Ensemble Models: Bagging

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OUTLINE

Foundational Machine Learning

- You've learned about:
  - Traditional Regression
  - Logistic Regression
  - K-Nearest Neighbors
  - Discriminant Analysis
  - Support Vector Machines
  - Tree-Based Methods

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Remember *there's no free lunch*!

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    2. Stacking
    3. Boosting

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Today we will focus on **bagging**

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2. σ^2
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- $\implies$ Averaging a set of observations reduces variance

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- Average the resulting predictions!
  - $\implies$ Single low-variance statistical learning model

$$\hat{f}_{\text{avg}}(x) = \frac{1}{B} \sum_{b=1}^{B} \hat{f}^b(x)$$

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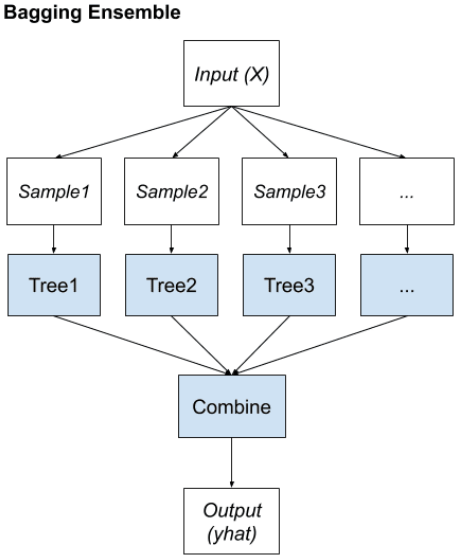
$$\hat{f}_{\text{bag}}(x) = \frac{1}{B} \sum_{b=1}^{B} \hat{f}^{*b}(x)$$

Bagging Visual



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  - Construct B trees (grown deep with no pruning) based on the B bootstrapped training sets, and then average the predictions
- **If we're performing classification instead, how should we obtain a prediction since we can't take an average?**

Out-of-Bag Error

- *Out-of-Bag* Error
  - On average, each bagged tree makes use of around 2/3 of the observations
  - The remaining 1/3 are referred to as the *out-of-bag* (OOB) observations
  - Make predictions for each observation using trees for which the observation was OOB
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- For sufficiently large B, OOB error is virtually equivalent to leave-one-out cross-validation error
- Much easier to compute than using cross-validation when dealing with a very large data set

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- Variable Importance
  - Bagging improves prediction accuracy at the expense of interpretability, since we have many trees now
  - For each variable, average the RSS (or Gini index) reduction across all B trees
  - **[Larger]/[Smaller]** values indicate an important predictor

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- Averaging highly correlated quantities does not reduce variance as much as averaging uncorrelated quantities

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- **What happens if $m = p$?**
- Typically we choose $m \approx \sqrt{p}$

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  - $\implies$ We just want B sufficiently large
- Different values of m could affect the performance: