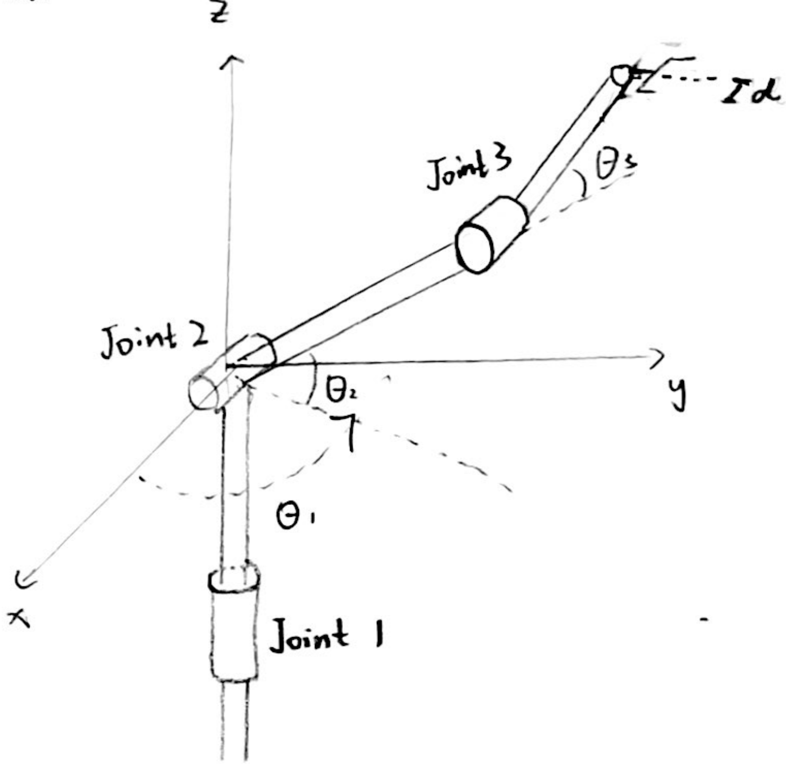
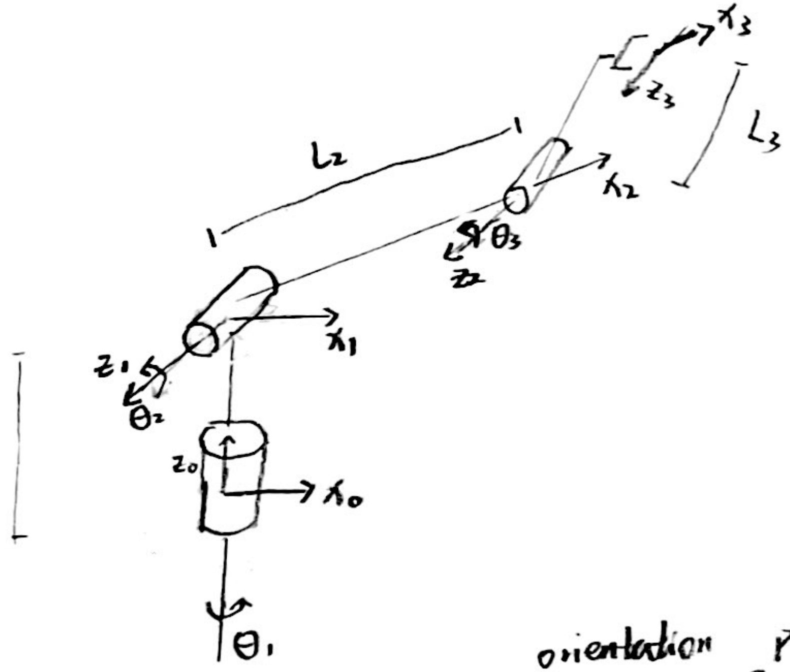
Forward Kinematics $(\theta_1, \theta_2, \theta_3, d) \longrightarrow (x, y, z)$

Inverse Kinematics : $(x, y, z) \longrightarrow (\theta_1, \theta_2, \theta_3, d)$

Forward:

(1)





Frame	d_i	θ_i	a_i	α_i
1	L_1	θ_1	0	90°
2	0	θ_2	L_2	0
3	0	θ_3	L_3	0

orientation position

$${}^0T_3 = {}^0T_1 \, {}^1T_2 \, {}^2T_3 \quad , \quad T = \left[\begin{array}{ccc|c} & R & & T \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$${}^{i-1}T_i = \left[\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\Rightarrow {}^0T_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
{}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_3\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_3\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_3 = {}^{0}T_1\,{}^{1}T_2\,{}^{2}T_3
$$

$$
= \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_1 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2C_3 - S_2S_3 & -(C_2S_3 + S_2C_3) & 0 & L_3C_2C_3 - L_3S_2S_3 + L_2C_2 \\ S_2C_3 + C_2S_3 & -S_2S_3 + C_2C_3 & 0 & L_3S_2C_3 + L_3C_2S_3 + L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1 & 0 & S_1 & 0 \\ S_1 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{23} & -S_{23} & 0 & L_2C_2 + L_3C_{23} \\ S_{23} & C_{23} & 0 & L_2S_2 + L_3S_{23} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1C_{23} & -C_1S_{23} & S_1 & C_1(L_2C_2 + L_3C_{23}) \\ S_1C_{23} & -S_1S_{23} & -C_2 & S_1(L_2C_2 + L_3C_{23}) \\ S_{23} & C_{23} & 0 & L_2S_2 + L_3S_{23} + L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$*$ $C_1 = \cos\theta_1$

$S_1 = \sin\theta_1$

$C_{23} = \cos(\theta_2 + \theta_3)$

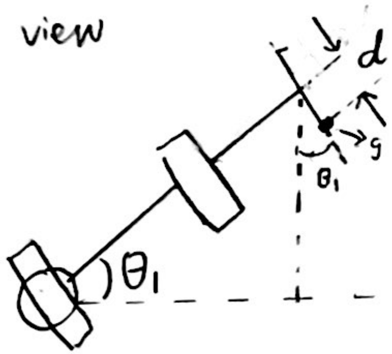
$S_{23} = \sin(\theta_2 + \theta_3)$

The position of end effector without gripper:

$$ {}^{0}T_3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{aligned} x_e &= C_1(L_2C_2 + L_3C_{23}) \\ y_e &= S_1(L_2C_2 + L_3C_{23}) \\ z_e &= L_1 + L_2S_2 + L_3S_{23} \end{aligned} $$

Taking gripper into consideration:

From Top view



position of gripper:

$$ \boxed{\begin{aligned} x_g &= d\sin\theta_1 + x_e \\ &= d\sin\theta_1 + C_1(L_2C_2 + L_3C_{23}) \\ y_g &= y_e - d\cos\theta_1 \\ &= S_1(L_2C_2 + L_3C_{23}) - d\cos\theta_1 \\ z_g &= z_e = L_1 + L_2S_2 + L_3S_{23} \end{aligned}} $$