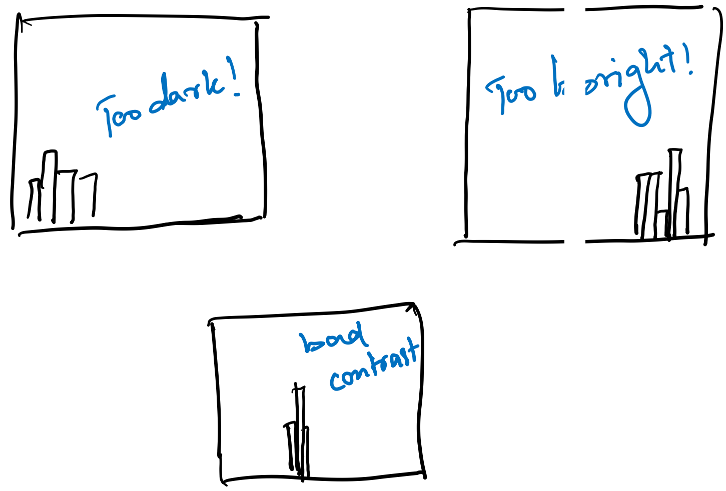
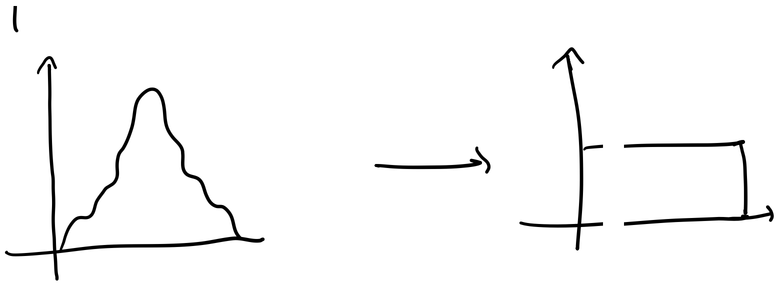
Histogram equalization

Friday, August 6, 2021 3:34 PM

Intensity histogram



equalization



1. convert to pmf

2. Assume distribution is continuous & find T such that

   $s = T(r)$ and s has a uniform distr.

3. we look at the transform

   $s = T[r] = (L-1)\int_0^r f_r(w)\,dw$ — increasing function lemma: for any one-one transform,

$s = T[r]$

$$f_s(s) = \frac{f_r(r)}{\left|\frac{ds}{dr}\right|}\Bigg|_{r=T^{-1}(s)}$$

proof:

If T is either increasing or decreasing:

I) Increasing ⟹ inv is increasing

$$F_s(s_0) = \mathbb{P}(s < s_0) = \mathbb{P}(T(r) < s_0)$$

$$= \mathbb{P}(r < T^{-1}(s_0))$$

$$= F_r(T^{-1}(s_0))$$

$$\Rightarrow \quad f_s(s_0) = \frac{d}{ds} F_r(r)\Big|_{s=s_0}$$

$$= \frac{d F_r(r)}{dr} \cdot \frac{dr}{ds}\Bigg|_{s=s_0}$$

II) Decreasing ⟹ inv is decreasing

$$F_s(s_0) = \mathbb{P}(s < s_0) = \mathbb{P}(T(r) < s_0)$$

$$= \mathbb{P}(r > T^{-1}(s_0))$$

$- \text{''} \;($

$= 1 - F(T^{-1}(s_0))$

$$\Rightarrow f_s(s_0) = -\frac{dF_r}{dr} \cdot \frac{dr}{ds}\bigg|_{s=s_0}$$

but $\frac{dr}{ds}$ is +ve for I (increasing)

& -ve for II (decreasing)

$$\Rightarrow f_s(s_0) = \frac{f_r(r)}{\underbrace{\left|\frac{ds}{dr}\right|}}\Bigg|_{r=T^{-1}(s_0)}$$

↳ since 1st degree differential

So in our case,

$$f_s(s_0) = \frac{\cancel{f_r(r)}}{(L-1)\cancel{f_r(r)} \cdot \cancel{\frac{dr}{dr}}^{1}}\Bigg|_{r=T^{-1}(s)}$$

$$= \frac{1}{L-1}$$

④ We apply this to discrete domain

* Note that T is not one-one any more & it is an <u>approximation</u>

$$\boxed{s_k = (L-1) \sum_{j=0}^{k} p_r(r_j)}$$

Q. What if we keep on applying this transform?

→ Continuous won't make a difference as the process exactly maps to Uniform.

In discrete, we will stop after some time.

As it is a many → one map, # non-zero lvls

one → one.

decreases till it becomes

Histogram Specification:

Try to find a transform to a specific distribution.

1. $r \rightarrow s$ (Uniform)
2. $s \rightarrow z$ (required)

We know $z \rightarrow s$ is $G(z) = (L-1) \int_0^z f_z(w)\,dw$

$$\Rightarrow \quad z = G^{-1}[T(r)]$$

?

CLAHE: