MOVING PARALLEL WITH OPENMP

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Agradecimientos a la Universidad del Rosario

- A brief introduction on computation
 - Evolution of infrastructure
 - Why HPC?
- 2 How does parallelization works
 - Strategies
 - Ahmdal's Law
- OpenMP: Preliminaries
 - Compiling
 - Examples
 - Practice
- Real Scenarios
- Task parallelism
- 6 Bibliography

Computation

What is computation?

Computation

What is computation?

Key infrastructure component

- Storage
- RAM
- Processing block: registries, instruction sets and clock
- FPGA's, GPU's, accelerators and other alternate processing units (RaspBerries, portable devices ... ARM)

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Compilers - Translator to Machine language

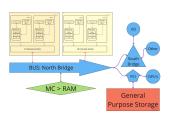


Figure: Von Neumann Architechture

BUS: North Bridge RC > RAM General Purpose Storage

Figure: Von Neumann Architechture

Computation

What is computation?

Key infrastructure component

- Storage
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- FPGA's, GPU's, accelerators and other alternate processing units (RaspBerries, portable devices ... ARM)
- Compilers Translator to Machine language

Limitations & Complications

- 1. All of the above
- Education: infrastucture topology, coding strategies, profiling & optimization
- 3. Interpreted languages
- 4. Unix like systems
- 5. Time accelerating technologies and real-time applications
- 6. Threats ⇒ Cybersecurity (https://meltdownattack.com/)

BUS: North Bridge MC > RAM General Purpose Storage

Figure: Von Neumann Architechture

History of parallelism

- 1. Origin dates back to the 80's
- 2. ILP + Vectorization: the superscalar architecture
- 3. Memory complexity:

CPU $\sim 0.5ns$ 1×

Table: By Jeff Dean@Google: http://research.google.com/people/jeff/

- 4. Memory coherency: (i.) HW with ECC (ii.) Software
- 5. Memory topology: UMA & NUMA & cc-NUMA

History of parallelism

 1996 SGI bought CRAY and soon after formed the ARB. 1997 OpenMP was born and announced at the <u>New York Times</u>.

- 8. CPU processor development stalled: (i.) Quantum limit $\sim 9nm$ (ii.) Energy efficiency per FLOP kept dropping
- 9. A full scale development of multi-core processors
- 10. Later: GPGPU's & MIC's & FPGA's

using HPC infrastructure



Figure: A cluster

Why the cluster?

- † Larger Storage
- † Bigger available RAM space (V-space)
- † More CPU cores per computing unit
- † Heterogeneous computing provisioning
- Software
- ‡ Fault-safe checks (ECC)
- ‡ Connectivity
- ‡ Service availability (power & hardware vendor support)
- ★ Efficient use of resources (power cost per FLOP)
- Research on topological improvements for high cost/effective throughput

- △ People, science & culture
- *'s and △ is HPC

using HPC infrastructure

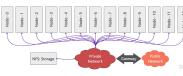


Figure: A cluster

Why the cluster?

- † Larger Storage
- † Bigger available RAM space (V-space)
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- △ People, science & culture
- *'s and △ is HPC

Advantages

- 1. Size of problems RealTime Operations
- 2. Encryption, Meteorology, Machine Learning, Block Chain
- 3. Smarter code, faster or better calculations
- 4. Precision (Accelerators)
- 5. Do you need to continuously upgrade your computer?

JP Mallarino (U Andes) learning OpenMP

Basics

Definition

A parallel computer is a system that is able to execute simultaneously multiple processing elements cooperatively to solve a computational problem

Requirements

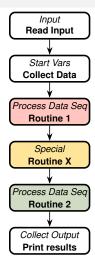
- Hardware
- OS
- Libraries: PThreads, TBB, OpenMP
- For HPC: Understand the process and data distribution model

Terminology

- Concurrent: A program is one in which multiple tasks can be in progress at any instant. Or in multiple-THREADS!
- △ Parallel: A program is one in which multiple tasks *cooperate closely* to solve a problem.
- △ Distributed: A program may need to cooperate with other programs to solve a problem.

Pacheto An instroduction to Parallel Programming, Elsevier (2011)

The idea?

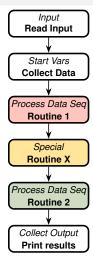


- The idea is to identify opportunities of parallelism
- Develop the application to exploit parallelism
- Run the application: identify Bugs and Improvements

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► Use resources efficiently

The idea?

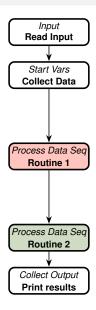


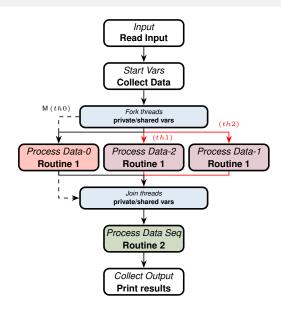
- The idea is to identify opportunities of parallelism
- Develop the application to exploit parallelism
- Run the application: identify Bugs and Improvements
- Use resources efficiently

API and Libraries

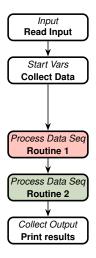
- * Pthreads (HARD): routines & variables
- ⋆ OpenMP: pragmas, routines & variables
- * Comes for C/C++ and Fortran

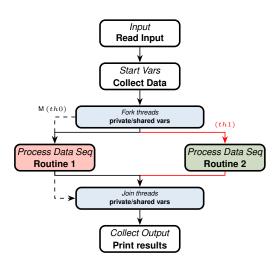
The idea parallelized: Data





The idea parallelized: Task





Ahmdal's Law

Due to the overheads, parallelism is only achieved at a certain level. Let t be the execution time of a sequential application, then

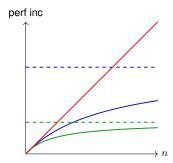
$$t = t_s + t_p$$

with t_s and t_p the time of sequential and parallel parts. If the data or tasks are going to be split equally into n threads, then the new time t' is written as,

$$t' = t_s + n \, \delta t_{\mathsf{oh}} + \frac{t_p}{n},$$

where $\delta t_{\rm oh}$ is the overhead time required to fire up each thread (approximately linear). Defining $f:=\frac{t_s}{t_n}$,

$$\mathrm{perf\,inc} = \frac{t}{t'} = n \frac{1+f}{1+n\,f+n^2\delta t_{\mathrm{oh}}} \leqslant 1 + \frac{1}{f}$$



Data environment/Declarations

- 1. #pragma omp
- 2. #pragma omp threadprivate
- 3. shared/*private
- 4. clauses
- 5. master/threads

Parallel construct structures

1. omp parallel

clauses / options

- **1.** if(...)
- 2. num threads(...)
- **3**. private(...)
- **4.** shared(...)
- 5. firstprivate(...): x(t = 0) = x before construct
- 6. default(...): none|shared
- **7.** copyin(...):
 - x private on master copied to threads privates
- 8. reduction(operator|list)

Data environment/Declarations

- 1. #pragma omp
- 2. #pragma omp threadprivate
- 3. shared/*private
- 4 clauses
- 5. master/threads

Work sharing

- 1. omp [parallel] for
- 2. omp [parallel] sections
- 3. Only Fortran: omp [parallel] workshare
- 4. omp single

loop clauses / options

Can be combined with the parallel construct

- 1. private(...)
- 2. firstprivate(...)
- 3. lastprivate(...): $x(t = t_f) = x$ last "loop" value
- 4. reduction(operator|list)
- 5. ordered(...):
- if ordered construct inside parallel region!
- 6. schedule(kind[, chunk size]): static.dvnamic.guided.runtime.auto
- 7. nowait:

sections clauses / options

Can be combined with the parallel construct private+firstprivate)+lastprivate)+ reduction+nowait

single clauses / options

- 1. private+firstprivate)+nowait
- **2.** copyprivate(...): x private in thread to threads

Data environment/Declarations

- #pragma omp
- 2. #pragma omp threadprivate
- 3. shared/*private
- 4. clauses
- 5. master/threads

atomic clauses / options

- 1. read(...)
- $\textbf{2.} \quad \text{write}(\dots)$
- **3**. update(...)
- capture(...)

Synchronization

- omp barrier: Look semaphores
- 2. omp ordered: only for loops!
- 3. omp critical [(name)]:
 is a block!
 Could use hints!
- 4. omp atomic: only a statement < 3.1!
- 5. omp master:
 no barrier at the end!
- 6. omp flush: enforces data consistency relaxed consistency model
- 7. omp task: called from within single construct
- 8. omp taskwait

Data environment/Declarations

- 1. #pragma omp
- 2. #pragma omp threadprivate
- 3. shared/*private
- 4. clauses
- 5. master/threads

Data environment

- 1. Routines (or functions)
- 2. ENVIRONMENT VARIABLES

useful routines

- omp_set_max_threads()
- 2. omp_get_max_threads()
- 3. omp_get_num_threads()
- 4. omp_get_num_devices()
- 5. omp get thread num()
- 6. omp get thread limit()
- 7. omp_set_nested()
- 8. omp_get_nested()
- 9. omp_get_schedule()
- 10. omp_get_wtime()
- 11. omp_in_parallel()
- 12. omp_init_lock()
- 13. omp_init_nest_lock()
- 14. omp_destroy_lock()
- 15. omp_destroy_nest_lock()
- 16. omp_test_lock()
- 17. omp_test_nest_lock()
- 18. omp_set_lock()
- 19. omp_set_nest_lock()
- 20. omp_unset_lock()
- 21. omp_unset_nest_lock()

Data environment/Declarations

- #pragma omp
- 2. #pragma omp threadprivate
- 3. shared/*private
- 4. clauses
- 5. master/threads

useful ENV VARS

- 1. OMP_THREAD_LIMIT
- 2. OMP_NUM_THREADS
- 3. OMP_DYNAMIC
- 4. OMP_NESTED
- OMP_SCHEDULE

Data environment

- 1. Routines (or functions)
- 2. ENVIRONMENT VARIABLES

Look at the reference guide!

How do we compile?

OpenMP version

"As of GCC 4.2, the compiler implements version 2.5 of the OpenMP specification, as of 4.4 it implements version 3.0 and since GCC 4.7 it supports the OpenMP 3.1 specification. GCC 4.9 supports OpenMP 4.0 for C/C++, GCC 4.9.1 also for Fortran. GCC 5 adds support for Offloading."

```
GCC: gcc -std=c++11 ex6-loop-reduction.cpp -o exe
-fopenmp
Intel Compiler: icc -std=c++11 ex6-loop-reduction.cpp -o exe
-gopenmp
```

JP Mallarino (U Andes) | learning OpenMP | June 10, 2019 | 11 / 37

Simple example #1: ex1-hostname

Source Code 1: Printing hostname with Master or Single!

```
#define INFO BUFFER SIZE 1024
   int main(int argc, char *argv[]) {
   #ifdef _OPENMP
       printf("**MESSAGE** OpenMP enabled\n");
4
       (void) omp_set_dynamic(FALSE);
5
       (void) omp set num threads(4);
6
   #e1se
       printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
       char hostname[INFO BUFFER SIZE];
10
11
       char username[INFO BUFFER SIZE];
12
       #pragma omp parallel
13
       gethostname (hostname, INFO BUFFER SIZE);
14
       getlogin r(username, INFO BUFFER SIZE);
15
       printf("Hostname %s in thread
16

→ %d\n", hostname, omp_get_thread_num());
17
       printf("Username %s in thread
        ⇔ %d\n", username, omp_get_thread_num());
18
       return 0;
19
20
```

Notice

- 1. The region enclosed in #ifdef
- Manually setting the # of threads
- 3. Delared the parallel construct (I.12-18)
- 4. We are using printf
- 5. Prints the thread numbers [0, 1, 2, 3]

Simple example #1: ex1-hostname

Source Code 2: Printing hostname with Master or Single!

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- 3. Delared the parallel construct (I.12-18)
- 4. We are using printf
- 5. Prints the thread numbers [0, 1, 2, 3]

Problem

1. Disordered output! std::cout



Simple example #2: ex2-master-single

Source Code 3: Printing thread number and Sleep!

```
#define INFO BUFFER SIZE 1024
   int main(int argc, char *argv[]){
   #ifdef OPENMP
       printf("**MESSAGE** OpenMP enabled\n");
4
       (void) omp_set_dynamic(FALSE);
5
       (void) omp set num threads(4);
6
   #e1se
       printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
       #pragma omp parallel
10
11
12
           int TID = omp_get_thread_num();
13
           sleep(omp_get_thread_num());
14
           printf("In parallel region - Thread ID is
15
             \hookrightarrow %d\n",TID);
       } /*-- End of parallel region --*/
16
       return 0:
18
```

Notice

- 1. The region enclosed in #ifdef
- Manually setting the # of threads
- 3. Delared the parallel construct (l.10-16)
- 4. Sets the sleep according to thread #

Simple example #2: ex2-master-single

Source Code 4: Printing thread number and Sleep!

```
int extra_time = 0;
        #pragma omp parallel shared(extra time)
3
            int TID = omp get thread num();
4
            #pragma omp master
5
6
                printf("\tInside Block - Thread ID is
                  \hookrightarrow %d\n",TID);
                sleep(1);
8
                extra time = 1;
10
            sleep(TID+extra_time);
11
12
            printf("In parallel region - Thread ID is
             \hookrightarrow %d\n",TID);
        } /*-- End of parallel region --*/
13
```

Notice

- 1. The region enclosed in #ifdef
- Manually setting the # of threads
 Delared the parallel construct
- (l.10-16)
- 4. Sets the sleep according to thread #

Problem

Cause: value not updated soon enough

Simple example #2: ex2-master-single

Source Code 5: Printing thread number and Sleep!

```
int extra time = 0:
        #pragma omp parallel shared(extra time)
3
            int TID = omp get thread num();
4
            #pragma omp single
5
6
                printf("\tInside Block - Thread ID is
                  \hookrightarrow %d\n",TID);
                sleep(1);
8
                extra time = 1;
10
            sleep(TID+extra_time);
11
12
            printf("In parallel region - Thread ID is
             \hookrightarrow %d\n",TID);
        } /*-- End of parallel region --*/
13
```

Notice

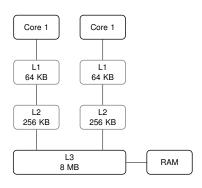
- 1. The region enclosed in #ifdef
- Manually setting the # of threads
 Delared the parallel construct
- (l.10-16)
- 4. Sets the sleep according to thread #

Problem

1. Corrected

What is the issue?

- ► Each thread is a copy of the original th0
- Private variables have redundant "private" memmory addresses
- Used values are load to cache! Not all cache is shared
- Updates have to be enforced for data coherency!
- ▶ How? flush or barrier.



Simple example #3: ex3-copy

Source Code 6: Private vars and firstprivate!

```
#define INFO BUFFER SIZE 1024
  int main(int argc, char *argv[]){
   #ifdef OPENMP
      printf("**MESSAGE** OpenMP enabled\n");
4
       (void) omp_set_dynamic(FALSE);
5
       (void) omp set num threads(4);
6
   #e1se
      printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
      int x = 10;
10
11
      printf("thread %d original value:
        12
       #pragma omp parallel private(x)
13
14
          int TID = omp get thread num();
15
          printf("thread %d value: %d\n", TID, x);
       } /*-- End of parallel region --*/
16
17
      printf("thread %d after value:
        ⇔ %d\n", omp_get_thread_num(), x);
18
      return 0;
19
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- 3. Delared the parallel construct (l.11-19)
- 4. Prints zero inside parallel construct!
- Prints again 10 after parallel construct!

Problem

1. Private value restarts at ZERO!

Simple example #3: ex3-copy

Source Code 7: Private vars and firstprivate!

```
#pragma omp parallel firstprivate(x)

{

int TID = omp_get_thread_num();

printf("thread %d value: %d\n",TID,x);

} /*-- End of parallel region --*/
```

Notice

- 1. The region enclosed in #ifdef
- Manually setting the # of threads
- 3. Delared the parallel construct (I.11-19)
- 4. Prints zero inside parallel construct!
- Prints again 10 after parallel construct!

Problem

1. Corrected, prints x = 10.

Simple example #4: ex4-shared

Source Code 8: Updating variable!

```
#define INFO BUFFER SIZE 1024
   int main(int argc, char *argv[]){
   #ifdef OPENMP
       printf("**MESSAGE** OpenMP enabled\n");
4
       (void) omp_set_dynamic(FALSE);
5
       (void) omp set num threads(4);
6
   #e1se
       printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
       int x = 10;
10
11
       #pragma omp parallel
12
13
           int TID = omp_get_thread_num();
           x += 5;
14
           printf("thread %d value: %d\n", TID, x);
15
16
       } /*-- End of parallel region --*/
17
       printf("Value: %d\n",x);
18
       return 0:
19
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Delared the parallel construct (I.11-16)
- 4. We are using printf
- 5. All threads add to \boldsymbol{x}
- 6. x is shared

Problem

- Every execution prints different values
- 2. RACE CONDITION

Simple example #5: ex5-loop

Source Code 9: Adding arrays!

```
#define INFO BUFFER SIZE 1024
   int main(int argc, char *argv[]){
   #ifdef _OPENMP
       printf("**MESSAGE** OpenMP enabled\n");
4
        (void) omp_set_dynamic(FALSE);
5
        (void) omp set num threads(4);
6
   #e1se
       printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
       Crandom r(10);
10
11
       double *v1.*v2.*res:
12
       CREATEARRAY (v1, double, N, r, gaussian (1, 2));
13
       CREATEARRAY (v2, double, N, -1);
       CREATEARRAY (res, double, N, 0);
14
15
       PRINTARRAY (v1, N);
16
       PRINTARRAY (v2, N);
       for (int j = 0; j < N; j++)
18
            res[i] = v1[i] + v2[i];
       PRINTARRAY (res.N);
19
20
       return 0;
21
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. No parallel construct
- 5. Prints the arrays v1, v2 and v1 + v2

Simple example #5: ex5-loop

Source Code 10: Adding arrays!

```
SimpleTimer_t t1;
       PRINTARRAY (v1, N);
3
       PRINTARRAY (v2, N);
       SimpleTimer start ( &t1 );
4
       #pragma omp parallel shared(j,N,v1,v2,res)
5
6
7
           #pragma omp for
           for (int j = 0; j < N; j++)
8
                res[i] = v1[i] + v2[i];
9
10
       SimpleTimer_stop( &t1 );
11
12
       SimpleTimer_print( &t1 );
13
       PRINTARRAY (res.N);
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. No parallel construct
- 5. Prints the arrays v1, v2 and v1 + v2

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Problem

1. Any comments?

Simple example #6: ex6-loop-reduction

Source Code 11: Norm of a vector!

```
#define INFO BUFFER SIZE 1024
   int main(int argc, char *argv[]){
   #ifdef OPENMP
       printf("**MESSAGE** OpenMP enabled\n");
4
        (void) omp_set_dynamic(FALSE);
5
        (void) omp set num threads(4);
6
   #e1se
       printf("**MESSAGE** OpenMP disabled\n");
8
   #endif
       int j, N = 200;
10
11
       Crandom r(10):
12
       double *v1;
13
       CREATEARRAY (v1, double, N, r.gaussian (0, 0.5));
       PRINTARRAY (v1, N);
14
       double norm v1 = 0;
15
16
       for (j = 0; j < N; j++)
17
            norm_v1 += v1[j]*v1[j];
18
       printf("v1 norm = %lf\n", norm v1);
       return 0:
19
20
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- No parallel construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

Simple example #6: ex6-loop-reduction

Source Code 12: Norm of a vector!

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. Parallel workshare construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

Problem

Norm doesn't provide reliable results!

Source Code 13: Norm of a vector!

```
3
            double tmp = 0;
4
            #pragma omp for
            for (j = 0; j < N; j++) {
5
                tmp = v1[j]*v1[j];
6
                #pragma omp critical
8
                    norm_v1 += tmp;
9
10
11
12
        } /*-- End of parallel region --*/
13
       // OR...
        #pragma omp parallel for shared(N, norm v1, v1)
14
       for(j = 0; j < N; j++)
15
16
            #pragma omp critical
            norm_v1 += v1[j]*v1[j];
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. Parallel workshare construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

Problem

1. Solved with: critical

Source Code 14: Norm of a vector!

```
3
           double tmp = 0;
4
           #pragma omp for
           for (j = 0; j < N; j++) {
5
                tmp = v1[j]*v1[j];
6
                #pragma omp atomic
                norm v1 += tmp;
8
9
       } /*-- End of parallel region --*/
10
11
       // OR...
12
       #pragma omp parallel for shared(N, norm_v1, v1)
13
       for (j = 0; j < N; j++)
14
           #pragma omp atomic
           norm v1 += v1[i]*v1[i];
15
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. Parallel workshare construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

Problem

1. Solved with: atomic

Source Code 15: Norm of a vector!

```
3
           #pragma omp for reduction(+:norm v1)
           for (j = 0; j < N; j++)
4
                norm_v1 += v1[j]*v1[j];
5
       } /*-- End of parallel region --*/
6
       // OR...
       #pragma omp parallel for shared(N, norm v1, v1)
8

    reduction(+:norm v1)

       for (j = 0; j < N; j++)
9
10
           norm_v1 += v1[j] *v1[j];
```

Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. Parallel workshare construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

Problem

1. Solved with: reduction

Source Code 16: Norm of a vector!

```
3
            double tmp = 0;
            #pragma omp for ordered
4
            for (j = 0; j < N; j++) {
5
                tmp = v1[j]*v1[j];
6
                #pragma omp ordered
8
9
                    norm v1 += tmp;
10
11
12
       } /*-- End of parallel region --*/
13
       // OR...
       #pragma omp parallel for shared(N, norm v1, v1)
14
         → ordered
15
       for (j = 0; j < N; j++)
            #pragma omp ordered
16
            norm_v1 += v1[j] *v1[j];
```

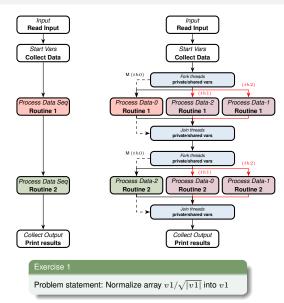
Notice

- 1. The region enclosed in #ifdef
- 2. Manually setting the # of threads
- Notice the strange definitions! defs.h
- 4. Parallel workshare construct
- 5. Prints the quadratic norm of array |v1| = 59.725...

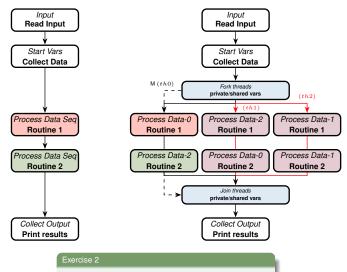
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Problem

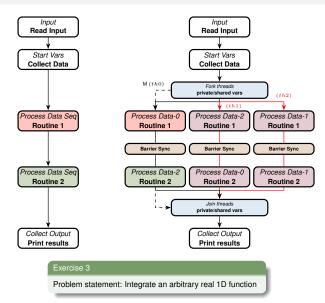
1. Solved with: ordered

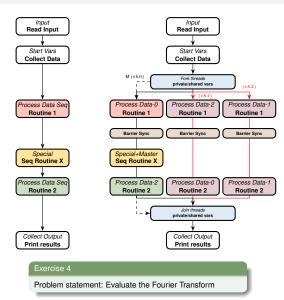


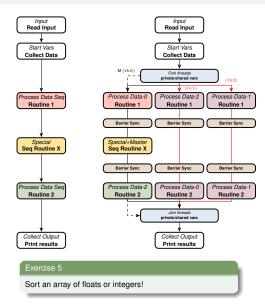
JP Mallarino (U Andes) June 10, 2019

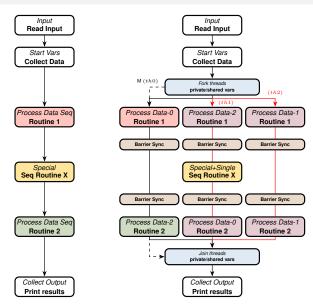


Problem statement: Find the determinant and inverse of a matrix



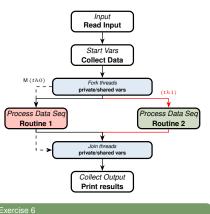






Back to the parallelized Task idea: ex7-task-palindrome/ex8-fibonacci

Source Code 17: The word palindrome!



Exercise 6 Problem statement: Find random numbers using all threads

```
printf("A ");
       printf("race ");
       printf("car ");
       printf("is fun to watch.\n");
       #pragma omp parallel
8
                printf("A ");
10
                #pragma omp task
                { printf("race "); }
13
                #pragma omp task
                { printf("car "); }
14
                #pragma omp taskwait
15
                printf("is fun to
16

    watch.\n");

17
18
```

Bibliography

Special thanks to ALL of you!

- ▶ USING OpenMP The Next Step, Ruud van der Pas, et. al., MIT Press (2017)
- Introduction to Parallel Programming, Peter S. Pacheco, Elsevier (2011)
- Deep Learning, Ian Goodfellow, et. al., MIT Press (2016)
- ▶ Structured Parallel Programming, Michael McCool, et. al., Elsevier (2012)

(Bonus-Challenge): Machine Learning RBM's

Problem statement: Evaluate $\langle E \rangle$ for a distribution $\rho = \frac{1}{\overline{Z}} e^{-E}$ with

$$E(\mathbf{v}, \mathbf{h}) = -\mathbf{b}^{\mathsf{T}} \mathbf{v} - \mathbf{c}^{\mathsf{T}} \mathbf{h} - \mathbf{v}^{\mathsf{T}} \mathbf{W} \mathbf{h}.$$