Homework 7 - November 3, 2016. Questions.

Intermediate Statistics (Carnegie Mellon University)

Homework 7
36-705
Due: Thursday November 3 by 3:00

1. Let $X_1, \ldots, X_n \sim N(\mu, \sigma^2)$. Let $\theta = \mu/\sigma$. Construct an asymptotic $1-\alpha$ confidence interval for θ.

2. Suppose that $X_1, \ldots, X_n \sim N(\mu, \Sigma)$ are multivariate Normal, where $X_i \in \mathbb{R}^k$. Assume that Σ is known. Let
$$C_n = \Big\{\mu : \ (\overline{X}_n - \mu)^T \Sigma^{-1} (\overline{X}_n - \mu) \leq t\Big\}.$$
Find t so that C_n is a $1-\alpha$ confidence set for μ.
Hint: We can write $X_i = \mu + \Sigma^{1/2}\epsilon_i$ where $\epsilon_i \sim N(0, I)$ and I is the $k \times k$ identity matrix.

3. Let $X_1, \ldots, X_n \sim F$. Recall that the empirical cdf is
$$F_n(t) = \frac{1}{n}\sum_{i=1}^n I(X_i \leq t).$$
Suppose that Let $t_1 < t_2 < \cdots < t_k$ be k fixed points on the real line. Let
$$Z_n = \sqrt{n}\Big(F_n(t_1) - F(t_1), \ldots, F_n(t_k) - F(t_k)\Big).$$
Show that
$$Z_n \rightsquigarrow N(\mathbf{0}, \Sigma)$$
where $\mathbf{0} = (0, \ldots, 0)$ and Σ is a $k \times k$ matrix with $\Sigma_{jk} = F(t_j \wedge t_k) - F(t_j)F(t_k)$.

4. Let $X_1, \ldots, X_n \sim \text{Exponential}(\theta)$. Find the size α, asymptotic, LRT test for
$$H_0 : \theta = \theta_0 \quad \text{versus} \quad H_1 : \theta \neq \theta_0.$$
By inverting the test, construct an asymptotic $1-\alpha$ confidence set for θ. Now construct the $1-\alpha$ Wald confidence interval for θ.

5. Let $X_1, \ldots, X_n \sim \text{Possion}(\lambda_1)$ and let $Y_1, \ldots, Y_m \sim \text{Possion}(\lambda_2)$. Find an asymptotic $1-\alpha$ confidence interval for $\lambda_1 - \lambda_2$.