PETUUM

Carnegie Mellon University
School of Computer Science

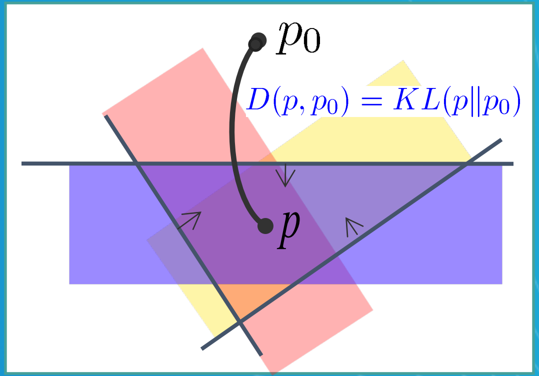
Probabilistic Graphical Models

Integrative Paradigms of GMs: Regularized Bayesian Methods

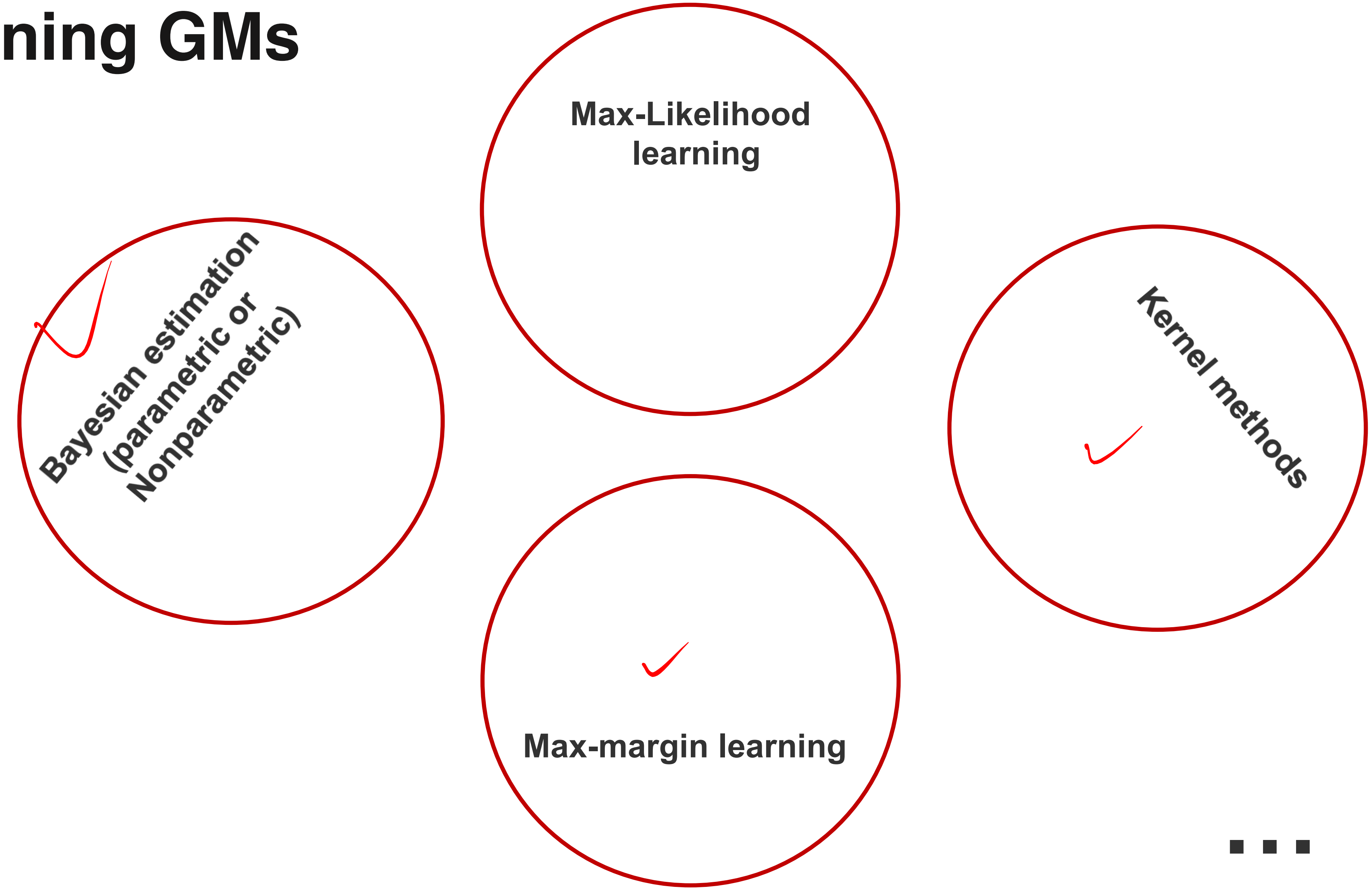
Eric Xing

Lecture 24, April 15, 2019

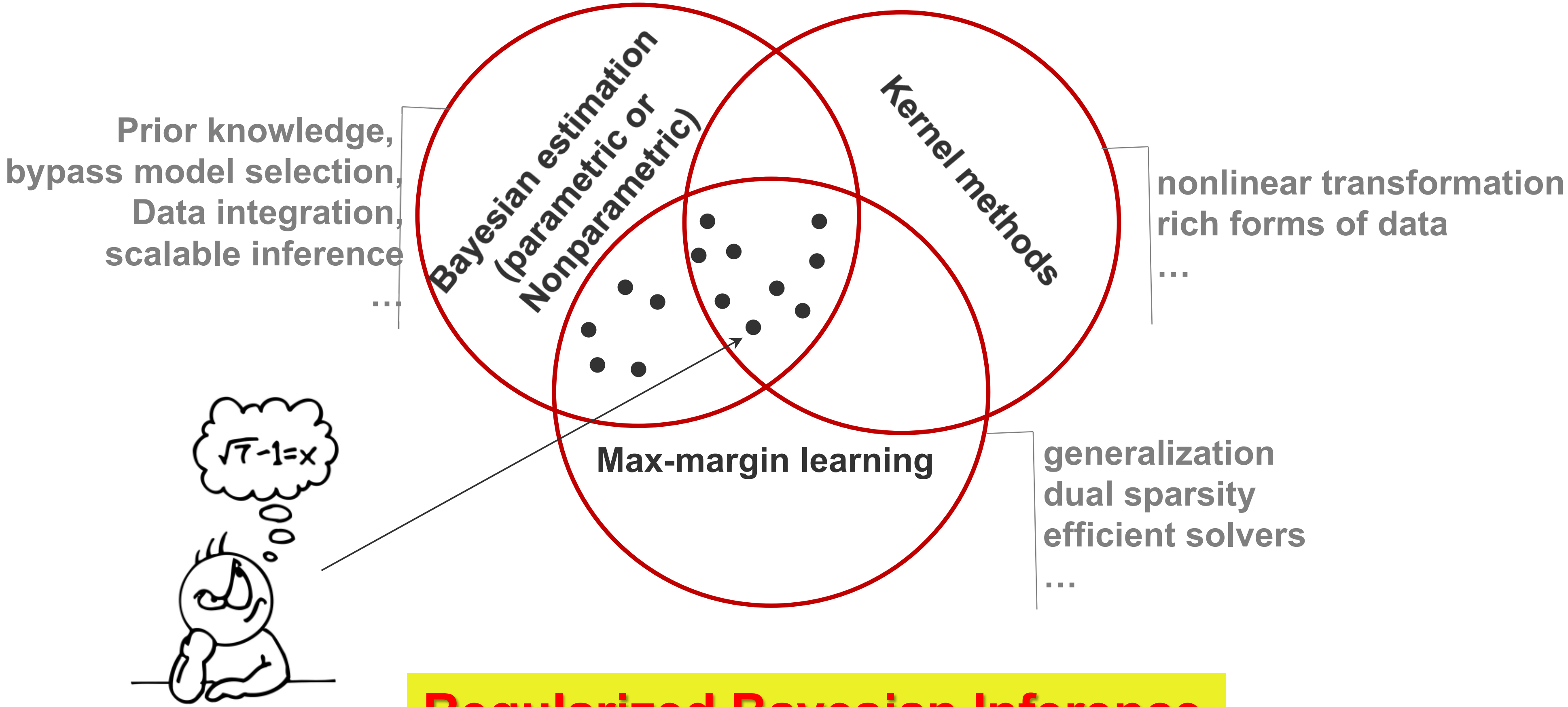
Reading: see class homepage



Learning GMs



Learning GMs



Regularized Bayesian Inference

Bayesian Inference

- A coherent framework of dealing with uncertainties

$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

  - ***M*: a model from some hypothesis space**
  - **x: observed data**

Thomas Bayes (1702 – 1761)

- Bayes' rule offers a mathematically rigorous computational mechanism for combining prior knowledge with incoming evidence

Parametric Bayesian Inference

$\mathcal{M}$ **is represented as a finite set of parameters** θ

- A parametric likelihood: $\mathbf{x} \sim p(\cdot|\theta)$
- Prior on θ : $\pi(\theta)$
- Posterior distribution

$$p(\theta|\mathbf{x}) = \frac{p(\mathbf{x}|\theta)\pi(\theta)}{\int p(\mathbf{x}|\theta)\pi(\theta)d\theta} \propto p(\mathbf{x}|\theta)\pi(\theta)$$

Examples:

- **Gaussian distribution prior + 2D Gaussian likelihood → Gaussian posterior distribution**
- **Dirichilet distribution prior + 2D Multinomial likelihood → Dirichlet posterior distribution**
- **Sparsity-inducing priors + some likelihood models → Sparse Bayesian inference**

Nonparametric Bayesian Inference

$\mathcal{M}$ is a richer model, e.g., with an infinite set of parameters

- A nonparametric likelihood: $\mathbf{x} \sim p(\cdot|\mathcal{M})$
- Prior on $\mathcal{M}$: $\pi(\mathcal{M})$
- Posterior distribution

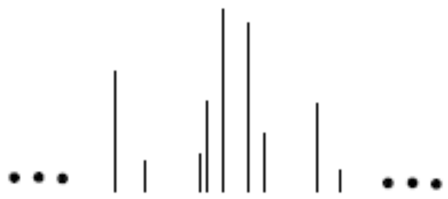
$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}} \propto p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})$$

Examples:

→ see next slide

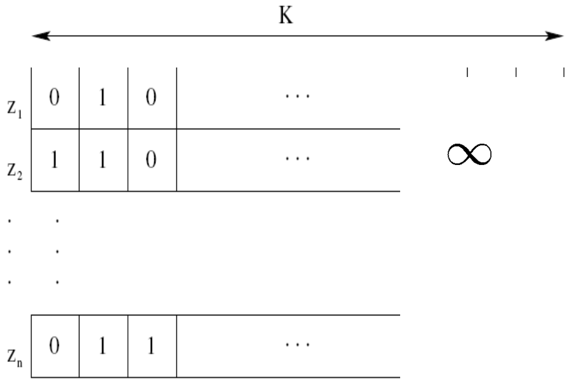
Nonparametric Bayesian Inference

probability measure



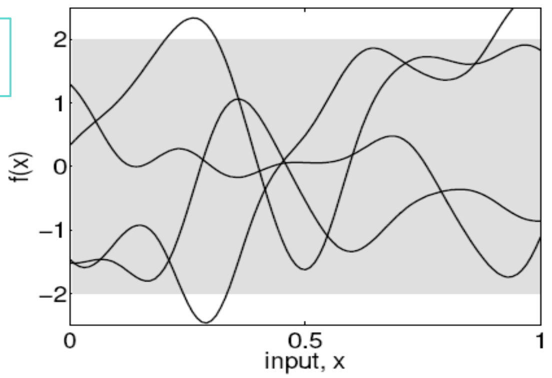
Dirichlet Process Prior [Antoniak, 1974]
+ Multinomial/Gaussian/Softmax likelihood

binary matrix



Indian Buffet Process Prior [Griffiths & Gharamani, 2005]
+ Gaussian/Sigmoid/Softmax likelihood

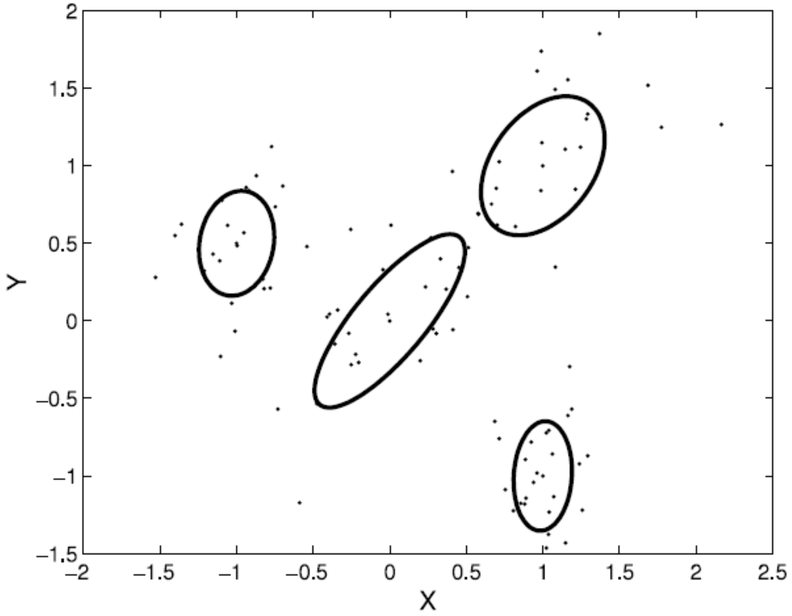
function



Gaussian Process Prior [Doob, 1944; Rasmussen & Williams, 2006]
+ Gaussian/Sigmoid/Softmax likelihood

Why Bayesian Nonparametrics?

- Let the data speak for themselves
- Bypass the model selection problem
  - let data determine model complexity (e.g., the number of components in mixture models)
  - allow model complexity to grow as more data observed



A reformulation of Bayesian inference

posterior　　likelihood model　　prior

$$p(\mathcal{M}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})}{\int p(\mathbf{x}|\mathcal{M})\pi(\mathcal{M})d\mathcal{M}}$$

- Bayes' rule is equivalent to:

$$\begin{aligned} \min_{p(\mathcal{M})} \quad & \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \mathbb{E}_{p(\mathcal{M})}[\log p(\mathbf{x}|\mathcal{M})] \\ \text{s.t.}: \quad & p(\mathcal{M}) \in \mathcal{P}_{\text{prob}}, \end{aligned}$$

A direct but trivial constraint on the posterior distribution

E.T. Jaynes (1988): "this fresh interpretation of Bayes' theorem could make the use of Bayesian methods more attractive and widespread, and stimulate new developments in the general theory of inference"

[Zellner, Am. Stat. 1988]

Regularized Bayesian Inference

$$\inf_{q(\mathrm{M}),\xi} \; \mathrm{KL}(q(\mathrm{M})\|\pi(\mathrm{M})) - \int_{\mathcal{M}} \log p(\mathcal{D}|\mathrm{M})q(\mathrm{M})d\mathrm{M} + U(\xi)$$
$$\text{s.t.}: \; q(\mathrm{M}) \in \mathcal{P}_{\text{post}}(\xi),$$

where, e.x.,

$$\mathcal{P}_{\text{post}}(\xi) \stackrel{\text{def}}{=} \left\{ q(\mathrm{M}) | \; \forall t = 1, \cdots, T, \; h(Eq(\psi_t; \mathcal{D})) \leq \xi_t \right\},$$

and

$$U(\xi) = \sum_{t=1}^{T} \mathbb{I}(\xi_t = \gamma_t) = \mathbb{I}(\xi = \gamma)$$

Solving such constrained optimization problem needs convex duality theory

So, where does the constraints come from?

MLE versus max-margin learning

$w \to p(w)$

- Likelihood-based estimation
  - Probabilistic (joint/conditional likelihood model)
  - Easy to perform Bayesian learning, and incorporate prior knowledge, latent structures, missing data
  - Bayesian or direct regularization
  - Hidden structures or generative hierarchy

- Max-margin learning
  - Non-probabilistic (concentrate on input-output mapping)
  - Not obvious how to perform Bayesian learning or consider prior, and missing data
  - Support vector property, sound theoretical guarantee with limited samples
  - Kernel tricks

- Maximum Entropy Discrimination (MED) (Jaakkola, et al., 1999)
  - Model averaging $\hat{y} = \text{sign} \int p(\mathbf{w}) F(x; \mathbf{w})\, \mathrm{d}\mathbf{w}$ $\quad (y \in \{+1, -1\})$
  - The optimization problem (binary classification)

$$\min_{p(\Theta)} \; KL(p(\Theta) || p_0(\Theta))$$

$$\text{s.t.} \quad \int p(\Theta)[y_i F(x; \mathbf{w}) - \xi_i]\, \mathrm{d}\Theta \geq 0, \forall i,$$

where Θ *is the parameter* $\mathbf{w}$ *when* ξ *are kept fixed or the pair* $(\mathbf{w}, \xi)$ *when we want to optimize over* ξ

Classical Predictive Models

- Input and output space: $\mathcal{X} \triangleq \mathbb{R}^{M_x}$ $\quad \mathcal{Y} \triangleq \{-1, +1\}$
- Learning: $$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}\in\mathcal{W}} \ell(\mathbf{x}, y; \mathbf{w}) + \lambda R(\mathbf{w})$$

where $\ell(\cdot)$ represents a convex loss, and $R(\mathbf{w})$ is a regularizer preventing overfitting

– **Logistic Regression**

- Max-likelihood (or MAP) estimation

$$\max_{\mathbf{w}} \mathcal{L}(\mathcal{D}; \mathbf{w}) \triangleq \sum_{i=1}^{N} \log p(y^i | \mathbf{x}^i; \mathbf{w}) + \mathcal{N}(\mathbf{w})$$

- Corresponds to a **Log loss** with L2 R

$$\ell_{LL}(\mathbf{x}, y; \mathbf{w}) \triangleq \ln \sum_{y' \in \mathcal{Y}} \exp\{\mathbf{w}^\top \mathbf{f}(\mathbf{x}, y')\} - \mathbf{w}^\top \mathbf{f}(\mathbf{x}, y)$$

– **Support Vector Machines (SVM)**

- Max-margin learning

$$\min_{\mathbf{w}, \xi} \frac{1}{2} \mathbf{w}^\top \mathbf{w} + C \sum_{i=1}^{N} \xi_i$$
$$\text{s.t. } \forall i, \forall y' \neq y^i : \mathbf{w}^\top \Delta \mathbf{f}_i(y') \geq 1 - \xi_i, \; \xi_i \geq 0.$$

- Corresponds to a **hinge loss** with L2 R

$$\ell_{MM}(\mathbf{x}, y; \mathbf{w}) \triangleq \max_{y' \in \mathcal{Y}} \mathbf{w}^\top \mathbf{f}(\mathbf{x}, y') - \mathbf{w}^\top \mathbf{f}(\mathbf{x}, y) + \ell'(y', y)$$

Advantages:

1. Full probabilistic semantics
2. Straightforward Bayesian or direct regularization
3. Hidden structures or generative hierarchy

Advantages:

1. Dual sparsity: few support vectors
2. Kernel tricks
3. Strong empirical results

$P(y|x)$

Structured Prediction Graphical Models

- Input and output space: $\mathcal{X} \triangleq \mathbb{R}_{X_1} \times, \ldots, \mathbb{R}_{X_K} \quad \mathcal{Y} \triangleq \mathbb{R}_{Y_1} \times, \ldots, \mathbb{R}_{Y_{K'}}$

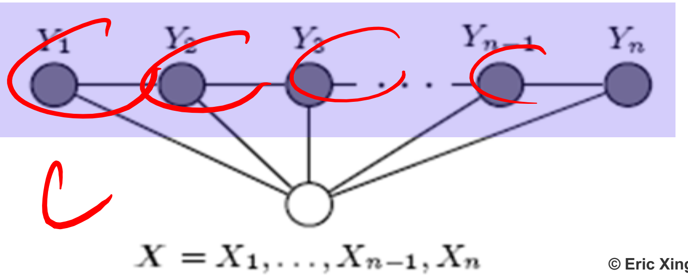
Conditional Random Fields (CRFs) (Lafferty et al 2001)
- Based on a Logistic Loss (LR)
- Max-likelihood estimation (point-estimate)

$$\mathcal{L}(\mathcal{D}; \mathbf{w}) \triangleq \log \sum_{\mathbf{y}'} \exp(\mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y}')) - \mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y}) + R(\mathbf{w})$$

Max-margin Markov Networks (M^3Ns) (Taskar et al 2003)
- Based on a Hinge Loss (SVM)
- Max-margin learning (point-estimate)

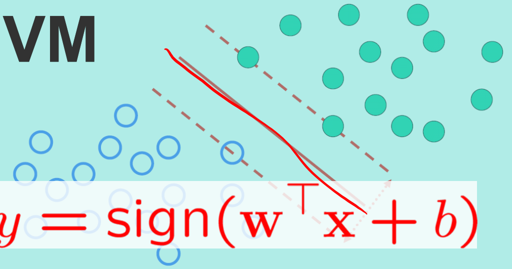
$$\mathcal{L}(\mathcal{D}; \mathbf{w}) \triangleq \log \max_{\mathbf{y}'} \mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y}') - \mathbf{w}^\top \mathbf{f}(\mathbf{x}, \mathbf{y}) + \ell(\mathbf{y}', \mathbf{y}) + R(\mathbf{w})$$

- Markov properties are encoded in the feature functions $\mathbf{f}(\mathbf{x}, \mathbf{y})$



Max-Margin Learning Paradigms

SVM



$$y = \text{sign}(\mathbf{w}^\top \mathbf{x} + b)$$

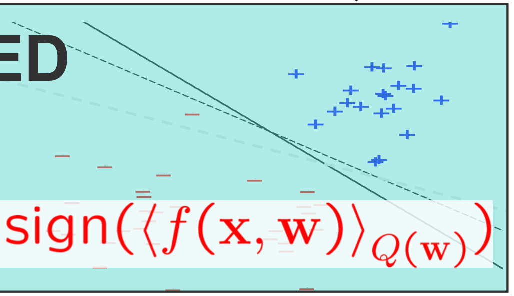
$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{m}\xi_i$$
$$y^i(\mathbf{w}^\top \mathbf{x}^i + b) \geq 1 - \xi_i, \quad \forall i$$

M³N

$$y = \arg\max_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} F(\mathbf{x}, \mathbf{y}; \mathbf{w})$$

$$\min_{\mathbf{w},\xi} \quad \frac{1}{2}||\mathbf{w}||^2 + C\sum_{i=1}^{m}\xi_i$$
$$\mathbf{w}^\top[\mathbf{f}(\mathbf{x}^i, \mathbf{y}^i) - \mathbf{f}(\mathbf{x}^i, \mathbf{y})] \geq \ell(\mathbf{y}^i, \mathbf{y}) - \xi_i, \quad \forall i, \forall \mathbf{y} \neq \mathbf{y}^i$$

MED



$$y = \text{sign}(\langle f(\mathbf{x}, \mathbf{w})\rangle_{Q(\mathbf{w})})$$

$$\min_{Q} \quad \text{KL}(Q||Q_0)$$
$$y^i\langle f(\mathbf{x}^i)\rangle_Q \geq \xi_i, \quad \forall i$$

MED-MN

= SMED + "Bayesian" M³N

Maximum Entropy Discrimination Markov Networks

(Zhu et al, ICML 2008)

- Structured MaxEnt Discrimination (SMED):

$$\text{P1}: \quad \min_{p(\mathbf{w}),\xi} \boxed{KL(p(\mathbf{w})||p_0(\mathbf{w})) + U(\xi)}$$

$$\text{s.t.} \quad p(\mathbf{w}) \in \mathcal{F}_1, \; \xi_i \geq 0, \forall i.$$

generalized maximum entropy or *regularized* KL-divergence

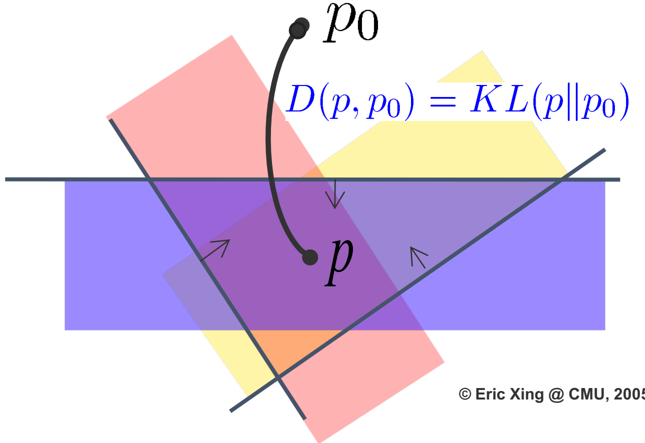
- Feasible subspace of weight distribution:

$$\mathcal{F}_1 = \{p(\mathbf{w}) : \boxed{\int p(\mathbf{w})[\Delta F_i(\mathbf{y};\mathbf{w}) - \Delta\ell_i(\mathbf{y})]\,\mathrm{d}\mathbf{w} \geq -\xi_i}, \forall i, \forall \mathbf{y} \neq \mathbf{y}^i\},$$

expected margin constraints.

- Average from distribution of M³Ns

$$h_1(\mathbf{x}; p(\mathbf{w})) = \arg\max_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \int p(\mathbf{w}) F(\mathbf{x},\mathbf{y};\mathbf{w})d\mathbf{w}$$



Solution to MaxEnDNet

$W_i \Rightarrow \alpha_i$

dim   data

- Theorem:
  - Posterior Distribution:

$$p(\mathbf{w}) = \frac{1}{Z(\alpha)} p_0(\mathbf{w}) \exp\left\{ \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y}) [\Delta F_i(\mathbf{y};\mathbf{w}) - \Delta \ell_i(\mathbf{y})] \right\}$$

  - Dual Optimization Problem:

$$\begin{aligned} \mathrm{D1}: \quad & \max_{\alpha} \; -\log Z(\alpha) - U^{\star}(\alpha) \\ & \text{s.t. } \alpha_i(\mathbf{y}) \geq 0, \; \forall i, \; \forall \mathbf{y}, \end{aligned}$$

$U^{\star}(\cdot)$ is the conjugate of the $U(\cdot)$, i.e., $U^{\star}(\alpha) = \sup_{\xi} \left(\sum_{i,y} \alpha_i(y)\xi_i - U(\xi) \right)$

Algorithmic issues of solving M³Ns

- **Primal problem:**

$$\text{P0 } (\text{M}^3\text{N}): \min_{\mathbf{w},\xi} \frac{1}{2}\|\mathbf{w}\|^2 + C\sum_{i=1}^{N} \xi_i$$

$$\text{s.t. } \forall i, \forall \mathbf{y} \neq \mathbf{y}^i: \mathbf{w}^\top \Delta\mathbf{f}_i(\mathbf{y}) \geq \Delta\ell_i(\mathbf{y}) - \xi_i,$$
$$\xi_i \geq 0,$$

- **Algorithms**
  - Cutting plane
  - Sub-gradient
  - …

- **Dual problem:**

$$\text{D0 } (\text{M}^3\text{N}): \max_{\alpha} \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y})\Delta\ell_i(\mathbf{y}) - \frac{1}{2}\eta^\top\eta$$

$$\text{s.t. } \forall i, \ \forall \mathbf{y}: \sum_{\mathbf{y}} \alpha_i(\mathbf{y}) = C; \ \alpha_i(\mathbf{y}) \geq 0.$$

$$\text{where } \eta = \sum_{i,\mathbf{y}} \alpha_i(\mathbf{y})\Delta\mathbf{f}_i(\mathbf{y}).$$

- **Algorithms:**
  - SMO
  - Exponentiated gradient
  - …

- **Nonlinear Features with Kernels**
  - Generative entropic kernels [Martins et al, JMLR 2009]
  - Nonparametric RKHS embedding of rich distributions [on going]
- **Approximate decoders for global features**
  - LP-relaxed Inference (polyhedral outer approx.) [Martins et al, ICML 09, ACL 09]
  - Balancing Accuracy and Runtime: Loss-augmented inference

Variational Learning of LapMEDN

- Exact primal or dual function is hard to optimize

$$\min_{\mu,\xi} \sqrt{\lambda}\sum_{k=1}^{K}\Big(\sqrt{\mu_k^2+\frac{1}{\lambda}}-\frac{1}{\sqrt{\lambda}}\log\frac{\sqrt{\lambda\mu_k^2+1}+1}{2}\Big)+C\sum_{i=1}^{N}\xi_i$$
$$\text{s.t.}\ \ \mu^\top\Delta\mathbf{f}_i(\mathbf{y})\geq\Delta\ell_i(\mathbf{y})-\xi_i;\ \xi_i\geq 0,\ \ \forall i,\ \forall\mathbf{y}\neq\mathbf{y}^i.$$

$$\max_{\alpha}\ \sum_{i,\mathbf{y}}\alpha_i(\mathbf{y})\Delta\ell_i(\mathbf{y})-\sum_{k=1}^{K}\log\frac{\lambda}{\lambda-\eta_k^2}$$
$$\text{s.t.}\ \ \sum_{\mathbf{y}}\alpha_i(\mathbf{y})=C;\ \alpha_i(\mathbf{y})\geq 0,\ \forall i,\ \forall\mathbf{y}.$$

- Use the hierarchical representation of Laplace prior, we get:

$$KL(p||p_0) = -H(p) - \langle\log\int p(\mathbf{w}|\tau)p(\tau|\lambda)\,\mathrm{d}\tau\rangle_p$$
$$\leq -H(p) - \langle\int q(\tau)\log\frac{p(\mathbf{w}|\tau)p(\tau|\lambda)}{q(\tau)}\,\mathrm{d}\tau\rangle_p \triangleq \mathcal{L}(p(\mathbf{w}),q(\tau))$$

- We optimize an upper bound:

$$\min_{p(\mathbf{w})\in\mathcal{F}_1;q(\tau);\xi}\ \mathcal{L}(p(\mathbf{w}),q(\tau))+U(\xi)$$

- Why is it easier?
  - Alternating minimization leads to nicer optimization problems

Keep $q(\tau)$ fixed	**Keep $p(\mathbf{w})$ fixed**
- The effective prior is normal $\forall k:\ p_0(w_k\|\tau_k)=\mathcal{N}(w_k\|0,\langle\frac{1}{\tau_k}\rangle_{q(\tau)}^{-1})$	- Closed form solution of $q(\tau)$ and its expectation $\langle\frac{1}{\tau_k}\rangle_q=\sqrt{\frac{\lambda}{\langle w_k^2\rangle_p}}.$

An M^3N optimization problem!

Closed-form solution!

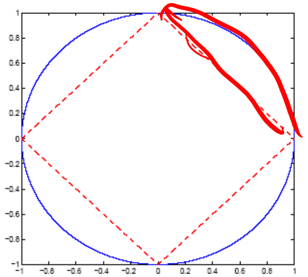
The 3 advantages of MEDN

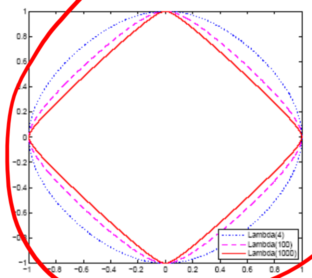
- An averaging Model: PAC-Bayesian prediction error guarantee (Theorem 3)

$$\Pr_Q(M(h,\mathbf{x},\mathbf{y}) \leq 0) \leq \Pr_{\mathcal{D}}(M(h,\mathbf{x},\mathbf{y}) \leq \gamma) + O\left(\sqrt{\frac{\gamma^{-2}KL(p||p_0)\ln(N|\mathcal{Y}|) + \ln N + \ln \delta^{-1}}{N}}\right).$$

- Entropy regularization: Introducing useful biases
  - Standard Normal prior => reduction to standard M3N (we've seen it)
  - Laplace prior => Posterior shrinkage effects (sparse M3N)

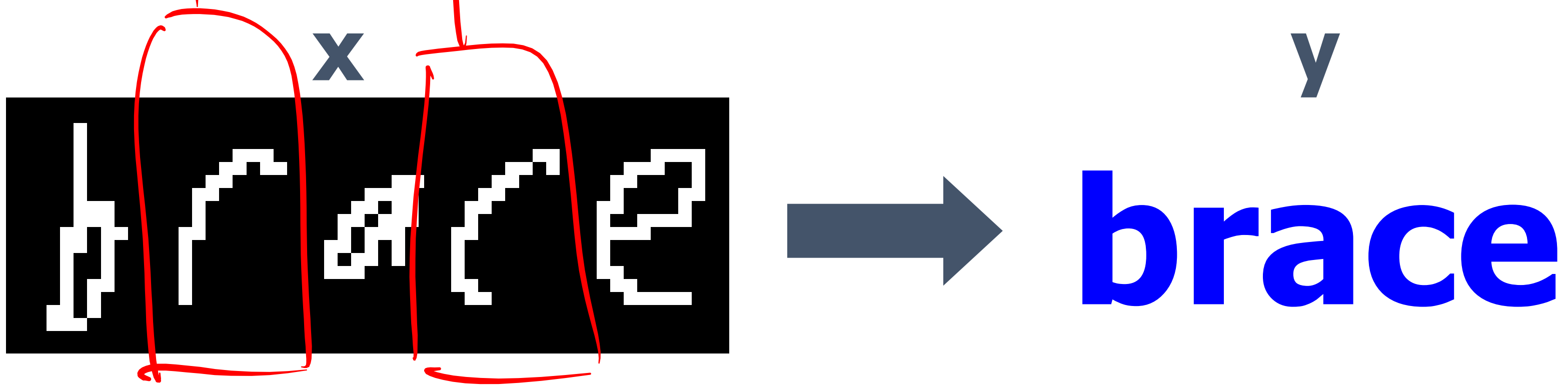
$$\min_{\mu,\xi} \sqrt{\lambda}\sum_{k=1}^{K}\left(\sqrt{\mu_k^2 + \frac{1}{\lambda}} - \frac{1}{\sqrt{\lambda}}\log\frac{\sqrt{\lambda\mu_k^2+1}+1}{2}\right) + C\sum_{i=1}^{N}\xi_i$$
$$\text{s.t. } \mu^\top \Delta\mathbf{f}_i(\mathbf{y}) \geq \Delta\ell_i(\mathbf{y}) - \xi_i;\ \xi_i \geq 0,\ \forall i,\ \forall \mathbf{y} \neq \mathbf{y}^i.$$



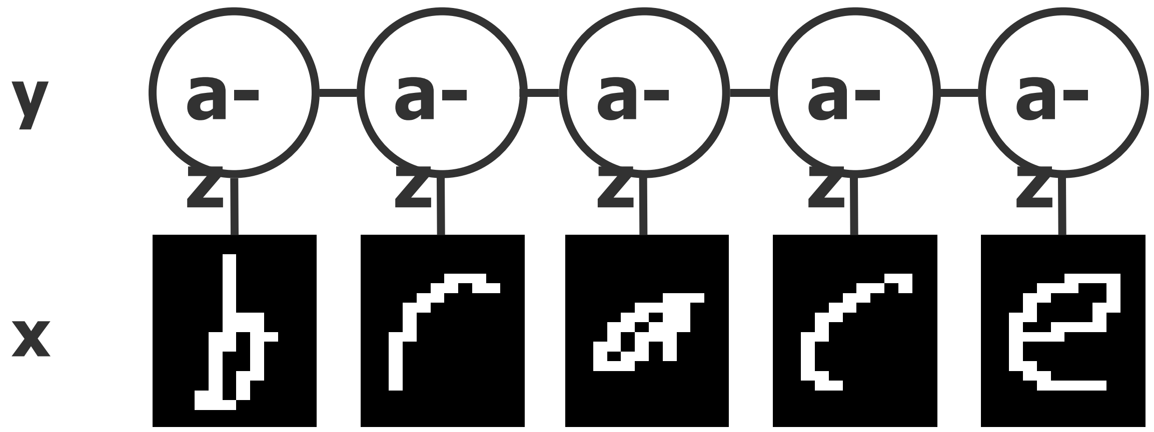


- Integrating Generative and Discriminative principles
- Incorporate ~~latent variables and structures (PoMEN)~~
  - Semisupervised ~~learning (with partially labeled data)~~

Experimental results on OCR datasets



Structured output

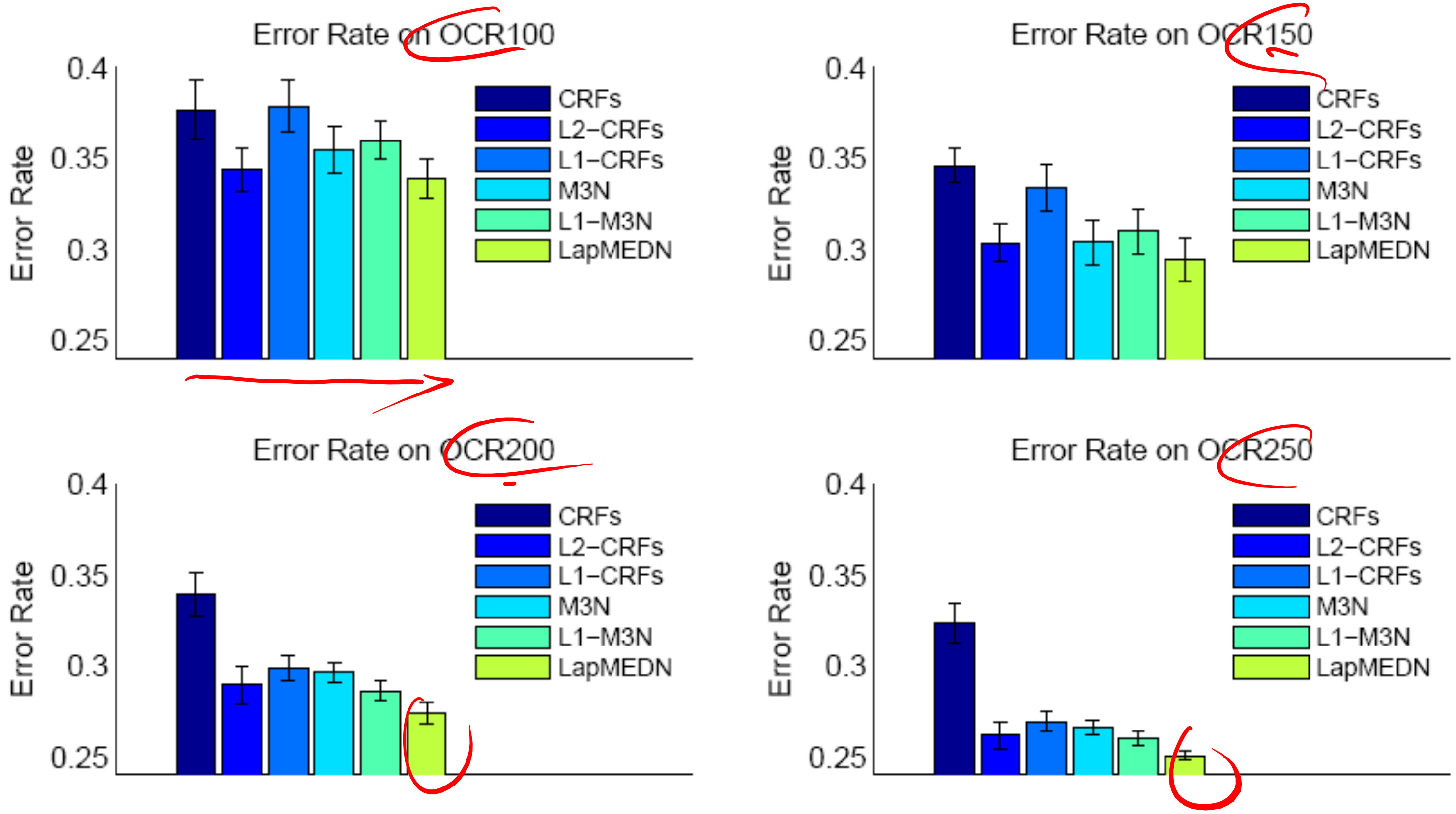


Experimental results on OCR datasets

$L + \lambda R(w)$

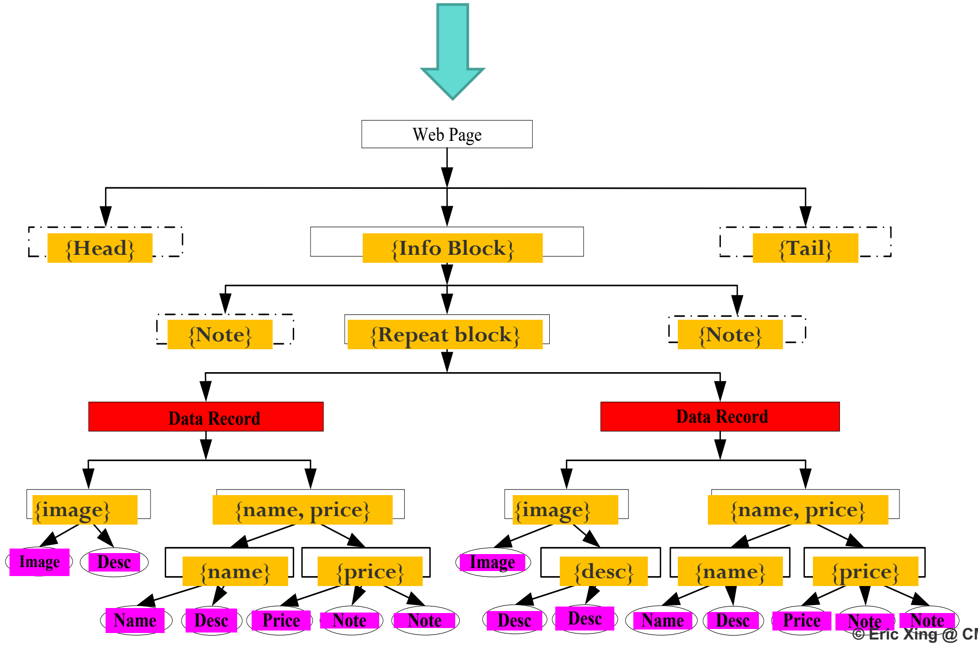
$$\text{(CRFs, } L_1 - \text{CRFs, } L_2 - \text{CRFs, } \mathrm{M^3Ns, } L_1 - \mathrm{M^3Ns, \text{ and LapMEDN)}}$$

- We randomly construct OCR100, OCR150, OCR200, and OCR250 for 10 fold CV.



Latent Hierarchical MaxEnDNet

- Web data extraction
  - Goal: *Name*, *Image*, *Price*, *Description*, etc.

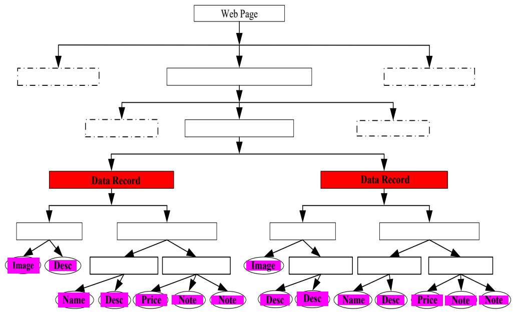


- **Hierarchical labeling**
- **Advantages:**
  - Computational efficiency
  - Long-range dependency
  - Joint extraction

Partially Observed MaxEnDNet (PoMEN)

(Zhu et al, NIPS 2008)

- Now we are given partially labeled data: $\mathcal{D} = \{ < \mathbf{x}^i, \mathbf{y}^i, \mathbf{z}^i > \}_{i=1}^N$



  - PoMEN: learning $p(\mathbf{w}, \mathbf{z})$

$$\mathrm{P2(PoMEN)}: \quad \min_{p(\mathbf{w},\{\mathbf{z}\}),\xi} \; KL(p(\mathbf{w},\{\mathbf{z}\})||p_0(\mathbf{w},\{\mathbf{z}\})) + U(\xi)$$

$$\text{s.t.} \;\; p(\mathbf{w},\{\mathbf{z}\}) \in \mathcal{F}_2, \;\; \xi_i \geq 0, \forall i.$$

$$\mathcal{F}_2 = \left\{ p(\mathbf{w},\{\mathbf{z}\}) : \sum_{\mathbf{z}} \int p(\mathbf{w},\mathbf{z})[\Delta F_i(\mathbf{y},\mathbf{z};\mathbf{w}) - \Delta\ell_i(\mathbf{y})]\, d\mathbf{w} \geq -\xi_i, \;\; \forall i, \forall \mathbf{y} \neq \mathbf{y}^i \right\},$$

  - Prediction:

$$h_2(\mathbf{x}) = \arg\max_{\mathbf{y}\in\mathcal{Y}(\mathbf{x})} \sum_{\mathbf{z}} \int p(\mathbf{w},\mathbf{z}) F(\mathbf{x},\mathbf{y},\mathbf{z};\mathbf{w})\, d\mathbf{w}$$

Alternating Minimization Alg.

- Factorization assumption:

$$p_0(\mathbf{w}, \{\mathbf{z}\}) = p_0(\mathbf{w}) \prod_{i=1}^{N} p_0(\mathbf{z}_i) \qquad p(\mathbf{w}, \{\mathbf{z}\}) = p(\mathbf{w}) \prod_{i=1}^{N} p(\mathbf{z}_i)$$

- Alternating minimization:
  - Step 1: keep $p(\mathbf{z})$ fixed, optimize over $p(\mathbf{w})$

$$\min_{p(\mathbf{w}),\xi} \; KL(p(\mathbf{w})||p_0(\mathbf{w})) + C\sum_i \xi_i$$
$$\text{s.t.} \;\; p(\mathbf{w}) \in \mathcal{F}_1', \;\; \xi_i \geq 0, \forall i.$$
$$\mathcal{F}_1' = \{p(\mathbf{w}) : \int p(\mathbf{w}) E_{p(\mathbf{z})}[\Delta F_i(\mathbf{y},\mathbf{z};\mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, \mathrm{d}\mathbf{w} \geq -\xi_i, \; \forall i, \; \forall \mathbf{y}\}$$

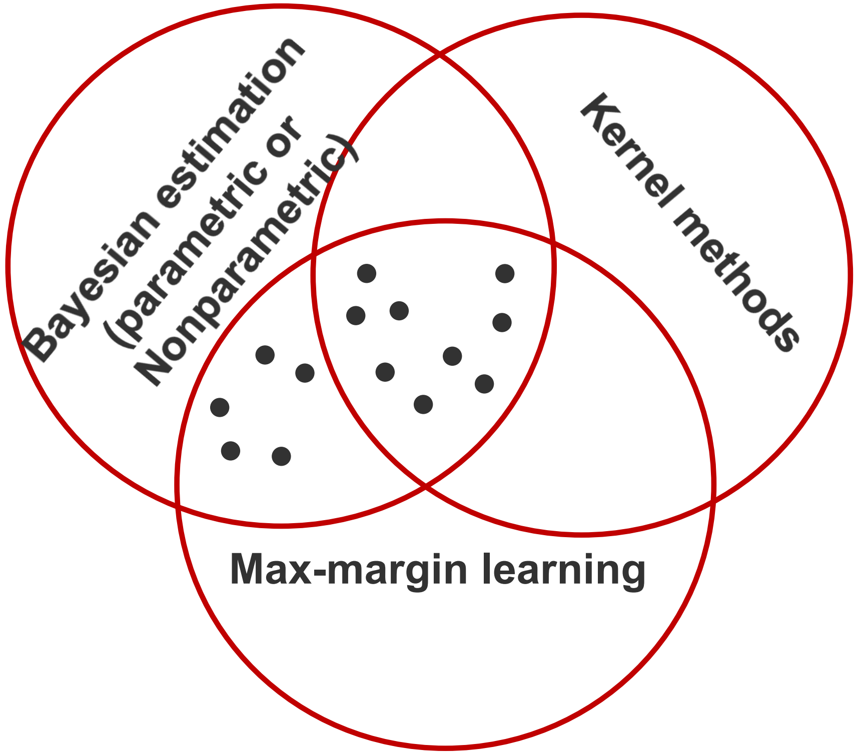
  - Normal prior
    - M^3N problem (QP)
  - Laplace prior
    - Laplace M^3N problem (VB)

  - Step 2: keep $p(\mathbf{w})$ fixed, optimize over $p(\mathbf{z})$

$$\min_{p(\mathbf{w}),\xi} \; KL(p(\mathbf{z})||p_0(\mathbf{z})) + C\xi_i$$
$$\text{s.t.} \;\; p(\mathbf{z}) \in \mathcal{F}_1^\star, \;\; \xi_i \geq 0.$$
$$\mathcal{F}_1^\star = \{p(\mathbf{z}) : \sum_{\mathbf{z}} p(\mathbf{z}) \int p(\mathbf{w})[\Delta F_i(\mathbf{y},\mathbf{z};\mathbf{w}) - \Delta \ell_i(\mathbf{y})] \, \mathrm{d}\mathbf{w} \geq -\xi_i, \; \forall i, \; \forall \mathbf{y}\}$$

Equivalently reduced to an LP with a polynomial number of constraints

An integrative paradigm for learning GM --- RegBayes



$$\inf_{q(\mathrm{M}),\xi} \mathrm{KL}(q(\mathrm{M})\|\pi(\mathrm{M})) - \int_{\mathcal{M}} \log p(\mathcal{D}|\mathrm{M})q(\mathrm{M})d\mathrm{M} + U(\xi)$$
$$\text{s.t.}: q(\mathrm{M}) \in \mathcal{P}_{\mathrm{post}}(\xi),$$

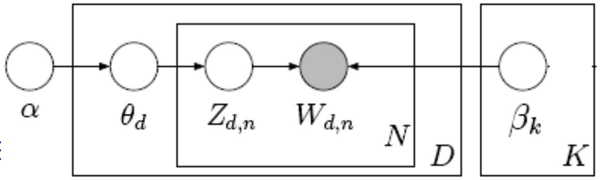
Predictive Latent Subspace Learning

via a large-margin approach

… where M is any subspace model and p is a parametric Bayesian prior

Unsupervised Latent Subspace Discovery

- Finding latent subspace representations (an old topic)
  - Mapping a high-dimensional representation into a latent low-dimensional representation, where each dimension can have some interpretable meaning, e.g., a semantic topic
- Examples:
  - Topic models (aka LDA) [Blei et al 2003]



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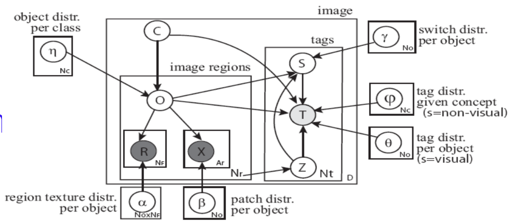
  - Total scene latent spa



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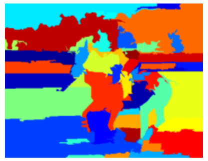
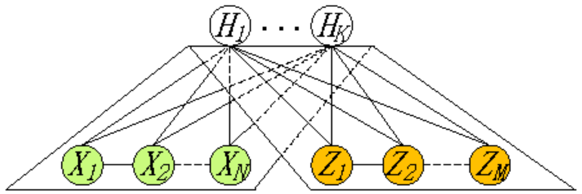
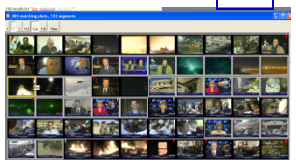


  - Multi-view latent Markov m
  - PCA, CCA, …





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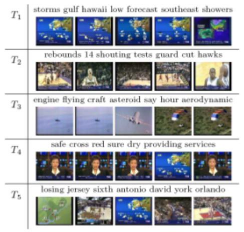


Predictive Subspace Learning with *Supervision*

- Unsupervised latent subspace representations are generic but can be sub-optimal for predictions
- Many datasets are available with supervised side information

- **Tripadvisor Hotel Review** (http://www.tripadvisor.com)

"Lovely welcomming staff, good rooms that give a good nights sleep, downtown location"

Meramees Hostel

SheikhSahib 10 contributions London

Save Review

Jul 7, 2009 | Trip type: Friends getaway

This hotel is just of the side streets of Talat Harb, one of the main arteries to downtown Cairo. It is walking distance to the Nile, riverfront hotels, Egyptian Museum, and there are many eateries in the area at night when it is still bustling. Only a short cab ride away from the Old Fatimid Cairo.

The staff are young and very friendly and able to sort out things like mobile chargers, internet, and they have skype installed on their computers which is brilliant. The rooms are nicer then the Luna (nearby) and much quieter as well.

My ratings for this hotel

Value
Rooms
Location
Cleanliness
Service

Date of stay February 2009

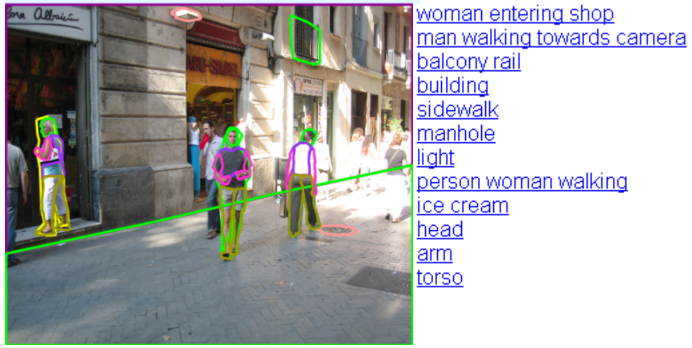
Visit was for Leisure

Traveled with With Friends

Member since July 03, 2009

Would you recommend this hotel to a friend? Yes

- **LabelMe** http://labelme.csail.mit.edu/



- **Many others**

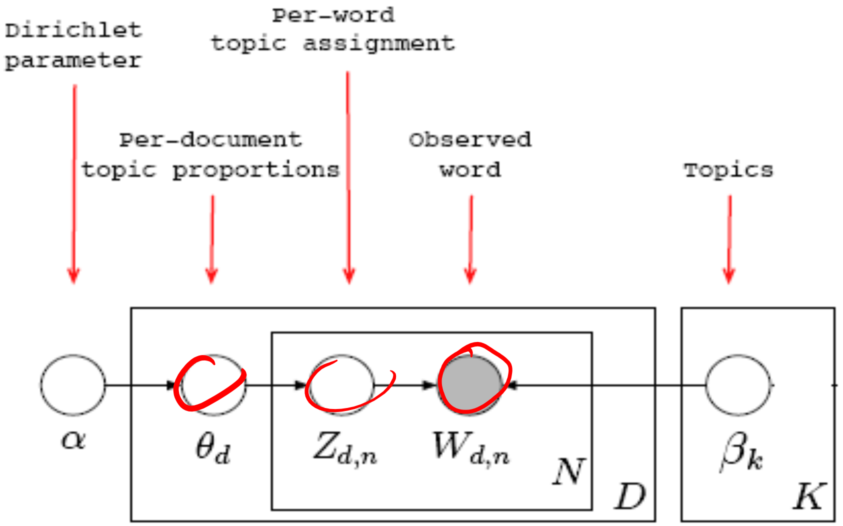
Flickr (http://www.flickr.com/)

IMAGENET

- Can be noisy, but n… & Na…
  - labels & rating scores are usually assigned based on some intrinsic property of the data
  - helpful to suppress noise and capture the most useful aspects of the data
- Goals:
  - Discover latent subspace representations that are both *predictive* and *interpretable* by exploring weak supervision information

I. LDA: Latent Dirichlet Allocation

(Blei et al., 2003)



- **Generative Procedure:**
  - **For each document *d*:**
    - **Sample a topic proportion** $\theta_d \sim \mathrm{Dir}(\alpha)$
    - **For each word:**
      - **Sample a topic** $Z_{d,n} \sim \mathrm{Mult}(\theta_d)$
      - **Sample a word** $W_{d,n} \sim \mathrm{Mult}(\beta_{z_{d,n}})$

- Joint Distribution:

$$p(\theta, \mathbf{z}, \mathbf{W}|\alpha, \beta) = \prod_{d=1}^{D} p(\theta_d|\alpha)\left(\prod_{n=1}^{N} p(z_{dn}|\theta_d)p(w_{dn}|z_{dn}, \beta)\right)$$

exact inference intractable!

- Variational Inference with $q(\mathbf{z}, \theta) \sim p(\mathbf{z}, \theta|\mathbf{W}, \alpha, \beta)$.

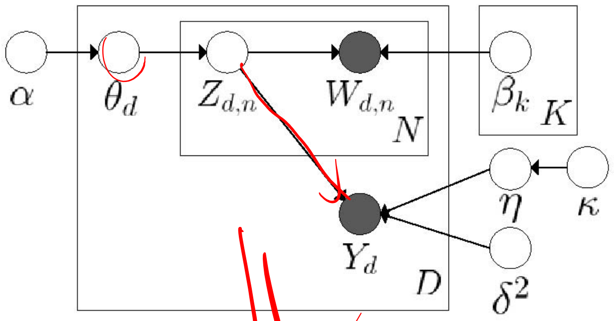
- Minimize the variational bound to estimate parameters and infer the posterior distribution

$$\mathcal{L}(q) \triangleq -E_q[\log p(\theta, \mathbf{z}, \mathbf{W}|\alpha, \beta)] - \mathcal{H}(q(\mathbf{z}, \theta)) \geq -\log p(\mathbf{W}|\alpha, \beta)$$

Maximum Entropy Discrimination LDA (MedLDA)

(Zhu et al, ICML 2009)

- Bayesian sLDA:



- MED Estimation:
  - MedLDA Regression Model

$$\mathrm{P1}(\mathrm{MedLDA}^r): \quad \min_{q,\alpha,\beta,\delta^2,\xi,\xi^\star} \mathcal{L}(q) + C\sum_{d=1}^{D}(\xi_d + \xi_d^\star)$$

$$\text{s.t. } \forall d: \begin{cases} y_d - E[\eta^\top \bar{Z}_d] \le \epsilon + \xi_d, & \mu_d \\ -y_d + E[\eta^\top \bar{Z}_d] \le \epsilon + \xi_d^\star, & \mu_d^\star \\ \xi_d \ge 0, & v_d \\ \xi_d^\star \ge 0, & v_d^\star \end{cases}$$

model fitting

predictive accuracy

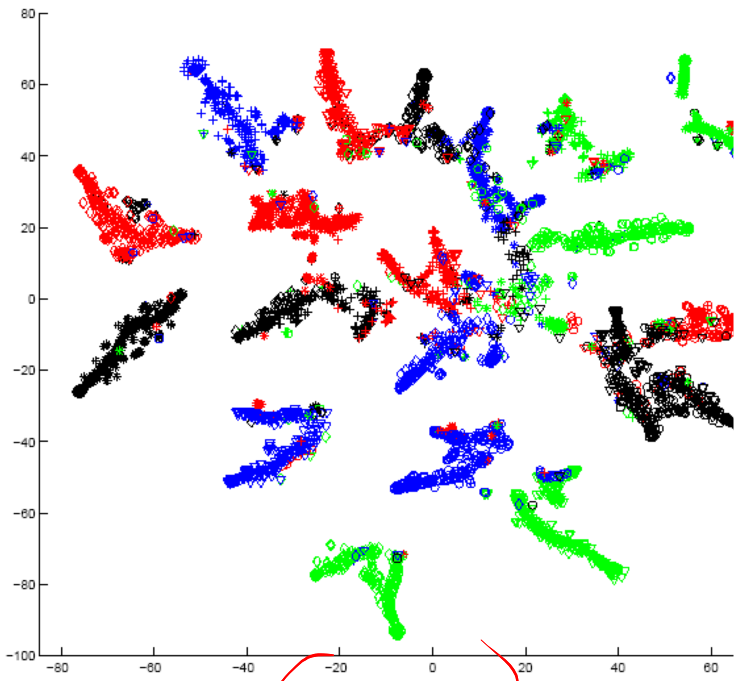
  - MedLDA Classification Model

$$\mathrm{P2}(\mathrm{MedLDA}^c): \quad \min_{q,q(\eta),\alpha,\beta,\xi} \mathcal{L}(q) + C\sum_{d=1}^{D}\xi_d$$

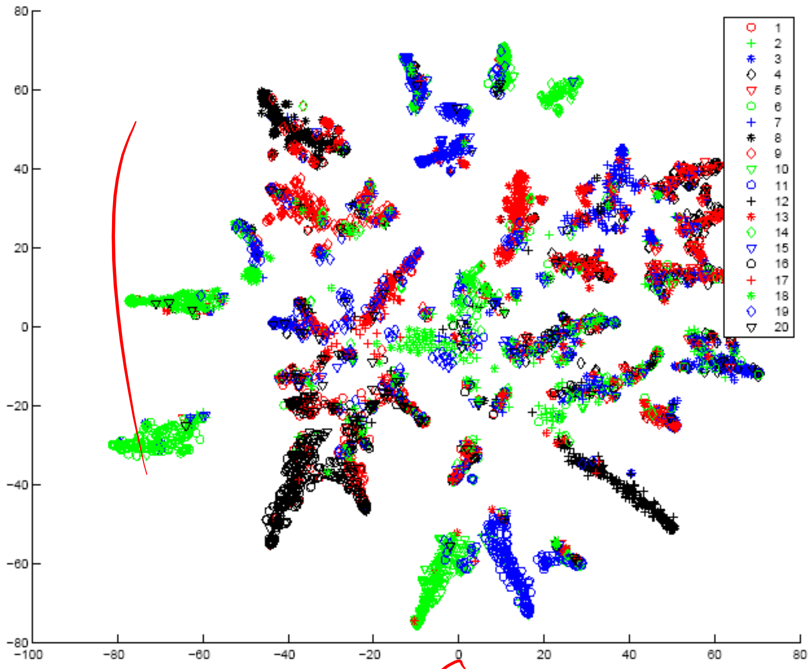
$$\text{s.t. } \forall d,\ y \neq y_d: \quad E[\eta^\top \Delta \mathbf{f}_d(y)] \ge 1 - \xi_d;\ \xi_d \ge 0.$$

Document Modeling

- Data Set: 20 Newsgroups
- 110 topics + 2D embedding with t-SNE (var der Maaten & Hinton, 2008)



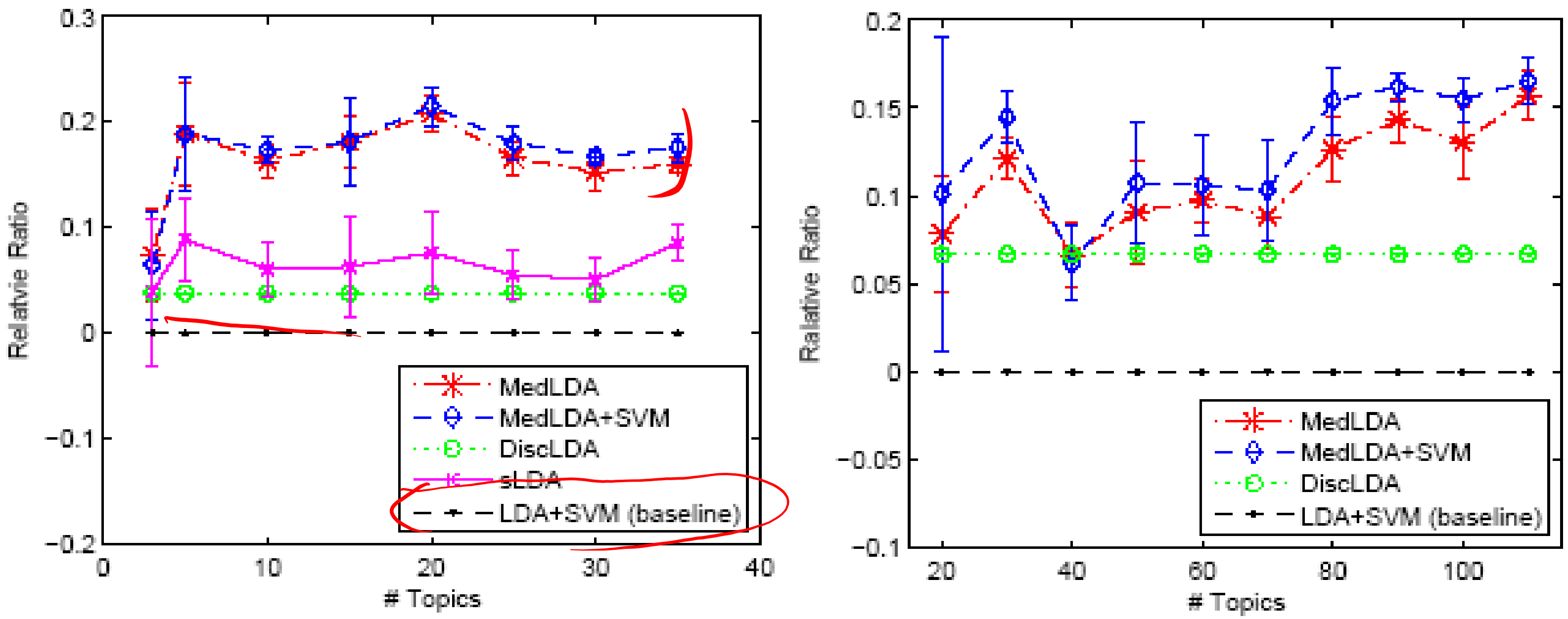
MedLDA



LDA

Classification

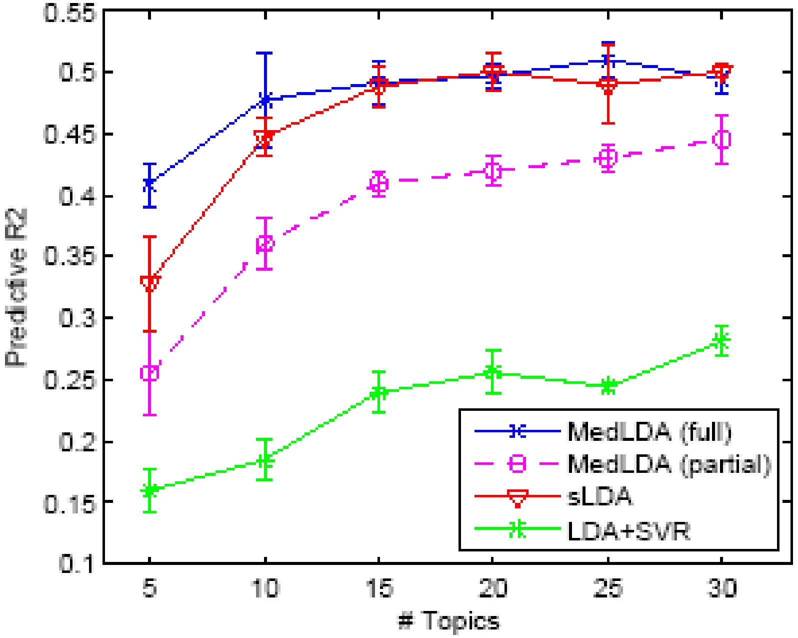
- Data Set: 20Newsgroups
  - Binary classification: “alt.atheism” and “talk.religion.misc” (Simon et al., 2008)
  - Multiclass Classification: all the 20 categories
- Models: DiscLDA, sLDA (Binary ONLY! Classification sLDA (Wang et al., 2009)), LDA+SVM (baseline), MedLDA, MedLDA+SVM
- Measure: Relative Improvement Ratio $$RR(\mathcal{M}) = \frac{precision(\mathcal{M})}{precision(LDA+SVM)} - 1$$

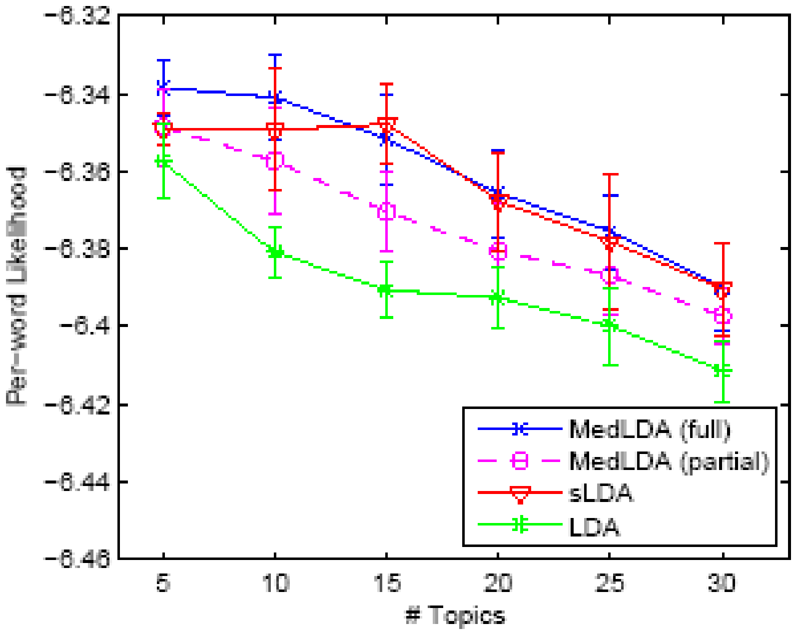


Regression

- **Data Set**: Movie Review (Blei & McAuliffe, 2007)
- **Models**: MedLDA(*partial*), MedLDA(*full*), sLDA, LDA+SVR
- **Measure**: predictive R^2 and per-word log-likelihood

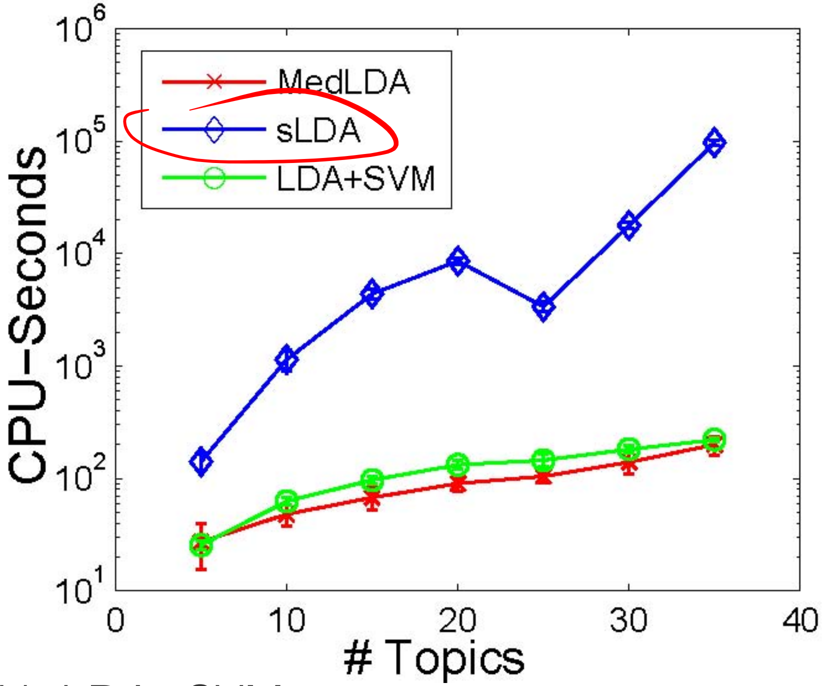
$$pR^2 = 1 - \frac{\sum_d (y_d - \hat{y}_d)^2}{\sum_d (y_d - \bar{y}_d)^2}$$





Time Efficiency

- Binary Classification



- Multiclass:
  - MedLDA is comparable with LDA+SVM
- Regression:
  - MedLDA is comparable with sLDA

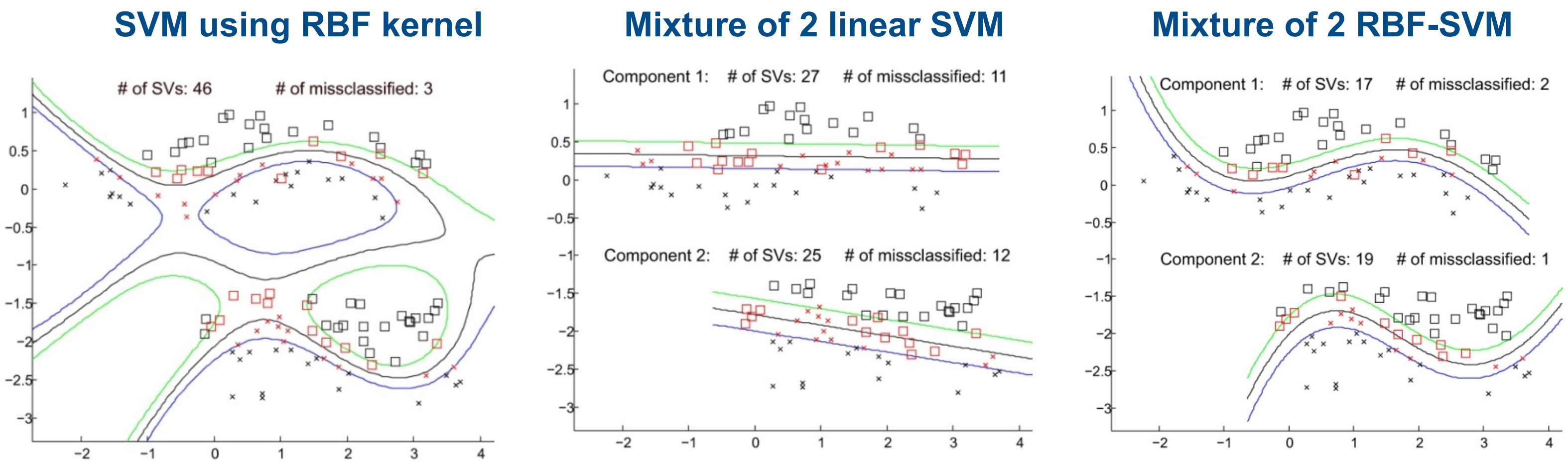
Infinite SVM and infinite latent SVM:

-- where SVMs meet NB for classification and feature selection

… where M is any combinations of classifiers and p is a nonparametric Bayesian prior

Mixture of SVMs

- Dirichlet process mixture of large-margin kernel machines
- Learn flexible non-linear local classifiers; potentially lead to a better control on model complexity, e.g., few unnecessary components



- The first attempt to integrate Bayesian nonparametrics, large-margin learning, and kernel methods

Infinite SVM

- RegBayes framework:

$$\min_{p(\mathcal{M}),\xi} \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \sum_{n=1}^{N}\int \log p(\mathbf{x}_n|\mathcal{M})p(\mathcal{M})d\mathcal{M} + U(\xi)$$

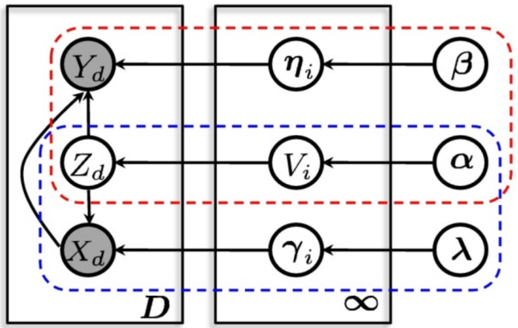
$$\text{s.t.}: \quad p(\mathcal{M}) \in \mathcal{P}_{\text{post}}(\xi),$$

convex function

direct and rich constraints on posterior distribution

- Model – latent class model
- Prior – Dirichlet process
- Likelihood – Gaussian likelihood
- Posterior constraints – max-margin constraints

Infinite SVM



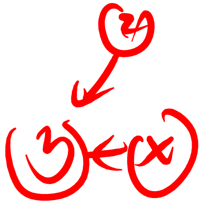
Graphical model with stick-breaking construction of DP

- DP mixture of large-margin classifiers

process of determining which classifier to use:

1. draw $V_i | \alpha \sim \mathrm{Beta}(1, \alpha),\ i \in \{1, 2, \cdots\}$.
2. draw $\eta_i | G_0 \sim G_0,\ i \in \{1, 2, \cdots\}$.
3. for the dth data point:

   (a) draw $Z_d | \{v_1, v_2, \cdots\} \sim \mathrm{Mult}(\pi(\mathbf{v}))$



- Given a component classifier:

$$F(y, \mathbf{x}; z, \boldsymbol{\eta}) = \eta_z^\top \mathbf{f}(y, \mathbf{x}) = \sum_{i=1}^{\infty} \delta_{z,i} \eta_i^\top \mathbf{f}(y, \mathbf{x})$$

- Overall discriminant function:

$$F(y, \mathbf{x}) = \mathrm{E}_{q(z, \boldsymbol{\eta})}[F(y, \mathbf{x}; z, \boldsymbol{\eta})] = \sum_{i=1}^{\infty} q(z = i) \mathrm{E}_q[\eta_i]^\top \mathbf{f}(y, \mathbf{x})$$

- Prediction rule:

$$y^* = \arg\max_y F(y, \mathbf{x})$$

- Learning problem:

$$\min_{q(\mathbf{z}, \boldsymbol{\eta})} \mathrm{KL}(q(\mathbf{z}, \boldsymbol{\eta}) \| p_0(\mathbf{z}, \boldsymbol{\eta})) + C_1 \mathcal{R}(q(\mathbf{z}, \boldsymbol{\eta})),$$

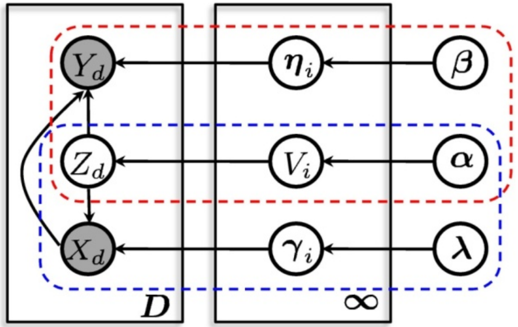
$$\mathcal{R}(q(\mathbf{z}, \boldsymbol{\eta})) = \sum_d \max_y (\ell_d^\Delta(y) + F(y, \mathbf{x}_d) - F(y_d, \mathbf{x}_d))$$

Infinite SVM

- Assumption and relaxation
  - Truncated variational distribution

$$q(\mathbf{z}, \boldsymbol{\eta}, \boldsymbol{\gamma}, \mathbf{v}) = \prod_{d=1}^{D} q(z_d) \prod_{t=1}^{T} q(\eta_t) \prod_{t=1}^{T} q(\gamma_t) \prod_{t=1}^{T-1} q(v_t)$$

  - Upper bound the KL-regularizer



Graphical model with stick-breaking construction of DP

- Opt. with coordinate descent
  - For $q(\boldsymbol{\eta})$, we solve an SVM learning problem
  - For $q(\mathbf{z})$, we get the closed update rule

$$q(z_d = t) \propto \exp\left\{(\mathbb{E}[\log v_t] + \sum_{i=1}^{t-1} \mathbb{E}[\log(1-v_i)]) + \rho(\mathbb{E}[\gamma_t]^\top \mathbf{x}_d - \mathbb{E}[A(\gamma_t)]) + (1-\rho) \sum_{y} \omega_d^y \mu_t^\top \mathbf{f}_d^\Delta(y)\right\}$$

    - The last term regularizes the mixing proportions to favor prediction
  - For $q(\boldsymbol{\gamma}), q(\mathbf{v})$, the same update rules as in (Blei & Jordan, 2006)

Experiments on high-dim real data

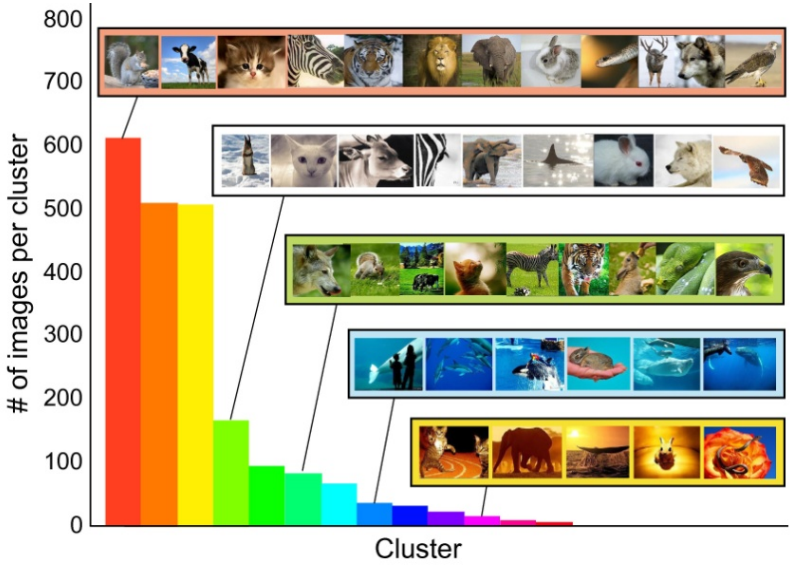
- Classification results and test time:

Table 4. Classification accuracy (%), F1 score (%), and test time (sec) for different models on the Flickr image dataset. All methods except dpMNL are implemented in C.

	Accuracy	F1 score	Test time
~~MNL~~	49.8 ± 0.0	48.4 ± 0.0	$\mathbf{0.02} \pm 0.00$
~~MMH~~	51.7 ± 0.0	50.1 ± 0.0	0.33 ± 0.01
RBF-SVM	52.2 ± 0.0	48.4 ± 0.0	7.58 ± 0.06
~~dpMNL~~-EFH70	51.2 ± 0.9	49.9 ± 0.8	42.1 ± 7.39
dpMNL-PCA50	51.9 ± 0.7	49.9 ± 0.8	27.4 ± 2.08
linear-iSVM	53.2 ± 0.4	51.3 ± 0.4	0.22 ± 0.01
RBF-iSVM	$\mathbf{54.2} \pm 0.5$	$\mathbf{51.6} \pm 0.7$	6.67 ± 0.05

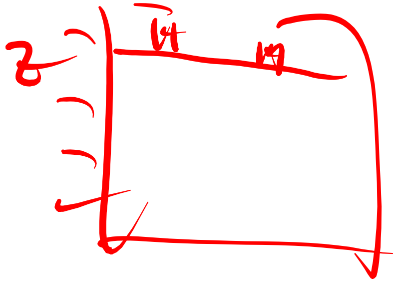
For training, linear-iSVM is very efficient (~200s); RBF-iSVM is much slower, but can be significantly improved using efficient kernel methods (Rahimi & Recht, 2007; Fine & Scheinberg, 2001)

- Clusters:
  - simiar backgroud images group
  - a cluster has fewer categories



Learning Latent Features

- Infinite SVM is a Bayesian nonparametric latent class model
  - discover clustering structures
  - each data point is assigned to a single cluster/class



- Infinite Latent SVM is a Bayesian nonparametric latent feature/factor model
  - discover latent factors
  - each data point is mapped to a set (can be infinite) of latent factors

  - Latent factor analysis is a key technique in many fields; Popular models are FA, PCA, ICA, NMF, LSI, etc.

Infinite Latent SVM

- RegBayes framework:

$$\min_{p(\mathcal{M}),\xi} \quad \mathrm{KL}(p(\mathcal{M})\|\pi(\mathcal{M})) - \sum_{n=1}^{N}\int \log p(\mathbf{x}_n|\mathcal{M})p(\mathcal{M})d\mathcal{M} + U(\xi)$$
$$\text{s.t.}: \quad p(\mathcal{M}) \in \mathcal{P}_{\text{post}}(\xi),$$

convex function

direct and rich constraints on posterior distribution

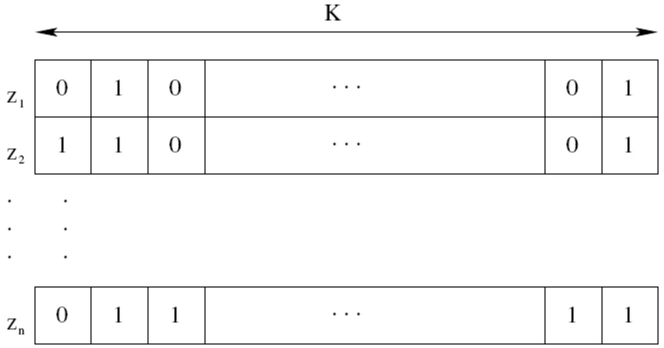
- Model – latent feature model
- Prior – Indian Buffet process
- Likelihood – Gaussian likelihood
- Posterior constraints – max-margin constraints

Beta-Bernoulli Latent Feature Model

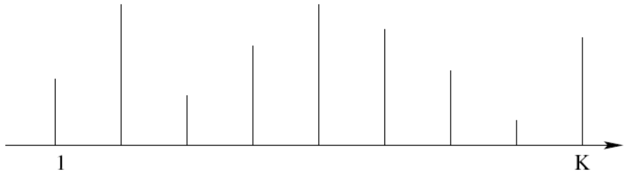
- A random finite binary latent feature models

$$\pi_k | \alpha \sim \mathrm{Beta}(\frac{\alpha}{K}, 1)$$

$$z_{ik} | \pi_k \sim \mathrm{Bernoulli}(\pi_k)$$



  - π_k is the relative probability of each feature being on, e.g.,

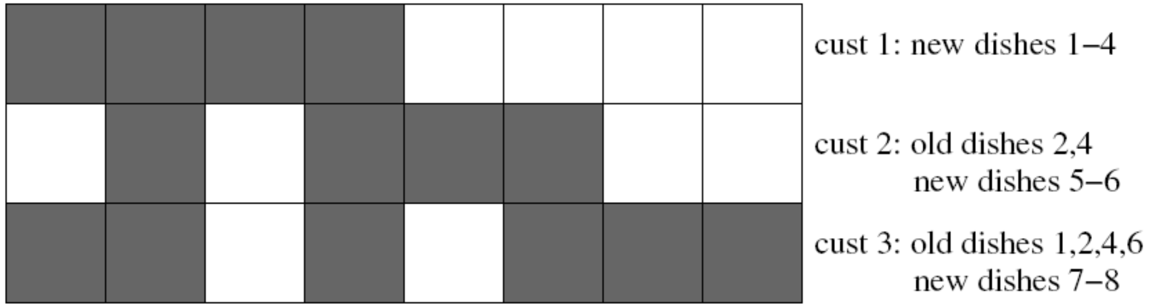


  - $z_{i\cdot}$ are binary vectors, giving the latent structure that's used to generate the data, e.g.,

$$\mathbf{x}_i \sim \mathcal{N}(\eta^\top z_{i\cdot}, \delta^2)$$

Indian Buffet Process

- A stochastic process on infinite binary feature matrices
- Generative procedure:
  - Customer 1 chooses the first K_1 dishes: $K_1 \sim \mathrm{Poisson}(\alpha)$
  - Customer *i* chooses:
    - Each of the existing dishes with probability $\frac{m_k}{i}$
    - K_i additional dishes, where $K_i \sim \mathrm{Poisson}(\frac{\alpha}{i})$



$$Z_{i\cdot} \sim \mathcal{IBP}(\alpha)$$

Posterior Constraints – classification

- Suppose latent features **z** are given, we define *latent discriminant function*:

$$f(y, \mathbf{x}, \mathbf{z}; \boldsymbol{\eta}) = \boldsymbol{\eta}^\top \mathbf{g}(y, \mathbf{x}, \mathbf{z})$$

- Define *effective discriminant function* (reduce uncertainty):

$$f(y, \mathbf{x}; p(\mathbf{Z}, \boldsymbol{\eta})) = \mathbb{E}_{p(\mathbf{Z}, \boldsymbol{\eta})}[f(y, \mathbf{x}, \mathbf{z}; \boldsymbol{\eta})] = \mathbb{E}_{p(\mathbf{Z}, \boldsymbol{\eta})}[\boldsymbol{\eta}^\top \mathbf{g}(y, \mathbf{x}, \mathbf{z})]$$

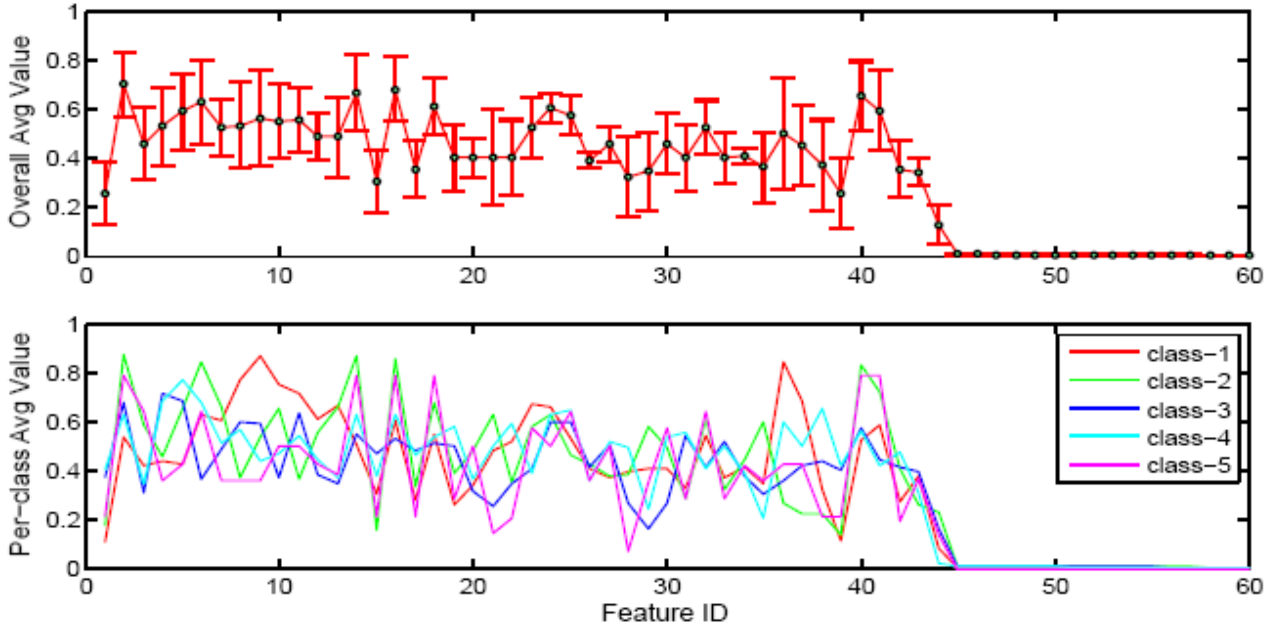
- Posterior constraints with max-margin principle

$$\forall n \in \mathcal{I}_{\mathrm{tr}}, \forall y: \quad f(y_n, \mathbf{x}_n; p(\mathbf{Z}, \boldsymbol{\eta})) - f(y, \mathbf{x}_n; p(\mathbf{Z}, \boldsymbol{\eta})) \geq \ell(y, y_n) - \xi_n$$

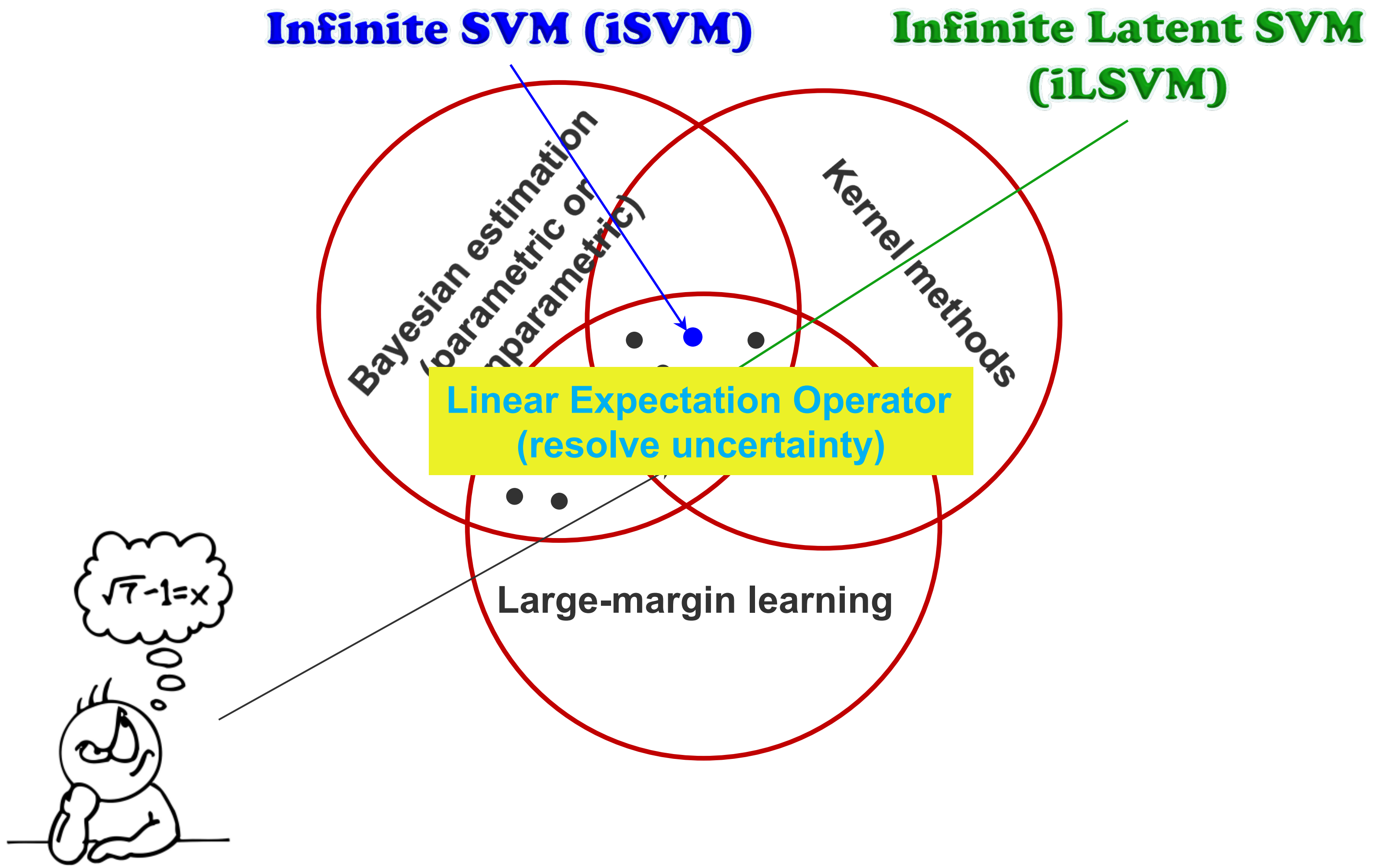
Experimental Results

- Classification
  - Accuracy and F1 scores on TRECVID2003 and Flickr image datasets

Model	TRECVID2003 Accuracy	TRECVID2003 F1 score	Flickr Accuracy	Flickr F1 score
EFH+SVM	0.565 ± 0.0	0.427 ± 0.0	0.476 ± 0.0	0.461 ± 0.0
MMH	$\mathbf{0.566} \pm 0.0$	0.430 ± 0.0	$\mathbf{0.538} \pm 0.0$	$\mathbf{0.512} \pm 0.0$
IBP+SVM	0.553 ± 0.013	0.397 ± 0.030	0.500 ± 0.004	0.477 ± 0.009
iLSVM	0.563 ± 0.010	$\mathbf{0.448} \pm 0.011$	0.533 ± 0.005	0.510 ± 0.010



Summary



Infinite SVM (iSVM)
Infinite Latent SVM (iLSVM)
Bayesian estimation
(parametric or
nparametric)
Kernel methods
Linear Expectation Operator
(resolve uncertainty)
Large-margin learning
√7-1=x

Summary

- A general framework of MaxEnDNet for learning structured input/output models
  - Subsumes the standard M^3Ns
  - Model averaging: PAC-Bayes theoretical error bound
  - Entropic regularization: sparse M^3Ns
  - Generative + discriminative: latent variables, semi-supervised learning on partially labeled data, fast inference
  - PoMEN
    - Provides an elegant approach to incorporate latent variables and structures under max-margin framework
    - Enable Learning arbitrary graphical models discriminatively
- Predictive Latent Subspace Learning
  - MedLDA for text topic learning
  - Med total scene model for image understanding
  - Med latent MNs for multi-view inference
- Bayesian nonparametrics meets max-margin learning
- Experimental results show the advantages of max-margin learning over likelihood methods in EVERY case.