Chapter 7 – Autopilot Models for Course and Heading Control

7.1 Autopilot Models for Course Control
7.2 Autopilot Models for Heading Control

Automatic pilot, “also called autopilot, device for controlling an aircraft or other vehicle without constant human intervention” (Encyclopedia Britannica)

Automatic pilot, or autopilot, is a system that controls the trajectory and behavior of a vehicle or machine without constant manual input, using pre-programmed instructions and sensors (ChatGPT – AI Language Model)

- The first aircraft autopilot was developed by the Sperry Corporation in 1912. It permitted the aircraft to fly straight and level on a compass course without the pilot's attention, greatly reducing the pilot's workload.
- Lawrence Sperry, the son of the famous inventor Elmer Sperry, demonstrated it in 1914 at an aviation safety contest held in Paris. Sperry showcased the credibility of the invention by flying the aircraft with his hands away from the controls. He was killed in 1923 when his aircraft crashed in the English Channel.
- In the early 1920s, the Standard Oil tanker J.A. Moffet became the first ship to use an autopilot (Wikipedia)

Elmer Ambrose Sperry, Sr. (1860–1930)
Alchetron - Free Social Encyclopedia for the World

Lawrence Burst Sperry (1892—1923)
Wiki Commons

Chapter Goals

- Be able to explain the differences between course and heading controlled marine craft. In what applications are they used?
- Understand what the crab angle is for a marine craft:

$$\chi = \psi + \beta_c$$

- Understand why heading control is used instead of course control during stationkeeping.
- Be able to compute the COG using two waypoints
- Know what kind of sensors directly measure SOG and COG underwater and on the surface.
- Understand the well-celebrated Nomoto model for heading and course control, and it is extension to nonlinear theory (maneuvering characteristics)

$$\frac{r}{\delta}(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)} \qquad \frac{r}{\delta}(s) = \frac{K}{Ts + 1} \qquad \chi(s) = \frac{K}{s(Ts + 1)}\tau_6(s) + \beta_c(s)$$

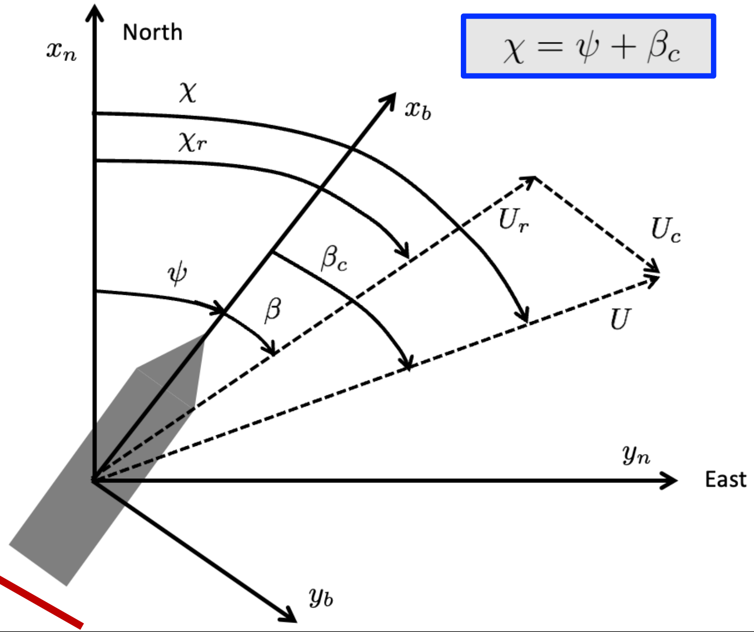
- Be able to explain what the pivot point is.

Recap: 2-Dimensional Amplitude-Phase Form

We want to prove the famous relationship (from the figure)

Course angle = heading (yaw) angle + crab angle

$$\boxed{\chi = \psi + \beta_c} \qquad \beta_c = \tan^{-1}\left(\frac{v}{u}\right)$$



Proof:

$$\dot{x}^n = u\cos(\psi) - v\sin(\psi)$$
$$\dot{y}^n = u\sin(\psi) + v\cos(\psi)$$

These equations can be expressed in amplitude-phase form

$$\dot{x}^n = U\cos(\psi + \beta_c) := U\cos(\chi)$$
$$\dot{y}^n = U\sin(\psi + \beta_c) := U\sin(\chi)$$

Amplitude (speed):

$$U = \sqrt{u^2 + v^2}$$

Phase (crab angle):

$$\beta_c = \tan^{-1}\left(\frac{v}{u}\right)$$

$$\begin{aligned} y_1 &= c_1\sin(x) + c_2\cos(x) \\ &= A\sin(x)\cos(\varphi) + A\cos(x)\sin(\varphi) \\ &= A\sin(x + \varphi) \end{aligned}$$

$$c_1 = A\cos(\varphi), \; c_2 = A\sin(\varphi) \qquad \begin{aligned} A &= (c_1^2 + c_2^2)^{1/2} \\ \varphi &= \tan^{-1}(c_2/c_1) \end{aligned}$$

Chapter 7 – Autopilot Models for Course and Heading Control

It is important to stress the concepts for course and heading control since there are many conceptual misunderstandings regarding the course and heading of an aircraft and marine craft.

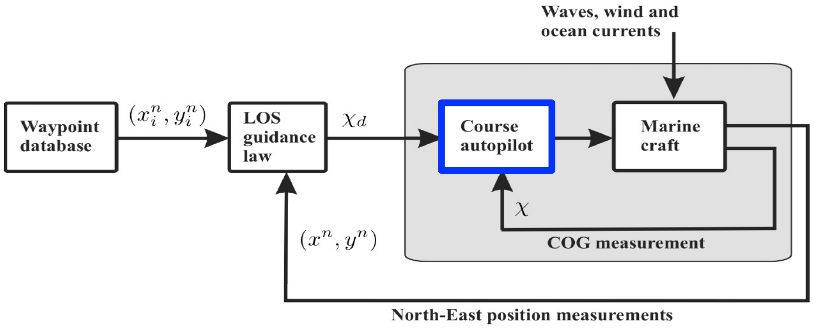
- The course angle χ of a vehicle is the cardinal direction in which the vehicle is moving, $\chi = \psi + \beta_c$
- The heading angle ψ, is the direction the craft's bow (x_b axis) is pointed

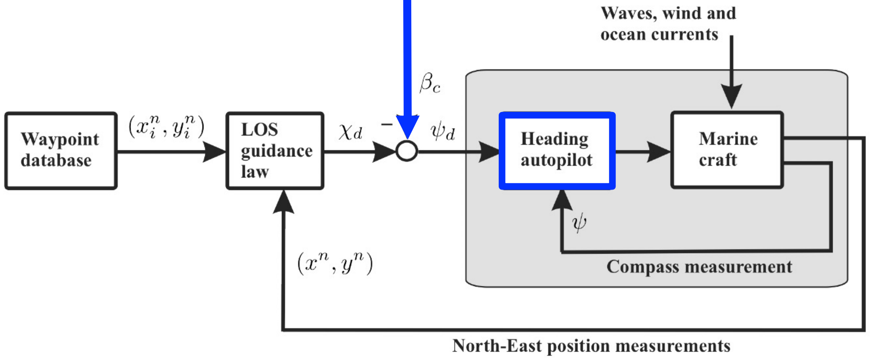
If your vehicle is exposed to an environmental force, the sway velocity v will be non-zero. This gives a non-zero crab angle β_c and the vehicle is said to **sideslip**.

$$\boxed{\beta_c = \tan^{-1}\left(\frac{v}{u}\right)}$$

How does this relate to bearing?

- Absolute bearing is the direction of the craft determined by a navigation system, usually a magnetic compass or a gyrocompass. Hence, a heading autopilot use "bearing" as a measurement to control its heading angle.
- Relative bearing refers to the angle between the craft's forward direction and the location of another object.





7.1 Autopilot Models for Course Control

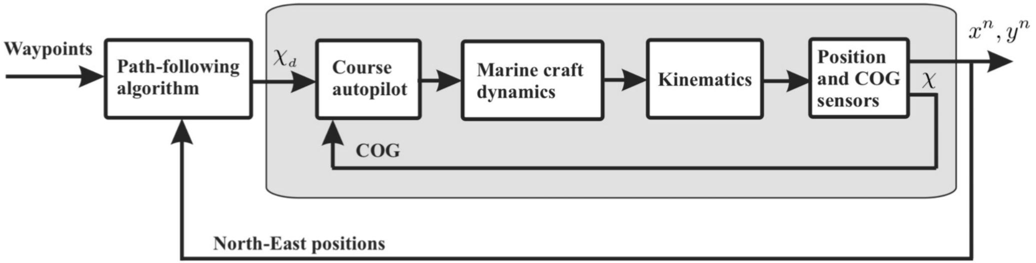
Surface craft are usually equipped with a **global navigation satellite system (GNSS)** receiver, which measures

- COG course over ground, χ
- SOG speed over ground, U

Underwater vehicles, however, use **hydroacoustic reference systems** to determine their position, velocity and course.

Modern GNSS receivers compute the SOG using the Euclidean distance between the receiver's previous position and its current position, divided by the time interval between the two positions. The COG represents the direction of the receiver's movement over the Earth's surface, measured clockwise from true North. It's calculated using the difference between the receiver's previous position and its current position. This difference is then converted into an angle using trigonometric calculations.

The SOG and COG can easily be computed from the track using a five-state extended Kalman filter (EKF) where the only measurements are the North-East positions or the latitude-longitude pair (Fossen and Fossen 2021).



$$\frac{\chi}{\chi_d} \approx 1$$

Inner-loop control objective

S. Fossen and T. I. Fossen (2021). Five-State Extended Kalman Filter for Estimation of Speed over Ground (SOG), Course over Ground (COG), and Course Rate of Unmanned Surface Vehicles (USVs): Experimental Results, Sensors 21, 7910. https://doi.org/10.3390/s21237910

7.1 Autopilot Models for Course Control

Five-state EKF Model for estimation of COG, SOG and course rate from North-East positions

$$
\begin{aligned}
\dot{x}^n &= U\cos(\chi) \\
\dot{y}^n &= U\sin(\chi) \\
\dot{U} &= -\alpha_1 U + w_1 \\
\dot{\chi} &= \omega_\chi \\
\dot{\omega}_\chi &= -\alpha_2 \omega_\chi + w_2 \\
y_1 &= x^n + \varepsilon_1 \\
y_2 &= y^n + \varepsilon_2
\end{aligned}
$$

$$
\begin{aligned}
x^n[k+1] &= x^n[k] + h\,U[k]\cos(\chi[k]) \\
y^n[k+1] &= y^n[k] + h\,U[k]\sin(\chi[k]) \\
U[k+1] &= (1-h\alpha_1)U[k] + h\,w_1[k] \\
\chi[k+1] &= \chi[k] + h\,\omega_\chi[k] \\
\omega_\chi[k+1] &= (1-h\alpha_2)\omega_\chi[k] + h\,w_2[k] \\
y_1[k] &= x^n[k] + \varepsilon_1[k] \\
y_2[k] &= y^n[k] + \varepsilon_2[k]
\end{aligned}
$$

The latitude-longitude pair can be used instead of the NE-positions in the EKF, see Fossen and Fossen (2021).

Discrete-time state-space model

$$
\begin{aligned}
\boldsymbol{x}[k+1] &= \boldsymbol{A}_d\,\boldsymbol{x}[k] + \boldsymbol{E}_d \boldsymbol{w}[k] \\
\boldsymbol{y}[k] &= \boldsymbol{C}_d\,\boldsymbol{x}[k] + \boldsymbol{\varepsilon}[k]
\end{aligned}
$$

$$
\boldsymbol{C}_d = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\boldsymbol{A}_d = \begin{bmatrix}
1 & 0 & h\cos(\hat{x}_4[k]) & -h\,\hat{x}_3[k]\sin(\hat{x}_4[k]) & 0 \\
0 & 1 & h\sin(\hat{x}_4[k]) & h\,\hat{x}_3[k]\cos(\hat{x}_4[k]) & 0 \\
0 & 0 & 1-h\alpha_1 & 0 & 0 \\
0 & 0 & 0 & 1 & h \\
0 & 0 & 0 & 0 & 1-h\alpha_2
\end{bmatrix}, \quad
\boldsymbol{E}_d = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ h & 0 \\ 0 & 0 \\ 0 & h \end{bmatrix},
$$

S. Fossen and T. I. Fossen (2021). Five-State Extended Kalman Filter for Estimation of Speed over Ground (SOG), Course over Ground (COG), and Course Rate of Unmanned Surface Vehicles (USVs): Experimental Results, Sensors 21, 7910. https://doi.org/10.3390/s21237910

7.1 Autopilot Models for Course Control

State-space model and measurement equations

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}$$

$$\dot{\boldsymbol{\nu}} = \begin{bmatrix} r v_c \\ -r u_c \\ 0 \end{bmatrix} + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}\boldsymbol{\nu}_r - \boldsymbol{D}_n(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r\right)$$

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - [u_c,\, v_c,\, 0]^\top$$

$$y_1 = x^n$$

$$y_2 = y^n$$

$$y_3 = \sqrt{u^2 + v^2}$$

$$y_4 = \psi + \operatorname{asin}\left(\frac{v}{U}\right)$$

Measurements:

- GNSS x position
- GNSS y position
- SOG (optionally)
- COG (optionally)

$$\beta_c = \sin^{-1}(v/U)$$

Not defined for $U = 0$. This term can be modeled as a disturbance/bias corresponding to a (nearly) constant crab angle β_c in the Kalman filter

The control law must compensate the drift force, that is the ocean current velocities, by adding integral action.

The drift force must also be estimated by the Kalman filter by augmenting the current velocities as unknown states to the state-space model.

7.1 Autopilot Models for Course Control

Decoupled course/yaw dynamics (mass—damper system)

$$\dot{\chi} = r + \dot{\beta}_c$$
$$(I_z - N_{\dot{r}})\dot{r} - N_r r = \tau_6$$

β_c is treated as a disturbance,

The control input τ_6 is a yaw moment generated by propellers/rudders

Yaw angle transfer function (Nomoto et al. 1957)

$$\frac{r}{\tau_6}(s) = \frac{K}{Ts+1}$$

$$K = 1/(-N_r)$$
$$T = (I_z - N_{\dot{r}})/(-N_r)$$

Course angle transfer function

$$\chi(s) = \frac{K}{s(Ts+1)}\tau_6(s) + \beta_c(s)$$

The control law can compensate the crab angle β_c by direct measurements, but this solution is not robust at small speeds. Integral action is still needed due to model uncertainty, so the preferred method is to model the crab angle as a disturbance (bias term) and use integral action to compensate the disturbance.

Nomoto, K., T. Taguchi, K. Honda and S. Hirano (1957). On the Steering Qualities of Ships. Technical Report. International Shipbuilding Progress, Vol. 4.

7.2 Autopilot Models for Heading Control

Second-order Nomoto model

$$\boxed{\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{N}\boldsymbol{\nu}_r = \boldsymbol{b}\,\delta}$$

$$\boldsymbol{\nu}_r = [v_r,\ r]^\top$$

$$r = \boldsymbol{c}^\top \boldsymbol{\nu}_r, \quad \boldsymbol{c}^\top = [0,\ 1]$$

$$\boldsymbol{M} = \begin{bmatrix} m - Y_{\dot{v}} & mx_g - Y_{\dot{r}} \\ mx_g - Y_{\dot{r}} & I_z - N_{\dot{r}} \end{bmatrix}$$

$$\boldsymbol{N} = \begin{bmatrix} -Y_v & (m - X_{\dot{u}})U - Y_r \\ (X_{\dot{u}} - Y_{\dot{v}})U - N_v & (mx_g - Y_{\dot{r}})U - N_r \end{bmatrix}$$

$$\boldsymbol{b} = \begin{bmatrix} -Y_\delta \\ -N_\delta \end{bmatrix}$$

$$\frac{r}{\delta}(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)}$$

Measurements:

- Compass (ψ)
- SOG (U)

The normalized model (see Appendix D) can be used as basis for gain-scheduling control by choosing *U* as scheduling variable. Consequently, the PID controller gains will be functions of the model parameters and the direct measurement *U*.

$$\left(\frac{L}{U}\right)^2 T_1' T_2' \psi^{(3)} + \left(\frac{L}{U}\right)(T_1' + T_2')\,\ddot{\psi} + \dot{\psi} = K' T_3' \dot{\delta} + \left(\frac{U}{L}\right) K' \delta$$

Nomoto, K., T. Taguchi, K. Honda and S. Hirano (1957). On the Steering Qualities of Ships. Technical Report. International Shipbuilding Progress, Vol. 4.

7.2 Autopilot Models for Heading Control

Second-order Nomoto model

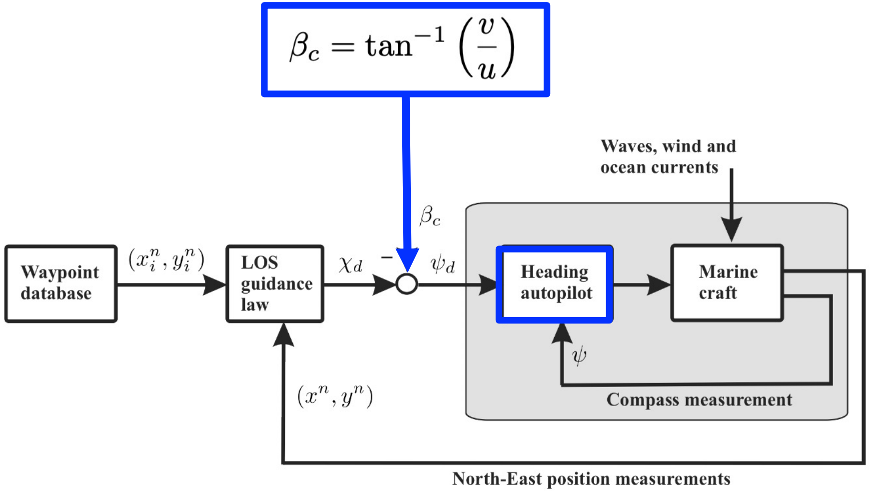
$$\frac{r}{\delta}(s) = \frac{K(T_3 s + 1)}{(T_1 s + 1)(T_2 s + 1)}$$

$$T := T_1 + T_2 - T_3$$

First-order Nomoto model

$$\frac{r}{\delta}(s) = \frac{K}{Ts + 1}$$

$$\frac{\psi}{\delta}(s) = \frac{K}{s(Ts + 1)}$$



The normalized model (see Appendix D) can be used as basis for gain-scheduling control by choosing *U* as scheduling variable

$$\left(\frac{L}{U}\right) T' \ddot{\psi} + \dot{\psi} = \left(\frac{U}{L}\right) K' \delta$$

Nomoto, K., T. Taguchi, K. Honda and S. Hirano (1957). On the Steering Qualities of Ships. Technical Report. International Shipbuilding Progress, Vol. 4.

7.2 Autopilot Models for Heading Control

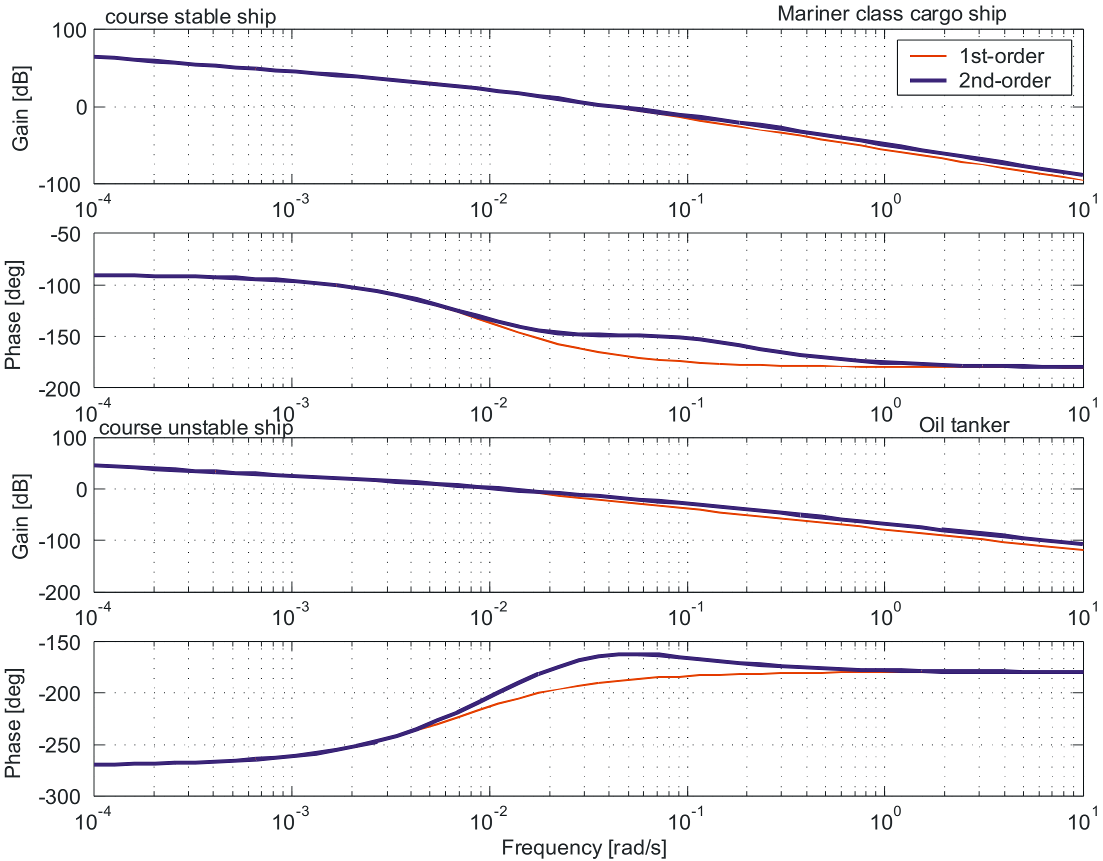
Matlab:
The Bode diagram is generated by using the MSS toolbox commands:

```
T1 = 118; T2 = 7.8; T3 = 18.5; K = 0.185;
nomoto(T1, T2, T3, K)

T1 = -124.1; T2 = 16.4; T3 = 46.0; K = -0.019;
nomoto(T1, T2, T3, K);

function nomoto(T1,T2,T3,K);
% NOMOTO(T1,T2,T3,K) generates the Bode plots for
%              K                          K (1+T3s)
% H1(s) = ------------      H2(s) = --------------------
%             s(1+Ts)                    s(1+T1s)(1+T2s)

T = T1 + T2 - T3;

d1 = [T 1 0]; n1 = K;
d2 = [T1*T2 T1+T2 1 0]; n2 = K*[T3 1];
[mag1,phase1,w] = bode(n1,d1);
[mag2,phase2]   = bode(n2,d2,w);

% shift phase with 360 deg for course-unstable ship
if K < 0
    phase1 = phase1-360;
    phase2 = phase2-360;
end

subplot(211),semilogx(w,20*log10(mag1)),grid;
xlabel('Frequency [rad/s]'),title('Gain [dB]');
hold on,semilogx(w,20*log10(mag2),'--'),hold off;

subplot(212),semilogx(w,phase1),grid;
xlabel('Frequency [rad/s]'),title('Phase [deg]');
hold on,semilogx(w,phase2,'--'),hold off;
```

Table 7.1: Parameters for a cargo ship and a fully loaded oil tanker

	L (m)	u_0 (m/s)	∇ (dwt)	K (1/s)	T_1 (s)	T_2 (s)	T_3 (s)
Cargo ship	161	7.7	16622	0.185	118.0	7.8	18.5
Oil tanker	350	8.1	389100	−0.019	−124.1	16.4	46.0

7.2 Autopilot Models for Heading Control



7.2 Autopilot Models for Heading Control

The linear Nomoto models can be extended to include nonlinear effects by adding static nonlinearities referred to as maneuvering characteristics.

Nonlinear Extension of Nomoto's 1st-order Model (Norrbin 1963)

$$T\dot{r} + H_N(r) = K\delta$$

$$H_N(r) = n_3 r^3 + n_2 r^2 + n_1 r + n_0$$

where $H_N(r)$ is a nonlinear function describing the maneuvering characteristics.
For $H_N(r) = r$, the linear model is obtained.

Nonlinear Extension of Nomoto's 2nd-order Model (Bech and Wagner-Smith 1969)

$$T_1 T_2 \ddot{r} + (T_1 + T_2)\dot{r} + K H_B(r) = K(\delta + T_3\dot{\delta})$$

$$H_B(r) = b_3 r^3 + b_2 r^2 + b_1 r + b_0$$

where $H_B(r)$ can be found from Bech's reverse spiral maneuver. The linear equivalent is obtained for $H_B(r) = r$

- **Norrbin, N. H. (1963).** On the Design and Analyses of the Zig-Zag Test on Base of Quasi Linear Frequency Response. Technical Report B 104-3. The Swedish State Shipbuilding Experimental Tank (SSPA). Gothenburg, Sweden.
- **Bech, M. I. and L. Wagner Smith (1969).** Analogue Simulation of Ship Maneuvers. Technical Report Hy-14. Hydro- and Aerodynamics Laboratory. Lyngby, Denmark.

7.2 Pivot Point

The pivot point of a ship, also known as the turning point or the point of rotation, is a critical concept in ship navigation and maneuvering. It refers to the specific location within the ship around which the ship rotates or pivots when making turns.

Definition (Pivot Point) A ship's pivot point x_{np} is a point on the centerline measured from the CG from the CG at which sway, and yaw completely cancel each other (Tzeng 1998)

$$v_{np} = v_{ng} + x_p r \equiv 0$$

The **pivot point** is not a physical point on the ship but a theoretical point that represents the average location of all the forces acting on the ship's hull as it turns.

The pivot point can be computed by measuring the sway velocity $v_{ng}(t)$ at the CG and the yaw rate $r(t)$ *such that*

$$x_p(t) = -\frac{v_{ng}(t)}{r(t)}, \quad r(t) \neq 0$$

This expression is not defined for a zero-yaw rate corresponding to a straight-line motion. This means that the pivot point is located at infinity when moving on a straight line or in a pure sway motion.

It is well known to the pilots that the pivot point of a turning ship is located at about 1/5~1/4 ship length aft of bow

Tzeng, C. Y. (1998). Analysis of the Pivot Point for a Turning Ship. *Journal of Marine Science and Technology* 6(1), 39–44.

NTNU

7.2 Pivot Point

The Nomoto transfer functions in sway and yaw gives the steady-state gain

$$\frac{v}{r} = \frac{K_v(1 + T_v s)}{K(1 + T_3 s)} \overset{s=0}{=} \frac{K_v}{K}$$

$$\frac{v}{\delta}(s) = \frac{K_v(1+T_v s)}{(1+T_1 s)(1+T_2 s)}$$

$$\frac{r}{\delta}(s) = \frac{K(1 + T_3 s)}{(1 + T_1 s)(1 + T_2 s)}$$

Steady-state location of the pivot point

$$x_{p,ss} = -\frac{K_v}{K}$$

$$x_p(t) = -\frac{v_{ng}(t)}{r(t)}, \quad r(t) \neq 0$$

$$\boxed{x_{p,ss} = -\frac{N_r Y_\delta - (Y_r - mU)N_\delta}{Y_v N_\delta - N_v Y_\delta}}$$

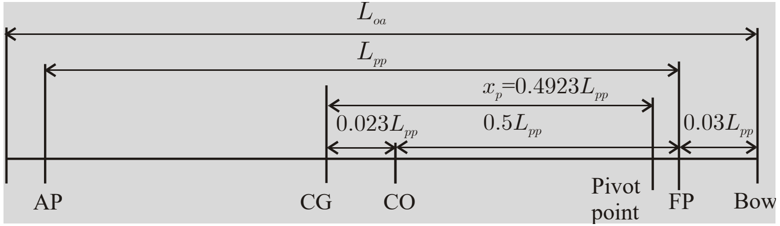
Location of the pivot point: $x_{p(s,s)} = 0.4923 L_{pp}$

Example: The Mariner Class vessel where the nondimensional linear maneuvering coefficients are given as:

$$Y'_v = -1160 \cdot 10^{-5} \qquad N'_v = -264 \cdot 10^{-5}$$

$$Y'_r - m' = -499 \cdot 10^{-5} \qquad N'_r = -166 \cdot 10^{-5}$$

$$Y'_\delta = 278 \cdot 10^{-5} \qquad N'_\delta = -139 \cdot 10^{-5}$$



Chapter Goals – Revisited

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- Be able to explain what the pivot point is.