Description of the problem

A risk-averse steel company, minimizing the discounted nested mean-CVaR risk measure, decides on ways of covering their emissions $Y_1, \ldots, Y_T$ by a single type of allowances. At each $t = 1, \ldots, T$, r_t allowances is given (grandfathered) to the company for free. Further, allowances may be bought (sold) at a secondary market at any time $t = 0, \ldots, T$. The allowances may be saved (banked) for future periods.

In addition to the spots, at each $t = 0, \ldots, T-1$, futures with maturities $t+1, t+2. \ldots .T$, may be bought. For simplicity, it is assumed that the futures margin (which has to be deposited upon purchasing the future) is equal to the future-spot spread (this assumption may be relaxed).

Moreover, at each $t = 0, \ldots, T-1$, call options with maturities $t+1, t+2. \ldots .T$, and strike prices $K_1, \ldots, K_\kappa$ may be bought at t, premiums computed by the Black-Scholes formula (possibly adjusted for smile) for $t > 0$. At the time of their maturity, the options need not be exercised; however, as exercising only some options is always no worse than excising all options and possibly selling the difference, we may assume all the options are exercised.

The company may fund their emission trading by loans with an interest rate ϱ. The insufficiency of cash at T is penalized by a prohibitive interest rate ι.

Problem definition

$$\min_{x_t \in \mathcal{X}_t, 0 \leq t \leq T} \rho\left(-z_0, \ldots, -z_T\right)$$

Here,

$$\xi_t = (X_t, Y_t, P_t, Q_t, B_t)$$

where $X_t \in \mathbb{R}_+$ is the profit from the production at time t, $Y_t \in \mathbb{R}_+$ are the emissions at t, $P_t \in \mathbb{R}_+$ and $Q_t \in \mathbb{R}_+^{T-t}$, $B_t \in \mathbb{R}_+^{T-t\times\kappa}$ are spot prices, future-spot margins, option prices, respectively, at time t. In particular, Q_t^τ is the difference of the future- and the spot price with maturity at τ and $B_t^{\tau,i}$ is the premium paid at t for the call option with the strike price K_i with maturity τ.

The decision variables are as follows:

$$x_t = (e_t, f_t, \phi_t, z_t, c_t)$$

where, s_t is the number of the spot allowances held at t, $f_t = (f_t^{t+1}, \ldots, f_t^T)$ are the numbers of the futures with maturities $t+1, \ldots, T$ held and

$$\phi_t = \begin{bmatrix} \phi_t^{1,t+1} & \ldots & \phi_t^{1,T} \\ \vdots & & \vdots \\ \phi_t^{\kappa,t+1} & \ldots & \phi_t^{\kappa,T} \end{bmatrix}$$

are the numbers of call potions held. Further, z_t is the cash-flow at and c_t is the debt after t.

Further,

$$\mathcal{X}_0 = \{(s_0, f_0, \phi_0, z_0, c_0) : s_0 \geq 0, f_0 \geq 0, \phi_t \geq 0,$$
$$z_0 = -P_0 s_0 - \sum_{\tau=1}^{T} (\rho^\tau P_0 + Q_0^\tau) f_0^\tau - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} B_0^{\tau,i} \phi_0^{\tau,i},$$
$$c_0 = [z_0]_-\}.$$

For any $0 < t < T$,

$$\mathcal{X}_t(s_{t-1}, f_{t-1}, \phi_{t-1}, c_{t-1}) = \{(s_t, f_t, \phi_t, z_t, c_t) \in \mathcal{F}_t : e_t \geq 0, f_t \geq 0, \phi_t \geq 0,$$
$$z_t = X_t - P_t[s_t + Y_t - (s_{t-1} + r_t + f_{t-1}^t + \sum_{i=1}^{\kappa} \phi_{t-1}^{t,i})] - \sum_{i=1}^{\kappa} \min(P_t, K_i) \phi_{t-1}^{t,i}$$
$$- \sum_{\tau=t+1}^{T} (\rho^{\tau-t} P_t + Q_t^\tau) \Delta f_t^{\tau-t} - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\kappa} B_t^{\tau,i} \Delta \phi_t^{\tau-t,i} - \varrho c_{t-1},$$
$$c_t = [z_t - c_{t-1}]_-\}.$$

Finally,
$$\mathcal{X}_T(s_{T-1}, f_{T-1}, \phi_{T-1}, c_{T-1}) = \{z_T : z_T = e_T - \iota[e_T]_-\}$$

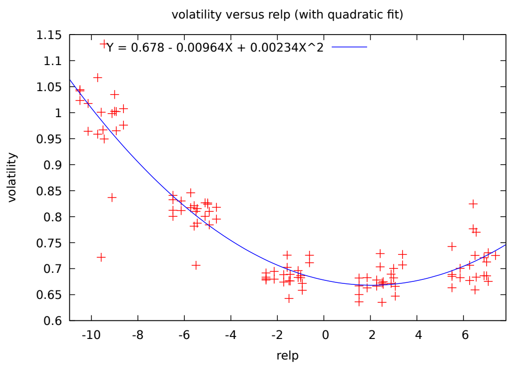
where

$$e_T = X_T - P_T[Y_T - (s_{T-1} + r_T + f_{T-1}^T + \sum_{i=1}^{\kappa} \phi_T^{T-1,i})] - \sum_{i=1}^{\kappa} \min(P_T, K_i) \phi_T^{T-1,i} - \varrho c_{T-1}.$$

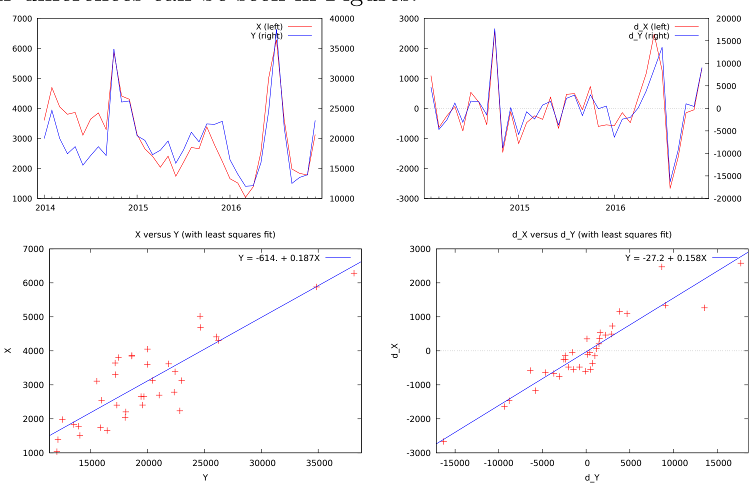
1 Data

- $T = 0$ odpovídá začátku 2018, $T = 3$ (end of 2020)
- P, budeme modelovat stejně jako v ANORu s6ř45 (bez diskretizace)
- Q taktéž (s7ř26) - bez předpokladu, že je spread deterministický

Option prices $B_t^{\tau,i}$ are computed according to the Black-Scholes formula where the implied volatility is a quadratic function of the strike price relative to the spot price. The shape of the function, depicted in Picture XX, has been estimated using 110 observations of actual option prices on ???market. As the risk-less rate ??? was taken.



Model for (X, Y). As both the correlations of X_t, Y_t with P_t, P_{t-1} and correlations $\Delta X_t, \Delta Y_t$ with ΔP_t, ΔP_{t-1} are insignificant, we model X, Y alone, without dependence on P. The time series– plots and xy–plots of X, Y and their differences can be seen in Figures:



As it could have been expected, the values of X and Y "go along". As the ADF tests rejected unit root hypothesis for both the series, we chose VAR model with a single lag to fit their time dependence:

$$X_t = 2306.45 + 1.26237X_{t-1} - 0.158943Y_{t-1} + \epsilon_t$$

$$Y_t = 12852.2 + 2.24683X_{t-1} + \varepsilon_t$$

with $\mathrm{stdev}(\epsilon_1) = 809.6647$, $\mathrm{stdev}(\varepsilon_1) = 4882.004$ and $\mathrm{corr}(\epsilon_1, \varepsilon_1) = 0.907$. Highly improbable negative observations are truncated to zero, if they appear during the computation.