




## KIT205 Data Structures and Algorithms Assignment 3

Due 15th October, 11:55pm

### Introduction

[Ticket to Ride](#)  is a popular board game that involves connecting cities in a network. In this assignment you will prototype some potential approaches for a player for this game (since the AI players for the computerised version are c

The basic gameplay of Ticket to Ride requires players to fulfill "tickets" that are randomly generated. Each ticket consists of two cities that need to be connected. Adjacent cities are connected by placing train tokens on the track. While there are other complications in the full game, the basis of the game is to fulfill your tickets using the least number of train tokens - this in turn allows you to fulfill more tickets with the number of train tokens available.

### Data Structures and Input

We will represent the map using the following data structures, as used in tutorials.

```
typedef struct edge{
    int to_vertex;
    int weight;
} Edge;

typedef struct edgeNode{
    Edge edge;
    struct edgeNode *next;
} *EdgeNodePtr;

typedef struct edgeList{
    EdgeNodePtr head;
} EdgeList;

typedef struct graph{
    int num_vertices;
```

```

    int v;
    EdgeList *edges;
} Graph;

```

In this case, the vertices represent the cities on the map, and the edge weight will be the number of roads required to connect adjacent cities.

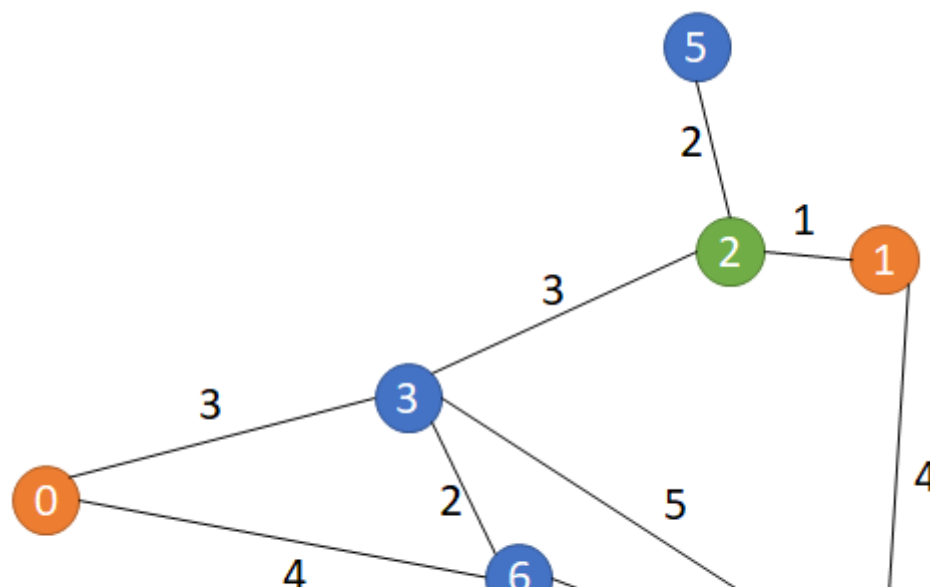
We will also use the same redirected input method used in tutorials to create the graph. In this case is *undirected*, so **two** edges need to be added for each pair of adjacent cities. i.e. if cities 2 and 7 are adjacent, an edge needs to be added from vertex 2 to 7 *and* from vertex 7 to 2. The graph input will include edges from cities with a lower index to cities with a higher index, but you must also add the edges in the opposite direction.

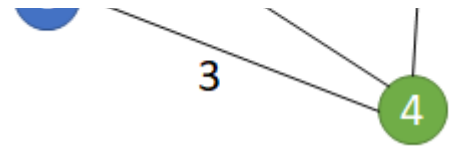
The graph input will then be followed by input for a given number of tickets. For example, create a graph of **7** cities, with **2** tickets from cities 2 to 4 and 0 to 1:

```

7
2
6, 4 3, 3
2
4, 4 2, 1
2
5, 2 3, 3
2
6, 2 4, 5
1
6, 3
0
0
2
2, 4 0, 1

```





## Part A

The problem of finding the cheapest way to fulfill tickets is obviously related to the minimum spanning tree problem. So part A is a warm-up exercise where you will implement Prim's MST algorithm. The following pseudocode should be followed:

### Prim's Minimal Spanning Tree ([from Wikipedia](#) )

1. Associate with each vertex  $v$  of the graph a number  $C[v]$  (the cheapest connection to  $v$ ) and an edge  $E[v]$  (the edge providing that cheapest connection). Initialize these values, set all values of  $C[v]$  to  $+\infty$  (or to any number greater than the maximum edge weight) and set each  $E[v]$  to a special flag value indicating no edge connecting  $v$  to earlier vertices.
2. Initialize an empty forest  $F$  and a set  $Q$  of vertices that have not yet been included in  $F$ .
3. Repeat the following steps until  $Q$  is empty:
  - a. Find and remove a vertex  $v$  from  $Q$  having the minimum possible value of  $C[v]$ .
  - b. Add  $v$  to  $F$  and, if  $E[v]$  is not the special flag value, also add  $E[v]$  to  $F$ .
  - c. Loop over the edges  $vw$  connecting  $v$  to other vertices  $w$ . For each such edge, if  $w$  belongs to  $Q$  and  $vw$  has smaller weight than  $C[w]$ , perform the following:
    - i. Set  $C[w]$  to the cost of edge  $vw$ .
    - ii. Set  $E[w]$  to point to edge  $vw$ .
4. Return  $F$ .

Your implementation must follow the above pseudocode and use the following C function:

```
Graph prims_mst(Graph *self);
```

The following assumptions and hints will be useful.

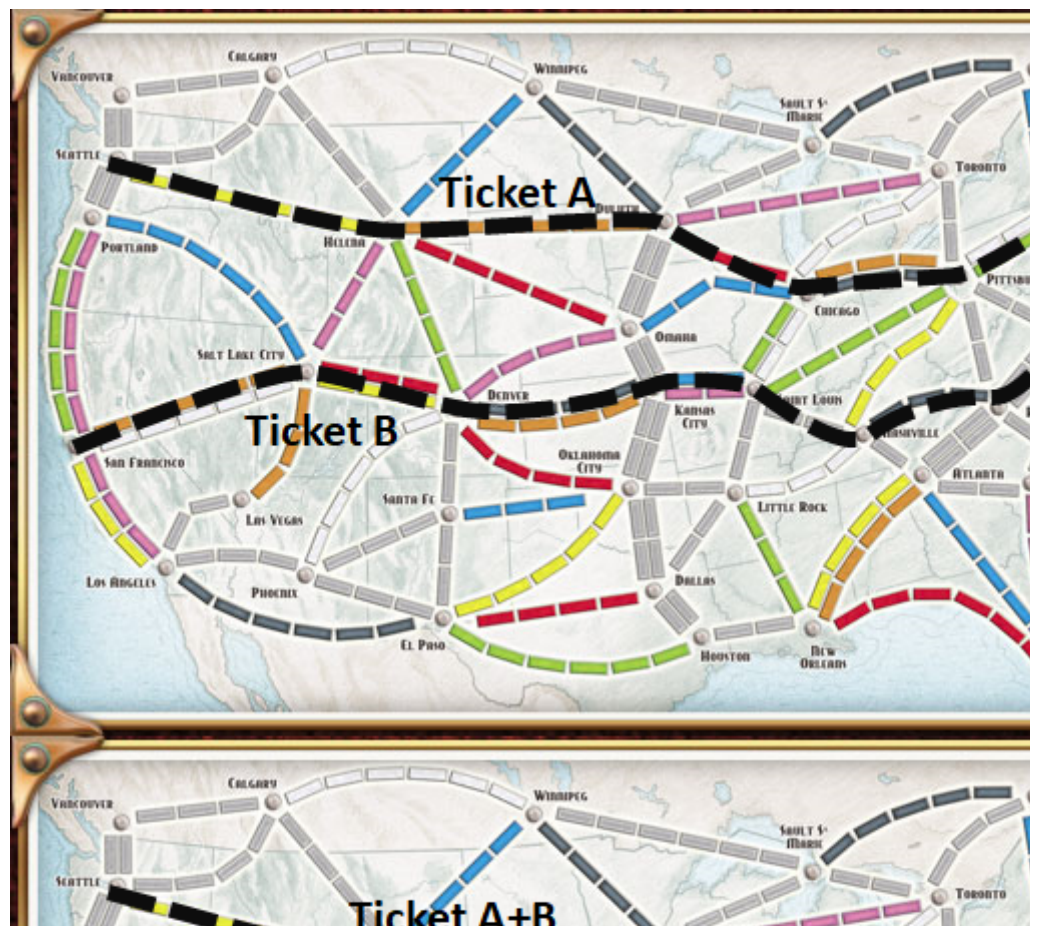
## Hints

- In the pseudocode  $F$  will be a `Graph` data structure
- Since you know that the graph is connected, you may start by initialising  $F$  in state containing all vertices (but no edges). This means that you can skip the part "Add all vertices to  $F$ " as you will only need to add the edge  $E[v]$  to  $F$
- You can combine the  $C[v]$  and  $E[v]$  lists by using an array of `Edges`
- $Q$  can also be implemented as a membership array (i.e. array representation of a set)
- You can use a simple search for step 3a. You do not need to use any of the more complex heap-based approaches.

For part A, you can ignore the tickets. You just need to return the MST for the given fully connected graph, so when you have returned the graph, you should print all of the edges that have been added and the total cost.

## Part B

You will now write an algorithm to find the cheapest way to fulfill your tickets. This is not a simple problem. For example, a naive solution might be to find the shortest path for each ticket and then add all of the edges from those paths. However, for certain tickets, this may be very wasteful, as illustrated below. In this case the top map shows the shortest paths for tickets A and B. The bottom map shows one way that these tickets can be fulfilled efficiently.





So, the ticket fulfillment problem is related to the MST problem and also to the [Steiner tree](#) problem, which is an MST for a subset of the vertices in the graph. While there are many polynomial time algorithms for the MST problem, there are surprisingly no known efficient algorithms for the Steiner tree problem - it is an [NP-hard](#) problem.

Like the Steiner tree problem, I suspect that the problem of finding the most efficient way to fulfill all tickets is NP-hard. Luckily, I do not expect you to find an exact solution. Your task is to find a good approximation. Some of the approaches for finding an [approximate solution](#) to the Steiner tree problem are listed in the following approaches that you can use here.

Any solution that finds a set of edges to fulfill all your tickets (including a correct combination of edges) will receive an HD grade for this part of the assessment. However, to get full marks for this part, you need to find a good approximation of the *optimal* set of edges. One example of a solution that would get full marks is:

1. Find an approximate solution to the Steiner tree problem of finding the minimal spanning tree of the vertices that are destinations in any of your tickets
2. Remove any edges that are not required to fulfill any ticket.

This solution would get full marks, but I am sure that you can do better!

The output for Part B should be the edges that were added and the total cost.

## Assignment Submission

Assignments will be submitted via MyLO (an Assignment 2 dropbox will be created). You should follow the following procedure to prepare your submission:

- Make sure that your project has been thoroughly tested using the School's lab computers
- Choose "Clean Solution" from the "Build" menu in Visual Studio. This step is very important as the version that the marker runs will be the same as the version that you believe the marker runs
- Quit Visual Studio and zip your entire project folder
- Upload a copy of the zip file to the MyLO dropbox

History tells us that mistakes frequently happen when following this process, so you should

- Unzip the folder to a new location
- Open the project and confirm that it still compiles and runs as expected
  - If not, repeat the process from the start

## Learning Outcomes and Assessment

This assignment is worth 15% of your overall mark for KIT205. Your submission will be assessed by running your code and by running additional tests in Visual Studio. If your submission does not run in Visual Studio on Windows, it cannot be assessed.

Grades will be assigned in accordance with the [Assignment 3 Rubric](#)

The assignment contributes to the assessment of learning outcomes:

- LO1 Transform a real-world problem into a simple abstract form that is suitable for a computer**
  - LO2 Implement common data structures and algorithms using a common programming language**
  - LO4 Use common algorithm design strategies to develop new algorithms when there are no known solutions**
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