Ripser examples

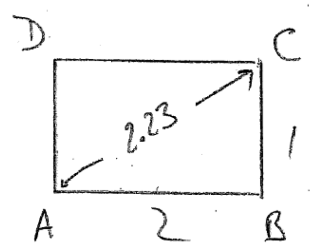
Consider a simplicial complex C. The Betti numbers b_0, b_1, b_2, … are ranks of the corresponding homology groups H_0, H_1, … We can interpret b_0 as counting the number of connected components of C, b_1 the number of 1-dimensional circular holes of C, b_2 the number of 2-dimensional voids, ….. Here, a 1-dimensional hole/cycle is a sum of edges with the boundary of this sum equal to 0.

How can we think about barcodes for a Vietoris-Rips complex? I will use d as the distance (we increase) to create our sequence of Vietoris-Rips complexes. A bar in the barcode represents a generator of the homology group H_i. A bar starts when a particular generator, i-cyle, is born and ends when this generator, i-cycle, dies. So, bars represent i-cycles generating the homology groups, not individual simplexes in the complex. For each homology group H_i, the number of bars (for a particular fixed distance d) equals to the rank of H_i = the Betti number b_i.

Reading: R. Ghrist, Barcodes: The persistent topology of data
https://www.math.upenn.edu/~ghrist/preprints/barcodes.pdf

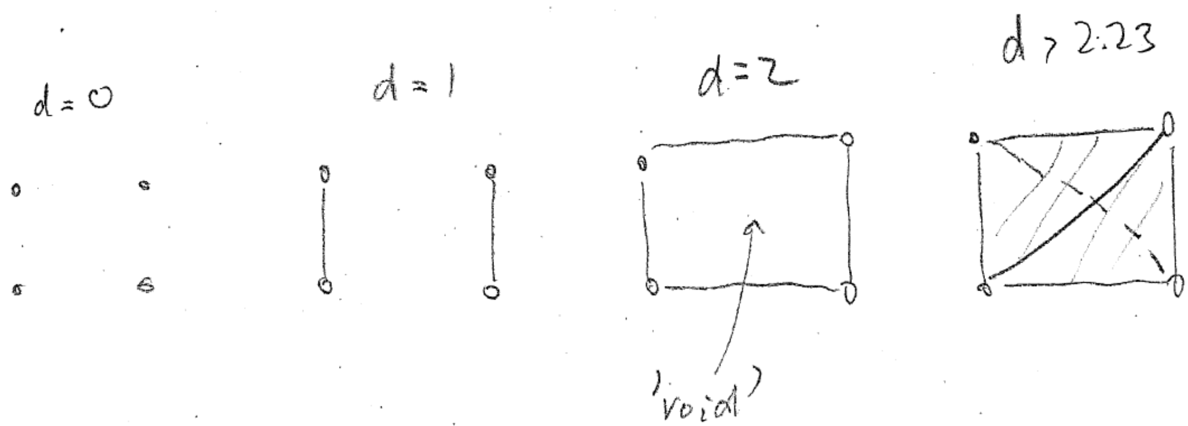
Notation of intervals: [0,) means [0,infinity) intervals, that is, a particular i-cycle which generates H_i is born at 0 and never dies. However, the barcode Ripser generates always ends at the maximal distance.

Example 1. Let us consider 4 points which are vertices of a rectangle in the plane with sides of length 2 and 1




```
2.0,
2.23,1.0,
1.0,2.23,2.0,
```

Above is the distance matrix of the point set. The Vietoris-Rips complexes at different distances d:



This agrees with the barcode below. The blue lines represent the components, starting with 4. The orange bar corresponds to the 'void'.

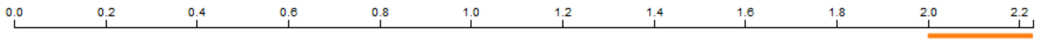
Ripser

Load a lower dist matrix to compute Vietoris–Rips persistence barcodes in dimensions 0 to 4 and up to distance :

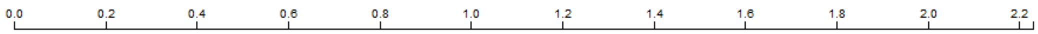
Browse... RipserTest3.tex

Persistence intervals in dimension 0:

Persistence intervals in dimension 1:



Persistence intervals in dimension 2:



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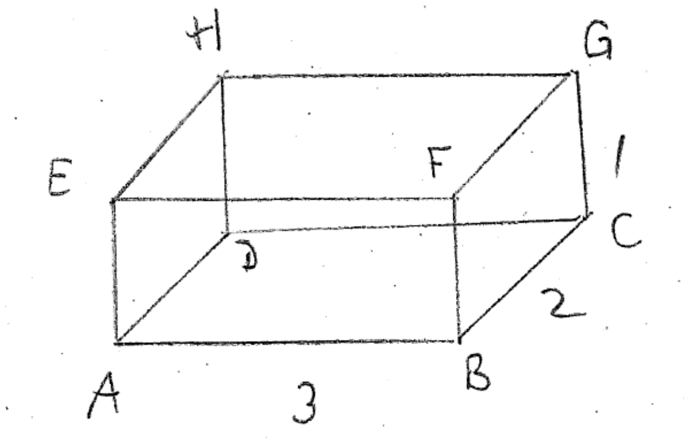
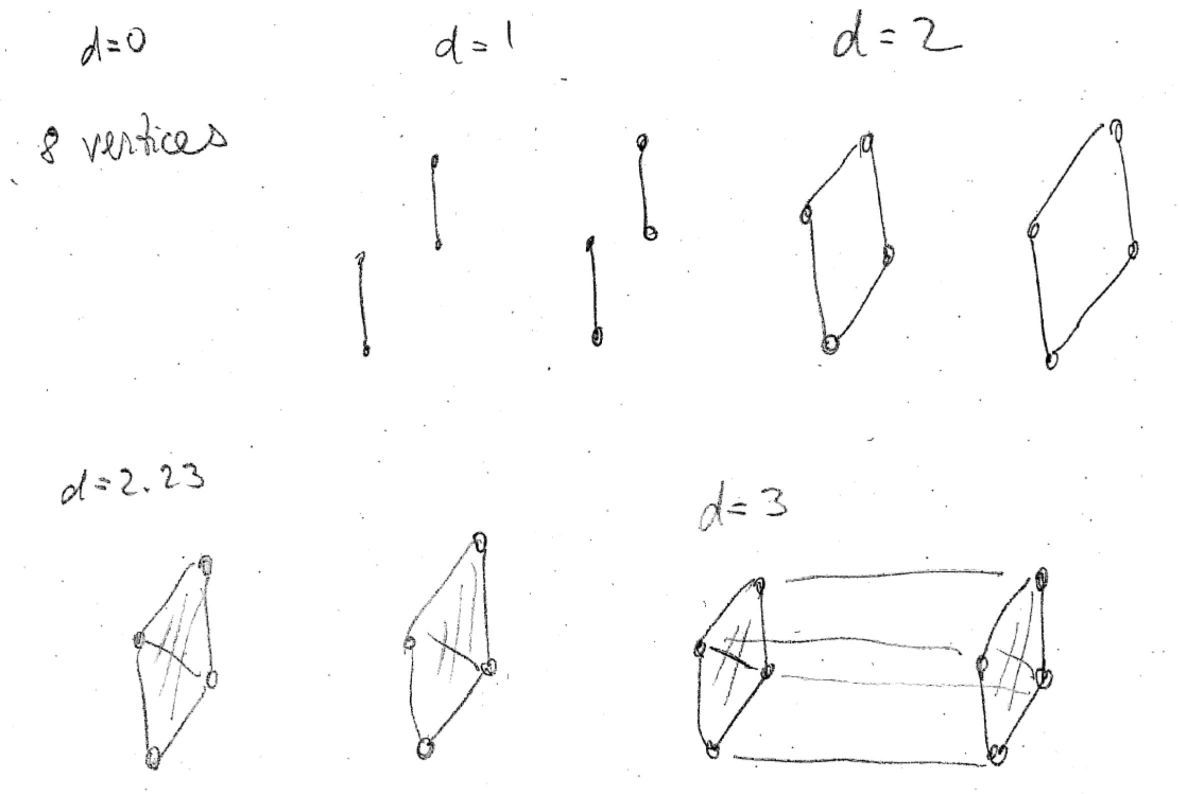
Example 2: Consider the 8 vertices of a rectangular box with sides of length 3,2, and 1.



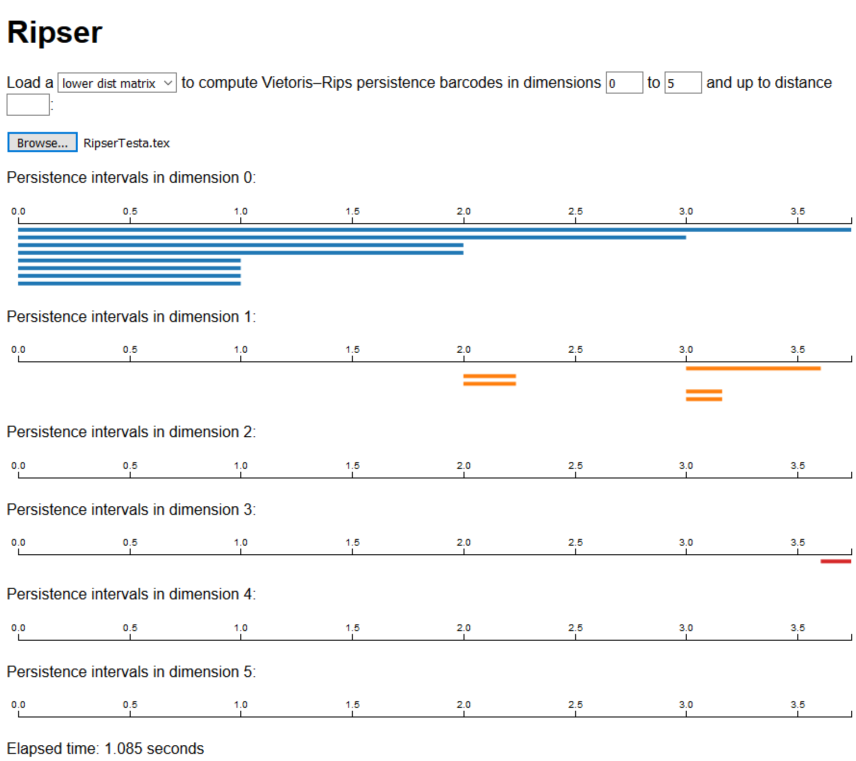
The distance matrix:

```
3.0,
3.605,2.0,
2.0,3.605,3.0,
1.0,3.162,3.7416,2.236,
3.605,1.0,2.236,3.7416,3.0,
3.7416,2.236,1.0,3.162,3.605,2.0,
2.236,3.7416,3.162,1.0,2.0,3.605,3.0,
```

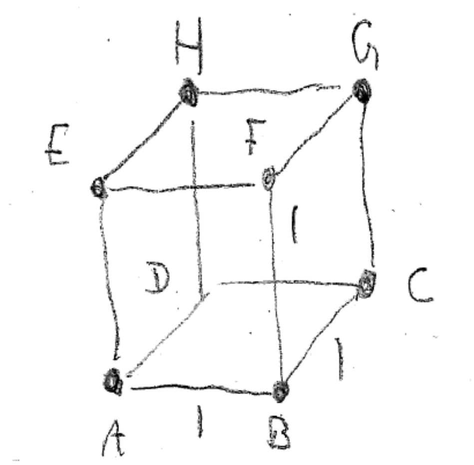
Now, few Vietoris-Rips complexes for different values of distance d:



And, the barcode



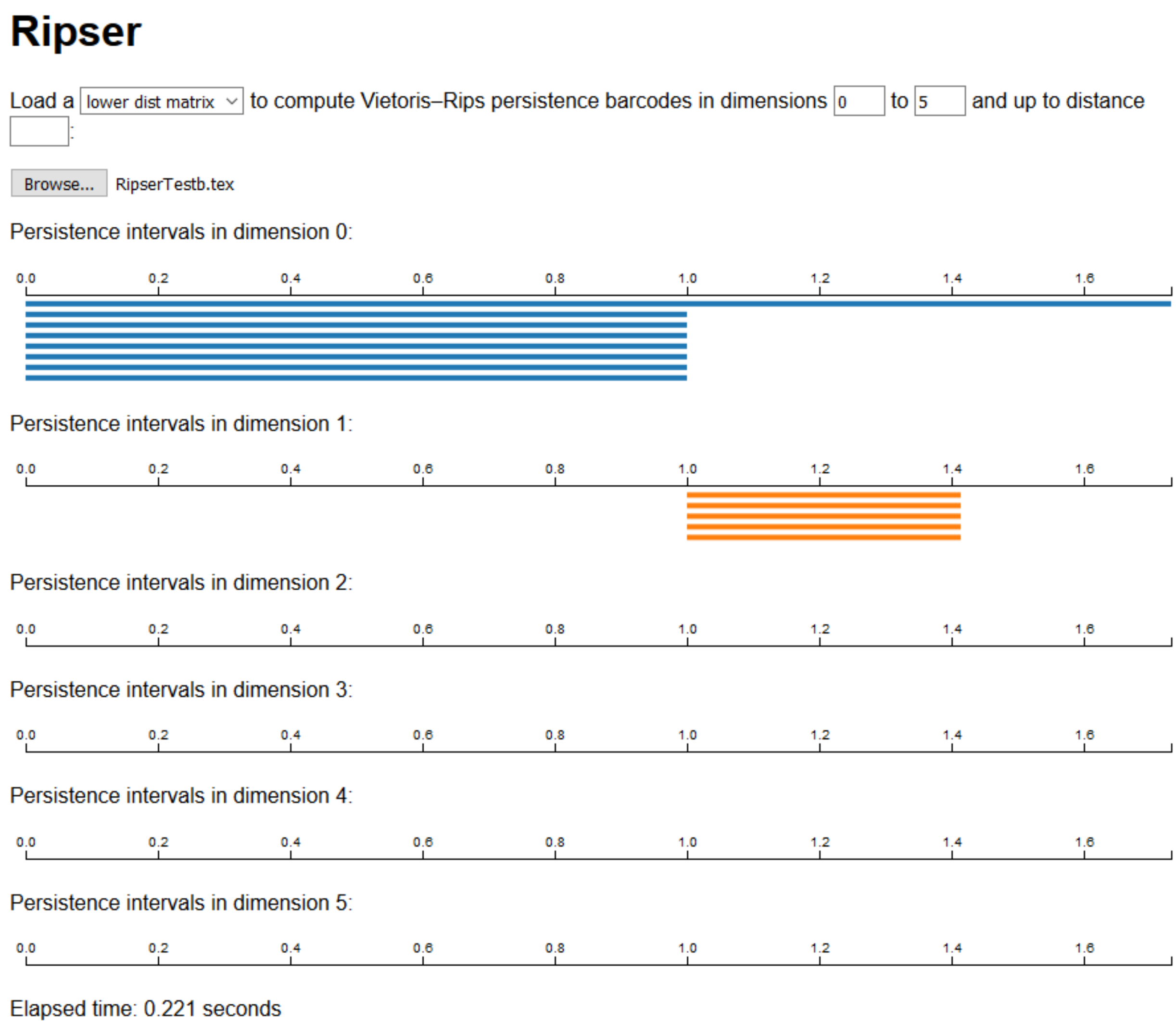
Example 3: Let us modify the previous rectangular box and add symmetries and decrease the number of distinct distances in the point set. Consider the unit cube, all sides of length 1:



and its distance matrix

```
1.0,
1.414,1.0,
1.0,1.732,1.0,
1.0,1.414,1.732,1.414,
1.414,1.0,1.414,1.732,1.0,
1.732,1.414,1.0,1.414,1.414,1.0,
1.414,1.732,1.414,1.0,1.0,1.414,1.0,
```

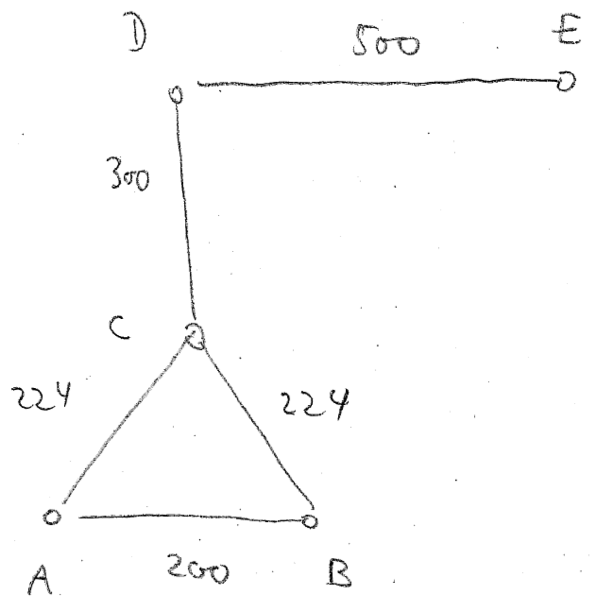
Here is the barcode for this example:



We see in some sense a simpler barcode then for the first rectangular box. The reason is that we have fewer distances between the points in the cube case. Also, once d reaches 1.414 (the root of 2), the sides of the cube are filled up by triangles. But also, by the definition of the Vietoris-Rips complex, the inside of the cube is filled up with simplexes and we do not get any voids.

Note: we should keep in mind the difference between examples 2 and 3. I assume we will see a similar effect (a simpler barcode) in the case of the binary vectors in Z^40,000 as we talked about yesterday (7/18) since the number of distinct distances in that point set is in some sense limited by the used encoding.

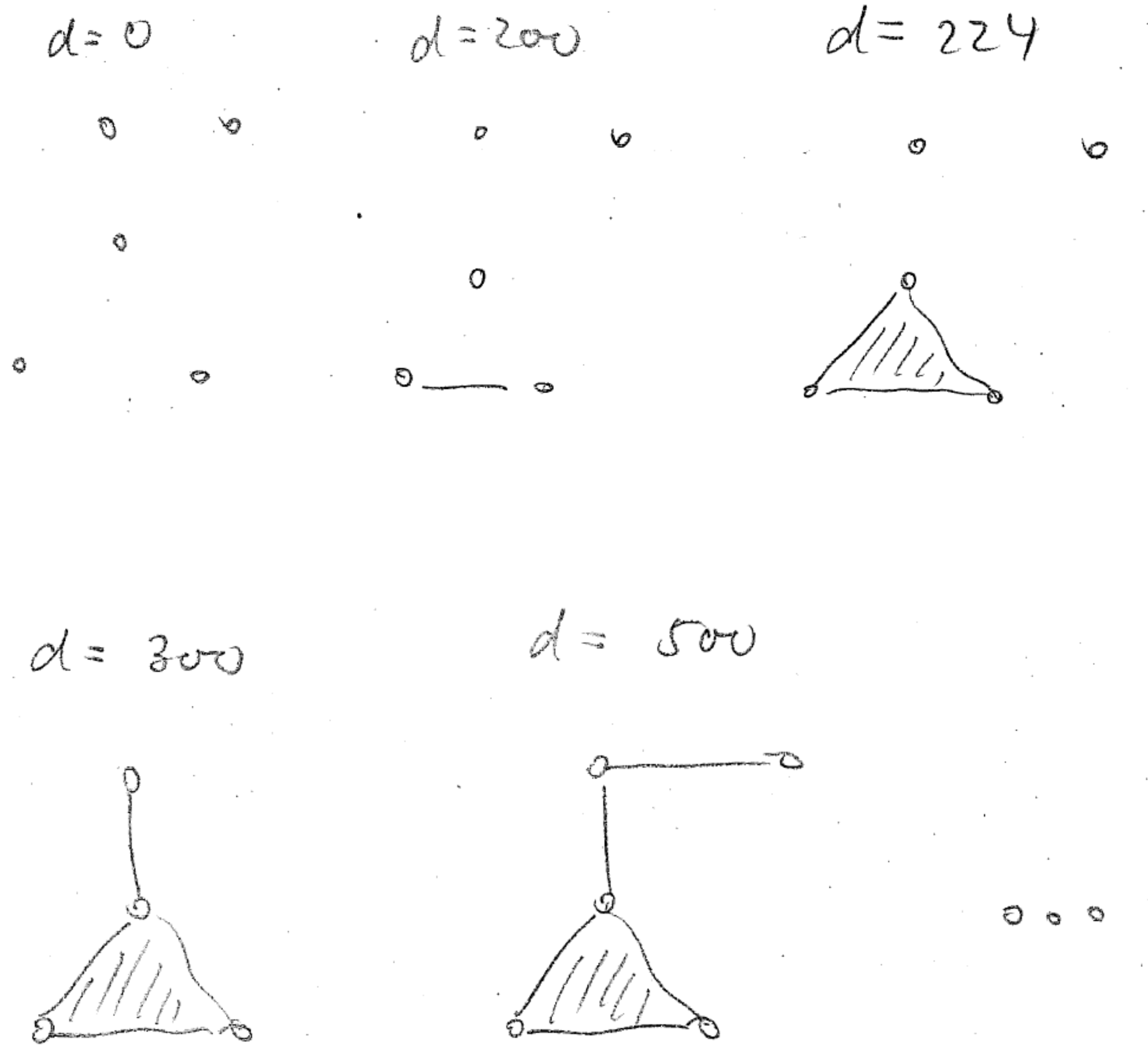
Example 4: This is the example discussed in the meeting yesterday (7/18):



and the distance matrix (by Matt)

```
200.0,
224.0,224.0,
510.0,510.0,300.0,
781.0,640.0,583.0,500.0,
```

Few pictures of the Vietoris-Rips complex at different distance d:



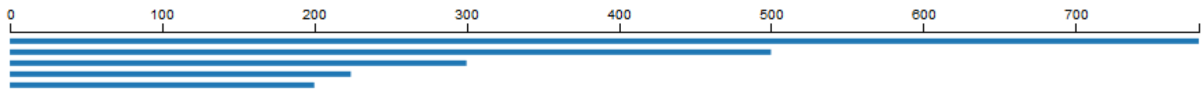
These pictures show that we do not expect any ‘voids’ in the complexes as we increase the distance d. Hence, the barcode will have only bars for dimension 0. Here is the barcode:

Ripser

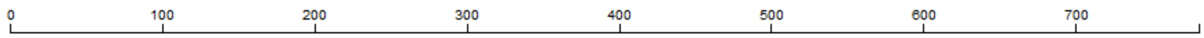
Load a lower dist matrix to compute Vietoris–Rips persistence barcodes in dimensions 0 to 5 and up to distance :

Browse... RipserTestaa.tex

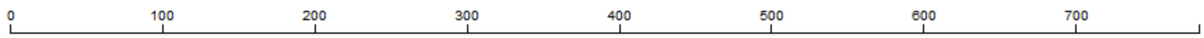
Persistence intervals in dimension 0:



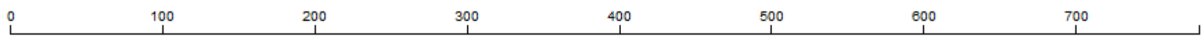
Persistence intervals in dimension 1:



Persistence intervals in dimension 2:



Persistence intervals in dimension 3:



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