A *BAD* Rendezvous of Probability and Logarithm

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1 Introduction

Logarithm has been indispensable tool in mathematics ever since its inception in the early 17-th century by John Napier. Developed originally to ease calculations involving heavy divisions and multiplications, it has transformed the way we approach calculations. While probability was not developed as a "tool", it was a mere formulation of the game of chance. Probability in the recent era, has proven to play a pivotal role in almost all domains of mathematics and engineering.

I have seen several algorithms using logarithms to represent probabilities in a more efficient way (what is called "log-likelihood), rather than working directly with a probability density function or a probability mass function, especially, when the computation of probabilities involves a large number of calculations and storing the probabilities in a compressed format can save a significant amount of memory.

In this paper, I tried linking the following formulation of logarithms with that of probability, because I believed in some way, logarithms can be used to "represent" probability .

Claim:

If A and B are two disjoint discrete random variables on $\mathbb{R}$, then:

$P(A \cup B) = P(A) + P(B)$

and for any pair of values A, B on $\mathbb{R}$, we can also say that:

$ln(AB) = ln(A) + ln(B)$

In this paper, I tried proving that there is a *deep* connection besides the two formulations.

2 The Math

The statement that we have to prove:

If A and B are two discrete random variables, such that A, B are mutually exclusive, then:

$$P(A \cup B) = P(A) + P(B)$$

Now, notice that if A and B belong to the real number line, then both A, B are in some essence "mutually disjoint", so if I have to find $ln(AB)$ I can write this as $ln(A) + ln(B)$.

Comparing the two formulations, and looking at $P(A \cup B)$ can we say something about $ln(AB)$, in this context? Can we relate the two formulations?

A good way to establish the correlation would be what we call "log likelihood", which is $ln(P(A \cup B)) = ln(P(A) + P(B)) = ln(P(A)) + ln(1 + \frac{P(B)}{P(A)})$.

If $P(A)/P(B)$ is a very small quantity then I can write $ln(1 + \frac{P(B)}{P(A)}) \sim \frac{P(B)}{P(A)}$. If A is very small compared to B, then we can say $P(B)/P(A)$ is approximately $ln(P(B))$.

Thus, proving our claim.

3 Conclusion

What we did above is indeed *bad mathematics*. Just because two identities look similar, there cannot be a necessarily a connection between them as they represent different structures. Hence, correlation does not imply causation. So, while one could say probability does leverage the power of logarithm to make it more computationally efficient, they are, however, independent events.