Description of the problem

A steel company decides on ways of covering their emissions $Y_1, \dots, Y_T$ stemming from their exogenously given production, at times $1, \dots, T$, by a single type of allowances. At each $t = 1, \dots, T$, r_t allowances is given (grandfathered) to the company for free. Further, allowances may be bought (sold) at a secondary market at any time $t = 0, \dots, T$. The allowances may be saved (banked) for future periods.

In addition to the spots, at each $t = 0, \dots, T-1$, futures with maturities $t+1, t+2. \dots .T$, may be bought. Moreover, at each $t = 0, \dots, T-1$, call options with maturities $t+1, t+2. \dots .T$, and strike prices $K_1, \dots, K_\kappa$ may be bought at t. In principle, the options need not be exercised at the time of their maturity; however, as we neglect transaction costs, exercising only some options is always no worse than exercising all options and possibly selling the difference, so we may assume all the options are exercised. The company may trade with the allowances and their derivatives freely; however, they may no take short positions.

The company is risk-averse minimizing the discounted nested mean-CVaR risk measure, applied to the differences of profits from the production and the costs of emission trading.

Problem definition

The subject of decision is the way of optimal emission trading. In particular, at t, it is the number Δs_t of the spot allowances purchased/sold at t, the numbers

$$\Delta f_t = (\Delta f_t^{t+1}, \dots, \Delta f_t^T)$$

of the futures with maturities $t+1, \dots, T$ purchased/sold, and the numbers

$$\Delta \phi_t = \begin{bmatrix} \Delta \phi_t^{t+1,1} & \dots & \Delta \phi_t^{T,1} \\ \vdots & & \vdots \\ \Delta \phi_t^{t+1,\kappa,} & \dots & \Delta \phi_t^{T,\kappa} \end{bmatrix}$$

of the call options purchased/sold; here, $\Delta \phi_t^{\tau,i}$ denotes the number of call options with maturity τ and strike price K_i.

At $t = 0$, the income of the company is

$$y_0 = -P_0 \Delta s_0 - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} B_0^{\tau,i} \Delta \phi_0^{\tau,i},$$

while, at any $0 < t \le T$, the income is

$$y_t = X_t - P_t \Delta s_t - \sum_{i=1}^{\kappa} \min(P_t, K_i) \phi_{t-1}^{t,i} - \sum_{\tau=0}^{t-1} Q_\tau^t \Delta f_\tau^t - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\kappa} B_t^{\tau,i} \Delta \phi_t^{\tau-t,i}.$$

Here, $f_t = \sum_{\tau=0}^{t} \Delta f_\tau^t$ and $\phi_t = \sum_{\tau=0}^{t} \Delta \phi_\tau^t$. Further, $X_t \in \mathbb{R}_+$ is the profit from the production at time t, $Y_t \in \mathbb{R}_+$ is the amount of the emissions at t, $P_t \in \mathbb{R}_+$ is the spot price at time t, $Q_t^\tau \in \mathbb{R}_+$ is the price of the future with maturity τ at time t, and $B_t^{\tau,i} \in \mathbb{R}_+$ is the premium paid at t for the call option with strike price K_i and maturity τ.

The aim of the company is to minimize $\rho(-y_0, \ldots, -y_T)$ where ρ is the nested mean-CVaR criterion; in particular, for any real $c_0, \ldots, c_T$,

$$\rho(c_0, \ldots, c_T) = \mu_1(\mu_2(\ldots \mu_T(c_0 + \varrho c_1 + \cdots + \varrho^T c_T) \ldots));$$

where $\mu_t(Z) = (1-\lambda)\mathbb{E}(Z|\mathcal{F}_{t-1}) + \lambda \mathrm{CVaR}_\alpha(Z|\mathcal{F}_{t-1})$, $0 \le \lambda \le 1$ and $0 < \alpha < 1$ are constants and ϱ is a discount factor.

Next we reformulate the decision problem into a form, which is more convenient for computation.

Denote e_t the amount of the allowances held (immediately after) t. Clearly, $e_0 = \Delta s_0$ and

$$e_t = e_{t-1} + \Delta s_t + r_t + f_{t-1}^t + \sum_{i=1}^{\kappa} \phi_{t-1}^{t,i} - Y_t, \tag{1}$$

As $\Delta f_\tau^t = f_\tau^t - f_{\tau-1}^t$, and similarly ϕ, and as Δs may be expressed from (1), we can alternatively take e, f, ϕ as decision variables. Further, as, at any t, the payment for futures with maturity t is equal to $\sum_{\tau=0}^{t-1} b_\tau$ where $b_\tau \in \mathcal{F}_\tau$, we may assume, without a change of the decision criterion's value, that $\varrho^{t-\tau} c_\tau$ is paid at each $0 \le \tau < T$ instead of the whole amount at t.

Consequently, the problem may be reformulated as

$$\min_{x_t \in \mathcal{X}_t, x_t \in \mathcal{F}_t 0 \le t \le T} \rho(-z_0, \ldots, -z_T)$$

where $\mathcal{F}_t$ is the filtration induced by process $\xi_t = (X_t, Y_t, P_t, Q_t, B_t)$, where

$$\mathcal{X}_t = \{(e_t, f_t, \phi_t) : e_0 \ge 0, f_0 \ge 0, \phi_0 \ge 0.\}, \qquad 0 \le t < T,$$

$$\mathcal{X}_T = \{e_T : e_T = 0\},$$

and

$$z_0 = -P_0 e_0 - \sum_{\tau=1}^{T} \varrho^\tau Q_0^\tau f_0^\tau - \sum_{\tau=1}^{T} \sum_{i=1}^{\kappa} B_0^{\tau,i} \phi_0^{\tau,i},$$

$$\begin{aligned} z_t = X_t - P_t & \left(e_t - e_{t-1} - r_t - f_{t-1}^t - \sum_{i=1}^{\kappa} \phi_{t-1}^{t,i} + Y_t \right) \\ & - \sum_{i=1}^{\kappa} \min(P_t, K_i) \phi_{t-1}^{t,i} - \sum_{\tau=t+1}^{T} \varrho^{\tau-t} Q_t^\tau (f_t^{\tau-t} - f_{t-1}^{\tau-t}) \\ & \qquad - \sum_{\tau=t+1}^{T} \sum_{i=1}^{\kappa} B_t^{\tau,i} (\phi_t^{\tau-t,i} - \phi_{t-1}^{\tau-t,i}), \qquad 0 < t \le T. \end{aligned}$$

1 Data

$T = 0$ odpovídá začátku 2018, $T = 3$ (end of 2020)

For the spot prices P and the spreads Q we adopt model from (anor). In particular, we fit the evolution of P by

$$P_t = P_0 \exp\left\{\sum\nolimits_{\tau=1}^{t} u_\tau\right\}, \qquad u_t \sim \mathcal{N}\left(-\frac{\sigma^2}{2}, \sigma^2\right), \qquad 1 \le t \le T,$$

with $\sigma = 0.439$ where $u_1, \dots, u_T$ are i.i.d., and

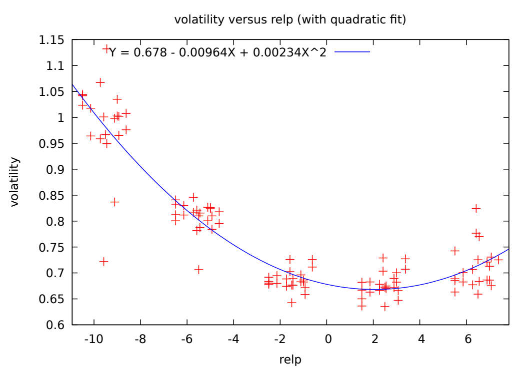
$$Q_t^\tau = P_t \exp\{(\tau - t)(0.00974 + v_t^\tau)\}, \qquad v_t^\tau \sim \mathcal{N}(0, \varsigma^2), \qquad 1 \le t < \tau, \quad 1 \le \tau \le T,$$

with $\varsigma = 0.010$ where $v_1^2, v_1^3, v_2^3, \dots$ are i.i.d., independent of $u_1, \dots, u_T$.

The initial prices are equal to

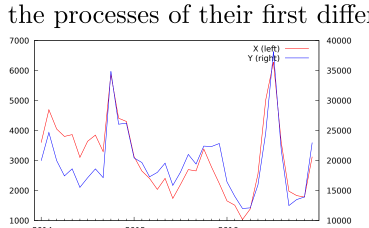
$$P_0 = 7.77, \quad Q_0^1 = 7.81, \quad Q_0^2 = 7.87, \quad Q_0^3 = 7.97$$

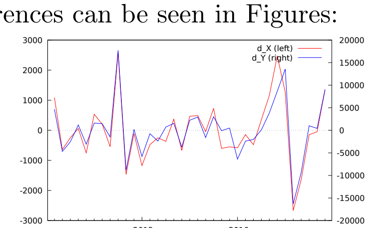
Option prices $B_t^{\tau,i}$ are computed by the Black-Scholes formula with the implied volatility being a quadratic function of the strike price relative to the spot price. The shape of the function, depicted in Picture XX, has been estimated using 110 observations of actual option prices on ???market. As the risk-less rate ??? was taken.

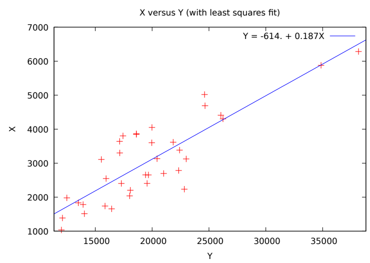


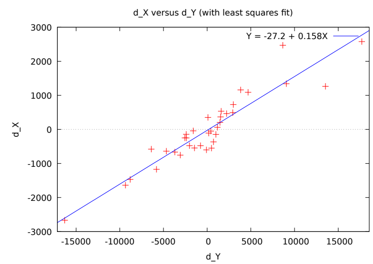
Our model of profits X and emissions Y was estimated using their monthly hypothetical historical values from 2014 to 2016, which were constructed as follows: TBD FRANTA.

As all the correlations of any of X_t, Y_t with any of P_t, P_{t-1} and correlations of any of $\Delta X_t, \Delta Y_t$ with any of ΔP_t, ΔP_{t-1} are insignificant, we model X, Y alone, independently of P. The time series plots and xy–plots of processes X_t, Y_t and the processes of their first differences can be seen in Figures:









In can be clearly seen that the values of X_t and Y_t "go along" as well as their first differences, so it is worth to model their evolution jointly. As the ADF tests rejected unit root hypothesis for both the series, we chose VAR model with the single lag to fit their time evolution:

$$\begin{aligned} X_t &= 2306.45 + 1.26237 X_{t-1} - 0.158943 Y_{t-1} + \epsilon_t \\ Y_t &= 12852.2 + 2.24683 X_{t-1} + \varepsilon_t \end{aligned} \tag{2}$$

with $\mathrm{stdev}(\epsilon_1) = 809.6647$, $\mathrm{stdev}(\varepsilon_1) = 4882.004$ and $\mathrm{corr}(\epsilon_1, \varepsilon_1) = 0.907$. Highly improbable negative observations are truncated to zero, if they appear during the computation.

The initial values

$$X_0 = 2744.83, \qquad Y_0 = 18982.64$$

were obtained by a year-ahead forecast in (2).