DOCUMENT

GSM-G-CPL.002

DOCUMENT TITLE

AERODYNAMICS 1

CHAPTER 9 – AIRCRAFT IN FLIGHT (TURNING)

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TURNING

INTRODUCTION

The topic of turning leads to the concept of manoeuvring.

Manoeuvring can be divided into two categories:

- **General turning performance.** This involves constant speed ***(but not velocity)*** turns and climbing, descending or level turns. General turning performance is common to all aircraft.

- **Energy manoeuvring.** This involves tactical manoeuvres such as air-to-air and air-to-ground combat manoeuvres.

The lessons on turning will only deal with general turning performance

CENTRIPETAL FORCE (CPF)

Newton 1:

Newton 1 states that: "A body will remain in a state of rest or uniform motion in a straight line unless acted upon by an external, unbalanced force."

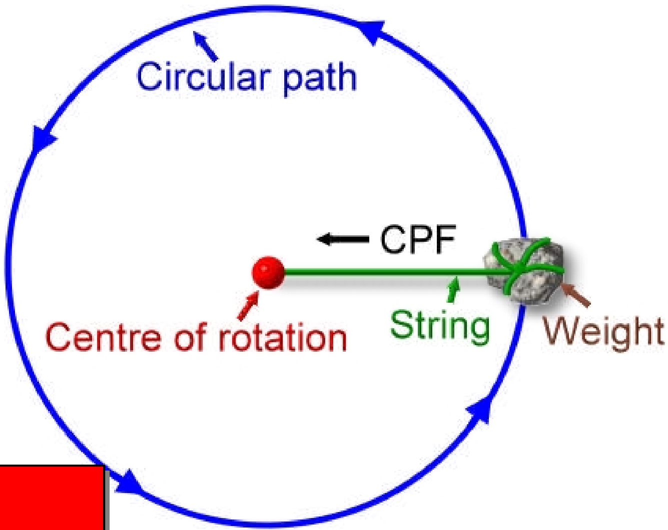
To change this state ***(either speed or direction)*** a force must be exerted on the body. Any change in speed or direction will be acceleration.

In the case of a turning aircraft, a force is required to accelerate the aircraft towards the centre of the turn. This force is called **centripetal force (CPF)**.

Consider the example of a small weight swinging around on a piece of string.

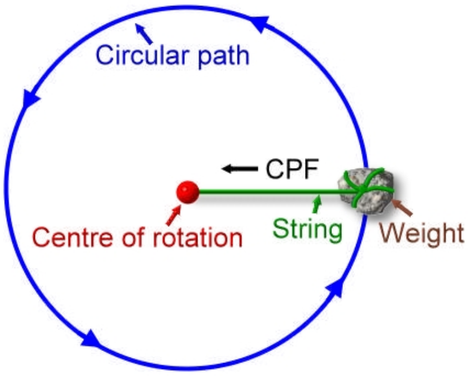
The graphic shows the top view of a **weight** on a **string** swinging around a **point**, thus following a **circular path**.

The tension in the **string** provides the CPF and acts towards the **centre of the circle**. Thus the weight is accelerating towards the **centre of rotation** ***(according to Newton 2)***.



Newton 2:

To change the state of rest or uniform motion of a body an external force is required. The rate of change in speed or direction is directly proportional to the force applied.

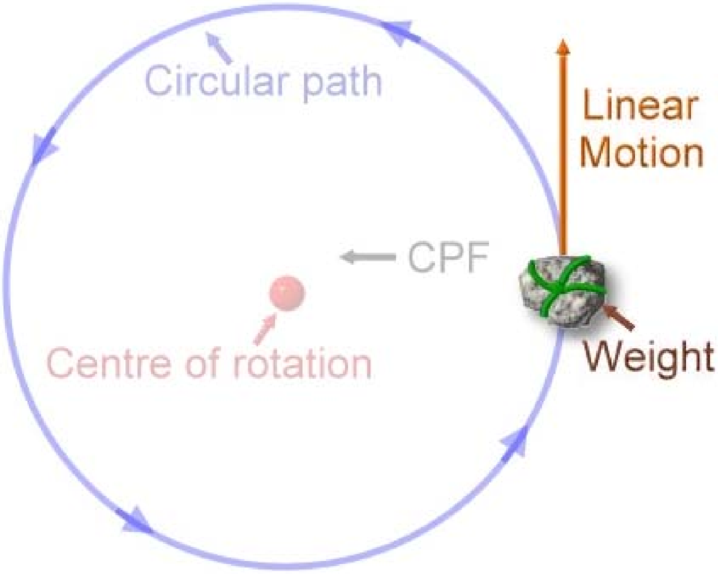


Note that this does not cause the weight to move directly towards the **centre** ***(Newton 3)*** but it causes a continual change in direction resulting in the **circular motion**.

Newton 3:

To every action there is an equal and opposite reaction.

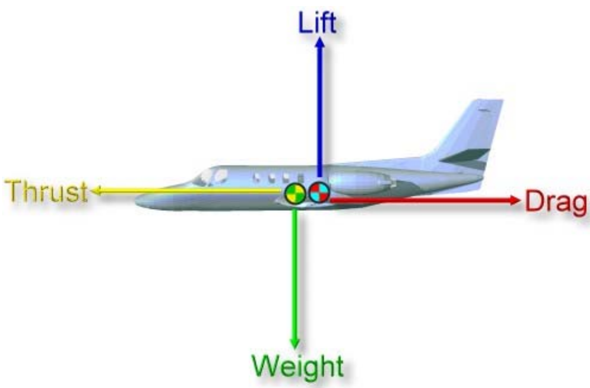
If the **CPF** is removed ***(string released)*** there will be no external unbalanced force. There will now be no acceleration and the weight will resume **linear** motion.



Linear:

Motion in a straight line at a constant speed.

FORCES IN THE TURN



During straight and level flight these forces are in equilibrium.

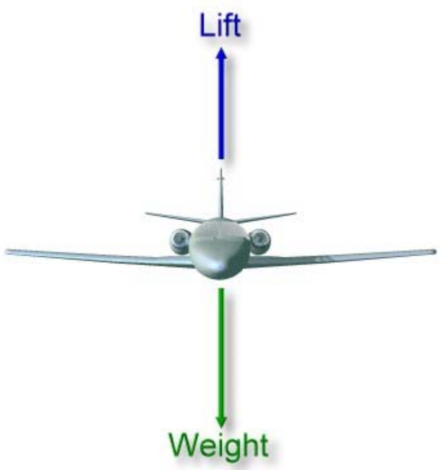
As was mentioned previously, **centripetal force (CPF)** was defined as the force required to accelerate the aircraft towards the centre of the turn.

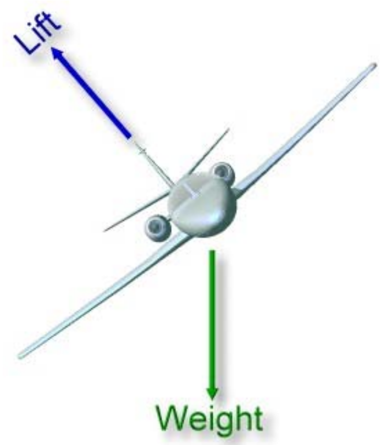
The formula used to calculate CPF is:

$$CPF = \frac{MV^2}{r}$$

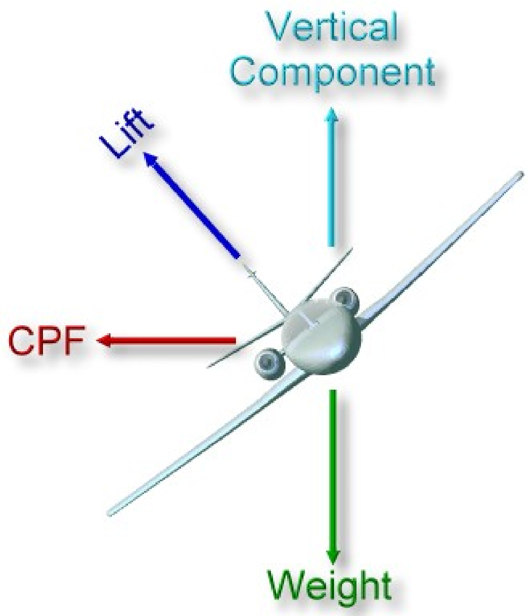
where **M** is the mass of the body, **V** the velocity and **r** the radius of the circle.

During straight and level flight, **lift** will balance **weight**.

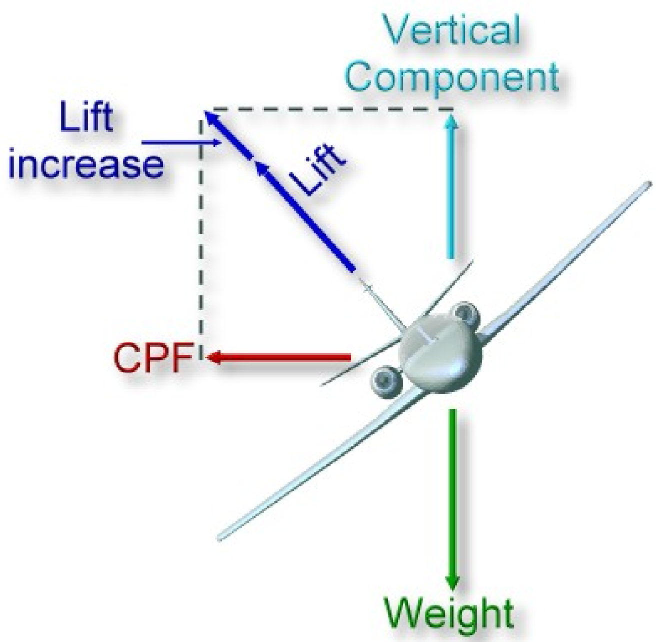




To turn the aircraft the **CPF** can be supplied by **tilting** the **lift** vector sideways into the turn.

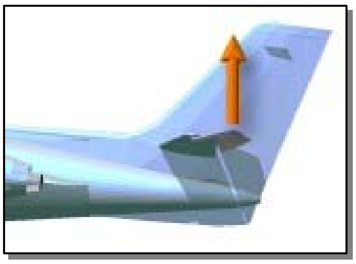


To maintain level flight, when the **lift vector** is tilted it must still supply a **vertical component** (force) to balance the **weight** in addition to the **CPF** it supplies.



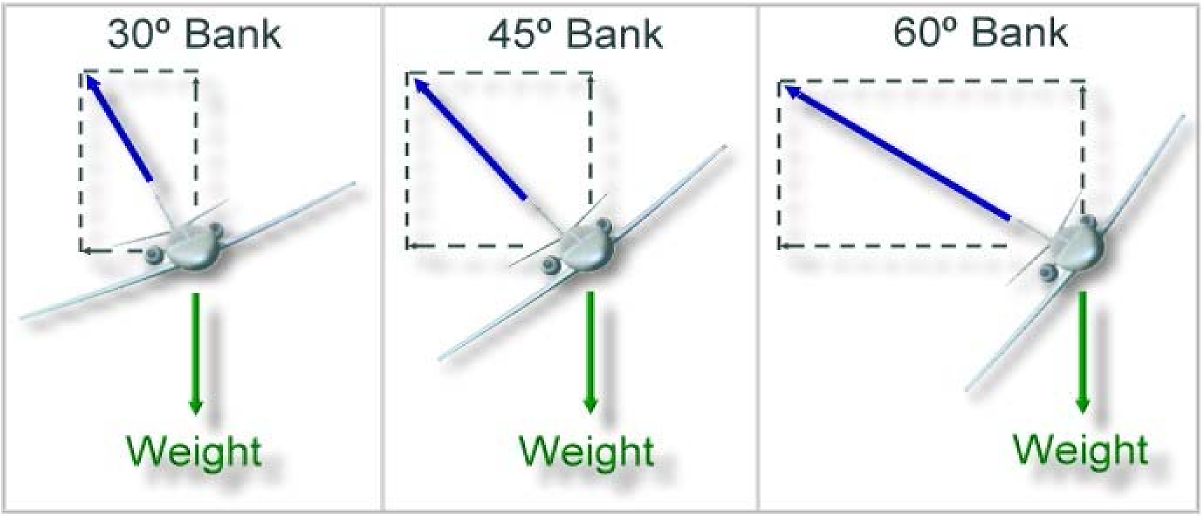
When the vector diagram is completed it is clear that the value of the **lift vector** must **increase** to supply the **vertical component** to balance **weight**.

For lift to increase, C_L must also increase. From the lift formula, $\mathbf{L = C_L \frac{1}{2} \rho V^2 S}$, if density, TAS and area remain constant, the angle of attack must thus **increase** in order to maintain constant height.



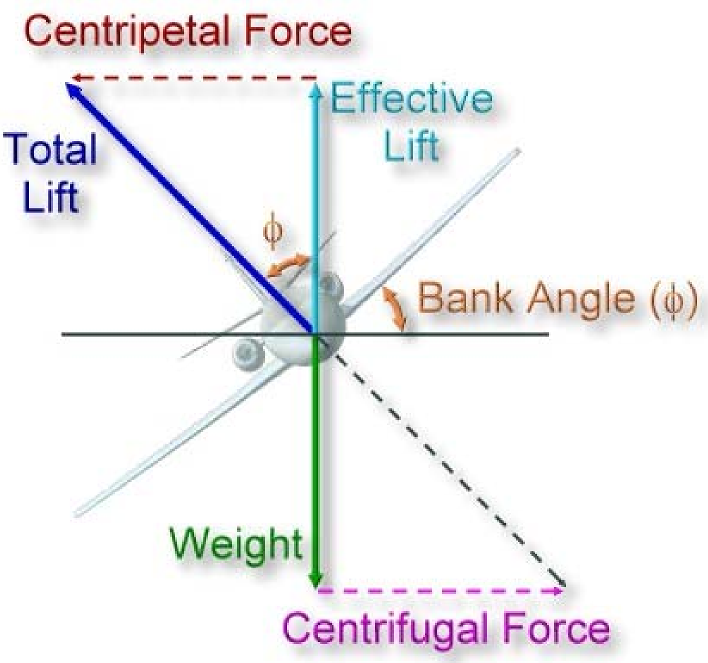
To increase the angle of attack the elevator is deflected upwards.

The steeper the bank during the level turn, the greater the extra lift force required:



Because the pilot turns with the aircraft they experience this increase in lift as an "apparent" increase in weight. This is known as "g-force".

The resultant forces on an aircraft in a level turn can be depicted graphically as follows:



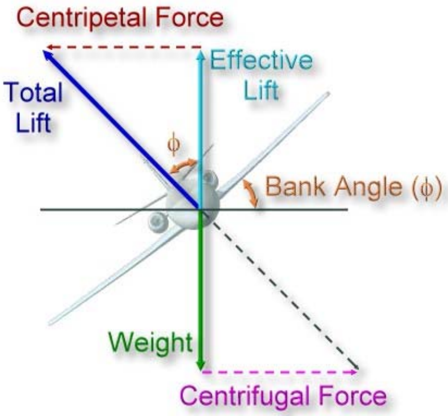
The **total lift** vector divided into the **effective lift** (balancing **weight**) and the **centripetal force**, acting towards the centre of the turn.

The angular difference between the **total** and **effective lift** is equal to the **bank angle**.

The **centrifugal force** is equal to, but acts in the opposite direction to the **centripetal force** ***(Newton 3)***.

There are a number of formulae that can be deduced from the forces in the turn diagram.

1. In the upper triangle:



$$\text{Cos } \phi = \frac{\text{Effective Lift}}{\text{Total Lift}} = \frac{\text{Weight}}{\text{Lift}} = \frac{W}{L}$$

Previously, in the lesson Aerodynamic Symbols and definitions it was stated that load factor is equal to:

$$n = \frac{L}{W}$$

By inverting the formula and substituting the value of $\frac{L}{W}$:

$$n = \frac{1}{\text{Cos } \phi} = \text{Sec } \phi$$

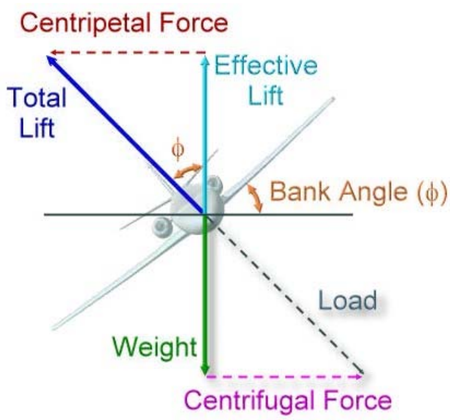
The centripetal force is also found in the upper triangle.

$$\text{Sin } \phi = \frac{\text{Centripetal Force}}{\text{Total Lift}}$$

and when the formula is rearranged

$$\text{Centripetal Force} = L \text{ Sin } \phi$$

2. The bottom of the diagram shows the reaction forces. **Weight** opposing **effective lift**, **centrifugal force** opposing **centripetal force** and **load** opposing the **total lift**.



Centripetal Force = L Sin ϕ

and Radial acceleration = $\frac{V^2}{r}$ **(from mechanics)**

$$CPF = \frac{MV^2}{r}$$

$$\text{Weight} = mg$$

$$\text{Thus } m = \frac{W}{g}$$

$$CPF = \frac{WV^2}{gr}$$

$$L \text{ Sin } \phi = \frac{WV^2}{gr}$$

$$\text{but } L \text{ Cos } \phi = W$$

$$L \text{ Sin } \phi = \frac{L \text{ Cos } \phi \, V^2}{gr}$$

$$\frac{L \text{ Sin } \phi}{L \text{ Cos } \phi} = \frac{V^2}{gr} \quad \text{But} \left(\frac{\text{Sin } \phi}{\text{Cos } \phi} = \text{Tan } \phi \right)$$

$$\text{Thus Tan } \phi = \frac{V^2}{gr}$$

$$r = \frac{V^2}{G \text{ Tan } \phi}$$

The formula to calculate **turn radius** is derived from the **centripetal force** formula.

The formulae applicable to the level turn are therefore:

$$n = \frac{1}{\text{Cos } \phi} = \text{Sec } \phi$$

$$\text{Centripetal Force} = L \text{ Sin } \phi$$

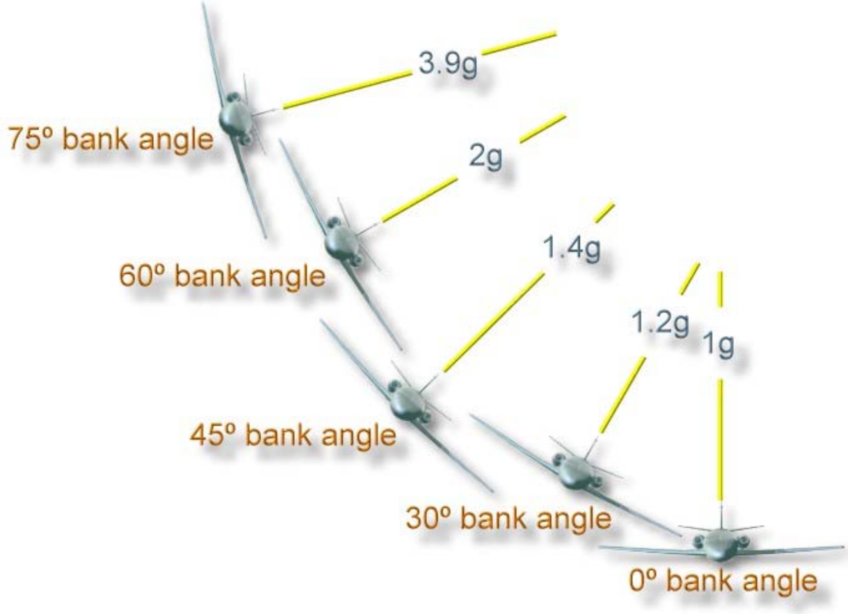
$$r = \frac{V^2}{G \text{ Tan } \phi}$$

LOAD FACTOR

By using the formula

$$n = \frac{1}{\cos\phi} = \sec\phi$$

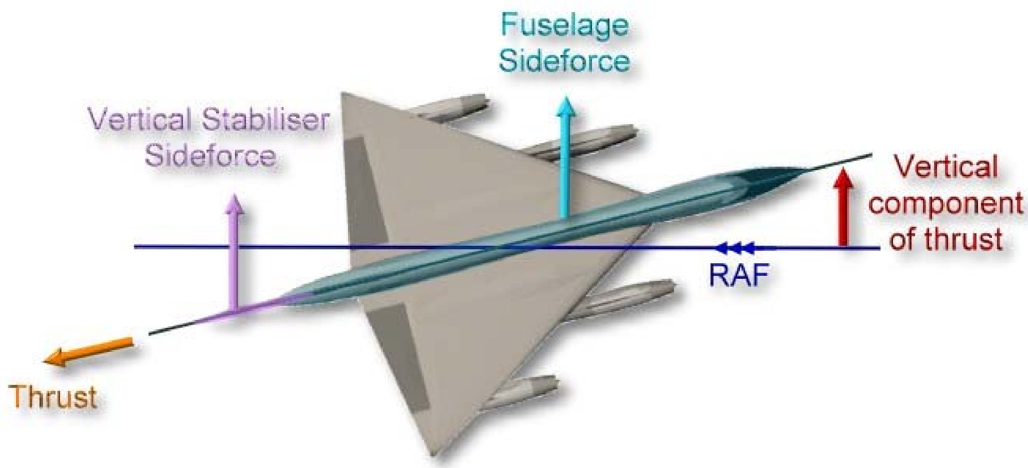
the different **load factors** at specified **bank angles** can be calculated.



It must be noted that the load factors required to maintain a constant altitude in the turn were determined by assuming that the wings provided all the lift.

According to the formula, at 90º bank the lift required will be infinite. This is clearly impossible, yet we have all seen an aircraft do a four-point roll, maintaining altitude at 90º of roll.

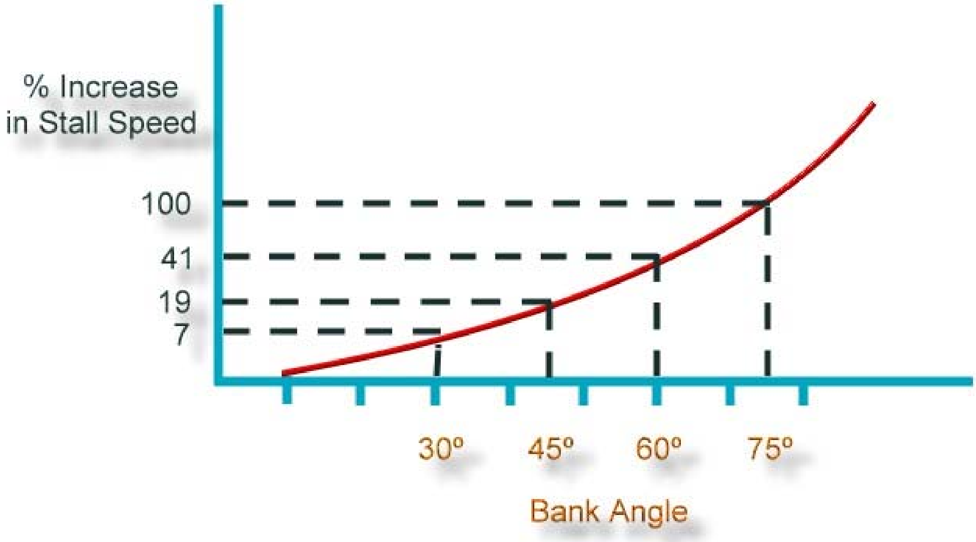
The secret is that additional "lift" is provided by the **vertical component of thrust**, the side force on the **vertical stabiliser** and the **fuselage** as a whole.



EFFECT ON STALL SPEED

The formula for manoeuvre stall speed is: $V_M = V_B \sqrt{n}$

When **n** is substituted by $\frac{1}{\cos \phi}$ the formula can be used to calculate the % increase in stall speed for specific bank angles. The values can be plotted on a graph.

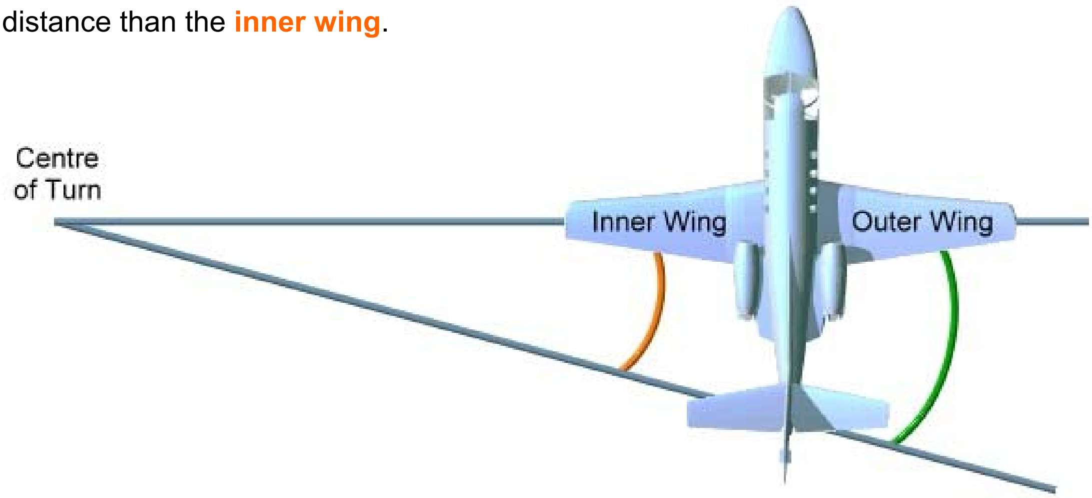


Note that in a 75 degree banked turn the stall speed is doubled and at 60 degrees of bank it is increased by nearly 50%.

EFFECT ON BANK ANGLE

To initiate the turn the ailerons are deflected to roll the aircraft to the desired bank angle. Once this bank angle is reached the ailerons are neutralised to maintain this bank angle.

If the turning aircraft is viewed from above it is clear that the **outer wing** will travel a longer distance than the **inner wing**.

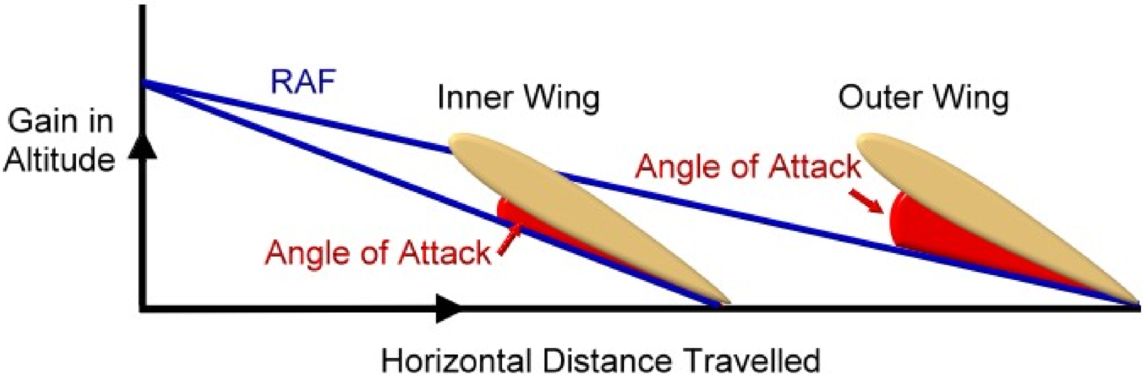


This results in a higher speed, and thus lift, on the outer wing. The aircraft will therefore tend to **roll into the turn (overbanking)**.

During a level turn the ailerons must thus be deflected slightly out-of-turn to maintain the desired bank angle.

The same effect ***(rolling into the turn)*** occurs when the aircraft is in a climbing turn. But in the case of a climbing turn there is also a second factor to consider.

For the same gain in altitude the outer wing will travel a longer horizontal distance. This will increase the **effective angle of attack** on the outer wing, thereby increasing the lift generated by that wing.

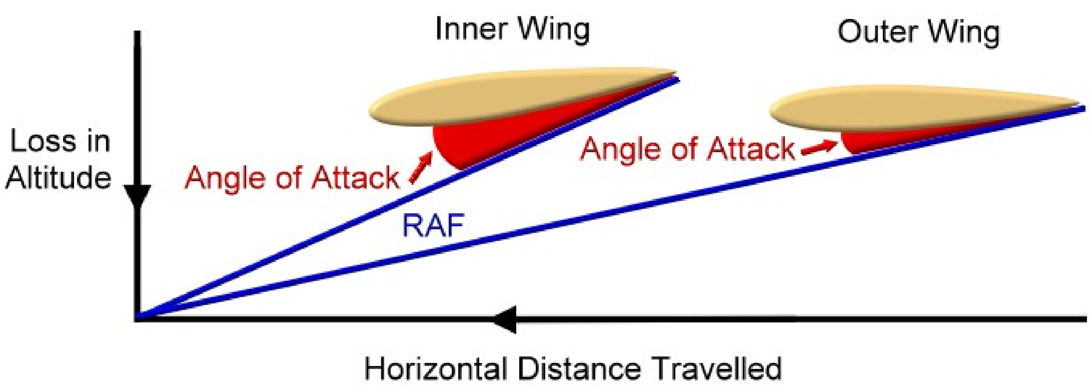


This will aid the roll due to the difference in wing speed.

This means that during a climbing turn the ailerons must be deflected slighty out-of-turn to maintain the desired bank angle.

In a descending turn the outer wing travels faster, producing more lift, thus rolling the aircraft into the turn.

But the inner wing has a larger **angle of attack** because it travels a shorter distance. The inner wing will therefore produce more lift, rolling the aircraft out of the turn.



The two effects thus tend to cancel each other. The actual effect **(overbank or underbank)** will depend on the specific aircraft.

BALANCING THE TURN

Refer to the lesson that dealt with the aileron as a primary control surface. One of the negative effects of aileron deflection was adverse aileron yaw. One way of minimising adverse aileron yaw is to couple the rudder to the ailerons so that the rudder deflects to provide the required yaw **(coupled controls)**.

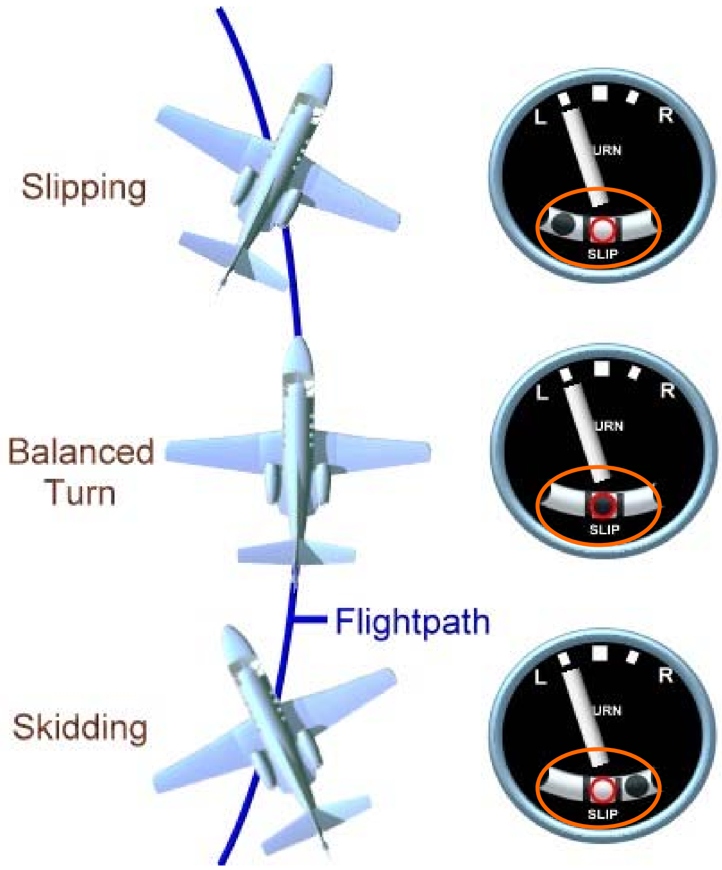
If the aircraft is not fitted with coupled controls, as are most light and military aircraft, the pilot must deflect the rudder himself to oppose the adverse aileron yaw **(the deflection being mainly during the roll into or out of the turn)**. This is called **balancing the turn**.

There are three cases to consider when balancing the turn:

- When the rudder deflection is too small to balance the adverse aileron yaw the aircraft will be slipping into the turn. This is **indicated** by the slip ball on the turn-and-slip indicator being to the inside of the **central** position.

- When the rudder deflection balances the adverse aileron yaw the aircraft will be balanced. This is **indicated** by the slip ball on the turn-and-slip indicator being **central**.

- When the rudder deflection is too large to balance the adverse aileron yaw the aircraft will be skidding out of the turn. This is **indicated** by the slip ball on the turn-and-slip indicator being to the outside of the **central** position.

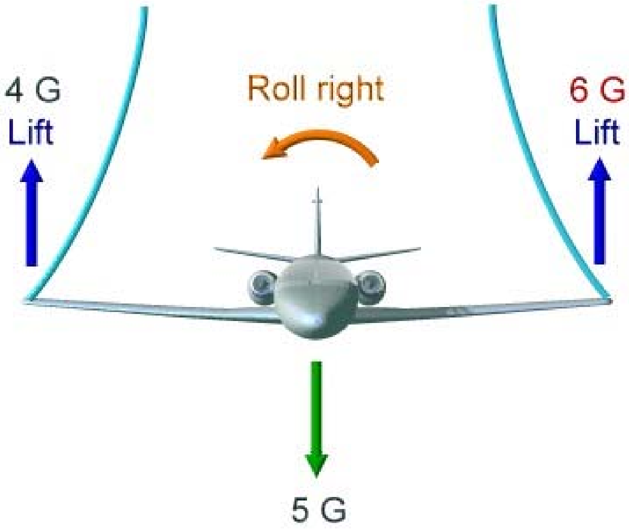


ASYMMETRICAL LOADING

An aircraft is designed to withstand a certain load factor (n). If this load factor is exceeded structural failure may occur.

Consider an aircraft recovering out of a dive at its maximum design load factor (5 n or 5 "g"). The load factor is calculated with the formula:

$$n = \frac{L}{W}$$



To **roll** the aircraft to the **right** the **lift** on the left wing must be increased. But according to the formula for n, this will further increase the load factor.

The load factor on the up going wing will thus increase beyond the maximum design load factor. It is for this reason there is also a maximum rolling "g" limit stipulated for aircraft.

CONCLUSIONS

The conclusions that can be derived from the turning radius formula are:

$$r = \frac{V^2}{G \tan \phi}$$

where V^2 = TAS

$$g = \frac{L}{W}$$

ϕ = bank angle

- **Turn radius is independent** of **weight** at any specific TAS (**V**) and bank angle (ϕ). Thus if a Boeing 747 and a Cessna 150 has the same TAS they will have the same turning radius at a given bank angle.
- If **TAS is increased** at a constant bank angle, **radius will increase**.
- If TAS is held constant but **bank increased**, **radius will decrease**.
- If **TAS is decreased** at a constant bank angle, **radius will decrease**.
- To **maintain** the same turn radius if **TAS is decreased**, **bank angle must decrease**.

For instrument flying especially, the rate of turn is also important. Usually the standard turn used in instrument flying is the "rate one" turn.

A rate one turn is a turn at 3 degrees per second. The aircraft will thus turn through 180 degrees in one minute and 360 degrees in two minutes.

Remember that, to maintain a rate one turn, if the **TAS increases** the **bank angle must also increase**.

An easy way to estimate the bank angle required for a rate one turn is: 1/10th of the airspeed in knots, plus 7.

For example at 120 knots it is 12 degrees (1/10) plus 7, giving us 19 degrees of bank.

TURNING PERFORMANCE

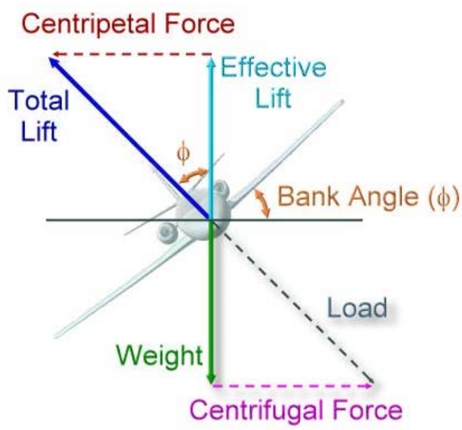
INTRODUCTION

Two aspects of general turning performance will be studied in this lesson, namely the minimum radius turn and the maximum rate turn.

Although these turns may have limited practical application ***(confidence exercises, air shows and collision avoidance)*** it lays the foundation for further understanding of aircraft manoeuvring, both theoretical and practical.

MINIMUM RADIUS TURN

From the previous lesson the forces in the turn can be represented as in the graphic.



Also in the previous lesson it was shown that the turn radius could be calculated with the formula:

$$r = \frac{V^2}{g \tan \phi}$$

The **V** in the formula represents manoeuvre stall speed (**V_M**).

Because not all aircraft can achieve the same angle of bank at the same TAS the equation for turn radius must be broken down further.

The formula for manoeuvre stall speed is: $V_M = V_s \sqrt{n}$

But $n = \frac{1}{\cos \phi}$ ***(from previous lesson)*.**

Thus $V_M^2 = V_s^2 \times \frac{1}{\cos \phi}$ ***(cancelling the square root)***

Giving $$r = \frac{Vs^2}{g \tan \phi} \times \frac{1}{\cos \phi}$$ *(substituting in normal radius formula)*

Thus $$r = \frac{Vs^2}{g \sin \phi}$$

The effect of load factor (n) on the stall:

$$L = C_L \tfrac{1}{2}\rho V^2 S$$

At Stall $$L = C_L \tfrac{1}{2}\rho V^2{}_{STALL}\, S$$

At stall $$V^2{}_{STALL} = \frac{L}{C_{L\,MAX} \tfrac{1}{2} \rho S}$$

$$V_{STALL} = \sqrt{\frac{L}{C_{L\,MAX} \tfrac{1}{2} \rho S}}$$ Equation 1

From load factor $$n = \frac{L}{W}$$

$$L = nW$$ Equation 2

Substituting (2) into (1) $$V_{STALL} = \sqrt{\frac{nW}{C_{L\,MAX} \tfrac{1}{2} \rho S}}$$

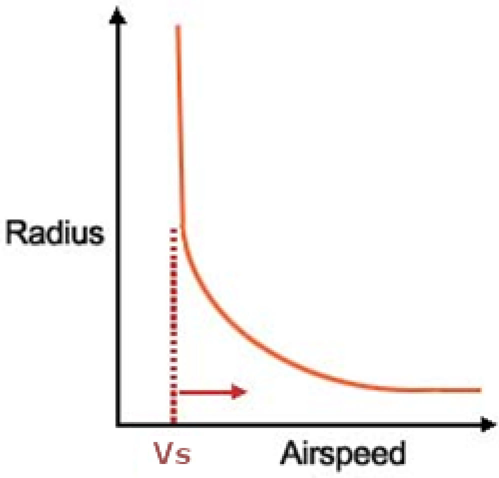
$$V_{STALL} = \sqrt{n} \times \sqrt{\frac{W}{C_{L\,MAX} \tfrac{1}{2} \rho S}}$$

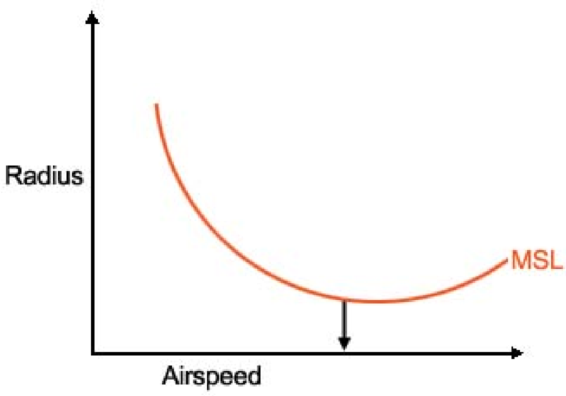
Thus $$V_s \propto \sqrt{n} \times \sqrt{W/S}$$

The **relationship** between TAS and the theoretical radius can be shown graphically.

The radius will decrease as the speed is increased above the basic stalling speed (**V_S**) until the minimum radius is achieved at an ***(theoretical)*** infinite airspeed.

At higher airspeeds the effect of an increase in Mach number is to decrease $C_{L\ MAX}$. This will change the graph ***(measured at sea level)*** to look as follows:





Instead of being an infinite speed, the speed to achieve the smallest turn radius is now read of at the **lowest point on the graph**.

Therefore it was found that to decrease the turn radius the bank angle must be increased. But to prevent the aircraft from descending the overall lift must also be increased. Maximum lift is generated when C_L is a maximum. Any further increase in bank angle will mean that the aircraft will start descending.

Thus a minimum radius level turn for any given airspeed is obtained when the aircraft is flown at CL MAX and maximum angle of bank.

RATE OF TURN

Rate of turn is a measure of angular displacement about the normal axis and is measured in the number degrees relative to time.

A rate 1 turn is when 360° is turned in a time period of 2 minutes. This equates to 3°/second.

The necessary angle of bank to achieve a rate one turn can be found by:

$$\text{Angle of Bank for Rate 1 turn} = \frac{TAS}{10} + 7°$$

It must be noted this is only an approximation and is accurate for small angles and relatively slow airspeeds.

The following relationships can be used to show the effects of speed (**V**), angle of bank (θ), radius of turn (**r**) and rate of turn (ROT).

$$\theta \propto \frac{V}{r}$$

- If ***V*** increases and ***r*** remains constant then θ must increase.
- If For a constant θ as ***V*** increases then ***r*** must increase.
- ***V*** is constant and θ increases then *r* reduces.

$$ROT \propto \frac{\theta}{V}$$

- For constant ROT as ***V*** increases then θ must increase.
- For a constant θ as ***V*** increases the ROT reduces.
- For a constant ***V*** as θ increases then ROT increases.

FACTORS AFFECTING TURN PERFORMANCE

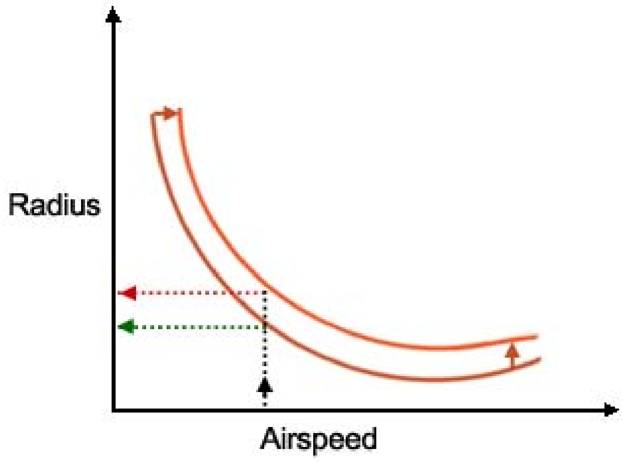
There are a set of factors that affect the radius and rate of turn:

- Wing loading
- Thrust
- Use of flap
- Altitude
- Maximum "G" loadings

WING LOADING

Wing loading directly relates to basic stalling speed. The higher the wing loading the higher the basic stalling speed.

The effect of an increase in the basic stalling speed on the graph showing the theoretical relationship between airspeed and radius of turn is to move the graph up and to the right.



At a fixed airspeed an aircraft with a **higher wing loading** ***(thus basic stalling speed)*** will have a larger radius of turn or slower rate of turn than an aircraft with a **low basic stalling speed**.

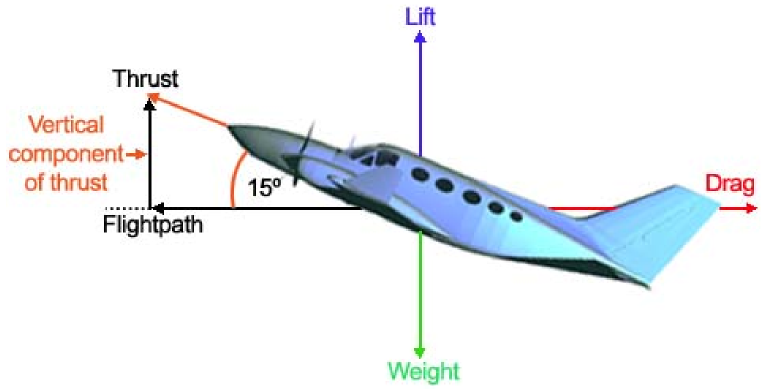
THRUST

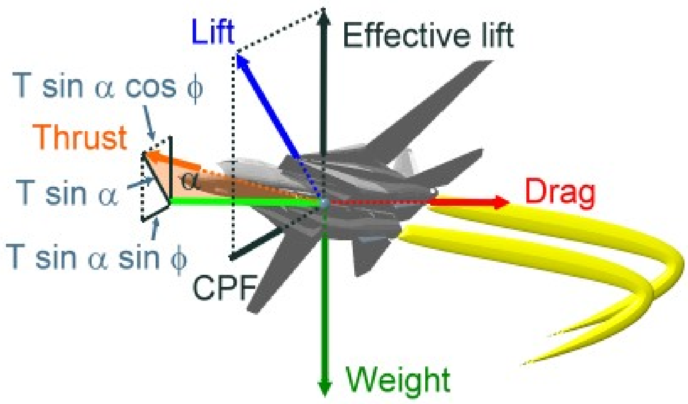
Thrust, or the lack of thrust, can be the determining factor as to whether the optimum speeds referred to earlier can be attained.

Even in level flight some aircraft have a component of thrust acting in the same direction as lift. This effect is more pronounced as the critical angle is approached.

The magnitude of the vertical component of thrust is equal to:

Thrust (T) x Sin α ***(angle of attack)***.





During a turn **lift** is broken up into effective lift ***(balancing weight)*** and the CPF ***(providing the turning force)***.

The **thrust** line is inclined above the **flight path** (angle α). This is especially true when the aircraft is operating at $C_{L\,MAX}$, as in the minimum radius and maximum rate turn.

The magnitude of the component of **thrust** acting in the same direction as **lift** is still **T Sin α**.

T Sin α is broken up into a component that aids the effective lift (T Sin α Cos ϕ) and a component that aids the CPF (T Sin α Sin ϕ).

The effect of thrust is to aid the CPF, thereby decreasing the turn radius or increasing the turn rate. The effect is a maximum at 90° of bank and zero at 0° of bank.

USE OF FLAP

The basic stalling speed of an aircraft decreases when flaps are lowered.

As long as the flap limiting speed is not a criterion, and thrust is available to counter the increased drag, then the use of flaps will be advantageous.

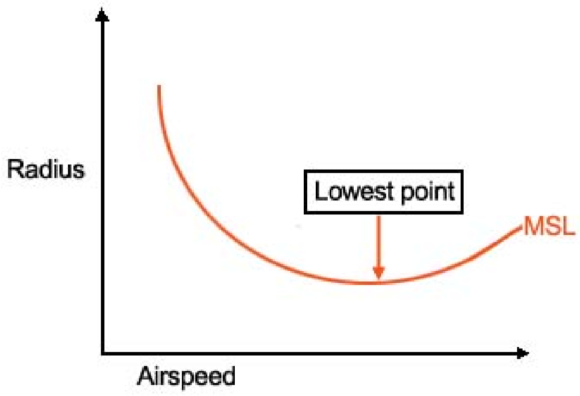
Flap limiting speed

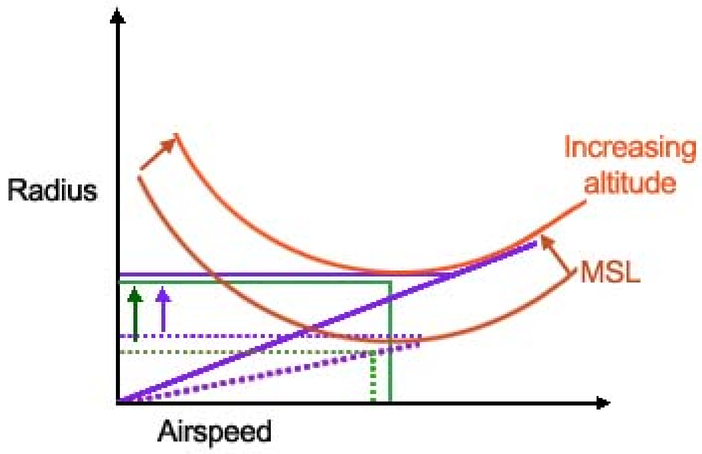
If the airspeed is increased with the flaps lowered, it increases the aerodynamic load on them, and this can lead to structural failure of the flaps. A maximum airspeed limitation is thus imposed for the use of flaps.

Thus, an aircraft with a **higher basic stalling speed** will have a larger radius of turn or slower rate of turn than an aircraft with a **low basic stalling speed**.

ALTITUDE

The speed for minimum radius is read off at the lowest point on the practical radius/airspeed graph.





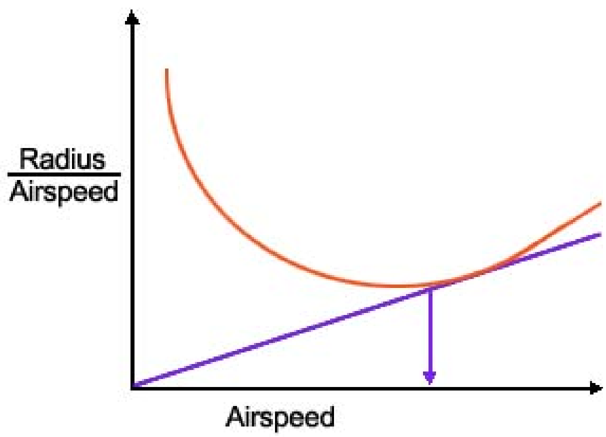
Due to the fact that TAS increases for a fixed IAS as altitude increases, as well as the reduction in $C_{L\ MAX}$ with an increase in altitude, it will mean that the graph will move up and to the right.

The effect of an increase in altitude is thus to **increase the radius** of turn and **decrease the rate of turn**.

MAXIMUM "G" LOADINGS

If $C_{L\ MAX}$ were constant the speed for minimum radius, maximum rate and maximum g loading would be the same.

But $C_{L\ MAX}$ decreases with an increase in Mach number. Because the g load depends on the acceleration towards the centre of the turn ***(provided by lift)*** the maximum g loading will be where this acceleration is a maximum.



The speed for maximum g load can be found graphically by plotting radius/airspeed against airspeed.

The actual speed for maximum g loading is read of at the **tangent to the curve**. This speed will be higher than both the speed for minimum radius and maximum rate.

FACTORS LIMITING TURN PERFORMANCE

As only level turn performance is considered in this lesson, there is a set of factors that limit the radius and rate of turn achievable by any aircraft:

- Aerodynamic
- Structural
- Power plant

AERODYNAMIC

The wing on an aircraft produces maximum lift at its critical angle of attack.

> **Critical angle**
>
> **The angle of attack at which maximum lift is produced and after which there is a marked reduction in lift because of the breakdown of airflow over the wing.**

Remember that the aircraft can stall at any airspeed. The maximum achievable load factor is reached at the stall for that given airspeed. The load factor will thus be the greatest when $C_{L\ MAX}$ is reached at a high airspeed.

Since an aircraft cannot be satisfactorily manoeuvred when $C_{L\ MAX}$ is exceeded it means that every aircraft has this aerodynamic limitation to manoeuvring flight.

STRUCTURAL

Even though modern aircraft are of the strongest structures for their weight, there is still a point at which the stress from either load factor or forward airspeed will cause distortion or damage to the airframe.

There is thus a structural limit to the manoeuvring stress that any aircraft can endure.

The structural limitations, in the form of the manoeuvre envelope, will be discussed in the next lesson.

POWER PLANT

In order to maintain altitude as bank angle is increased, at a constant IAS, lift, and consequently angle of attack, must also be increased.

Bank Angle	Load Factor	% induced drag increase form level flight
0º	1.000	0%
15º	1.036	7.2%
30º	1.154	33.3%
45º	1.414	100.0%
60º	2.000	300.0%

But as the angle of attack increases so the induced drag also increases.

If excess thrust (power) is available this can be used to offset the increase in drag so as to maintain a constant speed as well as height.

The bank angle at which total drag generated exactly equals the maximum thrust available is known as the maximum sustainable angle of bank. This angle of bank is dependant on the excess power available in the specific aircraft.

Any amount of bank greater than the abovementioned angle of bank will result in either a decrease in speed or a descent ***(or both)***.

NOMOGRAM

When a linear equation consists of three variables, two of which are known, the third can easily be determined by using a nomogram. ***(as shown below)***

For example:

TAS = 400 kts, altitude is 30 000 feet. The Mach number will now be **0.68**.

Take again a TAS of 400 kts, and a bank angle of 60 degrees. This will give a rate of turn of **284 deg/min** (approximately **rate 1.5**).