## **Special Characters**

- Decimal point depending on the number format settings in the System Preferences
- Decimal comma depending on the number format settings in the System Preferences
- ; List separator
- () Parenthesis
- Number of decimal digits:  $\pi'6 \rightarrow 3.141593$
- Number of significant digits:  $10*\pi^4 \rightarrow 31.42$
- Initial of a comment:  $\sin(\pi/3)$  "a comment...
- ? Shows associated description: ?BRL → Brazilian Real of currency symbols: ?USD → U.S. Dollar
- % Percent postfix operator may be attached directly to any number and has the immediate effect of dividing that number by 100 of course, that is exactly what per cent means, e.g.:  $76\% \rightarrow 0.76$

If Lazy % Syntax is enabled in the InstantCalc Preferences Panel (default), then the % operator allows for lazy addition/subtraction of the percentage of the left operand to/from itself:  $100 + 5\% \rightarrow 105$ 

100 - 5% → 95

If Lazy % Syntax is disabled, additions and subtractions behave strictly mathematical:  $100 + 5\% \rightarrow 100.05$  $100 - 5\% \rightarrow 99.95$ 

## **Hexadecimal and Binary Conversion Operators**

\$ Hexadecimal number prefix:  $$20 + 5 \rightarrow 37$ **0x** Alternate hexadecimal prefix:  $$0x7FFF \rightarrow 32767$ 

# Binary number prefix:  $#11100101+5 \rightarrow 234$ 

No Binary result output:  $123 \rightarrow #1111011$ 

Note: Above hexadecimal and binary result output formatting operators (&, §, \) must prefix the expression and work on integer values only, i.e., floating point values are not formatted:  $\$12.3 = \12.3 \rightarrow 12.3$ 

## **Display of Fractions**

f convert result to a fraction: f0.1875 → 3/16 f must be the first character of the expression.

Note: " " (<alt><space> = NO BREAK SPACE) substitutes "+"  $f11.5 \rightarrow 11 1/2$   $11 1/2 \rightarrow 11.5$ 

## **Operator Precedence**

InstantCalc basically adheres to the standard calculation rules as nowadays trained in the math courses of elementary and/or secondary schools, e.g.:  $1 + 2 * 3 = 3 * 2 + 1 = 1 + (2 * 3) \rightarrow 7$ .

According to these rules, the multiplication has a higher precedence than the addition, and consequently 2 \* 3 is evaluated be-

fore the multiplication result is added to 1. Therefore, there is no need of grouping the multiplication by parenthesis in the above example. The natural result is 7 in each notation.

For forcing another than the natural evaluation order, parenthesis must be used, therefore, for forcing the addition to being evaluated first, the expression needs to be  $(1 + 2) * 3 \rightarrow 9$ .

Besides honouring the standard precedence rules, InstantCalc evaluates everything strictly from left to the right. Note, that in some cases left to right evaluation is the same as right to left evaluation, e.g.:  $1 + 2 + 3 = 3 + 2 + 1 \rightarrow 6$ .

In other cases, it makes a difference, and we need to use parenthesis for forcing a different evaluation sequence, if desired.

$$100 + 25\% + 10\% \rightarrow 137.5$$
  
 $100 + (25\% + 10\%) \rightarrow 135$   
 $2^3^4 = (2^3)^4 \rightarrow 4096$   
 $2^(3^4) \rightarrow 2.417851639e+24$ 

Take care!

#### **Dyadic Operators** – by ascending precedence level

Level 0 – weakest binding of the operands to the operator

 $\stackrel{:=}{}$  Variable assignment operator: x := 5; (x + 2)\*3 → 21

Variables keep their definition even upon restart until they are undefined with x := none.

Variables can also be added, removed, and changed in the User Variables panel.

#### Level 1 – for equations

**E** Equation operator, i.e. subtraction with lowest precedence. This is especially useful in combination with the

Newton Solver, in order to subtract the right side of an equation from the left side, e.g.:

$$solve(x := 3; x^2 - 3*x + 2 = 12) \rightarrow 5$$

Level 2

or Logical OR: true or false → true  $3.4 \parallel 0 \rightarrow \text{true}$ 

Level 3

<u>xor</u> Logical XOR: true xor true → false

 $2 \text{ xor } 0 \rightarrow \text{true}$ 

Level 4

<u>and</u> Logical AND: true && false → false

<u>&&</u> 2 and 1 → true

Level 5

 $\perp$  Bitwise OR:  $5 \mid 3 \rightarrow 7$ 

Level 6

 $\geq$  Bitwise XOR:  $5 \approx 3 \rightarrow 6$ 

Level 7

& Bitwise AND:  $5 \& 3 \rightarrow 1$ 

Level 8 (Comparisons)

== is equal:  $\sin(\pi) == \cos(\pi) \rightarrow \text{false}$ 

!= is not equal:  $0 != 0.0 \rightarrow \text{false}$ <> 1 <> 2 → true

 $\leq$  is less than: have(3;6;7;9) < ave(3;6;7;9)  $\rightarrow$  true

$$\stackrel{\checkmark=}{}$$
 is less or equal than:  $\exp(2) \le 10$  → true

$$\rightarrow$$
 is greater than:  $5 > 3 \rightarrow$  true

is greater or equal than: 
$$stdev(3;6;7;9) \ge 2 \rightarrow true$$

#### Level 9

Sitwise SHL (shift left): 
$$128 \ll 2 \rightarrow 512$$

$$\rightarrow$$
 Bitwise SHR (shift right):  $32 >> 4 \rightarrow 2$ 

#### Level 10

$$+$$
 Addition  $1+2 \rightarrow 3$ 

\_ Subtraction 
$$178 - 13.5 \rightarrow 164.5$$

#### Level 11

\* 
$$\cdot$$
 Multiplication: 1\*1  $\rightarrow$  1 or 2.2  $\rightarrow$  4

 $17 \text{ div } 5 \rightarrow 3$ 

mod Modulo: 
$$17 \mod 5 \rightarrow 2$$

Integer division:

#### Level 12

div

Power: 
$$3.5^5 \rightarrow 525.21875$$
 evaluates left to right:  $2^3^2 = (2^3)^2 \rightarrow 64$ 

Level 13 – very strong binding of the operands to the operator

#### Monadic Operators – ultimate precedence Level 14

Negation: 
$$x := 6; -x \rightarrow -6$$

Square Root:  $\sqrt{3} = \text{sqrt}(3) = 2\sqrt{3} \rightarrow 1.73205...$ 

! true  $\rightarrow$  false
not false  $\rightarrow$  true
! $0 \rightarrow$  true
! $0.0 \rightarrow$  true
not  $5.5 \rightarrow$  false
!inf  $\rightarrow$  false

!nan → nan

*Angle conversions* – useful for the trigonometric functions:

Dots per inch conversions – useful in metric applications:

Bitwise inverse:

c pt 
$$\rightarrow$$
 cm:  $\zeta x = x \cdot 2.54/72$   $\zeta 96 \rightarrow 3.386666667$  cm  $\rightarrow$  pt:  $\P x = x \cdot 72/2.54$   $\P 1.5 \rightarrow 42.51968504$ 

All the bitwise operators can only be used with the 64 bit integer operands, and it affect ALL bits, including the sign bit and the leading zeros. For this reason a bitwise inverse of a positive number gives allways a negative number and vice versa:

 $\sim 5 \rightarrow -6$ 

## **Common Functions with one Argument**

abs(x) Absolute value: 
$$abs(-5) = abs(5) \rightarrow 5$$

**round(x)** to integer: round(1.5) 
$$\rightarrow$$
 2

$$round(1.3) \rightarrow 1$$
$$round(2.5) \rightarrow 3$$

round(-3.5) = -round(3.5) 
$$\rightarrow$$
 -4

trunc(x) Truncate to integer: 
$$trunc(1.5) \rightarrow 1$$

$$trunc(-3.5) = -trunc(3.5) \rightarrow -3$$

sqrt(x) Square root: 
$$\operatorname{sqrt}(2) = \sqrt{2} \rightarrow 1.414213562$$

**sqr(x)** Power of 2 
$$x^2$$
:  $sqr(1 + 2) = 3*3 \rightarrow 9$ 

**cube(x)** Power of 3 
$$x^3$$
: **cube(-5)** = -5^3  $\rightarrow$  -125

$$sign(x)$$
 Signum function {-1 for x < 0; 0 for x = 0; 1 for x > 0}

$$sign(-123) \rightarrow -1$$
  
 $sign(ln(1)) \rightarrow 0$   
 $sign(inf) \rightarrow 1$ 

$$sign(nan) \rightarrow nan$$

# **Exponential Functions**

**exp(x)** Base e 
$$e^x$$
: exp(1) → 2.718281828

expm1(x) Base e 
$$e^x$$
 - 1: expm1(1) → 1.718281828

**exp10(x)** Base 10 
$$10^{x}$$
: exp10(4.5) → 31622.7766

exp2(x) Base 2 
$$2^{x}$$
: exp2(12) → 4096

## Logarithms

$$ln(x)$$
 Base e (logarithmus naturalis):  $ln(2) \rightarrow 0.6931471806$ 

ln1p(x) Base e: 
$$ln1p(5) = ln(5 + 1) → 1.7917595$$

lg(x) Base 10: 
$$lg(1000) → 3$$

**lb(x)** Base 2: 
$$lb(64) \rightarrow 6$$

## Trigonometric Functions – Arguments in Radian

$$\sin(x)$$
 Sine:  $\sin(\pi/3)$  → 0.866025404

$$\cos(x)$$
 Cosine:  $\cos(\text{@45})$  → 0.707106781

$$\underline{\tan(x)}$$
 Tangent:  $\tan(\pi/8) \rightarrow 0.414213562$ 

cot(x) Cotangent: 
$$1/\tan(\pi/8) = \cot(\pi/8) \rightarrow 2.414213562$$

#### **Inverse Trigonometric Functions** – Results in Radian

$$\underline{asin(x)}$$
 Inverse Sine:  $asin(0.5) \rightarrow 0.5235987756$ 

$$a\cos(x)$$
 Inverse Cosine: °acos(0.5) → 60

atan(x) Inverse Tangent: 
$$^{\circ}$$
atan(inf) → 90

$$acot(x)$$
 Inverse Cotangent: °acot(1) → 45

atan2(y; x) Inverse tan with 2 arg's: atan2(1.5;3) 
$$\rightarrow$$
 0.463...

# **Hyperbolic Functions**

 $\frac{\sinh(x)}{\sinh(x)}$  Hyperbolic Sine:  $\sinh(-2.5) \rightarrow -6.050204481$ 

 $sinh(2.5) \rightarrow 6.050204481$ 

cosh(x) Hyperbolic Cosine: cosh(-1.5) → 2.352409615

 $cosh(1.5) \rightarrow 2.352409615$ 

tanh(x) Hyperbolic Tangent:  $tanh(0.5493061443) \rightarrow 0.5$ 

 $tanh(-inf) \rightarrow -1$  $tanh(inf) \rightarrow 1$ 

<u>coth(x)</u> Hyperbolic Cotangent: coth(0.5493061443) → 2

# **Inverse Hyperbolic Functions**

<u>asinh(x)</u> Inv. Hyperbolic Sine:  $asinh(-6.05020448) \rightarrow -2.5$ 

 $asinh(6.050204481) \rightarrow 2.5$ 

 $a\cosh(x)$  Inv. Hyperbolic Cosine:  $a\cosh(2.352409615) \rightarrow 1.5$ 

<u>atanh(x)</u> Inv. Hyperbolic Tangent:  $atanh(0.5) \rightarrow 0.5493061...$ 

acoth(x) Inv. Hyperbolic Cotangent:  $acoth(2.0) \rightarrow 0.5493061...$ 

# Other Functions with one Argument

Temperature conversions

FtoC(f) Fahrenheit → Celsius: FtoC(86)  $\rightarrow$  30

**CtoF(c)** Celsius  $\rightarrow$  Fahrenheit: CtoF(37.5)  $\rightarrow$  99.5

*Angle conversions* – useful for the trigonometric functions

```
\frac{\text{dgr}(\mathbf{x})}{\text{rad}(\mathbf{x})} Radian → Degree x \cdot 180/\pi: \frac{\text{dgr}(a\sin(0.5))}{\text{dgr}(a\sin(0.5))} \rightarrow 30

\frac{\text{rad}(\mathbf{x})}{\text{rad}(\mathbf{x})} Degree → Radian: x \cdot \pi/180: \cos(\text{rad}(60)) \rightarrow 0.5
```

Factorial/Permutation functions – the results of are straight integer numbers for integer arguments  $x \le 20$  only, otherwise, the non-integer real results of the gamma functions gamma(x + 1) or gammaLn(x + 1) are returned

fact(x)Factorial x!:fact(6)  
fact(6.01)→ 720  
→ 733.617factLn(x)Log-Factorial:factLn(18) = ln(fact(18))→ 36.3954perm(n)Permutations 
$$n!$$
:fact(8) = perm(8)→ 40320

**gamma(x)** Gamma function:  $gamma(5.5) \rightarrow 52.343$ 

**gammaLn(x)** Log-Gamma function:

 $gammaLn(21.5) = ln(gamma(21.5)) \rightarrow 43.85192586$ 

erf(x) Error function:  $erf(3) \rightarrow 0.9999779095$ 

prob(s) Probability function, i.e. the integral of the Standard Normal (Gaussian) Distribution in the range of  $\pm s$ , see also gauss():  $prob(1) \rightarrow 0.6826894921$   $prob(2) \rightarrow 0.9544997361$ 

Standard deviation that covers 95 % of all occasions: solve(s := 2; prob(s) = 95%)  $\rightarrow$  1.959963985

## **Functions with Multiple Number of Arguments**

<u>root(n; x)</u> nth Root  $\sqrt[n]{x}$ : root(5; 243) → 3

<u>scalb(x; n)</u> Binary scale  $x \cdot 2^n$ : scalb(64; 10) → 65536

pyth2D(x; y) Geometric distance  $\sqrt{(x^2+y^2)}$  – 2 dimensions Pythagoras formular: pyth2D(1; 2)  $\rightarrow$  2.2361

**pyth3D(x;y;z)** Geometric distance  $\sqrt{(x^2 + y^2 + z^2)} - 3$  dimensions Pythagoras formular: pyth3D(1;2;3)  $\rightarrow$  3.7417

gauss(x; m; s) Normal (Gaussian) distribution density function around mean m with standard deviation s:
gauss(3; 2; 1)  $\rightarrow$  0.2419707245

Note, this is the value at abscissa x of the gauss function, characterized by its mean m and standard deviation s. If we are looking for a probability value of events falling into a given range, then we need to integrate the Gaussian function in that range:

$$\int (z := 1; z; gauss(z; 2; 1)) \rightarrow 0.6826894921 \approx 68.3\%$$

As a matter of fact, 68.3 % is the well known probability of an event falling into  $\pm 1$  standard deviation. The Gaussian Distribution plays an essential role in all sorts of statistical analysis. For further reading see the respective Wikipedia Article.

**gaussLn(x;m;s)** Log-Gaussian distribution density function around mean m with standard deviation s, evaluated at ln(x): gaussLn(3;2;1)  $\rightarrow$  0.088584

gcd(m; n) Greatest common divisor: gcd(528;112) → 16

<u>lcm(m; n)</u> Least common multiple:  $lcm(528;112) \rightarrow 3696$ 

<u>comb(n; k)</u> Combinations (k out of n): comb(49;6) → 13983816 i.e. binomial coefficient  $n!/[k! \cdot (n-k)!]$ 

```
var(n; k)
              Variations n!/(n-k)!:
                                              var(4;2) \rightarrow 12
                    Compounded interest (1 + r)^n:
compound(r; n)
                    $20000 at 4.5 %/year after 60 month:
           e.g.
                    20000*compound(4.5\%/12; 60) \rightarrow $25035.92
annuity(r; n)
                    Annuity (down payment) (1 - (1+r)^{-n})/r:
                    $20000 at 4.5 %/year to be down paid in 60 m:
           e.g.
                    20000/\text{annuity}(4.5\%/12; 60) \rightarrow \$372.86/\text{month}
if(cnd; r1; r2)
                    Conditional evaluation
                    • r1 if condition is true: if(true; 1; 2) \rightarrow 1
                    • r2 if condition is false: if(false: 1: 2) \rightarrow 2
                                   if(rand < 0.5; 0; H*2+O) \rightarrow 18.015
                                   if(rand < 0.5: 0: H^*2+O) \rightarrow 0
                                   if(rand < 0.5; 0; H*2+O) \rightarrow 0
                                   if(rand < 0.5: 0: H*2+O) \rightarrow 18.015
```

#### <u>linpol()</u> Linear interpolation linpol(x; x1;y1; x2;y2)

Results in the value y at the position x on the straight line defined by the two points x1/y1 and x2/y2. This is a shortcut of the general linear interpolation formula (y2 - y1)/(x2 - x1)\*(x - x1) + y1:

```
linpol(80.37; 60;5.41; 100;4.79) \rightarrow 5.094265
(4.79 - 5.41)/(100 - 60)*(80.37 - 60) + 5.41 \rightarrow 5.094265
```

## **Functions with Variable Number of Arguments**

```
Explicit summation sum(1;2;3;4;5;6;7;8;9) \rightarrow 45
sum()
prod()
           Explicit product
                                prod(1;2;3;4;5;6;7;8;9) \rightarrow 362880
          Arithmetic average ave(1;2;3;4;5;6;7;8;9) \rightarrow 5
ave()
gave()
           Geometric average gave(1;2;3;4;5;6;7;8;9) \rightarrow 4.14717
have()
           Harmonic average have(1;2;3;4;5;6;7;8;9) \rightarrow 3.18137
stdev()
           Standard deviation stdev(1;2;3;4;5;6;7;8;9) \rightarrow 2.73861
linreg()
           Linear regression
                                linreg(x1;y1; x2;y2; ...)
Returns the correlation coefficient as its direct result. In addition,
it populates user variables with the other results of the regression
analysis:
             intercept
  а
  h
             slope
             standard deviation of y estimated by using a + b \cdot x
  stdev y
  stdev_a
             standard deviation of the intercept
  stdev_b standard deviation of the slope
linreg(0;1; 5;1.0538; 12;1.1309; 20;1.2191; 28;1.3064; 34;
1.3696; 40;1.43; 44;1.4685; 48;1.5065; 50;1.5253) \rightarrow 0.99969...
              \rightarrow 1.004350452
   a
  h
             \rightarrow 0.01055727928
```

stdev\_y  $\rightarrow$  0.005058603831 stdev\_a  $\rightarrow$  0.003066204759 stdev b  $\rightarrow$  1.713096898e-06

#### **Numerical Mathematical Methods**

Numerical accuracy can be adjusted in the InstantCalc Preferences Panel. The numerical methods are invoked by function calls with two or three arguments which are separated by semicolon. The first argument denotes the control variable for the method, and usually it shall be set to the initial/start value by using the variable assignment operator :=. The last argument is the expression, that should be evaluated with respect to the control variable of the first argument. In the case of Integration, Summation, and Serial Product, the expression in the second argument should evaluate the stop value.

#### Newton Equation Solver solve(var := init; eqL(var) = eqR(var))

```
solve(u := 3; \sin(u) = 0) \rightarrow 3.141592654
solve(v := 2; 3 \cdot v \wedge 2 + 6 \cdot v = 45) \rightarrow 3
```

The right side of the equation may be omitted, in which case the expression is resolved to zero. Therefore, the first example may be rewritten as  $solve(u := 3; sin(u)) \rightarrow 3.141592654$ .

The solver may fail in cases, i.e. the result is Not a Number:

- neither the left side, nor the right side of the equation is dependent on the control variable revise the equation!
- poor initial guess try to improve the initial guess, by playing around with different initial values of the control variable!
- in the course of the iteration, either side of the equation evaluate to non finite values, i.e. inf or nan try to improve the initial guess and/or rewrite the equation, perhaps by using the conditional function if(;;) in order to avoid invalid function evaluations!

#### Numerical Differentiation $\triangle$ (var := poe; expression(var))

$$\Delta(u := 1; \sin(u)) \rightarrow 0.5403023059 \leftarrow \cos(1)$$
  
 $\Delta(v := 2; 3 \cdot v \land 2 + 6 \cdot v) \rightarrow 18$ 

The expression shall be dependent on the control variable, other-

wise, the result is simply 0. At the *poe* (point of evaluation)  $\pm$  the relative tolerance (s. Preferences Panel) the expression must result in finite numbers, otherwise the numerical differentiation may fail, i.e. it reports  $\mathbf{nan} = \mathbf{not} \ \mathbf{a} \ \mathbf{number}$ .

```
Romberg Integration \underline{(var := start; stop; expression(var))}
```

```
\int (u := 0; \pi; \sin(u)) \rightarrow 2
\int (p := -1; 1; 1/(1 + \text{sqr}(p))) \rightarrow 1.570796327 \leftarrow \pi/2
\int (q := -2; 2; \text{gauss}(q; 0; 1)) \rightarrow 0.9544997361
```

# Double Integration $\int (p := a; b; \int (q := c; d; f(p; q)))$

```
\int (p :=-3;3; \int (q :=-2;2; gauss(p;0;1) \cdot gauss(q;0;1)))

\rightarrow 0.9519227815
```

#### Summation (implicit) $\sum$ (var := start; stop; expression(var))

```
\sum (k := 1; 9; k) \rightarrow 45
```

Saving \$750 per month over 10 years at 6.25 %/year  $\Sigma(k := 0; 119; 750*(1 + 6.25\%/12)^k) \rightarrow 124591.42$ 

the above gives the same result as:

750\*annuity(6.25%/12;120)\*compound(6.25%/12;120)

 $\rightarrow 124591.42$ 

## Serial Product (implicit) $\prod$ (var := start; stop; expression(var))

$$\Pi(k := 1; 9; k) \rightarrow 362880$$

Encapsulation of the numerical methods into each other, for example for double integration, or for using integration, differentiation, summation, and serial product together with the Newton Solver, is well possible:

$$\int (u := -1; 1; \int (v := -1; 1; pyth2D(u; v)))$$

$$\rightarrow 3.060782866$$

$$solve(p := 1.5; \int (q := -p; p; gauss(q; 0; 1)) = 0.95)$$

```
→ 1.959963985

solve(x := 2; \Delta(d := x; \pi/2·sqr(d) + 4/d) = 0)

→ 1.08385214

solve(dp := 1000; 50000·(1+6.25%/12)^48

- dp·\Sigma(k := 0; 47; (1+6.25%/12)^k)) → 1179.990993

50000/annuity(6.25%/12; 48) → 1179.990993
```

## **A Function without Argument**

rand Equally distributed random numbers between 0 and 1  $\sum (k := 1; 1000000; rand)/k \approx 0.5 \pm 0.0002$ 

## **Complex Number Calculations**

Complex numbers are usually denoted as a sum of their real and imaginary part, i.e.:  $re + i \cdot im$ 

For a calculator like InstantCalc, that deals with any sort of numbers, this text book notation is not quite appropriate, as the "+" sign within the complex context, while it is related, does not exactly correspond to an arithmetic addition operation.

For InstantCalc an alternative notation scheme for complex numbers has been defined. Real and imaginary part are separated by a semicolon and embraced by curly brackets: {re; im}

Some mathematical functions in real space are defined only for their arguments lying in a certain range. For example, the real space square root is not defined for negative values:

$$\sqrt{-1}$$
  $\rightarrow$  Not a Number (nan)

On the other hand, the complex variant of the square root is defined for negative arguments:

$$\sqrt{\{-2;0\}} \rightarrow \{0; 1.414213562\}$$

Examples for other functions that are undefined for certain ranges of arguments in the real space but may give results in the complex space are general roots of even integer power or of any real power, as well as asin(), acos(), atan(), logarithms, etc.

InstantCalc may automatically do a real to complex conversion for said kind of functions if the real arguments lie outside of the defined ranges. The automatic real to complex conversion can be enabled or disabled in the InstantCalc Preferences Panel. By default, InstantCalc does no automatic real to complex or complex to real conversions.

The following special complex functions exist. The examples use the variable definition  $z := \{3; 1.5\}.$ 

**real(z)** Real part of complex numbers:  $real(z) \rightarrow 3$ 

<u>imag(z)</u> Imaginary part of complex numbers:  $imag(z) \rightarrow 1.5$ 

```
norm(z) Euclidean Norm (distance/val.) of z: norm(z) → 3.354...

= sqrt(sqr(real(z)) + sqr(imag(z))) → 3.354...

conj(z) Complex Conjugate of z: conj(z) → {3;-1.5}

arg(z) Complex Argument (angle) of z: arg(z) → 0.463...
```

=  $atan2(imag(z); real(z)) \rightarrow 0.463...$ 

#### **Vector and Matrix Calculations**

InstantCalc calculates expressions, each on one line of pure text. *Vectors* and *Matrices* usually consist of multiple rows and/or columns, and some definitions need to be made in order to fold the rows and columns onto a single line.

The elements of *Vectors* are separated by semicolons and all together is embraced by square brackets: [a; b; c; ...]

Vectors may be of any dimension greater or equal to 1.

*Matrices* consist of rows of vectors whereby the rows are separated by semicolons and all together is embraced by square brackets. The following is an example for a

```
3×3 matrix: [[a11; a12; a13]; [a21; a22; a23]; [a31; a32; a33]]
```

Matrices may be of any dimensions  $M \times N$  whereby M (number of rows) and N (number of columns) must be greater or equal to 1. Special cases are:

```
1×1 matrix: [[a11]]
```

1×N matrix: [[a11; a12; ...; a1N]]

*M*×1 matrix: [[a11]; [a21]; ...; [aM1]]

Where appropriate, InstantCalc converts 1xN or Mx1 matrices into a row vector with N elements or a column vector with M elements.

Each vector/matrix element may be of any type, including complex type, vector type, or matrix type. However, not every mathematical operation is defined with any kind of vector or matrix, and the error message "Type Mismatch" may appear when appropriate.

All scalar operators and scalar functions operate on each element of vectors and matrices, e.g.:

```
[1; 2; 3]*5 \rightarrow [5; 10; 15]

[[3; 2]; [0; 1]]*[[1; 2]; [3; 1]] \rightarrow [[3; 4]; [0; 1]]

\exp([1; 2; 3]) \rightarrow [2.718...; 7.389...; 20.085...]
```

Note that +, -, \*, / are scalar operators in this respect. Special operators for Vectors and Matrices do exist:

- Operator for Vector Dot product and Matrix Product  $[1;3;-5] \bullet [4;-2;-1] \rightarrow 3$   $[[3;2;1]; [1;0;2]] \bullet [[1;2]; [0;1]; [4;0]] \rightarrow [[7;8]; [9;2]]$
- $\underline{x}$  Operator for Vector Cross Product [1;2;3] x [-7;8;9] → [-6; -30; 22]
- Operator for Matrix Division, which is not generally defined as such. In InstantCalc, Matrix Division is the shortcut for a sequence of two operations, a Matrix Inversion on the left operand (needs to be a matrix), and then the Matrix Product from the inversed matrix with the right operand (a vector or a matrix of proper dimensions). If A is a matrix and b is a vector then  $A \mid b$  would give exactly the same result as  $inv(A) \cdot b$  for examples, see the chapter Matrix Inversion below.

The Following special vector and matrix functions exist. The examples use the variable definition A := [[1;2;0]; [2;3;0]; [3;4;1]]

**simpl(W[; thresh])** simplify a vector or matrix, i.e. recursively set all elements that are less than a threshold value to 0, and convert unidimensional vectors/matrices to scalar values, and reduce complex values whoes imaginary part is zero to real values. If the parameter *thresh* is omitted, then the machine precision *prec* is used instead:

```
\begin{array}{lll} simpl([[1;2;3]]) & \rightarrow & [1;2;3] \\ simpl([[1];[2];[3]]) & \rightarrow & [1;2;3] \\ simpl([1;\{2;0.05\};3];0.1) & \rightarrow & [1;2;3] \end{array}
```

elem(W; m[; n]) extract an element from a vector or a matrix:

```
elem(A; 2) \rightarrow [2;3;0]
elem(A; 2; 1) \rightarrow 2
```

<u>norm(W)</u> Euclidean Vector or Matrix Norm: norm(A) → 6.633...

 $\underline{\operatorname{trans}(M)}$  Matrix Transpose:  $\underline{\operatorname{trans}(A)} \rightarrow [[1;2;3];[2;3;4];[0;0;1]]$ 

 $\underline{\det(M)}$  Determinant of a square matrix:  $\det(A) \rightarrow -1$ 

**svd(M)** Singular Value Decomposition of M, i.e. the factorization of the given matrix into  $M = U \sum V^*$ . Returns the vector S of the singular values on the diagonal positions of the diagonal matrix  $\Sigma$ :

 $svd(A) \rightarrow [6.591520852; 0.7116199857; 0.2131897072]$ 

inv(M) Matrix Inversion: inv(A)  $\rightarrow$  [[-3;2;0];[2;-1;0];[1;-2;1]] inv(A)•[1; -2; 5]  $\rightarrow$  [-7; 4; 10] A\[1; -2; 5]  $\rightarrow$  [-7; 4; 10]

**Caution:** the following is not a matrix operation, but simply an element by element scalar division:

$$A/[1; -2; 5] \rightarrow [[1;2;0];[-1;-1.5;0];[0.6;0.8;0.2]]$$

vec(x; n) return a vector of dimension n whose elements are set to x:  $vec(0.19; 3) \rightarrow [0.19; 0.19; 0.19]$ 

<u>diag(x; n)</u> return a  $n \times n$  matrix whose diagonal elements are set to x: diag(1; 3) → [[1;0;0];[0;1;0];[0;0;1]]

mat(x; m; n) return a  $m \times n$  matrix whose elements are set to x: mat(2; 3; 4) → [[2;2;2;2];[2;2;2;2]]

## **Indexed References into the Calculation History**

Result) in the history table at index sn

**Esn** E(xpression) in the history table at index sn

#### **Predefined Constants**

<u>true</u> Boolean values:

<u>false</u> 0

**<u>c0</u>** Speed of Light in vacuum: 299792458 m/s

<u>e</u> Euler's Number:

2.71828182845904523536028747135266

<u>т</u>, <u>рі</u> 3.14159265358979323846264338327948

<u>ln10</u> 2.30258509299404590109361379290931

**gold** Golden Ratio:

1.61803398874989484820458683436564

 $\underline{\infty}$ , <u>inf</u> IEEE754-infinity  $1/0 \rightarrow \text{inf}$ 

<u>nan</u> Not a Number (type 0)  $\sqrt{-1}$  → Not a Number

**none** Not a Number (type 255)

maxshort 32767

<u>maxint</u> 2147483647

**maxlong** 9223372036854775807

minreal 3.645199531882474603e-4951

<u>maxreal</u> 1.189731495357231765e+4932

**prec** Numerical precision, i.e. machine epsilon:

1.084202172485504434e-19

# Table of Atomic Weights of the Elements in g/mol

Compiled from:

IUPAC Technical Report - Atomic weights of the elements 2009, Pure Appl. Chem., Vol. 83, No. 2, pp. 359–396, 2011.

- \* Note: The standard atomic weights of 10 elements having 2 or more stable isotopes have been changed to reflect this variability of atomic weight values in natural terrestrial materials. These elements are *H*, *Li*, *B*, *C*, *N*, *O*, *Si*, *S*, *Cl*, *Tl*. For these selected elements, the conventional atomic weights are taken.
- \*\* Note: For radioactive/artificial elements, the atomic weights of the most stable isotope has been taken.

#### **Usage**

```
→ 18.015 molecular weight of water
H*2+O
                  → 46.069 molecular weight of ethanol
C*2+H*5+O+H
H*2+S+O*4
                  → 98.072 molecular weight of sulfuric acid
250*Cr/(Cr+O*3)'4 \rightarrow 129.9992 Cr content in 250 g chromic acid
1
         н
                   1.008 [1.00784; 1.00811]
                                              Hydrogen*
         He
                   4.002602
                                Helium
2
         Li
                   6.94
                         [6.938; 6.997]
                                               Lithium*
                                Beryllium
         Be
                   9.012182
         В
                 10.81
                         [10.806; 10.821]
                                               Boron*
         C
                 12.011 [12.0096; 12.0116]
                                               Carbon*
                 14.007 [14.00643; 14.00728] Nitrogen*
         N
                 15.999 [15.99903; 15.99977] Oxygen*
         O
         F
                 18.9984032
                                Fluorine
              → 20.1797
         Ne
                                 Neon
```

3	<u>Na</u>	$\rightarrow$	22.98976928	3	Sodium	
	<u>Mg</u>	$\rightarrow$	24.3050		Magnesium	
	<u>Al</u>	$\rightarrow$	26.9815386		Aluminium	
	<u>Si</u>	$\rightarrow$	28.085 [28.0	084	4; 28.086]	Silicon*
	<u>P</u>	$\rightarrow$	30.973762		Phosphorus	
	<u><b>S</b></u>	$\rightarrow$	32.06 [32.0	059	9; 32.076]	Sulfur*
	<u>Cl</u>	$\rightarrow$	35.45 [35.4	446	6; 35.457]	Chlorine*
	<u>Ar</u>	$\rightarrow$	39.948		Argon	
4	<u>K</u>	$\rightarrow$	39.0983		Potassium	
	<u>Ca</u>	$\rightarrow$	40.078		Calcium	
	<u>Sc</u>	$\rightarrow$	44.955912		Scandium	
	<u>Ti</u>	$\rightarrow$	47.867		Titanium	
	<u>V</u>	$\rightarrow$	50.9415		Vanadium	
	<u>Cr</u>	$\rightarrow$	51.9961		Chromium	
	<u>Mn</u>	$\rightarrow$	54.938045		Manganese	
	<u>Fe</u>	$\rightarrow$	55.845		Iron	
	<u>Co</u>	$\rightarrow$	58.933195		Cobalt	
	<u>Ni</u>	$\rightarrow$	58.6934		Nickel	
	<u>Cu</u>	$\rightarrow$	63.546		Copper	
	<u>Zn</u>	$\rightarrow$	65.38		Zinc	
	<u>Ga</u>	$\rightarrow$	69.723		Gallium	
	<u>Ge</u>	$\rightarrow$	72.63		Germanium	
	<u><b>As</b></u>	$\rightarrow$	74.92160		Arsenic	
	<u>Se</u>	$\rightarrow$	78.96		Selenium	
	<u>Br</u>	$\rightarrow$	79.904		Bromine	
	<u>Kr</u>	$\rightarrow$	83.798		Krypton	

5	<u>Rb</u>	→ 85.4678	Rubidium
	<u>Sr</u>	<b>→</b> 87.62	Strontium
	<u>Y</u>	→ 88.90585	Yttrium
	<u>Zr</u>	<b>→</b> 91.224	Zirconium
	<u>Nb</u>	→ 92.90638	Niobium
	<u>Mo</u>	→ 95.96	Molybdenum
	<u>Tc</u>	→ 97.9072	Technetium**
	<u>Ru</u>	<b>→</b> 101.07	Ruthenium
	<u>Rh</u>	→ 102.90550	Rhodium
	<u>Pd</u>	→ 106.42	Palladium
	<u>Ag</u>	→ 107.8682	Silver
	<u>Cd</u>	<b>→</b> 112.411	Cadmium
	<u>In</u>	<b>→</b> 114.818	Indium
	<u>Sn</u>	<b>→</b> 118.710	Tin
	<u>Sb</u>	→ 121.760	Antimony
	<u>Te</u>	→ 127.60	Tellurium
	1	<b>→</b> 126.90447	Iodine
	<u>Xe</u>	<b>→</b> 131.293	Xenon
6	<u>Cs</u>	→ 132.9054519	Cesium
	<u>Ba</u>	<b>→</b> 137.327	Barium
	<u>La</u>	<b>→</b> 138.90547	Lanthanum
	<u>Ce</u>	<b>→</b> 140.116	Cerium
	<u>Pr</u>	→ 140.90765	Praseodymium
	<u>Nd</u>	<b>→</b> 144.242	Neodymium
	<u>Pm</u>	<b>→</b> 144.9127	Promethium**
	<u>Sm</u>	<b>→</b> 150.36	Samarium

<u>Eu</u>	<b>→</b> 151.964	Europium	
<u>Gd</u>	<b>→</b> 157.25	Gadolinium	
<u>Tb</u>	→ 158.92535	Terbium	
<u>Dy</u>	→ 162.500	Dysprosium	
<u>Ho</u>	→ 164.93032	Holmium	
<u>Er</u>	→ 167.259	Erbium	
<u>Tm</u>	→ 168.93421	Thulium	
<u><b>Yb</b></u>	→ 173.054	Ytterbium	
<u>Lu</u>	<b>→</b> 174.9668	Lutetium	
<u>Hf</u>	<b>→</b> 178.49	Hafnium	
<u>Ta</u>	→ 180.94788	Tantalum	
$\underline{\mathbf{W}}$	<b>→</b> 183.84	Tungsten	
<u>Re</u>	→ 186.207	Rhenium	
<u>Os</u>	<b>→</b> 190.23	Osmium	
<u>lr</u>	<b>→</b> 192.217	Iridium	
<u>Pt</u>	<b>→</b> 195.084	Platinum	
<u>Au</u>	<b>→</b> 196.966569	Gold	
<u>Hg</u>	<b>→</b> 200.59	Mercury	
<u>Tl</u>	→ 204.38 [204.3	82; 204.385]	Thallium <sup>3</sup>
<u>Pb</u>	<b>→</b> 207.2	Lead	
<u>Bi</u>	→ 208.98040	Bismuth	
<u>Po</u>	→ 208.9824	Polonium**	
<u>At</u>	→ 209.9871	Astatine**	
<u>Rn</u>	→ 222.0176	Radon**	
<u>Fr</u>	→ 223.0197	Francium**	
<u>Ra</u>	→ 226.0254	Radium**	

<u>Np</u> → 237.0482	Neptunium**
<u>Pu</u> → 244.0642	Plutonium**
<u>Am</u> → 243.0614	Americium**
<u>Cm</u> → 247.0704	Curium**
<u><b>Bk</b></u> → 247.0703	Berkelium**
$\underline{\mathbf{Cf}} \rightarrow 251.0796$	Californium**
$\underline{Es} \rightarrow 252.0830$	Einsteinium**
<u>Fm</u> → 257.0951	Fermium**
$Md \rightarrow 258.0984$	Mendelevium**
<u>No</u> → 259.1010	Nobelium**
<u>Lr</u> → 262.1096	Lawrencium**
<u>Rf</u> → 265.1167	Rutherfordium**
<u>Db</u> → 268.125	Dubnium**
<u>Sg</u> → 271.133	Seaborgium**
<u>Bh</u> → 267.1277	Bohrium**
$\underline{Hs} \rightarrow 277.150$	Hassium**
$Mt \rightarrow 276.151$	Meitnerium**
<u>Ds</u> → 281.162	Darmstadtium**
$Rg \rightarrow 280.164$	Roentgenium**
<u>Cn</u> → 285.174	Copernicium**
<u>Uut</u> → 284.178	Ununtrium**
<u>FI</u> → 289.187	Flerovium**
<u>Uup</u> → 288.192	Ununpentium**

<u>Ac</u> → 227.0278

<u>Pa</u> → 231.03588

**→** 232.03806

 $U \rightarrow 238.02891$  Uranium

Th

Actinium\*\*

Protactinium

Thorium

 $\underline{\text{Lv}}$  → 292.200 Livermorium\*\*  $\underline{\text{Uus}}$  → 293.2 Ununseptium\*\*  $\underline{\text{Uuo}}$  → 294.2 Ununoctium\*\*

## **Error Messages**

Syntax Error The expression does not conform

to the pertinent calculation rules.

Check the selected range for missing or extra parenthesis or look out for extra operators at inappro-

priate places.

Semantic Error The expression contains unknown

identifiers.

Check the spelling of the selected

word or symbol.

Wrong Number of Parameters The number of arguments/para-

meters in a function call does not

match the function definition.

Error in Logical Expression The logical expression does not

conform to the pertinent evalua-

tion rules.

Look out for invalid chaining of

comparison operations, like:

term1 == term2 != term3

Type Mismatch Some functions work only with

selected types of operands, e.g. gcd() and lcm() expect integers.

Not a Number A valid numeric result does not

exist for an expression and its arguments, for example, in the case of automatic real to complex conversion is disabled, *sqrt(-1)* results

in Not a Number.

Singular Matrix Matrix inversion cannot be perfor-

med because the given matrix is

singular.

Invalid Row Index The row index sn of a R(esult) or

*E*(xpression) reference does not exist in the calculation history.

Invalid Recursion A reference to an E(xpression) in

the calculation history leads back

to itself.