

- Question 1(a) Indicate which of the following applications of the β -reduction rule are correct and which are incorrect.

$$1. \lambda x.((\lambda y.y + 1) (x + 1)) \rightarrow \lambda x.(x + 1) + 1$$

$$2. (\lambda y.y + x) (x + x) \rightarrow (x + x) + x$$

$$3. (\lambda y.\lambda x.x + y) (x + x) \rightarrow \lambda x.x + (x + x)$$

$$4. (\lambda f.\lambda x.f(fx)) (\lambda y.y + 1) \rightarrow \lambda x.(\lambda y.y + 1) ((\lambda y.y + 1) x)$$

- Question 1(b) Reduce the following lambda expression to a normal form. Show all steps, and underline the expression that is reduced at each step.

$$(\lambda x.((\lambda y.y + 1) (x + 1)))(\lambda z.z) 3)$$