cycleke(菜鸡)的 XCPC 模板





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cycleke

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1 数学

1.1 素数

素数的数目有近似 $\pi(x) \sim \frac{x}{\ln(x)}$, 判定如下:

```
inline 11 mmul(const 11 &a, const 11 &b, const 11 &mod) {
 1
 2
      11 k = (11)((1.0L * a * b) / (1.0L * mod)), t = a * b - k * mod;
3
      for (t -= mod; t < 0; t += mod) {}</pre>
      return t;
4
 5
   inline 11 mpow(11 a, 11 b, const 11 &mod) {
 6
7
      11 \text{ res} = 1;
      for (; b; b >>= 1, a = mmul(a, a, mod)) (b & 1) && (res = mmul(res, a, mod));
 8
9
      return res;
10
11
    inline bool check(const 11 &x, const 11 &p) {
12
13
      if (!(x % p) || mpow(p % x, x - 1, x) ^ 1) return false;
      for (11 k = x - 1, t; \sim k \& 1;) {
14
        if (((t = mpow(p \% x, k >>= 1, x)) ^ 1) && (t ^ (x - 1))) return false;
15
        if (!(t ^(x - 1))) return true;
16
17
      }
18
      return true;
19
20
21
   inline bool Miller_Rabin(const 11 &x) {
22
      if (x < 2) return false;
      static const int p[12] = {2, 3, 5, 7, 11, 13, 17, 19, 61, 2333, 4567, 24251};
23
      for (int i = 0; i < 12; ++i) {
24
25
        if (!(x ^ p[i])) return true;
26
        if (!check(x, p[i])) return false;
      }
27
28
      return true;
   }
```

1.2 Pollard Rho

```
mt19937_64 rnd(chrono::high_resolution_clock::now().time_since_epoch().count());
    inline 11 rand64(11 x) { return rnd() % x + 1; }
 2
 3
    inline 11 Pollard_rho(const 11 &x, const int &y) {
 4
      11 \ v0 = rand64(x), \ v = v0, \ d, \ s = 1;
 5
 6
      for (int t = 0, k = 1;;) {
        v = (mmul(v, v, x) + y) % x, s = mmul(s, abs(v - v0), x);
 7
 8
        if (!(v ^ v0) || !s) return x;
        if (++t == k) 
 9
          if ((d = \_gcd(s, x)) ^ 1) return d;
v0 = v, k <<= 1;
10
11
12
        }
13
      }
    }
14
15
16
    vector<11> factor;
17
    void findfac(ll n) {
      if (Miller_Rabin(n)) {
18
        factor.push_back(n);
19
20
        return;
21
22
      11 p = n;
23
      while (p >= n) p = Pollard_rho(p, rand64(n));
      findfac(p), findfac(n / p);
24
```

1.3 欧拉函数

欧拉函数的性质:

- 欧拉函数是积性函数;
- $n = \sum_{d|n} \varphi(d)$;
- 若 $n = p^k$, 其中 p 是质数, 那么 $\varphi(n) = p^k p^{k-1}$;
- $\Xi \gcd(a,m) = 1$, $M = a^{\varphi(m)} \equiv 1 \pmod{m}$;

```
拓展欧拉定理: a^b \equiv \begin{cases} a^{b \bmod \varphi(p)} & \gcd(a, p) = 1 \\ a^b & \gcd(a, p) \neq 1, b < \varphi(p) \pmod{p} \\ a^{b \bmod \varphi(p) + \varphi(p)} & \gcd(a, p) \neq 1, b \geq \varphi(p) \end{cases}
```

```
int euler_phi(int n) {
1
2
     int ans = n;
3
      for (int i = 2; i * i <= n; i++)
        if (n % i == 0) {
4
         ans = ans / i * (i - 1);
5
         while (n \% i == 0) n /= i;
6
     if (n > 1) ans = ans / n * (n - 1);
8
9
     return ans;
10
```

1.4 线性筛

lpf 为最小质因子的标号; mu 为莫比乌斯函数; phi 为欧拉函数; e 为质因子最高次幂, d 为因数个数; f 为因数和, g 为最小质因子的幂和, 即 $p+p^1+p^2+\cdots+p^k$ 。理论上积性函数都可以线性筛。

```
const int MAXN = 1e7 + 5;
   int prime[MAXN / 15], prime_cnt;
2
   int lpf[MAXN], e[MAXN], d[MAXN], mu[MAXN], phi[MAXN];
    void sieve() {
4
      prime[lpf[1] = 0] = 1, e[1] = 0, d[1] = 1, mu[1] = 1, phi[1] = 1;
5
      for (int i = 2; i < MAXN; ++i) {</pre>
6
        if (!lpf[i]) {
7
          prime[lpf[i] = ++prime_cnt] = i;
          mu[i] = -1, phi[i] = i - 1;
9
          e[i] = 1, d[i] = 2;
10
          g[i] = f[i] = i + 1;
11
12
        for (int j = 1, x; j \le lpf[i] && (x = i * prime[j]) < MAXN; ++j) {
13
          lpf[x] = j;
14
15
          if (j < lpf[i]) {</pre>
            mu[x] = -mu[i], phi[x] = phi[i] * (prime[j] - 1);
16
            e[x] = 1, d[x] = d[i] * 2;
17
            g[x] = 1 + prime[j], f[x] = f[i] * f[prime[j]];
18
          } else { // i % prime[j] == 0
19
            mu[x] = 0, phi[x] = phi[i] * prime[j];
20
            e[x] = e[i] + 1, d[x] = d[i] / e[x] * (e[x] + 1);
21
22
            g[x] = g[i] * prime[j] + 1, f[x] = f[i] / g[i] * g[x];
23
24
        }
25
      }
   }
26
```

1.5 拓展欧几里得算法

```
int exgcd(int a, int b, int &x, int &y) {
   if (b == 0) return x = 1, y = 0, a;
   int g = exgcd(b, a % b, y, x);
   y -= a / b * x;
   return g;
}
```

1.6 莫比乌斯反演

常见积性函数 (f(ab) = f(a)f(b), (a, b) = 1):

- 单位函数: $\epsilon(n) = [n = 1]$ (完全积性)
- 恒等函数: $id_k(n) = n^k$, $id_1(n)$ 通常简记作 id(n) (完全积性)。
- 常数函数: 1(n) = 1 (完全积性)
- 除数函数: $\sigma_k(n) = \sum_{d|n} d^k$, $\sigma_0(n)$ 通常简记作 d(n) 或 $\tau(n)$, $\sigma_1(n)$ 通常简记作 $\sigma(n)$ 。
- 欧拉函数: $\varphi(n) = \sum_{i=1}^{n} [\gcd(i, n) = 1]$
- 莫比乌斯函数: $\mu(n) = \begin{cases} 1 & n=1 \\ 0 & \exists d>1, d^2 \mid n, \text{ 其中 } \omega(n) \text{ 表示 } n \text{ 的本质不同质因子个数,它是一个加性函数 } (\omega(ab) = \omega(a) + \omega(b)). \end{cases}$

Dirichlet 卷积满足交换率、结合率和分配率, 常见 Dirichlet 卷积:

• $\varepsilon = \mu * 1$

• $\varphi = id * \mu$

• $f \cdot d = f * f$ (f 为完全积性)

• d = 1 * 1

• $id = \varphi * 1$

• $\sigma = id *1$

• $\operatorname{id}_{k+1} = (\operatorname{id}_k \cdot \varphi) * \operatorname{id}_k$

莫比乌斯反演: $f = g * 1 \iff g = \mu * f$ 。

拓展:对于数论函数 f,g 和完全积性函数 t 且 t(1) = 1:

$$f(n) = \sum_{i=1}^{n} t(i)g\left(\left\lfloor \frac{n}{i} \right\rfloor\right) \iff g(n) = \sum_{i=1}^{n} \mu(i)t(i)f\left(\left\lfloor \frac{n}{i} \right\rfloor\right)$$

常用结论:

- $\sum_{i=x}^n \sum_{j=y}^m [\gcd(i,j) = k] = \sum_{d=1} \mu(d) \lfloor \frac{n}{kd} \rfloor \lfloor \frac{m}{kd} \rfloor$
- $d(ij) = \sum_{x|i} \sum_{y|j} [\gcd(x,y) = 1] = \sum_{p|i,p|j} \mu(p) d\left(\frac{i}{p}\right) d\left(\frac{j}{p}\right)$

1.7 杜教筛

设 $S(n)=\sum_{i=1}^n f(i)$,则 $\sum_{i=1}^n (f*g)(i)=\sum_{i=1}^n g(i)S(\lfloor \frac{n}{i} \rfloor)\Rightarrow g(1)S(n)=\sum_{i=1}^n (f*g)(i)-\sum_{i=2}^n g(i)S(\lfloor \frac{n}{i} \rfloor)$ 。直接分块复杂度 $O(n^{\frac{3}{4}})$,预处理出前 $O(n^{\frac{2}{3}})$ 项复杂度为 $O(n^{\frac{2}{3}})$ 。常用结论:

- 莫比乌斯函数前缀和: $S(n) = \sum_{i=1}^n \epsilon(i) \sum_{i=2}^n S(\lfloor \frac{n}{i} \rfloor)$
- 欧拉函数前缀和: $S(n) = \sum_{i=1}^{n} id(i) \sum_{i=2}^{n} S(\lfloor \frac{n}{i} \rfloor)$

```
map<int, int> mp_mu;
 2
    int S_mu(int n) {
 3
      if (n < MAXN) return sum_mu[n];</pre>
      if (mp_mu[n]) return mp_mu[n];
 5
 6
      int ret = 1;
      for (int i = 2, j; i <= n; i = j + 1) {
  j = n / (n / i);</pre>
 7
 8
 9
         ret -= S_mu(n / i) * (j - i + 1);
10
11
      return mp_mu[n] = ret;
12
13
    // 使用莫比乌斯反演
14
    11 S_phi(int n) {
15
      11 \text{ res} = 0;
16
      for (int i = 1, j; i \le n; i = j + 1) {
17
         j = n / (n / i);
18
         res += 1LL * (S_mu(j) - S_mu(i - 1)) * (n / i) * (n / i);
19
20
21
      return (res - 1) / 2 + 1;
22
```

1.8 Min 25 筛

要求: 积性函数 f(p) 是一个关于 p 的项数较少的多项式或可以快速求值; $f(p^c)$ 可以快速求值。时间复杂度: $O\left(\frac{n^{\frac{3}{4}}}{\log n}\right)$ 。部分符号和结论:

- $F_{\text{prime}}(n) = \sum_{2 \le p \le n} f(p)$
- $F_k(n) = \sum_{i=2}^n [p_k \le lpf(i)] f(i)$, 答案为 $F_1(n) + f(1)$
- $F_k(n) = \sum_{\substack{k \le i \ p_i^2 \le n}} \sum_{\substack{c \ge 1 \ p_i^{c+1} \le n}} (f(p_i^c) F_{i+1}(n/p_i^c) + f(p_i^{c+1})) + F_{\text{prime}}(n) F_{\text{prime}}(p_{k-1})$
- $f(p) = \sum p^{s_i}, \ \ \ \ \mathcal{U}(p) = p^s, G_k(n) = \sum_{i=1}^n [p_k < \mathrm{lpf}(i) \lor \mathrm{isprime}(i)]g(i)$
- $G_k(n) = G_{k-1}(n) [p_k^2 \le n]g(p_k)(G_{k-1}(n/p_k) G_{k-1}(p_{k-1})), G_0 = \sum_{i=2}^n g(i)$

LOJ 例题: 给定
$$f(n)$$
: $f(n) = \begin{cases} 1 & n=1 \\ p \oplus c & n=p^c \\ f(a)f(b) & n=ab \land a \perp b \end{cases}$, 求 $f(n)$ 的和。因 $f(p) = p-1+2[p=2]$,

可以按照筛 φ 的方法来处理。

```
constexpr int MOD = 1e9 + 7;
    constexpr int INV2 = MOD / 2 + 1;
3
    template <typename X, typename Y> void inc(X &x, const Y &y) { x += y, (x >= MOD) && (x -= MOD); }
 4
    template <typename X, typename Y> void dec(X &x, const Y &y) { x -= y, (x < 0) && (x += MOD); }
    template <typename X, typename Y> int sum(X x, Y y) { return x + y < MOD ? x + y : x + y - MOD; }
 6
 7
    template <typename X, typename Y> int sub(X x, Y y) \{ return x < y ? x + MOD - y : x - y; \}
8
    constexpr int MAX_SIZE = 2e5 + 3;
9
   int prime[MAX_SIZE / 10], lpf[MAX_SIZE], spri[MAX_SIZE], prime_cnt;
10
11
    void sieve(int n) {
12
     for (int i = 2; i <= n; ++i) {
13
14
        if (lpf[i] == 0) {
          prime[lpf[i] = ++prime_cnt] = i;
15
16
          spri[prime_cnt] = sum(spri[prime_cnt - 1], i);
17
        for (int j = 1, x; j \le lpf[i] && (x = i * prime[j]) \le n; ++j) lpf[x] = j;
18
```

```
}
19
20
21
    11 g_n, lis[MAX_SIZE];
22
    int G[MAX_SIZE][2], Fprime[MAX_SIZE], cnt;
    int lim, le[MAX_SIZE], ge[MAX_SIZE];
24
    #define idx(x) (x <= lim ? le[x] : ge[g_n / x])
25
26
27
    void init(ll n) {
28
      for (ll i = 1, j, x; i \le n; i = n / j + 1) {
        j = n / i, x = j % MOD;
29
        lis[++cnt] = j, idx(j) = cnt;
30
        G[cnt][0] = sub(x, 1);
31
32
        G[cnt][1] = (x + 211) * (x - 111) % MOD * INV2 % MOD;
33
      }
   }
34
    void calcFPrime() {
35
      for (int k = 1; k <= prime_cnt; ++k) {</pre>
36
37
        const int p = prime[k];
38
        const ll sqrp = 111 * p * p;
        for (int i = 1; lis[i] >= sqrp; ++i) {
39
          const 11 \times = 1is[i] / p;
40
          const int id = idx(x);
41
          dec(G[i][0], sub(G[id][0], k - 1));

dec(G[i][1], 111 * p * sub(G[id][1], spri[k - 1]) % MOD);
42
43
44
      }
45
      // f(p) = g_1(p) - g_0(p)
46
      for (int i = 1; i <= cnt; ++i) Fprime[i] = sub(G[i][1], G[i][0]);
47
48
49
50
    int f_p(int p, int c) { return p ^ c; }
    int F(int k, ll n) {
51
      if (n < prime[k] || n <= 1) return 0;</pre>
52
53
      const int id = idx(n);
      int res = sub(Fprime[id], sub(spri[k - 1], k - 1));
54
55
      // F_prime(p_{k-1}) = spri[k-1] - (k-1)
      if (k == 1) res += 2; // 特殊处理 f(2)
56
57
      for (int i = k; i <= prime_cnt && 1ll * prime[i] * prime[i] <= n; ++i) {</pre>
        ll pw = prime[i], pw2 = pw * pw;
58
59
        for (int c = 1; pw2 <= n; ++c, pw = pw2, pw2 *= prime[i])</pre>
          inc(res, (111 * f_p(prime[i], c) * F(i + 1, n / pw) + f_p(prime[i], c + 1)) % MOD);
60
61
      }
62
      return res;
    }
63
```

新版 $\min 25$ 筛,复杂度 $O\left(n^{\frac{2}{3}}\right)$ 。

```
/**********
   f()函数中(31-37行)填函数在质数幂次处的表达式
2
   pow_sum()函数中(38-43行)填幂和函数(如果需要更高次的话可以在这里添加)
3
   202-205行按要求填写
4
   f_p[][0/1/2/3/...]分别代表质数个数/质数和/质数平方和/质数三次方和/...根据自己需要添加
5
   例:如果该函数在质数处表达式为f(p) =
6
7
   p^2+3*p+1,则表明需要质数个数/质数和/质数平方和,即f_p[][0],f_p[][1],f_p[][2]
8
   ***********
9
10
   inline 11 f(11 p, int e) { // return f(p^e)
    if (p == 1 || e == 0) return 1;
11
    11 \text{ res} = mpow(p, e);
12
    return res * res + 3 * res + 1;
13
14
   }
   ll pow_sum(ll n, int k) { return sum(i^k),i from 1 to n.
15
    if (k == 0) return n;
16
    if (k == 1) return n * (n + 1) / 2;
17
18
    if (k == 2) return n * (n + 1) * (2 * n + 1) / 6;
19
20 | 11 n, f_p[maxn][3]; // F_prime(id(n/i))
```

```
21 | int n_2, n_3, n_6; // sqrt(n), sqrt3(n), sqrt6(n);
                         // give the id, return the id-th number like 'n/i' ,(val_id[1] = 1)
   11 val_id[maxn];
   int val_id_num;
                         // how many numbers like 'n/i'
   int val_id_num_3;
                         // how many numbers like 'n/i' below n/n_3;
24
    int p[200000 + 100];
    bool isp[maxn];
26
27
    int p_sz_2, p_sz_3, p_sz_6; // pi(n_2), pi(n_3), pi(n_6)
28
    void init() {
29
      n_2 = (int) sqrt(n);
      n_3 = (int)pow(n, 1.0 / 3.0);

n_6 = (int)pow(n, 1.0 / 6.0);
30
31
      val_id_num = 0;
32
      for (11 i = 1; i <= n;) {
33
34
        val_id[++val_id_num] = i;
35
        if (i < n) i = n / (n / (i + 1));
36
37
      memset(isp, 1, sizeof isp);
      isp[1] = 0;
38
      for (int i = 2; i <= n_2; i++) {
39
40
        if (isp[i]) {
          p[++p_sz_2] = i;
41
          if (i <= n_3) p_sz_3++;
42
          if (i <= n_6) p_sz_6++;</pre>
43
44
        for (int j = 1; j \le p_sz_2 \& p[j] * i \le n_2; j++) {
45
          isp[i * p[j]] = 0;
46
47
          if (i % p[j] == 0) break;
48
49
      }
    }
50
    inline int get_id(ll k) { // give a number like 'n/i', return the id of it
51
52
      return k > n_2 ? val_id_num - n / k + 1 : k;
53
54
    11 c[maxn];
55
    void add(int x, 11 d) { for (; x < maxn; x += x & -x) c[x] += d; }
    11 sum(int x) {
56
57
      11 \text{ ans} = 0;
58
      for (; x; x &= x - 1) ans += c[x];
59
      return ans;
    }
60
61
    struct node {
62
      int k_max;
63
64
      ll val, f_val;
65
   };
66
    void update_bfs(int k, int type) {
      queue<node> q;
67
      while (!q.empty()) q.pop();
68
69
      int e = 1;
      for (ll i = p[k]; i < n / n_3; i *= p[k], ++e)
70
71
        q.emplace(k, i, type == -1 ? f(p[k], e) : mpow(i, type));
      while (!q.empty()) {
72
        node hd = q.front(); q.pop();
73
74
        if ((hd.val != p[hd.k_max] \&\& type >= 0) || type == -1) {
          11 w = n / hd.val;
75
76
          w = n / w;
77
          if (type == -1) {
78
            add(get_id(w), hd.f_val);
            add(val_id_num + 1, -111 * hd.f_val);
79
80
          } else {
            add(get_id(w), -111 * hd.f_val);
81
82
            add(val_id_num + 1, hd.f_val);
83
84
        for (int i = hd.k max + 1; hd.val * p[i] < n / n 3 & i <= p sz 2; i++) {
85
86
          ll res = p[i];
87
          for (int e = 1;; e++) {
            if (hd.val * res < n / n_3) {</pre>
88
```

```
q.emplace(i, hd.val * res, type == -1 ? hd.f_val * f(p[i], e) : hd.f_val * mpow(res,
 89
          type));
 90
              } else break;
              res *= p[i];
 91
 92
         }
 93
       }
 94
 95
     }
 96
     void get_f_p(ll n, int times) {
 97
       for (int i = 1; i <= val_id_num; i++)</pre>
          for (int j = 0; j <= times; j++)</pre>
 98
            f_p[i][j] = pow_sum(val_id[i], j) - 1;
 99
100
       int now:
       for (now = 1; p[now] <= n_6; now++) {
  for (int j = val_id_num; j >= 1; j--) {
101
102
           ll w = val_id[j] / p[now];
103
            if (w < p[now]) break;</pre>
104
            11 \text{ val} = 1;
105
106
            for (int k = 0; k \le times; k++) {
107
              f_p[j][k] = f_p[j][k] - val * (f_p[get_id(w)][k] - f_p[p[now - 1]][k]);
108
              val *= p[now];
           }
109
         }
110
111
112
       int nnow = now, val = 1;
       for (int tt = 0; tt <= times; tt++) {</pre>
113
         now = nnow;
114
         memset(c, 0, sizeof c);
115
116
          add(1, f_p[1][tt]);
         for (int i = 2; val_id[i] < n / n_3; i++) add(i, f_p[i][tt] - f_p[i - 1][tt]);
117
          for (; p[now] <= n_3; now++) {</pre>
118
119
            for (int j = val_id_num; j >= 1; j--) {
              ll w = val_id[j] / p[now];
120
121
              if (val_id[j] < n / n_3) break;</pre>
              if (w < p[now]) break;</pre>
122
123
              f_p[j][tt] = w < n / n_3
124
                ? (f_p[j][tt] - (sum(get_id(w)) - sum(p[now - 1])) * mpow(p[now], tt))
125
                : f_p[j][tt] = f_p[j][tt] - (f_p[get_id(w)][tt] - sum(p[now - 1])) * mpow(p[now], tt);
126
           update_bfs(now, tt);
127
128
          for (int i = 1; i <= val_id_num && val_id[i] < n / n_3; i++) f_p[i][tt] = sum(i);</pre>
129
130
          for (; now <= p_sz_2; now++) {</pre>
131
            for (int j = val_id_num; j >= 1; j--) {
              ll w = val_id[j] / p[now];
132
133
              if (val_id[j] < n / n_3) break;</pre>
              if (w < p[now]) break;</pre>
134
135
              f_p[j][tt] = (f_p[get_id(w)][tt] - f_p[p[now - 1]][tt]) * mpow(p[now], tt);
136
            }
         }
137
138
       }
139
140
       for (int i = 1; i <= val_id_num; i++) {</pre>
          // if f(p) = p^2+3p+1, then write: f_p[i][0] = f_p[i][2] + 3*f_p[i][1] + f_p[i][0];
141
         f_p[i][0] = f_p[i][2] + 3 * f_p[i][1] + f_p[i][0];
142
143
       }
144
145
     11 F[2000000 + 100];
     void get_f_3(ll n) { // V(F_{pi(n^{(1/3))+1},n)}
146
147
       ll q = p[p_sz_3 + 1];
148
       for (int now = 1; now <= val_id_num; now++) {</pre>
         if (val_id[now] < q) {</pre>
149
150
            F[now] = 1;
         } else if (val_id[now] < q * q) {
151
152
           F[now] = 1 + (f_p[now][0] - f_p[q - 1][0]);
153
          } else {
            F[now] = 1 + (f_p[now][0] - f_p[q - 1][0]);
154
            for (int pp = p_sz_3 + 1; p[pp] \le (int)(sqrt(val_id[now])) && pp <= <math>p_sz_2; pp++) {
155
```

```
F[now] += f(p[pp], 2) + (f(p[pp], 1)) * (f_p[get_id(val_id[now] / p[pp]))[0] - f_p[get_id(val_id[now] / p[pp])][0] - f_p[get_id[now] / p[pp])[0] - f_p[get_id[now] / p[pp]][0] - f_p[get_id[now] / p[pp])[0] - f_p[get_id[now] / p[pp]][0] - f_p[get_id[now] / p[get_id[now] / p[get
156
                       p[pp])][0]);
157
                       }
158
159
                 }
            }
160
            void get_f_6(ll n) { // V(F_{pi(n^{(1/6))+1},n)}
161
162
                 memset(c, 0, sizeof c), add(1, F[1]);
163
                  for (int i = 2; val_id[i] < n / n_3; i++) add(i, F[i] - F[i - 1]);
                  for (int k = p_sz_3; k > p_sz_6; k--) {
164
                       int now = val_id_num;
165
                       for (; val_id[now] >= n / n_3; now--) {
166
167
                           int e = 1;
168
                           11 _p = p[k];
                           while (val_id[now] / _p) {
   if (val_id[now] / _p >= n / n_3) {
169
170
                                      F[now] += F[get_id(val_id[now] / _p)] * f(p[k], e);
171
                                 } else {
172
173
                                      F[now] += sum(get_id(val_id[now] / _p)) * f(p[k], e);
174
                                 _p *= p[k], e++;
175
                           }
176
                       }
177
178
                       if (k == 1) break;
                       update_bfs(k, -1); // bfs to update [lpf(i)==P\{k-1\}]f(i)
179
180
181
                 for (int i = 1; i <= val_id_num && val_id[i] < n / n_3; i++) F[i] = sum(i);</pre>
182
            }
            void get_f(ll n) {
183
                 for (int k = p_sz_6; k >= 1; k--) {
184
                       for (int now = val_id_num; now >= 1; now--) {
185
186
                            int e = 1;
187
                           11 _p = p[k];
188
                            while (val_id[now] / _p) {
189
                                F[now] += F[get_id(val_id[now] / _p)] * f(p[k], e);
                                 _p *= p[k], e++;
190
191
192
                      }
193
                 }
194
195
            int main() { // n = 1000000000; 1e10:455052511,0.83s/0.58s; 1e12:37607912018 9.224s/5.105s
196
                 cin >> n;
197
                 init();
198
                 get_f_p(n, 2), get_f_3(n);
                 get_f_6(n), get_f(n);
199
200
                 for (int i = 1; i <= val_id_num; i++) cout << val_id[i] << " : " << F[i] << endl;</pre>
201
```

1.9 类欧几里德算法

```
const 11 P = 998244353;
   11 i2 = 499122177, i6 = 166374059;
2
   struct data {
3
4
     data() { f = g = h = 0; }
   ll f, g, h;
}; // 三个函数打包
5
6
   data calc(ll n, ll a, ll b, ll c) {
7
     ll ac = a / c, bc = b / c, m = (a * n + b) / c, n1 = n + 1, n21 = n * 2 + 1;
8
9
      data d;
10
      if (a == 0) { // 迭代到最底层
11
        d.f = bc * n1 \% P;
        d.g = bc * n % P * n1 % P * i2 % P;
12
        d.h = bc * bc % P * n1 % P;
13
14
        return d;
15
     }
```

```
if (a >= c || b >= c) { // 取模
16
        d.f = n * n1 \% P * i2 \% P * ac \% P + bc * n1 \% P;
17
18
        d.g = ac * n % P * n1 % P * n21 % P * i6 % P + bc * n % P * n1 % P * i2 % P;
        d.h = ac * ac % P * n % P * n1 % P * n21 % P * i6 % P +
19
20
              bc * bc % P * n1 % P + ac * bc % P * n % P * n1 % P;
        d.f %= P, d.g %= P, d.h %= P;
21
22
        data e = calc(n, a % c, b % c, c); // 迭代
23
24
25
        d.h += e.h + 2 * bc % P * e.f % P + 2 * ac % P * e.g % P;
        d.g += e.g, d.f += e.f;
26
        d.f %= P, d.g %= P, d.h %= P;
27
28
        return d;
29
      }
30
      data e = calc(m - 1, c, c - b - 1, a);
      d.f = n * m % P - e.f, d.f = (d.f % P + P) % P;
31
      d.g = m * n \% P * n1 \% P - e.h - e.f, d.g = (d.g * i2 \% P + P) \% P;
32
      d.h = n * m \% P * (m + 1) \% P - 2 * e.g - 2 * e.f - d.f;
33
      d.h = (d.h \% P + P) \% P;
34
35
      return d;
36
```

1.10 中国剩余定理

```
11 inv(ll a, ll p) {
2
      11 x, y;
      exgcd(a, p, x, y);
3
      return (x + p) \% p;
 4
5
   11 CRT(11 n, 11 *a, 11 *m) {
7
      11 lcm = 1, res = 0;
      for (11 i = 0; i < n; ++i) lcm *= m[i];</pre>
8
      for (11 i = 0; i < n; ++i) {
9
        11 t = 1cm / m[i], x = inv(t, m[i]);
10
11
        res = (res + a[i] * t % lcm * x) % lcm;
      }
12
13
      return res;
   }
14
```

```
模数不互质的情况 \begin{cases} x \equiv a_1 \pmod{m_1} \\ x \equiv a_2 \pmod{m_2} \end{cases}, 则转换为 m_1p - m_2q = a_2 - a_1,最终解(若有解)为 x \equiv m_1p + a_1 \pmod{\operatorname{lcm}(m_1, m_2)}。 拓展中国剩余定理:
```

```
int exctr(int n, int *a, int *m) {
2
      int M = m[0], res = a[0];
3
      for (int i = 1; i < n; ++i) {
        int a = M, b = m[i], c = (a[i] - res % b + b) % b, x, y;
4
        int g = exgcd(a, b, x, y), bg = b / g;
5
        if (c % g != 0) return -1;
6
7
        x = 1LL * x * (c / g) % bg;
8
        res += x * M;
       M *= bg;
9
10
        res = (res % M + M) % M;
11
      return res;
12
   }
13
```

1.11 原根

• 阶: 若 (a,m)=1, 使 $a^l\equiv 1\pmod m$ 成立的最小的 l, 称为 a 关于模 m 的阶, 记为 $\operatorname{ord}_m a_o$

- 原根: 若 (g,m) = 1, ord $_m g = \varphi(m)$, 则称 g 为 m 的一个原根。若 m 有原根,则 m 一定是下列形式: $\{2,4,p^a,2p^a\}$ (这里 p 为奇素数,a 为正整数)。
- 求原根: 设 p_1, p_2, \dots, p_k 是 $\varphi(m)$ 的所有不同的素因数,则 g 是 m 的原根 $\iff \forall 1 \leq i \leq k$,有 $g^{\frac{\varphi(m)}{p_i}} \not\equiv 1 \pmod{m}$,集合 $S = g^s \mid 1 \leq s \leq \varphi(m), (s, \varphi(m)) = 1$ 给出 m 的全部原根。

1.12 BGSG

大步小步算法用来求离散对数 $x^k \equiv a \pmod{p}$ 。求解 $a^x \equiv b \pmod{p}$ 则,令 $x = A\lceil \sqrt{p} \rceil - B, 0 \le A, B \le \lceil \sqrt{p} \rceil$,有 $a^{A\lceil \sqrt{p} \rceil} \equiv ba^B \pmod{p}$,先枚举 A 之后在哈希表中查找 B 就行。

```
// Finds the primitive root modulo p
2
    int generator(int p) {
3
      vector<int> fact;
      int phi = p - 1, n = phi;
4
      for (int i = 2; i * i <= n; ++i) {
5
 6
        if (n \% i == 0) {
7
          fact.push_back(i);
8
          while (n \% i == 0) n /= i;
        }
9
10
      }
      if (n > 1) fact.push_back(n);
11
12
      for (int res = 2; res <= p; ++res) {
13
        bool ok = true;
        for (int factor : fact)
14
          if (mpow(res, phi / factor, p) == 1) {
15
            ok = false;
16
17
            break;
18
        if (ok) return res;
19
20
21
      return -1;
22
23
    vector<int> BSGS(int n, int k, int a) {
24
      if (a == 0) return vector<int>({0});
25
26
      int g = generator(n);
27
      // Baby-step giant-step discrete logarithm algorithm
      int sq = (int) sqrt(n + .0) + 1;
28
      vector<pair<int, int>> dec(sq);
29
      for (int i = 1; i \le sq; ++i)
30
31
        dec[i-1] = \{mpow(g, i * sq * k % (n-1), n), i\};
32
33
      sort(dec.begin(), dec.end());
34
      int any ans = -1;
      for (int i = 0; i < sq; ++i) {
35
        int my = mpow(g, i * k % (n - 1), n) * a % n;
36
37
        auto it = lower_bound(dec.begin(), dec.end(), make_pair(my, 0));
        if (it != dec.end() && it->first == my) {
38
          any_ans = it -> second * sq - i;
39
40
          break;
41
42
      if (any_ans == -1) return vector<int>();
43
44
      // Print all possible answers
      int delta = (n - 1) / \_gcd(k, n - 1);
45
46
      vector<int> ans;
      for (int cur = any_ans % delta; cur < n - 1; cur += delta)</pre>
47
       ans.push_back(mpow(g, cur, n));
48
49
      sort(ans.begin(), ans.end());
50
      return ans;
51
```

1.13 自适应 Simpson

计算 $\int_a^b f(x)dx$ 。

```
double simpson(double a, double b) {
1
      double c = a + (b - a) / 2;
2
 3
      return (f(a) + 4 * f(c) + f(b)) * (b - a) / 6;
   }
4
   double integral(double a, double b, double eps, double A) {
  double c = a + (b - a) / 2;
5
6
7
      double L = simpson(a, c), R = simpson(c, b);
      if (fabs(L + R - A) \le 15 * eps) return L + R + (L + R - A) / 15;
9
      return integral(a, c, eps / 2, L) + integral(c, b, eps / 2, R);
10
   double integral(double a, double b, double eps) {
11
     return integral(a, b, eps, simpson(a, b));
12
13
```

1.14 卢卡斯定理

卢卡斯定理: $\binom{n}{m} \bmod p = \binom{\lfloor n/p \rfloor}{\lfloor m/p \rfloor} \cdot \binom{n \bmod p}{m \bmod p} \bmod p$

```
11 lucas(11 n, 11 m, int p) {
1
2
      11 \text{ ret} = 1;
3
      while (n && m) {
        11 nn = n \% p, mm = m \% p;
4
5
        if (nn < mm) return 0;</pre>
        ret = ret * fac[nn] % p * inv_fac[mm] % p * inv_fac[nn - mm] % p;
6
7
        n /= p, m /= p;
8
      }
9
      return ret;
10
   }
```

拓展卢卡斯定理:用于处理 p 不是质数的情况。

对于 $C_n^m \mod p$,我们将其转化为 r 个形如 $a_i \equiv C_n^m \pmod{q_i^{\alpha_i}}$ 的同余方程并分别求解;对于 $a_i \equiv C_n^m \pmod{q_i^{\alpha_i}}$,将 C_n^m 转化为 $\frac{\frac{n!}{q!}}{\frac{n!}{n!}\frac{(n-m)!}{(n-m)!}}q^{x-y-z}$,可求逆元;对于 $\frac{m!}{q^y}$ 和 $\frac{(n-m)!}{q^z}$,将其变换整理,可递归求解。

```
1
    ll calc(ll n, ll x, ll p) {
     if (!n) return 1;
2
3
      11 s = 1;
     for (ll i = 1; i \le p; ++i) (i \% x) && (s = s * i \% p);
 4
      s = mpow(s, n / p, p);
5
 6
      for (ll i = n / p * p; i \le n; ++i) (i \% x) && (s = s * (i \% p) \% p);
 7
      return s * calc(n / x, x, p) % p;
9
   11 multi_lucas(ll n, ll m, ll x, ll p) {
10
11
      11 cnt = 0;
      for (ll i = n; i; i /= x) cnt += i / x;
12
      for (11 i = m; i; i /= x) cnt -= i / x;
13
14
      for (ll i = n - m; i; i /= x) cnt -= i / x;
      return mpow(x, cnt, p) * calc(n, x, p) % p * inv(calc(m, x, p), p) % p *
15
16
             inv(calc(n - m, x, p), p) % p;
17
18
   11 exlucas(11 n, 11 m, 11 P) {
19
20
      11 \text{ cnt} = 0;
      static 11 p[20], a[20];
21
      for (11 i = 2; i * i <= P; ++i) {
22
        if (P % i) continue;
23
24
        p[cnt] = 1;
25
        while (P % i == 0) p[cnt] *= i, P /= i;
26
        a[cnt] = multi_lucas(n, m, i, p[cnt]);
27
        ++cnt;
28
     if (P > 1) p[cnt] = P, a[cnt] = multi_lucas(n, m, P, P), ++cnt;
```

```
30 return CRT(cnt, a, p);
31 }
```

1.15 Burnside 引理

设 A 和 B 为有限集合, $X=B^A$ 表示所有从 A 到 B 的映射。G 是 A 上的置换群,X/G 表示 G 作用在 X 上产生的所有等价类的集合(若 X 中的两个映射经过 G 中的置换作用后相等,则它们在同一等价类中),则 $|X/G|=\frac{1}{|G|}\sum_{a\in G}|X^{g}|$ 。

Gym 101873B: m 边形, 每边是 $n \times n$ 的矩形, 用 c 种颜色染色, 可进行水平旋转, 问不同多边形个数。

```
for (int i = 1; i \le m; ++i) ans = (ans + mpow(c, n * n * \_gcd(i, m))) % MOD; ans = 1LL * ans * mpow(m, MOD - 2) % MOD;
```

1.16 Pólya 定理

前置条件与 Burnside 引理相同,内容修改为 $|X/G|=\frac{1}{|G|}\sum_{g\in G}|B|^{c(g)}$,其中 c(g) 表示置换 g 能拆分成的不相交的循环置换的数量。

1.17 高斯解线性方程

```
const double EPS = 1e-9;
    const int MAXN = MAX_NODE;
    double a[MAXN][MAXN], x[MAXN];
 3
    int equ, var;
 5
 6
    int gauss() {
 7
      int i, j, k, col, max_r;
      for (k = 0, col = 0; k < equ && col < var; k++, col++) {
 8
        max_r = k;
 9
        for (i = k + 1; i < equ; i++)
10
          if (fabs(a[i][col]) > fabs(a[max r][col])) max r = i;
11
        if (fabs(a[max_r][col]) < EPS) return 0;</pre>
12
13
14
        if (k != max_r) {
          for (j = col; j < var; j++) swap(a[k][j], a[max_r][j]);</pre>
15
          swap(x[k], x[max_r]);
16
17
18
19
        x[k] /= a[k][col];
        for (j = col + 1; j < var; j++) a[k][j] /= a[k][col];</pre>
20
21
        a[k][col] = 1;
22
23
        for (i = k + 1; i < equ; i++)
          if (i != k) {
24
25
            x[i] = x[k] * a[i][col];
26
             for (j = col + 1; j < var; j++) a[i][j] -= a[k][j] * a[i][col];</pre>
27
            a[i][col] = 0;
28
          }
29
30
      for (col = equ - 1, k = var - 1; \sim col; --col, --k) {
31
        if (fabs(a[col][k]) > 0) {
32
          for (i = 0; i < k; ++i) {
33
            x[i] = x[k] * a[i][col];
34
35
             for (j = col + 1; j < var; j++) a[i][j] -= a[k][j] * a[i][col];</pre>
36
            a[i][col] = 0;
37
38
        }
      }
39
41
      return 1;
42
```

线性回归 1.18

```
struct LinearRecurrence {
 1
2
      using int64 = long long;
3
      using vec = std::vector<int64>;
 4
5
      static void extand(vec &a, size_t d, int64 value = 0) {
        if (d <= a.size()) return;</pre>
 6
 7
        a.resize(d, value);
 8
9
      static vec BerlekampMassey(const vec &s, int64 mod) {
10
11
        std::function<int64(int64)> inverse = [&](int64 a) {
          return a == 1 ? 1 : (int64)(mod - mod / a) * inverse(mod % a) % mod;
12
13
        };
14
        vec A = \{1\}, B = \{1\};
        int64 b = s[0];
15
16
        for (size_t i = 1, m = 1; i < s.size(); ++i, m++) {
          int64 d = 0;
17
          for (size_t j = 0; j < A.size(); ++j) { d += A[j] * s[i - j] % mod; }</pre>
18
19
          if (!(d %= mod)) continue;
          if (2 * (A.size() - 1) <= i) {
20
21
            auto temp = A;
22
            extand(A, B.size() + m);
23
            int64 coef = d * inverse(b) % mod;
24
            for (size_t j = 0; j < B.size(); ++j) {</pre>
              A[j + m] -= coef * B[j] % mod;
25
26
              if (A[j + m] < 0) A[j + m] += mod;
27
            }
28
            B = temp, b = d, m = 0;
29
          } else {
30
            extand(A, B.size() + m);
31
            int64 coef = d * inverse(b) % mod;
32
            for (size_t j = 0; j < B.size(); ++j) {</pre>
33
              A[j + m] = coef * B[j] % mod;
              if (A[j + m] < 0) A[j + m] += mod;
34
            }
35
          }
36
37
        }
38
        return A;
39
40
      static void exgcd(int64 a, int64 b, int64 &g, int64 &x, int64 &y) {
41
42
        if (!b)
43
          x = 1, y = 0, g = a;
        else {
44
45
          exgcd(b, a % b, g, y, x);
          y = x * (a / b);
46
47
48
      }
49
50
      static int64 crt(const vec &c, const vec &m) {
        int n = c.size();
51
        int64 M = 1, ans = 0;
52
        for (int i = 0; i < n; ++i) M *= m[i];</pre>
53
        for (int i = 0; i < n; ++i) {
54
55
          int64 x, y, g, tm = M / m[i];
          exgcd(tm, m[i], g, x, y);
56
57
          ans = (ans + tm * x * c[i] % M) % M;
        }
58
59
        return (ans + M) % M;
60
61
62
      static vec ReedsSloane(const vec &s, int64 mod) {
        auto inverse = [](int64 a, int64 m) {
63
64
          int64 d, x, y;
65
          exgcd(a, m, d, x, y);
          return d == 1 ? (x % m + m) % m : -1;
66
```

cycleke

```
67
 68
         auto L = [](const vec &a, const vec &b) {
 69
           int da = (a.size() > 1 || (a.size() == 1 && a[0])) ? a.size() - 1 : -1000;
 70
           int db = (b.size() > 1) (b.size() == 1 && b[0])) ? b.size() - 1 : -1000;
 71
           return std::max(da, db + 1);
 72
         auto prime_power = [&](const vec &s, int64 mod, int64 p, int64 e) {
 73
 74
           // linear feedback shift register mod p^e, p is prime
 75
           std::vector<vec> a(e), b(e), an(e), bn(e), ao(e), bo(e);
 76
           vec t(e), u(e), r(e), to(e, 1), uo(e), pw(e + 1);
 77
           pw[0] = 1;
 78
           for (int i = pw[0] = 1; i \le e; ++i) pw[i] = pw[i - 1] * p;
 79
           for (int64 i = 0; i < e; ++i) {
 80
 81
             a[i] = {pw[i]}, an[i] = {pw[i]};
             b[i] = \{0\}, bn[i] = \{s[0] * pw[i] \% mod\};
 82
             t[i] = s[0] * pw[i] % mod;
 83
             if (t[i] == 0) {
 84
 85
                t[i] = 1, u[i] = e;
 86
             } else {
                for (u[i] = 0; t[i] \% p == 0; t[i] /= p, ++u[i])
 87
 88
             }
 89
 90
           for (size_t k = 1; k < s.size(); ++k) {</pre>
 91
             for (int g = 0; g < e; ++g) {
 92
 93
               if (L(an[g], bn[g]) > L(a[g], b[g])) {
 94
                  ao[g] = a[e - 1 - u[g]];
                  bo[g] = b[e - 1 - u[g]];
 95
                 to[g] = t[e - 1 - u[g]];
 96
                 uo[g] = u[e - 1 - u[g]];
 97
 98
                  r[g] = k - 1;
 99
               }
100
             }
101
             a = an, b = bn;
102
             for (int o = 0; o < e; ++o) {
103
                int64 d = 0;
104
                for (size_t i = 0; i < a[o].size() && i <= k; ++i) {</pre>
105
                  d = (d + a[o][i] * s[k - i]) % mod;
106
107
               if (d == 0) {
                 t[o] = 1, u[o] = e;
108
109
                } else {
                  for (u[o] = 0, t[o] = d; t[o] % p == 0; t[o] /= p, ++u[o])
110
111
112
                  int g = e - 1 - u[o];
                  if (L(a[g], b[g]) == 0) {
113
114
                    extand(bn[o], k + 1);
115
                    bn[o][k] = (bn[o][k] + d) \% mod;
                  } else {
116
117
                    int64 coef =
                        t[o] * inverse(to[g], mod) % mod * pw[u[o] - uo[g]] % mod;
118
119
                    int m = k - r[g];
120
                    extand(an[o], ao[g].size() + m);
121
                    extand(bn[o], bo[g].size() + m);
122
                    for (size_t i = 0; i < ao[g].size(); ++i) {</pre>
                      an[o][i + m] -= coef * ao[g][i] % mod;
123
124
                      if (an[o][i + m] < 0) an[o][i + m] += mod;
125
126
                    while (an[o].size() && an[o].back() == 0) an[o].pop_back();
127
                    for (size_t i = 0; i < bo[g].size(); ++i) {</pre>
                      bn[o][i + m] -= coef * bo[g][i] % mod;
128
129
                      if (bn[o][i + m] < 0) bn[o][i + m] -= mod;
130
                    while (bn[o].size() \&\& bn[o].back() == 0) bn[o].pop back();
131
                 }
132
133
               }
             }
134
```

```
135
136
           return std::make_pair(an[0], bn[0]);
137
138
139
         std::vector<std::tuple<int64, int64, int>> fac;
         for (int64 i = 2; i * i <= mod; ++i)</pre>
140
           if (mod % i == 0) {
141
142
             int64 cnt = 0, pw = 1;
             while (mod % i == 0) mod /= i, ++cnt, pw *= i;
143
             fac.emplace_back(pw, i, cnt);
144
145
         if (mod > 1) fac.emplace_back(mod, mod, 1);
146
         std::vector<vec> as;
147
148
         size_t n = 0;
149
         for (auto &&x : fac) {
           int64 mod, p, e;
150
151
           vec a, b;
152
           std::tie(mod, p, e) = x;
153
           auto ss = s;
154
           for (auto &&x : ss) x %= mod;
           std::tie(a, b) = prime_power(ss, mod, p, e);
155
           as.emplace_back(a);
156
           n = std::max(n, a.size());
157
158
         vec a(n), c(as.size()), m(as.size());
159
         for (size_t i = 0; i < n; ++i) {</pre>
160
161
           for (size_t j = 0; j < as.size(); ++j) {</pre>
162
             m[j] = std::get<0>(fac[j]);
             c[j] = i < as[j].size() ? as[j][i] : 0;
163
164
165
           a[i] = crt(c, m);
         }
166
167
         return a;
168
       }
169
       LinearRecurrence(const vec &s, const vec &c, int64 mod)
170
171
           : init(s), trans(c), mod(mod), m(s.size()) {}
172
173
       LinearRecurrence(const vec &s, int64 mod, bool is_prime = true) : mod(mod) {
         vec A = is_prime ? BerlekampMassey(s, mod) : ReedsSloane(s, mod);
174
175
         if (A.empty()) A = \{0\};
         m = A.size() - 1;
176
177
         trans.resize(m);
         for (int i = 0; i < m; ++i) { trans[i] = (mod - A[i + 1]) % mod; }
178
179
         std::reverse(trans.begin(), trans.end());
180
         init = {s.begin(), s.begin() + m};
181
182
183
       int64 calc(int64 n) {
         if (mod == 1) return 0:
184
185
         if (n < m) return init[n];</pre>
         vec v(m), u(m \ll 1);
186
187
         int msk = !!n;
188
         for (int64 m = n; m > 1; m >>= 1) msk <<= 1;
189
         v[0] = 1 \% mod;
190
         for (int x = 0; msk; msk >>= 1, x <<= 1) {
           std::fill_n(u.begin(), m * 2, 0);
191
192
           x = !!(n \& msk);
           if (x < m)
193
194
             u[x] = 1 \% mod;
195
           else { // can be optimized by fft/ntt
             for (int i = 0; i < m; ++i) {
196
197
               for (int j = 0, t = i + (x \& 1); j < m; ++j, ++t) {
                 u[t] = (u[t] + v[i] * v[j]) % mod;
198
               }
199
200
             }
201
             for (int i = m * 2 - 1; i >= m; --i) {
               for (int j = 0, t = i - m; j < m; ++j, ++t) {
202
```

```
u[t] = (u[t] + trans[j] * u[i]) % mod;
203
               }
204
205
             }
           }
206
207
             = {u.begin(), u.begin() + m};
208
209
         int64 ret = 0;
         for (int i = 0; i < m; ++i) { ret = (ret + v[i] * init[i]) % mod; }
210
211
         return ret;
212
213
214
       vec init, trans;
215
       int64 mod;
216
       int m;
217
```

1.19 线性规划

给定 n 个约束条件, m 个未知数, 求 $\sum (a[0][i] \times x[i])$ 的最大值, 约束条件: $\sum (-a[i][j] \times x[j]) \le a[i][0]$ 。若要求最小值,则进行对偶:即把目标函数的系数和约束条件右边的数交换,然后把矩阵转置。

```
const int MAXN = 3e3 + 3, MAXM = 3e3 + 3, INF = \sim 0U >> 2;
    int n, m, a[MAXN][MAXM], nxt[MAXM];
 2
 3
    void pivot(int 1, int e) {
      a[1][e] = -1;
 4
      int t = MAXM - 1;
 5
      for (int i = 0; i <= m; ++i)
 6
 7
        if (a[l][i]) nxt[t] = i, t = i;
      nxt[t] = -1;
 8
      for (int i = 0; i <= n; ++i)
9
        if (i != 1 && (t = a[i][e])) {
10
11
          a[i][e] = 0;
          for (int j = nxt[MAXM - 1]; \sim j; j = nxt[j]) a[i][j] += a[l][j] * t;
12
13
14
    int simplex() {
15
16
      for (;;) {
        int mi = INF, 1 = 0, e = 0;
17
18
        for (int i = 1; i <= m; ++i)
          if (a[0][i] > 0) {
19
20
            e = i;
21
            break;
22
23
        if (!e) return a[0][0];
24
        for (int i = 1; i <= n; ++i)
25
          if (a[i][e] < 0 && a[i][0] < mi) mi = a[i][0], l = i;</pre>
26
        pivot(1, e);
27
    }
28
```

1.20 实数线性规划

 $\vec{x} \max\{c\vec{x}|A\vec{x} \le b, \vec{x} >= 0\}$

```
typedef vector<double> VD;
1
2
   VD simplex(vector<VD> A, VD b, VD c) {
     int n = A.size(), m = A[0].size() + 1, r = n, s = m - 1;
3
     vector<VD> D(n + 2, VD(m + 1, 0));
     vector<int> ix(n + m);
5
6
     for (int i = 0; i < n + m; ++i) ix[i] = i;
      for (int i = 0; i <= n; ++i) {
7
       for (int j = 0; j < m - 1; ++j) D[i][j] = -A[i][j];
8
       D[i][m-1] = 1, D[i][m] = b[i];
10
        if (D[r][m] > D[i][m]) r = i;
11
```

```
for (int j = 0; j < m - 1; ++j) D[n][j] = c[j];
12
      D[n + 1][m - 1] = -1;
13
14
      for (double d;;) {
        if (r < n) {
15
16
          swap(ix[s], ix[r + m]);
          D[r][s] = 1 / D[r][s];
17
          vector<int> speed_up;
18
19
          for (int j = 0; j <= m; ++j)
20
            if (j != s) {
21
               D[r][j] *= -D[r][s];
22
               if (D[r][j]) speed_up.push_back(j);
23
            }
          for (int i = 0; i \le n + 1; ++i)
24
            if (i != r) {
25
26
               for (int j : speed_up) D[i][j] += D[r][j] * D[i][s];
               D[i][s] *= D[r][s];
27
28
            }
        }
29
        r = -1, s = -1;
30
        for (int j = 0; j < m; ++j)
  if ((s < 0 || ix[s] > ix[j]) &&
31
32
               (D[n + 1][j] > EPS || (D[n + 1][j] > -EPS && D[n][j] > EPS)))
33
34
            s = j;
35
        if (s < 0) break;
        for (int i = 0; i < n; ++i)
36
          if (D[i][s] < -EPS)
37
38
            if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS ||</pre>
39
                 (d < EPS && ix[r + m] > ix[i + m]))
40
               r = i;
        if (r < 0) return VD(); //无边界
41
42
43
      if (D[n + 1][m] < -EPS) return VD(); // 无解
44
      VD \times (m-1);
      for (int i = m; i < n + m; ++i)
45
        if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
46
      return x; // 最优值在D[n][m]
47
48
    }
```

UOJ 板题最快榜代码:

```
#include <bits/stdc++.h>
1
    using namespace std;
2
 3
 4
    #define eps 1e-7
    int simplex(vector<vector<double>> &a, vector<double> &b, vector<double> &c,
5
 6
                  vector<int> &basic) {
      int m = b.size(), n = c.size();
 7
 8
      while (true) {
9
        int k = -1;
        for (int j = 0; j < n; ++j)
  if (c[j] < -eps) {</pre>
10
11
12
             k = j;
13
             break;
          }
14
         if (k == -1) {
15
           double ans = 0;
16
           for (int i = 0; i < m; ++i) ans += c[basic[i]] * b[i];</pre>
17
18
           return 0;
        }
19
20
         int 1 = -1;
         for (int i = 0; i < m; ++i)
21
           if (a[i][k] > eps) {
22
23
             if (1 == -1)
24
               1 = i;
25
             else {
               double ti = b[i] / a[i][k], tl = b[l] / a[l][k];
26
               if (ti < tl - eps || (ti < tl + eps && basic[i] < basic[l])) l = i;</pre>
27
             }
28
          }
29
```

```
if (1 == -1) return -1;
30
        basic[1] = k;
31
32
        double tmp = 1 / a[1][k];
        for (int j = 0; j < n; ++j) a[1][j] *= tmp;</pre>
33
34
        b[1] *= tmp;
        for (int i = 0; i < m; ++i)
35
          if (i != 1) {
36
37
            tmp = a[i][k];
38
             for (int j = 0; j < n; ++j) a[i][j] -= tmp * a[1][j];</pre>
39
            b[i] -= tmp * b[1];
          }
40
41
        tmp = c[k];
        for (int j = 0; j < n; ++j) c[j] = tmp * a[1][j];
42
43
      }
    }
44
45
    int main() {
46
      ios::sync_with_stdio(false);
47
48
      int n, m, T;
49
      cin >> n >> m >> T;
      vector<double> c(n + m, 0);
50
      for (int i = 0; i < n; ++i) {
51
        cin >> c[i];
52
53
        c[i] *= -1;
      }
54
      auto C = c;
55
      vector<vector<double>> a(m, vector<double>(n + m, 0));
56
57
      vector<double> b(m);
      vector<int> basic(m, −1), tmp;
58
      for (int i = 0; i < m; ++i) {</pre>
59
        for (int j = 0; j < n; ++j) cin >> a[i][j];
60
61
        a[i][i + n] = 1;
        cin >> b[i];
62
        if (b[i] > -eps)
63
          basic[i] = i + n;
64
65
        else
66
          tmp.push_back(i);
67
68
      if (!tmp.empty()) {
        sort(tmp.begin(), tmp.end(), [&](int i, int j) { return b[i] > b[j]; });
69
70
        vector<vector<double>> A;
        vector<double> B, C(n + m + 1, 0);
71
        vector<int> Basic;
72
73
        for (int i : tmp) {
74
          vector<double> foo;
75
          for (int j = 0; j < n + m; ++j) foo.push_back(-a[i][j]);
          foo.push_back(1);
76
          double bar = -b[i];
77
78
          for (int i = 0; i < A.size(); ++i) {</pre>
            double tmp = foo[Basic[i]];
79
80
            for (int j = 0; j <= n + m; ++j) foo[j] -= tmp * A[i][j];
81
            bar -= tmp * B[i];
82
          for (int j = n + m; j >= 0; --j)
83
            if (-eps < foo[j] - 1 && foo[j] - 1 < eps) {</pre>
84
85
               Basic.push_back(j);
86
              break;
87
          for (int i = 0; i < A.size(); ++i) {</pre>
88
            double tmp = A[i][Basic.back()];
89
90
             for (int j = 0; j \le n + m; ++j) A[i][j] -= tmp * foo[j];
91
            B[i] = tmp * bar;
92
          A.push_back(foo);
93
94
          B.push_back(bar);
95
96
        for (int i = 0; i < A.size(); ++i)</pre>
97
          if (Basic[i] == n + m) {
```

```
for (int j = 0; j < n + m; ++j) C[j] = -A[i][j];
 98
           }
 99
100
         for (int i = 0; i < m; ++i)
           if (b[i] > -eps) {
101
102
             A.push_back(a[i]);
              A[A.size() - 1].push_back(0);
103
              B.push back(b[i]);
104
105
              Basic.push_back(basic[i]);
106
107
         simplex(A, B, C, Basic);
         bool flag = true;
108
         for (int i = 0; i < m; ++i)
109
           if (Basic[i] == n + m) {
110
111
             if (B[i] > eps) {
                cout << "Infeasible\n";</pre>
112
113
                return 0;
              }
114
115
              int k = -1;
              for (int j = 0; j < n + m; ++j)
116
117
                if (A[i][j] > eps || A[i][j] < -eps) {</pre>
                  k = j;
118
119
                  break;
                }
120
121
              if (k != -1) {
                double tmp = 1 / A[i][k];
122
123
                Basic[i] = k;
124
                for (int j = 0; j <= n + m; ++j) A[i][j] *= tmp;</pre>
                B[i] *= tmp;
125
126
                for (int 1 = 0; 1 < m; ++1)
127
                  if (1 != i) {
128
                    tmp = A[1][k];
                    for (int j = 0; j \le n + m; ++j) A[1][j] -= tmp * A[i][j];
129
                    B[1] = tmp * B[i];
130
131
132
              } else
133
                flag = false;
134
              break;
135
           }
136
         if (flag) {
           A.push_back(vector<double>(n + m, 0));
137
138
           A[A.size() - 1].push_back(1);
           B.push_back(0);
139
140
           Basic.push_back(n + m);
141
           for (int i = 0; i < A.size() - 1; ++i) {
              double tmp = A[i].back();
142
143
              for (int j = 0; j \le n + m; ++j) A[i][j] -= tmp * A[A.size() - 1][j];
              B[i] = tmp * B.back();
144
145
           }
         }
146
         a = A:
147
148
         b = B;
         basic = Basic;
149
150
         c.push_back(0);
151
         for (int i = 0; i < a.size(); ++i) {
           double tmp = c[basic[i]];
152
153
           for (int j = 0; j \le n + m; ++j) c[j] -= tmp * a[i][j];
154
         }
155
       auto foo = simplex(a, b, c, basic);
156
157
       if (foo == -1)
         cout << "Unbounded" << endl;</pre>
158
159
       else {
160
         double res = 0;
161
         vector<double> ans(n, 0);
         for (int i = 0; i < basic.size(); ++i)</pre>
162
163
           if (basic[i] < n) ans[basic[i]] = b[i];</pre>
164
         for (int j = 0; j < n; ++j) res -= C[j] * ans[j];</pre>
         cout << setprecision(8) << res << endl;</pre>
165
```

CCPC Final 2017 F: 有 N 组人,每组有 a_i 人,可进行若干次选择,每次选择一些至少有 M 人的组,这些人都中奖。现在要使每个人中奖概率相等,且中奖概率最大 $N \le 10, M, a_i \le 100$ 。两种 LP 写法:

```
const int MAXN = int(3e3);
    const int MAXM = int(3e3);
    const double INF = 1e20, EPS = 1e-9;
 3
 4
 5
 6
    double a[MAXM][MAXN], v;
    void pivot(int 1, int e) {
 8
9
      int i, j;
      a[l][e] = 1 / a[l][e];
10
      for (j = 0; j \le n; ++j)
11
12
        if (j != e) a[l][j] *= a[l][e];
      for (i = 1; i <= m; ++i)
13
14
        if (i != 1 && fabs(a[i][e]) > EPS) {
          for (j = 0; j \le n; ++j)
15
            if (j != e) a[i][j] -= a[i][e] * a[l][j];
16
17
          a[i][e] = -a[i][e] * a[1][e];
        }
18
19
      v += a[0][e] * a[1][0];
      for (j = 1; j \le n; ++j)
20
21
        if (j != e) a[0][j] -= a[0][e] * a[1][j];
22
      a[0][e] = -a[0][e] * a[1][e];
23
24
25
    double simplex() {
      int e, 1, i;
26
      double mn;
27
28
      v = 0;
      while (true) {
29
        for (e = 1; e <= n; ++e)
30
          if (a[0][e] > EPS) break;
31
        if (e > n) return v;
32
        for (i = 1, mn = INF; i <= m; ++i)</pre>
33
          if (a[i][e] > EPS && mn > a[i][0] / a[i][e])
34
            mn = a[i][0] / a[i][e], l = i;
35
36
        if (mn == INF) return INF;
        pivot(1, e);
37
38
    }
39
40
41
    void solve() {
      static int n, m, g[10];
42
43
      static vector<int> con[10], able;
      scanf("%d %d", &n, &m);
44
      for (int i = 0; i < n; ++i) {
45
        scanf("%d", g + i);
46
47
        con[i].clear();
48
      if (n == 1) {
49
        printf("%.10f\n", m >= g[0] ? 1. : 0.);
50
51
        return;
52
53
      able.clear();
      for (int s = 0, S = 1 << n; s < S; ++s) {
54
        int sum = 0;
55
        for (int i = 0; i < n; ++i)</pre>
56
57
          if (s >> i & 1) sum += g[i];
```

```
if (sum > m) continue;
58
        able.push back(s);
59
60
        for (int i = 0; i < n; ++i)
          if (s >> i & 1) con[i].push_back(able.size());
61
62
      ::n = able.size();
63
      ::m = 0;
64
      static random_device rd;
65
66
      mt19937 gen(rd());
67
      shuffle(able.begin(), able.end(), gen);
      for (int step = 0; step < n; ++step) {</pre>
68
        int f = ++::m;
69
        for (int i = 0; i <= ::n; ++i) a[f][i] = 0;
70
71
        for (int x : con[step]) ++a[f][x];
72
        if (step + 1 < n) {
          for (int x : con[step + 1]) --a[f][x];
73
74
        } else {
          for (int x : con[0]) --a[f][x];
75
76
77
      }
78
79
      ++::m;
      a[::m][0] = 1;
80
81
      for (int i = 1; i <= ::n; ++i) a[::m][i] = 1;
82
83
84
      a[::m][0] = -1;
      for (int i = 1; i <= ::n; ++i) a[::m][i] = -1;
85
86
87
      for (int i = 0; i <= ::n; ++i) a[0][i] = 0;
      for (int x : con[0]) ++a[0][x];
88
      printf("%.10f\n", simplex());
89
90
```

```
const int MAXN = 3000;
1
   const int MAXM = 3000;
2
   const db EPS = 1e-9;
   const db INF = 1e200;
 4
5
   namespace LP {
6
   db a[MAXM][MAXN];
7
   int idA[MAXN], idB[MAXN];
8
9
   int m, n;
10
11
    void put_out(int x) {
     if(x == 0)
12
13
        printf("Infeasible\n");
14
        printf("Unbounded\n");
15
16
      exit(0);
   }
17
18
   void pivot(int xA, int xB) {
      swap(idA[xA], idB[xB]);
19
20
      static int next[MAXN];
      int i, j, last = MAXN - 1;
21
      db tmp = -a[xB][xA];
22
23
      a[xB][xA] = -1.0;
24
      for (j = 0; j <= n; j++)</pre>
25
       if (fabs(a[xB][j]) > EPS) a[xB][last = next[last] = j] /= tmp;
26
      next[last] = -1;
27
      for (i = 0; i <= m; i++)
28
29
        if (i != xB && fabs(tmp = a[i][xA]) > EPS)
30
          for (a[i][xA] = 0.0, j = next[MAXN - 1]; ~j; j = next[j])
            a[i][j] += tmp * a[xB][j];
31
32
   db calc() {
33
34 int xA, xB;
```

```
db Max, tmp;
35
36
       while (1) {
 37
         xA = n + 1, idA[xA] = n + m + 1;
38
         for (int i = 1; i <= n; i++)
 39
           if (a[0][i] > EPS && idA[i] < idA[xA]) xA = i;</pre>
40
         if (xA == n + 1) return a[0][0];
41
         xB = m + 1, idB[xB] = n + m + 1, Max = -INF;
42
43
         for (int i = 1; i <= m; i++)
 44
           if (a[i][xA] < -EPS && ((tmp = a[i][0] / a[i][xA]) > Max + EPS ||
                                      (tmp > Max - EPS \&\& idB[i] < idB[xB])))
45
             Max = tmp, xB = i;
 46
         if (xB == m + 1) put_out(1);
47
48
         pivot(xA, xB);
 49
50
       return a[0][0];
     }
 51
     db solve() {
52
       for (int i = 1; i <= n; i++) idA[i] = i;
for (int i = 1; i <= m; i++) idB[i] = n + i;
 53
 54
       static db tmp[MAXN];
55
       db Min = 0.0;
       int 1;
 57
 58
       for (int i = 1; i <= m; i++)
         if (a[i][0] < Min) Min = a[i][0], l = i;</pre>
 59
60
       if (Min > -EPS) return calc();
 61
       idA[++n] = 0;
62
       for (int i = 1; i <= m; i++) a[i][n] = 1.0;
 63
       for (int i = 0; i <= n; i++) tmp[i] = a[0][i], a[0][i] = 0.0;
64
       a[0][n] = -1.0;
 65
 66
       pivot(n, 1);
 67
       if (calc() < -EPS) put_out(0);</pre>
 68
       for (int i = 1; i <= m; i++)</pre>
69
         if (!idB[i]) {
 70
 71
            for (int j = 1; j <= n; j++)
             if (fabs(a[0][j]) > EPS) {
 72
 73
                pivot(j, i);
 74
                break;
             }
 75
 76
           break;
 77
         }
 78
 79
       int xA;
 80
       for (xA = 1; xA \le n; xA++)
         if (!idA[xA]) break;
 81
       for (int i = 0; i <= m; i++) a[i][xA] = a[i][n];
 82
 83
       idA[xA] = idA[n], n--;
 84
 85
       for (int i = 0; i <= n; i++) a[0][i] = 0.0;
       for (int i = 1; i <= m; i++)</pre>
 86
 87
         if (idB[i] <= n) {</pre>
           for (int j = 0; j <= n; j++) a[0][j] += a[i][j] * tmp[idB[i]];</pre>
 88
 89
 90
91
       for (int i = 1; i <= n; i++)
 92
         if (idA[i] <= n) a[0][i] += tmp[idA[i]];</pre>
93
       return calc();
94
     db ans[MAXN];
 95
96
     void findAns() {
       for (int i = 1; i <= n; i++) ans[i] = 0.0;
97
       for (int i = 1; i <= m; i++)
98
         if (idB[i] <= n) ans[idB[i]] = a[i][0];</pre>
99
     }
100
101
    void work() {
     for (int i = 1; i <= m; ++i)
```

```
for (int j = 1; j <= n; ++j) a[i][j] *= -1;
103
104
       printf("%.10f\n", -double(solve()));
105
     }
     } // namespace LP
106
107
     void solve() {
108
109
       static int n, m, g[10];
       static vector<int> con[10], able;
110
111
112
       scanf("%d %d", &n, &m);
       for (int i = 0; i < n; ++i) {
113
         scanf("%d", g + i);
114
         con[i].clear();
115
116
117
       if (n == 1) {
         printf("%.10f\n", m >= g[0] ? 1.0 : 0.0);
118
119
         return;
       }
120
       able.clear();
121
       for (int s = 0; s < (1 << n); ++s) {
122
         int sum = 0;
123
         for (int i = 0; i < n; ++i)
124
           if (s >> i & 1) sum += g[i];
125
126
         if (sum > m) continue;
127
         able.push_back(s);
         for (int i = 0; i < n; ++i)
128
129
           if (s >> i & 1) con[i].push_back(able.size());
130
131
       LP::n = able.size(), LP::m = 0;
132
133
       for (int step = 0; step < n; ++step) {</pre>
         int &f = ++LP::m;
134
         for (int i = 0; i <= LP::n; ++i) LP::a[f][i] = 0;
135
136
         for (int x : con[step]) ++LP::a[f][x];
         if (step + 1 < n) {
137
           for (int x : con[step + 1]) --LP::a[f][x];
138
139
         } else {
140
           for (int x : con[0]) --LP::a[f][x];
141
         }
       }
142
143
       ++LP::m;
144
145
       LP::a[LP::m][0] = 1;
146
       for (int i = 1; i <= LP::n; ++i) LP::a[LP::m][i] = 1;
147
148
       ++LP::m;
       LP::a[LP::m][0] = -1;
149
       for (int i = 1; i <= LP::n; ++i) LP::a[LP::m][i] = -1;
150
151
       for (int i = 0; i <= LP::n; ++i) LP::a[0][i] = 0;
152
153
       for (int x : con[0]) ++LP::a[0][x];
154
155
       static db a2[MAXM][MAXN];
156
       for (int i = 1; i <= LP::m; ++i)
         for (int j = 1; j <= LP::n; ++j) a2[i][j] = LP::a[i][j];</pre>
157
158
       for (int i = 1; i <= LP::m; ++i)</pre>
         for (int j = 1; j <= LP::n; ++j) LP::a[j][i] = a2[i][j];</pre>
159
160
       swap(LP::n, LP::m);
       for (int i = 1; i \le max(LP::n, LP::m); ++i) swap(LP::a[0][i], LP::a[i][0]);
161
162
       LP::a[0][0] = 0;
       for (int i = 1; i <= LP::m; ++i)
163
         for (int j = 1; j <= LP::n; ++j) LP::a[i][j] *= -1;
164
       for (int i = 1; i <= LP::m; ++i) LP::a[i][0] *= -1;
165
166
       for (int i = 1; i <= LP::n; ++i) LP::a[0][i] *= -1;
167
       LP::work();
168
169
     }
```

1.21 快速傅里叶变换

```
const int MAXN = 4 * 1e5 + 3;
   const double PI = acos(-1);
   complex<double> a[MAXN], b[MAXN];
3
 4
5
    int n, bit;
   int rev[MAXN];
6
    void fft(complex<double> *a, int sign) {
8
9
      for (int i = 0; i < n; ++i)
        if (i < rev[i]) swap(a[i], a[rev[i]]);</pre>
10
      for (int j = 1; j < n; j <<= 1) {</pre>
11
        complex < double > wn(cos(2 * PI / (j << 1)), sign * sin(2 * PI / (j << 1)));
12
        for (int i = 0; i < n; i += (j << 1)) {
13
          complex<double> w(1, 0), t0, t1;
14
          for (int k = 0; k < j; ++k, w *= wn) {
15
16
            t0 = a[i + k], t1 = w * a[i + j + k];
17
            a[i + k] = t0 + t1, a[i + j + k] = t0 - t1;
18
19
        }
20
      if (sign == -1) for (int i = 0; i < n; ++i) a[i] /= n;
21
22
23
24
    int main() {
25
      int n, m, x;
26
      cin >> n >> m;
      for (int i = 0; i <= n; ++i) cin >> x, a[i].real(x);
27
28
      for (int i = 0; i \le m; ++i) cin >> x, b[i].real(x);
29
30
      for (::n = 1, bit = 0; ::n <= n + m; ++bit) ::n <<= 1;
      rev[0] = 0;
31
      for (int i = 1; i < ::n; ++i) rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (bit - 1));
32
      fft(a, 1), fft(b, 1);
33
34
      for (int i = 0; i < ::n; ++i) a[i] *= b[i];
35
      fft(a, -1);
36
      for (int i = 0; i < n + m + 1; ++i) cout << int(a[i].real() + .5) << " \n"[i = n + m];
37
      return 0:
38
```

1.22 快速数论变换

998244353 原根为 3, 1004535809 原根为 3, 786433 原根为 10, 880803841 原根为 26。

```
void ntt(int *x, int n, int sign) {
1
       for (int i = 0; i < n; ++i)
2
         if (rev[i] < i) swap(x[i], x[rev[i]]);</pre>
3
       for (int j = 1; j < n; j <<= 1) {
4
         int gn = mpow(G, (P - 1) / (j << 1));
 5
6
         for (int i = 0; i < n; i += (j << 1)) {
           for (int k = 0, g = 1; k < j; ++k, g = 111 * g * gn % P) {
  int &a = x[i + j + k], &b = x[i + k], t = 111 * g * a % P;</pre>
7
8
              (a = b - t) < 0 ? a += P : 0;
9
10
              (b = b + t) >= P ? b -= P : 0;
           }
11
12
         }
13
      if (sign == -1) {
14
         reverse(x + 1, x + n);
15
16
         for (int i = 0, inv = mpow(n, P - 2); i < n; ++i)
17
           x[i] = 111 * x[i] * inv % P;
      }
18
    }
19
```

1.23 快速沃尔什变换

 $C_i = \sum_{i=j \oplus k} A_j B_k$,其中 \oplus 为位运算。

```
1
    void fwt(int *a, int n) {
2
      for (int d = 1; d < n; d <<= 1)
3
         for (int m = d << 1, i = 0; i < n; i += m)
 4
           for (int j = 0; j < d; ++j) {
             int x = a[i + j], y = a[i + j + d];
5
             a[i + j] = x + y;
                                                         // AND
 6
 7
             a[i + j + d] = x + y;
                                                         // OR
 8
             a[i + j] = x + y, a[i + j + d] = x - y; // XOR
 9
    }
10
    void ufwt(int *a, int n) {
11
12
      for (int d = 1; d < n; d <<= 1)
        for (int m = d << 1, i = 0; i < n; i += m)
for (int j = 0; j < d; ++j) {</pre>
13
14
             int x = a[i + j], y = a[i + j + d];
15
             a[i + j] = x - y;
                                                                       // AND
                                                                       // OR
             a[i + j + d] = y - x;
17
             a[i + j] = (x + y) / 2, a[i + j + d] = (x - y) / 2; // XOR
18
19
    }
20
```

2 动态规划

2.1 斜率优化

树上斜率优化,定义 dp_i 表示 i 节点传递到根节点的最短耗时,规定 $dp_{root} = -P$,有如下转移方程 $dp_u = dp_v + dist(u, v)^2 + P$,v 为 u 的祖先。

```
vector<pii> adj[MAXN];
    11 dp[MAXN], d[MAXN];
 2
    int n, p, q[MAXN], head, tail;
 4
    inline 11 S(int a, int b) { return (d[b] - d[a]) << 1; } inline 11 G(int a, int b) { return dp[b] - dp[a] + d[b] * d[b] - d[a] * d[a]; }
 5
 6
 7
    void dfs(int u, int from) {
 8
      vector<int> dhead, dtail;
9
      if (u ^ 1) {
10
11
        while (head + 2 <= tail &&
                S(q[head + 1], q[head]) * d[u] \leftarrow G(q[head + 1], q[head]))
12
13
           dhead.push_back(q[head++]);
        int v = q[head];
14
        dp[u] = dp[v] + p + (d[u] - d[v]) * (d[u] - d[v]);
15
16
17
      while (head + 2 <= tail &&</pre>
              G(u, q[tail - 1]) * S(q[tail - 1], q[tail - 2]) <=
18
                  G(q[tail - 1], q[tail - 2]) * S(u, q[tail - 1]))
19
        dtail.push_back(q[--tail]);
20
21
      q[tail++] = u;
      for (pii &e : adj[u]) {
22
23
         if (e.first == from) continue;
        d[e.first] = d[u] + e.second;
24
25
        dfs(e.first, u);
      }
26
27
      for (int i = dtail.size() - 1; ~i; --i) q[tail++] = dtail[i];
28
29
      for (int i = dhead.size() - 1; ~i; --i) q[--head] = dhead[i];
30
31
    void solve() {
32
33
      cin >> n >> p;
      for (int i = 1; i <= n; ++i) adj[i].clear();
```

```
35
      for (int i = 1, u, v, w; i < n; ++i) {
36
        cin >> u >> v >> w;
37
        adj[u].emplace_back(v, w);
38
        adj[v].emplace_back(u, w);
39
      dp[1] = -p;
40
      head = tail = 0;
41
      dfs(1, 1);
42
43
44
      11 ans = 0;
      for (int i = 1; i <= n; ++i)
45
       if (dp[i] > ans) ans = dp[i];
46
      cout << ans << '\n';
47
48
```

2.2 整数划分

 $f_{i,j}$ 表示选了 i 种不同的数字,和为 j 的方案数。 $f_{i,j}=f_{i-1,j-1}+f_{i,j-i}$,意义:要么新选一个 1,要么前面的数都加一。若每个数字最多一个, $f_{i,j}=f_{i-1,j-i}+f_{i,j-i}$ 。 求将 n 划分为若干整数的方案,则设 g_n 为答案,代码如下,时间复杂度 $O(n\sqrt{n})$ 。

```
int f[732], g[200001];
void init() {
    f[1] = 1, f[2] = 2, f[3] = 5, f[4] = 7;
    for (int i = 5; i < 732; ++i) f[i] = 3 + 2 * f[i - 2] - f[i - 4];
    for (int i = g[0] = 1; i <= n; ++i)
    for (int j = 1; f[j] <= i; ++j)
        g[i] = ((j + 1) >> 1 & 1) ? (g[i] + g[i - f[j]]) % MOD : (g[i] - g[i - f[j]] + MOD) % MOD;
}
```

3 数据结构

3.1 KD Tree

```
// 寻找近点
 1
    #include <bits/stdc++.h>
    using namespace std;
 4
    const int MAXN = 2e5 + 5;
 5
 6
    typedef long long 11;
    namespace KD_Tree {
 9
    const int DIM = 2;
10
11
12
    inline 11 sqr(int x) { return 1LL * x * x; }
13
    struct Point {
14
      int x[DIM], id, c;
15
16
      11 dist2(const Point &b) const {
17
18
        return sqr(x[0] - b.x[0]) + sqr(x[1] - b.x[1]);
      }
19
20
    };
    struct QNode {
21
      Point p;
22
23
      11 dis2;
24
25
      QNode() {}
26
      QNode(Point _p, ll _dis2) : p(_p), dis2(_dis2) {}
27
      bool operator<(const QNode &b) const {</pre>
28
        return dis2 < b.dis2 || (dis2 == b.dis2 && p.id < b.p.id);</pre>
29
```

```
}
30
    } ans;
31
32
    struct cmpx {
      int div;
33
34
      cmpx(int _div) : div(_div) {}
      bool operator()(const Point &a, const Point &b) {
35
         for (int i = 0; i < DIM; ++i)
36
           if (a.x[(i + div) \% DIM] != b.x[(i + div) \% DIM])
37
38
             return a.x[(i + div) \% DIM] < b.x[(i + div) \% DIM];
39
         return true;
      }
40
    };
41
42
    bool cmp(const Point &a, const Point &b, int div) {
43
44
      cmpx cp = cmpx(div);
      return cp(a, b);
45
46
47
48
    struct Node {
49
      Point e;
      Node *lc, *rc;
50
      int div;
51
    } node_pool[MAXN], *tail, *root;
52
53
    void init() { tail = node_pool; }
    Node *build(Point *a, int 1, int r, int div) {
54
      if (1 >= r) return nullptr;
55
56
      Node *p = tail++;
      p->div = div;
57
      int mid = (1 + r) >> 1;
58
      nth_element(a + 1, a + mid, a + r, cmpx(div));
59
60
      p->e = a[mid];
61
      p->lc = build(a, 1, mid, div ^ 1);
62
      p->rc = build(a, mid + 1, r, div ^ 1);
63
      return p;
    }
64
    void search(Point p, Node *x, int div) {
65
66
      if (!x) return;
67
      if (cmp(p, x->e, div)) {
68
         search(p, x \rightarrow lc, div ^ 1);
         if (ans.dis2 == -1) {
69
70
           if (x\rightarrow e.c \le p.c) ans = QNode(x\rightarrow e, p.dist2(x\rightarrow e));
           search(p, x->rc, div ^ 1);
71
72
         } else {
           QNode temp(x\rightarrow e, p.dist2(x\rightarrow e));
73
74
           if (x\rightarrow e.c \le p.c \&\& temp < ans) ans = temp;
75
           if (sqr(x->e.x[div] - p.x[div]) \le ans.dis2) search(p, x->rc, div ^ 1);
         }
76
77
      } else {
         search(p, x->rc, div ^ 1);
78
         if (ans.dis2 == -1) {
79
80
           if (x\rightarrow e.c \le p.c) ans = QNode(x\rightarrow e, p.dist2(x\rightarrow e));
           search(p, x->lc, div ^ 1);
81
82
         } else {
           QNode temp(x\rightarrow e, p.dist2(x\rightarrow e));
83
           if (x->e.c <= p.c && temp < ans) ans = temp;</pre>
84
85
           if (\operatorname{sqr}(x->e.x[\operatorname{div}] - p.x[\operatorname{div}]) \le \operatorname{ans.dis2}) search(p, x->lc, div ^ 1);
86
         }
87
      }
    }
88
89
    void search(Point p) {
90
      ans.dis2 = -1;
91
      search(p, root, 0);
92
93
    } // namespace KD_Tree
94
    void solve() {
95
96
      static KD_Tree::Point p[MAXN];
      int n, m;
```

```
98
       cin >> n >> m;
       for (int i = 0; i < n; ++i) {
 99
100
         p[i].id = i;
101
         cin >> p[i].x[0] >> p[i].x[1] >> p[i].c;
102
       KD_Tree::init();
103
       KD_Tree::root = KD_Tree::build(p, 0, n, 0);
104
105
106
       for (KD_Tree::Point q; m; --m) {
107
         cin >> q.x[0] >> q.x[1] >> q.c;
         KD_Tree::search(q);
108
         cout << KD_Tree::ans.p.x[0] << ' ' << KD_Tree::ans.p.x[1] << ' '
109
110
               << KD_Tree::ans.p.c << '\n';
111
       }
     }
112
     int main() {
113
       ios::sync_with_stdio(false);
114
       cin.tie(nullptr);
115
116
117
       int o_o;
       for (cin >> o_o; o_o; --o_o) solve();
118
119
120
       return 0;
     }
121
122
     // 寻找远点
123
     inline void cmin(int &a, int b) { b < a ? a = b : 1; }</pre>
     inline void cmax(int &a, int b) { a < b ? a = b : 1; }</pre>
125
     inline int ibs(int a) { return a < 0 ? -a : a; }</pre>
     struct D {
127
128
       int d[2], mx0, mx1, mi0, mi1;
129
       D *1, *r;
130
     } t[N], *rt;
     int cpd, ans;
131
132
     inline bool cmp(const D &a, const D &b) {
       return (a.d[cpd] ^ b.d[cpd]) ? a.d[cpd] < b.d[cpd]</pre>
133
134
                                       : a.d[cpd ^ 1] < b.d[cpd ^ 1];
135
136
     inline void kd_upd(D *u) {
       if (u->1) {
137
138
         cmax(u->mx0, u->1->mx0);
         cmax(u->mx1, u->l->mx1);
cmin(u->mi0, u->l->mi0);
139
140
141
         cmin(u->mi1, u->l->mi1);
142
143
       if (u->r) {
         cmax(u->mx0, u->r->mx0);
144
         cmax(u->mx1, u->r->mx1);
cmin(u->mi0, u->r->mi0);
145
146
         cmin(u->mi1, u->r->mi1);
147
148
       }
     }
149
150
     D *kd_bld(int 1, int r, int d) {
151
       int m = 1 + r >> 1;
       cpd = d;
152
153
       std::nth_element(t + 1 + 1, t + m + 1, t + r + 1, cmp);
       t[m].mx0 = t[m].mi0 = t[m].d[0];
154
155
       t[m].mx1 = t[m].mi1 = t[m].d[1];
       if (1 ^ m) t[m].1 = kd_bld(1, m - 1, d ^ 1);
156
157
       if (r ^ m) t[m].r = kd_bld(m + 1, r, d ^ 1);
       kd_upd(t + m);
158
159
       return t + m;
160
161
     inline void kd_ins(D *ne) {
       int cd = 0;
162
163
       D *u = rt;
164
       while (true) {
         cmax(u->mx0, ne->mx0), cmin(u->mi0, ne->mi0);
165
```

```
cmax(u->mx1, ne->mx1), cmin(u->mi1, ne->mi1);
166
          if (ne->d[cd] < u->d[cd]) {
167
168
            if (u->1)
              u = u \rightarrow 1;
169
170
            else {
              u->1 = ne;
171
172
              return;
            }
173
          } else {
174
175
            if (u->r)
              u = u \rightarrow r;
176
            else {
177
178
              u->r = ne;
179
              return;
            }
180
          }
181
182
          cd ^= 1;
       }
183
184
185
     inline int dist(int x, int y, D *u) {
       int r = 0;
186
       if (x < u->mi0)
187
188
          r = u - mi0 - x;
189
       else if (x > u->mx0)
190
         r = x - u - mx0;
       if (y < u->mi1)
191
192
         r += u -> mi1 - y;
193
       else if (y > u->mx1)
194
         r += y - u -> mx1;
195
       return r;
196
197
     inline void kd_quy(D *u, const int &x, const int &y) {
198
       int dl, dr, d0;
199
       d0 = ibs(u->d[0] - x) + ibs(u->d[1] - y);
200
       if (d0 < ans) ans = d0;
       dl = u->1 ? dist(x, y, u->1) : inf;
201
202
       dr = u \rightarrow r? dist(x, y, u \rightarrow r): inf;
203
       if (dl < dr) {</pre>
204
          if (dl < ans) kd_quy(u->1, x, y);
          if (dr < ans) kd_quy(u->r, x, y);
205
206
       } else {
          if (dr < ans) kd_quy(u->r, x, y);
207
208
          if (dl < ans) kd_quy(u->1, x, y);
209
       }
     }
210
```

3.2 zkw 线段树

```
1
    int tree[MAXN * 2], pre;
2
3
    void init(int n, int *a) {
      memset(tree, 0, sizeof(tree));
 4
      for (pre = 1; pre <= n; pre <<= 1) {}
5
      for (int i = 1; i <= n; ++i) tree[i + pre] = a[i];</pre>
 6
 7
      for (int i = pre; i; --i) tree[i] = max(tree[i << 1], tree[i << 1 | 1]);
   }
8
9
   void update(int pos, const int &val) {
10
     tree[pos += pre] = val;
11
12
      for (pos >>= 1; pos; pos >>= 1)
        tree[pos] = max(tree[pos << 1], tree[pos << 1 | 1]);</pre>
13
14
15
   int query(int s, int t) {
16
17
      int res = 0;
     for (s += pre - 1, t += pre + 1; s ^ t ^ 1; s >>= 1, t >>= 1) {
```

```
if (~s & 1) res = max(res, tree[s ^ 1]);
if (t & 1) res = max(res, tree[t ^ 1]);
}
return res;
}
```

3.3 Splay

```
struct Node {
 1
       long long sum;
 2
       int id, val, lazy, size;
 3
       Node *fa, *ch[2];
 4
 5
    } node_pool[MAXN], *pool_it, *root, *nil;
 6
 7
    Node *newnode(int id, int val) {
 8
       pool_it->id = id;
       pool_it->lazy = 0;
 9
       pool_it->size = 1;
10
11
       pool_it->sum = pool_it->val = val;
       pool_it->fa = pool_it->ch[0] = pool_it->ch[1] = nil;
12
13
       return pool_it++;
    }
14
15
16
    void maintain(Node *u) {
       if (u == nil) { return; }
17
       u->size = u->ch[0]->size + u->ch[1]->size + 1;
18
       u \rightarrow sum = u \rightarrow ch[0] \rightarrow sum + u \rightarrow ch[1] \rightarrow sum + u \rightarrow val;
19
20
21
22
    void push_down(Node *u) {
23
       if (u->lazy) {
24
         if (u->ch[0] != nil) {
25
            u->ch[0]->val += u->lazy;
26
            u - ch[0] - sum += 1LL * u - ch[0] - size * u - slazy;
27
            u\rightarrow ch[0]\rightarrow lazy += u\rightarrow lazy;
28
29
         if (u->ch[1] != nil) {
30
            u->ch[1]->val += u->lazy;
            u \rightarrow ch[1] \rightarrow sum += 1LL * u \rightarrow ch[1] \rightarrow size * u \rightarrow lazy;
31
            u\rightarrow ch[1]\rightarrow lazy += u\rightarrow lazy;
32
33
34
         u->lazy = 0;
35
       }
    }
36
37
    inline void rot(Node *u) {
38
39
       Node *f = u \rightarrow fa, *ff = f \rightarrow fa;
       int d = u == f->ch[1];
40
41
       push_down(f);
       push_down(u);
42
       if ((f->ch[d] = u->ch[d ^ 1]) != nil) f->ch[d]->fa = f;
43
44
       if ((u-)fa = ff) != nil) ff->ch[f == ff->ch[1]] = u;
       f->fa = u;
45
       u - ch[d ^ 1] = f;
46
47
       maintain(f);
       maintain(u);
48
49
50
    void splay(Node *u, Node *target) {
51
       for (Node *f; u->fa != target; rot(u))
52
53
         if ((f = u->fa)->fa != target) {
54
            ((u == f->ch[1]) ^ (f == f->fa->ch[1])) ? rot(u) : rot(f);
55
56
       if (target == nil) root = u;
    }
57
58
```

```
inline void insert(int id, int val) {
       if (root == nil) {
 60
 61
         root = newnode(id, val);
         return;
 62
 63
       Node *u = root;
 64
       while (u != nil) {
 65
         int d = id >= u -> id;
 66
 67
         ++u->size;
 68
         push_down(u);
         u->sum += val;
 69
 70
         if (u->ch[d] != nil) {
           u = u - > ch[d];
 71
 72
         } else {
           u->ch[d] = newnode(id, val);
 73
           u->ch[d]->fa = u;
 74
 75
           u = u - > ch[d];
 76
           break;
 77
 78
       }
 79
       splay(u, nil);
 80
     }
 81
 82
     inline Node *find_pred(int id) {
       Node *u = root, *ret = nil;
 83
 84
       while (u != nil) {
 85
         push_down(u);
         if (u->id < id) {</pre>
 86
 87
           ret = u;
 88
           u = u -> ch[1];
 89
         } else {
 90
           u = u - > ch[0];
 91
 92
       }
 93
       return ret;
     }
 94
 95
     inline Node *find_succ(int id) {
 96
 97
       Node *u = root, *ret = nil;
       while (u != nil) {
 98
 99
         push_down(u);
100
          if (u->id > id) {
101
           ret = u;
102
           u = u - ch[0];
103
         } else {
104
           u = u - ch[1];
105
106
       }
107
       return ret;
     }
108
109
     Node *find_kth(int k) {
110
111
       Node *u = root;
       while (u != nil) {
112
         push_down(u);
113
         if (u->ch[0]->size + 1 == k) {
114
115
            splay(u, nil);
116
           return u;
117
         if (u->ch[0]->size >= k) {
118
119
           u = u - > ch[0];
120
         } else {
121
           k = u - ch[0] - size + 1;
122
           u = u \rightarrow ch[1];
123
       }
124
125
       return nil;
126
```

```
127
128
     Node *range(int 1, int r) {
129
       Node *pred = find_pred(1);
       Node *succ = find_succ(r);
130
131
       splay(pred, nil);
132
133
       splay(succ, root);
       push_down(pred);
134
135
       push_down(succ);
136
       return root->ch[1]->ch[0];
     }
137
138
     void init() {
139
140
       pool_it = node_pool;
141
       nil = pool_it++;
       nil->ch[0] = nil->ch[1] = nil->fa = nil;
142
       nil->id = -1, nil->val = 0;
143
144
       root = nil;
145
146
       insert(INT_MIN, 0), insert(INT_MAX, 0);
147
```

3.4 LCT

```
struct LCT {
1
2
      struct node {
3
        int val, add;
 4
        node *fa, *ch[2];
        void modify(const int &x) {
5
6
          val += x, add += x;
7
      } node_mset[MaxS], *cnode, *null;
8
9
      LCT() {
10
        cnode = node_mset, null = cnode++;
        *null = (node){0, 0, null, {null, null}};
11
12
13
      inline node *newnode() {
14
        *cnode = (node){0, 0, null, {null, null}};
        return cnode++;
15
16
17
      inline bool isrt(node *u) const {
        return (u->fa->ch[0] != u) && (u->fa->ch[1] != u);
18
19
      inline bool which(node *u) const { return u->fa->ch[1] == u; }
20
21
      void push_down(node *u) {
        if (!isrt(u)) push_down(u->fa);
22
23
        if (u->add) {
24
          u->ch[0]->modify(u->add);
          u->ch[1]->modify(u->add);
25
26
          u->add = 0;
27
        }
28
29
      inline void rotate(node *u) {
        node *f = u - > fa;
30
31
        int d = which(u);
        f->ch[d] = u->ch[d ^ 1];
32
33
        f->ch[d]->fa=f;
        u - ch[d ^ 1] = f;
34
        u->fa = f->fa;
35
36
        if (!isrt(f)) f->fa->ch[which(f)] = u;
37
        f->fa = u;
38
39
      inline void splay(node *u) {
        push_down(u);
40
        for (node *f; !isrt(u); rotate(u))
41
          if (!isrt(f = u->fa)) rotate(which(u) == which(f) ? f : u);
42
```

```
43
      inline void access(node *x) {
44
45
        for (node *y = null; x != null; x = x->fa) {
          splay(x);
46
47
          x->ch[1] = y;
48
          y = x;
        }
49
50
      }
51
      inline void cut(node *u) {
52
        access(u), splay(u);
        u->ch[0]->fa = null;
53
        u->ch[0] = null;
54
55
56
      inline void link(node *u, node *v) {
57
        cut(u), u->fa = v;
58
    } tree;
```

4 字符串

4.1 KMP

```
void get_next(char *S, int *nxt, int n) {
      nxt[0] = -1;
2
 3
      int j = -1;
 4
      for (int i = 1; i < n; ++i) {
5
        while ((\sim j) \&\& S[j + 1] != S[i]) j = nxt[j];
        nxt[i] = (S[j + 1] == S[i]) ? (++j) : j;
 6
 7
      }
    }
 8
9
    int pattern(char *S, char *T, int *nxt, int n, int m) {
10
11
      int j = -1;
      for (int i = 0; i < m; ++i) {
12
13
        while ((\sim j) \&\& S[j + 1] != T[i]) j = nxt[j];
        j += S[j + 1] == T[i];
14
        if (j == n - 1) return i - n + 1;
15
16
      }
17
      return -1;
    }
```

4.2 拓展 KMP

next[i]: x[i...m-1] 与 x[0...m-1] 的最长公共前缀, extend[i]: y[i...n-1] 与 x[0...m-1] 的最长公共前缀

```
void prework(char x[], int m, int next[]) {
 1
 2
      next[0] = m;
3
      int j = 0;
      while (j + 1 < m \& x[j] == x[j + 1]) ++j;
 4
      next[1] = j;
 5
      int k = 1;
 6
 7
      for (int i = 2; i < m; ++i) {
        int p = next[k] + k - 1;
8
 9
        int L = next[i - k];
        if (i + L 
10
          next[i] = L;
11
12
        else {
13
          j = max(0, p - i + 1);
          while (i + j < m && x[i + j] == x[j]) j++;
next[i] = j, k = i;</pre>
14
15
16
      }
17
18 }
```

```
void exkmp(char x[], int m, char y[], int n, int next[], int extend[]) {
19
20
      prework(x, m, next);
21
      int j = 0;
      while (j < n \&\& j < m \&\& x[j] == y[j]) ++j;
22
23
      extend[0] = j;
      int k = 0;
24
      for (int i = 1; i < n; ++i) {
25
26
        int p = extend[k] + k - 1;
27
        int L = next[i - k];
28
        if (i + L 
          extend[i] = L;
29
30
        else {
          j = max(0, p - i + 1);
31
          while (i + j < n \&\& j < m \&\& y[i + j] == x[j]) j++;
32
33
          extend[i] = j, k = i;
34
35
      }
   }
36
```

4.3 AC 自动机

```
int tr[MAX_NODE][26], fail[MAX_NODE], dep[MAX_NODE], node_c;
 2
 3
    int add_char(int u, int id) {
      if (tr[u][id] < 0) tr[u][id] = node_c++;</pre>
 4
      return tr[u][id];
 5
    }
 6
 7
    void build_acam() {
 8
      queue<int> que;
      fail[0] = 0;
9
      for (int i = 0; i < 26; ++i)
10
11
        if (~tr[0][i]) {
          que.push(tr[0][i]);
12
13
          fail[tr[0][i]] = 0;
        } else {
14
15
          tr[0][i] = 0;
16
17
      while (!que.empty()) {
18
        int u = que.front(), f = fail[u];
        que.pop();
19
        for (int i = 0; i < 26; ++i)
20
          if (~tr[u][i]) {
21
22
            que.push(tr[u][i]);
23
             fail[tr[u][i]][i] = tr[f][i];
24
          } else {
25
             tr[u][i] = tr[f][i];
26
27
      for (int i = 1; i < node_c; ++i) adj[fail[i]].push_back(i);</pre>
28
    }
29
```

4.4 各种哈希

- 树哈希:将子树当作集合哈希,加入深度的影响。
- 集合哈希: 可以使用元素的哈希值映射为高进制的某一位, 也可以使用质数的积;

```
1 const unsigned int KEY = 6151; const unsigned int MOD = 1610612741; // 64 位哈希参数 KEY 随意 MOD 461168601842738784711 unsigned int hash[MAXN], p[MAXN];

6 unsigned int get_hash(int l, int r) { return (hash[r] + MOD - 1ull * hash[l - 1] * p[r - l + 1] % MOD) % MOD; }
```

```
7
8
void init(char *s, int n) {
9
p[0] = 1;
for (int i = 1; i <= n; ++i) {
    p[i] = 1ull * p[i - 1] * KEY % MOD;
    hash[i] = (1LL * hash[i - 1] * KEY + s[i]) % MOD;
}
13
}
14</pre>

void init(char *s, int n) {
    p[0] = 1;
    for (int i = 1; i <= n; ++i) {
        p[i] = 1ull * p[i - 1] * KEY % MOD;
        hash[i] = (1LL * hash[i - 1] * KEY + s[i]) % MOD;
}
</pre>
```

CCPC 秦皇岛 2020 J: 两次哈希

```
1
    const int MAXN = 3e5 + 3;
   const int MAX_PRIME = 8960453 + 3;
2
   const int MOD = 998244353;
   const 11 BASE = 709;
 4
5
    const 11 HASH MOD = 461168601842738784711;
 6
   char s[MAXN];
7
 8
   int fac[MAXN], inv[MAXN], fac_inv[MAXN], prime[MAXN * 2];
   11 ha[MAXN], p[MAXN], pref[MAXN], suff[MAXN], value[MAXN * 2];
9
10
   pair<11, int> bin[MAXN];
11
   int cnt[MAXN], pidx[MAXN], sidx[MAXN];
12
   bitset<MAX_PRIME> mark;
13
14
15
   11 fmul(ll a, ll b) {
      11 k = (11)((1.1 * a * b) / (1.1 * HASH_MOD)), t = a * b - k * HASH_MOD;
16
      for (t -= HASH_MOD; t < 0; t += HASH_MOD) {}</pre>
17
18
      return t;
19
20
   11 getRange(int 1, int r) {
     return (ha[r] - fmul(ha[l - 1], p[r - l + 1]) + HASH_MOD) % HASH_MOD;
21
22
23
24
   int gao(int n, int d) {
25
      int tot = 0;
26
      for (int l = 1, r = d; r \le n; l += d, r += d) value[tot++] = getRange(l, r);
      int bunch = n / d, rest = n % d;
27
28
      if (rest) {
29
        for (int r = n, l = n - d + 1; l >= 1; l -= d, r -= d)
30
          value[tot++] = getRange(1, r);
        sort(value, value + tot);
31
        tot = unique(value, value + tot) - value;
32
        pref[0] = pref[1] = suff[n] = suff[n + 1] = 1;
33
34
        for (int i = d; i <= n; i += d) {
35
          int idx = lower_bound(value, value + tot, getRange(i - d + 1, i)) - value;
          pidx[i] = idx;
36
37
          pref[i] = fmul(pref[i - d], prime[idx]);
38
39
        for (int i = n - d + 1; i >= 1; i -= d) {
40
          int idx = lower_bound(value, value + tot, getRange(i, i + d - 1)) - value;
41
          sidx[i] = idx;
42
          suff[i] = fmul(suff[i + d], prime[idx]);
43
44
        int sc = 0, cur = fac[bunch];
        memset(cnt, 0, tot * sizeof(int));
45
        for (int l = 1, r = rest; r \le n; l += d, r += d) {
46
47
          if (r + d \le n) {
            ++cnt[sidx[r + 1]];
48
            cur = 111 * cur * inv[cnt[sidx[r + 1]]] % MOD;
49
50
          bin[sc++] = \{fmul(pref[l-1], suff[r+1]), l\};
51
52
        }
        sort(bin, bin + sc);
53
54
        mark[bin[0].second] = 1;
        for (int i = 1; i < sc; ++i)
55
          mark[bin[i].second] = bin[i].first != bin[i - 1].first;
56
57
58
        int res = 0;
```

```
for (int l = 1, r = rest; r \le n; l += d, r += d) {
 59
           if (1 - 1 >= 1) {
 60
 61
             ++cnt[pidx[1 - 1]];
             cur = 111 * cur * inv[cnt[pidx[1 - 1]]] % MOD;
 62
 63
           if (mark[1]) {
 64
 65
             res += cur;
             if (res >= MOD) res -= MOD;
 66
 67
           }
 68
           if (r + 1 \le n) {
             cur = 1ll * cur * cnt[sidx[r + 1]] % MOD;
 69
             --cnt[sidx[r + 1]];
 70
           }
 71
 72
         }
 73
         return res;
 74
       } else {
 75
         sort(value, value + tot);
 76
         11 pre = value[0];
         int res = fac[bunch], cnt = 1;
 77
 78
         for (int i = 1; i < tot; ++i) {</pre>
           if (value[i] != pre) {
 79
             if (cnt > 1) res = 1ll * res * fac_inv[cnt] % MOD;
 80
             cnt = 1, pre = value[i];
 81
 82
           } else {
 83
             ++cnt;
 84
 85
         }
 86
         if (cnt > 1) res = 1ll * res * fac_inv[cnt] % MOD;
 87
         return res;
       }
 88
     }
 89
 90
     void solve() {
 91
 92
       cin >> (s + 1);
 93
       int n = strlen(s + 1);
       for (int i = 1; i <= n; ++i)
 94
         ha[i] = (fmul(ha[i - 1], BASE) + s[i]) % HASH_MOD;
 95
 96
 97
       int ans = 0;
       for (int d = 1; d <= n; ++d) {
 98
 99
         ans += gao(n, d);
100
         if (ans >= MOD) ans -= MOD;
101
       }
102
       cout << ans << "\n";
     }
103
104
     void prework() {
105
       p[0] = 1;
106
       for (int i = 1; i < MAXN; ++i) p[i] = fmul(p[i - 1], BASE);
107
       fac[0] = fac[1] = 1;
108
109
       fac_{inv}[0] = fac_{inv}[1] = inv[1] = 1;
110
       for (int i = 2; i < MAXN; ++i) {</pre>
111
         fac[i] = 111 * fac[i - 1] * i % MOD;
         inv[i] = 111 * (MOD - MOD / i) * inv[MOD % i] % MOD;
112
         fac_inv[i] = 111 * fac_inv[i - 1] * inv[i] % MOD;
113
114
115
116
       int pc = 0;
       for (int i = 2; i < MAX_PRIME; ++i) {</pre>
117
         if (!mark[i]) prime[pc++] = i;
118
119
         for (int j = 0; j < pc; ++j) {
120
           int t = i * prime[j];
121
           if (t >= MAX_PRIME) break;
           mark[t] = 1;
122
           if (i % prime[j] == 0) break;
123
124
125
       }
    }
126
```

```
127
128
     int main(int argc, char *argv[]) {
129
       ios::sync_with_stdio(false);
       cin.tie(nullptr), cout.tie(nullptr);
130
131
       prework();
132
133
       int T; cin >> T;
134
       for (int step = 1; step <= T; ++step) {</pre>
135
         cout << "Case #" << step << ": ";
136
         solve();
137
138
139
       return 0;
140
```

4.5 mancher

```
1
    void mancher(char *s, int n) {
2
      static char str[2 * MAX_LENGTH];
      str[0] = '~', str[1] = '!';
3
      int len = 2;
4
      for (int i = 0; i < n; ++i) {
5
        str[len++] = s[i], str[len++] = '!';
6
      }
8
      str[len] = 0;
9
      for (int i = 1, id = 0, mx = 0; i < len; ++i) {
        p[i] = i < mx ? min(p[2 * id - i], mx - i) : 1;
10
        while (str[i + p[i]] == str[i - p[i]]) ++p[i];
11
12
        if (mx < i + p[i]) id = i, mx = i + p[i];
13
     }
14
   }
```

4.6 回文树

- ch[chr][x]: x 两边添加字符 chr 后的回文串结点;
- fail[x]: x 代表的回文串的最长回文后缀;
- *l*[*x*]: *x* 代表的回文串的长度;
- *cnt*[x]: x 代表的回文串的出现次数。

```
struct PT {
1
2
      char s[MAXL];
3
      int fail[MAXL], ch[26][MAXL], 1[MAXL], dep[MAXL], cnt[MAXL], lst, nc, n;
      void init() {
4
5
        1[0] = 0, 1[1] = -1;
        fail[0] = fail[1] = 1;
6
        for (int i = 0; i < 26; ++i)
 7
         for (int j = 0; j < nc; ++j) ch[i][j] = 0;
8
        for (int i = 2; i < nc; ++i) l[i] = 0, fail[i] = 0;
9
10
        lst = 0, nc = 2, n = 0, s[0] = '#';
11
12
13
      int insert(char c) {
14
15
        int id = c - 'a';
        s[++n] = c;
16
17
        while (s[n - 1[1st] - 1] != s[n]) { 1st = fail[1st]; }
        if (ch[id][lst] == 0) {
18
          1[nc] = 1[1st] + 2;
19
          int f = fail[lst];
20
          while (s[n - 1[f] - 1] != s[n]) { f = fail[f]; }
21
```

```
fail[nc] = ch[id][f];
22
23
          dep[nc] = dep[fail[nc]] + 1;
24
          ch[id][1st] = nc;
25
          ++nc;
26
        ++cnt[lst = ch[id][lst]];
27
28
        return lst;
29
30
31
      void count() { for (int i = nc - 1; ~i; --i) cnt[fail[i]] += cnt[i]; }
    } pt;
32
33
    // 求最长双回文串
34
35
    char S[MAXL];
36
    int len[MAXL];
37
    int main() {
     ios::sync_with_stdio(false);
38
39
      cin.tie(0);
40
      cout.tie(0);
41
      cin >> S;
42
      int n = strlen(S);
43
      pt.init();
44
45
      for (int i = 0; i < n; ++i) { len[i] = pt.l[pt.insert(S[i])]; }</pre>
46
      pt.init();
47
      int ans = 0;
48
      for (int i = n - 1; i; ---i) {
49
        ans = \max(ans, len[i - 1] + pt.l[pt.insert(S[i])]);
50
      cout << ans << "\n";
51
52
53
      return 0;
    }
54
```

4.7 后缀数组(倍增)

```
#include <bits/stdc++.h>
 1
 2
    using namespace std;
 3
    const int MAXN = 1e5 + 3;
 5
    template <int MAX_LENGTH> class SuffixArray {
 6
 7
    public:
      int n, sa[MAX_LENGTH], rank[MAX_LENGTH], height[MAX_LENGTH];
8
 9
10
      void compute(char *s, int n, int m) {
11
        int i, p, w, j, k;
12
         this->n = n;
13
        if (n == 1) {
14
           sa[0] = rank[0] = height[0] = 0;
15
          return;
16
        memset(cnt, 0, m * sizeof(int));
17
         for (i = 0; i < n; ++i) ++cnt[rank[i] = s[i]];</pre>
18
19
         for (i = 1; i < m; ++i) cnt[i] += cnt[i - 1];
20
         for (i = n - 1; \sim i; --i) sa[--cnt[rank[i]]] = i;
21
        for (w = 1; w < n; w <<= 1, m = p) {
          for (p = 0, i = n - 1; i >= n - w; --i) id[p++] = i;
22
           for (i = 0; i < n; ++i)
23
24
            if (sa[i] >= w) id[p++] = sa[i] - w;
          memset(cnt, 0, m * sizeof(int));
for (i = 0; i < n; ++i) ++cnt[px[i] = rank[id[i]]];</pre>
25
26
          for (i = 1; i < m; ++i) cnt[i] += cnt[i - 1];</pre>
27
           for (i = n - 1; \sim i; --i) sa[--cnt[px[i]]] = id[i];
28
          memcpy(old_rank, rank, n * sizeof(int));
29
30
          for (i = p = 1, rank[sa[0]] = 0; i < n; ++i)
```

```
rank[sa[i]] = cmp(sa[i], sa[i-1], w) ? p - 1 : p++;
31
32
        }
33
        for (i = 0; i < n; ++i) rank[sa[i]] = i;</pre>
        for (i = k = height[rank[0]] = 0; i < n; height[rank[i++]] = k)
34
35
          if (rank[i])
            for (k > 0 ? --k : 0, j = sa[rank[i] - 1]; s[i + k] == s[j + k]; ++k) {}
36
37
38
39
      void init_st_table(int n) {
40
        int lgn = lg[n];
        for (int i = 0; i < n; ++i) table[0][i] = height[i];</pre>
41
        for (int i = 1; i <= lgn; ++i)
42
          for (int j = 0, l = 1 << (i - 1); j + l < n; ++j)
43
44
            table[i][j] = min(table[i - 1][j], table[i - 1][j + 1]);
45
      }
46
      int lcp(int i, int j) {
47
48
        if (i > j) swap(i, j);
        ++i;
49
50
        int lgl = lg[j - i + 1];
        return min(table[lgl][i], table[lgl][j - (1 << lgl) + 1]);</pre>
51
52
53
54
   private:
      int table[17][MAX_LENGTH], lg[MAX_LENGTH];
55
      int old_rank[MAX_LENGTH], id[MAX_LENGTH], px[MAX_LENGTH], cnt[MAX_LENGTH];
56
57
58
      bool cmp(int x, int y, int w) {
        return old_rank[x] == old_rank[y] && old_rank[x + w] == old_rank[y + w];
59
      }
60
61
   };
62
    char s[MAXN];
63
   SuffixArray<MAXN> sa;
64
65
66
   int main(int argc, char *argv[]) {
67
     int n = fread(s, 1, MAXN, stdin);
      while (s[n-1] - 97u > 25) --n;
68
69
      for (int i = 0; i < n; ++i) s[i] -= 'a';
      s[n] = '$';
70
71
      sa.compute(s, n, 26);
      for (int i = 0; i < n; ++i) printf("%d%c", sa.sa[i] + 1, " \n"[i == n - 1]);
72
73
      for (int i = 1; i < n; ++i) printf("%d%c", sa.height[i], " \n"[i == n - 1]);
74
      return 0;
75
```

4.8 后缀数组(SAIS)

UOJ 板题最快算法,字符串必须为正数,BUFFER_SIZE 要随 MAX_LENGTH 同步变化,1e6 为 25。

```
#include <bits/stdc++.h>
2
    const int BUFFER_SIZE = 1u << 23 | 1;</pre>
3
    char buffer[BUFFER_SIZE], *buffer_ptr = buffer;
 4
   #define alloc(x, type, len)
5
 6
     type *x = (type *)buffer_ptr;
     buffer_ptr += (len) * sizeof(type);
 7
   #define clear_buffer()
 8
     memset(buffer, 0, buffer_ptr - buffer), buffer_ptr = buffer;
9
10
   template <int MAX_LENGTH> class SuffixArray {
11
   #define L_TYPE true
12
   #define S_TYPE false
13
14
   public:
      int sa[MAX_LENGTH], rank[MAX_LENGTH], height[MAX_LENGTH];
15
16
      void compute(int n, int m, int *s) {
17
        sais(n, m, s, sa);
```

```
for (int i = 0; i < n; ++i) rank[sa[i]] = i;</pre>
18
        for (int i = 0, h = 0; i < n; ++i) {
19
20
          if (rank[i]) {
21
            int j = sa[rank[i] - 1];
22
            while (s[i + h] == s[j + h]) ++h;
23
            height[rank[i]] = h;
24
          } else {
25
            h = 0;
26
27
          if (h) --h;
28
        }
      }
29
30
   private:
31
32
      int l_bucket[MAX_LENGTH], s_bucket[MAX_LENGTH];
33
      void induce(int n, int m, int *s, bool *type, int *sa, int *bucket,
34
                   int *l_bucket, int *s_bucket) {
35
        memcpy(l_bucket + 1, bucket, m * sizeof(int));
memcpy(s_bucket + 1, bucket + 1, m * sizeof(int));
36
37
        sa[l_bucket[s[n - 1]]++] = n - 1;
38
        for (int i = 0; i < n; ++i) {
39
          int t = sa[i] - 1;
40
41
          if (t >= 0 && type[t] == L_TYPE) sa[l_bucket[s[t]]++] = t;
42
        for (int i = n - 1; i >= 0; --i) {
43
          int t = sa[i] - 1;
44
45
          if (t \ge 0 \& type[t] == S_TYPE) sa[--s_bucket[s[t]]] = t;
46
      }
47
      void sais(int n, int m, int *s, int *sa) {
48
49
        alloc(type, bool, n + 1);
50
        alloc(bucket, int, m + 1);
51
        type[n] = false;
        for (int i = n - 1; i >= 0; --i) {
52
          ++bucket[s[i]];
53
          type[i] = s[i] > s[i + 1] \mid | (s[i] == s[i + 1] && type[i + 1] == L_TYPE);
54
55
56
        for (int i = 1; i <= m; ++i) {
          bucket[i] += bucket[i - 1];
57
58
          s_bucket[i] = bucket[i];
59
60
        memset(rank, -1, n * sizeof(int));
61
        alloc(lms, int, n + 1);
62
63
        int n1 = 0;
        for (int i = 0; i < n; ++i) {
64
65
          if (!type[i] && (i == 0 || type[i - 1])) lms[rank[i] = n1++] = i;
66
67
        lms[n1] = n:
68
        memset(sa, -1, n * sizeof(int));
        for (int i = 0; i < n1; ++i) sa[--s_bucket[s[lms[i]]]] = lms[i];
69
70
        induce(n, m, s, type, sa, bucket, l_bucket, s_bucket);
71
        int m1 = 0;
72
        alloc(s1, int, n + 1);
73
        for (int i = 0, t = -1; i < n; ++i) {
          int r = rank[sa[i]];
74
75
          if (r != -1) {
            int len = lms[r + 1] - sa[i] + 1;
76
            m1 += t == -1 \mid \mid len != lms[rank[t] + 1] - t + 1 \mid \mid
77
78
                   memcmp(s + t, s + sa[i], len * sizeof(int)) != 0;
            s1[r] = m1;
79
80
             t = sa[i];
          }
81
        }
82
        alloc(sa1, int, n + 1);
83
84
        if (n1 == m1) {
          for (int i = 0; i < n1; ++i) sa1[s1[i] - 1] = i;
85
```

```
} else {
86
87
           sais(n1, m1, s1, sa1);
 88
         }
         memset(sa, -1, n * sizeof(int));
89
 90
         memcpy(s_bucket + 1, bucket + 1, m * sizeof(int));
         for (int i = n1 - 1; i >= 0; --i) {
91
           int t = lms[sa1[i]];
92
93
           sa[--s_bucket[s[t]]] = t;
94
 95
         induce(n, m, s, type, sa, bucket, l_bucket, s_bucket);
       }
96
     #undef S_TYPE
 97
    #undef L_TYPE
98
99
    };
100
    const int MAXN = 1e5 + 5;
101
    SuffixArray<MAXN> sa;
102
    char str[MAXN];
103
    int s[MAXN];
104
105
    int main() {
106
      int n = fread(str, 1, MAXN, stdin);
107
       while (str[n - 1] - 97u > 25) --n;
108
109
       for (int i = 0; i < n; ++i) s[i] = str[i] - 'a' + 1;
110
       sa.compute(n, 26, s);
       for (int i = 0; i < n; ++i) printf("%d%c", sa.sa[i] + 1, " \n"[i == n - 1]);
111
       for (int i = 1; i < n; ++i) printf("%d%c", sa.height[i], " \n"[i == n - 1]);
112
113
       return 0;
114
```

4.9 后缀自动机

SPOJ Lexicographical Substring Search: 求字典序第 k 大子串

```
#include <bits/stdc++.h>
    using namespace std;
3
    const int MAXN = 90000 + 3;
4
    const int ALPHABET = 26;
5
6
   struct Node {
 7
     int len, cnt;
8
9
      Node *link, *next[ALPHABET];
10
      void init(int len = 0) {
        link = nullptr;
11
12
        this->len = len, cnt = 0;
        memset(next, 0, sizeof(next));
13
14
   };
15
16
17
    template <int MAX_LENGTH> class SAM {
18
   public:
19
      Node *last, *root;
20
      void init() {
21
22
        pool_ptr = pool;
23
        last = root = new_node(0);
24
25
      void extend(int chr) {
26
        Node *p = last, *np = new_node(p->len + 1);
27
28
        for (last = np; p && !p->next[chr]; p = p->link) p->next[chr] = np;
29
        if (!p) {
30
          np->link = root;
        } else {
31
          Node *q = p->next[chr];
32
          if (q->len == p->len + 1) {
33
```

```
34
             np->link = q;
35
           } else {
36
              Node *nq = new_node(p->len + 1);
37
              memcpy(nq->next, q->next, sizeof(q->next));
38
              nq->link = q->link, q->link = np->link = nq;
              for (; p \&\& p-\text{next[chr]} == q; p = p-\text{link}) p-\text{next[chr]} = nq;
39
40
         }
41
42
       }
43
       void toposort() {
44
         int size = pool_ptr - pool;
45
         memset(cnt, 0, size * sizeof(int));
46
47
         for (Node *it = pool; it < pool_ptr; ++it) ++cnt[it->len];
         for (int i = 1; i < size; ++i) cnt[i] += cnt[i - 1];</pre>
48
         for (Node *it = pool; it < pool_ptr; ++it) order[--cnt[it->len]] = it;
49
         for (int i = size - 1; ~i; --i) {
50
           Node *u = order[i];
51
            for (int j = 0; j < ALPHABET; ++j)</pre>
52
53
              u \rightarrow cnt += u \rightarrow next[j] ? u \rightarrow next[j] \rightarrow cnt + 1 : 0;
         }
54
       }
55
56
57
       void find_kth(int k, char *str) {
58
         char *ptr = str;
         Node *u = root;
59
60
         while (k) {
            for (int j = 0; j < ALPHABET; ++j) {
61
62
              if (!u->next[j]) continue;
              if (u-\text{>next[j]}-\text{>cnt} + 1 < k) {
63
64
                k = u - next[j] - cnt + 1;
                continue;
65
              }
66
67
              --k, *ptr++ = j + 'a';
68
             u = u - \operatorname{next}[j];
69
              break:
70
           }
71
 72
         *ptr = 0;
       }
73
74
     private:
75
76
       int cnt[MAX_LENGTH * 2];
77
       Node pool[MAX_LENGTH * 2], *pool_ptr, *order[MAX_LENGTH * 2];
78
79
       Node *new_node(int len) {
80
         pool_ptr->init(len);
81
         return pool_ptr++;
       }
82
    };
83
84
     SAM<MAXN> sam;
85
86
     char str[MAXN];
87
     int main(int argc, char *argv[]) {
88
89
       ios::sync_with_stdio(false);
90
       cin.tie(nullptr), cout.tie(nullptr);
91
92
       cin >> str;
93
       sam.init();
       for (char *it = str; *it; ++it) sam.extend(*it - 'a');
94
95
       sam.toposort();
96
97
       for (cin >> q; q; --q) cin >> k, sam.find_kth(k, str), puts(str);
98
99
100
       return 0;
101
```

5 图论

5.1 Tarjan

对于无向图求边双连通,则添加两条有向边;若有重边,则在v == from处添加计数器。

```
vector<int> adj[MAXN];
    bitset<MAXN> instk, cut;
    int bridges, dfs_clk, top, scc, n, m, d;
    int dfn[MAXN], stk[MAXN], bel[MAXN], sz[MAXN];
 4
 5
    int tarjan(int u, int from) {
 6
 7
      int low = dfn[u] = ++dfs_clk;
      stk[top++] = u, instk[u] = 1;
 8
9
10
      int son = 0;
      for (int v : adj[u]) {
11
12
        if (v == from) continue;
        if (!dfn[v]) {
13
14
          ++son;
          int low_v = tarjan(v, u);
15
          (low_v < low) && (low = low_v);
16
          (low_v > dfn[u]) && (++bridges);
17
          (u != from \&\& low_v >= dfn[u]) \&\& (cut[u] = 1);
18
19
        } else if (instk[v] && low > dfn[v]) {
20
          low = dfn[v];
        }
21
22
      (u == from \&\& son > 1) \&\& (cut[u] = 1);
23
24
      if (low == dfn[u]) {
25
26
        int v, sz = 0;
27
        sz[++scc] = 0;
28
        do {
29
          ++sz;
          v = stk[--top];
30
          instk[v] = 0, bel[v] = scc;
31
        } while (u ^ v);
32
33
      }
34
      return low;
    }
35
```

5.2 Hopcroft 算法

pos 表示左边的点匹配右边哪一个, neg 反之, 时间复杂度 $O(m\sqrt{n})$ 。

```
vector<int> adj[MAXN];
 1
2
    int nl, nr, pos[MAXN], neg[MAXN], lx[MAXN], ly[MAXN];
3
 4
   bool dfs(int x) {
      int c = lx[x] + 1;
5
 6
      1x[x] = -1;
      for (int y : adj[x]) {
 7
        if (ly[y] != c) continue;
8
9
        ly[y] = -1;
        if (~neg[y] && !dfs(neg[y])) continue;
10
11
        pos[neg[y] = x] = y;
        return true;
12
13
14
      return false;
   }
15
16
   int match() {
17
18
      int cnt = 0;
      memset(pos, -1, sizeof(int) * nl);
19
      memset(neg, -1, sizeof(int) * nr);
20
```

```
for (int x = 0; x < n1; ++x)
21
22
         for (int y : adj[x]) {
23
           if (~neg[y]) continue;
24
           pos[neg[y] = x] = y, ++cnt;
25
26
27
      for (;;) {
28
         static int q[MAXN];
29
         int 1 = 0, r = 0, ok = 0;
        memset(lx, -1, sizeof(int) * nl);

memset(ly, -1, sizeof(int) * nr);
30
31
         for (int x = 0; x < n1; ++x)
32
           if (pos[x] < 0) lx[q[r++] = x] = 0;
33
34
         while (1 < r) {
35
           int x = q[1++];
           for (int y : adj[x]) {
36
             if (~ly[y]) continue;
37
38
             ly[y] = lx[x] + 1;
             if (~neg[y] && ~lx[neg[y]]) continue;
39
40
             (\text{-neg}[y]) ? lx[q[r++] = neg[y]] = ly[y] + 1 : ok = 1;
41
         if (!ok) return cnt;
43
44
         for (int x = 0; x < n1; ++x)
           if (pos[x] < 0 \&\& dfs(x)) ++cnt;
45
46
    }
47
```

5.3 KM 算法

点为 1..n(左为 1..nl,右为 1..nr),lk 表示左边的点匹配右边哪一个。 最大费用流时 NOT = 0,最大费用流最大流时 NOT = -11l* MAXN* ALPHA。

```
const int MAXN = 400 + 3;
   const int ALPHA = 1e9 + 10;
2
   const 11 NOT = 0;
    const 11 INF = 311 * MAXN * ALPHA;
 4
5
    struct KM {
      int n, nl, nr, lk[MAXN], pre[MAXN];
 6
      11 lx[MAXN], ly[MAXN], w[MAXN][MAXN], slack[MAXN];
7
      bitset<MAXN> vy;
8
9
      void init(int n) {
10
11
        this->n = n;
        memset(lk, 0, sizeof(int) * (n + 1));
12
13
        memset(pre, 0, sizeof(int) * (n + 1));
        memset(lx, 0, sizeof(ll) * (n + 1));
14
        memset(ly, 0, sizeof(ll) * (n + 1));
15
16
        memset(slack, 0, sizeof(11) * (n + 1));
17
        for (int i = 0; i <= n; ++i) fill(w[i], w[i] + n + 1, NOT);</pre>
18
19
20
      void add_edge(int x, int y, 11 z) {
21
        if (w[y][x] < z) w[y][x] = z;
22
23
24
      11 match() {
25
        for (int i = 1; i <= n; ++i)
          for (int j = 1; j <= n; ++j) lx[i] = max(lx[i], w[i][j]);</pre>
26
        for (int i = 1, py, p, x; i \le n; ++i) {
27
          for (int j = 1; j \le n; ++j) slack[j] = INF, vy[j] = 0;
28
29
          for (lk[py = 0] = i; lk[py]; py = p) {
30
            11 delta = INF;
            vy[py] = 1, x = lk[py];
31
            for (int y = 1; y <= n; ++y) {
32
33
              if (vy[y]) continue;
34
              if (1x[x] + 1y[y] - w[x][y] < slack[y])
```

```
slack[y] = 1x[x] + 1y[y] - w[x][y], pre[y] = py;
35
36
               if (slack[y] < delta) delta = slack[y], p = y;</pre>
37
             }
             for (int y = 0; y \le n; ++y)
38
39
               if (vy[y]) {
                 lx[lk[y]] = delta, ly[y] += delta;
40
41
               } else {
42
                 slack[y] -= delta;
43
44
          for (; py; py = pre[py]) lk[py] = lk[pre[py]];
45
46
47
        11 \text{ ans} = 0;
48
49
        for (int i = 1; i <= n; ++i) {
          ans += lx[i] + ly[i];
50
          if (w[lk[i]][i] == NOT) ans -= NOT;
51
52
53
        return ans;
54
      }
    } km;
55
56
    int main() {
57
58
      int nl, nr, m;
59
      cin >> nl >> nr >> m;
60
      km.init(max(nl, nr));
      for (int x, y, z; m; --m) {
61
62
        cin >> x >> y >> z;
63
        km.add_edge(x, y, z);
64
65
      cout << km.match() << "\n";</pre>
66
      for (int i = 1; i <= nl; ++i)
67
        cout << (km.w[km.lk[i]][i] == NOT ? 0 : km.lk[i]) << " \n"[i == nl];</pre>
68
69
      return 0;
    }
70
```

5.4 一般图最大匹配

```
class GeneralMatch {
2
   public:
3
      int n;
 4
      vector<vector<int>> g;
      vector<int> match, aux, label, orig, parent;
5
 6
      queue<int> q;
7
      int aux_time;
8
9
      GeneralMatch(int n)
          : match(n, -1), aux(n, -1), label(n), orig(n), parent(n, -1),
10
11
            aux_time(-1) {
        this->n = n;
12
13
        g.resize(n);
14
15
16
      void add_edge(int u, int v) {
        g[u].push_back(v);
17
18
        g[v].push_back(u);
19
20
      int find(int x) { return x == orig[x] ? x : orig[x] = find(orig[x]); }
21
22
23
      int lca(int u, int v) {
24
        ++aux_time;
25
        u = find(u), v = find(v);
26
        for (;; swap(u, v)) {
          if (~u) {
27
```

```
28
             if (aux[u] == aux_time) return u;
29
             aux[u] = aux_time;
30
             if (match[u] == -1) {
               u = -1;
31
32
             } else {
               u = find(parent[match[u]]);
33
34
           }
35
36
        }
37
      }
38
39
      void blossom(int u, int v, int o) {
        while (find(u) != o) {
40
           parent[u] = v;
41
42
           v = match[u];
           q.push(v);
43
44
           label[v] = 0;
           orig[u] = orig[v] = o;
45
           u = parent[v];
46
47
         }
      }
48
49
      int bfs(int x) {
50
        iota(orig.begin(), orig.end(), 0);
fill(label.begin(), label.end(), -1);
51
52
53
         while (!q.empty()) q.pop();
54
         q.push(x);
         label[x] = 0;
55
56
         while (!q.empty()) {
           int u = q.front();
57
           q.pop();
58
           for (int v : g[u]) {
59
             if (label[v] == -1) {
60
61
               parent[v] = u;
               label[v] = 1;
62
               if (match[v] == -1) {
63
64
                  while (v != -1) {
                    int pv = parent[v];
65
                    int next_v = match[pv];
match[v] = pv;
66
67
68
                    match[pv] = v;
69
                    v = next_v;
70
                 }
71
                 return 1;
72
               }
73
               q.push(match[v]);
               label[match[v]] = 0;
74
75
             } else if (label[v] == 0 && find(u) != find(v)) {
                int o = lca(u, v);
76
77
               blossom(u, v, o);
78
               blossom(v, u, o);
             }
79
80
           }
         }
81
82
         return 0;
83
84
85
      int find_max_match() {
86
         int res = 0;
87
         for (int i = 0; i < n; ++i) {
88
           if (~match[i]) continue;
89
           res += bfs(i);
90
91
         return res;
      }
92
    };
93
```

5.5 SAP

```
struct MaxFlow {
 1
 2
      struct Edge {
        int to, rest;
 3
      } edges[MAXM * 4];
 4
 5
      vector<int> adj[MAXN];
 6
 7
      int n, edges_c, dep[MAXN], depc[MAXN], s, t, last[MAXN];
 8
 9
      void init(int _n) {
10
        n = _n, edges_c = 0;
         for (int i = 1; i <= n; ++i) adj[i].clear();</pre>
11
12
13
14
      void add_edge(int u, int v, int cap) {
        edges[edges_c] = {v, cap, 0};
15
16
         adj[u].push_back(edges_c++);
17
         edges[edges_c] = \{u, 0, 0\};
18
        adj[v].push_back(edges_c++);
19
20
      int dfs(int u, int flow) {
  if (u == t || !flow) return flow;
21
22
23
         int v, e, temp, res = 0;
24
         for (int &i = last[u]; i < (int)adj[u].size(); ++i) {</pre>
25
           e = adj[u][i], v = edges[e].to;
26
           if (!edges[e].res || dep[v] != dep[u] - 1) continue;
          temp = dfs(v, min(flow, edges[e].cap - edges[e].flow));
27
28
          res += temp, flow -= temp;
29
           edges[e].rest -= temp, edges[e ^ 1].rest += temp;
          if (!flow || !dep[s]) return res;
30
31
         last[u] = 0;
32
        if (!(--depc[dep[u]])) dep[s] = n + 1;
33
34
         ++depc[++dep[u]];
35
        return res;
36
37
      int max_flow(int s, int t) {
        this->s = s, this->t = t;
38
39
40
        static queue<int> que:
41
        memset(dep + 1, 0, sizeof(int) * n);
        memset(depc + 1, 0, sizeof(int) * n);
42
43
        memset(last + 1, 0, sizeof(int) * n);
        while (!que.empty()) que.pop();
44
        dep[t] = 1, que.push(t);
45
46
47
        while (!que.empty()) {
          int u = que.front();
48
           que.pop();
49
           ++depc[dep[u]];
50
           for (int i = 0, v; i < (int)adj[u].size(); ++i) {</pre>
51
            v = edges[adj[u][i]].to;
52
53
             if (dep[v]) continue;
54
             dep[v] = dep[u] + 1;
             que.push(v);
55
56
57
58
59
         int res = 0;
60
         while (dep[s] <= n) res += dfs(s, INT_MAX);</pre>
61
         return res;
62
      }
    };
```

5.6 dinic

```
struct MaxFlow {
 1
 2
      struct Edge {
        int to, rest;
 3
      } edges[MAXM * 4];
 4
 5
      vector<int> adj[MAXN];
 6
 7
      int n, edges_c, dep[MAXN], s, t, last[MAXN];
 8
 9
      void init(int _n) {
10
        n = _n, edges_c = 0;
         for (int i = 1; i <= n; ++i) adj[i].clear();</pre>
11
12
13
14
      void add_edge(int u, int v, int cap) {
        edges[edges_c] = \{v, cap, 0\};
15
16
        adj[u].push_back(edges_c++);
17
        edges[edges_c] = \{u, 0, 0\};
18
        adj[v].push_back(edges_c++);
19
20
21
      bool bfs() {
        memset(dep + 1, -1, sizeof(int) * n);
22
23
        static queue<int> q;
24
        q.push(s);
        dep[s] = 0;
25
26
        while (!q.empty()) {
27
           int u = q.front();
28
           q.pop();
29
           for (int i = 0; i < adj[u].size(); ++i) {</pre>
             Edge &e = edges[adj[u][i]];
30
31
             if ((~dep[e.to]) || !e.rest) continue;
             dep[e.to] = dep[u] + 1;
32
             q.push(e.to);
33
          }
34
35
        }
36
        return ~dep[t];
37
      }
38
39
      int dfs(int u, int flow) {
        if (u == t || flow == 0) return flow;
40
        int res = 0, temp, e, v;
for (int &i = last[u]; i < adj[u].size(); ++i) {</pre>
41
42
43
           e = adj[u][i], v = edges[e].to;
           if (dep[v] == dep[u] + 1 \&\& edges[e].rest) {
44
             temp = dfs(v, min(edges[e].rest, flow));
45
46
             res += temp, flow -= temp;
47
             edges[e].rest -= temp, edges[e ^ 1].rest += temp;
48
             if (!flow) break;
           }
49
50
51
        return flow;
52
53
54
      int max_flow(int s, int t) {
        this->s = s, this->t = t;
55
56
         int res = 0;
57
        while (bfs()) {
58
           memset(last + 1, 0, sizeof(int) * n);
59
           res += dfs(s, INF);
60
61
        return res;
62
      }
63
    };
```

5.7 高标预流推进

```
const int N = 1e4 + 4, M = 2e5 + 5, INF = 0x3f3f3f3f;
 1
 2
    int n, m, s, t;
3
4
   struct qxx {
 5
     int nex, t, v;
6
   };
   qxx e[M * 2];
   int h[N], cnt = 1;
8
   void add_path(int f, int t, int v) { e[++cnt] = (qxx)\{h[f], t, v\}, h[f] = cnt; }
10
   void add_flow(int f, int t, int v) {
     add_path(f, t, v);
add_path(t, f, 0);
11
12
   }
13
14
   int ht[N], ex[N], gap[N]; // 高度; 超额流; gap 优化
15
   bool bfs_init() {
16
17
     memset(ht, 0x3f, sizeof(ht));
     queue<int> q;
18
     q.push(t), ht[t] = 0;
19
     while (q.size()) { // 反向 BFS, 遇到没有访问过的结点就入队
20
21
       int u = q.front();
22
        q.pop();
       for (int i = h[u]; i; i = e[i].nex) {
23
         const int &v = e[i].t;
24
         if (e[i ^ 1].v \& ht[v] > ht[u] + 1) ht[v] = ht[u] + 1, q.push(v);
25
26
27
     }
     return ht[s] != INF; // 如果图不连通, 返回 0
28
29
   }
30
   struct cmp {
31
    bool operator()(int a, int b) const { return ht[a] < ht[b]; }</pre>
   };
32
                                             // 伪装排序函数
   priority_queue<int, vector<int>, cmp> pq; // 将需要推送的结点以高度高的优先
33
                                             // 是否在优先队列中
   bool vis[N];
   int push(int u) { // 尽可能通过能够推送的边推送超额流
35
36
     for (int i = h[u]; i; i = e[i].nex) {
       const int &v = e[i].t, &w = e[i].v;
37
        if (!w || ht[u] != ht[v] + 1) continue;
38
       int k = min(w, ex[u]); // 取到剩余容量和超额流的最小值
39
40
       ex[u] -= k, ex[v] += k, e[i].v -= k, e[i ^ 1].v += k; // push
41
        if (v != s && v != t && !vis[v])
         pq.push(v), vis[v] = 1; // 推送之后, v 必然溢出, 则入堆, 等待被推送
42
       if (!ex[u]) return 0; // 如果已经推送完就返回
43
44
     }
45
     return 1;
46
   }
47
   void relabel(int u) { // 重贴标签(高度)
     ht[u] = INF;
48
49
      for (int i = h[u]; i; i = e[i].nex)
        if (e[i].v) ht[u] = min(ht[u], ht[e[i].t]);
50
51
     ++ht[u];
   }
52
                                // 返回最大流
53
   int hlpp() {
     if (!bfs_init()) return 0; // 图不连通
54
55
     ht[s] = n;
56
     memset(gap, 0, sizeof(gap));
      for (int i = 1; i <= n; i++)
57
       if (ht[i] != INF) gap[ht[i]]++; // 初始化 gap
58
      for (int i = h[s]; i; i = e[i].nex) {
59
       const int v = e[i].t, w = e[i].v; // 队列初始化
60
61
       if (!w) continue;
62
        ex[s] -= w, ex[v] += w, e[i].v -= w, e[i ^ 1].v += w; // 注意取消 w 的引用
       if (v != s && v != t && !vis[v]) pq.push(v), vis[v] = 1; // \lambda M
63
64
     while (pq.size()) {
65
66
       int u = pq.top();
```

cycleke

```
pq.pop(), vis[u] = 0;
67
        while (push(u)) { // 仍然溢出
68
69
          // 如果 u 结点原来所在的高度没有结点了, 相当于出现断层
          if (!--gap[ht[u]])
70
            for (int i = 1; i <= n; i++)
  if (i != s && i != t && ht[i] > ht[u] && ht[i] < n + 1) ht[i] = n + 1;</pre>
71
72
73
          relabel(u):
          ++gap[ht[u]]; // 新的高度, 更新 gap
74
75
76
      }
77
      return ex[t];
    }
78
```

5.8 最小费用流

```
class MinCostFlow {
2
    public:
3
      struct Result {
 4
        int flow, cost;
5
 6
      struct Edge {
 7
        int to, next, rest, cost;
8
9
10
      vector<bool> inq;
11
      vector<int> head, dist, from, flow;
      vector<Edge> edges;
12
13
      MinCostFlow(int n, int m) : inq(n), head(n, -1), dist(n), from(n), flow(n) {
14
        edges.reserve(2 * m);
15
16
17
      void add_edge(int u, int v, int capacity, int cost) {
18
        internal_add_edge(u, v, capacity, cost);
19
        internal_add_edge(v, u, 0, -cost);
20
21
22
      void internal_add_edge(int u, int v, int capacity, int cost) {
23
        edges.push_back((Edge){v, head[u], capacity, cost});
24
25
        head[u] = edges.size() - 1;
26
27
28
      Result augment(int source, int sink) {
29
        fill(dist.begin(), dist.end(), INT_MAX);
30
        dist[source] = 0;
        flow[source] = INT_MAX;
31
32
        queue<int> q;
33
        q.push(source);
34
        while (!q.empty()) {
35
          int u = q.front();
          q.pop();
36
37
          inq[u] = false;
          for (int it = head[u]; ~it; it = edges[it].next) {
38
39
            auto &e = edges[it];
40
            int v = e.to;
            if (e.rest > 0 && dist[u] + e.cost < dist[v]) {</pre>
41
42
              from[v] = it;
              dist[v] = dist[u] + e.cost;
43
              flow[v] = min(e.rest, flow[u]);
44
45
              if (!inq[v]) {
                q.push(v);
46
47
                inq[v] = true;
              }
48
            }
49
          }
50
        }
51
```

```
52
        if (dist[sink] == INT_MAX) return {0, 0};
53
54
        int min_flow = flow[sink];
        for (int u = sink; u != source; u = edges[from[u] ^ 1].to) {
55
56
          edges[from[u]].rest -= min_flow;
          edges[from[u] ^ 1].rest += min_flow;
57
58
59
        return {min_flow, dist[sink]};
60
61
      Result min_cost_flow(int source, int sink) {
62
        int flow = 0, cost = 0;
63
        for (;;) {
64
65
          auto result = augment(source, sink);
66
          if (!result.flow) break;
67
          flow += result.flow, cost += result.cost;
68
        return {flow, cost};
69
70
      }
71
   };
```

5.9 上下界费用流

```
const int MAXN = 53;
   const int MAX_NODE = 113;
   const int MAX_EDGE = 1e5 + 5;
   const int INF = 0x3f3f3f3f;
 4
 5
   int n, s, t, ss, tt, tote;
 6
 7
   int R[MAXN], C[MAXN], board[MAXN][MAXN];
 8
9
   struct Edge {
10
     int to, cap, flow, cost;
   } edges[MAX_EDGE];
11
    vector<int> adj[MAX_NODE];
12
13
    int from[MAX_NODE], in[MAX_NODE];
14
    void add_edge(int from, int to, int 1, int r, int cost) {
15
      in[to] += 1, in[from] -= 1;
16
      edges[tote] = (Edge){to, r - 1, 0, cost};
17
      adj[from].push_back(tote++);
18
      edges[tote] = (Edge)\{from, 0, 0, -cost\};
19
20
      adj[to].push_back(tote++);
21
   }
22
   bool spfa(int s, int t) {
23
24
      static queue<int> q;
      static bool inq[MAX_NODE];
25
      static int dist[MAX_NODE];
26
27
      memset(inq + 1, 0, sizeof(bool) * tt);
      memset(dist + 1, 0x3f, sizeof(int) * tt);
28
29
      memset(from + 1, -1, sizeof(int) * tt);
30
      dist[0] = 0, from[0] = -1;
31
      q.push(0);
32
      while (!q.empty()) {
        int u = q.front();
33
34
        q.pop();
        inq[u] = false;
35
        for (int e : adj[u]) {
36
          if (edges[e].cap == edges[e].flow) continue;
37
38
          int v = edges[e].to, d = dist[u] + edges[e].cost;
39
          if (d >= dist[v]) continue;
          dist[v] = d;
40
          from[v] = e;
41
          if (!inq[v]) {
42
43
            q.push(v);
```

```
44
             inq[v] = true;
           }
 45
 46
         }
 47
       }
 48
       return dist[t] < INF;</pre>
     }
 49
 50
     pair<int, int> min_cost_max_flow(int s, int t) {
 51
 52
       int flow = 0, cost = 0;
 53
       while (spfa(s, t)) {
         int mi = INF;
 54
          for (int it = from[t]; ~it; it = from[edges[it ^ 1].to])
 55
           mi = min(mi, edges[it].cap - edges[it].flow);
 56
         flow += mi;
 57
         for (int it = from[t]; ~it; it = from[edges[it ^ 1].to]) {
  edges[it].flow += mi, edges[it ^ 1].flow -= mi;
 58
 59
 60
            cost += mi * edges[it].cost;
         }
 61
 62
       }
 63
       return make_pair(flow, cost);
     }
 64
 65
     void solve() {
 66
 67
       tote = 0;
       s = 2 * n + 1, t = 2 * n + 2, ss = 0, tt = 2 * n + 3;
 68
       for (int i = 0; i <= tt; ++i) adj[i].clear(), in[i] = 0;</pre>
 69
 70
 71
       memset(R + 1, 0, sizeof(int) * n);
 72
       memset(C + 1, 0, sizeof(int) * n);
 73
 74
       for (int i = 1; i <= n; ++i)
         for (int j = 1; j \le n; ++j) {
 75
 76
           cin >> board[i][j];
 77
           R[i] += board[i][j];
 78
           C[j] += board[i][j];
 79
 80
 81
       for (int i = 1; i <= n; ++i) {
 82
          add_edge(s, i, R[i], R[i], 0);
         add_edge(s, i + n, C[i], C[i], 0);
 83
 84
 85
 86
       for (int i = 1, l, r; i \le n; ++i) {
         cin >> 1 >> r;
 87
 88
         add_edge(i, t, 1, r, 0);
 89
 90
       for (int i = 1, 1, r; i <= n; ++i) {
         cin >> 1 >> r;
 91
 92
         add_edge(i + n, t, l, r, 0);
 93
 94
       for (int step = n * n / 2, x1, y1, x2, y2; step; --step) {
 95
 96
         cin >> x1 >> y1 >> x2 >> y2;
         if (board[x1][y1] == board[x2][y2]) continue;
 97
 98
          if (board[x2][y2]) swap(x1, x2), swap(y1, y2);
 99
         if (x1 == x2)
100
            add_edge(y1 + n, y2 + n, 0, 1, 1);
101
          else
           add_edge(x1, x2, 0, 1, 1);
102
103
       add_edge(t, s, 0, INF, 0);
104
105
       int sum = 0;
       for (int i = 1; i < tt; ++i) {
106
         if (in[i] > 0) {
107
           sum += in[i];
108
            add_edge(ss, i, 0, in[i], 0);
109
110
         } else if (in[i] < 0) {</pre>
            add_edge(i, tt, 0, -in[i], 0);
111
```

```
112
113
114
       pair<int, int> ans = min_cost_max_flow(ss, tt);
115
116
       if (sum != ans.first) {
         cout << "-1\n";
117
118
       } else {
         cout << ans.second << '\n';</pre>
119
120
121
     }
```

6 计算几何

7 Java

7.1 进制转换

```
import java.io.*;
 2
   import java.util.*;
   import java.math.*;
 3
 4
5
    public class Main {
      public static void main(String[] args) {
 6
 7
        InputStream inputStream = System.in;
8
        OutputStream outputStream = System.out;
9
        Scanner in = new Scanner(inputStream);
        PrintWriter out = new PrintWriter(outputStream);
10
11
        Solver solver = new Solver();
        int testCount = Integer.parseInt(in.next());
12
13
        for (int i = 1; i <= testCount; i++)</pre>
14
          solver.solve(i, in, out);
        out.close();
15
      }
16
17
18
      static class Solver {
        public void solve(int testNumber, Scanner in, PrintWriter out) {
19
          int a = in.nextInt();
20
21
          int b = in.nextInt();
          String num = in.next();
22
23
          BigInteger value = BigInteger.ZERO;
24
25
          for (int i = 0; i < num.length(); ++i) {</pre>
            value = value.multiply(BigInteger.valueOf(a));
26
            value = BigInteger.valueOf(getValue(num.charAt(i))).add(value);
27
28
          out.println(a + " " + num);
29
30
          if (value.equals(BigInteger.ZERO)) {
31
32
            out.println(b + " 0");
33
            out.println();
34
            return;
35
36
37
          out.print(b + " ");
38
          char[] ans = new char[1000];
39
          int length = 0;
40
          while (!value.equals(BigInteger.ZERO)) {
41
42
            int digit = value.mod(BigInteger.valueOf(b)).intValue();
43
            value = value.divide(BigInteger.valueOf(b));
            ans[length] = getChar(digit);
44
            ++length;
45
46
47
```

```
for (int i = length - 1; i >= 0; --i) {
48
49
            out.print(ans[i]);
50
          out.println("\n");
51
52
53
        private int getValue(char ch) {
54
          if (ch >= 'A' && ch <= 'Z') {
55
56
            return ch - 'A' + 10;
57
          if (ch >= 'a' && ch <= 'z') {
58
            return ch - 'a' + 36;
59
          }
60
61
          return ch - '0';
62
63
        private char getChar(int x) {
64
65
          if (x < 10) {
            return (char) ('0' + x);
66
67
          } else if (x < 36) {</pre>
            return (char) ('A' + x - 10);
68
69
            return (char) ('a' + x - 36);
70
71
        }
72
      }
73
74
    }
```

8 杂项

8.1 I/O 优化

```
namespace FastIO {
 1
    struct Control {
 2
 3
      int ct, val;
      Control(int Ct, int Val = -1) : ct(Ct), val(Val) {}
 4
      inline Control operator()(int Val) { return Control(ct, Val); }
 5
    } _endl(0), _prs(1), _setprecision(2);
 6
 7
    const int IO_SIZE = 1 << 16 | 127;</pre>
 8
9
10
    struct FastIO {
      char in[IO_SIZE], *p, *pp, out[IO_SIZE], *q, *qq, ch[20], *t, b, K, prs;
11
12
      FastIO(): p(in), pp(in), q(out), qq(out + IO_SIZE), t(ch), b(1), K(6) {}
      ~FastIO() { fwrite(out, 1, q - out, stdout); }
13
14
      inline char getc() {
15
        return p == pp && (pp = (p = in) + fread(in, 1, IO_SIZE, stdin), p == pp)
16
                   ? (b = 0, EOF)
17
                    : *p++;
18
19
      inline void putc(char x) {
        q == qq \&\& (fwrite(out, 1, q - out, stdout), q = out), *q++ = x;
20
21
22
      inline void puts(const char str[]) {
        fwrite(out, 1, q - out, stdout), fwrite(str, 1, strlen(str), stdout),
23
24
            q = out;
25
      inline void getline(string &s) {
26
        s = "";
27
28
        for (char ch; (ch = getc()) != '\n' && b;) s += ch;
29
30
    #define indef(T)
      inline FastIO &operator>>(T &x) {
31
        x = 0;
32
33
        char f = 0, ch;
```

```
while (!isdigit(ch = getc()) && b) f |= ch == '-';
34
        while (isdigit(ch)) x = (x << 1) + (x << 3) + (ch^ 48), ch = getc();
35
36
        return x = f ? -x : x, *this;
37
38
      indef(int);
      indef(long long);
39
40
      inline FastIO &operator>>(string &s) {
41
42
        s = "";
43
        char ch;
        while (isspace(ch = getc()) && b) {}
44
        while (!isspace(ch) && b) s += ch, ch = getc();
45
46
        return *this;
47
48
      inline FastIO &operator>>(double &x) {
        x = 0;
49
        char f = 0, ch;
50
        double d = 0.1;
51
52
        while (!isdigit(ch = getc()) && b) f |= (ch == '-');
        while (isdigit(ch)) x = x * 10 + (ch ^ 48), ch = getc();
53
        if (ch == '.')
54
          while (isdigit(ch = getc())) x += d * (ch ^ 48), d *= 0.1;
55
        return x = f ? -x : x, *this;
56
57
    #define outdef(_T)
58
      inline FastIO &operator<<(_T x) {</pre>
59
        !x && (putc('0'), 0), x < 0 && (putc('-'), x = -x);
60
        while (x) *t++ = x % 10 + 48, x /= 10;
61
        while (t != ch) *q++ = *--t;
62
63
        return *this;
64
65
      outdef(int);
      outdef(long long);
66
      inline FastIO &operator<<(char ch) { return putc(ch), *this; }</pre>
67
      inline FastIO &operator<<(const char str[]) { return puts(str), *this; }</pre>
68
      inline FastIO &operator<<(const string &s) { return puts(s.c_str()), *this; }</pre>
69
70
      inline FastIO &operator<<(double x) {</pre>
71
        int k = 0;
72
        this->operator<<(int(x));
        putc('.');
73
74
        x = int(x);
        prs && (x += 5 * pow(10, -K - 1));
75
76
        while (k < K) putc(int(x *= 10) ^ 48), x -= int(x), ++k;
77
        return *this;
78
79
      inline FastIO &operator<<(const Control &cl) {</pre>
        switch (cl.ct) {
80
81
        case 0: putc('\n'); break;
82
        case 1: prs = cl.val; break;
        case 2: K = cl.val; break;
83
84
        }
85
        return *this;
86
      inline operator bool() { return b; }
87
   };
88
    } // namespace FastIO
```

8.2 优化 STL 内存申请

```
// useage: vector<int, myalloc<int>> L;
static char space[10000000], *sp = space;
template <typename T> struct myalloc : allocator<T> {
  myalloc() {}
template <typename T2> myalloc(const myalloc<T2> &a) {}
template <typename T2> myalloc<T> &operator=(const myalloc<T2> &a) {
  return *this;
```

```
8
9
      template <typename T2> struct rebind { typedef myalloc<T2> other; };
10
      inline T *allocate(size_t n) {
        T * result = (T *) sp;
11
12
        sp += n * sizeof(T);
        return result;
13
14
     inline void deallocate(T *p, size_t n) {}
15
   };
16
```

8.3 Emacs 配置

```
(defun myc++ ()
      (c-set-style "stroustrup")
 2
 3
      (setq tab-width 2)
 4
      (setq indent-tabs-mode nil)
      (setq c-basic-offset 2)
 5
 6
      (c-toggle-hungry-state)
      (defun compile-and-run()
 7
 8
        (interactive)
 9
        (setq file-name (file-name-sans-extension (file-name-nondirectory buffer-file-name)))
10
        (compile
         (format "g++ %s.cpp -o %s -Wall -Wextra -Wshadow -O2 && ./%s < in.txt"
11
                  file-name file-name file-name)))
12
      (local-set-key (kbd "C-c C-c") 'compile-and-run)
13
      (local-set-key (kbd "C-c C-k") 'kill-compilation))
14
    (add-hook 'c++-mode-hook 'myc++)
15
16
    (global-set-key [(meta ?o)] 'other-window)
(global-set-key [(meta ?/)] 'hippie-expand)
17
18
    (global-set-key [(control tab)] ' senator-completion-menu-popup)
19
    (setq hippie-expand-try-functions-list
20
          '(try-expand-dabbrev
21
22
            try-expand-dabbrev-visible
23
             try-expand-dabbrev-all-buffers
            try-expand-dabbrev-from-kill
24
            try-complete-file-name-partially
25
            try-complete-file-name
26
27
            try-expand-all-abbrevs
28
             try-expand-list
            try-expand-line))
29
30
31
    (setq auto-save-mode nil)
    (setq make-backup-files nil)
32
33
34
    (ido-mode t)
35
    (show-paren-mode 1)
    (delete-selection-mode t)
36
37
    (global-linum-mode t)
    (global-auto-revert-mode t)
38
```

8.4 Vim 配置

```
syntax on
set tabstop=2 softtabstop=2 shiftwidth=2 expandtab smarttab autoindent
set incsearch ignorecase smartcase hlsearch
set ruler laststatus=2 showcmd showmode
set nobackup noswapfile noundofile autowrite noerrorbells
set wrap breakindent encoding=utf-8 number title cursorline

autocmd filetype c nnoremap <F5> :w <bar> exec '!gcc '.shellescape('%').' -o '.shellescape('%:r').
    ' -std=c11 -Wall && ./'.shellescape('%:r')<CR>
autocmd filetype cpp nnoremap <F5> :w <bar> exec '!g++ '.shellescape('%').' -o '.shellescape('%:r').' -std=c++17 -Wall && ./'.shellescape('%:r')<CR>
```

```
autocmd filetype java nnoremap <F5> :w <bar> exec '!javac -encoding UTF-8 -sourcepath . -d . '.
        shellescape('%').' && java '.shellescape('%:r')<CR>
11
   autocmd filetype c nnoremap <F6> :w <bar> exec '!gcc '.shellescape('%').' -o '.shellescape('%:r').
12
        ' -std=c11 -Wall -g && gdb '.shellescape('%:r')<CR>
    autocmd filetype cpp nnoremap <F6> :w <bar> exec '!g++ '.shellescape('%').' -o '.shellescape('%:r'
13
        ).' -std=c++17 -Wall -g && gdb '.shellescape('%:r')<CR>
14
15
    function! TrimWhitespace()
16
        let 1:save = winsaveview()
        %s/\\\@<!\s\+$//e
17
        call winrestview(1:save)
18
   endfunction
19
20
21
   imap jk <Esc>
   nmap t :call TrimWhitespace()<CR>
22
   nmap <M-1> 1gt
24
   nmap < M-2 > 2gt
25
26
   nmap <M-3> 3gt
   nmap <M-4> 4gt
27
   nmap < M-5 > 5gt
   nmap <M-6> 6gt
29
30
   nmap < M-7 > 7gt
   nmap <M-8> 8gt
31
   nmap <M-9> 9gt
32
   nmap <M-t> :tabnew<CR>
   nmap <M-w> :close<CR>
34
   nmap <Tab> :tabnext<CR>
   nmap <S-Tab> :tabprevious<CR>
```

8.5 对拍

*unix 下对拍:

```
while true; do
2
     python gen.py > in.txt
3
     time ./my < in.txt > out.txt
     time ./std < in.txt > ans.txt
4
     if diff out.txt ans.txt; then
5
6
        echo AC
     else
7
8
        echo WA
        exit 0
9
10
     fi
11
   done
```

Windows 下对拍:

```
1   Qecho off
2   :loop
3   python gen.py > in.txt
4   my.exe < in.txt > out.txt
5   std.exe < in.txt > ans.txt
6   fc out.txt ans.txt
7   if not errorlevel 1 goto loop
8   pause
9   :end
```