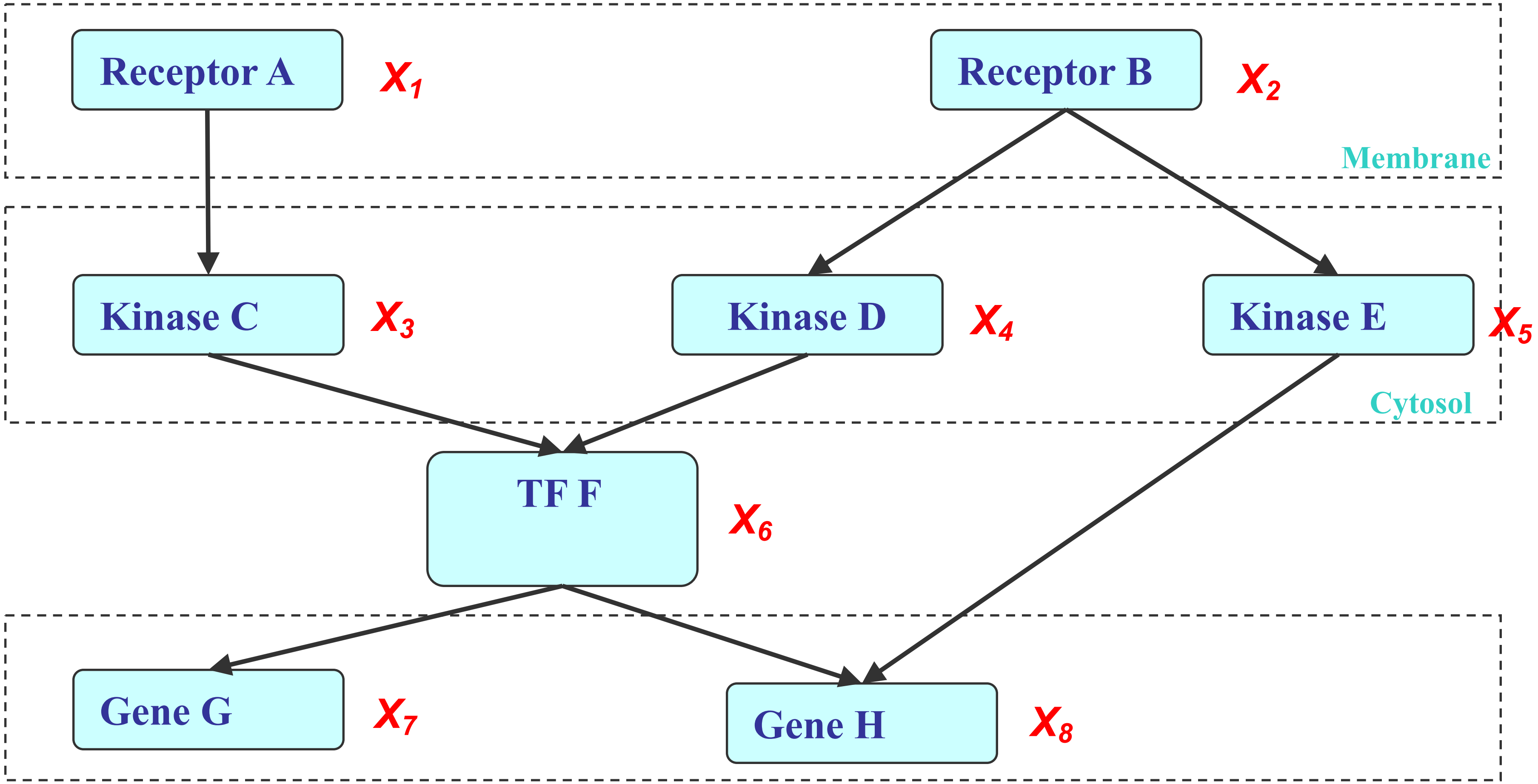
--- Multivariate Distribution in High-D Space

- A possible world for cellular signal transduction:



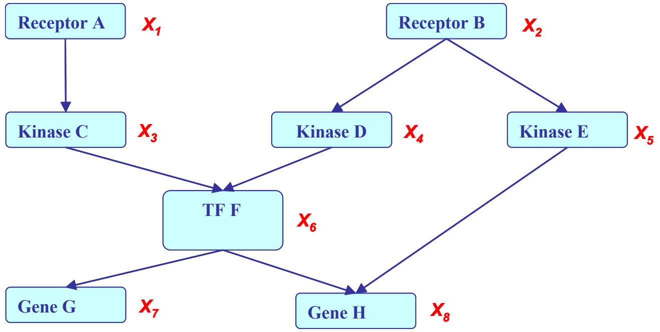
# GM: Structure Simplifies Representation

- Dependencies among variables



# Probabilistic Graphical Models

- If $X_i$'s are conditionally independent (as described by a PGM), the joint can be factored to a product of simpler terms, e.g.,




$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$

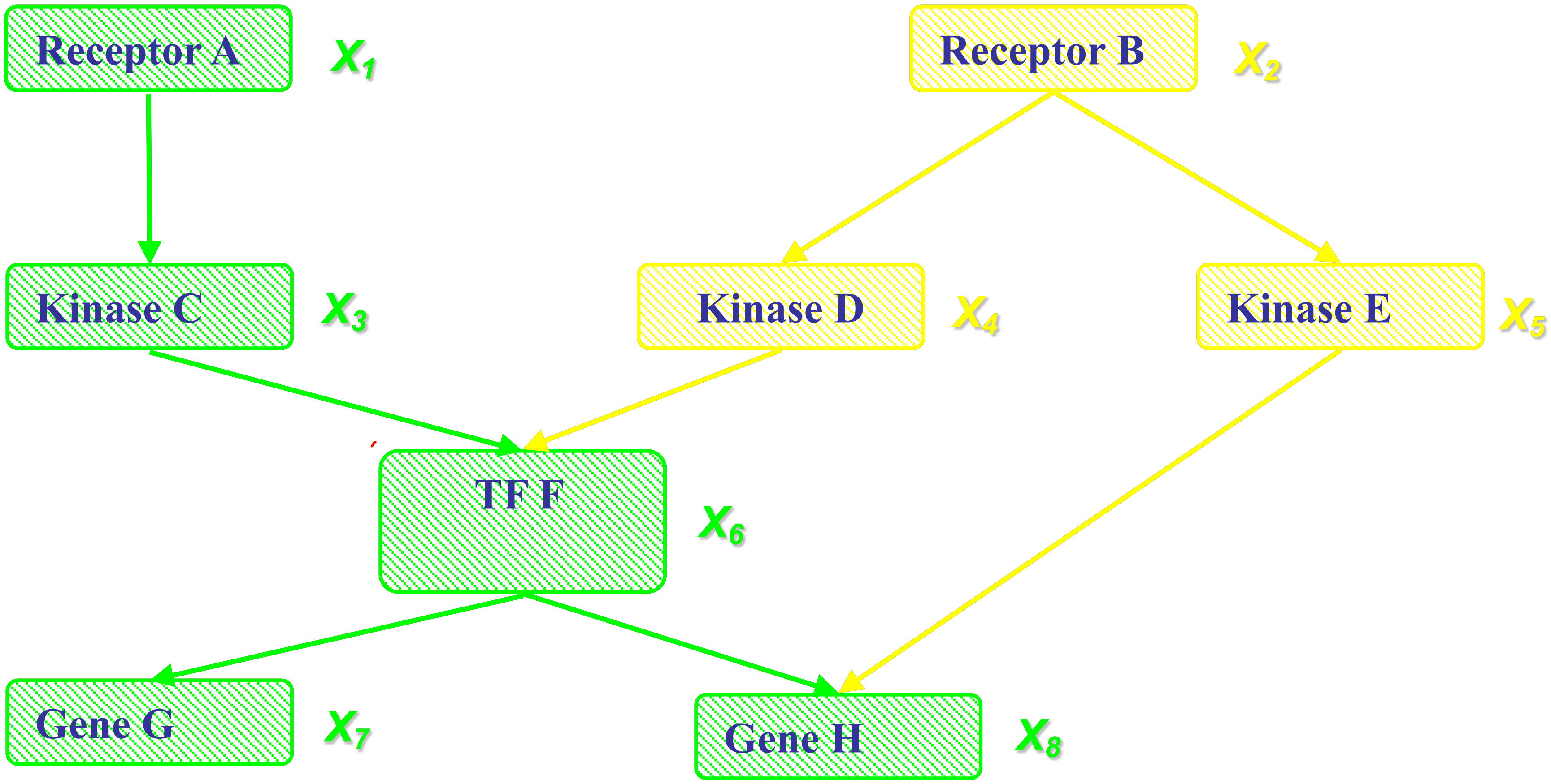
$$= P(X_1)\, P(X_2)\, P(X_3|\, X_1)\, P(X_4|\, X_2)\, P(X_5|\, X_2)\, P(X_6|\, X_3, X_4)\, P(X_7|\, X_6)\, P(X_8|\, X_5, X_6)$$

**Stay tune for what are these independencies!**

- Why we may favor a PGM?
  - Incorporation of domain knowledge and causal (logical) structures

1+1+2+2+2+4+2+4=18, a 16-fold reduction from $2^8$ in representation cost !

# GM: Data Integration



# More Data Integration

- Text + Image + Network ➔ Holistic Social Media

- Genome + Proteome + Transcriptome + Phenome + … ➔ PanOmic Biology

# Probabilistic Graphical Models

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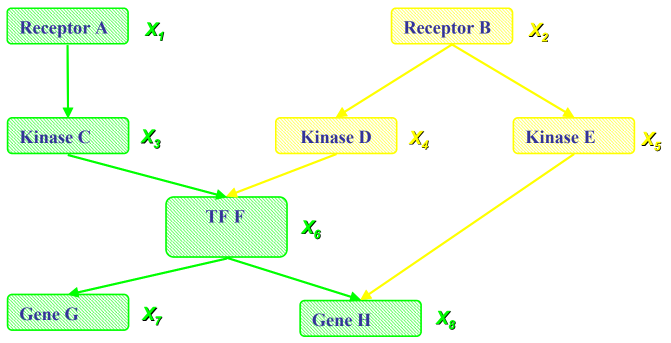



$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$
$$= P(X_2)\, P(X_4 | X_2)\, P(X_5 | X_2)\, P(X_1)\, P(X_3 | X_1)\, P(X_6 | X_3, X_4)\, P(X_7 | X_6)\, P(X_8 | X_5, X_6)$$

- Why we may favor a PGM?
  - Incorporation of domain knowledge and causal (logical) structures

    2+2+4+4+4+8+4+8=36, an 8-fold reduction from $2^8$ in representation cost !

  - Modular combination of heterogeneous parts – data fusion

# Rational Statistical Inference

## The Bayes Theorem:

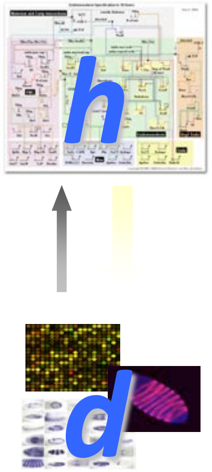
Posterior probability

Likelihood

Prior probability

$$p(h \mid d) = \frac{p(d \mid h)\, p(h)}{\sum_{h' \in H} p(d \mid h')\, p(h')}$$

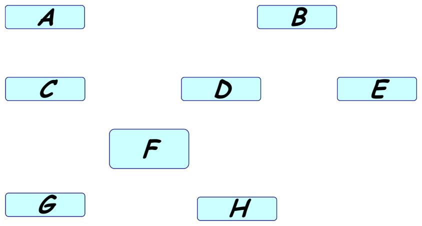
Sum over space of hypotheses




- This allows us to capture uncertainty about the model in a principled way
- But how can we specify and represent a complicated model?
  - <u>Typically the number of genes need to be modeled are in the order of thousands!</u>

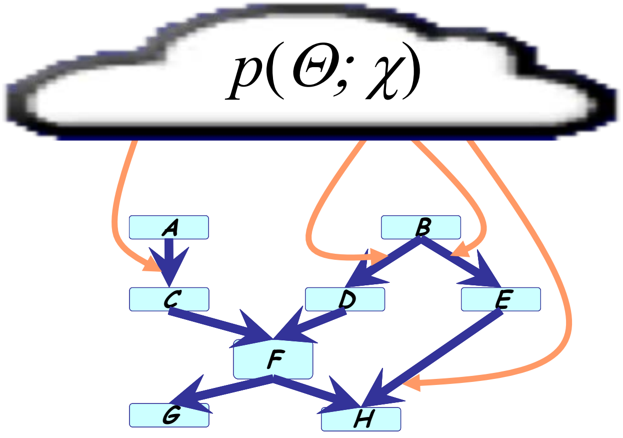
# GM: MLE and Bayesian Learning

- Probabilistic statements of $\Theta$ is conditioned on the values of the observed variables $A_{obs}$ and prior $p(\ |\chi)$




$$\mathbf{A} = \begin{array}{l} (A,B,C,D,E,\ldots)=(T,F,F,T,F,\ldots) \\ (A,B,C,D,E,\ldots)=(T,F,T,T,F,\ldots) \\ \ldots\ldots.. \\ (A,B,C,D,E,\ldots)=(F,T,T,T,F,\ldots) \end{array}$$

$$\boxed{\Theta_{Bayes} = \int \Theta\, p(\Theta \mid \mathbf{A}, \chi)\, d\Theta}$$




| C | D | P(F / C,D) | |
|---|---|---|---|
| $c$ | $d$ | 0.9 | 0.1 |
| $c$ | $\bar{d}$ | 0.2 | 0.8 |
| $\bar{c}$ | $d$ | 0.9 | 0.1 |
| $\bar{c}$ | $\bar{d}$ | 0.01 | 0.99 |

$$\underbrace{p(\Theta \mid \mathbf{A}; \chi)}_{\text{posterior}} \propto \underbrace{p(\mathbf{A} \mid \Theta)}_{\text{likelihood}}\, \underbrace{p(\Theta; \chi)}_{\text{prior}}$$

# Probabilistic Graphical Models

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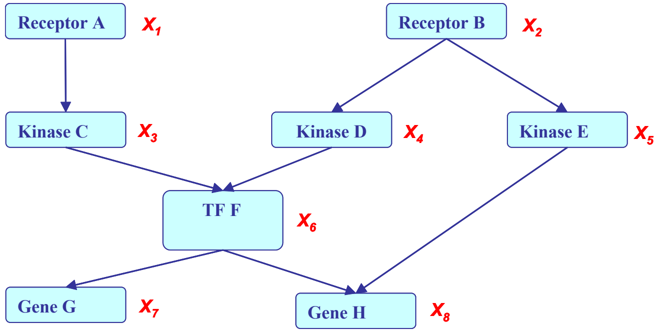


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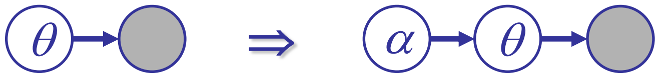
- Why we may favor a PGM?
  - Incorporation of domain knowledge and causal (logical) structures
  - Modular combination of heterogeneous parts – data fusion

2+2+4+4+4+8+4+8=36, an 8-fold reduction from $2^8$ in representation cost !

  - Bayesian Philosophy
    - Knowledge meets data



## So What Is a PGM After All?

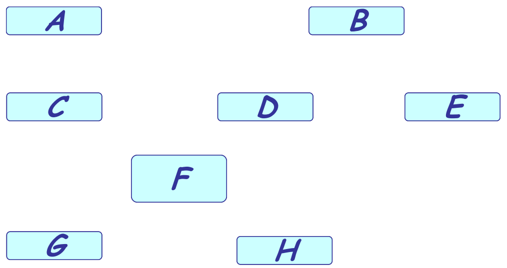
# In a nutshell:

**PGM = Multivariate Statistics + Structure**

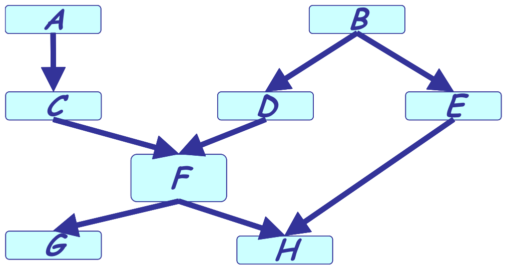
**GM = Multivariate Obj. Func. + Structure**

# So What Is a PGM After All?

- The informal blurb:
  - It is a smart way to write/specify/compose/design exponentially-large probability distributions without paying an exponential cost, and at the same time endow the distributions with *structured semantics*




$$P(X_1,X_2,X_3,X_4,X_5,X_6,X_7,X_8)$$



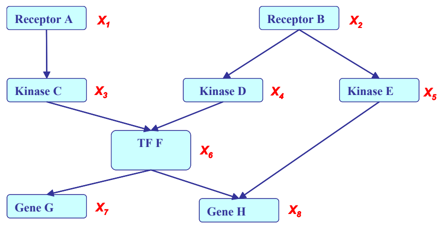

$$P(X_{1:8}) = P(X_1)P(X_2)P(X_3 \mid X_1X_2)P(X_4 \mid X_2)P(X_5 \mid X_2)$$
$$P(X_6 \mid X_3,X_4)P(X_7 \mid X_6)P(X_8 \mid X_5,X_6)$$

- A more formal description:
  - It refers to a family of distributions on a set of random variables that are compatible with all the probabilistic independence propositions encoded by a graph that connects these variables

# Two types of GMs

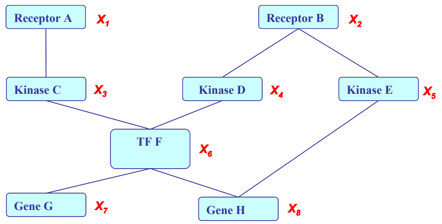
- Directed edges give causality relationships (Bayesian Network or Directed Graphical Model):

$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$
$$= P(X_1)\, P(X_2)\, P(X_3|\, X_1)\, P(X_4|\, X_2)\, P(X_5|\, X_2)\, P(X_6|\, X_3, X_4)\, P(X_7|\, X_6)\, P(X_8|\, X_5, X_6)$$



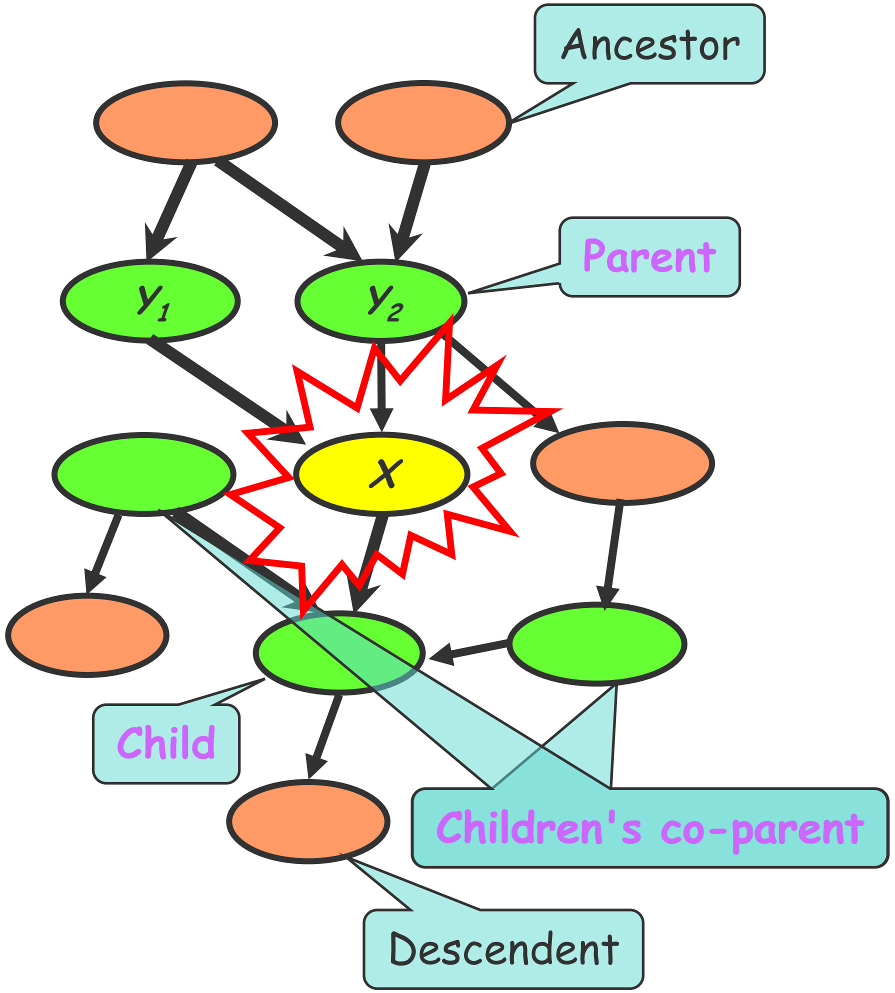

- Undirected edges simply give correlations between variables (Markov Random Field or Undirected Graphical model):

$$P(X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)$$
$$= 1/Z \exp\{E(X_1)+E(X_2)+E(X_3, X_1)+E(X_4, X_2)+E(X_5, X_2) + E(X_6, X_3, X_4)+E(X_7, X_6)+E(X_8, X_5, X_6)\}$$



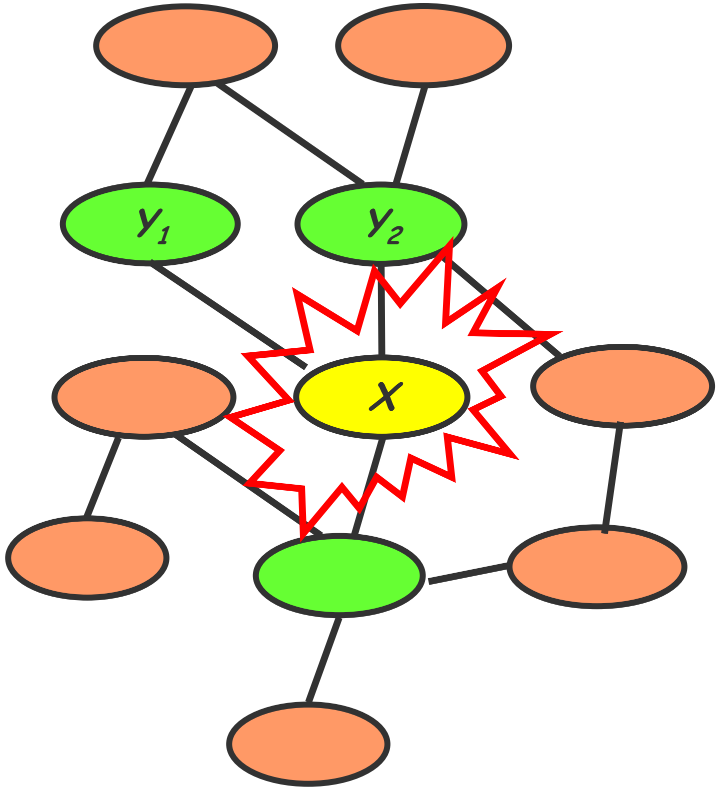
# Bayesian Networks

- Structure: *DAG*
  - Meaning: a node is conditionally independent of every other node in the network outside its Markov blanket
  - Local conditional distributions (CPD) and the DAG completely determine the joint dist.
  - Give causality relationships, and facilitate a generative process



# Markov Random Fields

- Structure: *undirected graph*
  - Meaning: a node is conditionally independent of every other node in the network given its Directed neighbors
  - Local contingency functions (potentials) and the cliques in the graph completely determine the joint dist.
  - Give correlations between variables, but no explicit way to generate samples



# Towards structural specification of probability distribution

- Separation properties in the graph imply independence properties about the associated variables
- For the graph to be useful, any conditional independence properties we can derive from the graph should hold for the probability distribution that the graph represents

- The Equivalence Theorem

  For a graph G,

  Let $\mathbf{D}_1$ denote the family of all distributions that satisfy I(G),

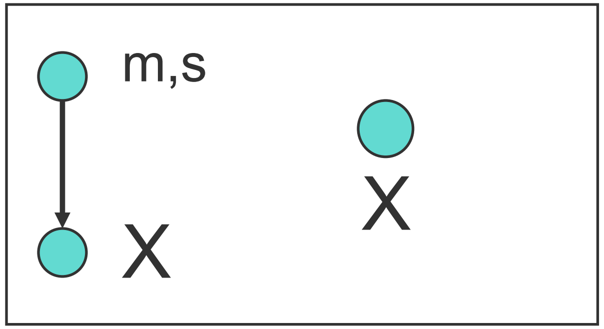
  Let $\mathbf{D}_2$ denote the family of all distributions that factor according to G,

  Then $\mathbf{D}_1 \equiv \mathbf{D}_2$.

# GMs are your old friends

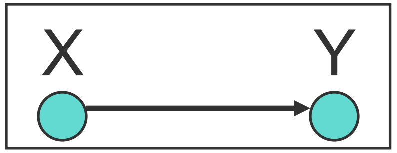
Density estimation

Parametric and nonparametric  methods



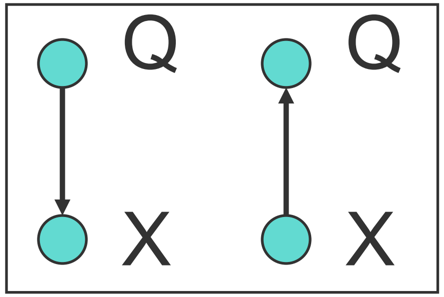

Regression

Linear, conditional mixture, nonparametric



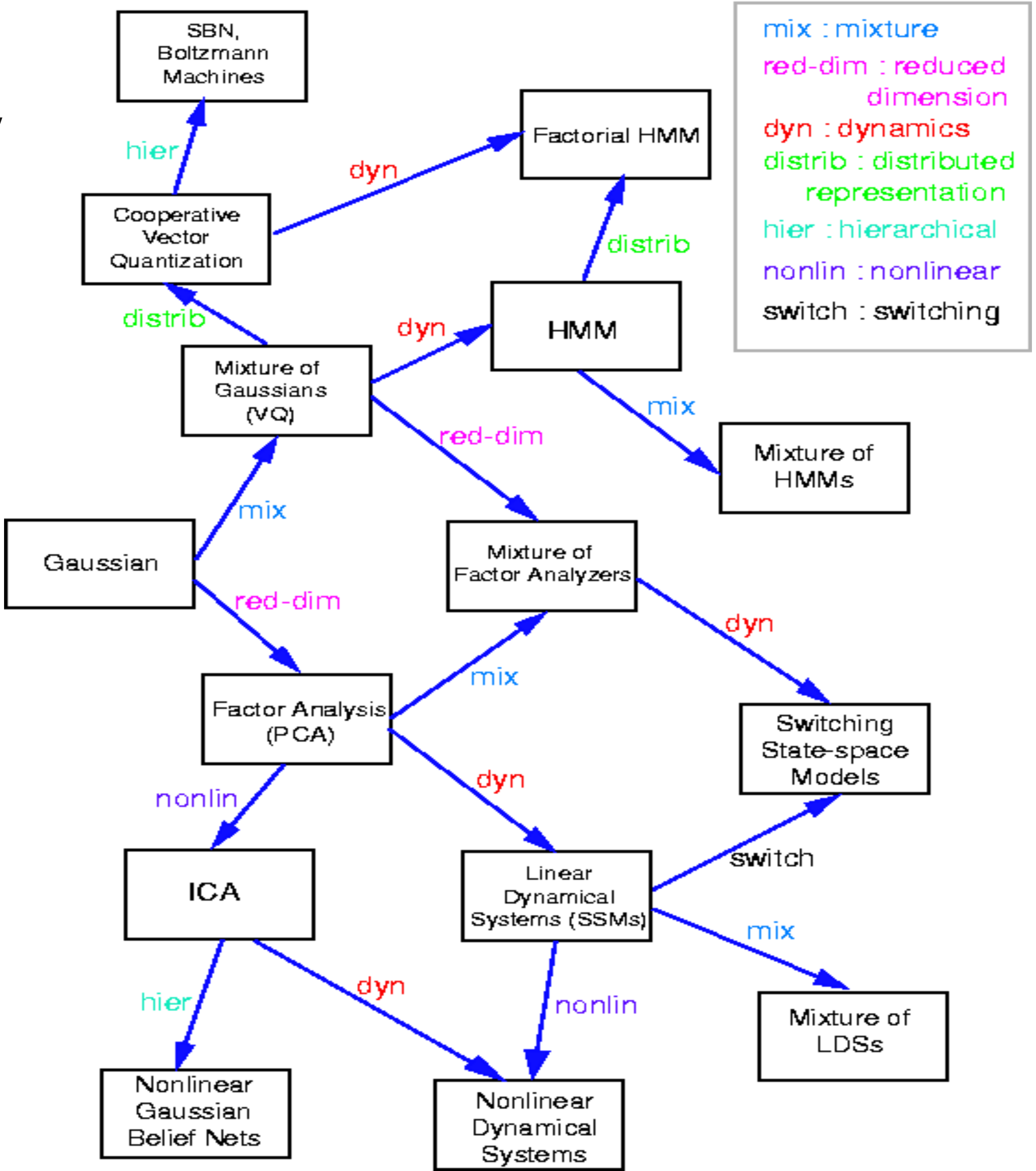

Classification

Generative and discriminative approach




Clustering

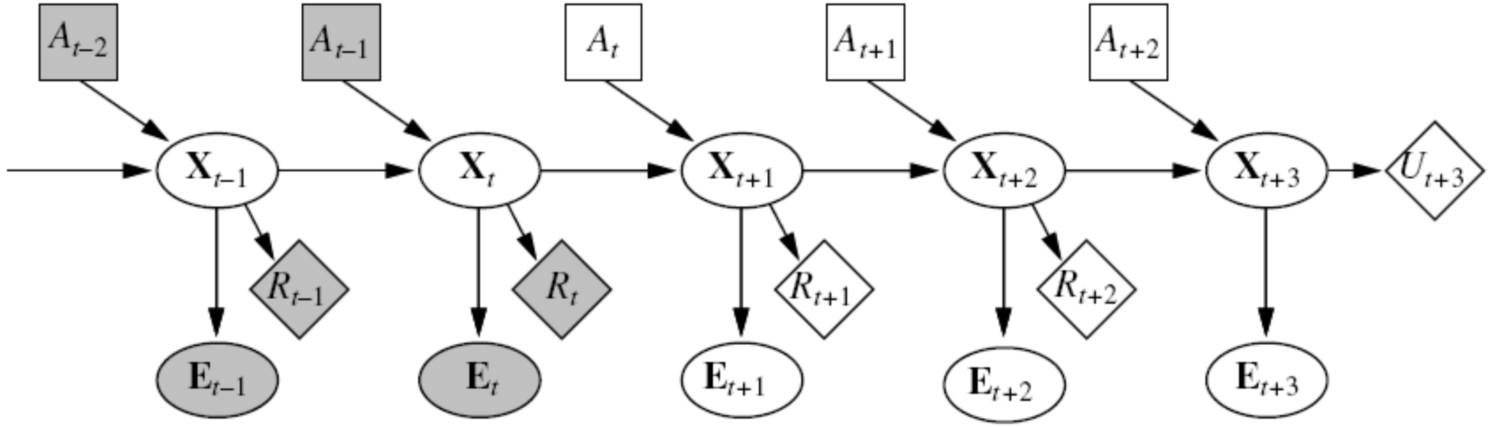
# An (incomplete) genealogy of graphical models



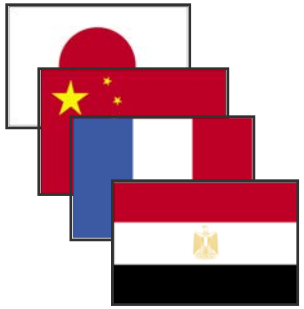

(Picture by Zoubin Ghahramani and Sam Roweis)

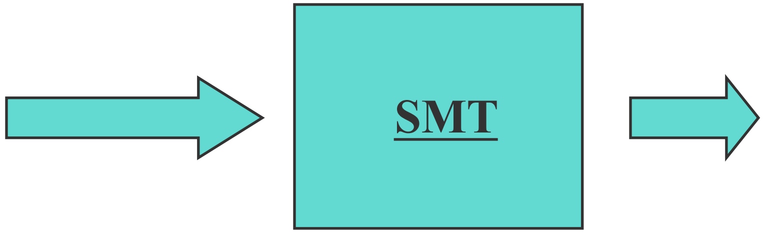
# Fancier GMs: reinforcement learning

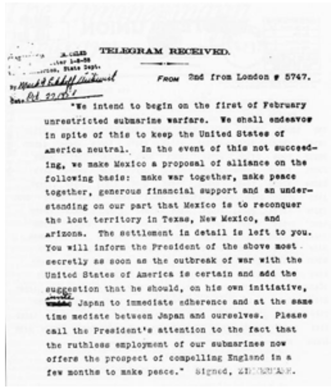
- Partially observed Markov decision processes (POMDP)




# Fancier GMs: machine translation



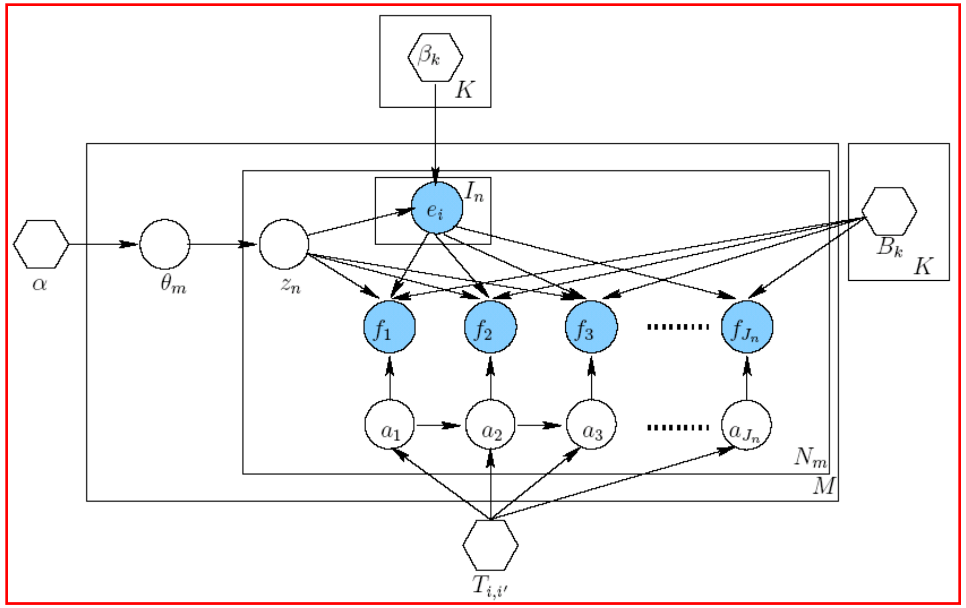


TELEGRAM RECEIVED.

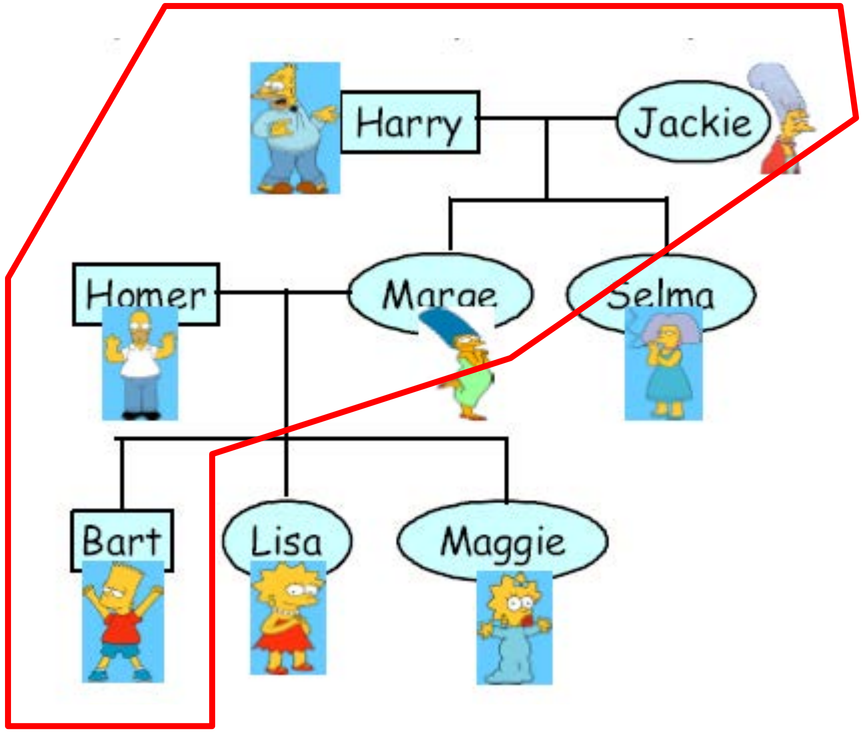
FROM 2nd from London # 5747.

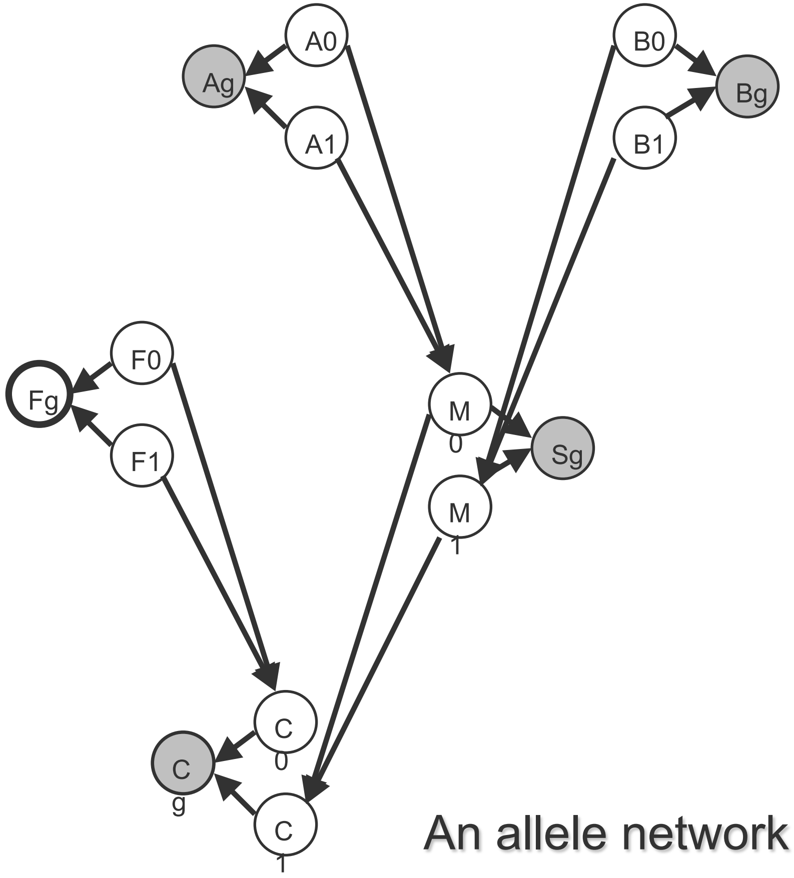
"We intend to begin on the first of February unrestricted submarine warfare. We shall endeavor in spite of this to keep the United States of America neutral. In the event of this not succeeding, we make Mexico a proposal of alliance on the following basis: make war together, make peace together, generous financial support and an understanding on our part that Mexico is to reconquer the lost territory in Texas, New Mexico, and Arizona. The settlement in detail is left to you. You will inform the President of the above most secretly as soon as the outbreak of war with the United States of America is certain and add the suggestion that he should, on his own initiative, invite Japan to immediate adherence and at the same time mediate between Japan and ourselves. Please call the President's attention to the fact that the ruthless employment of our submarines now offers the prospect of compelling England in a few months to make peace." Signed, ZIMMERMANN.




The HM-BiTAM model (B. Zhao and E.P Xing, ACL 2006)

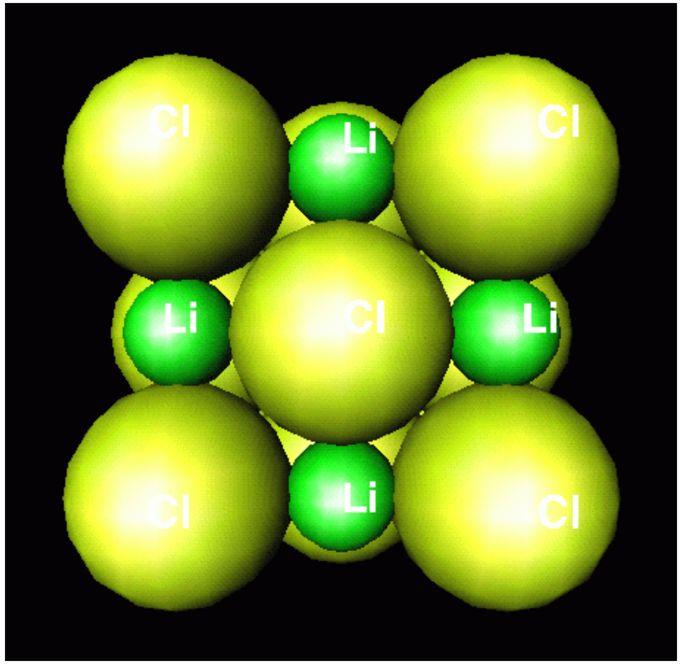
# Fancier GMs: genetic pedigree

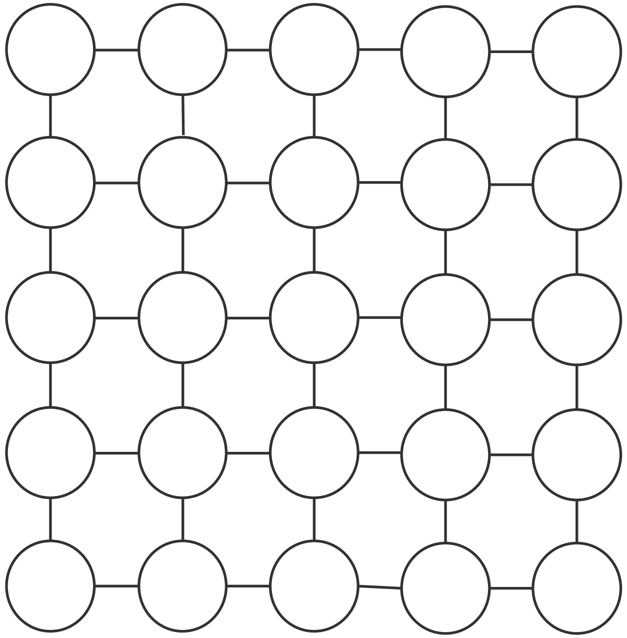



An allele network

# Fancier GMs: solid state physics



Ising/Potts model

# Application of GMs

- Machine Learning
- Computational statistics

- Computer vision and graphics
- Natural language processing
- Informational retrieval
- Robotic control
- Decision making under uncertainty
- Error-control codes
- Computational biology
- Genetics and medical diagnosis/prognosis
- Finance and economics
- Etc.

# Why graphical models

- A language for communication
- A language for computation
- A language for development

- Origins:
  - Wright 1920's
  - Independently developed by Spiegelhalter and Lauritzen in statistics and Pearl in computer science in the late 1980's

# Why graphical models

- Probability theory provides the glue whereby the parts are combined, ensuring that the system as a whole is consistent, and providing ways to interface models to data.
- The graph theoretic side of graphical models provides both an intuitively appealing interface by which humans can model highly-interacting sets of variables as well as a data structure that lends itself naturally to the design of efficient general-purpose algorithms.
- Many of the classical multivariate probabilistic systems studied in fields such as statistics, systems engineering, information theory, pattern recognition and statistical mechanics are special cases of the general graphical model formalism
- The graphical model framework provides a way to view all of these systems as instances of a common underlying formalism.

**--- M. Jordan**

# Plan for the Class

- Fundamentals of Graphical Models:
  - Bayesian Network and Markov Random Fields
  - Discrete, Continuous and Hybrid models, exponential family, GLIM
  - Basic representation, inference, and learning
- Advanced topics and latest developments
  - Approximate inference
    - Monte Carlo algorithms
    - Variational methods and theories
  - “Infinite” GMs: nonparametric Bayesian models
  - Optimization-theoretic formulations for GMs, e.g., Structured sparsity
  - Deep Nets vs. GMs
  - Nonparametric and spectral graphical models, where GM meets kernels and matrix algebra
  - Alternative GM learning paradigms,
    - e.g., Margin-based learning of GMs (where GM meets SVM)
    - e.g., Regularized Bayes: where GM meets SVM, and meets Bayesian, and meets NB …
- Case studies: popular GMs and applications
    - Multivariate Gaussian Models
    - Conditional random fields
    - Mixed-membership, aka, Topic models

# Questions ?