Chapter 4 | Soil Temperatures and Heat Transfer

4.1 Surface Temperature

An 'ideal' surface, defined in Chapter 2, may be considered to have a uniform surface skin temperature (T_s) which varies with time only in response to the time-dependent energy fluxes at the surface. An uneven or nonhomogeneous surface is likely to have spatially varying surface temperature. The surface temperature at a given location is essentially given by the surface energy balance, which in turn depends on the radiation balance, atmospheric exchange processes in the immediate vicinity of the surface, presence of vegetation or plant cover, and thermal properties of the subsurface medium. Here, we distinguish between the surface skin temperature and the near-surface air temperature. The latter is measured at standard meteorological stations at the screen height of 1–2 m. The temperature at the air–soil interface will be referred to as the surface skin temperature or simply as the surface temperature (T_s).

The direct *in situ* measurement of surface temperature is made very difficult by the extremely large temperature gradients that commonly occur near the surface in both the air (temperature gradients of 10–20 K mm^{-1} in air are not uncommon very close to a bare heated surface) and the soil media, by the finite dimensions of the temperature sensor, and by the difficulties of ventilating and shielding the sensor when it is placed at the surface. Therefore, the surface skin temperature is often determined by extrapolation of measured temperature profiles in soil and air, with the knowledge of their expected theoretical behaviors. Another, perhaps better, method of determining the surface temperature, when emissivity of the surface is known, is through remote sensors, such as a downward-looking radiometer which measures the flux of outgoing longwave radiation from the surface and, hence, T_s, using the modified Stefan–Boltzmann Equation (3.7). If the surface emissivity is not known with sufficient accuracy, this method will give the apparent (equivalent blackbody) surface temperature. In this way, surface or apparent surface temperatures are being routinely monitored by weather satellites. Measurements are made in narrow regions of the spectrum in which water vapor and CO_2 are transparent. The technique has proved to be fairly reliable for measuring sea-surface temperatures, but not so reliable for land-surface temperatures.

The times of minimum and maximum in surface temperatures as well as the diurnal range are of considerable interest to micrometeorologists. On clear days, the maximum surface temperature is attained typically an hour or two after the time of maximum insolation, while the minimum temperature is reached in early morning hours. The maximum diurnal range is achieved for a relatively dry and bare surface, under relatively calm air and clear skies. For example, on bare soil in summer, midday surface temperatures of 50–60°C are common in arid regions, while early morning temperatures may be only 10–20°C. The bare-surface temperatures also depend on the texture of the soil. Fine-textured soils (e.g., clay) have greater heat capacities and smaller diurnal temperature range, as compared to coarse soils (e.g., sand).

The presence of moisture at the surface and in the subsurface soil greatly moderates the diurnal range of surface temperatures. This is due to the increased evaporation from the surface, and also due to increased heat capacity and thermal conductivity of the soil. Over a free water surface, on the average, about 80% of the net radiation is utilized for evaporation. Over a wet, bare soil a substantial part of net radiation goes into evaporation in the beginning, but this fraction reduces as the soil surface dries up. Increased heat capacity of the soil further slows down the warming of the upper layer of the soil in response to radiative heating of the surface. The ground heat flux is also reduced by evaporation.

The presence of vegetation on the surface also reduces the diurnal range of surface temperatures. Part of the incoming solar radiation is intercepted by plant surfaces, reducing the amount reaching the surface. Therefore surface temperatures during the day are uniformly lower under vegetation than over a bare soil surface. At night the outgoing longwave radiation is also partly intercepted by vegetation, but the latter radiates energy back to the surface. This slightly slows down the radiative cooling of the surface. Vegetation also enhances the latent heat exchange due to evapotranspiration. It increases turbulence near the surface which provides more effective exchanges of sensible and latent heat between the surface and the overlying air. The combined effect of all these processes is to significantly reduce the diurnal range of temperatures of vegetative surfaces.

4.2 Subsurface Temperatures

Subsurface soil temperatures are easier to measure than the skin surface temperature. It has been observed that the range or amplitude of diurnal variation of soil temperatures decreases exponentially with depth and becomes

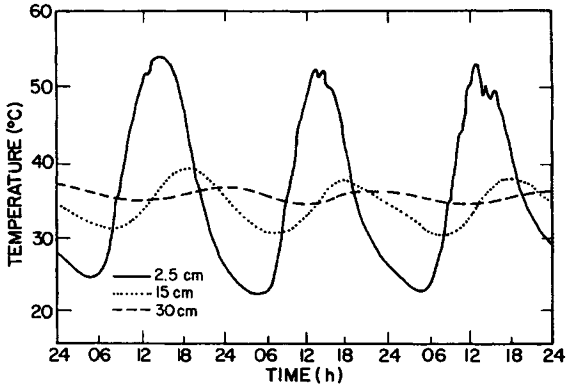

Figure 4.1 Observed diurnal course of subsurface soil temperatures at various depths in a sandy loam soil with bare surface. [From Deacon (1969); after West (1952).]

insignificant at a depth of the order of 1 m or less. An illustration of this is given in Figure 4.1, which is based on measurements in a dry, sandy loam soil.

Soil temperatures depend on a number of factors, which also determine the surface temperature. The most important are the location (latitude) and the time of the year (month or season), net radiation at the surface, soil texture and moisture content, ground cover, and surface weather conditions. Temperature may increase, decrease, or vary nonmonotonically with depth, depending on the season and the time of day.

In addition to the 'diurnal waves' present in soil temperatures in the top layer, daily or weekly averaged temperatures show a nearly sinusoidal 'annual wave' which penetrates to much greater depths (~ 10 m) in the soil. This is illustrated in Figure 4.2.

A simple theory for explaining the observed diurnal and annual temperature waves in soils and variations of their amplitudes and phase with depth will be presented later. First, we define and discuss the thermal properties of soils which influence soil temperatures and heat transfer.

4.3 Thermal Properties of Soils

Thermal properties relevant to the transfer of heat through a medium and its effect on the average temperature or distribution of temperatures in the medium are the mass density, specific heat, heat capacity, thermal conductivity, and thermal diffusivity.

The specific heat (c) of a material is defined as the amount of heat absorbed or released in raising or lowering the temperature of a unit mass of the material by

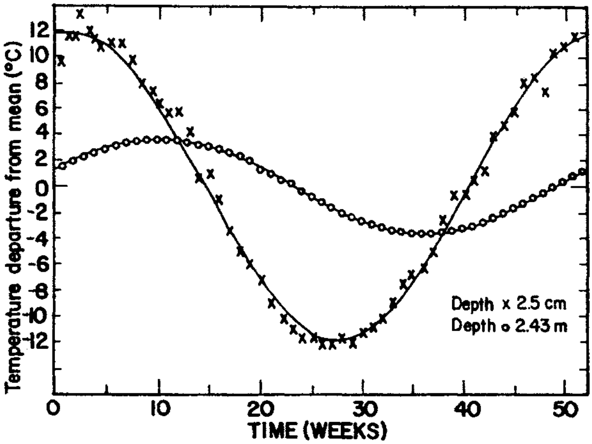

Figure 4.2 Annual temperature waves in the weekly averaged subsurface soil temperatures at two depths in a sandy loam soil. Fitted solid curves are sine waves. [From Deacon (1969); after West (1952).]

1°. The product of mass density (ρ) and specific heat is called the heat capacity per unit volume, or simply heat capacity (C).

Through solid media and still fluids, heat is transferred primarily through conduction, which involves molecular exchanges. The rate of heat transfer or heat flux in a given direction is found to be proportional to the temperature gradient in that direction, i.e., the heat flux in the z direction

$$H = -k(\partial T/\partial z) \tag{4.1a}$$

in which the proportionality factor k is known as the thermal conductivity of the medium. The ratio of thermal conductivity to heat capacity is called the thermal diffusivity α_h. Then, Equation (4.1) can also be written as

$$H/\rho c = -\alpha_h(\partial T/\partial z) \tag{4.1b}$$

where $\alpha_h = k/\rho c = k/C$.

Heat is transferred through soil, rock, and other subsurface materials (except for water in motion) by conduction, so that the above-mentioned molecular thermal properties also characterize the submedium. Table 4.1 gives typical values of these for certain soils, as well as for the reference air and water media, for comparison purposes.

Note that air has the lowest heat capacity, as well as the lowest thermal conductivity of all the natural materials, while water has the highest heat

Table 4.1 Molecular thermal properties of natural materials[a].

Material	Condition	Mass density ρ ($kg\,m^{-3} \times 10^3$)	Specific heat c ($J\,kg^{-1}\,K^{-1} \times 10^3$)	Heat capacity C ($J\,m^{-3}\,K^{-1} \times 10^6$)	Thermal conductivity k ($W\,m^{-1}\,K^{-1}$)	Thermal diffusivity α_h ($m^2\,s^{-1} \times 10^{-6}$)
Air	20°C, Still	0.0012	1.01	0.0012	0.025	20.5
Water	20°C, Still	1.00	4.18	4.18	0.57	0.14
Ice	0°C, Pure	0.92	2.10	1.93	2.24	1.16
Snow	Fresh	0.10	2.09	0.21	0.08	0.38
Snow	Old	0.48	2.09	0.84	0.42	0.05
Sandy soil	Fresh	1.60	0.80	1.28	0.30	0.24
(40% pore space)	Saturated	2.00	1.48	2.96	2.20	0.74
Clay soil	Dry	1.60	0.89	1.42	0.25	0.18
(40% pore space)	Saturated	2.00	1.55	3.10	1.58	0.51
Peat soil	Dry	0.30	1.92	0.58	0.06	0.10
(80% pore space)	Saturated	1.10	3.65	4.02	0.50	0.12
Rock	Solid	2.70	0.75	2.02	2.90	1.43

[a]After Oke (1987) and Garratt (1992).

capacity. Thermal diffusivity of air is very large because of its low density. Thermal properties of both air and water depend on temperature. Most soils consist of particles of different sizes and materials with a significant fraction of pore space, which may be filled with air and/or water. Thermal properties of soils, therefore, depend on the properties of the solid particles and their size distribution, porosity of the soil, and the soil moisture content. Of these, the soil moisture content is a short-term variable, changes of which may cause significant changes in heat capacity, conductivity, and diffusivity of soils (Oke, 1987, Chapter 2; Rosenberg *et al.*, 1983, Chapter 2).

Addition of water to an initially dry soil increases its heat capacity and conductivity markedly, because it replaces air (a poor heat conductor) in the pore space. Both the heat capacity and thermal conductivity are monotonic increasing functions of soil moisture content. The former increases almost linearly with the soil moisture content, while the latter approaches a constant value with increasing soil moisture. Their ratio, thermal diffusivity, for most soils increases with moisture content initially, attains a maximum value, and then falls off with further increase in soil moisture content. It will be seen later that thermal diffusivity is a more appropriate measure of how rapidly surface temperature changes are transmitted to deeper layers of submedium. Soils with high thermal diffusivities allow rapid penetration of surface temperature changes to large depths. Thus, for the same diurnal heat input or output, their temperature ranges are less than for soils with low thermal diffusivities.

4.4 Theory of Soil Heat Transfer

Here we consider a uniform conducting medium (soil), with heat flowing only in the vertical direction. Let us consider the energy budget of an elemental volume consisting of a cylinder of horizontal cross-section area ΔA and depth Δz, bounded between the levels z and $z + \Delta z$, as shown in Figure 4.3.

Heat flow in the volume at depth $z = H\Delta A$.

Heat flow out of the volume at $z + \Delta z = [H + (\partial H/\partial z)\,\Delta z]\Delta A$.

The net rate of heat flow in the control volume $= -(\partial H/\partial z)\Delta z\,\Delta A$.

The rate of change of internal energy within the control volume $= (\partial/\partial t)(\Delta A \Delta z C_s T)$, where C_s is the volumetric heat capacity of soil.

According to the law of conservation of energy, if there are no sources or sinks of energy within the elemental volume, the net rate of heat flowing in the volume should equal the rate of change of internal energy in the volume, so that

$$(\partial/\partial t)(C_s T) = -\partial H/\partial z \tag{4.2}$$

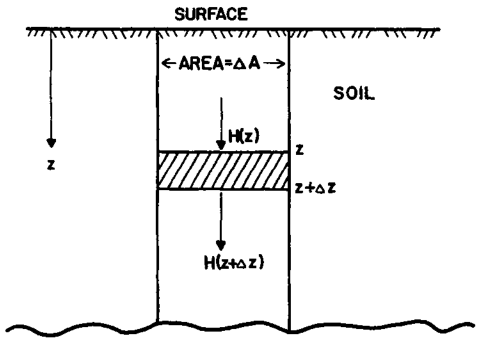

Figure 4.3 Schematic of heat transfer in a vertical column of soil below a flat, horizontal surface.

Further, assuming that the heat capacity of the medium does not vary with time, and substituting from Equation (4.1) into (4.2), we obtain Fourier's equation of heat conduction:

$$\partial T/\partial t = (\partial/\partial z)\,[(k/C_s)(\partial T/\partial z)] = (\partial/\partial z)[\alpha_h(\partial T/\partial z)] \tag{4.3}$$

The one-dimensional heat conduction equation derived here can easily be generalized to three dimensions by considering the net rate of heat flow in an elementary control volume $\Delta x \Delta y \Delta z$ from all the directions. In our applications involving heat transfer through soils, however, we will be primarily concerned with the one-dimensional Equation (4.2) or (4.3).

Equation (4.2) can be used to determine the ground heat flux H_G in the energy balance equation from measurements of soil temperatures as functions of time, at various depths below the surface. The method is based on the integration of Equation (4.2) from $z = 0$ to D

$$H_G = H_D + \int_0^D \frac{\partial}{\partial t}(C_s T)\, dz \tag{4.4}$$

where D is some reference depth where the soil heat flux H_D is either zero (e.g., if at $z = D$, $\partial T/\partial z = 0$) or can be easily estimated [e.g., using Equation (4.1)]. The former is preferable whenever feasible, because it does not require a knowledge of thermal conductivity, which is more difficult to measure than heat capacity.

4.5 Thermal Wave Propagation in Soils

The solution of Equation (4.3), with given initial and boundary conditions, is used to study theoretically the propagation of thermal waves in soils and other substrata. For any arbitrary prescription of surface temperature as a function of time and soil layers, the solution to Equation (4.3) can be obtained numerically. Much about the physics of thermal wave propagation can be learned, however, from a simple analytic solution which is obtained when the surface temperature is specified as a sinusoidal function of time and the subsurface medium is assumed to be homogeneous throughout the depth of wave propagation

$$T_s = T_m + A_s \sin[(2\pi/P)(t - t_m)] \tag{4.5}$$

Here, T_m is the mean temperature of the surface or submedium, A_s and P are the amplitude and period of the surface temperature wave, and t_m is the time when $T_s = T_m$, as the surface temperature is rising.

The solution of Equation (4.3) satisfying the boundary conditions that at $z = 0$, $T = T_s(t)$, and as $z \to \infty$, $T \to T_m$, is given by

$$T = T_m + A_s \exp(-z/d) \sin[(2\pi/P)(t - t_m) - z/d] \tag{4.6}$$

which the reader may verify by substituting in Equation (4.3). Here, d is the damping depth of the thermal wave, defined as

$$d = (P\alpha_h/\pi)^{1/2} \tag{4.7}$$

Note that the period of thermal wave in the soil remains unchanged, while its amplitude decreases exponentially with depth ($A = A_s \exp(-z/d)$); at $z = d$ the wave amplitude is reduced to about 37% of its value at the surface and at $z = 3d$ the amplitude decreases to about 5% of the surface value. The phase lag relative to the surface wave increases in proportion to depth (phase lag = z/d), so that there is a complete reversal of the wave phase at $z = \pi d$. The corresponding lag in the time of maximum or minimum in temperature is also proportional to depth (time lag = $zP/2\pi d$).

The results of the above simple theory would be applicable to the propagation of both the diurnal and annual temperature waves through a homogeneous submedium, provided that the thermal diffusivity of the medium remains constant over the whole period, and the surface temperature wave is nearly sinusoidal. For the diurnal period, the latter condition is usually not satisfied especially during the nighttime period when the wave is observed to be more asymmetric around its minimum value (Rosenberg *et al.*, 1983, Chapter 2). Also, disturbed weather conditions with clouds and precipitation are likely to alter the surface temperature wave, as well as thermal diffusivity due to changes in the soil moisture content. For the annual period, the assumption of constant thermal diffusivity may be even more questionable, except for bare soils in arid regions. Note that the damping depth, which is a measure of the extent of thermal wave propagation, for the annual wave is expected to be $\sqrt{365} = 19.1$ times the damping depth for the diurnal wave.

Example Problem 1

Calculate the damping depth and the depth of thermally active soil layer where there is a complete reversal of the phase of the diurnal thermal wave from that at the surface for the following types of soils:

(a) dry sandy soil; (b) saturated sandy soil (40% pore space);
(c) dry clay soil; (d) dry peat soil.

What will be the amplitude of the thermal wave at the depth of the phase reversal relative to that at the surface?

Solution

Using the thermal diffusivities given in Table 4.1, Equation (4.7) for damping depth d, and the depth of phase reversal as πd, one can obtain the following results:

(a) For dry sandy soil, $\alpha_h = 0.24 \times 10^{-6}\ m^2\ s^{-1}$.
Damping depth $d = (24 \times 3600 \times 0.24 \times 10^{-6}/\pi)^{1/2} = 0.081$ m.
Depth of phase reversal $= \pi d = 0.25$ m.
(b) For saturated sandy soil, $\alpha_h = 0.74 \times 10^{-6}\ m^2\ s^{-1}$.
Damping depth $d = 0.143$ m.
Depth of phase reversal $= \pi d = 0.45$ m.
(c) For dry clay soil, $\alpha_h = 0.18 \times 10^{-6}\ m^2\ s^{-1}$.
Damping depth $d = 0.070$ m.
Depth of phase reversal $= \pi d = 0.22$ m.
(d) For dry peat soil, $\alpha_h = 0.10 \times 10^{-6}\ m^2\ s^{-1}$.
Damping depth $d = 0.052$ m.
Depth of phase reversal $= \pi d = 0.16$ m.

In all the cases the amplitude of the thermal wave at $z = \pi d$, relative to that at the surface, is

$$A/A_s = \exp(-\pi\ d/d) = \exp(-\pi) \cong 0.042$$

One can conclude that dry peat soil offers the maximum resistance (minimum d) and saturated sandy soil the minimum resistance (maximum d) to the propagation of the diurnal thermal wave.

The observed temperature waves (Figures 4.1 and 4.2) in bare, dry soils are found to conform well to the pattern predicted by the theory. In particular, the observed annual waves show nearly perfect sinusoidal forms. The plots of wave amplitude (on log scale) and phase or time lag as functions of depth (both on linear scale) are well represented by straight lines (Figure 4.4) whose slopes determine the damping depth and, hence, thermal diffusivity of the soil.

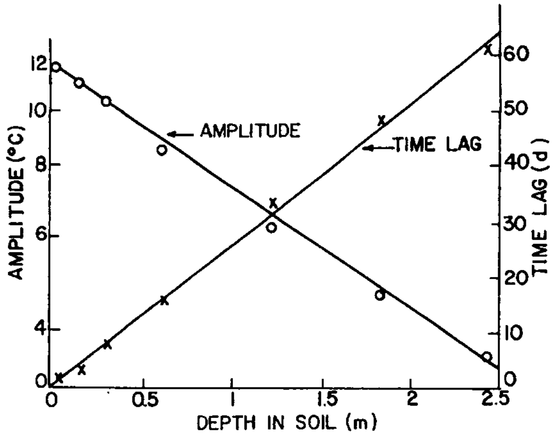

Figure 4.4 Variations of amplitude and time lag of the annual soil temperature waves with depth in the soil. [From Deacon (1969).]

Example Problem 2

Using the weekly averaged data on soil temperatures at different depths obtained by West (1952), the amplitude and time lag of thermal waves are plotted as functions of depth in soil in Figure 4.4. Estimate the damping depth and thermal diffusivity of the soil from the best-fitted lines through the data points.

Solution

Note that according to Equation (4.6), the amplitude of thermal wave decreases exponentially with depth, i.e.,

$$A = A_s \exp(-z/d)$$

or

$$\ln A = \ln A_s - \frac{z}{d}$$

Therefore, a plot of ln A (A on a log scale) against z should result in a straight line with a slope of $-1/d$. The slope of the best-fitted straight line through the amplitude data in Figure 4.4 can be estimated, from which $d \cong 2.05$ m.

Thermal diffusivity $\alpha_h = \pi d^2/P \cong 0.42 \times 10^{-6}$ m^2 s^{-1}.

Also, according to the thermal wave equation (Equation 4.6),

$$\text{Time lag} = Pz/2\pi d$$

Therefore, a plot of time lag versus z should also result in a straight line with a slope of $P/2\pi d$. The slope of the best-fitted line throughout the time lag data points in Figure 4.4 can be estimated as 25.6 days m^{-1}, from which

$$d = P/(2\pi \times \text{slope}) \cong 2.27 \text{ m}$$

$$\alpha_h = \pi d^2/P \cong 0.51 \times 10^{-6} \text{ m}^2 \text{ s}^{-1}$$

The two estimates of d, based on amplitude and time lag data, are in fairly good agreement. The agreement between the estimated damping depths based on the top 0.30 m of the soil temperature data for the diurnal period is found to be much better (Deacon, 1969).

4.6 Measurement and Parameterization of Ground Heat Flux

4.6.1 Measurement by heat flux plate

There is no direct method of measuring the ground heat flux H_G (the heat flux through soil at the surface). Theoretically, the heat conduction equation [Equation (4.1a)] is valid right up to the surface. In practice, however, the difficulty of accurately measuring the near-surface soil temperature gradient and soil thermal conductivity renders this approach impractical. Even at a small depth below the surface, where the soil temperature gradient can be accurately measured, thermal conductivity of the soil is highly variable and not easy to measure. However, heat flux plates are often used to measure the soil heat flux directly. The plate consists of material of known thermal conductivity, and temperature difference across its lower and upper faces is measured by a differential thermopile. The faces are often covered by thin metal plates in order to protect them and to ensure good thermal contact with the soil. The heat flux plate should be small, so that it does not disrupt the soil and has a low heat capacity for quick response to any changes with time. In a field experiment, several heat flux plates may be buried horizontally in the soil at a depth of 10–50 mm. Thus, the average soil heat flux H_1 at a small depth z_1 can be measured.

In order to estimate H_G from the measurement of H_1, one can use Equation (4.4) with $D = z_1$ so that

$$H_G = H_1 + \int_0^{z_1} \frac{\partial}{\partial t}(C_s T)dz = H_1 + \Delta H_s \tag{4.8}$$

where ΔH_s is the rate of heat storage in the top soil layer of depth z_1. For the thin soil layer, the above heat storage term will be significant only when there is rapid warming or cooling of the top soil layer (e.g., around the times of sunrise and sunset). The average warming or cooling rate can be determined from the continuous measurement of soil temperature as a function of time in the middle of the layer.

An indirect method of estimating H_G from the measurement of soil temperatures at various depths, at regular time intervals, has already been discussed in Section 4.4. It is based on the application of Equation (4.4) in which the last term represents the rate of heat storage in a deeper layer of depth D. The soil heat flux at the bottom of this layer is also estimated, rather than measured with a heat flux plate.

4.6.2 Simple parameterizations

In some climate and general circulation models, the ground heat flux is simply neglected. The rationale for this is provided by the observation that, averaged over the entire diurnal cycle, the net ground heat flux is often near zero. This is because the heating of the thermally active soil layer during the day is approximately balanced by the cooling at night. For shorter periods used in micrometeorology, however, H_G is a significant component of the energy balance near the surface, especially at night. A simple parameterization of the same, discussed in Chapter 2, is to specify H_G as a constant fraction of the net radiation R_N during the day and a different (larger) fraction at night. The actual values of the ratio H_G/R_N for the day and night periods depend on the surface and soil characteristics and are determined empirically.

Alternatively, the ground heat flux can also be assumed as a constant fraction of the sensible heat flux into air. Both of these simple parameterizations imply that the sign of the ground heat flux is always the same as that of net radiation or sensible heat flux in air. This is not particularly true during the morning and evening transition periods when different energy fluxes change signs at different times. These simple parameterizations are also not applicable over water surfaces.

4.6.3 Soil heat transfer models

An expression for the ground heat flux can be obtained from Equation (4.6) as

$$H_G = -k\left(\frac{\partial T}{\partial z}\right)_{z=0} = (2\pi C_s k/P)^{1/2} A_s \sin\left[\frac{2\pi}{P}(t - t_m) + \frac{\pi}{4}\right] \qquad (4.9)$$

which predicts that the maximum in surface temperature should lag behind the maximum in ground heat flux by $P/8$, or 3 h for the diurnal period and 1.5 months for the annual period. The above equation also indicates that the amplitude of ground heat flux wave is proportional to the square root of the product of heat capacity and thermal conductivity and inversely proportional to the square root of the period; it is also proportional to the amplitude of the surface temperature wave.

The above predicted sinusoidal variation of the ground heat flux is not usually consistent with direct measurements or indirect estimates of H_G from the energy budget equation. In particular, the nighttime variation of H_G with time is much flatter or more nearly uniform (see Munn, 1966, Chapter 5). There are several reasons for this disagreement. First, the assumed sinusoidal variation of the surface temperature in Equation (4.5), which constitutes the upper boundary condition for the solution of Equation (4.3), may not be realistic. Secondly, the simplifying assumption of the homogeneity of the submedium is often not realized. Thermal properties may vary with depth because of the layer structure of the submedium, variations in the soil moisture content, and presence of roots and surface vegetation. Thermal properties may also vary with time in response to irrigation, precipitation, and evaporation. In such cases, numerical models of soil heat and moisture transfer with varying degrees of complication are used, in conjunction with the surface energy balance equation.

4.6.4 Force–restore method

A more widely used parameterization of the energy exchange with the subsurface medium in atmospheric models is based on the two-layer approximation of the submedium, in which a shallow thermally active layer of soil overlies a thick constant-temperature slab. The depth d_g of the near-ground layer is carefully chosen, based on knowledge of the damping depth. The lower layer is assumed to be of sufficient thickness that, over the time scales of interest, the soil heat flux at the bottom is zero. The soil heat flux at $z = d_g$ can be parameterized as

$$H_g = \mu_g(T_g - T_m) \tag{4.10}$$

where T_g and T_m are the layer-averaged temperatures of the upper and lower layers, respectively, and μ_g is a heat transfer coefficient.

The energy balance equation near the surface, including the near-ground soil layer of depth d_g, can be written as

$$R_N = H + H_L + H_g + \Delta H_s \tag{4.11}$$

in which the rate of energy storage can be expressed as

$$\Delta H_s = C_s d_g \frac{\partial T_g}{\partial t} = C_g \frac{\partial T_g}{\partial t} \tag{4.12}$$

where $C_g = C_s d_g$ is the heat capacity per unit area of the near-ground soil layer.

Substituting from Equations (4.10) and (4.12) into (4.11), one obtains a prognostic equation for the temperature of the near-ground layer as

$$C_g \frac{\partial T_g}{\partial t} = (R_N - H - H_L) - \mu_g (T_g - T_m) \tag{4.13}$$

This prognostic equation for the ground temperature is generally referred to as the force–restore method, because the net radiative and atmospheric forcing $(R_N - H - H_L)$ at the surface is modified by the deep soil flux which tends to restore the near-ground temperature to that of deep soil. Note that, if the surface forcing term is removed, the restoring term in Equation (4.13) will cause T_g to move exponentially towards T_m.

Equation (4.13) has simply been extended to predict the diurnal variation of the ground surface temperature as

$$C_g \frac{\partial T_s}{\partial t} = (R_N - H - H_L) = \mu_s (T_s - T_m) \tag{4.14}$$

in which the soil properties are appropriately chosen or specified as (Garratt, 1992, Chapter 8)

$$C_g = C_s (\alpha_h / 2\Omega)^{1/2} \tag{4.15}$$

$$\mu_s = \Omega C_g \tag{4.16}$$

where Ω is the angular rotational speed of the earth. Note that Equation (4.15) with $C_g = C_s d_g$ implies that $d_g = d/2$. Thus, the most appropriate depth of the near-ground layer in the above two-layer model is half of the damping depth for the diurnal period $P = 2\pi/\Omega$.

The above expressions for C_g and μ_s are strictly based on the assumption of a sinusoidal surface temperature and, hence, ground heat flux forcing, as expressed in Equations (4.5) and (4.9), respectively. Since the surface forcing is not really sinusoidal in nature, slightly modified expressions

$$C_g = 0.95 C_s (\alpha_h / 2\Omega)^{1/2} \tag{4.17}$$

$$\mu_s = 1.18 \Omega C_g \tag{4.18}$$

are more commonly used in numerical prediction models of the ground surface temperature (Garratt, 1992).

4.7 Applications

Some of the applications of the knowledge of surface and soil temperatures and soil heat transfer are as follows:

- Study of surface energy budget and radiation balance.
- Prediction of surface temperature and frost conditions.
- Determination of the rate of heat storage or release by the submedium.
- Study of microenvironment of plant cover, including the root zone.
- Environmental design of underground structures.
- Determination of the depth of permafrost zone in high latitudes.

Problems and Exercises

1. You want to measure the temperature of a bare soil surface using fine temperature sensors on a summer day when the sensible heat flux to air might be typically 600 W m^{-2} and the ground heat flux 100 W m^{-2}. Will you do this by extrapolation of measurements near the surface in air or in soil? Give reasons for your choice. Also estimate temperature gradients near the surface in both the air ($k = 0.03$ W m^{-1} K^{-1}) and the soil ($k = 0.30$ W m^{-1} K^{-1}).

2. Discuss the effect of vegetation or plant cover on the diurnal range of surface temperatures, as compared to that over a bare soil surface, and explain why green lawns are much cooler than roads and driveways on sunny afternoons.

3. What is the physical significance of the damping depth in the propagation of thermal waves in a submedium and on what factors does it depend?

4. What are the basic assumptions underlying the simple theory of soil heat transfer presented in this chapter? Discuss the situations in which these assumptions may not be valid.

5. By substituting from Equation (4.6) into (4.3), verify that the former is a solution of Fourier's equation of heat conduction through a homogeneous soil medium, and then, derive the expression [Equation (4.9)] for the ground heat flux.

6. Compare and contrast the alternative methods of determining the ground heat flux from measurements of soil temperatures and thermal properties.

7. The following soil temperatures (°C) were measured during the Great Plains Field Program at O'Neil, Nebraska:

		Depth (m)				
Time (h)	Day	0.025	0.05	0.10	0.20	0.40
0435	August 31	25.54	26.00	26.42	26.32	24.50
0635		24.84	25.30	25.84	25.97	24.48
0835		25.77	25.35	25.42	25.56	24.36
1035		29.42	27.36	25.98	25.39	24.34
1235		33.25	30.32	27.62	25.57	24.27
1435		35.25	32.63	29.52	26.11	24.24
1635		34.84	33.20	30.62	26.88	24.26
1835		32.63	32.05	30.62	27.41	24.32
2035		30.07	30.20	29.91	27.68	24.47
2235		28.42	28.74	28.84	27.57	24.64
0035	September 1	27.09	27.50	27.84	27.22	24.73
0235		26.09	26.60	27.06	26.87	24.78
0435		25.30	25.83	26.40	26.53	24.84

(a) Plot on a graph temperature waves as functions of time and depth.
(b) Plot on a graph the vertical soil temperature profiles at 0435, 0835, 1235, 1635, 2035, and 0035 h.
(c) Determine the damping depth and thermal diffusivity of the soil from the observed amplitudes, as well as from the times of temperature maxima (taken from the smoothed temperature waves) as functions of depth.
(d) Estimate the amplitude of the surface temperature wave and the time of maximum surface temperature from extrapolation of the soil temperature data.

8. From the soil temperature data given in the above problem, calculate the ground heat flux at 0435 and 1635 h, using Equation (4.4) with the measured soil heat capacity of 1.33×10^6 J m^{-3} K^{-1} and the average value of thermal diffusivity determined in Problem 7(c) above.