Chapter 8 | Fundamentals of turbulence

8.1 Instability of Flow and Transition to Turbulence

8.1.1 Types of instabilities

In Chapter 5 we introduced the concept of static (gravitational) stability or instability and showed how the parameter $s = (g/T_v)(\partial\Theta_v/\partial z \cong -(g/\rho)(\partial\rho/\partial z)$ may be used as a measure of static stability of an atmospheric (fluid) layer. This was based on the simple criterion whether vertical motions of fluid parcels were suppressed or enhanced by the buoyancy force arising from density differences between the parcel and the environment. In particular, when $s < 0$, the fluid layer is gravitationally unstable; the parcel moves farther and farther away from its equilibrium position.

Another type of flow instability is the dynamic or hydrodynamic instability. A flow is considered dynamically stable if perturbations introduced in the flow, either intentionally or inadvertently, are found to decay with time or distance in the direction of flow, and eventually are suppressed altogether. It is dynamically unstable if the perturbations grow in time or space and irreversibly alter the nature of the basic flow. A flow may be stable with respect to very small (infinitesimal) perturbations but might become unstable if large (finite) amplitude disturbances are introduced in the flow.

A simple example of dynamically unstable flow is that of two inviscid fluid layers moving initially at different velocities so that there is a sharp discontinuity at the interface. The development of this inviscid shear instability can be demonstrated by assuming a slight perturbation (e.g., in the form of a standing wave) of the interface and the resulting perturbations in pressure (in accordance with the Bernoulli equation $u^2/2 + p/\rho$ = constant) at various points across the interface. The forces induced by pressure perturbations tend to progressively increase the amplitude of the wave, no matter how small it was to begin with, thus irreversibly modifying the interface and the flow in its vicinity (Tritton, 1988, Chapter 17). Shear instability at the interface between two parallel streams of different velocities and densities, with the heavier fluid at the bottom, or of continuously stratified flows is called the Kelvin–Helmholtz (KH) instability.

The KH instability is caused by the destabilizing effect of shear, which overcomes the stabilizing effect of stratification. This type of instability is ubiquitous in the atmosphere and oceans and is considered to be a major source of internal gravity waves in these environments (Kundu, 1990, Chapter 11). In real fluids, of course, the processes leading to the development of mixing layers are more complicated, but the initiation of the Kelvin–Helmholtz instability at the interface is qualitatively similar to that in an inviscid fluid.

Laminar flows in channels, tubes, and boundary layers, introduced in Chapter 7, are found to be dynamically stable (i.e., they can be maintained as laminar flows) only under certain restrictive conditions (e.g., Reynolds number less than some critical value Re_c and a relatively disturbance-free environment), which can be realized only in carefully controlled laboratory facilities. For example, by carefully eliminating all extraneous disturbances, laminar flow in a smooth tube has been realized up to $Re \cong 10^5$, while under ordinary circumstances the critical Reynolds number (based on mean velocity and tube diameter) is only about 2000. Similarly, the critical Reynolds numbers for the instability of laminar channel and boundary layer flows are also of the same order of magnitude. For other flows, such as jets and wakes, instabilities occur at much lower Reynolds numbers. Thus, all laminar flows are expected to become dynamically unstable and, hence, cannot be maintained as such, at sufficiently large Reynolds numbers ($Re > Re_c$) and in the presence of disturbances ordinarily present in the environment (Kundu, 1990, Chapter 11).

The above Reynolds number criterion for dynamic stability does not mean that all inviscid flows are inherently unstable, because zero viscosity implies an infinite Reynolds number. Actually, many nonstratified (uniform density) inviscid flows are found to be stable with respect to small perturbations, unless their velocity profiles have discontinuities or inflexion points. The development of Kelvin–Helmholtz instability along the plane of discontinuity in the velocity profile has already been discussed. Even without a sharp discontinuity, the existence of an inflexion point in the velocity profile constitutes a necessary condition for the occurrence of instability in a neutrally stratified flow; it is also a sufficient condition for the amplification of disturbances. This inviscid instability mechanism also operates in viscous flows having points of inflexion in their velocity profiles. It is for this reason that laminar jets, wakes, mixing layers, and separated boundary layers become unstable at low Reynolds numbers. The laminar Ekman layers are also inherently unstable because their velocity profiles have inflexion points. In all such flows, both the viscous and inviscid shear instability mechanisms operate simultaneously and consequently, the stability criteria are more complex (Kundu, 1990, Chapter 11).

Gravitational instability is another mechanism through which both the inviscid and viscous streamlined flows can become turbulent. Flows with fluid density increasing with height become dynamically unstable, particularly at large Reynolds numbers (Re > Re_c) or Rayleigh numbers (Ra > Ra_c). In the lower atmosphere, this criterion can essentially be expressed in terms of virtual potential temperature decreasing with height, because Reynolds and Rayleigh numbers are much larger than their estimated critical values. Any statically unstable layers, based on either local or nonlocal stability criteria, are also expected to be dynamically unstable and, hence, generally turbulent. The daytime unstable or convective boundary layer is a prime example of gravitational instability.

Stably stratified flows with weak negative density gradients and/or strong velocity gradients can also become dynamically unstable, if the Richardson number is less than its critical value $Ri_c = 0.25$ (this value is based on a number of theoretical as well as experimental investigations). The Richardson number criterion is found to be extremely useful for identifying possible regions of active or incipient turbulence in stably stratified atmospheric and oceanic flows. If the local Richardson number in a certain region is less than 0.25 and there is also an inflexion point in the velocity profile (e.g., in jet streams and mixing layers), the flow there is likely to be turbulent. This criterion is found to be generally valid in stratified boundary layers, as well as in free shear flows, and is often used in forecasts of clear-air turbulence in the upper troposphere and lower stratosphere for aircraft operations. Dynamic instability promotes the growth of wave-like disturbances which may break down to produce turbulence. The instability-generated wave motion may also become dynamically stable and not always result in a turbulent flow. The nighttime stable boundary layer is a prime example of the shear instability of a stratified flow.

8.1.2 Methods of investigation

The stability of a physically realizable laminar flow can be investigated experimentally and the approximate stability criteria can be determined empirically. This approach has been used by experimental fluid dynamists ever since the classical experiments in smooth glass tubes carried out by Osborne Reynolds. Systematic and carefully controlled experiments are required for this purpose (Monin and Yaglom, 1971, Chapter 1). Empirical observations to test the validity of the Richardson number criterion have also been made by meteorologists and oceanographers.

Another, more frequently used approach is theoretical, in which small-amplitude wavelike perturbations are superimposed on the original laminar or inviscid flow and the conditions of stability or instability are examined through analytical or numerical solutions of perturbation equations (Monin and Yaglom, 1971, Chapter 1; Kundu, 1990, Chapter 11). The analysis is straightforward and simple, so long as the amplitudes of perturbations are small enough for the nonlinear terms in the equations to be dropped out or linearized. It becomes extremely complicated and even intractable in the case of finite-amplitude perturbations. Thus, most theoretical investigations give only some weak criteria of the stability of flow with respect to small-amplitude (in theory, infinitesimal) perturbations. Investigations of flow stability with respect to finite-amplitude perturbations are being attempted now using new approaches to the numerical solution of nonlinear perturbation equations on large computers. Comprehensive reviews of the literature on stability analyses of viscous and inviscid flows are given by Chandrasekhar (1961), Monin and Yaglom (1971), and Drazin and Reid (1981).

The importance of stability analysis arises from the fact that any solution to the equations of motion, whether exact or approximate, cannot be considered physically realizable unless it can be shown that the flow represented by the solution is also stable with respect to small perturbations. If the basic flow is found to be unstable in a certain range of parameters, it cannot be realized under those conditions, because naturally occurring infinitesimal disturbances will cause instabilities to develop and alter the flow field irreversibly.

8.1.3 Transition to turbulence

Experiments in controlled laboratory environments have shown that several distinctive stages are involved in the transition from laminar flow (e.g., the flat-plate boundary layer) to turbulence. The initial stage is the development of primary instability which, in simple cases, may be two-dimensional. The primary instability produces secondary motions which are generally three-dimensional and become unstable themselves. The subsequent stages are the amplification of three-dimensional waves, the development of intense shear layers, and the generation of high-frequency fluctuations. Finally, 'turbulent spots' appear more or less randomly in space and time, grow rapidly, and merge with each other to form a field of well-developed turbulence. The above stages are easy to identify in a flat-plate boundary layer and other developing flows, because changes occur as a function of increasing Reynolds number with distance from the leading edge. In fully developed two-dimensional channel and pipe flows, however, transition to turbulence occurs more suddenly and explosively over the whole length of the channel or pipe as the Reynolds number is increased beyond its critical value.

Mathematically, the details of transition from initially laminar or inviscid flow to turbulence are rather poorly understood. Much of the theory is linearized, valid for small disturbances, and cannot be used beyond the initial stages. Even the most advanced nonlinear theories dealing with finite-amplitude disturbances cannot handle the later stages of transition, such as the development of 'turbulent spots.' A rigorous mathematical treatment of transition from turbulent to laminar flow is also lacking, of course, due to the lack of a generally valid and rigorous theory or model of turbulence.

Some of the turbulent flows encountered in nature and technology may not go through a transition of the type described above and are produced as such (turbulent). Flows in ordinary pipes, channels, and rivers, as well as in atmospheric and oceanic boundary layers, are some of the commonly occurring examples of such flows. Other cases, such as clear air turbulence in the upper atmosphere and patches of turbulence in the stratified ocean, are obviously the results of instability and transition processes. Transitions from laminar flow to turbulence and vice versa continuously occur in the upper parts of the stable boundary layer, throughout the night. Micrometeorologists have a deep and abiding interest in understanding these transition processes, because turbulent exchanges of momentum, heat and mass in the SBL and other stable layers occur intermittently during episodes of turbulence generation.

8.2 The Generation and Maintenance of Turbulence

It is not easy to identify the origin of turbulence. In an initially nonturbulent flow, the onset of turbulence may occur suddenly through a breakdown of streamlined flow in certain localized regions (turbulent spots). The cause of the breakdown, as pointed out in the previous section, is an instability mechanism acting upon the naturally occurring disturbances in the flow. Since turbulence is transported downstream in the manner of any other fluid property, repeated breakdowns are required to maintain a continuous supply of turbulence, and instability is an essential part of this process.

Once turbulence is generated and becomes fully developed in the sense that its statistical properties achieve a steady state, the instability mechanism is no longer required (although it may still be operating) to sustain the flow. This is particularly true in the case of shear flows, where shear provides an efficient mechanism for converting mean flow energy to turbulence kinetic energy (TKE). Similarly, in unstably stratified flows, buoyancy provides a mechanism for converting potential energy of stratification into turbulence kinetic energy (the reverse occurs in stably stratified flows). The two mechanisms for turbulence generation become more evident from the so-called shear production (S) and buoyancy production (B) terms in the TKE equation

$$d(\mathrm{TKE})/dt = S + B - D + T_r \tag{8.1}$$

which is written here in a short-form notation for the various terms; a more complete version is given later in Chapter 9. The TKE equation also shows that there is a continuous dissipation (D) of energy by viscosity in any turbulent flow and there may be transport (T_r) of energy from or to other regions of flow. Thus, in order to maintain turbulence, one or more of the generating mechanisms must be active continuously. For example, in the daytime unstable or convective boundary layer, turbulence is produced both by shear and buoyancy (the relative contribution of each depends on Richardson number). Shear is the only effective mechanism for producing turbulence in the nocturnal stable boundary layer, in low-level jets, and in regions of the upper troposphere and stratosphere where negative buoyancy actually suppresses turbulence. Near the earth's surface wind shears become particularly intense and effective, because wind speed must vanish at the surface and air flow has to go around the various surface inhomogeneities. For this reason, shear-generated turbulence is always present in the atmospheric surface layer.

Richardson (1920) originally proposed a particularly simple condition for the maintenance of turbulence in stably stratified flows, viz., that the rate of production of turbulence by shear must be equal to or greater than the rate of destruction by buoyancy. That is, the Richardson number, which turns out to be the ratio of the buoyancy destruction to shear production, must be smaller than or equal to unity. An important assumption implied in deriving this condition was that there is no viscous dissipation of energy on reaching the critical condition for the sudden decay of turbulence. On the other hand, subsequent observations of the transition from turbulent to laminar flow have indicated viscous dissipation to remain significant; they also suggest a much lower value of the critical Richardson number ($\mathrm{Ri_c}$), in the range of 0.2 to 0.5 [it has not been possible to determine $\mathrm{Ri_c}$ very precisely (see Arya, 1972)]. There is no requirement that this should be the same as the critical Richardson number ($\mathrm{Ri_c} = 1/4$) for the transition from laminar to turbulent flow, which has been determined more rigorously and precisely from theoretical stability analyses. There is some observational evidence that the critical Richardson number for the transition from turbulent to laminar flow increases with height in the SBL and may be close to one at the top of the SBL.

8.3 General Characteristics of Turbulence

Even though turbulence is a rather familiar notion, it is not easy to define precisely. In simplistic terms, we refer to very irregular and chaotic motions as turbulent. But some wave motions (e.g., on an open sea surface) can be very irregular and nearly chaotic, but they are not turbulent. Perhaps it would be more appropriate to mention some general characteristics of turbulence.

1. *Irregularity or randomness.* This makes any turbulent motion essentially unpredictable. No matter how carefully the conditions of an experiment are reproduced, each realization of the flow is different and cannot be predicted in detail. The same is true of the numerical simulations (based on Navier–Stokes equations), which are found to be highly sensitive to even minute changes in initial and boundary conditions. For this reason, a statistical description of turbulence is invariably used in practice.
2. *Three-dimensionality and rotationality.* The velocity field in any turbulent flow is three-dimensional (we are excluding here the so-called two-dimensional or geostrophic turbulence, which includes all large-scale atmospheric motions) and highly variable in time and space. Consequently, the vorticity field is also three-dimensional and flow is highly rotational.
3. *Diffusivity or ability to mix properties.* This is probably the most important property, so far as applications are concerned. It is responsible for the efficient diffusion of momentum, heat, and mass (e.g., water vapor, CO_2, and various pollutants) in turbulent flows. Macroscale diffusivity of turbulence is usually many orders of magnitude larger than the molecular diffusivity. The former is a property of the flow while the latter is a property of the fluid. Turbulent diffusivity is largely responsible for the evaporation in the atmosphere, as well as for the spread (dispersion) of pollutants released in the atmospheric boundary layer. It is also responsible for the increased frictional resistance of fluids in pipes and channels, around aircraft and ships, and on the earth's surface.
4. *Dissipativeness.* The kinetic energy of turbulent motion is continuously dissipated (converted into internal energy or heat) by viscosity. Therefore, in order to maintain turbulent motion, the energy has to be supplied continuously. If no energy is supplied, turbulence decays rapidly.
5. *Multiplicity of scales of motion.* All turbulent flows are characterized by a wide range (depending on the Reynolds number) of scales or eddies. The transfer of energy from the mean flow into turbulence occurs at the upper end of scales (large eddies), while the viscous dissipation of turbulent energy occurs at the lower end (small eddies). Consequently, there is a continuous transfer of energy from the largest to the smallest scales. Actually, it trickles down through the whole spectrum of scales or eddies in the form of a cascade process. The energy transfer processes in turbulent flows are highly nonlinear and are not well understood.

Of the above characteristics, rotationality, diffusivity, and dissipativeness are the properties which distinguish a three-dimensional random wave motion from turbulence. The wave motion is nearly irrotational, nondiffusive, and nondissipative.

8.4 Mean and Fluctuating Variables

In a turbulent flow, such as the PBL, velocity, temperature and other variables vary irregularly in time and space; it is therefore common practice to consider these variables as sums of mean and fluctuating parts, e.g.,

$$\begin{aligned}\tilde{u} &= U + u \\ \tilde{v} &= V + v \\ \tilde{w} &= W + w \\ \tilde{\theta} &= \Theta + \theta\end{aligned} \tag{8.2}$$

in which the left-hand side represents an instantaneous variable (denoted by a tilde) and the right-hand side its mean (denoted by a capital letter) and fluctuating (denoted by a lower case letter) parts. The decomposition of an instantaneous variable in terms of its mean and fluctuating parts is called Reynolds decomposition, because it was first proposed by Reynolds (1894).

There are several types of means or averages used in theory and practice. The most commonly used in the analysis of observations from fixed instruments is the time mean, which, for a continuous record (time series), can be defined as

$$F = \frac{1}{T}\int_0^T \tilde{f}(t)dt$$

where $\tilde{f}$ is any variable or a function of variables, F is the corresponding time mean, and T is the length of record or sampling time over which averaging is desired. For the digitized data, mean is simply an arithmetic average of all the digitized values of the variable during the chosen sampling period. In certain flows, such as the atmospheric boundary layer, the choice of an optimum sampling time or the averaging period T is not always clear. It should be sufficiently long to ensure stable averages and to incorporate the effects of all the significantly contributing large eddies in the flow. On the other hand, T should not be too long to mask what may be considered as real trends (e.g., the diurnal variations) in the flow. In the analysis of micrometeorological observations, the optimum averaging time may range between 10^3 and 10^4 s, depending on the height of observation, the PBL height, and stability. Time averages are frequently used for micrometeorological variables measured by instruments placed on the ground or mounted on masts, towers, or tethered balloons.

Another type of averaging used, particularly in the analyses of aircraft, radar, and sodar observations, is the spatial average of all the observations over the covered space. The optimum length, area, or volume over which the spatial averaging is performed depends on the spatial scales of significant large eddies in the flow. Horizontal line and area averages are more meaningful, considering the inhomogeneity of the PBL in the vertical due to shear and buoyancy effects.

Finally, the type of averaging which is almost always used in theory, but rarely in practice, is the ensemble or probability mean. It is an arithmatical average of a very large (approaching infinity) number of realizations of a variable or a function of variables, which are obtained by repeating the experiment over and over again under the same general conditions. It is quite obvious that the ensemble averages would be nearly impossible to obtain under the varying weather conditions in the atmosphere, over which we have little or no control. Even in a controlled laboratory environment, it would be very time consuming to repeat an experiment many times to obtain ensemble averages.

The fact that different types of averages are used in theory and experiments requires that we should know about the conditions in which time or space averages might become equivalent to ensemble averages. It has been found that the necessary and sufficient conditions for the time and ensemble means to be equal are that the process (in our case, flow) be stationary (i.e., the averages be independent of time) and the averaging period be very large ($T \to \infty$) (for a more detailed discussion of these conditions, see Monin and Yaglom, 1971). It should suffice to say that these conditions cannot be strictly satisfied in the atmosphere so that the above-mentioned equivalence of averages can only be approximate. The corresponding conditions for the equivalence of spatial and ensemble averages are spatial homogeneity (independence of averages to spatial coordinates in one or more directions) and very large averaging paths or areas. These conditions are even more restrictive and difficult to satisfy in micro-meteorological applications. Quasi-stationarity over a limited period of time, or quasi-homogeneity over a limited length or area in the horizontal plane, is the best one can hope for in an idealized PBL over a homogeneous surface under undisturbed weather conditions. Under such conditions, one might expect an approximate correspondence between time or space averages used in experiments and observations and ensemble averages used in theories and mathematical models of the PBL.

Irrespective of the type of averaging used, it is obvious from Reynolds decomposition that a fluctuation is the deviation of an instantaneous variable from its mean. By definition, then, the mean of any fluctuating variable is zero ($\bar{u} = 0$, $\bar{v} = 0$, etc.), so that there are compensating negative fluctuations for positive fluctuations on the average. Here, an overbar over a variable denotes its mean.

The relative magnitudes of mean and fluctuating parts of variables in the atmospheric boundary layer depend on the type of variable, observation height relative to the PBL height, atmospheric stability, the type of surface, and other factors. In the surface layer, during undisturbed weather conditions, the magnitudes of vertical fluctuations, on the average, are much larger than the mean vertical velocity, the magnitudes of horizontal velocity fluctuations are of the same order or less than the mean horizontal velocity, and the magnitudes of the fluctuations in thermodynamic variables are at least two orders of magnitude smaller than their mean values. The relative magnitudes of turbulent fluctuations generally decrease with increasing stability and also with increasing height in the PBL. As an example, measured time series of velocity, temperature, and humidity fluctuations at a suburban site in Vancouver, Canada, during the daytime moderately unstable conditions are shown in Figure 8.1. These observations were taken at a height of 27.4 m above ground level, which falls within the unstable surface layer. The corresponding hourly averaged values of the variables were:

$$U = 3.66\,\mathrm{m\,s^{-1}};\; V = W = 0$$

$$T = 294.6\,\mathrm{K};\; Q = 8.3\,\mathrm{g\,m^{-3}}$$

Note the difference in the character of traces of horizontal and vertical velocity components, temperature, and absolute humidity fluctuations. Under unstable and convective conditions represented by these traces, buoyant plumes and thermals cause strong asymmetry between positive and negative

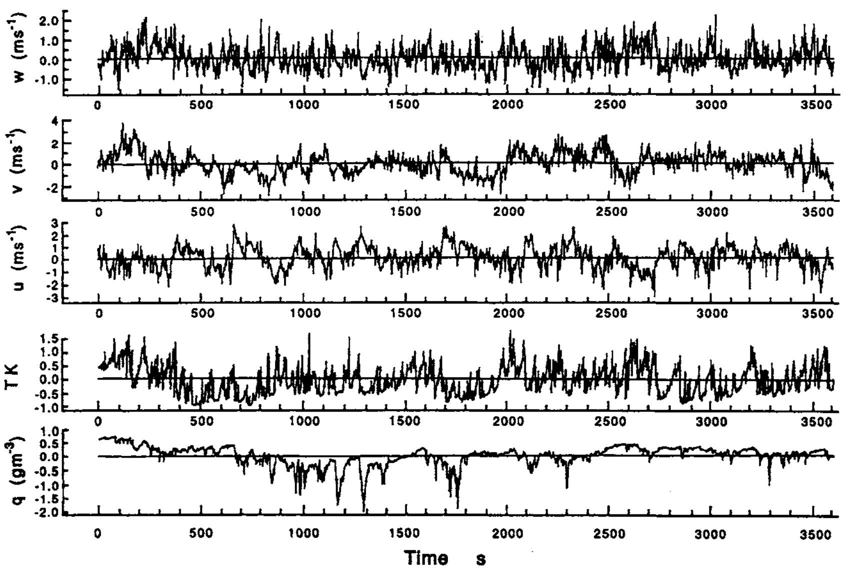

Figure 8.1 Observed time series of velocity, temperature, and absolute humidity fluctuations in the atmospheric surface layer at a suburban site in Vancouver, Canada, during moderately unstable conditions. [From Roth (1991).]

fluctuations, particularly in vertical velocity, temperature, and humidity. Under strong winds and near-neutral stability, on the other hand, fluctuations of the various variables tend to be more or less similar, with an approximate symmetry between positive and negative fluctuations.

8.5 Variances and Turbulent Fluxes

The vertical profiles of mean variables, such as mean velocity components, temperature, etc., can tell much about the mean structure of flow, but little or nothing about the turbulent structure and the various exchange processes taking place in the flow. Various statistical measures are used to study and represent the turbulence structure. These are all based on statistical analyses of turbulent fluctuations observed in the flow.

The simplest measures of fluctuation levels are the variances $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$ [these three are often combined to define the turbulence kinetic energy per unit mass $(1/2)(\overline{u^2} + \overline{v^2} + \overline{w^2})$], $\overline{\theta^2}$, etc., and standard deviations $\sigma_u = (\overline{u^2})^{1/2}$, etc. The ratios of standard deviations of velocity fluctuations to the mean wind speed are called turbulence intensities (e.g., $i_u = \sigma_u/|\mathbf{V}|$, $i_v = \sigma_v/|\mathbf{V}|$ and $i_w = \sigma_w/|\mathbf{V}|$), which are measures of relative fluctuation levels in different directions (velocity components). Note that the mean wind speed rather than the particular component velocity, is used in the definition of turbulence intensities. When the x axis is oriented with the mean wind, i_u, i_v, and i_w are designated as longitudinal, lateral, and vertical turbulence intensities, respectively.

Observations indicate that turbulence intensities are typically less than 10% in the nocturnal boundary layer, 10–15% in a near-neutral surface layer, and greater than 15% in unstable and convective boundary layers. Turbulence intensities are generally largest near the surface and decrease with height in stable and near-neutral boundary layers. Under convective conditions, however, the secondary maxima in turbulence intensities often occur at the inversion base and sometimes in the middle of the CBL.

Even more important in turbulent flows are the covariances $\overline{uw}$, $\overline{vu}$, $\overline{\theta w}$, etc., which are directly related to and sometimes referred to as turbulent fluxes of momentum, heat, etc. As covariances, they are averages of products of two fluctuating variables and depend on the correlations between the variables involved. These can be positive, negative, or zero, depending on the type of flow and symmetry conditions. For example, if u and w are the velocity fluctuations in the x and z directions, respectively, their product uw will also be a fluctuating quantity but with a nonzero mean; $\overline{uw}$ will be positive if u and w are positively correlated, and negative if the two variables are negatively correlated. A casual inspection of u and w time series in Figure 8.1 indicates that these variables are negatively correlated, so that $\overline{uw} < 0$.

A better measure of the correlation between two variables is provided by the correlation coefficient

$$r_{uw} = \overline{uw}/\sigma_u \sigma_w \tag{8.3}$$

whose value always lies between -1 and 1, according to Schwartz's inequality $|\overline{uw}| \leqslant \sigma_u \sigma_w$. Similarly, one can define correlation coefficients for all the covariances to get an idea of how well correlated different variables are in a turbulent flow. For example, observations in Figure 8.1 indicate that temperature fluctuations are well correlated with vertical velocity fluctuations with $r_{w\theta} \cong 0.6$. There is still some order to be found in an apparently chaotic motion which is responsible for all the important exchange processes taking place in the flow.

In order to see a clear connection between covariances and turbulent fluxes, let us consider the vertical flux of a scalar with a variable concentration $\tilde{c}$ in the flow. The flux of any scalar in a given direction is defined as the amount of the scalar per unit time per unit area normal to that direction. It is quite obvious that the velocity component in the direction of flux is responsible for the transport and hence for the flux. For example, it is easy to see that in the vertical direction the flux at any instant is $\tilde{c}\tilde{w}$ and the mean flux with which we are normally concerned is $\overline{\tilde{c}\tilde{w}}$. Further writing $\tilde{c} = C + c$ and $\tilde{w} = W + w$ and following the Reynolds averaging rules, it can be shown that

$$\overline{\tilde{c}\tilde{w}} = \overline{(C + c)(W + w)} = CW + \overline{cw} \tag{8.4}$$

Thus, the total scalar flux can be represented as a sum of the mean transport (transport by mean motion) and the turbulent transport. The latter, also called the turbulent flux, is usually the dominant transport term and always of considerable importance in a turbulent flow. If the scalar under consideration is the potential temperature $\tilde{\theta}$, or enthalpy $\rho c_p \tilde{\theta}$, the corresponding turbulent flux in the vertical is $\overline{\theta w}$, or $\rho c_p \overline{\theta w}$. In this way, the covariance $\overline{\theta w}$ may be interpreted as the turbulent heat flux (in kinematic units). Similarly, $\overline{qw}$ may be considered as the vertical turbulent flux of moisture or water vapor.

Turbulent fluxes of momentum involve covariances between velocity fluctuations in different directions. For example, by symmetry, $\rho\overline{uw}$ may be considered as the vertical flux of u momentum and also as the horizontal (x direction) flux of the vertical momentum. Since the rate of change (flux) of momentum is equal to the force per unit area, or the stress, turbulent fluxes of momentum can also be interpreted as turbulent stresses, that is,

$$\bar{\tau}_{zx} = -\rho\overline{wu}, \quad \bar{\tau}_{zy} = -\rho\overline{wv}, \text{ etc.} \tag{8.5}$$

Like the viscous stresses represented in Equation (7.2), the turbulent stresses above are designated by two direction subscripts. The first subscript to $\bar{\tau}$ denotes the direction normal to the plane of the stress and the second subscript denotes the direction of the stress. There are, in all, nine stress components acting on any cubical fluid element, of which three ($\bar{\tau}_{xx}$, $\bar{\tau}_{yy}$, and $\bar{\tau}_{zz}$) are normal stresses, which are proportional to velocity variances, and six ($\bar{\tau}_{xy}$, $\bar{\tau}_{xz}$, etc.) are shearing stresses which are proportional to covariances. When expressed in terms of variances and covariances of turbulent velocity fluctuations, these are called Reynolds stresses. These turbulent stresses are found to be much larger in magnitude than the corresponding mean viscous stresses

$$T_{zx} = \mu\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right), \quad T_{zy} = \mu\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right), \text{ etc.} \tag{8.6}$$

These viscous stresses can usually be ignored, except within extremely thin viscous sublayers near smooth surfaces.

Example Problem 1

The following mean flow and turbulence measurements were made at the 22.6 m height level of the micrometeorological tower during an hour-long run of the 1968 Kansas Field Program:

Mean wind speed $= 4.89$ m s^{-1}.
Velocity variances: $\overline{u^2} = 0.69$, $\overline{v^2} = 1.04$, $\overline{w^2} = 0.42$ m^2 s^{-2}.
Turbulent fluxes: $\overline{uw} = -0.081$ m^2 s^{-2}, $\overline{\theta w} = 0.185$ K m s^{-1},
$\overline{\theta u} = -0.064$ K m s^{-1}.

Calculate the following:
(a) standard deviations of velocity fluctuations;
(b) turbulence intensities;
(c) correlation coefficients;
(d) turbulence kinetic energy (TKE) and the ratio TKE/$\overline{uw}$.

Solution

(a) From the definition, standard deviations are

$$\sigma_u = (\overline{u^2})^{1/2} = 0.83 \text{ m s}^{-1}$$
$$\sigma_v = (\overline{v^2})^{1/2} = 1.02 \text{ m s}^{-1}$$
$$\sigma_w = (\overline{w^2})^{1/2} = 0.64 \text{ m s}^{-1}$$

(b) The corresponding turbulence intensities can be computed as

$$i_u = \sigma_u/|\mathbf{V}| = 0.17$$
$$i_v = \sigma_v/|\mathbf{V}| = 0.21$$
$$i_w = \sigma_w/|\mathbf{V}| = 0.13$$

(c) The correlation coefficients can be calculated as

$$r_{uw} = \overline{uw}/\sigma_u\sigma_w = -0.15$$
$$r_{w\theta} = \overline{\theta w}/\sigma_\theta\sigma_w = 0.59$$
$$r_{u\theta} = \overline{\theta u}/\sigma_\theta\sigma_u = -0.16$$

which indicate that θ and w are positively well correlated, while θ and u are only weakly correlated with a negative correlation coefficient of -0.16 in the convective surface layer.

(d) Turbulence kinetic energy can be calculated as

$$\text{TKE} = \tfrac{1}{2}(\overline{u^2} + \overline{v^2} + \overline{w^2}) = 1.075 \text{ m}^2 \text{ s}^{-2}$$
$$\text{TKE}/\overline{uw} = -13.27$$

8.6 Eddies and Scales of Motion

It is a common practice to speak in terms of eddies when turbulence is described qualitatively. An eddy is by no means a clearly defined structure or feature of the flow which can be isolated and followed through, in order to study its behavior. It is rather an abstract concept used mainly for qualitative descriptions of turbulence. An eddy may be considered akin to a vortex or a whirl in common terminology. Turbulent flows are highly rotational and have all kinds of vortexlike structures (eddies) buried in them. However, eddies are not simple two-dimensional circulatory motions of the type in an isolated vortex, but are believed to be complex, three-dimensional structures. Any analogy between turbulent eddies and vortices can only be very rough and qualitative.

On the basis of flow visualization studies, statistical analyses of turbulence data, and some of the well-accepted theoretical ideas, it is believed that a turbulent flow consists of a hierarchy of eddies of a wide range of sizes (length scales), from the smallest that can survive the dissipative action of viscosity to the largest that is allowed by the flow geometry. The range of eddy sizes increases with the Reynolds number of the overall mean flow. In particular, for the ABL, the typical range of eddy sizes is 10^{-3} to 10^3 m.

Of the wide and continuous range of scales in a turbulent flow, a few have special significance and are used to characterize the flow itself. One is the characteristic large-eddy scale or macroscale of turbulence, which represents the length (l) or time scale of eddies receiving the most energy from the mean flow. Another is the characteristic small-eddy scale or microscale of turbulence, which represents the length (η) or time scale of most dissipating eddies. The ratio l/η is found to be proportional to $\mathrm{Re}^{3/4}$ and is typically 10^5 (range: 10^4–10^6) for the atmospheric PBL.

The macroscale, or simply the scale of turbulence, is generally comparable (same order of magnitude) to the characteristic scale of the mean flow, such as the boundary layer thickness or channel depth, and does not depend on the molecular properties of the fluid. On the other hand, the microscale depends on the fluid viscosity ν, as well as on the rate of energy dissipation ε. Dimensional considerations further lead to the defining relationship

$$\eta \equiv \nu^{3/4}\varepsilon^{-1/4} \tag{8.7}$$

In stationary or steady-state conditions, the rate at which the energy is dissipated is exactly equal to the rate at which the energy is supplied from mean flow into turbulence. This leads to an inviscid estimate for the rate of energy dissipation $\varepsilon \sim u_\ell^3/l$, where u_ℓ is the characteristic velocity scale of turbulence (large eddies), which can be defined in terms of the turbulence kinetic energy as

$$u_\ell = \sqrt{\mathrm{TKE}} \tag{8.8}$$

Some investigators define the turbulence velocity scale as the square-root of $(\overline{u^2} + \overline{v^2} + \overline{w^2})$ which is twice the turbulence kinetic energy (Tennekes and Lumley, 1972).

8.7 Fundamental Concepts, Hypotheses, and Theories

Here, we briefly describe, in qualitative terms only, some of the fundamental concepts, hypotheses, and theories of turbulence, which were mostly proposed in the first-half of the twentieth century and subsequently refined and verified by experimental data on turbulence from the laboratory as well as the atmosphere. More comprehensive reviews of theories and models of turbulence are given elsewhere (Monin and Yaglom, 1971; 1975; Hinze, 1975; Panofsky and Dutton, 1984; McComb, 1990).

8.7.1 Energy cascade hypothesis

One of the fundamental concepts of turbulence is that describing the transfer of energy among different scales or eddy sizes. It has been recognized for a long time that in large Reynolds number flows almost all of the energy is supplied by mean flow to large eddies, while almost all of it is eventually dissipated by small eddies. The transfer of energy from large (energy-containing) to small (energy-dissipating) eddies occurs through a cascade-type process involving the whole range of intermediate size eddies. This energy cascade hypothesis was originally suggested by Lewis Richardson in 1922 in the form of a parody

> Big whorls have little whorls,
> Which feed on their velocity;
> And little whorls have lesser whorls,
> And so on to viscosity.

This may be considered to be the qualitative picture of turbulence structure in a nutshell and the concept of energy transfer down the scale. The actual mechanism and quantitative aspects of energy transfer are very complicated and are outside the scope of this text. Smaller eddies are assumed to be created through an instability and breakdown of larger eddies and the energy transfer from larger to smaller eddies presumably occurs during the breakdown process. The largest eddies are produced by mean flow shear and thermal convection. Their characteristic length scale l is of the same order as the characteristic length scale (e.g., the PBL height) of the mean flow, especially for the neutral, unstable, and convective boundary layers. In the stable boundary layer, on the other hand, large-eddy size is found to be severely restricted by negative buoyancy forces, and l is usually much smaller than the PBL height h. The stronger the stability (large Ri), the smaller is the large-eddy length scale, which becomes independent of the PBL height.

If the large-eddy Reynolds number $\mathrm{Re}_\ell = u_\ell l/\nu$ is sufficiently large, these large eddies become dynamically unstable and produce eddies of somewhat smaller size, which themselves become unstable and produce eddies of still smaller size, and so on further down the scale. This cascade process is terminated when the Reynolds number based on the smallest eddy scales becomes small enough (order of one) for the smallest eddies to become stable under the influence of viscosity. This qualitative concept of energy cascade provided an underpinning for subsequent theoretical ideas, as well as experimental studies of turbulence.

8.7.2 Statistical theory of turbulence

A sophisticated mathematical and statistical theory of turbulence has been developed for the simplest type of turbulence, called homogeneous and isotropic turbulence (Hinze, 1975; Monin and Yaglom, 1975). This idealized field of turbulence has no boundaries, no mean flow and shear, and no thermal or density stratification, that may cause deviations from perfect homogeneity and isotropy. Consequently, there are no momentum, heat, and mass fluxes in any direction and the variance of velocity fluctuations is the same in all directions. In the absence of any turbulence production mechanisms (shear and buoyancy), an isotropic turbulence decays with time. Such a turbulence can be approximately realized when a uniform grid is towed through a still fluid, as in a laboratory water channel or a wind tunnel (uniform flow passing through a grid is often used as a surrogate for a homogeneous and isotropic turbulence).

Atmospheric turbulence is not homogeneous and isotropic when one considers its large-scale structure. The presence of mean wind shears and thermal stratification, at least in the vertical direction, are responsible for the generation and maintenance of turbulence. Horizontal inhomogeneities of mean flow and turbulence are often caused by variations of surface roughness, temperature, and topography. It is not easy to extend the rigorous mathematical formulation of the statistical theory to such an inhomogeneous flow and turbulence field. For the horizontally homogeneous PBL, however, approximate formulations including the effects of vertical wind shear and buoyancy have been made, but their numerical solutions still depend on many closure hypotheses and assumptions. In short, theoretical rigor and simplicity are lost, whenever significant deviations from the complete homogeneity of turbulence have to be considered.

8.7.3 Kolmogorov's local similarity theory

Kolmogorov (1941) postulated that at sufficiently large Reynolds numbers, small-scale motions in all turbulent flows have similar universal characteristics. He proposed a local similarity theory to describe these characteristics. The basic foundation of his widely accepted theory lies in the energy cascade hypothesis. Kolmogorov argued that if the characteristic Reynolds number of the mean flow or that of the most energetic large eddies is sufficiently large, there will be many steps in the energy cascade process before the energy is dissipated by small-scale motions. The large-scale motions or eddies that receive energy directly from mean flow shear or buoyancy are expected to be inhomogeneous. But the small eddies, that are formed after many successive breakdowns of large and intermediate size eddies, are likely to become homogeneous and isotropic, because they are far enough removed from the original sources (shear and buoyancy) of inhomogeneity and have no memory of large-scale processes.

Following the above reasoning, Kolmogorov (1941) proposed his local isotropy hypothesis, which states that at sufficiently large Reynolds numbers, small-scale structure is locally isotropic whether large-scale motions are isotropic or not. Here 'local' refers to the particular range of small-scale eddy motions that may be considered isotropic.

The concept of local isotropy has proved to be very useful in that it applies to all turbulent flows with large Reynolds numbers. It also made possible for the sufficiently well-developed statistical theory of isotropic turbulence to be applied to small-scale motions in most turbulent flows encountered in practice. The condition of sufficiently large Reynolds number is particularly well satisfied in the atmosphere. Consequently, the local isotropy concept or hypothesis is found to be valid over a wide range of scales and eddies in the atmosphere.

For locally isotropic, small-scale turbulence Kolmogorov (1941) proposed a local similarity theory or scaling based on his well-known equilibrium range hypothesis. This states that at sufficiently high Reynolds numbers there is a range of small scales for which turbulence structure is in a statistical equilibrium and is uniquely determined by the rate of energy dissipation (ε) and kinematic viscosity (ν). Here the statistical equilibrium refers to the stationarity of small-scale turbulence, even when the large-scale turbulence may not be stationary. The above two parameters lead to the following Kolmogorov's microscales from dimensional considerations:

$$\begin{aligned} &\text{Length scale: } \eta = \nu^{3/4}\varepsilon^{-1/4} \\ &\text{Velocity scale: } \upsilon = \nu^{1/4}\varepsilon^{1/4} \end{aligned} \tag{8.9}$$

which are used to normalize any turbulence statistics and to examine their universal similarity forms (Hinze, 1975; Monin and Yaglom, 1975; Panofsky and Dutton, 1984).

8.7.4 Taylor's frozen turbulence hypothesis

While trying to relate the spatial distribution of turbulence and related statistics to temporal statistics obtained from measurements at a fixed point, Taylor (1938) proposed the following plausible hypothesis:

> If the velocity of air stream which carries the eddies is very much greater than the turbulent velocity, one may assume that the sequence of changes in u at the fixed point are simply due to the passage of an unchanging pattern of turbulent motion over the

point, i.e., one may assume that $u(t) = u(x/U)$ where x is measured upstream at time $t = 0$ from the fixed point where u is measured.

This so-called frozen-turbulence hypothesis was originally suggested for a homogeneous field of low-intensity turbulence in a uniform mean flow. It implies a direct correspondence between spatial changes in a turbulence variable in the mean flow direction and its temporal changes at a fixed point, assuming an unchanging or frozen pattern of turbulence. The hypothesis is frequently used for converting temporal statistics to spatial statistics in the direction of mean flow. Earlier, it provided a practical and convenient way of testing the results of the statistical theory of homogeneous and isotropic turbulence. Subsequently, the frozen-turbulence hypothesis has been extended and generalized to all turbulent variables and also to nonhomogeneous shear flows.

Taylor's hypothesis has been critically examined theoretically, as well as experimentally by many investigators. Two primary limitations are found on the validity of the hypothesis in the atmospheric boundary layer. The first arises from the sufficiently large turbulence intensities, especially under unstable and convective conditions, which are found to have the apparent effect of increasing the effective advection velocity above the actual mean velocity. The validity of the hypothesis becomes particularly questionable under free convection conditions when velocity fluctuations are of the same order or larger than the mean velocity. The second limitation arises from the mean flow shear. In the presence of shear, large eddies may not be advected at the same local mean velocity which advects small eddies, because large velocity differences are likely to exist across large eddies. According to a well-known theoretical criterion, only eddies with frequencies much greater than the magnitude of mean wind shear may be assumed to be transported by the local mean velocity, as implied by Taylor's hypothesis. Aside from turbulence, Taylor's hypothesis has widespread applications in atmospheric dispersion theories and models (Arya, 1999).

8.8 Applications

A knowledge of the fundamentals of turbulence is essential to any qualitative or quantitative understanding of the turbulent exchange processes occurring in the PBL, as well as in the free atmosphere. The statistical description in terms of variances and covariances of turbulent fluctuations, as well as in terms of eddies and scales of motion, provide the bases for further quantitative analyses of observations. At the same time the basic concepts of the generation and maintenance of turbulence, of supply, transfer, and dissipation of energy, and of local similarity are the foundation stones of turbulence theory.

Problems and Exercises

1. Discuss the various types of local or small-scale instability mechanisms that might be operating in the atmosphere and their possible effects on atmospheric motions.

2.
If you have available only the upper air velocity and temperature soundings, how would you use this information to determine which layers of the atmosphere might be turbulent?

3.
(a) What are the possible mechanisms of generation and maintenance of turbulence in the atmosphere?
(b) What criterion may be used to determine if turbulence in a stably stratified layer could be maintained or if it would decay?

4. What are the special characteristics of turbulence that distinguish it from a random wave motion?

5. Compare and contrast the time and ensemble averages. Under what conditions might the two types of averages be equivalent?

6. What is the possible range of turbulence intensities in the atmospheric PBL and under what conditions might the extreme values occur?

7. Write down all the Reynolds stress components and indicate which of these have the same magnitudes.

8. The following mean flow and turbulence measurements were made at a height of 22.6 m during the 1968 Kansas Field Program under different stability conditions:

Run no.	19	20	43	54	18	17	23
Ri	−1.89	−1.15	−0.54	−0.13	0.08	0.12	0.22
U (m s^{-1})	4.89	6.20	8.06	9.66	7.49	5.50	5.02
σ_u (m s^{-1})	0.83	1.04	1.27	1.02	0.73	0.45	0.20
σ_v (m s^{-1})	1.02	1.07	1.26	0.91	0.56	0.36	0.17
σ_w (m s^{-1})	0.65	0.73	0.73	0.64	0.48	0.28	0.10
σ_θ (K)	0.48	0.61	0.56	0.25	0.14	0.15	0.18
$\overline{uw}$ (m^2 s^{-2})	−0.081	−0.111	−0.244	−0.214	−0.118	−0.043	−0.005
$\overline{\theta w}$ (m s^{-1} K)	0.185	0.273	0.188	0.072	−0.029	−0.016	−0.005
$\overline{\theta u}$ (m s^{-1} K)	−0.064	−0.081	−0.175	−0.129	0.068	0.039	0.023

(a) Calculate and plot turbulence intensities as functions of Ri.
(b) Calculate and plot the magnitudes of correlation coefficients r_{uw}, $r_{w\theta}$, and $r_{u\theta}$ as functions of Ri. Explain their different signs under unstable and stable conditions.
(c) Calculate the turbulent kinetic energy (TKE) for each run and plot the ratio $-\mathrm{TKE}/\overline{uw}$ as a function of Ri.
(d) What conclusion can you draw about the variation of the above turbulence quantities with stability?

9.

(a) Calculate Kolmogorov's microscales of length and velocity for the following values of the rate of energy dissipation observed in the atmospheric boundary layer under different stability conditions and at different heights in the PBL:

$$\varepsilon = 10^{-4},\ 10^{-3},\ 10^{-2},\ \text{and}\ 10^{-1}\ \mathrm{m^2\,s^{-3}}$$

(b) Comment on the possible variation of η with height in the surface layer, considering that ε generally decreases with height (it is inversely proportional to z near the surface).