FOUNDATION

Type

Cost of misusing types

```
case class Country(value: String)

val UK: Country      = Country("United Kingdom")
val France: Country  = Country("France")
val Germany: Country = Country("Germany")
```

Cost of misusing types

```
case class Country(value: String)

val UK: Country      = Country("United Kingdom")
val France: Country  = Country("France")
val Germany: Country = Country("Germany")
```

```
def getCurrency(country: Country): String = ???
```

such as

```
getCurrency(Country("United Kingdom")) == "GBP"
getCurrency(Country("France"))         == "EUR"
getCurrency(Country("Germany"))        == "EUR"
```

Cost of misusing types

```scala
def getCurrency(country: Country): Option[String] =
  country.value match {
    case "United Kingdom"     => Some("GBP")
    case "France" | "Germany" => Some("EUR")
    case _                    => None
  }
```

Cost of misusing types

```
def getCurrency(country: Country): Option[String] =
  country.value match {
    case "United Kingdom"     => Some("GBP")
    case "France" | "Germany" => Some("EUR")
    case _                    => None
  }
```

```
scala> getCurrency(Country("UK"))
res0: Option[String] = None

scala> getCurrency(Country("GBR"))
res1: Option[String] = None

scala> getCurrency(Country("Royaume-Uni"))
res2: Option[String] = None
```

Cost of misusing types

```
sealed trait Country

object Country {
  case object UnitedKingdom extends Country
  case object France        extends Country
  case object Germany       extends Country
}
```

```
import Country._

def getCurrency(country: Country): String =
  country match {
    case UnitedKingdom    => "GBP"
    case France | Germany => "EUR"
  }
```

```
def parseCountry(country: String): Option[Country] = ???
```

Cost of misusing types

```
sealed trait Country
object Country {
  case object UnitedKingdom extends Country
  case object France        extends Country
  case object Germany       extends Country
}

sealed trait Currency
object Currency {
  case object BritishPound extends Currency
  case object Euro         extends Currency
}
```

```
import Country._, Currency._

def getCurrency(country: Country): Currency =
  country match {
    case UnitedKingdom    => BritishPound
    case France | Germany => Euro
  }
```

Plan

- What is the cost of misusing types
- How to use ADTs to encode data
- Learn how to measure impact of types and tests
- Explore relationship between types, algebra and logic

Exercise 1: Misused types

`exercises.types.TypeExercises.scala`

Type should exactly fit business requirements

Imprecise data lead to errors

and misleading documentation

How should we encode data?

Algebraic Data Type (ADT)

- OR, a `ConfigValue` is
  - a `String` OR
  - an `Int` OR
  - `Empty`

- AND, a `User` is
  - an `userId` AND
  - a `name` AND
  - an `address`

OR

- a `Boolean` is `true` OR `false`
- an Int is a `-10` OR `0` OR `1` OR …
- a `DayOfTheWeek` is `Monday` OR `Tuesday` OR `Wednesday` OR …
- an `Option` is a `Some` OR a `None`
- a `List` is a `Nil` OR a `Cons`
- a `Json` is a `JsonNumber` OR a `JsonString` OR a `JsonArray` OR a `JsonObject` OR …

How should we encode OR?

A `ConfigValue` is a `String` OR an `Int` OR `Empty`

OR Encoding

```
sealed trait ConfigValue

object ConfigValue {
  case class ConfigString(value: String) extends ConfigValue
  case class ConfigNumber(value: Double) extends ConfigValue
  case object ConfigEmpty extends ConfigValue
}
```

OR Encoding

```scala
sealed trait ConfigValue

object ConfigValue {
  case class ConfigString(value: String) extends ConfigValue
  case class ConfigNumber(value: Double) extends ConfigValue
  case object ConfigEmpty extends ConfigValue
}
```

In Scala 3

```scala
enum ConfigValue {
  case ConfigString(value: String)
  case ConfigNumber(value: Double)
  case ConfigEmpty
}
```

AND

- a `User` is a `userId` AND a `name` AND an `address`
- a `ZonedDateTime` is a `dateTime` AND a `timeZone`
- a `Cons` is a `head` AND a `tail`
- a `Tuple2` is a `_1` AND a `_2`

How should we encode AND?

A `User` is a `userId` AND a `name` AND an `address`

AND Encoding

```
import java.util.UUID

case class User(userId: UUID, name: String, address: Address)

case class Address(streetNumber: Int, streetName: String, postCode: String)
```

```
scala> User(UUID.randomUUID(), "John Doe", Address(108, "Cannon Street", "EC4N 6EU"))
res3: User = User(71e3b5b6-4392-42a1-a5f7-a5f84bc5707c,John Doe,Address(108,Cannon Street,EC4N 6EU))
```

Exercise 2: Data Encoding

`exercises.types.TypeExercises.scala`

Case class and sealed trait map exactly to business language AND and OR

Together, they form what is called Algebraic Data Types (ADTs)

Nested AND and OR can be used to encode data precisely

OR is generally underused

How can we compare two encodings?

```
def getCurrency(country: String): Option[String]
```

Is it better to reduce input or reduce output?

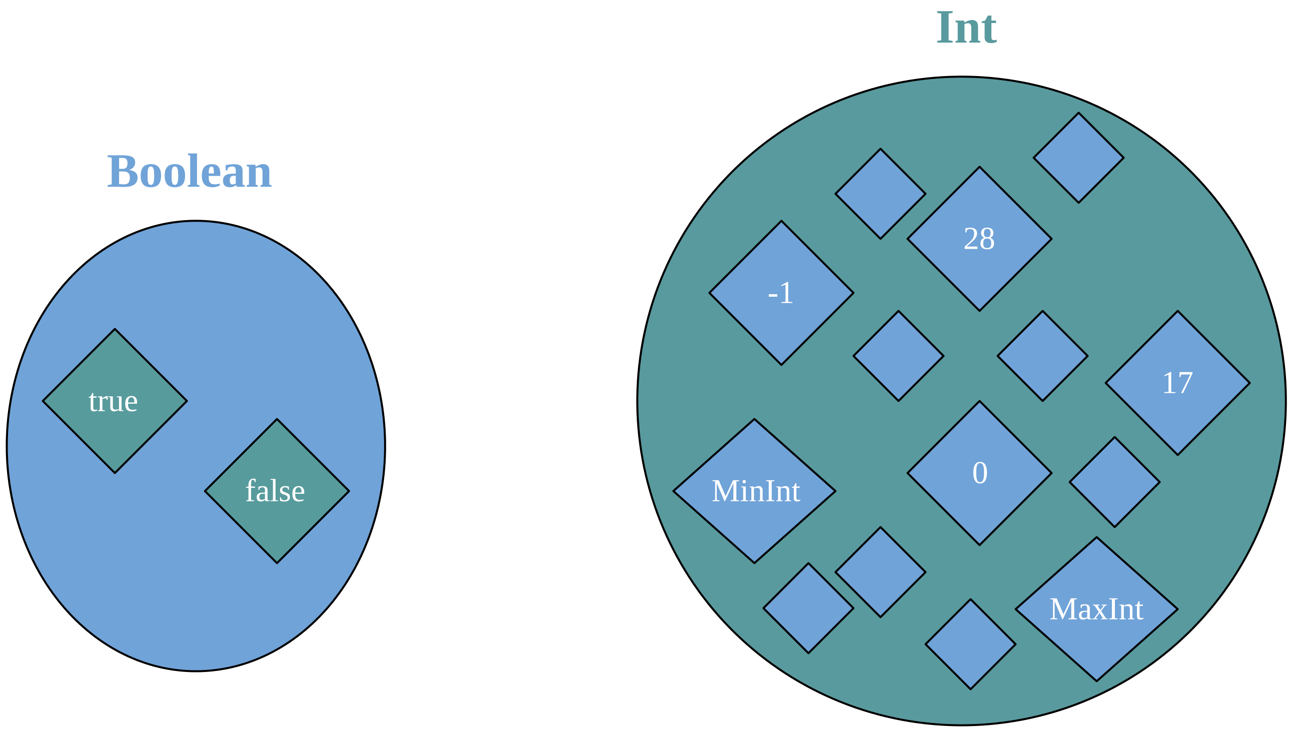
```
def getCurrency(country: Country): String
```

```
def getCurrency(country: String): Option[Currency]
```

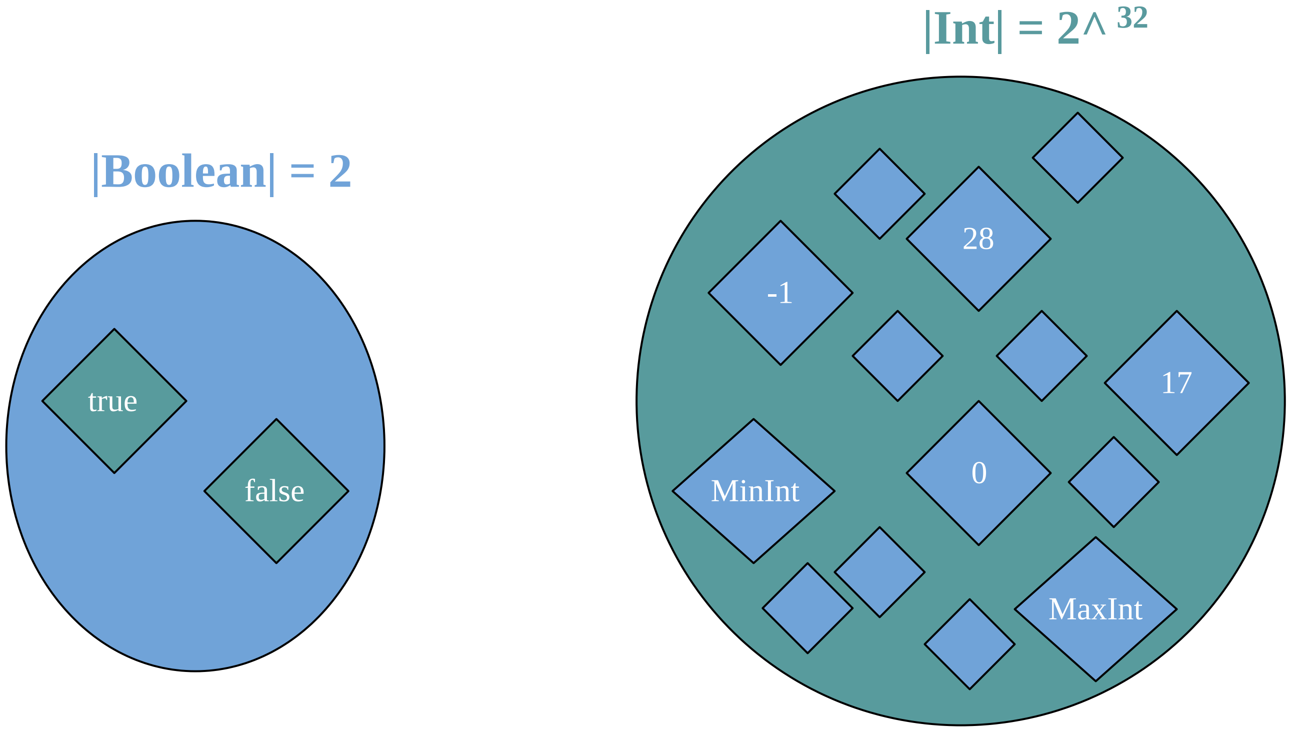
How much better it is to reduce both?

```
def getCurrency(country: Country): Currency
```

Type is a set



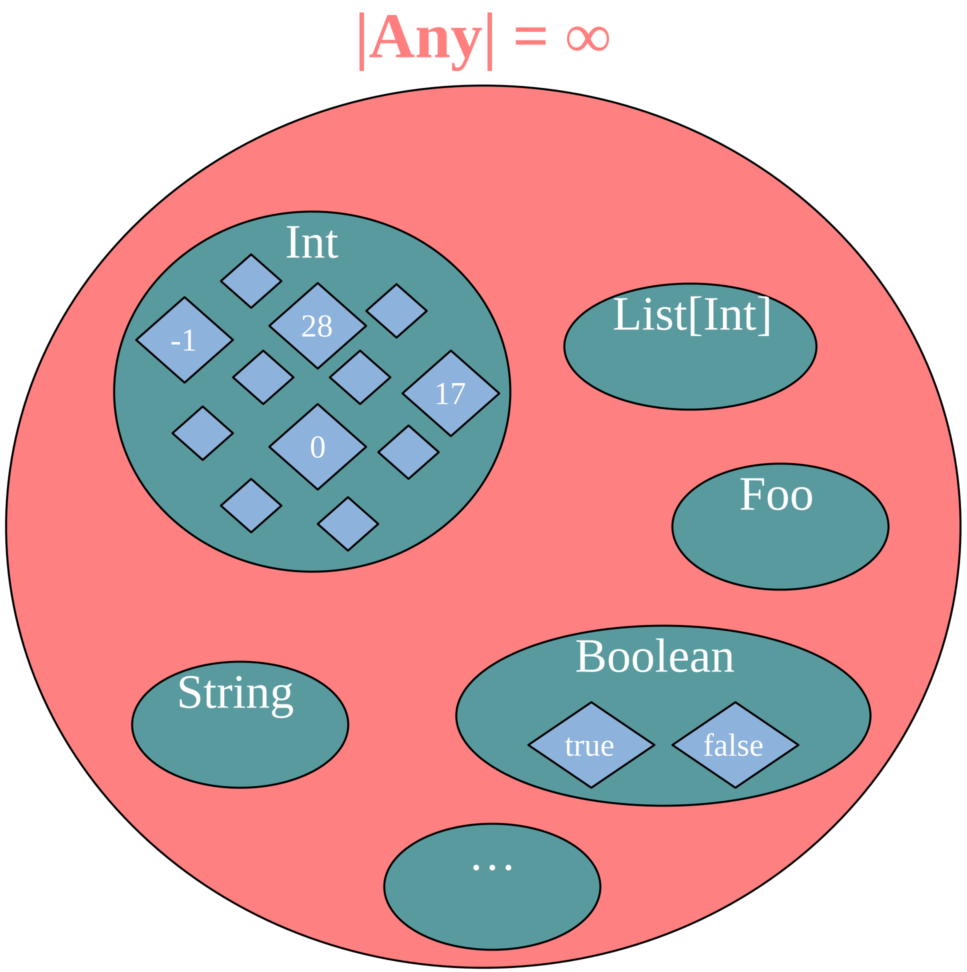
Cardinality



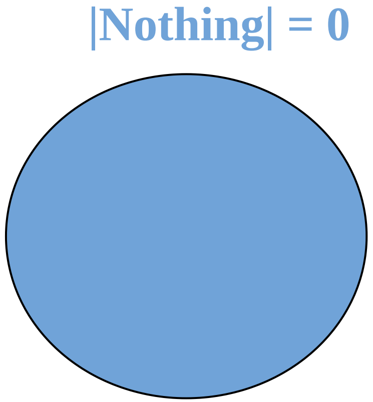
Cardinality: Any

```scala
val x: Int = 3

val hello: String = "hello"

case class User(name: String, age: Int)

val john: User = User("John", 53)
```

```
scala> x: Any
res4: Any = 3

scala> john: Any
res5: Any = User(John,53)

scala> List(x, hello, john)
res6: List[Any] = List(3, hello, User(John,53))
```

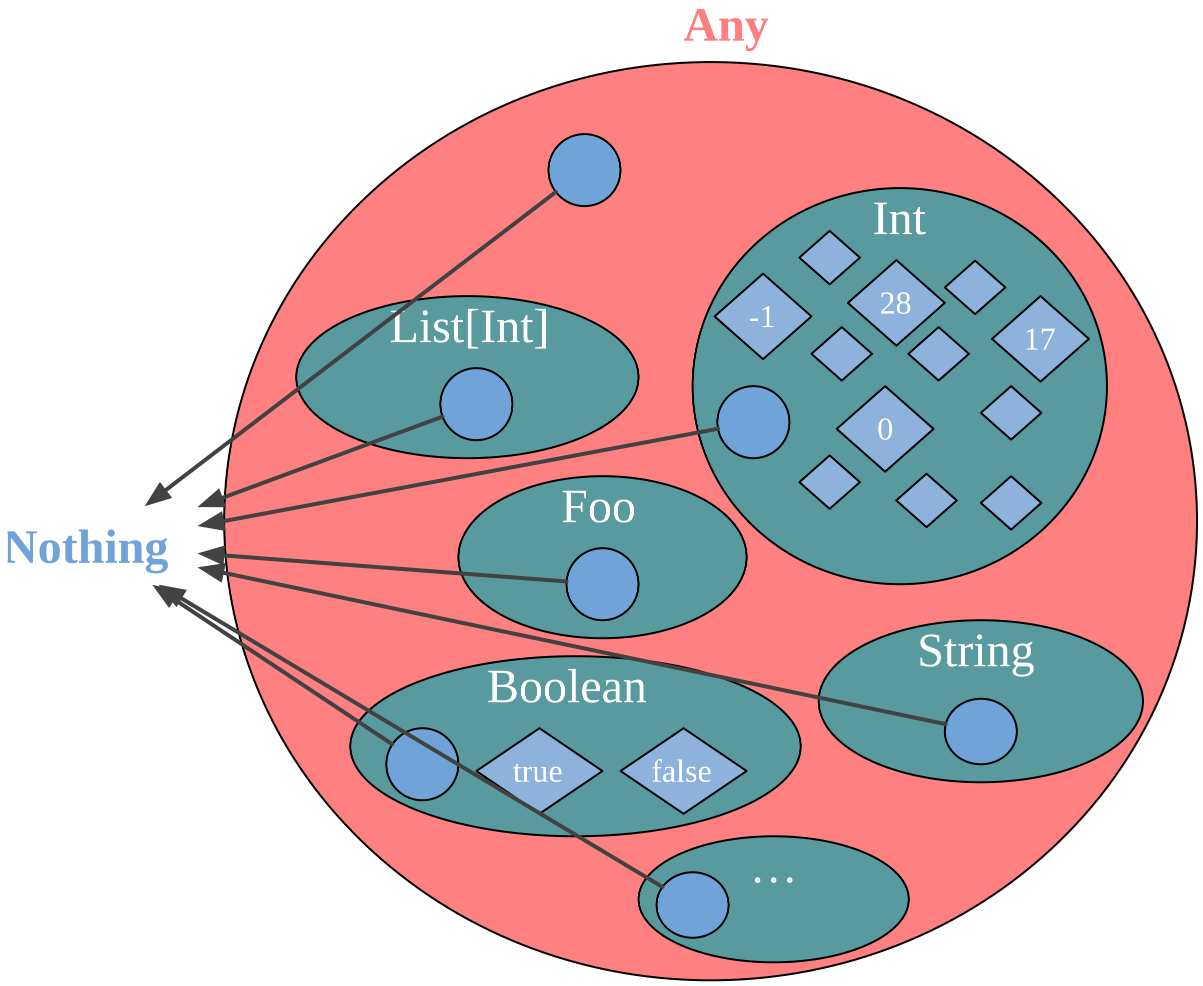


Cardinality: Nothing

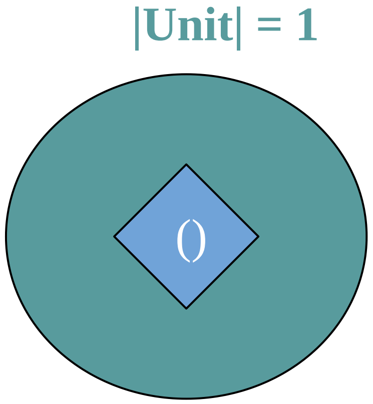


Cardinality: Nothing

```
sealed trait Nothing
  extends Int
  with    Boolean
  with    String
  with    Foo
  with    List[Int]
  with    Any
```

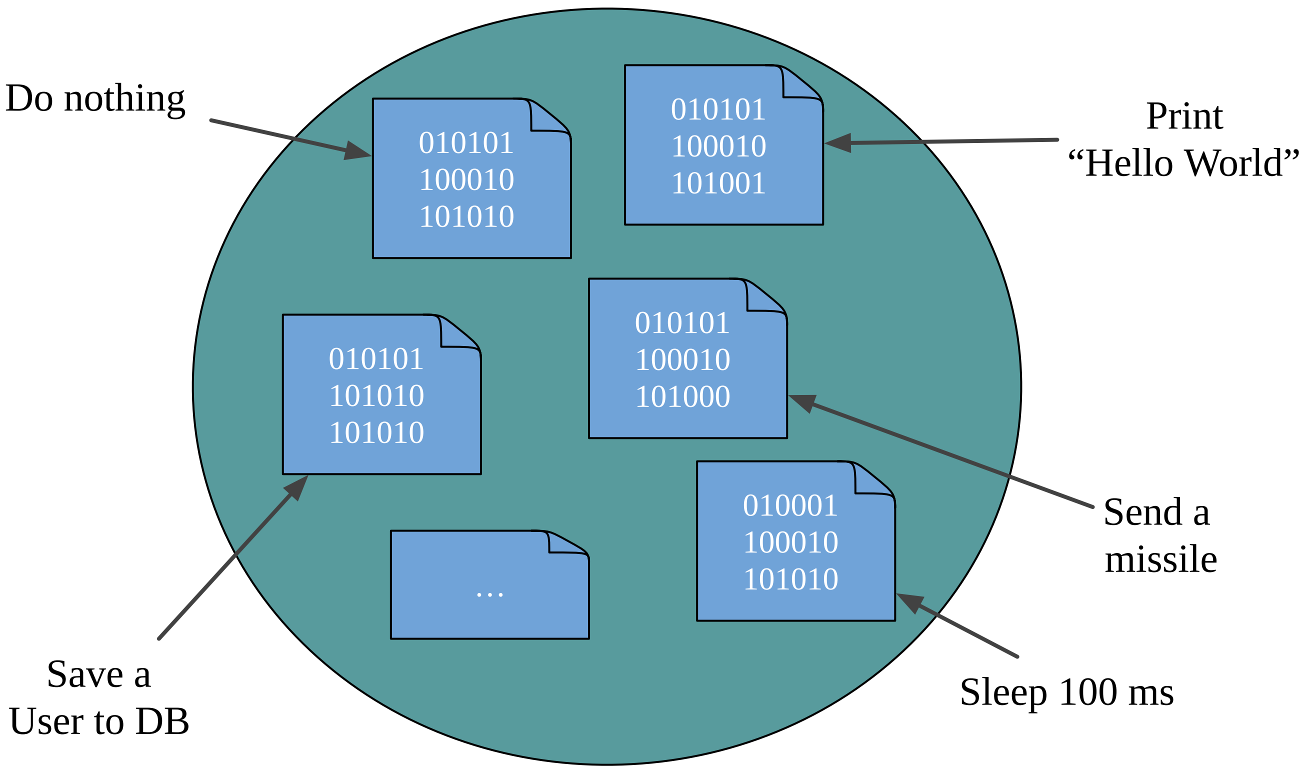


Cardinality: Unit

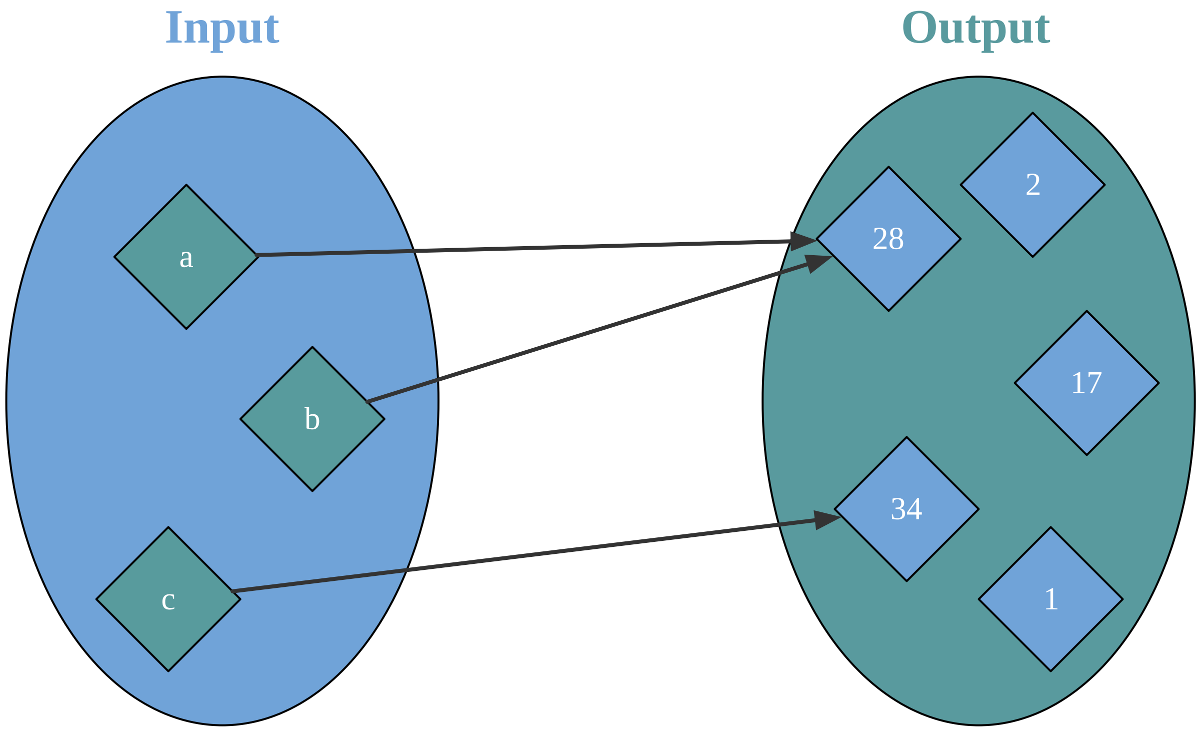


Cardinality: IO

|IO[Unit]| = ∞



Function is mappings between two sets



A => B is a type

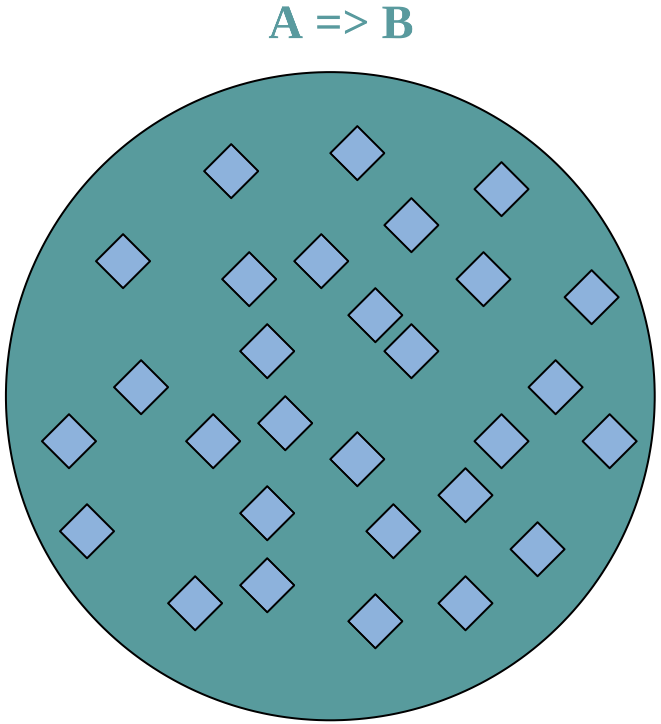
```
val isEven: Int => Boolean =
  (x: Int) => x % 2 == 0


val increment: Int => Int =
  (x: Int) => x + 1
```

A => B is a type

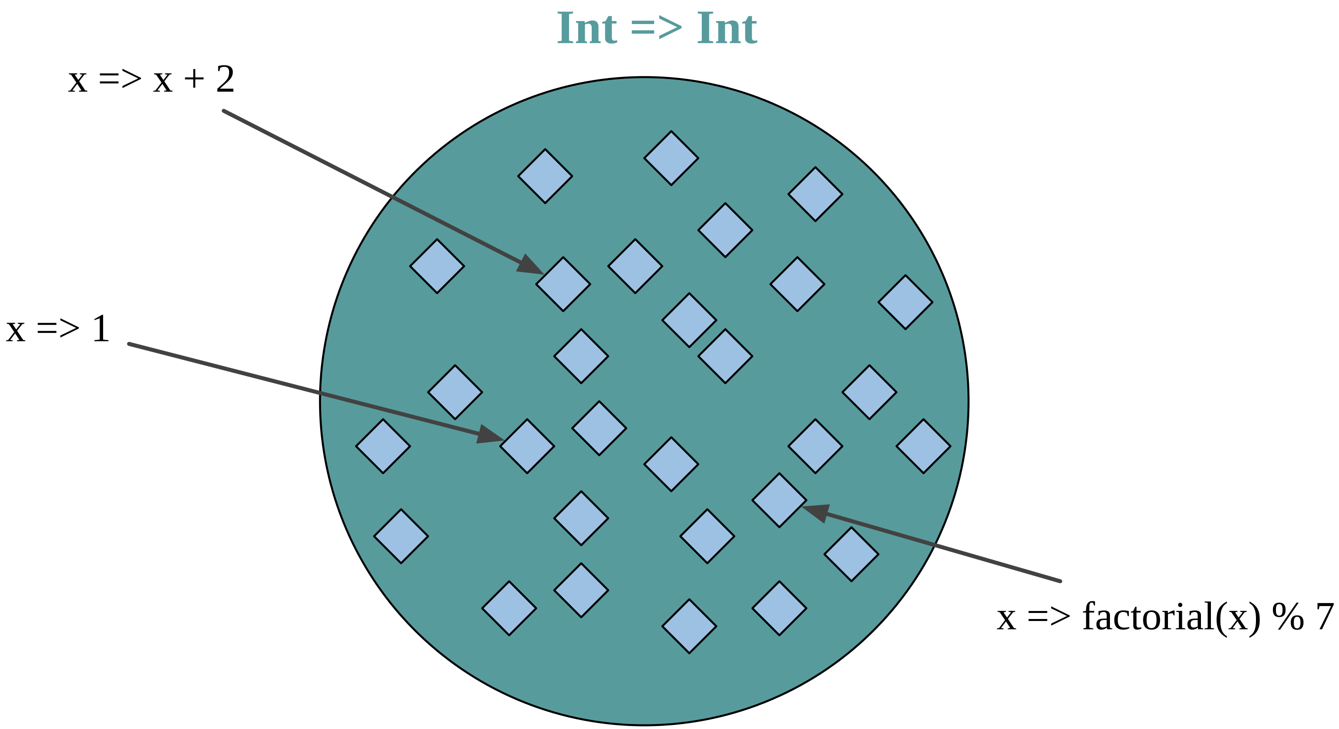
&&

A type is a set of values

A => B is a set of values!



A function type is a set of implementations!



How many elements are in the set A => B?

How many implementations satisfy the type checker?

|A => B|

The smaller |A => B|, the better

Perfect case is when |A => B| = 1

Exercise 3: Cardinality

`exercises.types.TypeExercises.scala`

Sealed trait

```
sealed trait IntOrBoolean

case class AnInt(value: Int) extends IntOrBoolean
case class ABoolean(value: Boolean) extends IntOrBoolean
```

```
AnInt(Int.MinValue) // ~ -2 billion
...
AnInt(0)
AnInt(1)
...
AnInt(Int.MaxValue) // ~ +2 billion

ABoolean(false)
ABoolean(true)
```

```
|IntOrBoolean| = |AnInt| + |ABoolean|
               = |Int|   + |Boolean|
```

Case class

```
case class IntAndBoolean(i: Int, b: Boolean)
```

```
IntAndBoolean(Int.MinValue, false)  // ~ -2 billion
...
IntAndBoolean(0, false)
IntAndBoolean(1, false)
...
IntAndBoolean(Int.MaxValue, false) // ~ +2 billion

IntAndBoolean(Int.MinValue, true)  // ~ -2 billion
...
IntAndBoolean(0, true)
IntAndBoolean(1, true)
...
IntAndBoolean(Int.MaxValue, true) // ~ +2 billion
```

```
|IntAndBoolean| = |Int| * |Boolean|
```

A sealed trait is called a sum type

A case class is called a product type

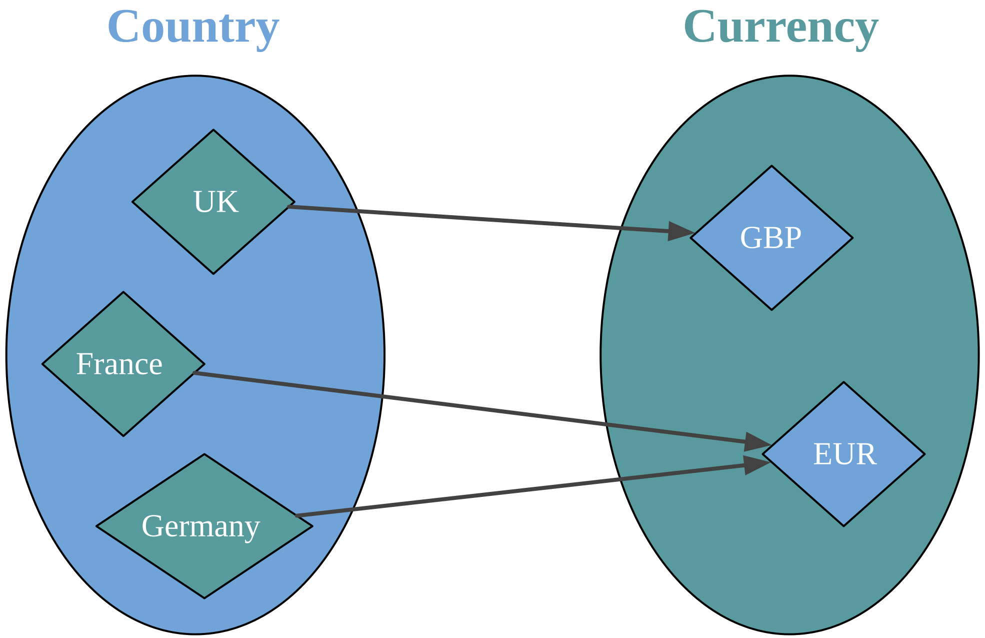
```
|A OR B OR C| = |A| + |B| + |C|
|A AND B AND C| = |A| * |B| * |C|
```

Exercise 4a-f: Advanced Cardinality

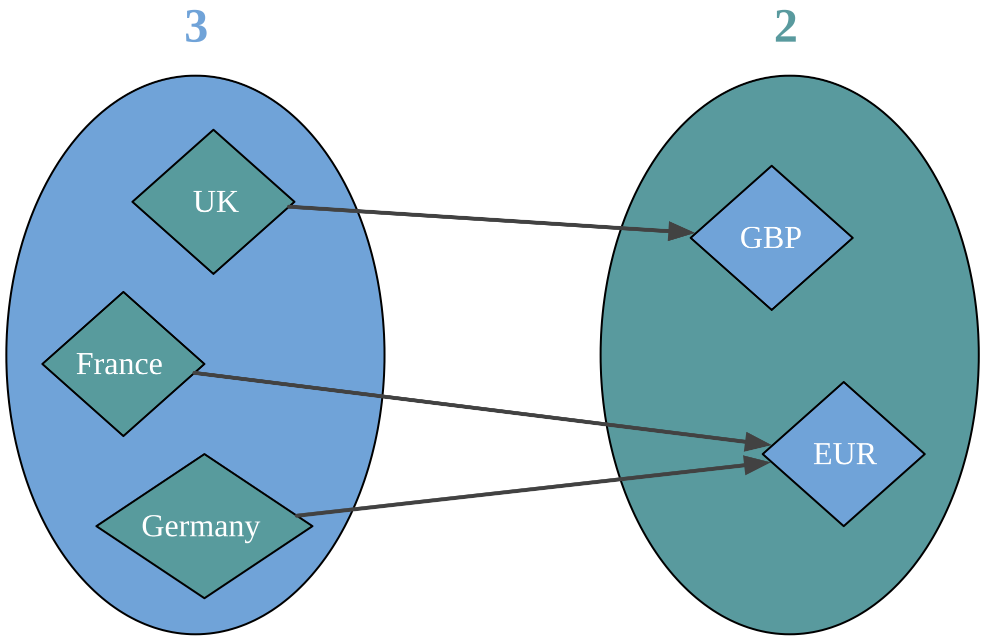
`exercises.types.TypeExercises.scala`

|A => B|

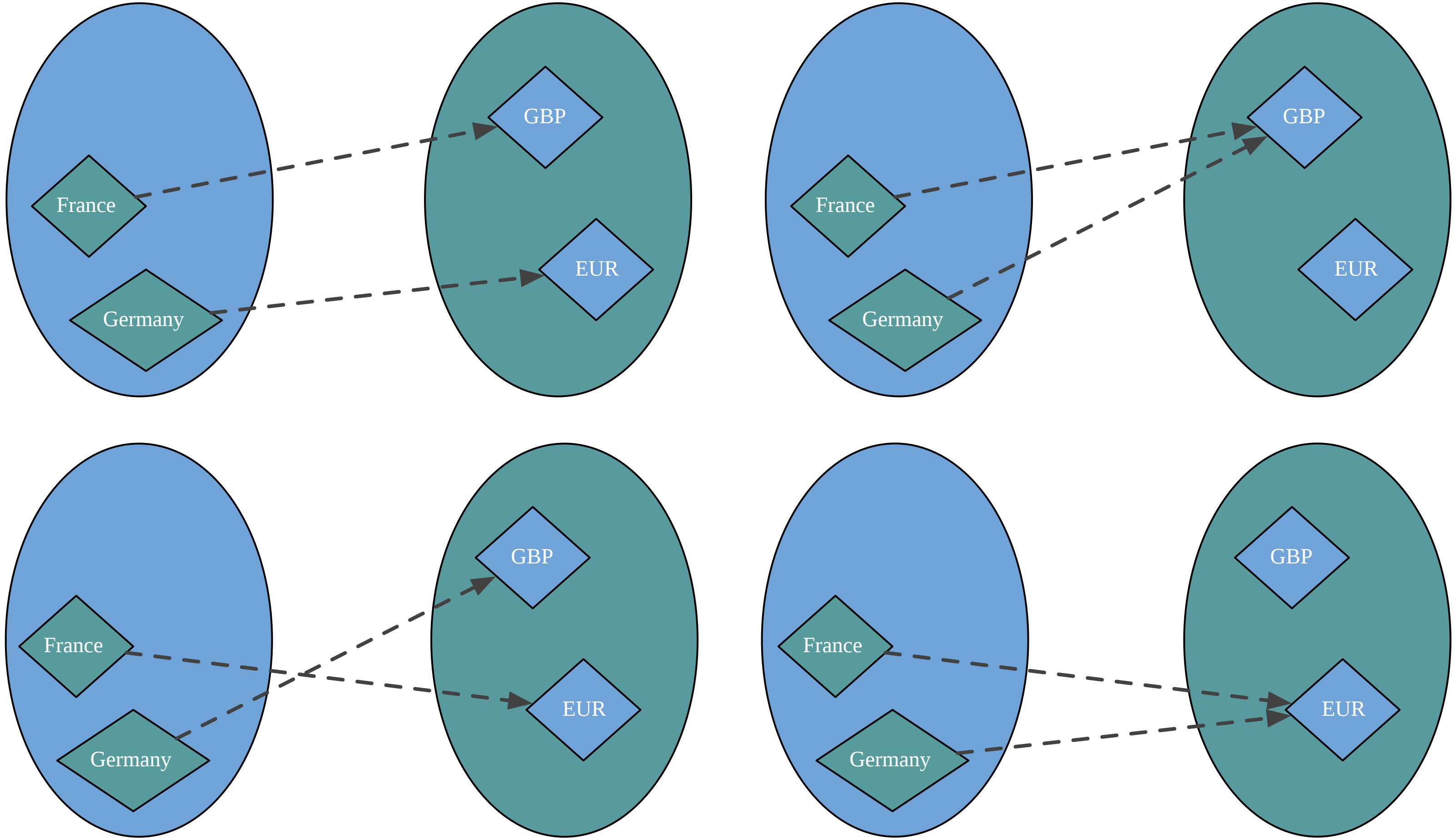
|Country => Currency|



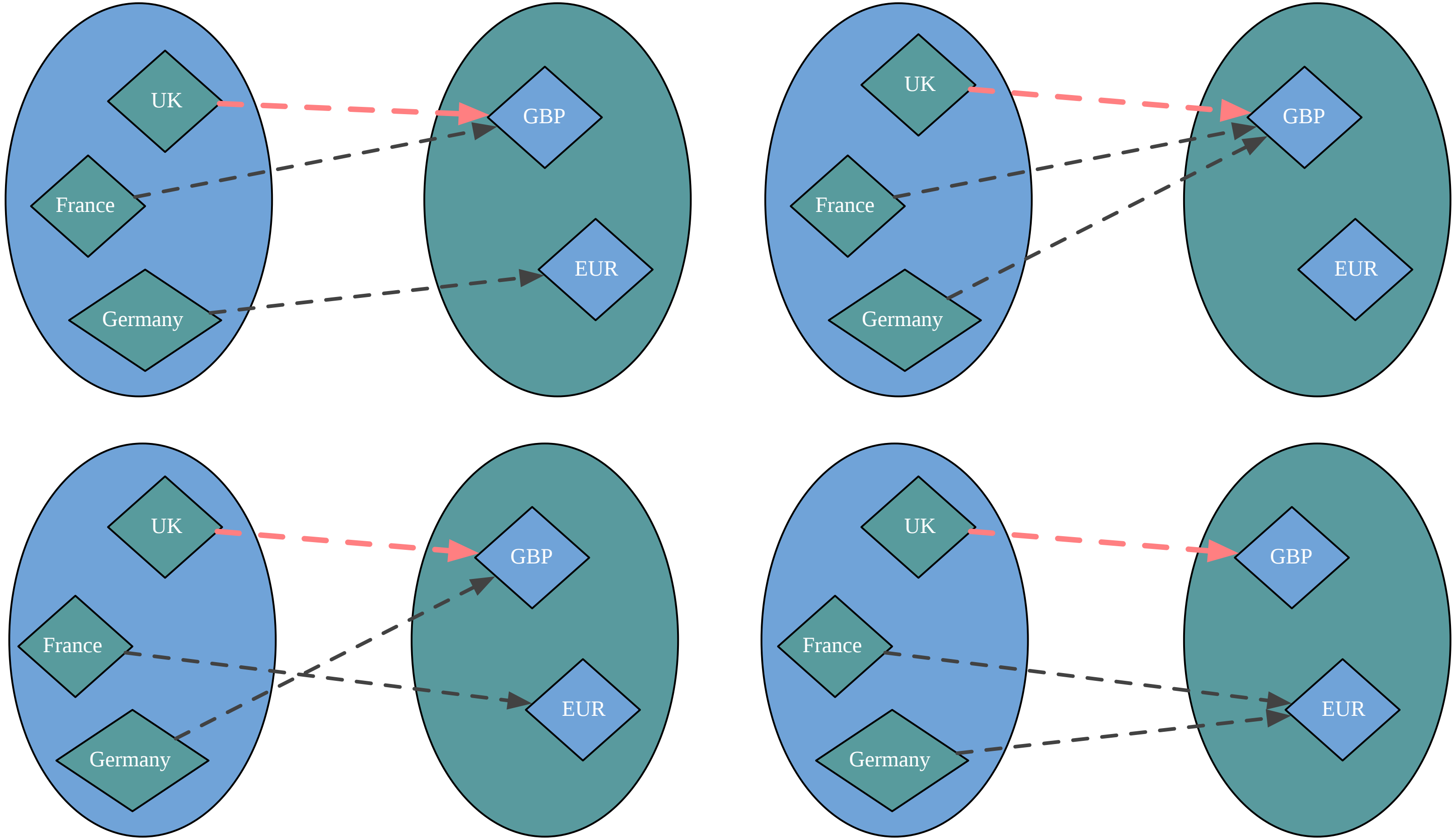
|3 => 2|



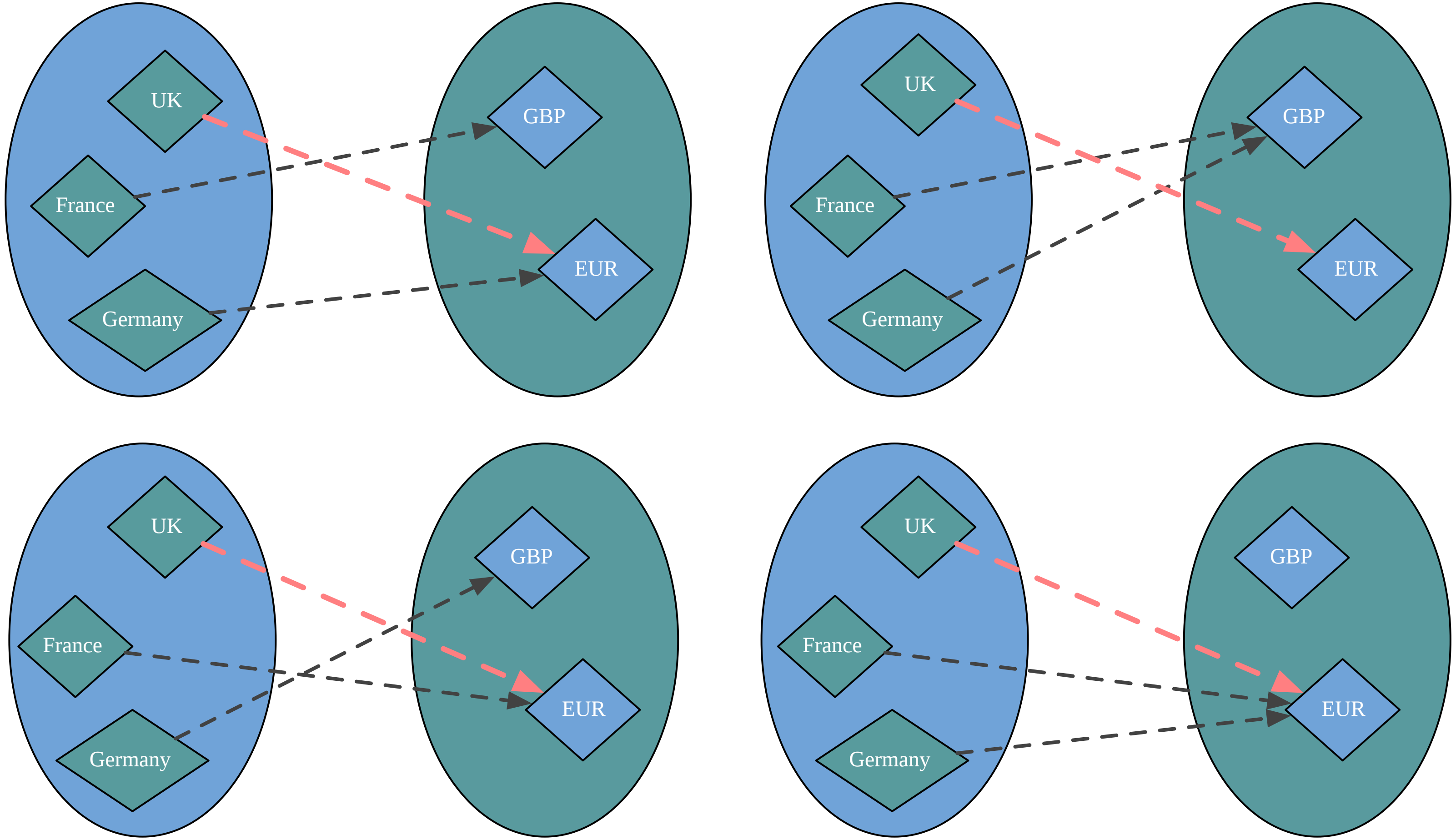
|2 => 2| = 4


France
Germany
GBP
EUR
France
Germany
GBP
EUR
France
Germany
GBP
EUR
France
Germany
GBP
EUR

|3 => 2|

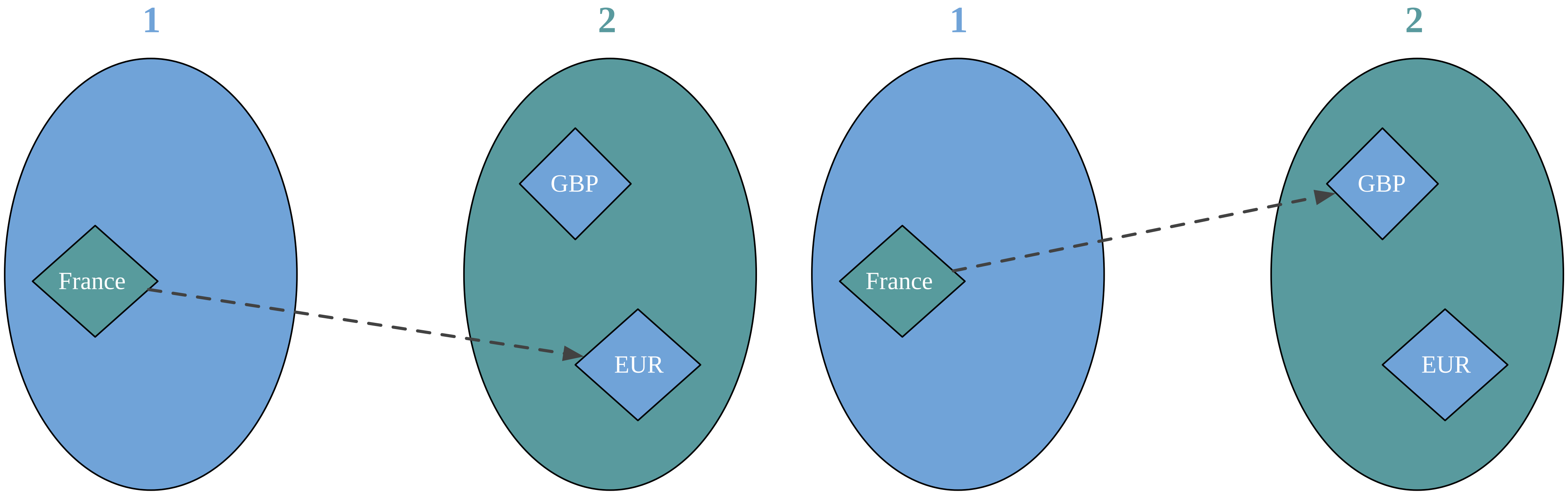

UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR

|3 => 2| = |2 => 2| + |2 => 2|


UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR
UK
France
Germany
GBP
EUR

```
|3 => 2| = 2 * |2 => 2|
|3 => 2| = 2 * |2 => 2|
         = 2 * 2 * |1 => 2|
```

|1 => 2| = 2



```
|3 => 2| = 2 * |2 => 2|
         = 2 * 2 * |1 => 2|
         = 2 * 2 * 2
```

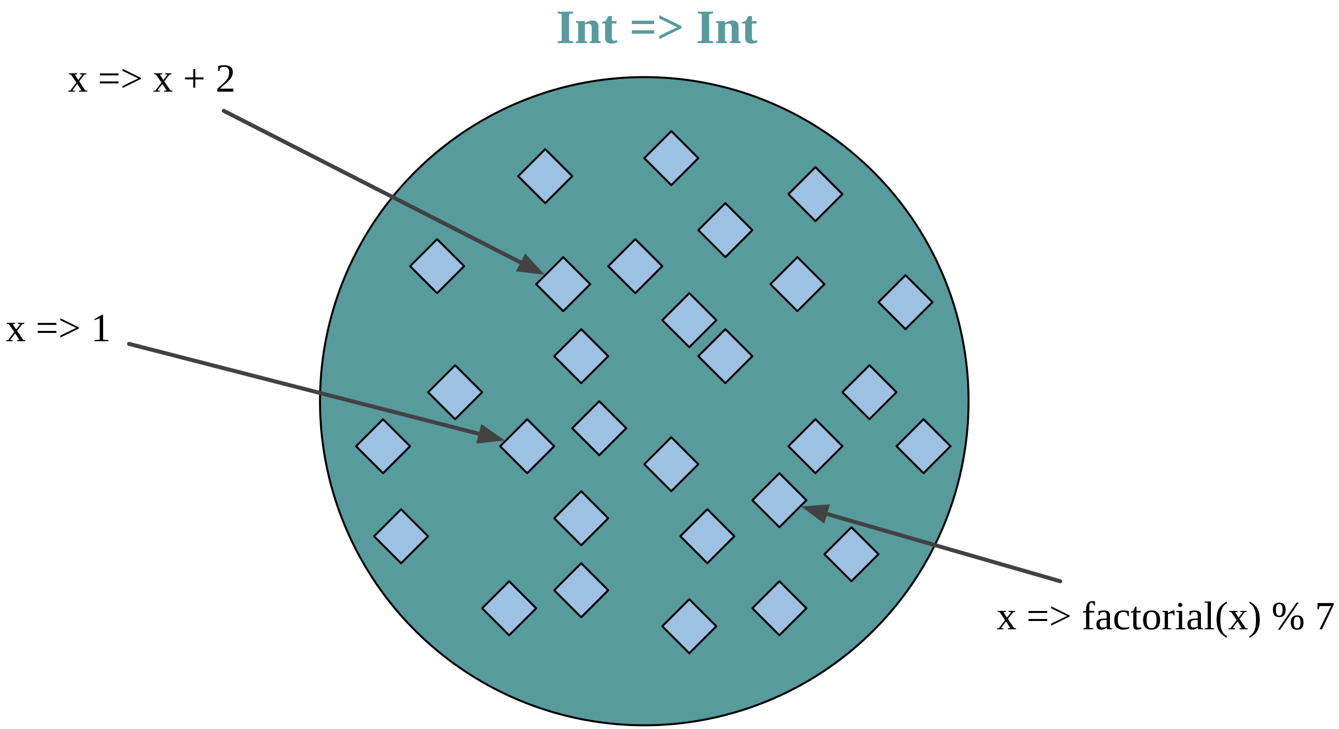
```
|3 => 2| = 2 * |2 => 2|
         = 2 * 2 * |1 => 2|
         = 2 * 2 * 2
         = 2 ^ 3
```

```
|A => B| = |B| ^ |A|
```

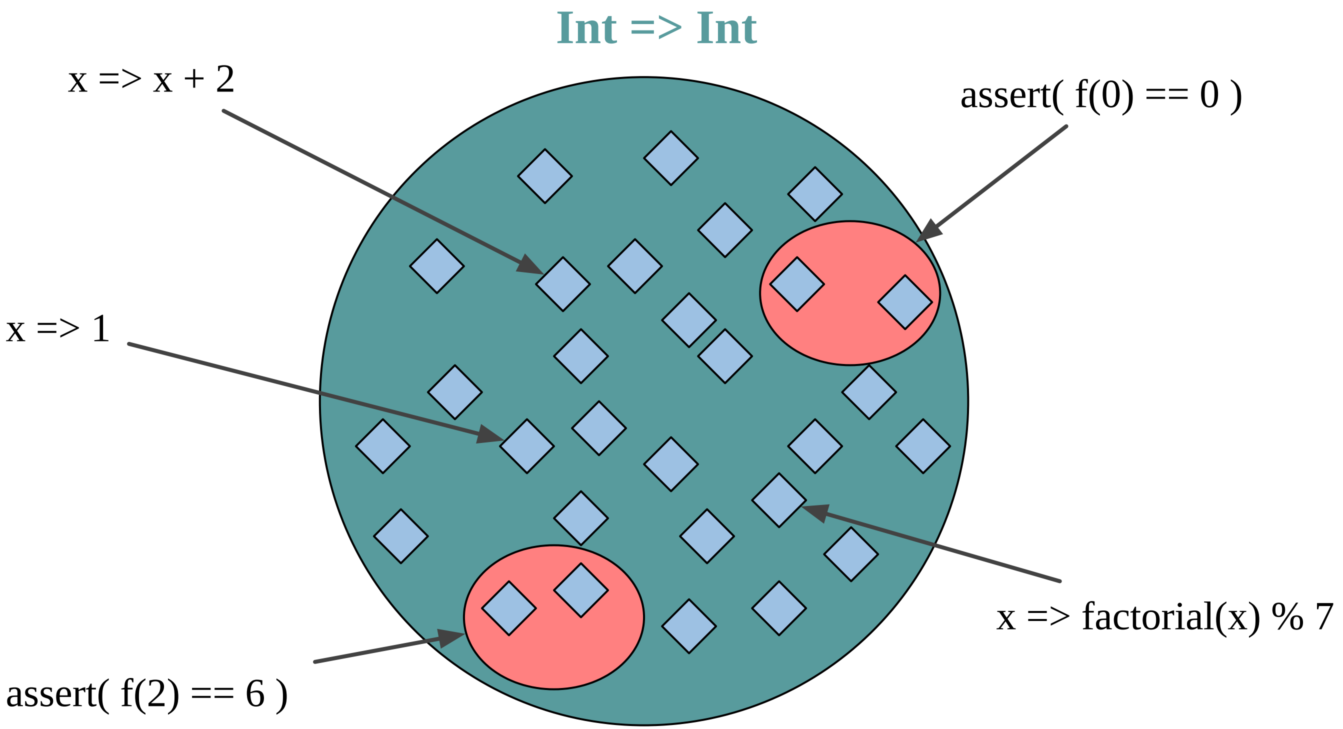
Finish Exercise 4

`exercises.types.TypeExercises.scala`

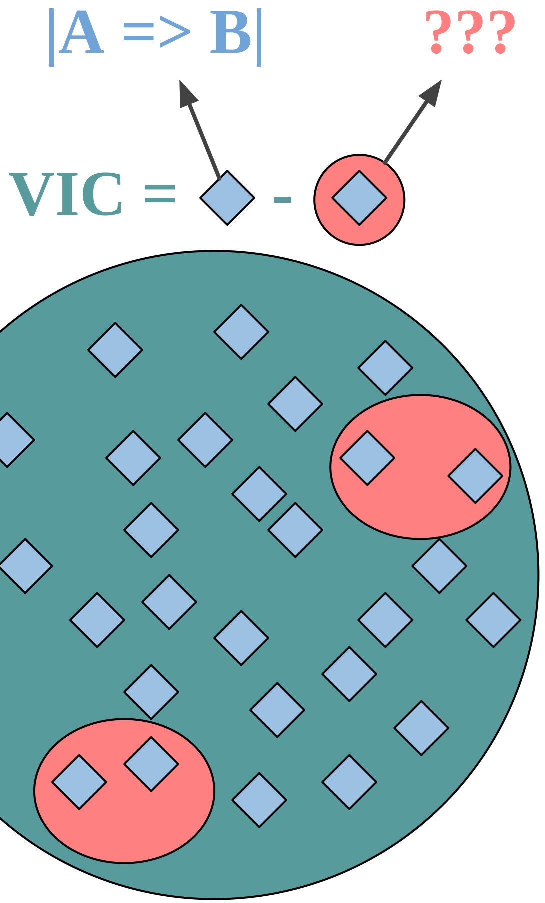
Functions are sets!



Unit tests



Valid Implementation Count (VIC)

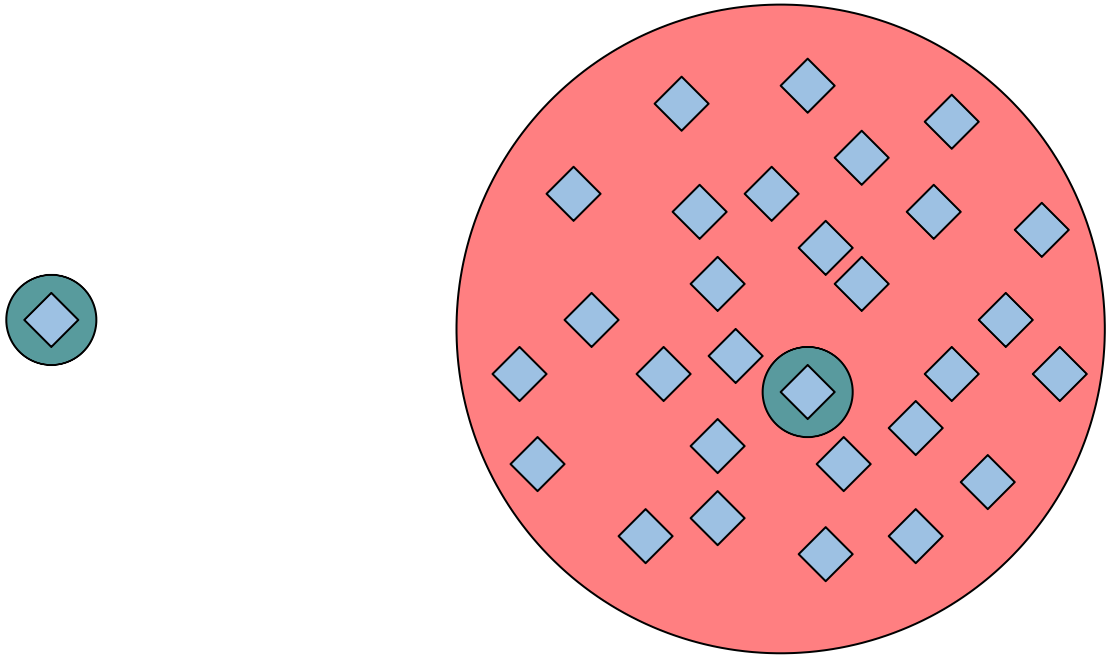


|A => B|

???

VIC = ◇ - ◇

$\mathrm{VIC}(f) = 1$



Exercise 5: Tests

`exercises.types.TypeExercises.scala`

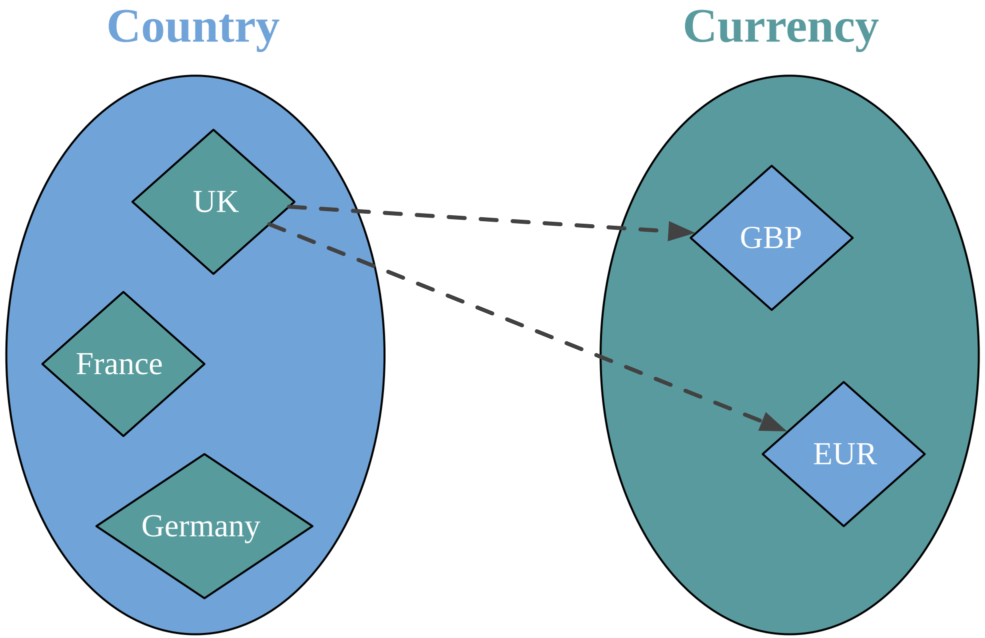
Unit Test

```
sealed trait Country
object Country {
  case object UnitedKingdom extends Country
  case object France        extends Country
  case object Germany       extends Country
}

sealed trait Currency
object Currency {
  case object GBP extends Currency
  case object EUR extends Currency
}

def getCurrency(country: Country): Currency = ???
```

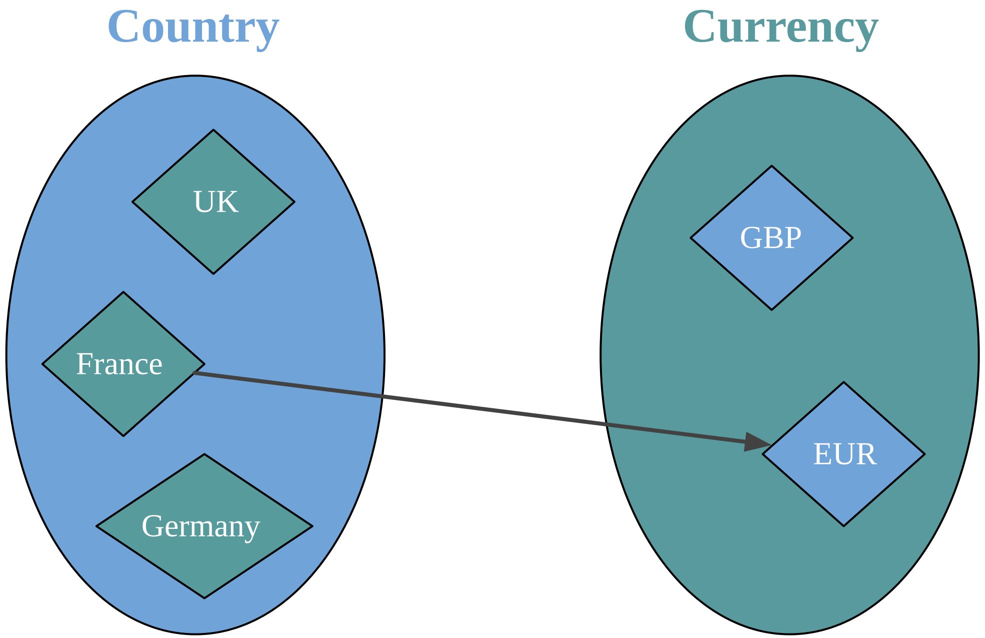
such as

```
assert(getCurrency(France) == EUR)
```

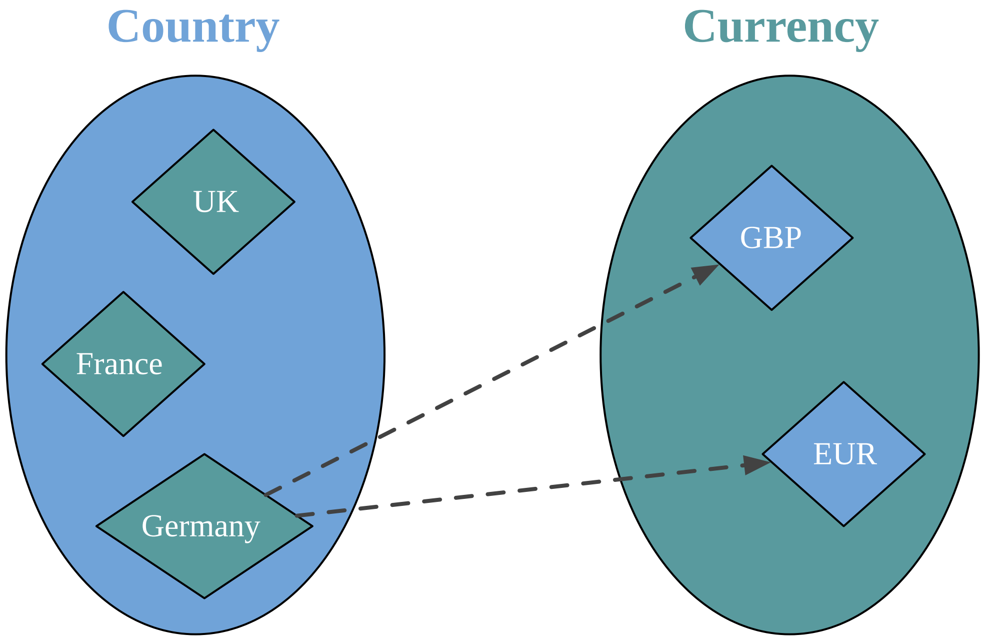
VIC(getCurrency) = 2 * . . .



VIC(getCurrency) = 2 * 1 * ...



VIC(getCurrency) = 2 * 1 * 2



```
VIC(f: A => B) = |B| ^ (|A| - n)
```

where `n` is the number of unit tests

Exercises 6 and 7

`exercises.types.TypeExercises.scala`

Type Algebra

Type	Algebra
Nothing	0
Unit	1
Either[A, B]	A + B
(A, B)	A * B
A => B	B ^ A
Isomorphism	A == B

Curry–Howard isomorphism

Propositions as types from Philip Wadler

Type Algebra Logic

Type	Algebra	Logic
Nothing	0	⊥
Unit	1	⊤
Either[A, B]	A + B	A ∨ B
(A, B)	A * B	A ∧ B
A => B	B ^ A	A → B
Isomorphism	A == B	A ⇔ B

```
Either[A, Nothing] == A
```

```
Either[A, Nothing] == A
```

```
A ∨ ⊥ ⇔ A
```

```
(A, Nothing) == Nothing
```

```
(A, Nothing) == Nothing
```

```
A ∧ ⊥ ⇔ ⊥
```

Find the representation that makes sense to you

```
Either[Int, String] => Boolean     <==>     (Int => Boolean, String => Boolean)
```

Find the representation that makes sense to you

```
Either[A, B] => C   <==>   (A => C, B => C)
```

Find the representation that makes sense to you

```
Either[A, B] => C    <==>    (A => C, B => C)
```

Algebra

```
Either[A, B] => C = C ^ (A + B)
                  = C ^ A * C ^ B
                  = (A => C, B => C)
```

Logic

```
Either[A, B] => C = (A ∨ B) → C
                  = (A → C) ∧ (B → C)
                  = (A => C, B => C)
```

Summary

- Cardinality of types matters
- Unit tests offer almost no benefit in term of correctness
- `VIC(f: A => B) = |B| ^ (|A| - n)`
- Two techniques to achieve correctness
    - Property based testing
    - Parametric polymorphism

All dynamic languages are static languages with a single type

Any

Any => Any

```
def inc(value: Any): Any = value match {
  case x: Int    => x + 1
  case x: Double => x + 1
  case x: Char   => x.toString + "1"
  case x: String => x + "1"
}
```

Any => Any

```
def inc(value: Any): Any = value match {
  case x: Int    => x + 1
  case x: Double => x + 1
  case x: Char   => x.toString + "1"
  case x: String => x + "1"
}
```

```
scala> inc(5)
res7: Any = 6

scala> inc(10.3)
res8: Any = 11.3

scala> inc('c')
res9: Any = c1
```

```
scala> inc(java.time.Instant.ofEpochMilli(0))
scala.MatchError: 1970-01-01T00:00:00Z (of class java.time.Instant)
  at .inc(<console>:2)
  ... 42 elided
```

VIC(Any => Any) = |Any| ^ (|Any| - n)

where n is the number of unit tests

Resources and further study

- Programming with Algebra: property based testing with storage
- Much Ado About Testing: property based testing best practices and pitfalls
- Choosing properties for property-based testing
- Property-Based Testing in a Screencast Editor
- Property-Based Testing The Ugly Parts: Case Studies from Komposition
- Types vs Tests
- Counting type inhabitants
- Thinking with types: type, algebra, logic
- Propositions as types: Curry–Howard isomorphism

Module 6: Typeclass