GOD'S ACTION IN THE WORLD: THE RELEVANCE OF QUANTUM MECHANICS

by Peter E. Hodgson

Abstract. It has been suggested that God can act on the world by operating within the limits set by Heisenberg's uncertainty principle (HUP) without violating the laws of nature. This requires nature to be intrinsically indeterministic. However, according to the statistical interpretation the quantum mechanical wavefunction represents the average behavior of an ensemble of similar systems and not that of a single system. The HUP thus refers to a relation between the spreads of possible values of position and momentum and so is consistent with a fully deterministic world. This statistical interpretation of quantum mechanics is supported by reference to actual measurements, resolves the quantum paradoxes, and stimulates further research. If this interpretation is accepted, quantum mechanics is irrelevant to the question of God's action in the world.

Keywords: God's action; Heisenberg uncertainty principle; statistical interpretation of quantum mechanics.

It is a basic Christian belief that we are all totally dependent on God, that God cares for us and guides our lives. And yet, according to the scientific account, the world is like a vast machine behaving in a deterministic manner following mathematical equations. If we maintain that God gave matter its properties and started its motions on the day of creation, this implies that it continues to act strictly in accord with these laws. How then can we also believe that God acts upon the world?

It has been suggested that this question can be answered by considering the properties of the quantum world (Pollard 1958, 139; Polkinghorne 1988, 333; Russell 1988, 343; Tracy 1995, 289; Murphy 1995, 325; Ellis

Peter E. Hodgson is Head of the Nuclear Physics Theoretical Group of the Nuclear Physics Laboratory, University of Oxford, and Emeritus Fellow of Corpus Christi College. He edited *Science and Religion Forum Reviews* from 1982 to 1998.

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1995, 359; Clayton 1997, 194).[1] Quantum mechanics, it is argued, has shown that the microworld is essentially indeterministic and so this provides the means whereby God, by acting within the limits of quantum uncertainty, can affect the world without violating the laws of physics. In this way we can maintain the rule of scientific law and also allow God to act freely.

This proposal raises a number of questions that deserve critical attention. To determine whether they are sufficient to achieve their object, we must first ask if the minute interventions constrained within the limits of the uncertainty principle are able to produce macroscopic effects. We need also to confirm that material reality is indeed an indeterministic system, and this raises the question of the relation between quantum mechanics and reality. In particular, does quantum mechanics give a complete or an incomplete account of the quantum world? We can also ask if this conception of God's action is consistent with our existing knowledge of divine intervention. Are there instances where other laws of nature are violated?

It is useful to recall that the history of the interpretations of quantum mechanics could well have been very different. Soon after the formulation of quantum mechanics, Louis de Broglie proposed the deterministic pilot wave theory, but he soon abandoned it following criticisms by Wolfgang Pauli, which were long afterwards shown by David Bohm to be unfounded (Cushing 1994, chap. 9). The difficulties of interpreting quantum mechanics were then avoided by the Copenhagen interpretation, at the expense of introducing the quantum paradoxes. Because of the persuasiveness of Niels Bohr, the Copenhagen interpretation was generally accepted and is now found in most textbooks and in the popular literature—and as a result it has been uncritically accepted by most physicists. It has however been strongly criticized by philosophers of science, and many books have been devoted to alternative deterministic interpretations (Bohm 1952; 1957; 1980; de Broglie 1954; Belinfante 1973; Bell 1987; Holland 1993). One of these could well have been accepted long ago, and then no one would have claimed that quantum mechanics provides evidence for radical indeterminacy of the world (Cushing 1994, chap. 10).

CHAOS THEORY

The uncertainty principle, as it is usually understood, sets very tight limits on the results of measurements, so we can ask whether such minute interventions are adequate to produce the macroscopic effects implied by God's action in the world. We could imagine God making billions of such minute interventions so that eventually they produce macroscopic effects (Clayton 1997, 194), though whether this is consistent with the omnipotence and dignity of God is another question. This may, however, be unnecessary given the effects studied in chaos theory (Russell, Murphy, and Peacocke 1995). It is well known that even in classical systems very small changes in the initial conditions often produce very different subsequent behavior. For example, a minute change in the trajectory of a gas molecule greatly affects the dynamics of a collision, and this is magnified in subsequent collisions. More picturesquely, it is referred to as the "butterfly effect" in climate predictions. Such effects have been studied in recent years because computers provide the means to make the lengthy calculations that are required. If we assume that God could foresee the ultimate effects of divine intervention at the quantum level, then a minute intervention could indeed produce a macroscopic effect. Furthermore, in certain circumstances a single quantum event can produce a macroscopic effect, as in the case of Schrödinger's cat.

Quantum Mechanics and the Material World

Quantum mechanics is a very successful theory and describes a wide range of phenomena in great detail and to a high degree of accuracy. In many cases it gives us a good understanding of what is taking place. However, we may be so impressed by the success of quantum mechanics that we overlook its defects. Its results are expressed in terms of probabilities. Thus, we cannot calculate in which direction a particle will scatter or when a particular nucleus will decay. Quantum mechanics is therefore incomplete. The question is whether this incompleteness is irreducible or whether there will eventually be a more fundamental theory that gives a more detailed account of reality.

There is another sense in which most physical theories are incomplete, especially quantum mechanics. We can describe some aspects of a process very well, but do we really understand what is happening and why? Is it sufficient to be able to calculate the results of measurements, or do we seek something more? We can describe pair production, but we do not understand how or why it happens as it does. This remark applies also to classical mechanics; we can describe gravitational phenomena extremely well, but we do not really understand them.

The first question to be answered is therefore whether the incompleteness of our understanding is compatible with the conclusions that are drawn.

Bohr maintained that quantum mechanics is the final theory, the end of the road for physics, and that no improvement is possible. It is "the last, the final, the never-to-be-surpassed revolution in physics," a claim described by Karl Popper as outrageous (1982, 6). The wavefunction of a system contains all that can possibly be known about it. Albert Einstein, however, maintained that quantum mechanics is but one step on the road to a full understanding of the world. In this view, the wavefunction gives the average behavior of an ensemble of similar systems.

This is illustrated by some examples in the following sections.

The Heisenberg Uncertainty Principle

The Heisenberg uncertainty principle is usually understood to say that, when we consider a fundamental particle such as an electron, we cannot simultaneously measure both its position and its momentum to any desired accuracy. The more accurately we determine the one, the less we know about the other. More precisely, the product of the uncertainties in position and momentum is always greater than Planck's constant divided by 4π.

This may be illustrated by the passage of a collimated beam of mono-energetic electrons through a narrow slit. If they fall on a screen after passing through the slit, we find that they fan out to produce the familiar diffraction pattern. If we consider motion in a direction perpendicular to that of the incident beam, then the uncertainty in position corresponds to the width of the slit, and the uncertainty in momentum is given by the angular spread of the electrons after passing through the slit. If we examine these uncertainties, we find that they indeed satisfy Heisenberg's uncertainty principle.

If, however, we consider a single electron emitted at a particular angle, we find a different result. We can allow this electron to hit a screen and thus determine its momentum in the direction perpendicular to that of the incident beam to a much higher accuracy than is allowed by the uncertainty principle. The technical details of this may be found in an article by Leslie Ballentine (1970; see also Popper 1982, 62).

This illustrates the essential point that quantum mechanics describes the statistical behavior of a large number of particles, not the behavior of a single particle. Heisenberg's uncertainty principle thus gives the relation between the spreads in position and momenta of a large number of particles. It does not mean that we cannot measure the position and momentum of a single particle with higher accuracy. We cannot, however, predict which way an electron will go, although it is conceivable that a more developed theory will allow this to be done. At present this seems unlikely, but it has happened many times in the history of science that what seemed impossible at one time became a familiar achievement. Bohr's Copenhagen interpretation would prevent us from even trying to find a new theory, while that of Einstein leaves the door open to future advances. There are several possibilities, and the recent work on stochastic electrodynamics (de la Pena and Cetto 1996) is particularly promising.

Quantum mechanics is thus essentially a statistical theory that describes the average behavior of a large number of similar systems but not the behavior of each individual system. In some ways this is like the distinction between actuarial statements about populations, which are statistical, and the behavior of a particular individual, which is not.

Concerning our ability to calculate the position and momentum of an electron to an accuracy far greater than that implied by the uncertainty principle, Werner Heisenberg said that "this knowledge of the past is of a purely speculative character, since it can never be used as the initial condition in any calculation of the future progress of the electron." Thus "it is a matter of personal belief whether such a calculation concerning the past history of the electron can be assigned any physical reality or not" (quoted in Popper 1982, 62). Popper, however, denied that it is a matter of personal belief, because

> to question whether the so ascertained "past history of the electron can be ascribed any physical reality or not" is to question the significance of an indispensable standard method of measurement (retrodictive, of course); indispensable, specially, for quantum physics. . . . But, once we ascribe physical reality to measurements for which, as Heisenberg admits, $\Delta x \Delta p << h$, the whole situation changes completely: for now there can be no question whether, according to the quantum theory, *an electron can 'have' a precise position and momentum. It can*. (Popper 1982, 63)

This fact has been continually denied by the supporters of the Copenhagen interpretation.

Quite generally, our inability to measure any physical quantity with unlimited accuracy does not imply that it does not have a precise value, unless, of course, one believes that reality can be attached only to the results of a measurement. Such a positivistic view not only has been thoroughly discredited philosophically but also is inimical to science.

Thus physics gives us no grounds for saying that the position and momentum of a particle are unknowable within the limits of the uncertainty principle, and still less for saying that it does not have position and momentum. Indeed, the uncertainty principle is perfectly compatible with each electron moving along a definite trajectory determined by forces in the vicinity of the slit that we are as yet unable to calculate or measure. The same type of explanation applies to the other paradoxes connected with the interference at two slits and with the Bell inequalities.

Radioactive Decay

Radioactive decay is often cited as an example of the statistical nature of reality. We can calculate the probability of decay per unit time but not the actual instant of decay. If this is combined with the belief that quantum mechanics gives a complete account of reality, then we must conclude that radioactive decay provides an example of an uncaused event. If, however, we do not accept this view of quantum mechanics, then we can say that, indeed, each decay has a cause that we do not yet know. There are many possibilities: maybe the decay happens when the motions of the constituent nucleons reach a suitable configuration, or perhaps it is due to some external influence. These are possibilities that could provide a subject for future research. Certainly, radioactive decay cannot be proved to be uncaused.

The Double Slit Experiment and Wave-Particle Duality

The wave-particle duality of fundamental particles is often cited as an example of the mysterious and counterintuitive nature of the quantum world. In the double slit experiment the electrons seem to behave like waves when they pass through the slits and like particles when they impinge on the screen. The observation of an interference pattern raises the question, How can an interference pattern be formed if the electron is a particle that goes through only one slit? According to the Copenhagen interpretation this question is meaningless and so must not even be asked. B. L. Bransden and C. J. Joachain (1989) conclude that "the particle is not localised before it is detected, and hence must be considered as having passed through both slits" (p. 53). And yet we know that an electron behaves like a point particle down to a very small distance. The problem seems to be insuperable and, indeed, contrary to the laws of logic. A detailed analysis by Arthur Fine led him to the conclusion that it is necessary to abandon the distributive law of logic (Fine 1972, 3). If this were the case, it would indeed be an example of a philosophical implication of modern physics, but closer analysis shows that this argument is invalid (Brody 1993).

According to the Einstein interpretation, the electron, being a pointlike particle, goes through only one of the slits, and its trajectory is influenced, as in the single-slit case, by its incident direction and by where it passes through that slit. But how can we explain the observed interference? If the electron passes through one slit, how is its motion affected by whether the other slit is open or not? This is because the electron interacts with the whole system, and it is the field that tells it, while it is going through one slit, whether the other is open or not (Popper 1982, 59). It is then possible to give a consistent account of the observations.

The wave-particle duality is thus simply a category confusion. On the one hand we have particles moving along definite trajectories with definite momenta, and on the other we recognize that, due to their interactions with the slits and with other matter and radiation, these trajectories have a certain probability distribution calculable from Schrödinger's equation.

Hidden Variables

The Copenhagen interpretation avoids the quantum paradoxes by concentrating on the observables and dismissing any questions about the underlying reality as meaningless. Thus, according to Heisenberg, "objective reality has evaporated" (1958, 95). Normally, however, a physicist faced with a phenomenon he does not understand will try to postulate some hidden mechanism to render it intelligible. The Copenhagen interpretation explicitly rejects any such "hidden variables." The impossibility of hidden variables was proved mathematically by John von Neumann in 1932 (von Neumann 1955), and this greatly strengthened the Copenhagen interpretation. What he actually proved was that, on the basis of some general assumptions, dispersionless ensembles cannot be incorporated into the formal structure of quantum mechanics. This leaves open the question, Are these assumptions unduly restrictive? and also the more fundamental question, Is quantum mechanics a complete account of reality?

Von Neumann's proof was believed for many decades to exclude the possibility of hidden variables. The situation changed when Bohm in 1952 succeeded in constructing a hidden variable theory (Bohm 1952, 166, 186), and although this was in some respects unappealing and unfruitful, it nevertheless showed that there was something wrong with von Neumann's proof. Subsequently, John Bell identified the key assumption in von Neumann's proof, namely that "any real linear combination of any two Hermitian operators represents an observable and the same linear combination of expectation values is the expectation value of the combination" (Bell 1966, 448). This is true for quantum-mechanical states, and von Neumann reasonably assumed that it is true of the hypothetical dispersion-free states. However, Bell showed by a single counterexample, namely, the measurement of the two spin orientations σ_x and σ_y, that this assumption is false. J. M. Jauch and C. Pirion (1963) proposed a new version of von Neumann's argument, but Bell showed that it is subject to the same objection.

The next development was A. M. Gleason's work (1957) on the axiomatic basis of quantum mechanics, which apparently yielded von Neumann's result, without any assumptions about noncommuting operators. However, Bell showed that Gleason assumed that "the measurement of an observable must yield the same value independently of what measurements may be made simultaneously" (Bell 1966, 451). Thus the whole experimental arrangement must be considered. The implication of this is that "the implicit assumption of the impossibility proof was essential to its conclusion" (Bell 1966, 451). There is thus no reason to believe that hidden variables are excluded and with them a fully determined theory of quantum mechanics.

Deterministic Theories of Quantum Mechanics

Although my remarks do not depend on the success of a deterministic theory of quantum mechanics, it is nevertheless useful to mention briefly some of the work that is in progress. We can distinguish between interpretations of the existing quantum mechanical formalism and new formalisms. I have already shown that the statistical interpretation clarifies the existing formalism and shows how the quantum paradoxes may be resolved. New formalisms in terms of hidden variables will be accepted by physicists only if they give predictions differing from those of quantum mechanics. Even if this is never achieved, it is still a great gain to have conceptual clarity together with a physical explanation in place of the positivistic obscurity of the Copenhagen interpretation.

Among the most promising of the deterministic theories are the pilot wave theory (de Broglie 1954; 1993; Belinfante 1973; Bell 1987; Holland 1993) and stochastic electrodynamics (de la Pena and Cetto 1996), and these will be briefly described. There are many others in various stages of development (see Goldstein 1998).

The pilot wave theory was put forward by Max Born in 1926 and further developed by de Broglie in his paper presented to the Solvay Conference in 1927 (Born 1926; Cushing 1994; de Broglie 1928). It was strongly criticized by Pauli, and Born and de Broglie abandoned it. Subsequently, Bohm showed how Pauli's criticisms could be answered; if this had been done at the time, quantum mechanics might have developed in a quite different way. According to this theory, a particle is guided by a pilot wave that acts on it through the quantum potential. If the initial position of the particle and the form of the wavefunction are specified, the equations may be solved to give the subsequent motion of the particle. Many calculations have been made with this theory, including the particle trajectories in the double slit experiment (Philippidis, Dewdney, and Hiley 1979). The formalism can be extended to include particles with spin and then accounts for all the fundamental features of nonrelativistic quantum mechanics (Bohm and Hiley 1993).

Stochastic electrodynamics recognizes that every physical system is subject to the influence of its surroundings through the medium of gravitational and electromagnetic forces and through bombardment by photons, electrons, neutrinos, and other particles. In classical physics we argue that most of these influences are irrelevant to the variables of interest, or average out, or can be allowed for, and this allows us to consider a system in isolation and to set up its equations of motion. Another possibility is to include some of the outside influences explicitly and to average over them subsequently; this is what we do in statistical physics. If neither of these methods were practicable, science would be impossible.

Quantum mechanics is an attempt to formulate the physics of a closed system (that is, to assume that all external influences are negligible) in a situation where the external influences are in fact not negligible. This is the root of the difficulties with quantum mechanics, and to solve the difficulties it is necessary to identify these external influences and to take them into account.

Stochastic electrodynamics identifies the external influence on quantum systems as the fluctuating background electromagnetic field. Any charged particle experiences this field as a result of the motions of all other accelerated charged particles. This field must account for the stability of atomic structures, because they would rapidly collapse if electrons are classical particles moving in the absence of any external background.

If we consider the motion of an electron, taking into account the possibilities of emission and absorption, its equation of motion is the Newtonian one with the addition of emission and absorption terms, giving the Braffort-Marshall equation. Unfortunately, this equation is difficult to solve. However, the corresponding harmonic oscillator problem has been solved and gives the same results as quantum mechanics does. Agreement is also found for the widths of the absorption and emission lines and for the case when a magnetic field is applied to the harmonic oscillator.

The stochastic theory thus provides a conceptually simple way of tackling a problem that, like quantum mechanics, has only one undetermined constant, that of Planck. It is unfortunately, and perhaps not surprisingly, complicated, so that only a few simple cases can be worked out. In this respect it is opposite to quantum mechanics, which is conceptually difficult but computationally elegant, allowing a large range of problems to be solved. Furthermore, stochastic electrodynamics is an open theory, in that further developments are very likely that will allow it to be extended to solve a range of problems.

The pilot wave theory and stochastic electrodynamics provide just two of many possible realist theories of quantum mechanics. There are several others, including decoherent histories and spontaneous localization. Thus,

> the Bohr-Einstein debate has already been resolved, and in favour of Einstein: what Einstein desired and Bohr deemed impossible—an observer-free formulation of quantum mechanics, in which the process of measurement can be analysed in terms of more fundamental concepts—does, in fact, exist. (Goldstein 1998 [March], 43)

Importance for Physics

It may well be thought that questions of the interpretation of quantum mechanics are all very well for those with a liking for such discussions but are not the concern of the practical physicist. There are several reasons why this is not so.

The first is the need for conceptual clarity. Many students of quantum mechanics are repelled by the quantum paradoxes. They cannot imagine an entity that is both a wave and a particle, they cannot understand the tunnels in alpha decay, and they are baffled when they are told that it is meaningless to ask which slit the electron went through in the double slit experiment. It is not unknown for students to be put off physics entirely by such talk.

Second, declaring that no further advance is possible and that certain questions must not be asked prevents all further progress. A notable example of this occurred when Ernest Rutherford was trying to find the structure of the nucleus. Bohr told him that the interior of the nucleus was just a structureless soup and so it was meaningless to try to find out about it. Rutherford was discouraged by this and abandoned his attempts (Wilson 1984). A decade or so later, evidence was found for the shell structure of the nucleus, thus confirming Rutherford's original intuition. As Popper has remarked:

> The metaphysical belief in causality seems more fertile in its various manifestations than any indeterminist metaphysics of the kind advocated by Heisenberg. Indeed we can see that Heisenberg's comments have had a crippling effect on research. Connections which are not far to seek may easily be overlooked if it is continually repeated that the search for such connections is "meaningless." (1959, 248)

The effects of this discouragement are still strong today.

The overwhelming majority of physicists, particularly those who struggle daily in the laboratory, instinctively reject these debilitating beliefs and continue to believe, in the words of Einstein, that "something deeply hidden had to be behind things" (Schilpp 1949, 9). To pursue scientific research within the framework of the opposing belief—that all we are doing is trying to correlate our sense-impressions—is to cut us off from the source of scientific creativity.

It is sometimes said that only Einstein in his old age supported the idea of a deterministic substratum to quantum mechanics, so it should be mentioned that he was not the only physicist to oppose the Copenhagen interpretation. Among others one may mention Planck, de Broglie, von Laue, Schrödinger, Dirac, Fermi, Feynman, and Bohm—hardly a negligible group.

Theological Reflections

The foregoing has shown that the success of quantum mechanics does not imply that the world is indeterminate and so does not provide the means whereby God can intervene. Even if it did provide those means, they would not be able to account for all recorded interventions, since they violate other physical principles. For example, the feeding of the five thousand is contrary to the law of the conservation of matter, and so are several miracles of healing in recent times. It is an impoverished conception of God to suppose that he is bound by his own laws. God is the supreme lord of nature, who can make and unmake its laws and bring it into being, modify it, or extinguish it at will. It is unnecessary to think of God trying to change the course of events by keeping within the limits of quantum indeterminacy.[2]

Notes

1. "The clear and determinate character of physical processes, as Sir Isaac understood it, has dissolved at its constituent roots into the cloudy and fitful quantum world" (Polkinghorne 1988, 333). "The overwhelming impression one gets from quantum physics is of the irreducibly statistical character of experience" (Russell 1988, 343).
2. Polkinghorne (1988, 339) pertinently asks whether "God is the ultimate Hidden Variable, skilfully exercising his room to maneuver at the rickety constituent roots of the world, whilst cleverly respecting the statistical regularity which his faithfulness imposes?"

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