- Question 1(a) Indicate which of the following applications of the β-reduction rule are correct and which are incorrect.

  1. $\lambda x.((\lambda y.y + 1)\ (x + 1)) \quad \rightarrow \quad \lambda x.(x + 1) + 1$
     correct

  2. $(\lambda y.y + x)\ (x + x) \quad \rightarrow \quad (x + x) + x$
     correct

  3. $(\lambda y.\lambda x.x + y)\ (x + x) \quad \rightarrow \quad \lambda x.x + (x + x)$
     incorrect (because of variable capture), the correct solution is:
     $(\lambda y.\lambda x.x + y)\ (x + x) \quad \rightarrow \quad (\lambda y.\lambda z.z + y)\ (x + x) \quad \rightarrow \quad \lambda z.z + (x + x)$

  4. $(\lambda f.\lambda x.f(fx))\ (\lambda y.y + 1) \quad \rightarrow \quad \lambda x.(\lambda y.y + 1)\ ((\lambda y.y + 1)\ x)$
     correct

- Question 1(b) Reduce the following lambda expression to a normal form. Show all steps, and underline the expression that is reduced at each step.

$$(\lambda x.((\lambda y.y + 1)\ (x + 1)))((\lambda z.z)\ 3)$$

Sample solution:

$(\lambda x.((\lambda y.y + 1)\ (x + 1)))\ \underline{((\lambda z.z)\ 3)} \rightarrow$

$(\lambda x.\underline{((\lambda y.y + 1)\ (x + 1))})\ 3 \rightarrow$

$\underline{(\lambda x.(x + 1) + 1)\ 3} \rightarrow$

$(3 + 1) + 1$