FIRST PRINCIPLES MODELING

Topics Covered

- Obtaining the equations of motion of a DC motor based rotary servo.
- Creating and validating a system model.
- Model validation.

Prerequisites

- QUBE-Servo Integration Lab.
- Filtering Lab.

1 Background

The Quanser QUBE-Servo is a direct-drive rotary servo system. Its motor armature circuit schematic is shown in Figure 1.1 and the electrical and mechanical parameters are given in Table 1.1. The DC motor shaft is connected to the *load hub*. The hub is a metal disk used to mount the disk or rotary pendulum and has a moment of inertia of J_h. A disk load is attached to the output shaft with a moment of inertia of J_d.

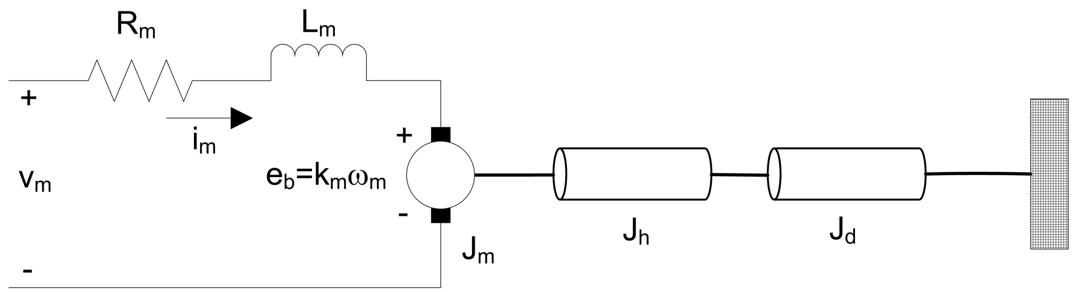


Figure 1.1: QUBE-Servo DC motor and load

The back-emf (electromotive) voltage $e_b(t)$ depends on the speed of the motor shaft, ω_m, and the back-emf constant of the motor, k_m. It opposes the current flow. The back emf voltage is given by:

$$e_b(t) = k_m \omega_m(t)$$

Symbol	Description	Value
DC Motor		
R_m	Terminal resistance	6.3 Ω
k_t	Torque constant	0.036 N-m/A
k_m	Motor back-emf constant	0.036 V/(rad/s)
J_m	Rotor inertia	4.0×10^{-6} kg-m^2
L_m	Rotor inductance	0.85 mH
m_h	Load hub mass	0.0087 kg
r_h	Load hub mass	0.0111 m
J_h	Load hub inertia	1.07×10^{-6} kg-m^2
Load Disk		
m_d	Mass of disk load	0.054 kg
r_d	Radius of disk load	0.0248 m

Table 1.1: QUBE-Servo system parameters

Using Kirchoff's Voltage Law, we can write the following equation:

$$v_m(t) - R_m i_m(t) - L_m \frac{di_m(t)}{dt} - k_m \omega_m(t) = 0.$$

Since the motor inductance L_m is much less than its resistance, it can be ignored. Then, the equation becomes

$$v_m(t) - R_m i_m(t) - k_m \omega_m(t) = 0.$$

Solving for $i_m(t)$, the motor current can be found as:

$$i_m(t) = \frac{v_m(t) - k_m \omega_m(t)}{R_m}. \tag{1.1}$$

The motor shaft equation is expressed as

$$J_{eq}\dot{\omega}_m(t) = \tau_m(t), \tag{1.2}$$

where J_{eq} is total moment of inertia acting on the motor shaft and τ_m is the applied torque from the DC motor. Based on the current applied, the torque is

$$\tau_m = k_m i_m(t)$$

The moment of inertia of a disk about its pivot, with mass m and radius r, is

$$J = \frac{1}{2}mr^2. \tag{1.3}$$

2 In-Lab Exercises

Based on the models already designed in QUBE-Servo Integration and Filtering labs, design a model that applies a 1-3 V 0.4 Hz square wave to the motor and reads the servo velocity using the encoder as shown in Figure 2.1.

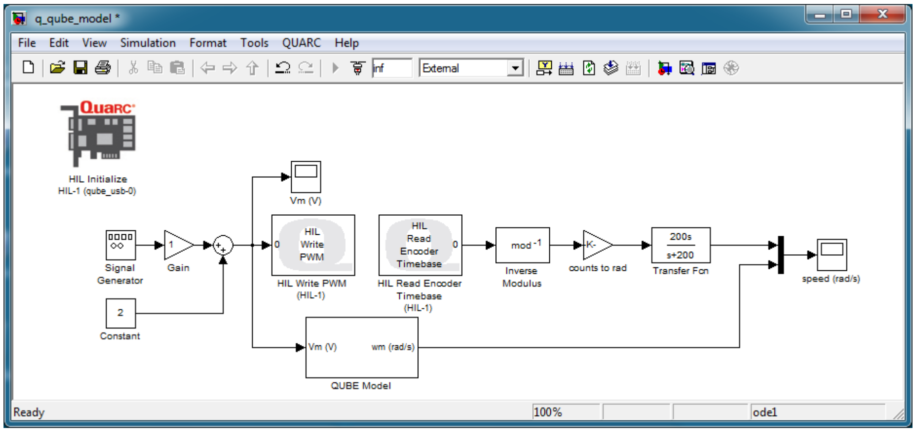


Figure 2.1: Applies a step voltage and displays measured and simulated QUBE-Servo speed.

Create subsystem called *QUBE-Servo Model*, as shown in Figure 2.1, that contains blocks to model the QUBE-Servo system. Thus using the equations given above, assemble a simple block digram in Simulink to model the system. You'll need a few Gain blocks, a Subtract block, and an Integrator block (to go from acceleration to speed). Part of the solution is shown in Figure 2.2.

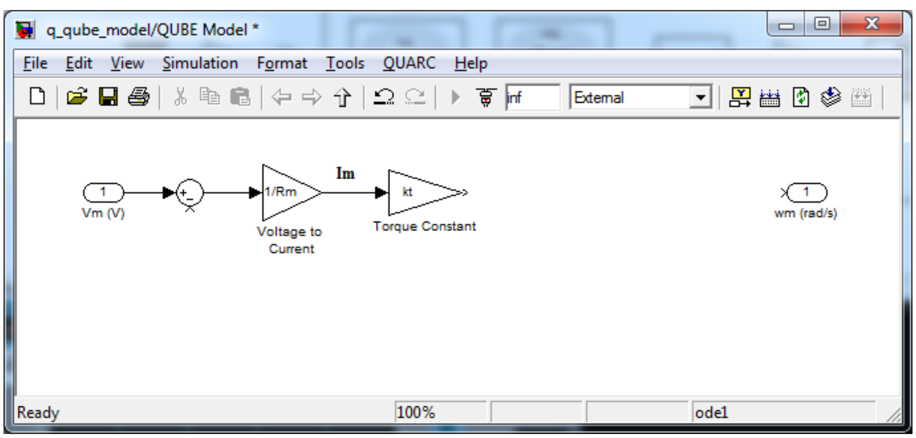


Figure 2.2: Incomplete *QUBE-Servo Model* subsystem.

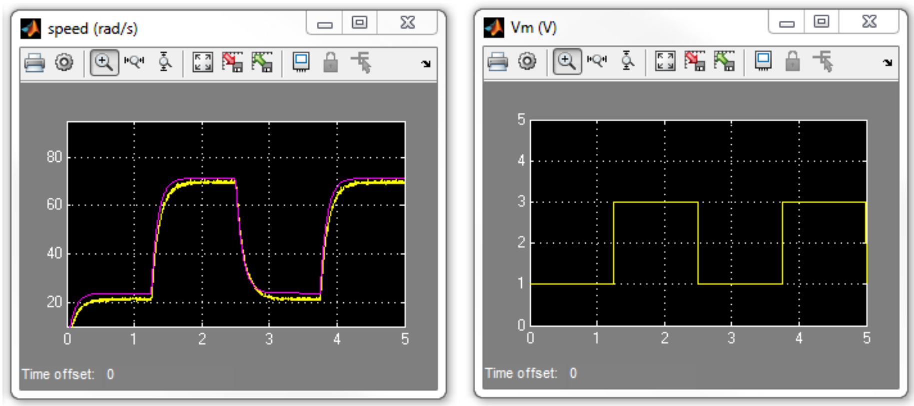
It may also help to write a short Matlab script that sets the various system parameters in Matlab, so you can use the symbol instead of entering the value numerically in the Gain blocks. In the example shown in Figure 2.2, we are using *Rm* for motor resistance and *kt* for the current-torque constant. To define these, write a script like:

```
% Resistance
Rm = 8.4;
% Current-torque (N-m/A)
kt = 0.042;
```

1. The motor shaft of the QUBE-Servo is attached to a *load hub* and a disk load. Based on the parameters given in Table 1.1, calculate the equivalent moment of inertia that is acting on the motor shaft.

2. Design the *QUBE-Servo Model* subsystem as described above. Attach a screen capture of your model and the Matlab script (if you used one).

3. Build and run the QUARC controller with your QUBE-Servo model. The scope response should be similar to Figure 2.3. Attach a screen capture of your scopes. Does your model represent the QUBE-Servo well? Explain.



(a) Motor Speed

(b) Motor Voltage

Figure 2.3: QUBE Response

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