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Problem 1

$$\underbrace{((\lambda f. \lambda g. f(g))}_{\text{main}} (\underbrace{\lambda x. x+4})_{f}) (\underbrace{\lambda y. 3-y}_{g})$$

(a) eliminate the λ as far to the left as possible:

$$\begin{aligned} & [(\lambda x. x+4)/f] (\lambda f. \lambda g. f(g)) (\lambda y. 3-y) \\ &= (\lambda g. (\lambda x. x+4)(g)) (\lambda y. 3-y) \\ &= [(\lambda y. 3-y)/g] ((\lambda x. x+4)(g)) \\ &= (\lambda x. x+4)((\lambda y. 3-y) 1) \\ &= [((\lambda y. 3-y) 1)/x] (\lambda x. x+4) \\ &= ((\lambda y. 3-y) 1) + 4 \\ &= [1/y](\lambda y. 3-y) + 4 \\ &= 3 - 1 + 4 \\ &= 6 \end{aligned}$$

(b) eliminate the λ as far as to the right as possible:

$$\begin{aligned} & [(\lambda y. 3-y)/g] ((\lambda f. \lambda g. f(g)) (\lambda x. x+4)) \\ &= (\lambda f. f((\lambda y. 3-y) 1)) (\lambda x. x+4) \\ &= [1/y] (\lambda f. f(\lambda y. 3-y)) (\lambda x. x+4) \\ &= (\lambda f. f(3-1)) (\lambda x. x+4) \\ &= [(\lambda x. x+4)/f] (\lambda f. f(3-1)) \\ &= (\lambda x. x+4)(3-1) \\ &= [(3-1)/x] (\lambda x. x+4) \\ &= (3-1) + 4 \\ &= 6 \end{aligned}$$

Problem 2

$$\begin{aligned} & (\lambda \text{compose} (\lambda h. \text{compose } h h 3) \lambda x. x+x) \lambda f. \lambda g. \lambda x. f(gx) \\ &= [\lambda f \lambda g. \lambda x. f(gx)/\text{compose}] (\lambda \text{compose} (\lambda h. \text{compose } h h 3) \lambda x. x+x) \\ &= (\lambda h. (\lambda f \lambda g. \lambda z. f(gz)) h h 3) \lambda x. x+x \quad (\text{Rename}) \\ &= [\lambda x. x+x/h] (\lambda h. (\lambda f \lambda g. \lambda z. f(gz)) h h 3) \\ &= (\lambda f \lambda g. \lambda z. f(gz)) (\lambda x. x+x) (\lambda x. x+x) 3 \\ &= [\lambda x. x+x/f] (\lambda f \lambda g. \lambda z. f(gz)) (\lambda x. x+x) 3 \\ &= (\lambda g. \lambda z. (\lambda y. y+y) (g z)) (\lambda x. x+x) 3 \quad (\text{Rename}) \\ &= [\lambda x. x+x/g] (\lambda g. \lambda z. (\lambda y. y+y) (g z)) 3 \\ &= (\lambda z. (\lambda y. y+y) ((\lambda x. x+x) z)) 3 \\ &= [3/z] (\lambda z. (\lambda y. y+y) ((\lambda x. x+x) z)) \\ &= (\lambda y. y+y) ((\lambda x. x+x) 3) \\ &= [3/x] ((\lambda y. y+y) (\lambda x. x+x)) \\ &= (\lambda y. y+y)(3+3) \\ &= [3+3]/y 3 (\lambda y. y+y) \\ &= (3+3)+(3+3) \\ &= 12 \end{aligned}$$

Problem 3

Grammar: $<n> \rightarrow 111$

$<\text{str}> \rightarrow <n> | <\text{str}> <\text{bit}> | <\text{bit}> <\text{str}>$

$<\text{bit}> \rightarrow 0 | 1$

Problem 4

Derivation of "1 b / 4 n +":

$$\begin{aligned} & <\text{arg}> \rightarrow <\text{expr RP}> \rightarrow <\text{arg}> <\text{arg}> <\text{binary op}> \rightarrow <\text{arg}> <\text{arg}> + \\ & \rightarrow <\text{arg}> <\text{expr PR}> + \rightarrow <\text{arg}> <\text{arg}> <\text{unary op}> + \rightarrow <\text{arg}> <\text{arg}> n + \\ & \rightarrow <\text{arg}> <\text{expr RP}> n + \rightarrow <\text{arg}> <\text{const}> n + \rightarrow <\text{arg}> <\text{digit}> n + \\ & \rightarrow <\text{arg}> 4 n + \rightarrow <\text{expr RP}> 4 n + \rightarrow <\text{arg}> <\text{arg}> <\text{binary op}> 4 n + \\ & \rightarrow <\text{arg}> <\text{arg}> / 4 n + \rightarrow <\text{arg}> <\text{expr RP}> / 4 n + \rightarrow <\text{arg}> <\text{const}> / 4 n + \\ & \rightarrow <\text{arg}> <\text{digit}> / 4 n + \rightarrow <\text{arg}> b / 4 n + \rightarrow <\text{expr RP}> b / 4 n + \\ & \rightarrow <\text{const}> b / 4 n + \rightarrow <\text{digit}> b / 4 n + \rightarrow 1 b / 4 n + \quad \square \end{aligned}$$

(continued on the next page)

- Derivation of "8 * 7 + 3 /" :

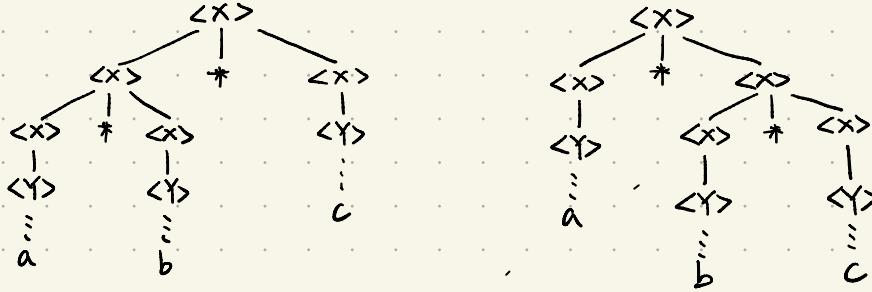
$\langle \text{arg} \rangle \rightarrow \langle \text{expr RP} \rangle \rightarrow \langle \text{arg} \rangle \langle \text{arg} \rangle \langle \text{binary op} \rangle \rightarrow \langle \text{arg} \rangle \langle \text{arg} \rangle /$
 $\rightarrow \langle \text{arg} \rangle \langle \text{expr RP} \rangle / \rightarrow \langle \text{arg} \rangle \langle \text{const} \rangle / \rightarrow \langle \text{arg} \rangle \langle \text{digit} \rangle / \rightarrow \langle \text{arg} \rangle 3 /$
 $\rightarrow \langle \text{expr RP} \rangle 3 / \rightarrow \langle \text{arg} \rangle \langle \text{arg} \rangle \langle \text{binary op} \rangle 3 /$
 $\rightarrow \langle \text{arg} \rangle \langle \text{arg} \rangle + 3 / \rightarrow \langle \text{arg} \rangle \langle \text{expr RP} \rangle + 3 / \rightarrow \langle \text{arg} \rangle \langle \text{const} \rangle + 3 /$
 $\rightarrow \langle \text{arg} \rangle \langle \text{digit} \rangle + 3 / \rightarrow \langle \text{arg} \rangle 7 + 3 / \rightarrow \langle \text{expr RP} \rangle 7 + 3 / \rightarrow$
 $\rightarrow \langle \text{arg} \rangle \langle \text{binary op} \rangle 7 + 3 / \rightarrow \langle \text{arg} \rangle * 7 + 3 / \rightarrow \langle \text{expr RP} \rangle * 7 + 3 /$
 $\rightarrow \langle \text{const} \rangle * 7 + 3 / \rightarrow \langle \text{digit} \rangle * 7 + 3 / \rightarrow 8 * 7 + 3 /$

Problem 5

- Consider the expression $a * b * c$ with the given grammar

There are two parse trees for it:

① This tree indicates $(a * b) * c$ ② This tree indicates $a * (b * c)$



- New Grammar :

$\langle S \rangle \rightarrow \langle X \rangle$
 $\langle X \rangle \rightarrow (\langle X \rangle * \langle X \rangle) | \langle Y \rangle$
 $\langle Y \rangle \rightarrow a | b | c$

We solve the ambiguity by introducing the parenthesis to the grammar to determine the precedence.