Parul University
Faculty of Engineering and Technology
Department of Applied Science & Humanities
Academic Year 2024-25
Subject: Quant and Reasoning (303105311)
Branch: CSE

Unit -1: Number system, LCM &. HCF simplification and approximation

1.1 Numbers

Types of Numbers:

a. Natural numbers

1, 2, 3 ...

b. Whole Numbers

0, 1, 2, 3 ...

c. Integers

-3, -2, -1, 0, 1, 2, 3 ...

d. Rational Numbers

Rational numbers can be expressed as a/b where a and b are integers and $b \neq 0$
Examples: 112112, 4242, 00, −811−811 etc.

All integers, fractions and terminating or recurring decimals are rational numbers.

e. Irrational Numbers

Any number which is not a rational number is an irrational number. In other words, an irrational number is a number which cannot be expressed as a/b where a and b are integers.

For instance, numbers whose decimals do not terminate and do not repeat cannot be written as a fraction and hence they are irrational numbers.

Example: π, $\sqrt{2}$, $(3+\sqrt{5})$, $4\sqrt{3}$ (meaning $4\times\sqrt{3}$), $3\sqrt{6}$ etc

Please note that the value of π = 3.14159 26535 89793 23846 26433 83279 50288 41971 69399 37510 58209 74944 59230 78164 06286 20899 86280 34825 34211 70679...

We cannot π as a simple fraction (The fraction 22/7 = 3.14.... is just an approximate value of π)

f. Real Numbers

Real numbers include counting numbers, whole numbers, integers, rational numbers and irrational numbers.

Divisibility

One whole number is divisible by another if the remainder we get after the division is zero.

Examples

36 is divisible by 4 because 36 ÷ 4 = 9 with a remainder of 0.

36 is divisible by 6 because 36 ÷ 6 = 6 with a remainder of 0.

36 is not divisible by 5 because 36 ÷ 5 = 7 with a remainder of 1.

Divisibility Rules

By using divisibility rules, we can easily find whether a given number is divisible by another number without actually performing the division. This saves time especially when working with numbers. Divisibility rules of numbers 1 to 20 are provided below.

1. Divisible by 2 : The last digit should be even (i.e. 0,2,4,6,8)
2. Divisible by 3: The sum of the digits should be divisible by 3.
3. Divisible by 4: The last two digits should be divisible by 4.
4. Divisible by 5: The last digit should either be 0 or 5.
5. Divisible by 6: The number should be divisible by both 2 and 3.
6. Divisible by 7: Do double of the last digit and subtract it from the rest of the number, the difference obtained should be divisible by 7.
7. Divisible by 8: The last three digits should be divisible by 8 or should be 000.
8. Divisible by 9 : The sum of the digits should be divisible by 9
9. Divisible by 10: The last digit should be 0.
10. Divisible by 11: The difference of the alternating sum of digits should be divisible by 11.
11. Divisible by 12: The number should be divisible by both 3 and 4.
12. Divisible by 13: Do four times of the last digit and add it to the rest of the number, the result obtained should be divisible by 13.
13. Divisible by 14: The number should be divisible by both 2 and 7.
14. Divisible by 15: The number should be divisible by both 3 and 5.

15. Divisible by 16:

Test - a) For 3 digit number:

Do 4 times of 100'th place digit and add to the last two digit of the number ; the number obtained should be divisible by 16.

Test - b) For 4 Digit Number:

1) If number's thousand place is even observe the last three digits of the number and do Test – a).
2) If number's thousand place is odd observe the last three digits of the number ; Add 8 to the last three digits ; the number obtained should be divisible by 16.

16. Divisible by 17: Do five times of the last digit, subtract it from the rest of the number, the difference obtained should be divisible by 17.
17. Divisible by 18: The number should be divisible by both 2 and 9.
18. Divisible by 19: Do double of the last digit, add it to the rest of the number, the result obtained should be divisible by 19.
19. Divisible by 20 : The **last two digits of the number are either 00 or must be divisible by 20.**

EXAMPLES:

1. Determine the following numbers are divisible by 2 or not?
   176, 221, 327, 90, 192, 64, 715632.
2. Determine the following numbers are divisible by 3 or not?
   987654321, 1001, 387, 123456789, 780, 54, 137, 231, 194, 153, 1368.
3. Determine the following numbers are divisible by 4 or not?
   100,104,108,117,124,204,93812,31520,2500,199416.
4. Determine the following numbers are divisible by 5 or not?
   120, 165, 1335, 2505,1000,10015.
5. Determine the following numbers are divisible by 6 or not?
   144,180,258,184,156,7182,4008,11190,96420.
6. Determine the following numbers are divisible by 7 or not?
   133,273,329,167,233,297,889,1617,2975.
7. Determine the following numbers are divisible by 8 or not?
   1792,1824,2000,2880,3320,564108,987048.
8. Determine the following numbers are divisible by 9 or not?
   99,198,171,3411,1277,1379,367821.
9. Determine the following numbers are divisible by 10 or not?
   110,200,317,50050,1250.
10. Determine the following numbers are divisible by 11 or not?
    2728: odd digit's sum: 8+7 = 15
    Even digit's sum: 2+2 = 4
    Diff: 15-4 = 11; the diff is 11 which is divisible by 11 so 2728 is divisible by 11.
    65678932, 86416, 9780, 536393,11011, 1210121.

For 3 Digit Number : To check if a 3 digit number is divisible by 3 or not, sum the digits at extreme positions and subtract it with middle term. If the difference is zero or multiple of 11, then the number is divisible by 11.

Check if 792 is divisible by 11?

Answer : Sum of 1^{st} and last digit is 7+2 = 9 and then subtract that sum i.e. 9 from middle digit 9 i.e. 9-9 = 0. So 792 is divisible by 11.

Check if 537 is divisible by 11? Not

11. Determine the following numbers are divisible by 12 or not? 844, 5844, 5864,936,720.
12. Determine the following numbers are divisible by 13 or not? 2795,1169,7884,736,2723.
13. Determine the following numbers are divisible by 14 or not? 154, 112, 252, 1568, 1554.
14. Determine the following numbers are divisible by 15 or not? 135, 165, 195, 270, 1665, 1995, 20025.
15. Determine the following numbers are divisible by 16 or not? 112, 176, 304, 1776, 1840, 2656.
16. Determine the following numbers are divisible by 17 or not? 187, 221, 425, 1887, 3009.
17. Determine the following numbers are divisible by 18 or not? 162, 234, 270, 1998, 3384, 9990.
18. Determine the following numbers are divisible by 19 or not? 171, 209, 247, 2565, 2109, 1919.
19. Determine the following numbers are divisible by 20 or not? 200,20040,4004,205060,440.

Prime Numbers and Composite Numbers

Prime Numbers

A prime number is a positive integer that is divisible by itself and 1 only. Prime numbers will have exactly two integer factors.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, etc.

Please note the following facts

Zero is not a prime number because zero is divisible by more than two numbers. Zero can be divided by 1, 2, 3 etc.
(0 ÷ 1 = 0, 0÷ 2 = 0 ...)

One is not a prime number because it does not have two factors. It is divisible by only 1

Composite Numbers

Composite numbers are numbers that have more than two factors. A composite number is divisible by at least one number other than 1 and itself.

Examples: 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, etc.

Please note that zero and 1 are neither prime numbers nor composite numbers.

Every whole number is either prime or composite, with two exceptions 0 and 1 which are neither prime nor composite.

Prime Factorization and Prime factors

Prime factor

The factors which are prime numbers are called prime factors

Prime factorization

Prime factorization of a number is the expression of the number as the product of its prime factors.

Example 1:
Prime factorization of 280 can be written as $280 = 2 \times 2 \times 2 \times 5 \times 7 = 2^3 \times 5 \times 7$ and the prime factors of 280 are 2, 5 and 7

Example 2:
Prime factorization of 72 can be written as $72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$ and the prime factors of 72 are 2 and 3.

Co-prime Numbers or Relatively Prime Numbers

Two numbers are said to be co-prime (also spelled co-prime) or relatively prime if they do not have a common factor other than 1. i.e., if their HCF is 1.

Example 1: 3, 5 are co-prime numbers (Because HCF of 3 and 5 = 1)

Example 2: 14, 15 are co-prime numbers (Because HCF of 14 and 15 = 1)

A set of numbers is said to be pairwise co-prime (or pairwise relatively prime) if every two distinct numbers in the set are co-prime

Example 3: The numbers 10, 7, 33, 13 are pairwise co-prime, because HCF of any pair of the numbers in this is 1.
HCF (10, 7) = HCF (10, 33) = HCF (10, 13) = HCF (7, 33) = HCF (7, 13) = HCF (33, 13) = 1.

Example 4: The numbers 10, 7, 33, 14 are not pairwise co-prime because HCF(10, 14) = $2 \neq 1$ and HCF(7, 14) = $7 \neq 1$.

Note:-
1. If a number is divisible by two co-prime numbers, then the number is divisible by their product also.

2. If a number is divisible by more than two pairwise co-prime numbers, then the number is divisible by their product also.

Example 5: Check if the number 14325 is divisible by 15.
3, 5 are co-prime numbers (Because HCF of 3 and 5 = 1)

14325 is divisible by 3 and 5.
$3 \times 5 = 15$
Hence 14325 is divisible by 15 also.

Example 6: Check if the number 1440 is divisible by 60.
The numbers 3, 4, 5 are pairwise co-prime because HCF of any pair of numbers in this is 1.
1440 is divisible by 3, 4 and 5.
$3 \times 4 \times 5 = 60$. Hence 1440 is also divisible by 60.

Exercise

1.
Which of the following number is divisible by 24 ?

A. 35718

B. 63810

C. 537804

D. 3125736

Explanation:

24 = 3 x8, where 3 and 8 co-prime.

Clearly, 35718 is not divisible by 8, as 718 is not divisible by 8.

Similarly, 63810 is not divisible by 8 and 537804 is not divisible by 8.

Consider option (D),

Sum of digits = (3 + 1 + 2 + 5 + 7 + 3 + 6) = 27, which is divisible by 3.

Also, 736 is divisible by 8.

$\therefore$ 3125736 is divisible by (3 x 8), *i.e.*, 24.

2.
On dividing a number by 5, we get 3 as remainder. What will the remainder when the square of the this number is divided by 5 ?

A. 0

B. 1

C. 2

D. 4

Explanation:
Let the number be x and on dividing x by 5, we get k as quotient and 3 as remainder.

$\therefore \quad x = 5k + 3$

$\Rightarrow \quad x^2 = (5k + 3)^2$

$= (25k^2 + 30k + 9)$

$= 5(5k^2 + 6k + 1) + 4$

$\therefore$ On dividing x^2 by 5, we get 4 as remainder.

3.
The difference of two numbers is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number?

A. 240

B. 270

C. 295

D. 360

Explanation:
Let the smaller number be x. Then larger number = $(x + 1365)$.

$\therefore \; x + 1365 = 6x + 15$

$\Rightarrow 5x = 1350$

$\Rightarrow x = 270$

$\therefore$ Smaller number = 270.

4.
If the number 517*324 is completely divisible by 3, then the smallest whole number in the place of * will be:

A. 0

B. 1

C. 2

D. None of these

Explanation:
Sum of digits = $(5 + 1 + 7 + x + 3 + 2 + 4) = (22 + x)$, which must be divisible by 3.

$\therefore \; x = 2$.

5.
On dividing a number by 357, we get 39 as remainder. On dividing the same number 17, what will be the remainder ?

A. 0

B. 3

C. 5

D. 11

Explanation:
Let x be the number and y be the quotient. Then,
$x = 357 \times y + 39$
$= (17 \times 21 \times y) + (17 \times 2) + 5$
$= 17 \times (21y + 2) + 5)$

$\therefore$ Required remainder = 5.

LCM and HCF

1.2.1 LCM and HCF

Least common multiple is a number which is multiple of two or more than two numbers.

For example: The common multiples of 3 and 4 are 12, 24 and so on. Therefore, l.c.m.is smallest positive number that is multiple of both. Here, l.c.m. is 12.

Highestcommon factors are those integral values of number that can divide that number.

For example: The common Factors of 24 and 36 are 3 and 4. Therefore, h.c.f .is highest positive number which is multiplication of all common factors that is 3*4=12.

Some important LCM and HCF tricks:

1) Product of two numbers = Their HCF × Their LCM.
2) HCF of given numbers always divides their LCM .

How to find H.C.F by Trick?

1) **H.C.F OF 2 NUMBERS:-** Find difference of 2 given two numbers ; check that obtained difference can divide that given two numbers or not?

If yes then that obtained difference is the H.C.F of given 2 numbers.
If not then find factors of that obtained difference; check can any factor divide given 2 numbers or not; if yes then that factors are the H.C.F of given 2 numbers.

For Example:
H.C.F of 6 & 12

Difference: 12-6 =6; 6 can divide both 6 & 12; so H.C.F (6,12) = 6.

H.C.F (5,15) = 5

Difference: 15-5 = 10 ; 10 can't divide 5 & 15; so factors of 10 are 10=5*2*1;

5 can divide both 5 & 10.

So H.C.F (5,10) = 5.

2) **H.C.F Of 3 Numbers:** Do the same process as mentioned in above just find now difference between each 3 numbers.
Then pick up the smallest difference and check can that obtained difference divide the each three numbers or not?
If yes then that the obtained difference is The H.C.F of 3 given numbers.
If not then find factors of that small obtained diff. and repeat the above procedure.

For Ex:
H.C.F (12,15,21)
Diff: 15-12=3, 21-15=6, 21-12=9. Smallest diff. is 3; Now 3 can divide all 12,15 & 21. So H.C.F(12,15,21) =3.

H.C.F(50,70,85)
Diff: 70-50=20 , 85-70=15; Smallest diff is 15; 15 can't divide all the 3 numbers.
So Factors of 15 are 1,3 & 5. Out of these 3 factors 5 can divide all 3.
Hence H.C.F(50,70,85) = 5.

Exercise:

1. The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:
   a) 101
   b) 107
   c) 111
   d) 185

   Answer: (c)

2. The G.C.D. of 1.08, 0.36 and 0.9 is:
   a) 0.03
   b) 0.9

c) 0.18

d) 0.108

Answer: (c)

3. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is:

a) 1

b) 2

c) 3

d) 4

Answer: (b)

4. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8 10 and 12 seconds respectively. In 30 minutes, how many times do they toll together ?

a) 4

b) 10

c) 15

d) 16

Answer: (d)

Explanation:

L.C.M. of 2, 4, 6, 8, 10, 12 is 120.

So, the bells will toll together after every 120 seconds(2 minutes).

In 30 minutes, they will toll together $\frac{30}{2} + 1 = 16$ Minutes.

5. The G.C.D. of 1.08, 0.36 and 0.9 is:

a) 0.03

b) 0.9

c) 0.18

d) 0.108

Answer: (c)

Explanation: Given numbers are 1.08, 0.36 and 0.90. H.C.F. of 108, 36 and 90 is 18,

$\therefore$ H.C.F. of given numbers = 0.18.

6. Find the numbers if the HCF of two numbers is 29 and their sum is 174.

Ans: Given that the HCF of two numbers is 29.

Let 29a and 29b be the two required numbers.

According to the given,

29a + 29b = 174

29(a + b) = 174

a + b = 174/29 = 6

The pair of values of co-primes with sum 6 is (1, 5).

So, the possible numbers are:

29 x 1 = 29

29 x 5 = 145

Verification:

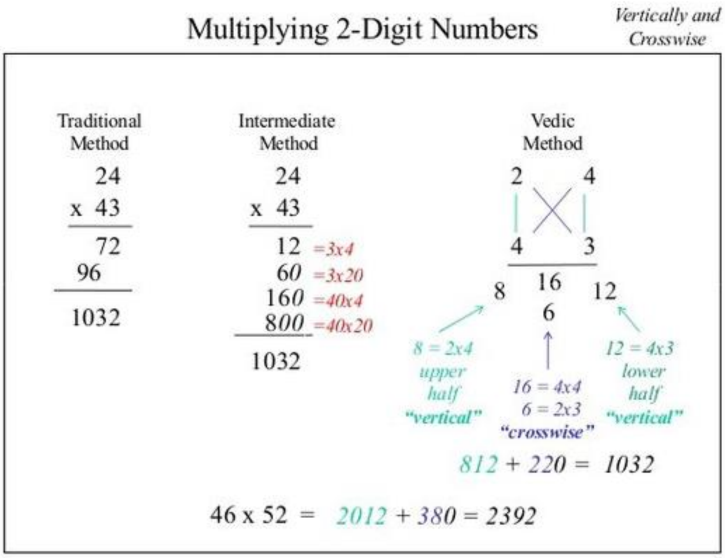
Sum of numbers = 29 + 145 = 174

Hence, the required numbers are 29 and 145.

Exercise:

1) H.C.F (15,35)	5) H.C.F (22,33,55)
2) H.C.F (30,24)	6) H.C.F (72,108,180)
3) H.C.F (33,55)	7) 7) H.C.F (12,18,24)
4) H.C.F (270,900)	8) H.C.F (64,136,238)

Multiplicaion of two digit numbers



Exercise:

1. 21x11 Ans: 231

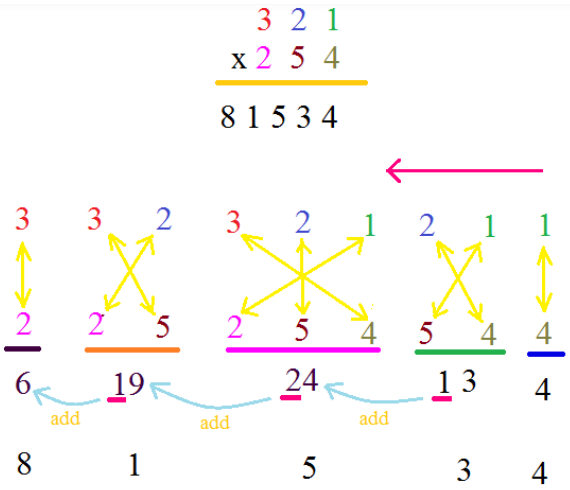
2. 43x22 Ans: 946

3.46x12 Ans: 552

4.23x76 Ans: 1748

5. 64x45 Ans: 2880

1.3.3 Multiplicaion of three digit numbers



Exercise:

1. 212x111	Ans: 23532
2. 413x212	Ans: 87556
3.146x112	Ans: 16352
4.231x176	Ans: 40656
5. 164x145	Ans: 23780

Que...

The difference of the squares of two consecutive odd integers is divisible by which of the following integers ?

A. 3

B. 6

C. 7

D. 8

: Option D

Explanation:

Let the two consecutive odd integers be $(2n + 1)$ and $(2n + 3)$. Then,

$(2n + 3)^2 - (2n + 1)^2 = (2n + 3 + 2n + 1)(2n + 3 - 2n - 1)$

$= (4n + 4) \times 2$

$= 8(n + 1)$, which is divisible by 8.