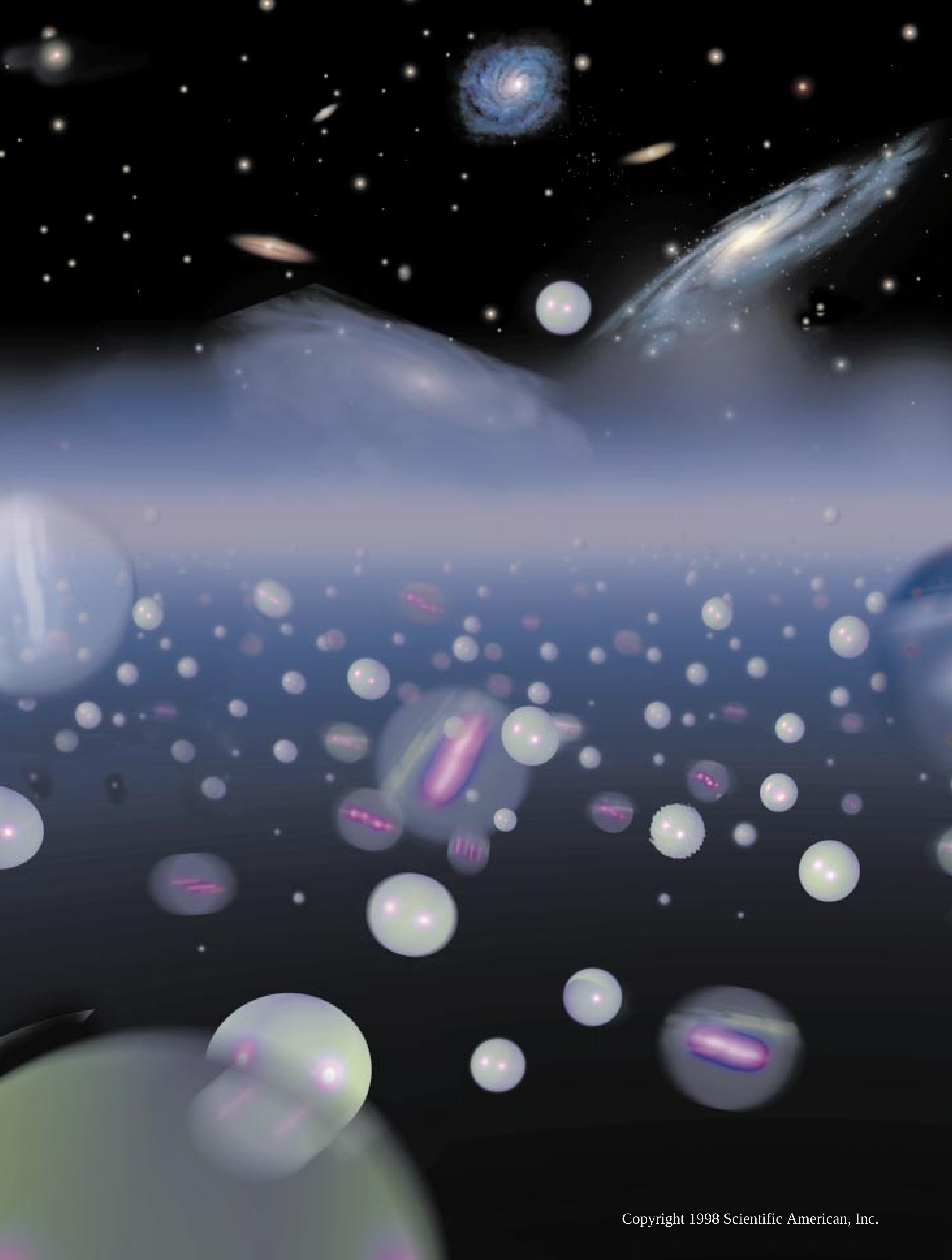
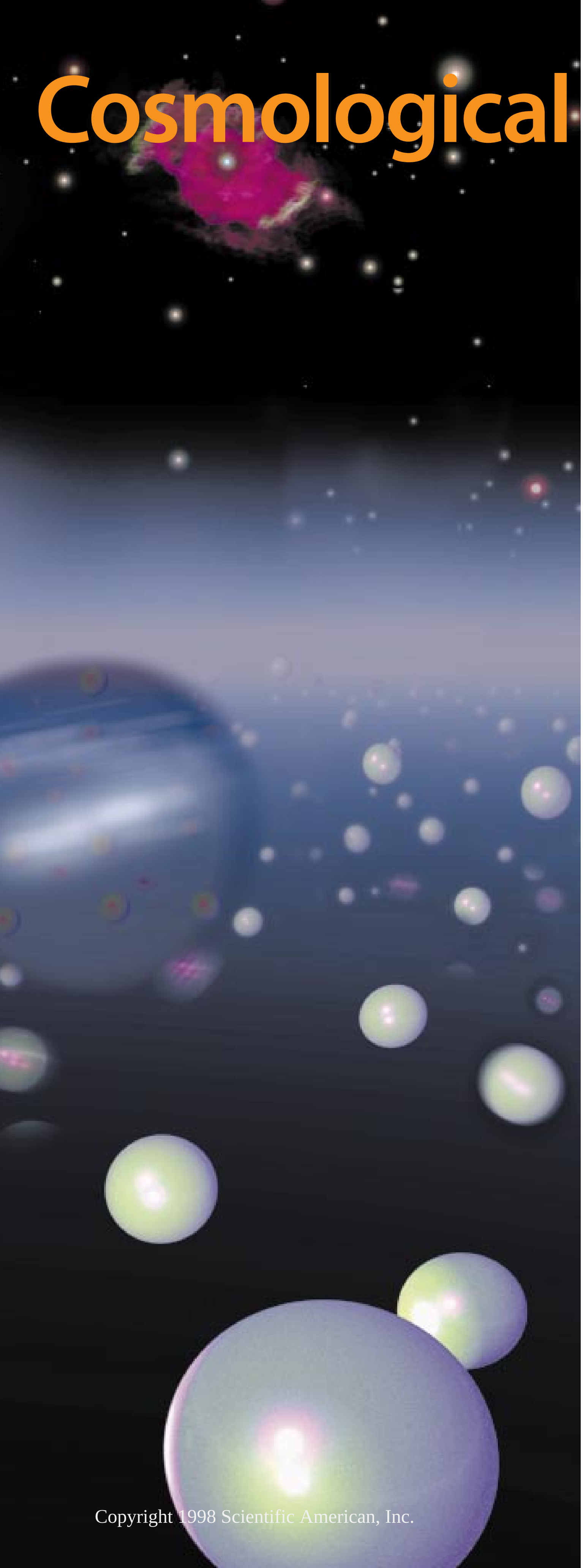


Cosmological Antigravity



The long-derided cosmological constant—a contrivance of Albert Einstein's that represents a bizarre form of energy inherent in space itself—is one of two contenders for explaining changes in the expansion rate of the universe

by Lawrence M. Krauss

Novelist and social critic George Orwell wrote in 1946, "To see what is in front of one's nose requires a constant struggle." These words aptly describe the workings of modern cosmology. The universe is all around us—we are part of it—yet scientists must sometimes look halfway across it to understand the processes that led to our existence on the earth. And although researchers believe that the underlying principles of nature are simple, unraveling them is another matter. The clues in the sky can be subtle. Orwell's adage is doubly true for cosmologists grappling with the recent observations of exploding stars hundreds of millions of light-years away. Contrary to most expectations, they are finding that the expansion of the universe may not be slowing down but rather speeding up.

Astronomers have known that the visible universe is expanding since at least 1929, when Edwin P. Hubble demonstrated that distant galaxies are moving apart as they would if the entire cosmos were uniformly swelling in size. These outward motions are counteracted by the collective gravity of galaxy clusters and all the planets, stars, gas and dust they contain. Even the minuscule gravitational pull of, say, a paper clip retards cosmic expansion by a slight amount. A decade ago a congruence of theory and observations suggested that there were enough paper clips and other matter in the universe to almost, but never quite, halt the expansion. In the geometric terms that Albert Einstein encour-

SO-CALLED EMPTY SPACE is actually filled with elementary particles that pop in and out of existence too quickly to be detected directly. Their presence is the consequence of a basic principle of quantum mechanics combined with special relativity: nothing is exact, not even nothingness. The aggregate energy represented by these "virtual" particles, like other forms of energy, could exert a gravitational force, which could be either attractive or repulsive depending on physical principles that are not yet understood. On macroscopic scales the energy could act as the cosmological constant proposed by Albert Einstein.

ALFRED T. KAMAJIAN

Types of Matter

Type	Likely Composition	Main Evidence	Approximate Contribution to Ω
Visible matter	Ordinary matter (composed mainly of protons and neutrons) that forms stars, dust and gas	Telescope observations	0.01
Baryonic dark matter	Ordinary matter that is too dim to see, perhaps brown or black dwarfs (massive compact halo objects, or MACHOs)	Big bang nucleosynthesis calculations and observed deuterium abundance	0.05
Nonbaryonic dark matter	Exotic particles such as "axions," neutrinos with mass or weakly interacting massive particles (WIMPs)	Gravity of visible matter is insufficient to account for orbital speeds of stars within galaxies and of galaxies within clusters	0.3
Cosmological "dark matter"	Cosmological constant (energy of empty space)	Microwave background suggests cosmos is flat, but there is not enough baryonic or nonbaryonic matter to make it so	0.6

CONTENTS OF THE UNIVERSE include billions and billions of galaxies, each one containing an equally mind-boggling number of stars. Yet the bulk seems to consist of "dark matter" whose identity is still uncertain. The cosmological constant, if its existence is confirmed, would act like a yet more exotic form of dark matter on cosmological scales. The quantity omega, Ω, is the ratio of the density of matter or energy to the density required for flatness.

aged cosmologists to adopt, the universe seemed to be "flat."

The flat universe is an intermediate between two other plausible geometries, called "open" and "closed." In a cosmos where matter does battle with the outward impulse from the big bang, the open case represents the victory of expansion: the universe would go on expanding forever. In the closed case, gravity would have the upper hand, and the universe would eventually collapse again, ending in a fiery "big crunch." The open, closed and flat scenarios are analogous to launching a rocket faster than, slower than or exactly at the earth's escape velocity—the speed necessary to overcome the planet's gravitational attraction.

EINSTEIN ARCHIVES, UNIVERSITY OF ZURICH

LETTER FROM EINSTEIN, then at the Prussian Academy of Sciences in Berlin, to German mathematician Hermann Weyl concedes that a universe of unchanging size would be prone to expansion or collapse: "In the De Sitter universe two fluid and unstable distinct points separate at an accelerated pace. If there is no quasi-static world, then away with the cosmological term!"

That we live in a flat universe, the perfect balance of power, is one of the hallmark predictions of standard inflationary theory, which postulates a very early period of rapid expansion to reconcile several paradoxes in the conventional formulation of the big bang. Although the visible contents of the cosmos are clearly not enough to make the universe flat, celestial dynamics indicate that there is far more matter than meets the eye. Most of the material in galaxies and assemblages of galaxies must be invisible to telescopes. Over a decade ago I applied the term "quintessence" to this so-called dark matter, borrowing a term Aristotle used for the ether—the invisible material supposed to permeate all of space [see "Dark Matter in the Universe," by Lawrence M. Krauss; Scientific American, December 1986].

Yet an overwhelming body of evidence now implies that even the unseen matter is not enough to produce a flat universe. Perhaps the universe is not flat but rather open, in which case scientists must modify—or discard—inflationary theory [see "Inflation in a Low-Density Universe," by Martin A. Bucher and David N. Spergel, on page 62]. Or maybe the universe really is flat. If that is so, its main constituents cannot be visible matter, dark matter or radiation. Instead the universe must be composed largely of an even more ethereal form of energy that inhabits empty space, including that which is in front of our noses.

Fatal Attraction

The idea of such energy has a long and checkered history, which began when Einstein completed his general theory of relativity, more than a decade before Hubble's convincing demonstration that the universe is expanding. By tying together space, time and matter, relativity promised what had previously been impossible: a scientific understanding not merely of the dynamics of objects within the universe but of the universe itself. There was only one problem. Unlike other fundamental forces felt by matter, gravity is universally attractive—it only pulls; it cannot push. The unrelenting gravitational attraction of matter could cause the universe to collapse eventually. So Einstein, who presumed the universe to be static and stable, added an extra term to his equations, a "cosmological term," which could stabilize the universe by producing a new long-range force throughout space. If its value were positive, the term would represent a repulsive force—a kind of antigravity that could hold the universe up under its own weight.

Alas, within five years Einstein abandoned this kludge, which he associated with his "biggest blunder." The stability offered by the term turned out to be illusory, and, more important, evidence had begun to mount that the universe is expanding. As early as 1923, Einstein wrote in a letter to mathematician Hermann Weyl that "if there is no quasi-static world, then away with the cosmological term!" Like the ether before it, the term appeared to be headed for the dustbin of history.

Physicists were happy to do without such an intrusion. In the general theory of relativity, the source of gravitational forces (whether attractive or repulsive) is energy. Matter is simply one form of energy. But Einstein's cosmological term is distinct. The energy associated with it does not depend on position or time—hence the name "cosmological constant." The force caused by the constant operates even in the complete absence of matter or radiation. Therefore, its source must be a curious energy that resides in empty space. The cosmological constant, like the ether, endows the void with an almost metaphysical aura. With its demise, nature was once again reasonable.

Or was it? In the 1930s glimmers of the cosmological constant arose in a completely independent context: the effort to combine the laws of quantum mechanics with Einstein's special theory of relativity. Physicists Paul A. M. Dirac and later Richard Feynman, Julian S. Schwinger and Shinichiro Tomonaga showed that empty space was more complicated than anyone had previously imagined. Elementary particles, it turned out, can spontaneously pop out of nothingness and disappear again, if they do so for a time so short that one cannot measure them directly [see "Exploiting Zero-Point Energy," by Philip Yam; Scientific American, December 1997]. Such virtual particles, as they are called, may appear as far-fetched as angels sitting on the head of a pin. But there is a difference. The unseen particles produce measurable effects, such as alterations to the energy levels of atoms as well as forces between nearby metal plates. The theory of virtual particles agrees with observations to nine decimal places. (Angels, in contrast, normally have no discernible effect on either atoms or plates.) Like it or not, empty space is not empty after all.

Virtual Reality

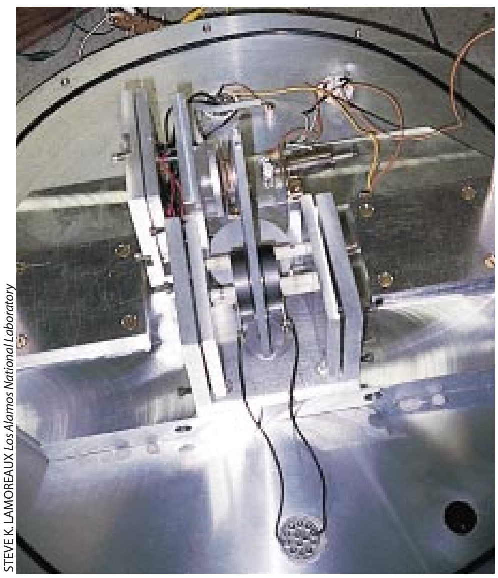
If virtual particles can change the properties of atoms, might they also affect the expansion of the universe? In 1967 Russian astrophysicist Yakov B. Zeldovich showed that the energy of virtual particles should act precisely as the energy associated with a cosmological constant. But there was a serious problem. Quantum theory predicts a whole spectrum of virtual particles, spanning every possible wavelength. When physicists add up all the effects, the total energy comes out infinite. Even if theorists ignore quantum effects smaller than a certain wavelength—for which poorly understood quantum gravitational effects presumably alter things—the calculated vacuum energy is roughly 120 orders of magnitude larger than the energy contained in all the matter in the universe.

What would be the effect of such a humongous cosmological constant? Taking a cue from Orwell's maxim, you can easily put an observational limit on its value. Hold out your hand and look at your fingers. If the constant were as large as quantum theory naively suggests, the space between your eyes and your hand would expand so rapidly that the light from your hand would never reach your eyes. To see what is in front of your face would be a constant struggle (so to speak), and you would always lose. The fact that you can see anything at all means that the energy of empty space cannot be large. And the fact that we can see not only to the ends of our arms but also to the far reaches of the universe puts an even more stringent limit on the cosmological constant: almost 120 orders of magnitude smaller than the estimate mentioned above. The discrepancy between theory and observation is the most perplexing quantitative puzzle in physics today [see "The Mystery of the Cosmological Constant," by Larry Abbott; Scientific American, May 1988].

The simplest conclusion is that some as yet undiscovered physical law causes the cosmological constant to vanish. But as much as theorists might like the constant to go away, various astronomical observations—of the age of the universe, the density of matter and the nature of cosmic structures—all independently suggest that it may be here to stay.

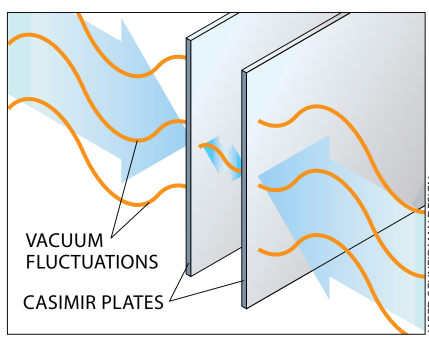
Determining the age of the universe is one of the long-standing issues of modern cosmology. By measuring the velocities of galaxies, astronomers can calculate how long it took them to arrive at their present positions, assuming they all started out at the same place. For a first approximation, one can ignore the deceleration caused by gravity. Then the universe would expand at a constant speed and the time interval would just be the ratio of the distance between galaxies to their measured speed of separation—that is, the reciprocal of the famous Hubble constant. The higher the value of the Hubble constant, the faster the expansion rate and hence the younger the universe.

Hubble's first estimate of his eponymous constant was almost 500 kilometers per second per megaparsec—which would mean that two galaxies separated by a distance of one megaparsec (about three million light-years) are moving apart, on average, at 500 kilometers per second. This value would imply a cosmic age of about two billion years, which is in painful contradiction with the known age of the earth—about four billion years. When the gravitational attraction of matter is taken into account,



STEVE K. LAMOREAUX *Los Alamos National Laboratory*

DEMONSTRATION OF CASIMIR EFFECT is one way that physicists have corroborated the theory that space is filled with fleeting "virtual particles." The Casimir effect generates forces between metal objects—for instance, an attractive force between parallel metal plates (*above*). Loosely speaking, the finite spacing of the plates prevents virtual particles larger than a certain wavelength from materializing in the gap. Therefore, there are more particles outside the plates than between them, an imbalance that pushes the plates together (*right*). The Casimir effect has a distinctive dependence on the shape of the plates, which allows physicists to tease it out from other forces of nature.



JARED SCHNEIDMAN DESIGN

the analysis predicts that objects moved faster early on, taking even less time to get to their present positions than if their speed had been constant. This refinement reduces the age estimate by one third, unfortunately worsening the discrepancy.

Over the past seven decades, astronomers have improved their determination of the expansion rate, but the tension between the calculated age of the universe and the age of objects within it has persisted. In the past decade, with the launch of the Hubble Space Telescope and the development of new observational techniques, disparate measurements of the Hubble constant are finally beginning to converge. Wendy L. Freedman of the Carnegie Observatories and her colleagues have inferred a value of 73 kilometers per second per megaparsec (with a most likely range, depending on experimental error, of 65 to 81) [see "The Expansion Rate and Size of the Universe," by Wendy L. Freedman; Scientific American, November 1992]. These results put the upper limit on the age of a flat universe at about 10 billion years.

The Age Crisis

Is that value old enough? It depends on the age of the oldest objects that astronomers can date. Among the most ancient stars in our galaxy are those found in tight groups known as globular clusters, some of which are located in the outskirts of our galaxy and are thus thought to have formed before the rest of the Milky Way. Estimates of their age, based on calculations of how fast stars burn their nuclear fuel, traditionally ranged from 15 to 20 billion years. Such objects appeared to be older than the universe.

To determine whether this age conflict was the fault of cosmology or of stellar modeling, in 1995 my colleagues—Brian C. Chaboyer, then at the Canadian Institute of Theoretical Astrophysics, Pierre Demarque of Yale University and Peter J. Kernan of Case Western Reserve University—and I reassessed the globular cluster ages. We simulated the life cycles of three million different stars whose properties spanned the existing uncertainties, and then compared our model stars with those in globular clusters. The oldest, we concluded, could be as young as 12.5 billion years old, which was still at odds with the age of a flat, matter-dominated universe.

But two years ago the Hipparcos satellite, launched by the European Space Agency to measure the locations of over 100,000 nearby stars, revised the distances to these stars and, indirectly, to globular clusters. The new distances affected estimates of their brightness and forced us to redo our analysis, because brightness determines the rate at which stars consume fuel and hence their life spans. Now it seems that globulars could, at the limit of the observational error bars, be as young as 10 billion years old, which is just consistent with the cosmological ages.

But this marginal agreement is uncomfortable, because it requires that both sets of age estimates be near the edge of their allowed ranges. The only thing left that can give is the assumption that we live in a flat, matter-dominated universe. A lower density of matter, signifying an open universe with slower deceleration, would ease the tension somewhat. Even so, the only way to lift the age above 12.5 billion years would be to consider a universe dominated not by matter but by a cosmological constant. The resulting repulsive force would cause the Hubble expansion to accelerate over time. Galaxies would have been moving apart slower than they are today, taking longer to reach their present separation, so the universe would be older.

The current estimates of age are merely suggestive. Meanwhile other pillars of observational cosmology have recently been shaken, too. As astronomers have surveyed ever larger regions of the cosmos, their ability to tally up its contents has improved. Now the case is compelling that the total amount of matter is insufficient to yield a flat universe.

This cosmic census first involves calculations of the synthesis of elements by the big bang. The light elements in the cosmos—hydrogen and helium and their rarer isotopes, such as deuterium—were created in the early universe in relative amounts that depended on the number of available protons and neutrons, the constituents of normal matter. Thus, by comparing the abundances of the various isotopes, astronomers can deduce the total amount of ordinary matter that was produced in the big bang. (There could, of course, also be other matter not composed of protons and neutrons.)

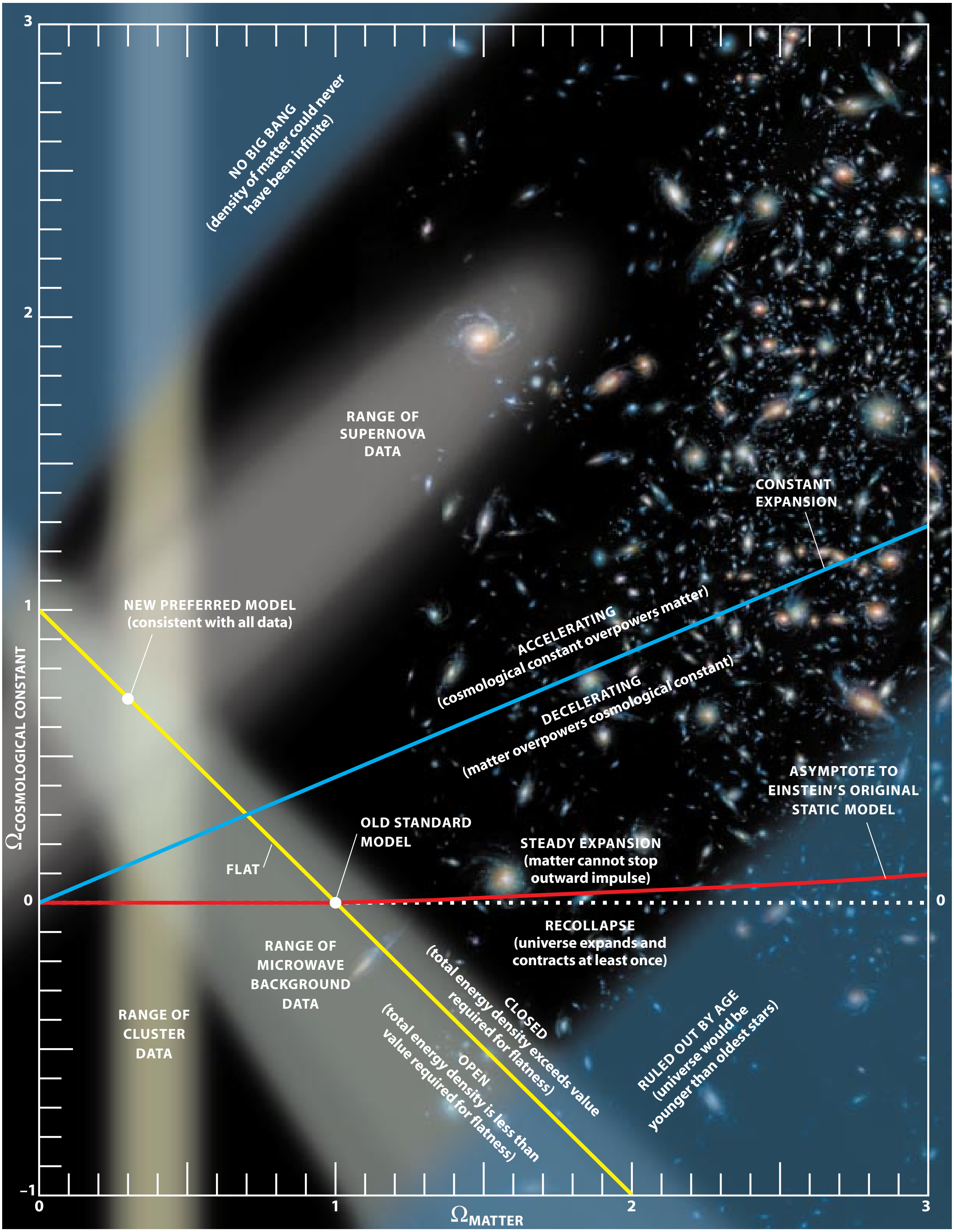
The relevant observations took a big step forward in 1996 when David R. Tytler and Scott Burles of the University of California at San Diego and their colleagues measured the primordial abundance of deuterium using absorption of quasar light by intergalactic hydrogen clouds. Because these clouds have never contained stars, their deuterium could only have been created by the big bang. Tytler and Burles's finding implies that the average density of ordinary matter is between 4 and 7 percent of the amount needed for the universe to be flat.

Astronomers have also probed the density of matter by studying the largest gravitationally bound objects in the universe: clusters of galaxies. These groupings of hundreds of galaxies account for almost all visible matter. Most of their luminous content takes the form of hot intergalactic gas, which emits x-rays. The temperature of this gas, inferred from the spectrum of the x-rays, depends on the total mass of the cluster: in more massive clusters, the gravity is stronger and hence the pressure that supports the gas against gravity must be larger, which drives the temperature higher. In 1993 Simon D. M. White, now at the Max Planck Institute for Astrophysics in Garching, Germany, and his colleagues compiled information about several different clusters to argue that luminous matter accounted for between 10 and 20 percent of the total mass of the objects. When combined with the measurements of deuterium, these results imply that the total density of clustered matter—including protons and neutrons as well as

Summary of Inferred Values of Cosmic Matter Density

Observation	Ω_{matter}
Age of universe	<1
Density of protons and neutrons	0.3–0.6
Galaxy clustering	0.3–0.5
Galaxy evolution	0.3–0.5
Cosmic microwave background radiation	$\lesssim 1$
Supernovae type Ia	0.2–0.5

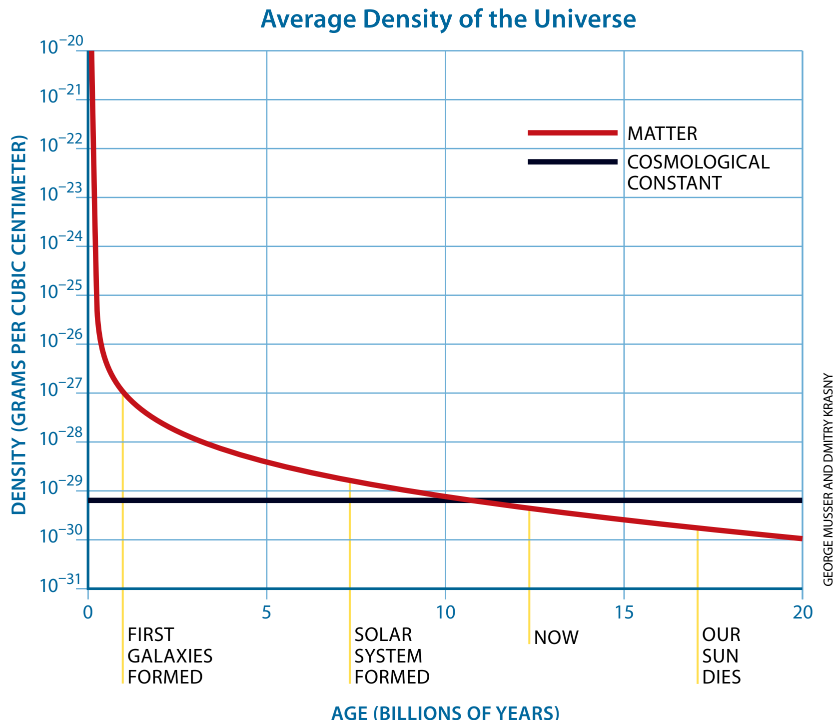
MEASUREMENTS of the contribution to Ω from matter are in rough concordance. Although each measurement has its skeptics, most astronomers now accept that matter alone cannot make Ω equal to 1. But other forms of energy, such as the cosmological constant, may also pitch in.



DON DIXON (*illustration*); DMITRY KRASNY (*chart*); SOURCE: LAWRENCE M. KRAUSS AND MAX TEGMARK

MAP OF MODELS shows how the unfolding of the universe depends on two key cosmological quantities: the average density of matter (*horizontal axis*) and the density of energy in the cosmological constant (*vertical axis*). Their values, given here in standard cosmological units, have three distinct effects. First, their sum (which represents the total cosmic energy content) determines the geometry of space-time (*yellow line*). Second, their difference (which represents the relative strength of expansion and gravity) determines how the expansion rate changes over time (*blue line*). These two effects have been probed by recent observations (*shaded regions*). The third, a balance of the two densities, determines the fate of the universe (*red line*). The three effects have many permutations—unlike the view of cosmology in which the cosmological constant is assumed to be zero and there are only two possible outcomes.

COSMIC COINCIDENCE is one of many mysteries swirling about the cosmological constant. The average density of ordinary matter decreases as the universe expands (*red*). The equivalent density represented by the cosmological constant is fixed (*black*). So why, despite these opposite behaviors, do the two have nearly the same value today? The consonance is either happenstance, a precondition for human existence (an appeal to the weak anthropic principle) or an indication of a mechanism not currently envisaged.



The Fate of the Universe

The cosmological constant changes the usual simple picture of the future of the universe. Traditionally, cosmology has predicted two possible outcomes that depend on the geometry of the universe or, equivalently, on the average density of matter. If the density of a matter-filled universe exceeds a certain critical value, it is "closed," in which case it will eventually stop expanding, start contracting and ultimately vanish in a fiery apocalypse. If the density is less than the critical value, the universe is "open" and will expand forever. A "flat" universe, for which the density equals the critical value, also will expand forever but at an ever slower rate.

Yet these scenarios assume that the cosmological constant equals zero. If not, it—rather than matter—may control the ultimate fate of the universe. The reason is that the constant, by definition, represents a fixed density of energy in space. Matter cannot compete: a doubling in radius dilutes its density eightfold. In an expanding universe the energy density associated with a cosmological constant must win out. If the constant has a positive value, it generates a long-range repulsive force in space, and the universe will continue to expand even if the total energy density in matter and in space exceeds the critical value. (Large negative values of the constant are ruled out because the resulting attractive force would already have brought the universe to an end.)

Even this new prediction for eternal expansion assumes that the constant is indeed constant, as general relativity suggests that it should be. If in fact the energy density of empty space does vary with time, the fate of the universe will depend on how it does so. And there may be a precedent for such changes—namely, the inflationary expansion in the primordial universe. Perhaps the universe is just now entering a new era of inflation, one that may eventually come to an end. *—L.M.K.*

more exotic particles such as certain dark-matter candidates—is at most 60 percent of that required to flatten the universe.

A third set of observations, ones that also bear on the distribution of matter at the largest scales, supports the view that the universe has too little mass to make it flat. Perhaps no other subfield of cosmology has advanced so much in the past 20 years as the understanding of the origin and nature of cosmic structures. Astronomers had long assumed that galaxies coalesced from slight concentrations of matter in the early universe, but no one knew what would have produced such undulations. The development of the inflationary theory in the 1980s provided the first plausible mechanism—namely, the enlargement of quantum fluctuations to macroscopic size.

Numerical simulations of the growth of structures following inflation have shown that if dark matter was not made from protons and neutrons but from some other type of particle (such as so-called WIMPs), tiny ripples in the cosmic microwave background radiation could grow into the structures now seen. Moreover, concentrations of matter should still be evolving into clusters of galaxies if the overall density of matter is high. The relatively slow growth of the number of rich clusters over the recent history of the universe suggests that the density of matter is less than 50 percent of that required for a flat universe [see "The Evolution of Galaxy Clusters," by J. Patrick Henry, Ulrich G. Briel and Hans Böhringer; Scientific American, December 1998].

Nothing Matters

These many findings that the universe has too little matter to make it flat have become convincing enough to overcome the strong theoretical prejudice against this possibility. Two interpretations are viable: either the universe is open, or it is made flat by some additional form of energy that is not associated with ordinary matter. To distinguish between these alternatives, astronomers have been pushing to measure the microwave background at high resolution. Initial indications now favor a flat universe. Meanwhile researchers studying distant supernovae have provided the first direct, if tentative, evidence that the expansion of the universe is accelerating, a telltale sign of a cosmological constant with the same value implied by the other data [see "Surveying Space-time with Supernovae," by Craig J. Hogan, Robert P. Kirshner and Nicholas B. Suntzeff, on page 46]. Observations of the microwave background and of supernovae illuminate two different aspects of cosmology. The microwave background reveals the geometry of the universe, which is sensitive to the total density of energy, in whatever form, whereas the supernovae directly probe the expansion rate of the universe, which depends on the difference between the density of matter (which slows the expansion) and the cosmological constant (which can speed it up).

Together all these results suggest that the constant contributes between 40 and 70 percent of the energy needed to make the universe flat [*see illustration on page 57*]. Despite the preponderance of evidence, it is worth remembering the old saw that an astronomical theory whose predictions agree with all observations is probably wrong, if only because some of the measurements or some of the predictions are likely to be erroneous. Nevertheless, theorists are already scrambling to understand what 20 years ago would have been unthinkable: a cosmological constant greater than zero yet much smaller than current quantum theories predict. Some feat of fine-tuning must subtract virtual-particle energies to 123 decimal places but leave the 124th untouched—a precision seen nowhere else in nature.

One direction, explored recently by Steven Weinberg of the University of Texas at Austin and his colleagues, invokes the last resort of cosmologists, the anthropic principle. If the observed universe is merely one of an infinity of disconnected universes—each of which might have slightly different constants of nature, as suggested by some incarnations of inflationary theory combined with emerging ideas of quantum gravity—then physicists can hope to estimate the magnitude of the cosmological constant by asking in which universes intelligent life is likely to evolve. Weinberg and others have arrived at a result that is compatible with the apparent magnitude of the cosmological constant today.

Most theorists, however, do not find these notions convincing, as they imply that there is no reason for the constant to take on a particular value; it just does. Although that argument may turn out to be true, physicists have not yet exhausted the other possibilities, which might allow the constant to be constrained by fundamental theory rather than by accidents of history [see "The Anthropic Principle," by George Gale; SCIENTIFIC AMERICAN, December 1981].

Another direction of research follows in a tradition established by Dirac. He argued that there is one measured large number in the universe—its age (or, equivalently, its size). If certain physical quantities were changing over time, they might naturally be either very large or very small today [see "P. A. M. Dirac and the Beauty of Physics," by R. Corby Hovis and Helge Kragh; SCIENTIFIC AMERICAN, May 1993]. The cosmological constant could be one example. It might not, in fact, be constant. After all, if the cosmological constant is fixed and nonzero, we are living at the first and only time in the cosmic history when the density of matter, which decreases as the universe expands, is comparable to the energy stored in empty space. Why the coincidence? Several groups have instead imagined that some form of cosmic energy mimics a cosmological constant but varies with time.

This concept was explored by P. James E. Peebles and Bharat V. Ratra of Princeton University a decade ago. Motivated by the new supernova findings, other groups have resurrected the idea. Some have drawn on emerging concepts from string theory. Robert Caldwell and Paul J. Steinhardt of the University of Pennsylvania have reproposed the term "quintessence" to describe this variable energy. It is one measure of the theoretical conundrum that the dark matter that originally deserved this term now seems almost mundane by comparison. As much as I like the word, none of the theoretical ideas for this quintessence seems compelling. Each is ad hoc. The enormity of the cosmological-constant problem remains.

How will cosmologists know for certain whether they have to reconcile themselves to this theoretically perplexing universe? New measurements of the microwave background, the continued analysis of distant supernovae and measurements of gravitational lensing of distant quasars should be able to pin down the cosmological constant over the next few years. One thing is already certain. The standard cosmology of the 1980s, postulating a flat universe dominated by matter, is dead. The universe is either open or filled with an energy of unknown origin. Although I believe the evidence points in favor of the latter, either scenario will require a dramatic new understanding of physics. Put another way, "nothing" could not possibly be more interesting. SA

The Author

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