Description of the problem

A steel company, decides on ways of covering their emissions $Y_1, \dots, Y_T$ stemming from their exogenously given production, at times $1, \dots, T$, by a single type of allowances. At each $t = 1, \dots, T$, r_t allowances is given (grandfathered) to the company for free. Further, allowances may be bought (sold) at a secondary market at any time $t = 0, \dots, T$. The allowances may be saved (banked) for future periods.

In addition to the spots, at each $t = 0, \dots, T-1$, futures with maturities $t+1, t+2. \dots .T$, may be bought. For simplicity, it is assumed that the futures margin (which has to be deposited upon purchasing the future) is equal to the future-spot spread (this assumption may be relaxed for the price of additional decision variables).

Moreover, at each $t = 0, \dots, T-1$, call options with maturities $t+1, t+2. \dots .T$, and strike prices $K_1, \dots, K_\kappa$ may be bought at t. At the time of their maturity, the options need not be exercised; however, as exercising only some options is always no worse than excising all options and possibly selling the difference, we may assume all the options are exercised.

The company may fund their emission trading by loans with an interest rate ϱ. The insufficiency of cash at T is penalized by a prohibitive interest rate ι.

The company is risk-averse minimizing the nested mean-CVaR risk measure, applied to the sum of profits $X_1, \dots, X_T$ from the production and the costs of emission trading.

Problem definition

$$\min_{x_t \in \mathcal{X}_t, 0 \le t \le T} \rho\left(-z_0, \dots, -z_T\right)$$

Here,

$$\xi_t = (X_t, Y_t, P_t, Q_t, B_t)$$

where $X_t \in \mathbb{R}_+$ is the profit from the production at time t, $Y_t \in \mathbb{R}_+$ are the emissions at t, $P_t \in \mathbb{R}_+$ and $Q_t \in \mathbb{R}_+^{T-t}$, $B_t \in \mathbb{R}_+^{T-t \times \kappa}$ are spot prices, future-spot margins, option prices, respectively, at time t. In particular, Q_t^τ is the future-spot difference of the future with maturity at τ, and $B_t^{\tau,i}$ is the premium paid at t for the call option with the strike price K_i with maturity τ.

The decision variables are

$$x_t = (s_t, f_t, \phi_t, z_t, c_t)$$

where, s_t is the number of the spot allowances held at t, $f_t = (f_t^{t+1}, \dots, f_t^T)$ are the numbers of the futures with maturities $t+1, \dots, T$ held, and

$$\phi_t = \begin{bmatrix} \phi_t^{1,t+1} & \dots & \phi_t^{1,T} \\ \vdots & & \vdots \\ \phi_t^{\kappa,t+1} & \dots & \phi_t^{\kappa,T} \end{bmatrix}$$

are the numbers of call potions held. Further, z_t is the cash-flow at and c_t is the debt after t.

Further,

$$\mathcal{X}_0 = \{(s_0, f_0, \phi_0, z_0, c_0) : s_0 \geq 0, f_0 \geq 0, \phi_t \geq 0,$$
$$z_0 = -P_0 s_0 - \sum_{\tau=1}^{T} (\rho^\tau P_0 + Q_0^\tau) f_0^\tau - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} B_0^{\tau,i} \phi_0^{\tau,i}, c_0 = [z_0]_- \}.$$

For any $0 < t < T$,

$$\mathcal{X}_t(s_{t-1}, f_{t-1}, \phi_{t-1}, c_{t-1}) = \{(s_t, f_t, \phi_t, z_t, c_t) \in \mathcal{F}_t : e_t \geq 0, f_t \geq 0, \phi_t \geq 0,$$
$$z_t = X_t - P_t[s_t + Y_t - (s_{t-1} + r_t + f_{t-1}^t + \sum_{i=1}^{\kappa} \phi_{t-1}^{t,i})] - \sum_{i=1}^{\kappa} \min(P_t, K_i) \phi_{t-1}^{t,i}$$
$$- \sum_{\tau=t+1}^{T} (\rho^{\tau-t} P_t + Q_t^\tau) \Delta f_t^{\tau-t} - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\kappa} B_t^{\tau,i} \Delta \phi_t^{\tau-t,i} - \varrho c_{t-1}, c_t = [z_t - c_{t-1}]_- \}.$$

Finally,

$$\mathcal{X}_T(s_{T-1}, f_{T-1}, \phi_{T-1}, c_{T-1}) = \{z_T : z_T = e_T - \iota[e_T]_- \}$$

where

$$e_T = X_T - P_T[Y_T - (s_{T-1} + r_T + f_{T-1}^T + \sum_{i=1}^{\kappa} \phi_T^{T-1,i})] - \sum_{i=1}^{\kappa} \min(P_T, K_i) \phi_T^{T-1,i} - \varrho c_{T-1}.$$

1 Data

$T = 0$ odpovídá začátku 2018, $T = 3$ (end of 2020)

For the spot prices P and the spreads Q we adopt model from (anor). In particular, we fit the evolution of P by

$$P_t = P_0 \exp\left\{\sum\nolimits_{\tau=1}^{t} u_\tau\right\}, \qquad u_t \sim \mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right), \qquad 1 \leq t \leq T,$$

with $\sigma = 0.439$ where $u_1, \ldots, u_T$ are i.i.d., and

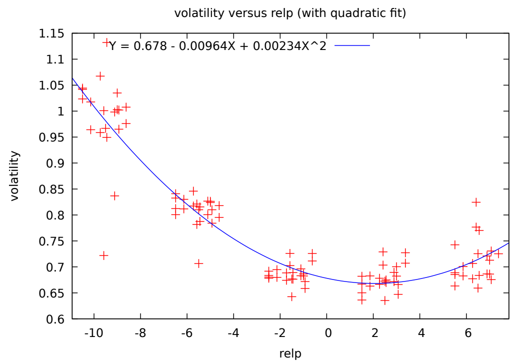
$$Q_t^\tau = P_t \exp\{(\tau - t)(0.00974 + v_t^\tau)\}, \qquad v_t^\tau \sim \mathcal{N}(0, \varsigma^2), \qquad 1 \leq t < \tau, \quad 1 \leq \tau \leq T,$$

with $\varsigma = 0.010$ where $v_1^2, v_1^3, v_2^3, \ldots$ are i.i.d., independent of $u_1, \ldots, u_T$.

The initial prices are equal to

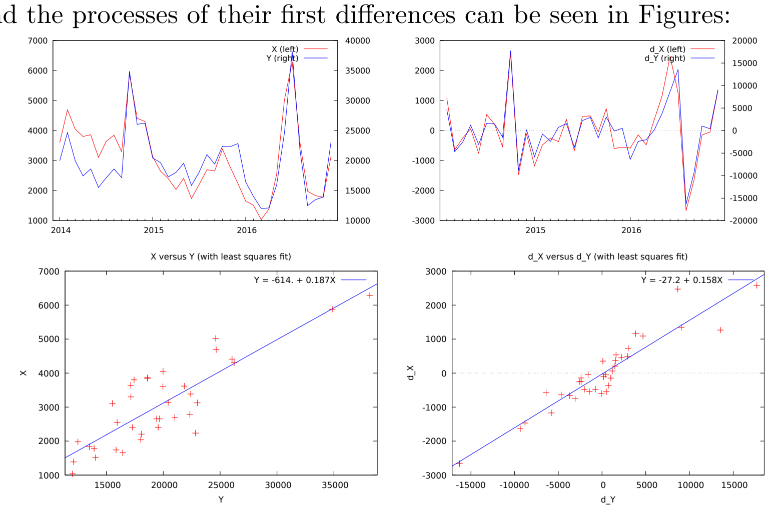
$$P_0 = 7.77, \quad Q_0^1 = 0.04, \quad Q_0^2 = 0.1, \quad Q_0^3 = 0.2$$

Option prices $B_t^{\tau,i}$ are computed by the Black-Scholes formula with the implied volatility being a quadratic function of the strike price relative to the spot price. The shape of the function, depicted in Picture XX, has been estimated using 110 observations of actual option prices on ???market. As the risk-less rate ??? was taken.



Our model of profits X and emissions Y was estimated using their monthly hypothetical historical values from 2014 to 2016, which were constructed as follows: TBD FRANTA.

As all the correlations of any of X_t, Y_t with any of P_t, P_{t-1} and correlations of any of $\Delta X_t, \Delta Y_t$ with any of ΔP_t, ΔP_{t-1} are insignificant, we model X, Y alone, independently of P. The time series plots and xy–plots of processes X_t, Y_t and the processes of their first differences can be seen in Figures:



In can be clearly seen that the values of X_t and Y_t “go along” as well as their first differences, so it is worth to model their evolution jointly. As the ADF tests rejected unit root hypothesis for both the series, we chose VAR model with the single lag to fit their time evolution:

$$\begin{aligned} X_t &= 2306.45 + 1.26237 X_{t-1} - 0.158943 Y_{t-1} + \epsilon_t \\ Y_t &= 12852.2 + 2.24683 X_{t-1} + \varepsilon_t \end{aligned} \tag{1}$$

with $\mathrm{stdev}(\epsilon_1) = 809.6647$, $\mathrm{stdev}(\varepsilon_1) = 4882.004$ and $\mathrm{corr}(\epsilon_1, \varepsilon_1) = 0.907$. Highly improbable negative observations are truncated to zero, if they appear during the computation.

The initial values

$$X_0 = 2744.83, \qquad Y_0 = 18982.64$$

were obtained by a year-ahead forecast in (1).