

Electronically Tunable Current-Mode Third-Order Square-Root-Domain Filter Design*

Ali Kircay[†]

*Electrical and Electronics Engineering,
Harran University, Sanliurfa, Turkey
kircay@harran.edu.tr*

M. Serhat Keserlioglu

*Electrical and Electronics Engineering,
Pamukkale University, Denizli, Turkey
mskeserlioglu@pau.edu.tr*

F. Zuhul Adalar

*Electrical and Electronics Engineering,
Harran University, Sanliurfa, Turkey
fzadalar@harran.edu.tr*

Received 22 March 2017

Accepted 26 October 2017

Published 7 December 2017

In this study, electronically-tunable, current-mode, square-root-domain, third-order low-pass filter is proposed. The study is carried out with three circuit designs. First circuit is third-order low-pass Butterworth filter, second circuit is third-order low-pass Chebyshev filter and the last circuit is third-order low-pass elliptic filter. All the input and output values of the filter circuit are current. Only grounded capacitors and MOSFETs are required in order to realize the filter circuit. Additionally, natural frequency f_0 of the current-mode filter can be adjusted electronically using outer current sources. To validate the theory and to demonstrate the performance of third-order filter, frequency and time domain simulations of PSPICE program are used. To that end, TSMC 0.35 μm Level 3 CMOS process parameters are utilized to realize the simulations of the filter.

Keywords: Current-mode filters; square-root-domain filters; state-space-synthesis; third-order filters.

*This paper was recommended by Regional Editor Piero Malcovati.

[†]Corresponding author.

1. Introduction

A branch of companding (COMpressing-ExPANDING) circuits is square-root-domain circuits. They are suitable to VLSI (Very Large Scale Integration) technologies, have a wide dynamic range, require only MOSFETs and grounded capacitors, provide low power under low voltage, operate at high frequencies and can be tuned electronically using outer current sources. Considering these properties, companding circuits are suitable for CMOS (Complementary Metal Oxide Semiconductor) VLSI technology. Companding method applications are logarithmic-domain and square-root-domain circuits. These circuits are the most widely used translinear circuits. Logarithmic-domain circuits are proposed by Adams¹ and then they have been studied by Frey.^{2,3} The BJTs' or MOSFETs' exponential I-V (current-voltage) characteristics in weak inversion region are used in basic translinear principle.^{4,5} The linear transconductor that was proposed by Bult⁶ is an elementary example of the quadratic law of MOSFETs. Seevinck⁷ derived the MOS translinear (MTL) principle from the bipolar translinear (BTL) principle.⁴ MTL principle uses quadratic relationship between the voltage and current of the MOSFETs in saturation and strong inversion region. Starting from state-space equation, likewise quadratic relationship between the voltage and current of the MOSFETs, filters performed by using analog processing circuit blocks like square-root and squarer/divider circuit are called square-root-domain filters.⁸⁻¹³

Square-root-domain first-order filter circuits,⁹⁻¹¹ second order filter circuits^{8,12,13} have been studied by various researchers. However, it has been seen that the studies on square-root-domain third-order filter circuits was found to be minimal. Third-order filter circuits obtained using OTA and OTRA are presented in Refs. 14-16. For square-root-domain filter design, a state-space-synthesis method, using Class AB structure based on the MOS transistors square law is proposed in Ref. 17. A third-order low pass elliptic filter using a square-root-domain differentiator is presented in Ref. 18. A third-order square-root-domain elliptic lowpass LC (Inductance-Capacitance) ladder filter is proposed in Ref. 19. A third-order filter circuit that provides band-pass, low-pass and high-pass filter functions is proposed in Ref. 20. A square-root-domain third-order voltage-mode low-pass Butterworth and Chebyshev filters are presented in Ref. 21.

In this study, square-root-domain, third-order, current-mode, low-pass Butterworth, Chebyshev and elliptic filters are designed. To design the filter circuit, state-space synthesis method is used. Designed filter circuit comprises two types of analog processing circuit blocks like square-root and squarer/divider circuits. In addition to these analog blocks, the proposed filter circuit is composed of MOSFET current mirrors, grounded capacitors, DC current sources and DC supply voltage. Since all the input and output signals of the proposed low-pass circuit are current, it can be defined as current-mode. The natural frequency f_0 of proposed low-pass filter can be adjusted electronically using outer current sources.

2. The Proposed Current-Mode Third-Order Low-Pass Filter

Third-order, current-mode, square-root-domain, low-pass filter transfer function can be expressed as follows:

$$N(s) = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{\omega_{01}^2 \omega_{02}}{\left(s^2 + \frac{\omega_{01}}{Q}s + \omega_{01}^2\right)(s + \omega_{02})}, \quad (1)$$

where, I_{in} denotes the input current, I_{out} denotes the output current, Q stands for the quality factor and ω_{01} and ω_{02} are the natural frequencies of the filter circuit.

Third-order, current-mode, square-root-domain, low-pass filter transfer function can be converted to the following state-space equations²¹:

$$\dot{x}_1 = -\frac{\omega_{01}}{Q}x_1 + \omega_{01}x_3, \quad (2)$$

$$\dot{x}_2 = -\omega_{02}x_2 + \omega_{02}u, \quad (3)$$

$$\dot{x}_3 = -\omega_{01}x_1 + \omega_{01}x_2. \quad (4)$$

The output equation is²¹

$$y_{\text{LP}} = x_1, \quad (5)$$

where y represents the output and u represents the input currents of the filter. Additionally, x_1 , x_2 and x_3 represent the state variables. They are drain currents of MOSFETs. Square mappings are used on the state variables, Eqs. (2)–(4) can be transformed into nodal equations set. Hence, Eq. (6) can be implemented to quantities in equations.¹⁰

$$I_i = \frac{\beta}{2}(V_i - V_{\text{th}})^2 \quad i = 1, 2, 3, \quad (6)$$

where I_i denotes drain current of MOSFET in saturation region, $\beta = \mu_0 C_{0x}(W/L)$ stands for transconductance, V_{th} represents the threshold voltage and V_i represents the gate–source voltage of MOSFETs.

If we take the derivative of I_1 , I_2 and I_3 , we get

$$\dot{I}_i = \dot{V}_i \sqrt{2\beta I_i} \quad i = 1, 2, 3. \quad (7)$$

The relation given above can be organized to yield the following nodal equations which are applied to Eqs. (2)–(4):

$$C\dot{V}_1 = -\frac{C\omega_{01}}{Q\sqrt{2\beta I_1}}I_1 + \frac{C\omega_{01}}{\sqrt{2\beta I_1}}I_3, \quad (8)$$

$$C\dot{V}_2 = -\frac{C\omega_{02}}{\sqrt{2\beta I_2}}I_2 + \frac{C\omega_{02}}{\sqrt{2\beta I_2}}I_U, \quad (9)$$

$$C\dot{V}_3 = -\frac{C\omega_{01}}{\sqrt{2\beta I_3}}I_1 + \frac{C\omega_{01}}{\sqrt{2\beta I_3}}I_2. \quad (10)$$

In these equations, C is a capacitor value resembling a multifunction factor. $C\dot{V}_1$, $C\dot{V}_2$ and $C\dot{V}_3$ in Eqs. (8)–(10) can be accepted as time-dependent currents that are grounded via three capacitors.

The following equations I_{01} , I_{02} and I_{03} can be defined for use in Eqs. (8)–(10).

$$\sqrt{I_{01}} = \frac{C\omega_{01}}{\sqrt{\beta}}, \quad (11)$$

$$\sqrt{I_{02}} = \frac{C\omega_{02}}{\sqrt{\beta}}, \quad (12)$$

$$\sqrt{I_{03}} = \frac{C\omega_{01}}{\sqrt{\beta}}. \quad (13)$$

Additionally, quality factor adjusting current I_Q defined as given below via Eqs. (8) and (11)

$$I_Q = I_{01}/Q^2. \quad (14)$$

Equations (8)–(10) can be arranged as follows:

$$C\dot{V}_1 = -\sqrt{\frac{I_Q I_1}{2}} + \sqrt{\frac{I_{01} I_3^2}{2I_1}}, \quad (15)$$

$$C\dot{V}_2 = -\sqrt{\frac{I_{02} I_2}{2}} + \sqrt{\frac{I_{02} I_U^2}{2I_2}}, \quad (16)$$

$$C\dot{V}_3 = -\sqrt{\frac{I_{03} I_1^2}{2I_3}} + \sqrt{\frac{I_{03} I_2^2}{2I_3}}. \quad (17)$$

Third-order, current-mode, square-root-domain, low-pass filter circuit shown in Fig. 1 has been actualized using Eqs. (15)–(17), where I_U denotes the input current and I_1 , I_2 and I_3 denote the output currents of filter circuit. Additionally, using Eqs. (11) and (12), ω_{01} and ω_{02} natural frequency of the filter circuit can be determined depending on the I_{01} , I_{02} , β and C .

$$\omega_{01} = \frac{\sqrt{\beta}\sqrt{I_{01}}}{C}, \quad (18)$$

$$\omega_{02} = \frac{\sqrt{\beta}\sqrt{I_{02}}}{C}. \quad (19)$$

Using Eqs. (2)–(4), output variables of the square-root-domain third-order low-pass filter circuit can be determined on ω_{01} , ω_{02} and Q .

$$I_1 = \frac{\omega_{01}^2 \omega_{02}}{\left(s^2 + \frac{\omega_{01}}{Q}s + \omega_{01}^2\right)(s + \omega_{02})} I_U, \quad (20)$$

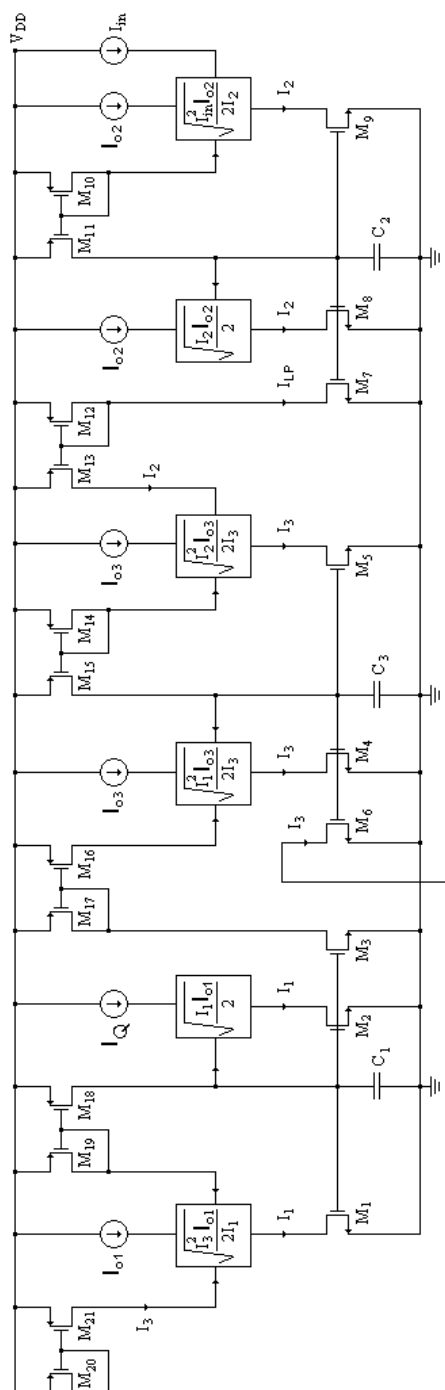


Fig. 1. Square-root-domain third-order current-mode low-pass filter circuit.

$$I_2 = \frac{\omega_{01}}{(s + \omega_{02})} I_U, \quad (21)$$

$$I_3 = \frac{\omega_{01}\omega_{02}\left(s + \frac{\omega_{01}}{Q}\right)}{\left(s^2 + \frac{\omega_{01}}{Q}s + \omega_{01}^2\right)(s + \omega_{02})} I_U. \quad (22)$$

In accordance with (20), the output of the circuit presented in Fig. 1 provides a third-order noninverting low-pass filter transfer function.

$$I_{LP} = I_1. \quad (23)$$

3. The Realization of Butterworth Output

Consequently, using the output of the circuit shown in Fig. 1 for $Q = 1$ and $\omega_{01} = \omega_{02} = \omega_0$, third-order Butterworth low-pass filter current transfer function is accomplished as defined in Eq. (24).

$$I_1 = \frac{\omega_0^3}{(s^2 + \omega_0 s + \omega_0^2)(s + \omega_0)} I_U. \quad (24)$$

As a result, DC control currents I_{01} , I_{02} and I_{03} can be chosen as below to obtain Butterworth output.

$$I_{01} = I_{02} = I_{03} = I_Q. \quad (25)$$

4. The Realization of Chebyshev Output

If we accept the quality factor as $Q \neq 1$ and $\omega_{01} \neq \omega_{02}$ in Eqs. (14)–(16), third-order square-root-domain low-pass Chebyshev filter circuit can be performed. In accordance with Eq. (22), the output of the circuit as shown in Fig. 1 provides a third-order inverting low-pass Chebyshev filter transfer function.

In case of $Q \neq 1$, the relation can be defined by using Eqs. (18) and (19) between I_{01} and I_{02} , as given by Eq. (26):

$$I_{02} = I_{01} \left(\frac{\omega_{02}}{\omega_{01}} \right)^2, \quad (26)$$

with 1 dB passband ripple and $\omega_0 = 1$ rad/s cut-off frequency, third-order normalized low-pass Chebyshev filter's ω_{01} , ω_{02} and Q values given in Eq. (14) are as shown below.²²

$$\omega_{01} = 0.997, \quad \omega_{02} = 0.494, \quad Q = 2.018. \quad (27)$$

In this case, DC control currents I_{01} , I_{02} and I_{03} can be chosen as below to obtain Chebyshev output.

$$I_{02} = 0.2425 I_{01}, \quad (28)$$

$$I_{03} = I_{01}, \quad (29)$$

$$I_Q = 0.2425 I_{01}. \quad (30)$$

5. The Realization of Elliptic Output

The transfer function of third-order low-pass elliptic filter can be written depending on ω_{01} , ω_{02} and Q as follows:

$$N(s) = \frac{I_{\text{out}}}{I_{\text{in}}} = \frac{\alpha \left(\frac{\omega_{01}}{Q} + \omega_{02} \right) s^2 + \omega_{01}^2 \omega_{02}}{\left(s^2 + \frac{\omega_{01}}{Q} s + \omega_{01}^2 \right) (s + \omega_{02})}, \quad (31)$$

where α is a positive constant. The transfer function of third-order low-pass elliptic filter can be signified by using a linear combination of filter outputs I_1 , I_2 and I_3 as shown in Eq. (32).

$$I_{\text{Elliptic}} = K_1 I_1 + K_2 I_2 + K_3 I_3, \quad (32)$$

where K_1 , K_2 and K_3 coefficients are defined as follows:

$$K_1 = 1 - \frac{\alpha}{\omega_{01} \omega_{02}} \left(1 - \frac{1}{Q^2} \right), \quad (33)$$

$$K_2 = \frac{\alpha}{\omega_{02}}, \quad (34)$$

$$K_3 = \frac{\alpha}{Q \omega_{02}}. \quad (35)$$

The normalized transfer function of third-order low-pass elliptic filter can be written as follows²³:

$$N(s) = \frac{P(s)}{Q(s)} = \frac{0.35225s^2 + 0.5987}{s^3 + 0.84929s^2 + 1.14586s + 0.5987}. \quad (36)$$

This transfer function can be rearranged by using factorization of denominator polynomial of transfer function as given below:

$$N(s) = \frac{P(s)}{Q(s)} = \frac{0.35225s^2 + 0.59870}{(s^2 + 0.24852s + 0.99655)(s + 0.60077)}. \quad (37)$$

Natural frequencies, quality factor and α can be determined as $\omega_{01} = 0.99827$, $\omega_{02} = 0.60077$, $Q = 4.01687$ and $\alpha = 0.35225$ by using Eqs. (37) and (31). Hence, K_1 , K_2 and K_3 coefficients can be calculated via Eqs. (33)–(35).

$$K_1 = 0.44905, \quad (38)$$

$$K_2 = 0.5863, \quad (39)$$

$$K_3 = -0.14597. \quad (40)$$

Then, the output current of third-order low-pass elliptic filter can be achieved by using linear summation as defined Eq. (41).

$$I_{\text{Elliptic}} = 0.44905I_1 + 0.5863I_2 - 0.14597I_3. \quad (41)$$

Finally, DC control currents I_{01} , I_{02} and I_{03} can be chosen as below to obtain elliptic output.

$$I_{02} = 0.3620I_{01}, \quad (42)$$

$$I_{03} = I_{01}, \quad (43)$$

$$I_Q = 0.06198I_{01}. \quad (44)$$

6. Simulation Results

TSMC 0.35 μm Level-3 CMOS transistor parameters are used in PSPICE simulations of the designed third-order square-root-domain current-mode low-pass Butterworth, Chebyshev and elliptic filters. Transistor dimensions are selected as $W/L = 10 \mu\text{m}/10 \mu\text{m}$ for $M_1 \sim M_9$ and $W/L = 220 \mu\text{m}/2 \mu\text{m}$ for $M_{10} \sim M_{21}$.

The supply voltage of the filter circuit is selected as $V_{DD} = 3 \text{ V}$. The values of three capacitances of the circuit are selected as $C = 40 \text{ pF}$. By changing the values of the outer current sources, the simulations are realized to adjust the natural frequency. The natural frequency changes between 195 kHz–693 kHz when the values of the DC current sources I_{01} are changed between $20 \mu\text{A}$ – $300 \mu\text{A}$. As a result, the natural frequency of the filter can be adjusted in about 500 kHz frequency range.

The gain responses obtained for different values of the DC current sources of the third-order low-pass Butterworth filter circuit are given in Fig. 2.

The phase responses obtained for the different values of the DC current sources of the third-order low-pass Butterworth filter circuit are given in Fig. 3.

The time-domain response of the Butterworth filter is shown in Fig. 4. The total harmonic distortion is measured as 0.95% when a 690 kHz sinus signal that has $20 \mu\text{A}$ peak value is applied to the input.

The natural frequencies of third-order current-mode low-pass Chebyshev filter are 260 kHz, 405 kHz and 675 kHz for I_{01} control currents $40 \mu\text{A}$, $100 \mu\text{A}$ and $300 \mu\text{A}$, respectively.

The gain responses obtained for the different values of the DC current sources of the third-order Chebyshev filter circuit are shown in Fig. 5.

The time-domain response of the Chebyshev filter is shown in Fig. 6. The total harmonic distortion is measured as 1.4%, when a 675 kHz sinus signal that has $20 \mu\text{A}$ peak value is applied to the input.

The natural frequencies of third-order current-mode low-pass elliptic filter are 175 kHz, 375 kHz and 650 kHz for I_{01} control currents $19.8 \mu\text{A}$, $100 \mu\text{A}$ and $300 \mu\text{A}$, respectively. The gain responses obtained for the different values of the DC current sources of the third-order elliptic filter circuit are shown in Fig. 7.

The time-domain response of the elliptic filter is shown in Fig. 8. The total harmonic distortion is measured as 1.0% when a 650 kHz sinus signal that has sine-wave input signal which has $10 \mu\text{A}$ peak value is applied to the input.

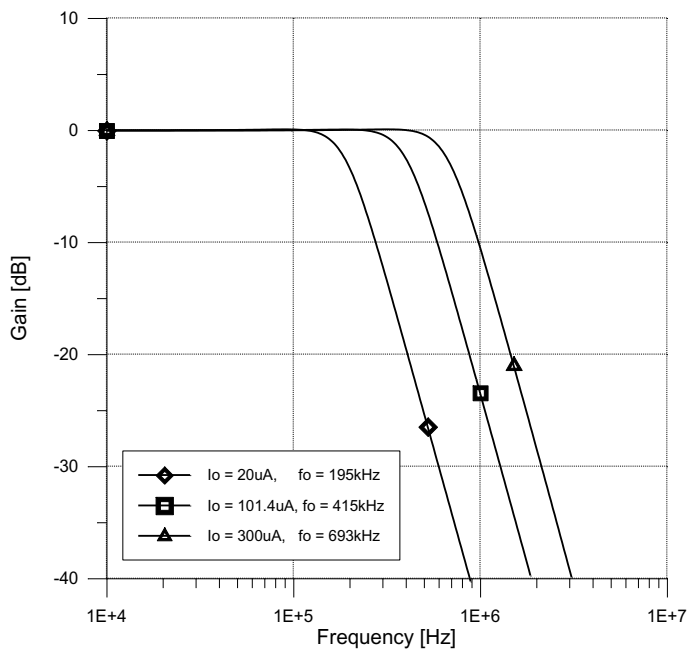


Fig. 2. Gain responses of third order low-pass Butterworth filter at different values of I_0 as a function of applied frequency.

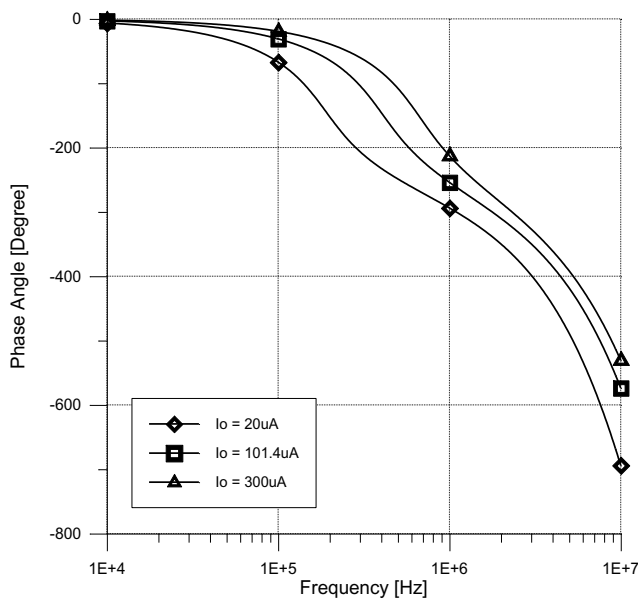


Fig. 3. Phase responses of third order low-pass Butterworth filter at different values of I_0 as a function of applied frequency.

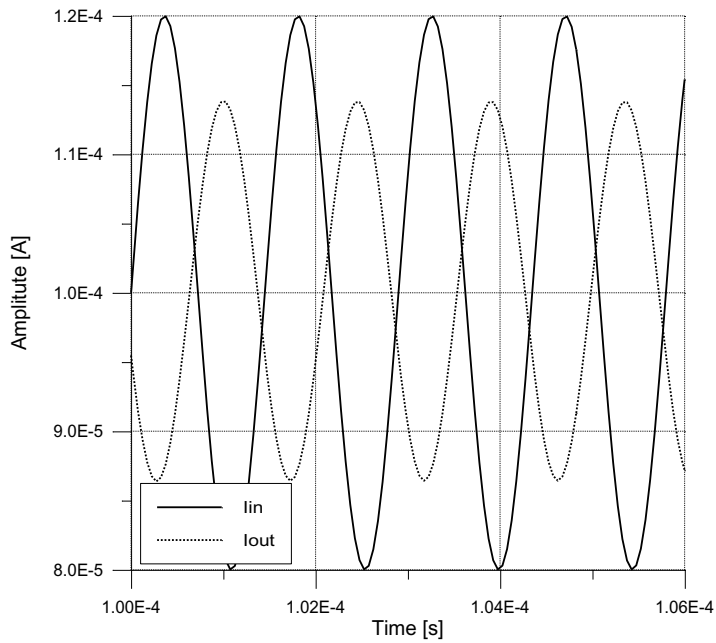


Fig. 4. Time domain response of third order low-pass Butterworth filter.

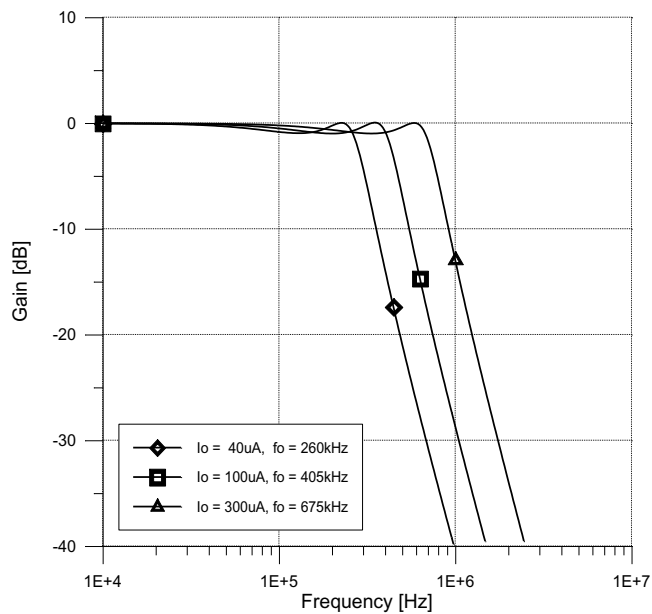


Fig. 5. Gain responses of third order low-pass Chebyshev filter at different values of I_0 as a function of applied frequency.

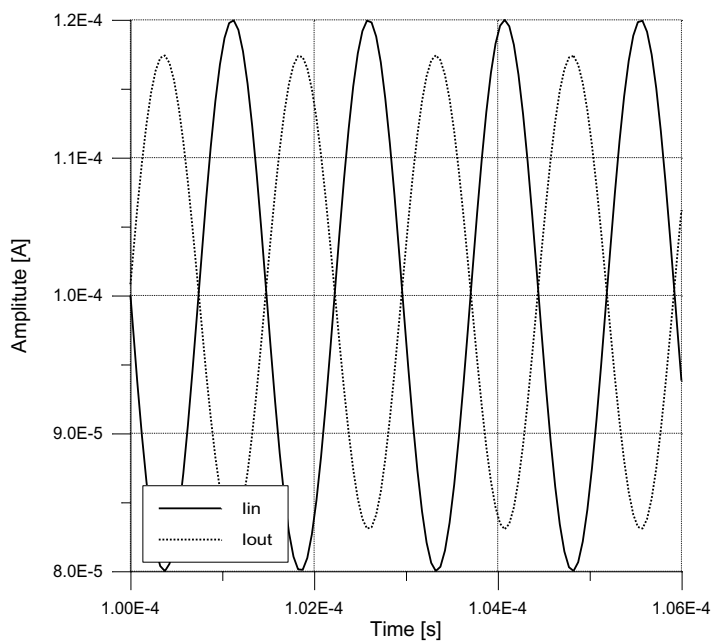


Fig. 6. Time domain response of third order low-pass Chebyshev filter.

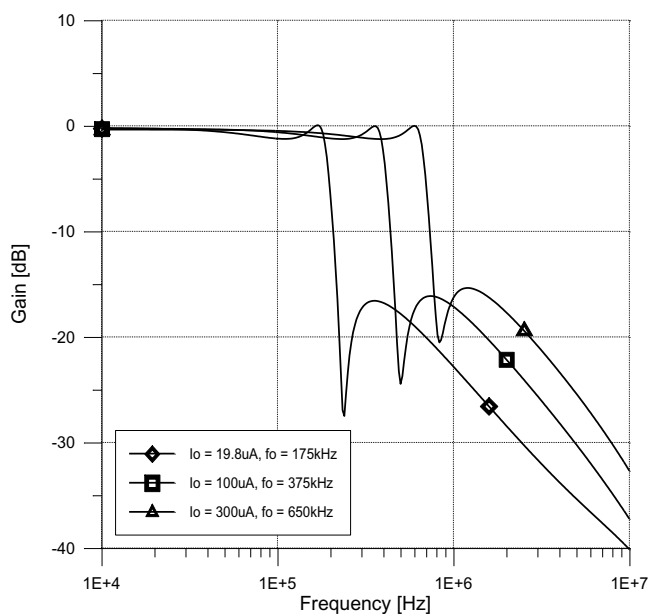


Fig. 7. Gain responses of third order low-pass Elliptic filter at different values of I_0 as a function of applied frequency.

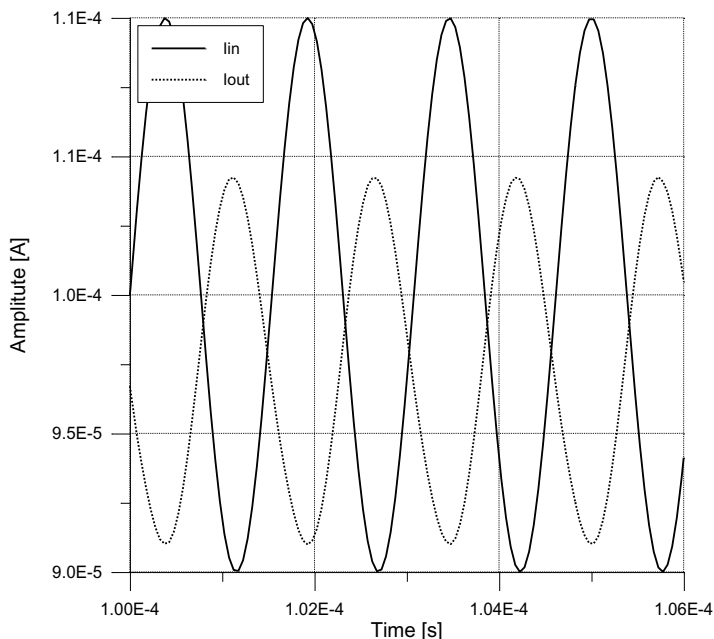


Fig. 8. Time domain responses of third order low-pass Elliptic filter.

7. Conclusion

Square-root-domain third-order current-mode low-pass Butterworth, Chebyshev and elliptic filters are proposed in this study. To design the third-order low-pass filter circuit, state-space synthesis method is used. This circuit consists of only MOSFETs and grounded capacitors. The natural frequency of the filter circuits can be adjusted electronically by changing the value of outer current sources. The proposed filter circuits have various advantages such as the ability to be adjusted electronically, requiring only MOSFETs and grounded capacitors, suitability to VLSI technologies, suitability to low voltage/power applications and the ability to operate at high frequencies. PSPICE simulations are given in order to confirm the theoretical analysis.

References

1. R. W. Adams, *Linear Filtering in the Log-Domain* (New York, 1979).
2. D. R. Frey, Log-domain filtering: An approach to current-mode filtering, *IEE Proc. G* **140** (1993) 406–416.
3. D. R. Frey, Exponential state-space filters: A generic current mode design strategy, *IEEE Trans. Circuits Syst. I* **43** (1996) 34–42.
4. B. Gilbert, Translinear circuits: A proposed classification, *Electron. Lett.* **11** (1975) 14–16.
5. J. Ngarmnil and C. Toumazou, Micropower Log-Domain Active Inductor, *Electron. Lett.* **32** (1996) 953–955.

6. K. Bult and H. Wallinga, A class of analog CMOS circuits based on the square-law characteristic of a MOS transistor in saturation, *IEEE J. Solid-State Circuits* **22** (1987) 357–365.
7. E. Seevinck and R. J. Wiegink, Generalized translinear circuit principle, *IEEE J. Solid-State Circuits* **26** (1991) 1098–1102.
8. S. Menekay, R. C. Tarcen and H. Kuntman, The second-order low-pass filter design with a novel higher precision square-root circuits, *Istanbul Univ. J. Electr. Electron.* **7** (2007) 323–329.
9. A. J. Lopez-Martin and A. Carlosena, Very low voltage CMOS companding filter based on the MOS translinear principle, *SSMSD, Southwest Symp. on Mixed-Signal-Design*, Texas, USA, 2001, pp. 105–109.
10. A. J. Lopez-Martin and A. Carlosena, 1.5 V CMOS companding filter, *Electron. Lett.* **38** (2002) 1346–1348.
11. M. S. Kesterlioglu and A. Kircay, The design of current-mode electronically tunable first-order square-root domain filters using state-space synthesis method, *Int. Rev. Model. Simul.* **2** (2009) 124–128.
12. A. Kircay and M. S. Kesterlioglu, Novel current-mode second-order square-root-domain highpass and allpass filter, *6th Int. Conf. Electrical and Electronics Eng.*, Bursa, Turkey, 5–8 November 2009.
13. A. Kircay, M. S. Kesterlioglu and U. Cam, A new current-mode square-root-domain general notch filter, *J. Circuits Syst. Comput.* **22** (2013) 1250072.
14. S. H. Ismail, E. A. Soliman and S. A. Mahmoud, Cascaded third-order tunable low-pass filter using low voltage low power OTA, *Int. Symp. Integr. Circuits*, Singapore, 12–14 December 2011.
15. M. Ghosh, S. K. Paul and R. K. Ranjan, Third-order universal filter using single operation transresistance amplifier, Hindawi Publishing Corporation, *J. Eng.* Article ID: 317296 (2013).
16. Y. Sun, B. Jefferies and J. Teng, Universal third order OTA-C filters, *Int. J. Electron.* **85** (1998) 597–609.
17. S. S. Yilmaz and A. T. Tola, A systematic Class AB state space synthesis method based on MOSFET square law and translinear square-root cells, *Int. J. Circuit Theory Appl.* **43** (2015) 1989–2010. doi: 10.1002/cta.2054.
18. K. O. M. Fouad and A. M. Soliman, Square root domain differentiator, *IEE Proc.-Circuits Devices Syst.* **152** (2005) 1350–2409.
19. C. Psychalinos, Square-root domain operational simulation of LC ladder elliptic filters, *Circuits Syst. Signal Process.* **26** (2007) 263–280.
20. G. N. Shindel and S. R. Bhagat, Tunable bandwidth third order switched-capacitor with multiple feedback filter for different center frequencies, *Sci. Res. Eng.* **2** (2010) 179–183.
21. A. Kircay, M. S. Kesterlioglu and F. Z. Sagi, Design of third order square-root-domain filters using state-space synthesis method, *9th Int. Conf. Elect. Electronics Eng.*, 26–28 November 2015, Bursa, Turkey, pp. 916–920.
22. S. Kendall, *Analog Filters*, 2nd edn. (Kluwer Academic Publishers, 2002).
23. S. H. Tu, C. M. Chang, J. N. Ross and M. N. S. Swamy, Analytical synthesis of current-mode high-order single-ended-input OTA and equal capacitor elliptic filter structures with the minimum number of components, *IEEE Trans. Circuits Syst. I* **54** (2007) 2195–2210.