Exercise 23 Section 2.2

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

Augmented Matrix

$$A = \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

$$R_2 \to R_2 - (-1)R_1 \quad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

$$\downarrow$$

$$R_2 \to \frac{R_2}{(-1)} \quad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & 9 \\ 0 & -10 & 2 & -14 \end{array}\right)$$

$$\to$$

$$R_3 \to \left(\frac{-1}{52}\right) R_3 \quad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\downarrow$$

$$R_2 \to R_2 + 5R_3 \quad \left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\downarrow$$

$$R_1 \to R_1 - R_2 \quad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\Leftarrow$$

$$\boxed{x_1, x_2, x_3 = (3, 1, 2)}$$

Exercise 43 Section 2.2

$x + 2y + 3z = \underline{\quad} 4$

$3x - \underline{\quad} y + \underline{\quad} 5z = \underline{\quad} 2$

$4x + \underline{\quad} y - 14z = a + 2$

Augmented matrix

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & -14 & a+2 \end{bmatrix}$$

$-3R_1 + R_2 \to R_2$

$-4R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -7 & -4 & -10 \\ 0 & -7 & -26 & a-14 \end{bmatrix}$$

$R_2 -$ $R_2 x - 7$

$R_2(x-1) + R_3 \to R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -7 & -4 & 10 \\ 0 & 0 & -22 & a-4 \end{bmatrix}$$

$z = \dfrac{a-4}{-22}$

the system has exactly one solution.

Exercise 15 section 2.2

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$x_1 - 3x_2 + 4x_3 = 7$

$x_2 + 2x_3 = 2$

$x_3 = 5$

$x_2 = 2 - 2x_3 = 2 - 2 \times 5 = -8$

$x_1 = 1 + 3x_2 - 4x_3 = 7 - 24 - 20 = -37$

$x_1 = -37 \quad x_2 = -8 \quad x_3 = 5.$

Exercise 25 (a) section 2.1

(a) $A = \begin{bmatrix} 1 & 3 & 5 \\ -2 & 2 & 1 \\ 3 & 4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 8 & 11 \\ 3 & 4 & -2 \end{bmatrix}$

$A \to B \quad 2 \times R_1 + R_2 \to R_2$

$B \to A$

$-2R_2 + R_1 \to R_1$

Exercise 17 Section 2.1

$$\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$2x_1 = 0$

$3x_1 - 4x_2 = 0$

$x_2 = 1$

Exercise 39 (a) Section 1.3

(a) $x = 9 - 5t \quad y - 1 - t \quad z = 3 + t$

$2x - 3y + 4z + 7 = 0$

find the point of intersection

$2(9-5t) - 3(-1-t) + 4(3+t) + 7 = 0$

$18 - 10t + 3 + 3t + 12 + 4t + 7 = 0$

$40 - 3t = 0 \qquad t = \frac{40}{3}$

point of intersection

$\left(9 - 5 \cdot \frac{40}{3},\ -1 - \frac{40}{3},\ 3 + \frac{40}{3}\right) = \left(-\frac{173}{3},\ -\frac{43}{3},\ \frac{49}{3}\right)$

Exercise 31(a) Section 1.3

(a) $x = -5 - 4t$, $y = 1 - t$, $z = 3 + 2t$;

$$x + 2y + 3z - 9 = 0$$

$$\begin{cases} x = -5 - 4t \\ y = 1 - t \\ z = 3 + 2t \end{cases}$$

$\vec{n} = (1, 2, 3)$ ← normal vector

$\vec{v} \cdot \vec{n} = (-4, -1, 2)(1, 2, 3) = -4 - 2 + 6 = 0$

orthogonals ∥ so plane and line are parallel

Exercise 24 Section 1.3

find Parallel plane to the plane $3x + y - 2z = 5$

$\vec{n} = (1, 2, -3)$

a) $\vec{n}_a = (1, 2, -3) = \vec{n}$

b) $\vec{n}_b = \left(\frac{1}{4}, \frac{1}{2}, -\frac{3}{4}\right) = \frac{1}{4}(1, 2, -3) = \frac{1}{4}\vec{n}$

a and b are parallel

c) $\vec{n}_c = (1, 2, 3) \neq \lambda(1, 2, -3) \quad \lambda \in \mathbb{R}$

Exercise 12 Section 1.3

$(3,2,1)$, $(-1,-1,-1)$ and $(6,0,2)$

$$\vec{X} = X_0 + t_1(\vec{X}_1 - \vec{X}_0) + t_2(\vec{X}_2 - \vec{X}_0)$$

$$\vec{X} = (3,2,1) + t_1(-9,-3,-2) + t_2(3,-2,1)$$

$$\vec{X} = (x,y,z)$$

$$(x,y,z) = (3,2,1) + t_1(-4,-3,-2) + t_2(3,-2,1)$$

$x = 3 - 9t_1 + 3t_2$
$y = 2 - 3t_1 - 2t_2$
$z = 1 - 2t_1 + t_2$

for $t_1 = 0$ $t_2 = 0$
$x = 6$, $y = 0$, $z = 2$

for $t_1 = 1$, $t_2 = 1$ $x = 2$, $y = -3$, $z = 0$ $P_0(2,-3,0)$

for $t_1 = -2$, $t_2 = 0$ $x = 11$, $y = 8$, $f = 5$ $P_1(11,8,5)$

for $t_1 = 0$, $t_2 = 2$ $x = 9$, $y = -2$, $z = 3$ $P_2(9,-2,3)$

Exercise 14(a) Section 1.3

$x = t, \quad y = -3+5t, \quad z = 1+t$

$(x, y, z) = (t, -3+5t, 1+t)$

answer

$\vec{x} = (0, -3, 1) + t(1, 5, 1)$

Exercise 6(a) Section 1.3

$(1,2)$ and $(-5,6)$

$(x, y) = (1-t)(1,2) + t(-5,6)$

$x = 1-6t \quad y = 2+4t$

Exercise 24 Section 1.2

$\vartheta_1 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right)$ $\vartheta_2 = \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right)$

$\vartheta_3 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right)$

$$\|\vec{\vartheta}_1\| = \left(\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\right)^{\frac{1}{2}} = \left(\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\right)^{\frac{1}{2}} = \left(\frac{3+1+2}{6}\right)^{\frac{1}{2}} = 1$$

$$\|\vec{\vartheta}_2\| = \left[0^2 + \left(-\frac{2}{\sqrt{6}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\right]^{\frac{1}{2}} = \left(\frac{4}{6} + \frac{1}{3}\right)^{\frac{1}{2}} = 1$$

$$\|V_3\| = \left[\frac{1}{2} + \frac{1}{6} + \frac{1}{3}\right]^{\frac{1}{2}} = \sqrt{1} = 1$$

Unitary vectors.

$$\vec{\vartheta}_1 \cdot \vec{\vartheta}_2 = \vec{\vartheta}_1 \cdot \vec{\vartheta}_3 = \vec{\vartheta}_2 \cdot \vec{\vartheta}_3 = 0$$

$$\vec{\vartheta}_1 \cdot \vec{\vartheta}_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) \cdot \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) = 0 - \frac{2}{6} + \frac{1}{3} = 0$$

$$\vec{\vartheta}_1 \cdot \vec{\vartheta}_3 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) = -\frac{1}{2} + \frac{1}{6} + \frac{1}{3} = 0$$

$$\vec{\vartheta}_2 \cdot \vec{\vartheta}_3 = \left(0, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}\right) = 0 - \frac{2}{6} + \frac{1}{3} = 0$$

So $\{\vec{\vartheta}_1, \vec{\vartheta}_2, \vec{\vartheta}_3\}$ is an orthonormal set.

Exercise 4 Section 1.2

(a) $\|u+v+w\| = \|(6,1,3)\| = \sqrt{6^2+1^2+3^2} = \sqrt{36+1+9} =$

$= \sqrt{46}$

answer

$u=(2,-2,3)$

$v=(1,-3,4)$

$w=(3,6,-4)$

Exercise 8 Section 1.2

$v = (1, 1, 2, -3, 1)$ $\qquad ||kv|| = 4$

$||v|| = \sqrt{1^2 + 1^2 + 2^2 + (-3)^2 + 1^2} = \sqrt{1+1+4+9+1} = \sqrt{16} = 4$

$||kv|| = |k|\,||v||$

answer

$||kv|| = 4 \qquad |k|\,||v|| = 4 \qquad |k| = \frac{4}{4} \qquad k = \underline{\pm 1}$

Exercise 10 (a) Section 1.2

(a)

$u = (1, 1, -2, 3) \qquad v = (-1, 0, 5, 1)$

$u \cdot v = -1 + 0 - 10 + 3 = -8$

$u \cdot u = 1 + 1 + 4 + 9 = 15$

$v \cdot v = 1 + 0 + 25 + 1 = 27$

exercise 19 Section 1.1

first we set up our system

$au + bv = (1, -4, 9, 18)$

$a(1, -1, 3, 5) + b(2, 1, 0, -3) = (1, -4, 9, 18)$

$(a + 2b, -a + b, 3a, 5a - 3b) = (1, -4, 9, 18)$

$a + 2b = 1$ $\quad -a + b = -4$ $\quad 3a = 9$ $\quad 5a - 3b = 18$

$b = -1$ $\quad -4 = -4$ $\quad a = 3$ $\quad 18 = 18$

$\underline{a = 3 \quad b = -1}$ answer

Section 1.1 exercise 8

(a) $P_1(-6, 2)$, $P_2(-4, -1)$

$\overrightarrow{P_1P_2} = (-4 + 6, -1 - 2) = \underline{(2, -3)}$ answer

(b) $P_1(0, 0, 0)$, $P_2(-1, 6, 1)$

$\overrightarrow{P_1P_2} = (-1 - 0, 6 - 0, 1 - 0) = \underline{(-1, 6, 1)}$ ← answer

The program I used to take pictures : CamScanner
Thank you

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