COSMOLOGICAL SINGULARITY AND THE CREATION OF THE UNIVERSE

by Michael Heller

Abstract. One of the most important and most frequently discussed theological problems related to cosmology is the creation problem. Unfortunately, it is usually considered in a context of a rather simplistic understanding of the initial singularity (often referred to as the Big Bang). This review of the initial singularity problem considers its evolution in twentieth-century cosmology and develops methodological rules of its theological (and philosophical) interpretations. The recent work on the "noncommutative structure of singularities" suggests that on the fundamental level (below the Planck's scale) the concepts of space, time, and localization are meaningless and that there is no distinction between singular and nonsingular states of the universe. In spite of the fact that at this level there is no time, one can meaningfully speak about dynamics, albeit in a generalized sense. Space, time, and singularities appear only in the transition process to the macroscopic physics. This idea, explored here in more detail, clearly favors an atemporal understanding of creation.

Keywords: atemporality; Big Bang; creation of the universe; singularities; temporality.

It would be difficult to find a popular book or an article on cosmology in which the author says nothing about the Big Bang and the creation of the universe. It would be even harder to find a book or an article in which this problem was dealt with in a responsible manner from the point of view of both science and theology. The goal of this article is to improve the statistics in this respect. However, this purpose cannot be achieved by repeating

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commonplace statements about the Big Bang as being a point with an infinite matter density at which the universe and time began. One must go a little bit deeper into a mathematical definition of the *initial singularity* (a geometric counterpart of the Big Bang) and the conditions of its existence, for only then can one correctly decipher its physical content and its philosophical (or theological) significance. We shall examine this question by starting with the first appearance of the singularity problem in twentieth-century cosmology and ending with the most recent results concerning the geometric nature of singularities.

EARLY DISPUTES

It is interesting to notice that the cosmological singularity started making trouble in our science of the universe even before it had been formally discovered. Its appearance was acknowledged not earlier than in the work by Alexander A. Friedman in 1922. It should be noted, however, that in his first paper on cosmology in 1917, Albert Einstein had met with the same difficulty that is responsible for the singular behavior of the universe in certain of its states. The problem at stake is that of gravitational instability, the same problem with which Isaac Newton had to cope when he pondered why "Matter evenly scattered through a finite Space would not convene in the midst" (Cohen 1958, 292). He gave as his opinion

> that there should be a central Particle, so accurately placed in the middle, as to be always equally attracted on all Sides, and thereby continue without Motion, seems to be a Supposition fully as hard as to make the sharpest Needle stand upright on its Point upon a Looking-Glass. And if the question is asked with respect to an infinite universe it becomes even harder: For I reckon this is as hard as to make not one Needle only, but an infinite number of them (so many as there are Particles in an infinite Space) stand accurately poised upon their Points. (Cohen 1958, 292)

Initially Einstein himself (and many other cosmologists after him) believed that the appearances of singularities in the cosmological models considered at that time were by-products of the over-strong simplifying assumptions that were required to construct these models. The suspicion was that the so-called cosmological principle (the assumption of spatial homogeneity and isotropy of the universe) was responsible. The understanding that this is not the case and that the real cause lies in gravitational instability emerged rather laboriously. An important hint was given by Georges Lemaître (1933), who demonstrated that in a certain class of anisotropic world models the tendency toward the appearance of singularities is even greater than in isotropic ones; but the decisive step was made much later by Roger Penrose and Stephen W. Hawking (and some others), who proved several theorems on the existence of singularities (see Hawking and Ellis 1973). It turned out that to remove singularities from the theory of gravitation is "as hard as to make an infinite number of Needles stand accurately poised upon their Points" (Cohen 1958, 292). However, this does not mean that this cannot be done, as we shall see.

Long before this stage of the dispute ended, heated philosophical and even theological polemics had begun. Lemaître (1958) tells us that when he was discussing with Einstein the primeval-atom hypothesis, Einstein's reaction was: *Non, pas cela; cela suggere trop la création* (No, not this; this too much suggests the creation). From the very beginning philosophical views interfered with doing cosmology. Many authors shared Einstein's reluctance to accept any kind of beginning of the universe. Some people shared his views for good methodological reasons, but some others never tried to hide their antireligious attitude. For instance, in William Bonnor's view, theologians had long awaited such an occasion as a superdense state in the beginning of the universe, for now they could claim that the biblical account of creation is right, and bishop James Ussher was mistaken by only a few years (Bonnor 1964, 134).

In these discussions arguments taken from science were mixed with those of purely theological origin. For instance, Arthur E. Milne argued that

> the creation of the universe demanded creation at a point-singularity. For the creation by God of an extended universe would require an impossibility, the impossibility of the fixation of simultaneity in the void—impossibility, that is, to a rational God. The paradox follows that the Deity himself, though in principle all-powerful, is yet limited by his very rationality. (Milne 1952, 157)

The strategy of this argumentation is rather transparent. We have a scientific result (the point singularity) and a theological idea of God (as a rational Being), and we make a post hoc deduction of this scientific result from this theological idea (God's rationality implies the point singularity). Apart from a methodological incoherence of such strategies, at least two implicit dangers are always involved in them: First, the results of science are of their very nature provisional—and in cosmology even more so than in other physical disciplines because of the high degree of extrapolation always present in cosmological theories. In fact, later on it emerged, as we shall see, that classical singularities are not parts or elements of space-time, and that consequently such concepts as that of "point" (in its usual meaning) are not applicable to them. Second, any "deduction" of a scientific result from a theological premise can be only apparent (for instance, one can overlook some other important premises); and, if the idea of God as a rational Being is true, the deduction in Milne's argumentation is indeed only apparent, inasmuch as the point-singularity concept, being meaningless, is not a scientific result.

As we look at these early polemics from the present perspective, we can quite clearly see that they were based, perhaps with a very few exceptions, on two often tacit presuppositions, one purely scientific and another purely theological. The scientific presupposition asserted that singularities (in particular the initial singularity) can be removed from cosmological models. Even those who claimed that the initial singularity is a permanent element of our image of the universe (among them Milne and Lemaître) were unable to prove this claim. The theological presupposition consisted in identifying the beginning of the universe with its creation: if the universe had a beginning, it had to be created by God; and vice versa, if it was created by God, it had a beginning. This presupposition was made by both defenders and opponents of the theological concept of Creation. However, one should distinguish the theological meaning of creation from other meanings, as when this term is used by cosmologists as a synonym for physical origin or even for initial singularity. The confusion of these meanings and the linguistic carelessness of many cosmologists significantly contributed to various misunderstandings. For instance, how is one to understand the following statements made by Jayant Narlikar:

> At an epoch, which we may denote by t=0, the Universe explodes into existence. . . . The epoch t=0 is taken as the event of "creation." Prior to this there existed no Universe, no physical laws. Everything suddenly appeared at t=0. (Narlikar 1977, 125)

It is important not to overlook the quotation marks around "creation." To many theologians and philosophers who are not experts in physics, this could appear to be a true theological account of creation ("the Universe explodes into existence"!). Strictly speaking, however, the issue here is the technical problem of the violation of the principle of energy conservation, as could be guessed from the following:

> The most fundamental question in cosmology is, "Where did the matter we see around us originate in the first place?" This point has never been dealt with in big-bang cosmologies in which, at t=0, there occurs sudden and fantastic violation of the law of conservation of matter and energy. After t=0 there is no such violation. (Narlikar 1977, 136–37)

However, the suspicion arises that confusions of meanings, like the one above, could be intended, at least to some extent and at least by some authors. Science sells better if it is shown to conquer realms traditionally controlled by theology.

The first of the foregoing presuppositions, that singularities can be removed from cosmological models, was later made the subject matter of intensive studies. The quite unexpected results significantly changed the atmosphere of the Creation dispute. Unfortunately, the second presupposition, that beginning = creation, is still "on the market" and continues to cause confusion in discussions of the philosophical and theological implications of cosmology. These problems are explored in the subsequent sections.

Classical Beginning

An important breakthrough in discussions about the beginning of the universe took place in the sixties, when the singularity theorems were proved. They are of purely mathematical character but have a natural physical interpretation. First, one defines a model of space-time. Its intended goal is to describe space-times encountered in the special and general theories of relativity; but from the mathematical point of view, such a model refers to any space considered in differential geometry, provided it satisfies the required conditions. On such a space one imposes further conditions, which geometrically mimic properties of a gravitational field. Then, a long chain of mathematical deductions shows that the foregoing sets of conditions (together with another geometric condition, which can be interpreted as saying that the considered space-time is singularity-free) inevitably lead to a contradiction.

The key point is how to define *singularity* and what is meant by saying that a space-time is singularity-free. This is a big issue in relativistic physics, and, as we shall see, it leads to sophisticated mathematical problems. Happily enough, what is needed to prove the singularity theorems is not the true singularity definition but only a working criterion for a given space-time to be free of singularities. What becomes apparent is that such a criterion is provided by the so-called geodesic completeness of space-time (in the null and timelike sense). Geodesics are "the straightest curves" in a given space-time.[1] In the theory of relativity, null and timelike geodesics describe the free motion of photons and massive particles (or observers), respectively. The geometric concept of geodesic completeness represents the situation in which such motions could be indefinitely continued (in both time directions). This means, of course, that the history of any photon or particle will never cease to happen and, consequently, that space-time has no edges or singularities.

This is a good working criterion for a given space-time's being singularity free without the necessity of knowing the physical nature of singularities. However, by reversing the former reasoning, we could guess something about singularities themselves. If a (null or timelike) geodesic cannot be prolonged,[2] a certain history of a photon or particle must break down, and this occurs exactly because of the singularity (quite often it happens that not only one, but all histories, break down at a singularity). If a history cannot be prolonged, it ceases to happen; and what does it mean that the history of a particle ceases to happen? It means that this particle emerges out of nothingness, or disappears into nothingness. For instance, at the initial singularity in the Friedman world model, all histories of photons and particles emerge from nothingness, in this sense. This concept strongly resembles the theological one of creation out of nothing (*creatio ex nihilo*). In such a context the last paragraph of the well-known monograph on the singularity theorems is hardly surprising:

> The creation of the Universe out of nothing has been argued, indecisively, from early times; see for example Kant's first Antinomy of Pure Reason and comments on it. . . . The results we have obtained support the idea that the universe began a finite time ago. However, the actual point of creation, the singularity, is outside the scope of presently known laws of physics. (Hawking and Ellis 1973, 364)

However, we should not forget that conclusions of this sort are always model dependent. The model in question is provided by the geometric model of space-time, mathematical conditions of the singularity theorems, and their physical interpretation. The latter is given by the theory of general relativity which, as a classical theory, does not take into account quantum gravity effects. Because there are strong reasons to believe that these effects play the decisive role in the early, superdense states of the universe, the problem of the existence of the initial singularity crucially depends on the future theory of quantum gravity. This conclusion does not come from any trustworthy theory but rather from a combination of the present classical theory of gravity and various methods of quantizing physical fields. Based on such knowledge, we conclude that when the energy density approaches that of the Planck threshold[3] quantum gravity effects become dominant. The singularity theorems are proven mathematical theorems; as such, they will always remain true. We can only hope that the future theory of quantum gravity will violate one of the conditions appearing in these theorems and in this way free cosmology from singularities (the so-called energy conditions are the most frequent candidates for being broken down by quantum gravity effects). Taking all these issues into account, it would be premature to claim that the singularity theorems prove the beginning of the universe, let alone its creation.

Strong reasons also prevent one from identifying the initial singularity with the "moment of creation." The nothingness out of which the histories of particles or observers emerge has nothing in common with the "metaphysical nonbeing" of philosophers and theologians. The singularity theorems have been proven within the conceptual environment of the precisely defined model of space-time, and saying that some histories suddenly end at the final singularity only means that the curves representing these histories have reached the edges of the model. It is true that, in the case of the initial singularity, these histories emerge out of nothingness, but it is nothingness from the point of view of the model. The *nothingness*, in this sense, is only what the model *says nothing* about. What is outside the model, the model itself does not specify.

This is the kind of interpretation I am calling the "exegesis of the mathematical structure" of a given physical theory (Heller 1993). It is a minimum interpretation closely following the mathematical formulae constituting the body of the theory under interpretation. Everybody who understands these formulae and their functioning in the given theory must accept this interpretation. Of course, one may superimpose any interpretative comments on this theory as long as they do not contradict its mathematical structure. However, strictly speaking, the theory itself remains neutral with respect to such comments. One may even superimpose on the same theory some other comments, which contradict the previous ones (provided they do not contradict the mathematical structure of the theory).

For instance, in our case, one can claim that the emergence of all histories of particles and observers from the initial singularity in the Friedman world model should be understood as the creation of the universe by God from nothing (*ex nihilo*), or alternatively that the universe lasted from minus temporal infinity and that the initial singularity in the Friedman model is but a geometrical expression of the fact that all information from the previous cycle of the world's evolution has been lost (the universe has forgotten its presingularity past). Neither of these interpretative comments contradicts the mathematical structure of the model, and any serious discussion between these two philosophies should look for support in departments of human speculation other than cosmology. The well-known criticism by Adolf Grünbaum (1989; 1990) strikes at the theological doctrine of creation only if one forgets the above methodological analysis. It is not true, however, that the theologian as theologian has nothing to learn about creation from modern cosmological theories. This problem is discussed in subsequent sections.

Space-Time Boundaries

For the time being let us set aside theological interpretations and stick to the "exegesis of mathematical structures." The singularity theorems are not the last word in the problem of the beginning of the physical universe. The major question concerning the nature of singularities remains open.

In the singularity theorems singularities are understood as "end points" of curves representing histories of particles or photons. Can we be more precise about that? The end point itself of such a curve is inaccessible for our research, because precisely at this "point" our model breaks down. However, we can use here a trick often used in geometry. Because a given end point is determined by all curves that end at it, in all calculations we can treat interchangeably the end point and all curves that end at it. Or, simply, we can identify end points with such classes of curves. This means that by investigating the geometry of such classes of curves we are, in fact, investigating the set of end points; and this, of course, is done from inside our model of space-time. The set of all end points, in this sense, forms what is called the *singular boundary* of space-time.

In such a manner all singularities of a given space-time are represented as points of its singular boundary. If there are no singularities, the space-time has no singular boundary. In fact, the singularity theorems reveal themselves as theorems concerning the existence of singular boundaries. Notice, however, that although the boundary points of a given space-time are defined from within, they do not belong to the space-time itself (they do belong to the space-time boundary), and consequently it is meaningless to speak about them as of points in the usual sense. In fact, they can have a highly complicated structure which strongly depends on details of the boundary construction.

There are several known constructions of space-time boundaries. One of the first belongs to Hawking (1966) and Robert Geroch (1968) and is called *g-boundary*; *g* is here an abbreviation for "geodesic," and it means that in this boundary construction only the geodesic curves have been taken into account. In general relativity, timelike and null geodesic curves represent histories of freely falling particles (or observers) and photons, respectively.[4] The importance of the g-boundary of space-time stems from the fact that in the singularity theorems only the completeness (or incompleteness) of space-time with respect to geodesic curves is taken into account. Precisely this issue is the source of another interesting problem. In the universe there can exist particles the histories of which are not geodesic curves. For instance, a rocket moving with a bounded acceleration is a perfectly physical body[5] (although it is not "freely falling"), and the space-times exist which are geodesically complete but incomplete with respect to curves of bounded acceleration. To have a reasonable singularity definition, a criterion concerning such curves should be included in the definition.

A space-time boundary construction satisfying this requirement was proposed by Bernard Schmidt (1971). It is mathematically elegant and physically appealing. Schmidt does not consider directly space-time itself but rather a larger space of all possible local reference frames that can be defined in this space-time. This larger space is called a *frame bundle* (over space-time). It is very much in the spirit of relativity theory for which reference frames are "more real" than points in space-time. One can meaningfully speak about curves in the frame-bundle space, and the standard notion of length refers to them correctly. The boundary points of a given space-time are defined in terms of classes of the frame-bundle curves having finite lengths. The corresponding space-time boundary is called a *bundle boundary* or, for short, *b-boundary*, of space-time; and it takes into account both geodesics and other curves in space-time.

Shortly after its publication Schmidt's b-boundary came to be viewed as the best available description of singularities. It had, however, one serious drawback: to compute b-boundaries of more interesting (nontrivial) space-times effectively was extremely difficult. Only a few years later were B. Bosshardt (1976) and R. A. Johnson (1977) able to say something more concrete about the structure of the b-boundaries of such important cases as the closed Friedman world model and the Schwarzschild solution (describing a symmetric black hole)—and their results proved disastrous. It turned out that in both cases the corresponding b-boundary consisted of a single point. This looks especially pathological in the case of the closed Friedman universe, in which there are two singularities: the initial singularity and the final singularity. In the b-boundary construction they coalesce to a single point, that is, the beginning of the Friedman universe is simultaneously its end. Moreover, in both the closed Friedman and the Schwarzschild solutions, from the topological point of view the entire space-times together with their b-boundaries reduce to a single point.[6] Something is really going wrong.

There were many attempts to cure the situation but with no substantial effect. (For more details see, for instance, Dodson 1978; 1980.) During the next several years the beautiful, but now useless, b-boundary construction, almost forgotten, waited on library shelves for a better time.

Malicious Singularities and a Demiurge

The situation described in the preceding section suggests that perhaps overly coarse tools were used to deal with a very subtle object. Is there any possibility of finding a subtler tool? The standard way of dealing with spaces in differential geometry is by means of local coordinate systems, but it has been demonstrated by L. Koszul (1960) that one can *equivalently* develop differential geometry in terms of functions defined on a considered space. Later on, Geroch (1972) showed that this method also works, in principle, when it is applied to space-times of general relativity. By using it we obtain nothing essentially new (and this method is usually much harder than the standard one), but it can be quite naturally generalized. With some refinement of technology, functions can be defined on spaces that are not necessarily smooth (in the traditional sense)[7]—that is, on spaces that contain some sorts of singularities. The suitable geometric technology has been elaborated, and the corresponding spaces are known under the name of *differential* or *structured spaces*.[8]

Now it may be possible to reconstruct Schmidt's b-boundary in terms of structured spaces. The results of this procedure are encouraging. Milder kinds of singularities can be fully analyzed if this new method is employed. For instance, a space-time containing the so-called cone singularity (which models an infinitely long one-dimensional "cosmic string") can be regarded as a structured space (the singularity is a part of this space), and effectively studied with the help of the theory of these spaces. As far as stronger singularities are concerned, such as the ones in the closed Friedman and Schwarzschild solutions, the situation is much subtler. We recall that now space-time is modeled by a family of functions defined on it, and to have the full description of space-time this family must be sufficiently rich. In the case of space-times with strong singularities, the family of functions defining them contains only constant functions, far from enough for a satisfactory model of space-time (with singularities). Constant functions, in particular, do not distinguish points, because the value of such a function on each point is the same. This explains why the space-times of the closed Friedman model and the Schwarzschild solution together with their b-boundaries collapse to the single points. Singularities that produce such pathologies have been called *malicious singularities*.

It is possible to say even more. Consider the closed Friedman world model. As long as one deals with its space-time without taking into account its two singularities (the initial and the final ones), a sufficiently rich family of functions is defined on it,[9] and everything is as it should be. If, however, one tries to "extend" these functions to the singular boundary, all functions, with the exception of the constant ones, vanish, and everything collapses to a single point.

The following interpretative comment illustrates the situation.[10] For beings living inside the closed Friedman model, everything is all right. By studying cosmology they can learn about the existence of the initial singularity in their past, and they can predict the final singularity in their future. Neither of these singularities is directly accessible to them; however, they have learned about the singularities by collecting information from within space-time in which they live. If they had directly "touched" one of the singularities (tried to "extend" to them the corresponding family of functions), space-time with singularities would immediately have been reduced to a single point. Suppose further that the world under consideration has been created by a Demiurge in the initial singularity. To create the world, the Demiurge must "touch" the singularity (must deal only with constant functions), and therefore for the Demiurge the beginning of the world is simultaneously its end. Theologians always claimed that God is atemporal and therefore everything happens instantaneously for God.

Remember, however, that such metaphoric interpretations are good only as didactic tools illuminating some aspects of the model. The story is evidently not yet finished; what we need are better tools to deal with the malicious nature of strong singularities. For the time being, one could summarize the situation by noting that although the method of structured spaces has provided insight into the nature of the problem and explained the source of difficulties in which the b-boundary construction is involved, it is still not powerful enough to solve completely the problem of malicious singularities.

Pointless Spaces

Happily enough, still one generalization of geometry is possible. It starts with a simple step. Multiplication of functions has a simple property: the order of functions in their product is irrelevant. Suppose two functions, f and g (say), are defined on a space X, and we want to multiply them. We do this point by point. Let x be a point of X. First we compute $f(x)$—the value of the function f at the point x. Then we compute $g(x)$—the value of g at x. These two values, $f(x)$ and $g(x)$, are numbers. We multiply them in the usual way and treat the result as the value of the product $f{\cdot}g$ at the point x, as $(f{\cdot}g)(x)$. We can express this rule in the short formula $(f{\cdot}g)(x)=f(x){\cdot}g(x)$. And we repeat the same for all points of the considered space X. In the multiplication of numbers the order is irrelevant ($7{\cdot}3 = 3{\cdot}7$), and the same is true as far as multiplication of functions (defined above) is concerned. We express this by saying that multiplication of functions is *commutative* or that it satisfies the *axiom of commutativity*.

If, in the theory of differential spaces, we rejected the axiom of commutativity, we would obtain a new generalization of geometry, called *noncommutative geometry*. The problem is, however, that the multiplication of functions is always commutative (with the foregoing definition of multiplication); and to pass from commutative (that is, standard) geometry to noncommutative geometry we must change from functions to some other mathematical objects (such as matrices or operators), which are not multiplied "pointwise." (This would lead back to commutativity.) In other words, such objects are not permitted to "feel" points; and if we define a space with the help of such mathematical objects, this space turns out to be a pointless space, that is, a space in which the concept of point has no meaning. In fact, noncommutative spaces are, in principle, purely global constructs in which local notions (such as that of point and its neighborhood) cannot even be defined.

It came as a surprise that in spite of this "strange" property, differential geometry could be done in noncommutative spaces, albeit in a highly generalized sense. The seminal work of Alain Connes (1994) soon matured into a new field of research in mathematics and mathematical physics (see, for instance, Madore 1999).

Noncommutative geometry has two sources. One is evidently the standard differential geometry of which it is a vast generalization. It should be remembered that the tendency toward generalizing concepts and methods has always been a powerful driving force of mathematical progress. The second source of noncommutative geometry is none other than quantum mechanics. This fact might surprise an outsider, but it is well known to every physicist that in this physical theory observable magnitudes are represented by mathematical objects called *operators in a Hilbert space*, which multiply in a noncommutative way. In fact, the famous Heisenberg uncertainty principles are but the consequence of this noncommutativity. These properties of quantum mechanics were certainly an inspiration for the creators of noncommutative geometry. However, the real connections between noncommutative geometry and the mathematical structure of quantum mechanics go further than that. Every noncommutative space can be represented as a theory of operators in a Hilbert space. Roughly speaking, this means that every noncommutative space can be described in terms of some operators in a certain Hilbert space. One could even suspect that the "strange behavior" met in quantum mechanics (such as nonlocality's manifesting itself in the Einstein-Podolsky-Rosen experiment) is the consequence of this structural affinity of quantum mechanics with noncommutative geometry (which from its very nature is nonlocal).

To formulate a mathematical problem correctly, very often one must define a space in which this problem unfolds. The structure of this space does not depend on the mathematician's will but is implied by the nature of the problem. It happens quite often that the structure of the "space of the problem" is highly sophisticated—sometimes even pathological (from the point of view of standard geometrical methods). The aim of generalizing the usual (commutative) concept of space to the concept of noncommutative space was to find a tool for dealing with such pathological spaces. In fact, the new geometry efficiently deals even with such spaces with which the standard geometrical methods are hopelessly ineffective.[11]

As explained in the preceding sections, relativistic space-times with malicious singularities (such as the Big Bang singularity in the closed Friedman world model) are, from the geometric point of view, highly pathological spaces. Therefore, it seems natural to apply noncommutative methods to their analysis.

Noncommutative Structure of Singularities

Indeed, relativistic space-times with all kinds of singularities can be described as noncommutative spaces. All major difficulties met so far in their study disappear in the degree of generalization that leads to the very concept of noncommutative space.[12] As already shown, in this generalization process local concepts, such as those of points and their neighborhoods, become meaningless. In many situations they are replaced by the concept of *state*, a global notion. Even within the commutative context, if we speak about a state of a given system, we do not mean a "well localized" part of it but rather a certain characteristic that refers to the system as a whole. Our everyday language also conforms to this way of speaking. If, for example, we speak about the state of an enterprise, we mean by that certain of its global characteristics, such as the increasing production rate or general income. The same intuitions are incorporated into the concept of the state of a physical system.

If we change to the noncommutative description of a space-time with singularities, we lose the possibility of distinguishing points and their neighborhoods, but we can still meaningfully speak about states of the system. Here, however, all states of the universe are on an equal footing; there is no longer any distinction between singular and nonsingular states. Moreover, each state can be described in terms of operators in a certain Hilbert space in an analogous way, as is usually done in quantum mechanics.

With the use of this method, it was possible to prove several theorems characterizing various types of singularities, including malicious singularities, which occur in relativistic cosmology and astrophysics (Heller and Sasin 1999a). These theorems are important also because they disclose the way the singularities originate. As we already know, in the noncommutative regime the question of the existence of singularities is meaningless: we can speak only about the states of the universe, and there is no distinction between singular and nonsingular states. However, when we change from the noncommutative description of the universe to its usual (commutative) description, the ordinary space-time, with its points and neighborhoods, emerges, and some states degenerate into singularities.

This opens a new conceptual possibility. We could speculate that noncommutative geometry is not an artificial tool to use in coping with classical singularities in general relativity, but it somehow reflects the structure of the quantum-gravity era. The fact that operators in a Hilbert space (which are mathematical objects typical for quantum mechanics) enter the very core of the noncommutative description of singularities could suggest that singularities "know something" about quantum effects. The tempting hypothesis is that below the Planck threshold there is the quantum-gravity era, which is modeled by a noncommutative geometry and, consequently, is totally nonlocal. In this era there is no space and time in their usual meaning. Only when the universe passes through the Planck threshold does a "phase transition" to the commutative geometry occur, and in this transition the standard space-time emerges together with its singular boundary. In fact, such a scenario of the very early universe has been proposed (Heller, Sasin, and Lambert 1997; Heller and Sasin 1998; 1999b).

One may ask, Will the future theory of quantum gravity remove the initial singularity from our image of the universe? Usually, either a yes or a no answer is given to this question, and the two are supposed to be mutually exclusive. In light of the proposed scenario, a third possibility should be taken into account: The Planck era is atemporal and aspatial, and the foregoing question, as referred to this era, becomes meaningless. From the point of view of noncommutative geometry, everything is regular, although drastically different from what we usually meet in space-time. As already shown, the singularities are formed in the process of the transition through the Planck threshold when space-time emerges out of noncommutative geometry. This process could also be explained in the following way.

Usually, we think of the Planck era as being hidden in the prehistory of the universe when its typical scale was of the order of 10^{-33} cm. However, the Planck era could be found even now if we were to delve deeper and deeper into the strata of the world's structure until we reached the threshold length of 10^{-33} cm. After crossing this threshold, we would find ourselves in the Planck "stratum" with its noncommutative regime. On this fundamental level, below the Planck scale, all states are on an equal footing; there is no distinction between singular states and nonsingular states. Only the macroscopic observer, situated in space-time (and thus well beyond the Planck threshold), can say that his universe started from the initial singularity in its finite past and possibly will meet the final singularity in its finite future.

Methodological Conclusions

Having reviewed the story of the singularity problem in twentieth-century cosmology, including some of its interpretations as the beginning of the universe, we can now draw out of it a few methodological conclusions referring to the relationship between theology and natural theology (or philosophy, in general), from one part, and scientific theories and models, from the other. Here and in the rest of the present section, by *interpretation* I do not mean the "exegesis of the mathematical structure" of a given theory or model but rather an interpretation that is "superimposed" on this theory or model.

1. As we have seen, it is usually the latest scientific theory or model that gives rise to theological or philosophical interpretations, and very often these interpretations are announced with such conviction as to suggest that the theories or models are indubitable results of science. For instance, after proving several theorems about the existence of classical singularities, Hawking (with his coauthor George Ellis) expressed the view that these theorems support "the idea that the universe began a finite time ago." Later on, when Hawking (with his colleague Jim Hartle) produced the now well-known model of the quantum origin of the universe, he switched to the interpretation that is best encapsulated in the following quotation: "So long as the universe had a beginning, we could suppose it had a creator. But if the universe is really completely self-contained, having no boundary or edge, it would have neither beginning nor end: it would simply be. What place, then, for a creator?" (Hawking 1988, 140–41)
2. One could do theology or natural theology without any contact with scientific theories or models, and in fact many theologians and philosophers prefer this way of pursuing their disciplines. However, in such a case, there is danger that instead of scientific theories or models, some pseudoscientific ideas or outdated concepts will serve as a background for theological or philosophical speculations. The point is that neither theology nor philosophy can be studied without a "cultural environment" of a given epoch, and a general image of the world constitutes a vital element of this environment. If the image of the world is not taken (critically) from the sciences, it will certainly infiltrate theological or philosophical speculations from various, intellectually suspect sources of human imagination.
3. In many theological or philosophical interpretations of cosmological theories or models, both theologians and cosmologists (especially the latter) often present the image of God's creating the world and playing with the laws of physics, for instance, by throwing dice in order to decide which model of the universe should be brought into exist-

ence. Incidentally, very often in such situations, the laws of physics seem to be exempt from God's omnipotence; in any case God is supposed to be constrained by the laws of probability and statistics. Sometimes this picture of the world plays the role of a metaphor or a heuristic tool in some abstract analyses. If such an image of the Creator is taken more seriously, one can hardly recognize the God of theology or Judaeo-Christian belief whose functions can by no means be reduced to those connected with "manufacturing" the world. It seems that such images of the Creator are laden more with the deistic concept of Deity than with the Judaeo-Christian idea of God. This is why in such contexts I prefer to speak of a Demiurge rather than of the authentic God. However, this does not mean that the theologian has no lesson about God to learn from scientific theories. Such theories can disclose some unexpected "ways of existing" (for example, the atemporal character of a noncommutative regime), which, by analogy, could be used in theological speculations about God.

4. A mathematical model could be of some importance for theological or philosophical analyses even if it is not yet empirically verified and even if it will never be. Any mathematical model, provided it is correctly constructed, shows that the set of assumptions upon which it is based is not contradictory, and as such it can either falsify or corroborate some philosophical idea. For example, our noncommutative model unifying general relativity and quantum mechanics tells us something philosophically interesting, even if it will never find any empirical support. Its message is that existence in space and time is not the necessary prerequisite for the unfolding of physics. In particular, this model falsifies the doctrine, common among philosophers, that existence in time is the *condition sine qua non* for the possibility of any change and dynamics. Change and dynamics in a generalized sense, however, are possible even in the absence of local concepts such as that of point or time instant (see the next section).

5. One should never forget about the temporary and transitory character of all physical theories and models. Even if some of them have successfully undergone the confrontation process with empirical data, they always can become a "special case" of a future more-general theory or of a model. The new conceptual environment could make their present philosophical or theological interpretation no longer attractive, or even highly artificial.

6. Scientific theories or models are per se neutral with respect to theological or philosophical interpretations. They can be interpreted in various ways as long as these interpretations do not contradict their mathematical structure. This does not mean that all such interpretations are on an equal footing, only that they cannot be refuted by

arguments taken from these theories or models alone (since we suppose that these interpretations are not contradictory with their mathematical structures). Theological or philosophical interpretations of scientific theories or models can, of course, critically compete with each other. Popper's "criterion of disputability" clearly applies to them: any rational interpretation should be open for discussion and criticism by its rivals.

Theological and Philosophical Conclusions

Having in mind all of these methodological warnings, we can finally ask about a theological (and philosophical) lesson to be learned from the singularity problem as it evolved in twentieth-century cosmology.

1. It has built a strong case against the Newtonian concept of creation (the idea of an absolute space and an absolute time existing "from forever," a kind of *sensoria Dei*) and of God's creating energy and matter at certain places of the absolute space and at the determined instant of the absolute time. Even classical singularities could hardly be reconciled with such an idea of creation. At classical singularities space-time breaks down, and the Newtonian concept of creation could be saved only by claiming that classical singularities are but an artifact of the method rather than the authentic element of the theory. Such a possibility, however, is practically excluded by the theorems about the existence of singularities.
2. The modern theologian should consider the possibility of going back to the traditional doctrine that the creation of the universe is an atemporal (and aspatial) act. Starting from the second half of the nineteenth century, some physical theories (thermodynamics, statistical mechanics, certain data from the theory of elementary particles) suggested that time is essentially a macroscopic phenomenon connected with statistical properties of a great number of physical individuals (particles); and many contemporary quantum-gravity proposals and models of the very early universe describe the Planck era as timeless and spaceless. The most radical in this respect seems to be the noncommutative model, described in the previous sections, in which all local concepts are excluded by the very nature of noncommutative geometry. It would be inconsistent to regard creation as the process immersed in time while simultaneously asserting that the beginnings of the universe are atemporal.
3. Some theologians and philosophers (especially those of the Whiteheadian school) claim that the existence "in transient time" is an ontological necessity. The fact that atemporal mathematical models of the physical world (in its Planck era) have been constructed falsi-

fies this claim. The main argument of these theologians and philosophers that a timeless God would be a static being with no possibility of acting, is erroneous. It is interesting to look in this respect at the noncommutative model of the Planck era. In physics we usually describe motion in terms of vector quantities. For instance, the velocity of a moving point is a tangent vector to the curve describing the trajectory of this point. The curve is parametrized by a time parameter; and to represent the velocity of the moving point at a given time instant, we choose the tangent vector to this curve at the point corresponding to this time instant. In physical models based on a noncommutative geometry there are no points and time instants and, consequently, no vectors tangent at a given point. All of these concepts are local, and as such they have no counterparts in the noncommutative setting. However, a standard (commutative) dynamical system (for instance, a body in motion) can be described in terms of vector fields; and vector field, being a global concept, has its noncommutative generalization, called the *derivation of a noncommutative algebra*. We cannot go into detail here, but it is enough to remember that vector fields on the usual (commutative) space are also derivations of a certain algebra (the algebra of smooth functions on this space). One can meaningfully speak of a noncommutative dynamics (without reference to local concepts such as that of point or time instant) provided that one describes noncommutative dynamics in terms of derivations of the corresponding algebra. Therefore, a generalized dynamics is possible even in the absence of the usual notion of time. Notice that this noncommutative dynamics is not the usual dynamics simply transferred to a new conceptual environment but rather dynamics in the truly generalized sense. One of the essential features of this generalization consists in replacing all local elements with their global counterparts (if they exist).[13]

As always in similar situations in physics, the correspondence with previous theories is important. This is the case as far as the temporal properties of our model are concerned. It can be shown (Heller and Sasin 1998) that, as we cross the Planck threshold, first some temporal order appears, and only then does space-time emerge, eventually with its singular boundary (depending on the model).

4. The latter property discloses a certain relativity of the concept of the beginning of the universe. If we regard the initial singularity as a physical counterpart of the theological notion of the beginning of the universe, we must say that from the perspective of the macroscopic observer the universe had its beginning a finite number of years ago; but from the perspective of the fundamental level (supposing it is essentially noncommutative), the very concept of the beginning is meaningless. In the light of this result, it could be interesting

to go back to another traditional doctrine, strongly defended by Saint Thomas Aquinas, that the beginning of the universe and the creation of the universe are two completely distinct concepts. Because the creation of the universe is but a dependence of the universe in its existence on the Prime Cause, one can think, without any danger of contradiction, about the created universe as existing from minus time infinity. The dependence in existence does not require the initiation of existence (see Baldner and Carroll 1997).

I emphasize again that the noncommutative model does not prove or imply this traditional doctrine concerning the beginning and creation; it only shows its logical consistency.

TIMELESSNESS AND TIME

This final section explores, as a corollary of the foregoing analyses, the traditional doctrine of the relationship between the temporal existence of the universe and God's eternity, and its consequences for our understanding of creation. To this end I cite the recent essay by Ernan McMullin (1997) in which this author goes back to the traditional doctrine on time and eternity in order to cope with another important problem—that of purpose and contingency in the evolutionary process.

In the Platonic myth of creation, a Demiurge transformed independently existing chaotic stuff into the ordered Cosmos. In Aristotle's physics the world always existed and needed only to be put into motion by the First Mover. Saint Augustine saw God in a totally different way, namely as a "Creator in the fullest sense, a Being from whom the existence of all things derives" (McMullin 1997, 104). Such a being must be above all constraints, and it cannot be denied that existence in time is one of the most far-reaching constraints. It limits one's existence to the transitory "now" surrounded by two kinds of nothingness: the nothingness of all those things that formerly existed but do not now exist and the nothingness of all those things that do not yet exist. Perhaps it is even immersed into the third kind of nothingness—that of all those things that could have existed but never will.

Therefore, God should be regarded as being "'outside' time created, though the metaphor is an imperfect one. Calling God 'eternal' is not a way of saying that God is without beginning or end, like Aristotle's universe. 'Eternal' does not mean unending duration; it means that temporal notions simply do not apply to the Creator as Creator" (McMullin 1997, 105). The objection that this would make God a "static," devoid of all dynamics, was neutralized by the famous Boetius "definition" of *eternity*. His formulation, "Eternity is the whole, simultaneous and perfect possession of boundless life" (quoted after McMullin 1997, 105), emphasizes the abundant activity of the perfect and unconstrained life. Accordingly, one should understand the act of creation:

> Time is a condition of the creature, a sign of dependence. It is created *with* the creature. . . . The act of creation is a single one, in which what is past, present or future from the perspective of the creature issues as a single whole from the Creator. . . . Creation continues at every moment, and each moment has the same relation of dependency on the Creator. (McMullin 1997, 105)

Such an understanding of God's eternity and creation has further consequences for many theological problems, including the problems of design and chance. If God exists outside time and space,

> God knows the past and the future of each creature, not by memory or by foretelling . . . as another creature might, but in the same direct way that God knows the creature's present. . . . Terms like "plan" and "purpose" obviously shift meaning when the element of time is absent. For God to plan is for the outcome to occur. There is no interval between decision and completion. (1997, 105–6)

In such a perspective, the concepts of chance and necessity also "shift their meanings." They are but two aspects of the same atemporal and totally global activity. God knows the outcomes of laws and chance not by calculating from the initial conditions but in the same direct way as God knows everything. What for us is a chance, for God is a detail of the picture that is simply present.

Such an approach solves so many theological questions that it ought not to be hastily dismissed (and, as we have seen, it is in consonance with contemporary trends in theoretical physics). Its unpopularity among some theologians may well stem from the fact that this approach strongly emphasizes the transcendence of God, whereas nowadays we prefer speaking about God's immanence. However, we should not forget that in Saint Augustine's teaching, God "is also immanent in every existent at every moment, sustaining it in being" (McMullin 1997, 105). Here again the noncommutative model of the atemporal Planck's era might be of some help to our imagination: macroscopic physics is only the result of some "averaging" of what happens on the noncommutative, fundamental level. Time is but an epiphenomenon of timeless existence.

NOTES

1. Every straight line in the Euclidean space is a geodesic. For the precise definition see any textbook on differential geometry.
2. Notice that the usual concept of length has no invariant meaning (the meaning independent of a coordinate system) in relativity theory; the term *prolongation* should here be understood in the precise technical meaning.
3. The Planck threshold is characterized by the Planck length: $(\hbar G/c^3)^{1/2} \approx 10^{33}$ cm; the Planck time: $(\hbar G/c^5)^{1/2} \approx 10^{44}$ s; and the Planck density: $c^5/\hbar G^2 \approx 10^{93}$ g/cm^3. All of these magnitudes are constructed out of fundamental constants: velocity of light c, Planck's constant $\hbar$, and the Newtonian gravitational constant G.
4. "Freely falling particles" is a technical term for particles that move in a given gravitational field under the influence of no other forces.
5. The condition that the acceleration should be bounded is essential. A rocket moving with unbounded acceleration would require an infinite amount of fuel, and such a condition could hardly be regarded as physical.

6. Technically, the singularities are not Hausdorff separated from the rest of space-time.
7. In this approach, in fact, the very notion of smoothness is generalized.
8. See Heller and Sasin 1995; strictly speaking, structured spaces are even more general than differential spaces.
9. This family contains all smooth functions (in the usual sense) on space-time.
10. It is only noncontradictory with the mathematical structure of our model.
11. A beautiful example of such a space is provided by the so-called Penrose's tiling, a tiling of the Euclidean plane that has two basic tiles: kites and darts. Every finite patch of these tiles occurs infinitely many times in any other tiling of the plane so that any two tilings are locally indistinguishable. The space of all such tilings is a noncommutative space.
12. This has been shown in Heller and Sasin 1996.
13. Theologians would certainly notice an analogy between this generalization process and the way of forming concepts referring to God in the traditional theology. I have in mind especially the so-called *via emminentiae*, a concept that is known from everyday usage is ascribed to God, but only after it has been purified from all negative connotations and after all of its positive connotations have been strengthened to their possible maximum.

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