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ATPL NAVIGATION (AUS)

CHAPTER 3 – MAPS AND CHARTS (GENERAL)

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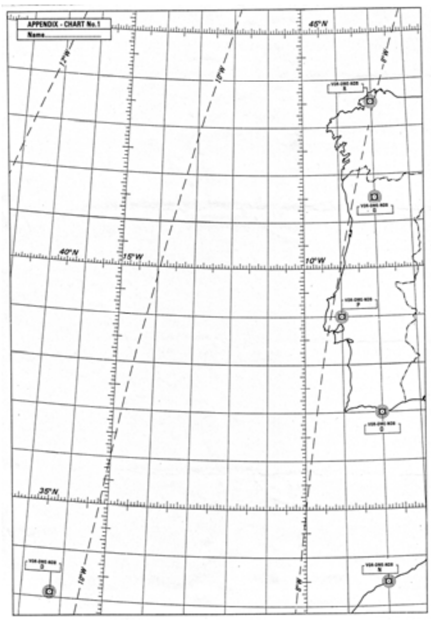
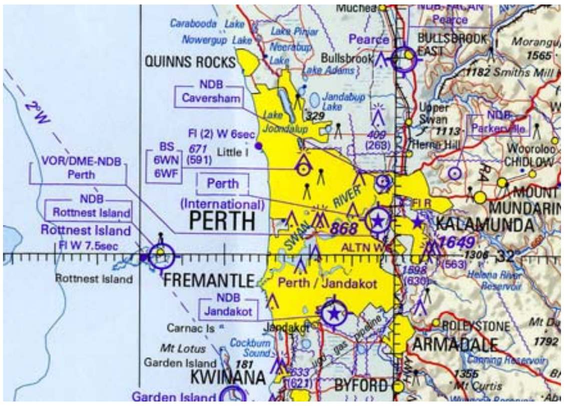
CHAPTER 3 - MAPS AND CHARTS (GENERAL)

A map is a depiction of the Earth's surface, usually produced on a flat sheet of paper. The construction of the map, and the selection of the information to be shown, depends entirely upon the purpose for which the map is designed.



Originally the term 'chart' referred to a map which showed an accurate graticule of latitude and longitude and a few major topographical features such as coastlines, but which otherwise was unencumbered with detail. A 'map' on the other hand, attempted to give an accurate and detailed picture of the terrain, and of the man-made features upon it. The chart was designed for navigation by plotting, while the map provided the detail for the navigator to plan his journey, and to fix his position as he proceeded along it.



These distinctions have now largely disappeared. The general public still uses the word 'map' almost exclusively, but professional aviators tend increasingly to use the term 'chart', even for publications that are clearly topographical maps, eg. the Operational Navigation Chart (ONC) and World Aeronautical Chart (WAC). Rather than prolong this semantic argument, we will usually treat the two words as synonymous.

SCALE

The major aim of any map is to provide a representation of a near-spherical Earth, or part thereof, on a small, flat and manageable surface. The most obvious result of the transformation is the small size of the map when compared with the real features that it represents. This difference leads directly to the vital concept of 'scale'.

The scale of a map is defined formally as the ratio between the linear dimensions of an object depicted on the map, and the dimensions of the real object on the ground, both dimensions being expressed in the same units.

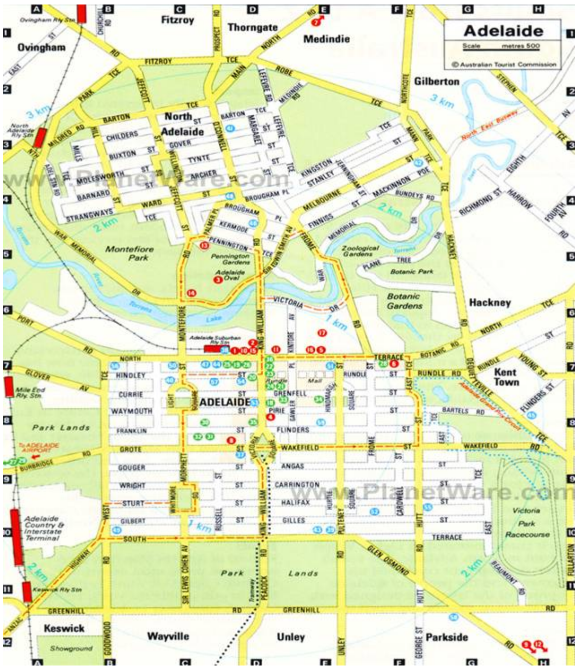
i.e.

$$\text{Scale} = \frac{\text{chart dimension of object}}{\text{Earth dimension of the same object}}$$

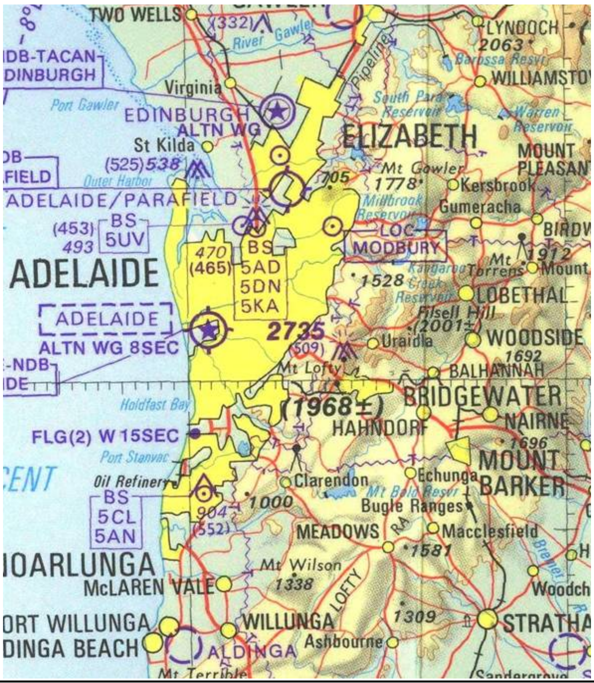
For example, if the scale is given as 1/50,000, it means that an object measuring 1 inch on the chart actually measures 50,000 inches on the ground: or alternatively, that the chart shows the object as 1/50,000th of its actual size.

This form of scale expression is called the ‘Representative Fraction’. Note that the representative fraction is a number, in this case 1/50,000, with all the usual mathematical characteristics of a number expressed in that form. The numerical nature of the representative fraction is sometimes lost, because we tend to speak of a scale of ‘1 in 50,000’ rather than ‘one fifty thousandth’, and we usually write the fraction as ‘1:50,000’.

Reference is often made to ‘large scale’ and small scale’ maps, and such reference relates to the numerical size of the whole fraction, not to the size of its denominator. For instance, a very large map scale is used in a street directory, typically 1:10,000 or 1:20,000, and is designed to depict a very small area of the Earth in very great detail. On the other hand, a wall chart of the world would usually be at a very small scale, perhaps 1:30,000,000. Such a chart shows a very large area in very little detail. The terminology can be confusing, but the confusion disappears with the realisation that 1/20,000 is a much larger number than 1/30,000,000, and hence represents a much larger scale.



Example of a *Large Scale Map*
Depicts small area to *Large Detail*



Example of a *Small Scale Map*
Depicts large area to *Small Detail*

The amount of additional detail that can be portrayed on a larger scale chart is evident when we consider charts of the same geographic area, but at different scales. Observe, for instance, the Visual Terminal Chart (VTC) of the Adelaide area, and compare it with the depiction of the same area on the WAC. The WAC scale is 1:1,000,000 and the VTC scale is 1:250,000, so we say that the VTC is drawn at four times the scale of the WAC. But this is one-dimensional, and when we compare the two charts we see that the VTC is 16-times larger. See Figure 1.

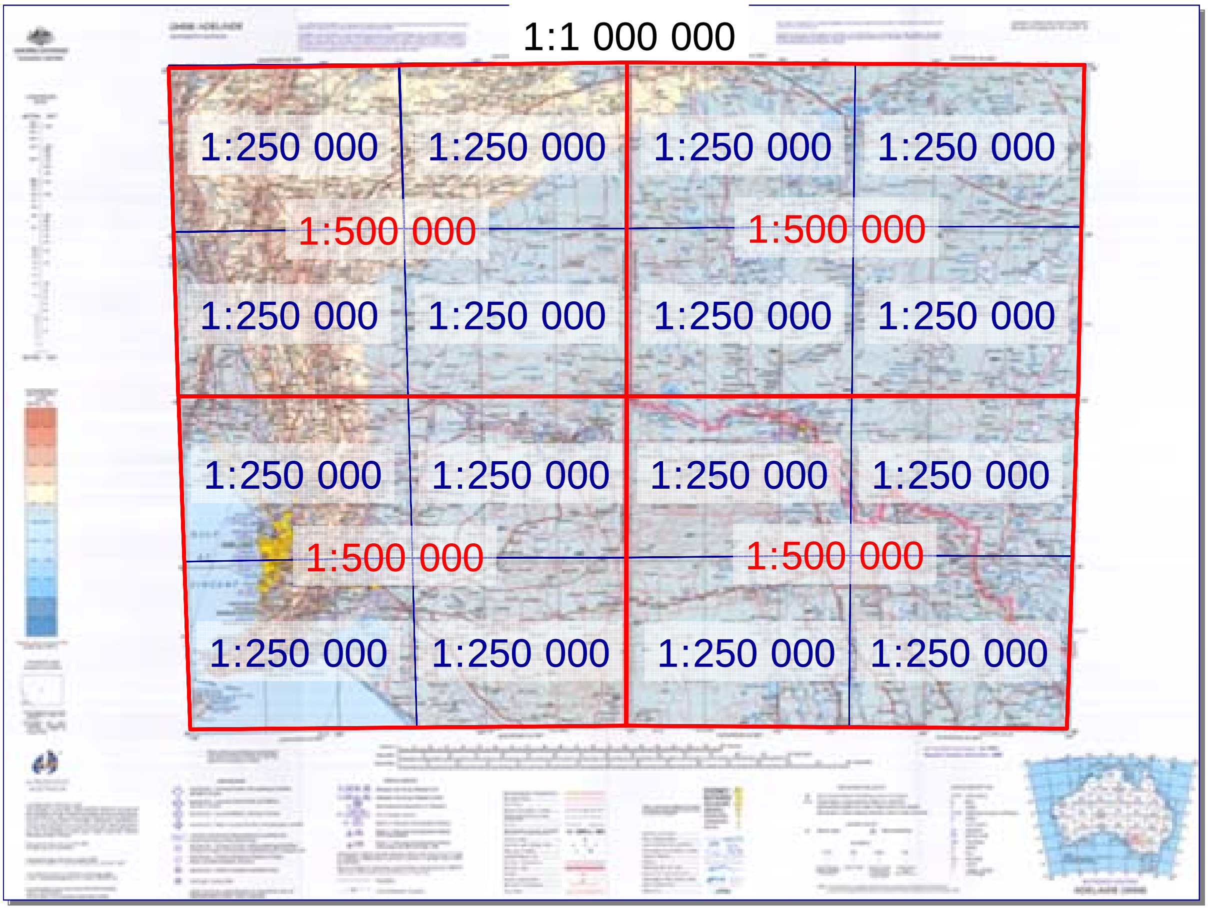


Figure 1

In general, if the scale of Chart A is n-times larger than that of Chart B, Chart A will be n^2 times larger than Chart B if both cover the same geographic area.

Scale is also frequently quoted in words, eg. ‘1cm represents 50nm’ or ‘1 inch represents 1 statute mile’, etc. This form is easier to visualise but is of less practical use. Since both forms are in common use, we must be able to change from one to the other.

Example 1

Change '1cm represents 50nm' to a representative fraction.

The method is simply to form a fraction with a numerator of 1 and the same units in the numerator and denominator. Thus:

$$\frac{1\text{cm}}{50\text{nm}} = \frac{1\text{cm}}{50 \times 1852 \times 100\text{cm}}$$

$$= \frac{1}{9{,}260{,}000}$$ so the chart scale is 1:9,260,000.

Example 2

Change the representative fraction 1:1,000,000 to the following forms:

'1cm represents __________ km' and,

'1 inch represents __________ nm'

Proceed as follows:

1:1,000,000 means $\frac{1\text{cm}}{1{,}000{,}000\text{cm}}$.

Change the denominator to kilometres:

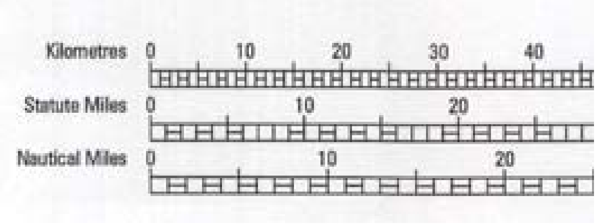
$1{,}000{,}000\text{cm} = \frac{1{,}000{,}000}{1000 \times 100} = 10\text{km}$ $\therefore$ *Scale in words is '1cm represents 10km'.*

Also, 1:1,000,000 means $\frac{1\text{inch}}{1{,}000{,}000\text{ inches}}$

Change the denominator to nm:

$1{,}000{,}000\text{ inches} = \frac{1{,}000{,}000}{12 \times 6{,}076} = 13.72\text{nm}$ $\therefore$ *Scale in words is '1 inch represents 13.72nm'*

The other method commonly used on maps to define scale is the 'scale line' or 'scale diagram'. This is a line graduated in units of distance against which given map distances can be measured. For instance, the WAC series presents three scale lines graduated in nautical miles, statute miles and kilometres respectively. Scale lines are little used on navigational charts because distance measurement is almost always required in nm, and this is read more conveniently and directly from the latitude scale on the map itself.



THE REDUCED EARTH

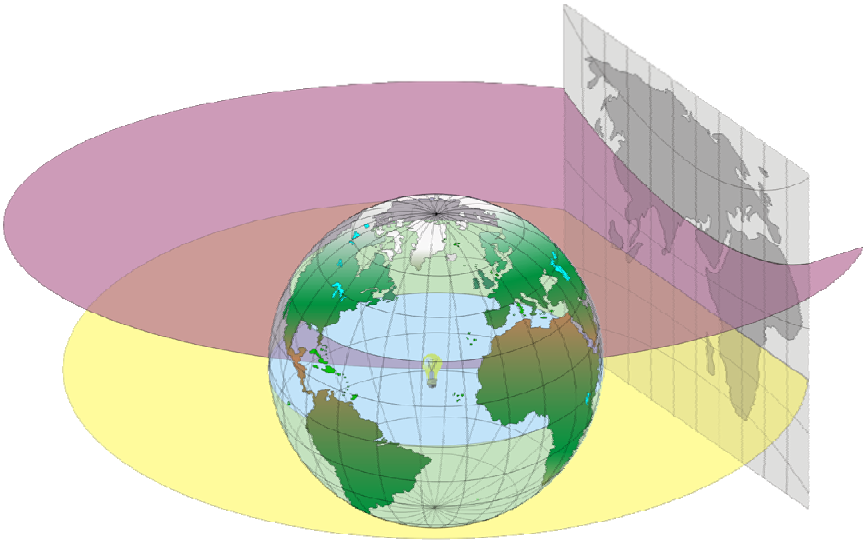
Another less obvious but particularly valuable means of presenting the concept of scale, is to consider a model of the Earth, such as a classroom globe. When such a model is used in the construction of a map it is called a 'Reduced Earth' (RE). The reduced Earth is itself a map, albeit in an inconvenient spherical form, but its scale is clearly linked to that of any flat map that we might make from it.

Note that the scale of the reduced Earth is constant over its entire surface. When we speak later of the scale of maps being 'constant and correct' we mean that:

1. The scale of the map exactly equals the scale of the particular reduced Earth on which the map was based, and

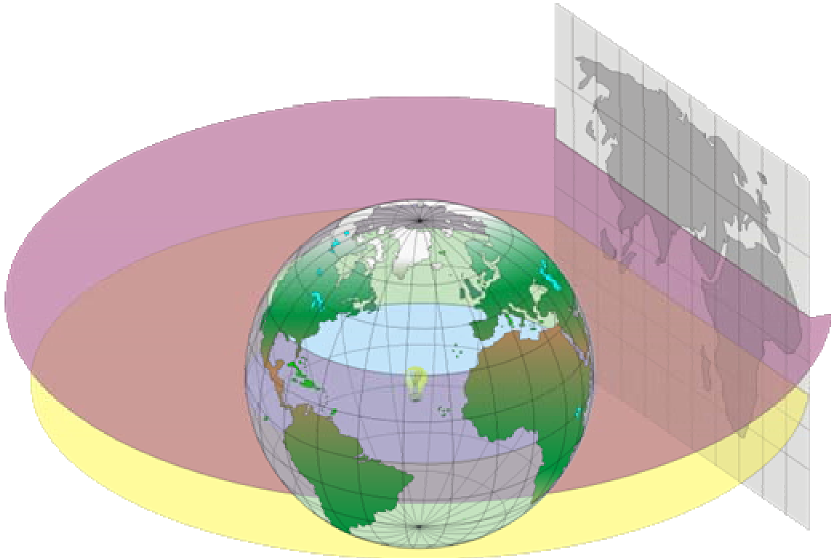
2. The scale of the map is constant over its entire area.

PROJECTIONS



We now come to the problem of changing our near-spherical reduced Earth, constructed to the required scale, into a map on a flat sheet of paper. Such a process is called projecting a map, or making a projection, the term originating with the close analogy between the map-making process and the projection of an image onto a screen. Imagine that the reduced Earth is made of glass, with the latitude/longitude graticule and required topographic and cultural detail etched onto its surface. Now imagine a point source of light at the centre of the reduced Earth. When the light is turned on and a suitable screen placed against the sphere, an image of the markings on the surface is projected onto the screen and could be used as the basis of a map.

It is most unlikely that maps were ever made by this process, but it does illustrate the geometric principles that underlie the production of at least some types of map. Indeed, projections that could, theoretically, be produced like this are called ‘Geometric’ or ‘Perspective’ projections. They rely upon the fact that the light source, any point on the reduced Earth, and the corresponding point on the map, are all co-linear, so that the line could be represented by a ray of light. Projections that lack this property are called ‘Non-perspective’ or ‘Mathematical’ projections, to indicate that the linearity of the system has been discarded in order to produce some other desired property.



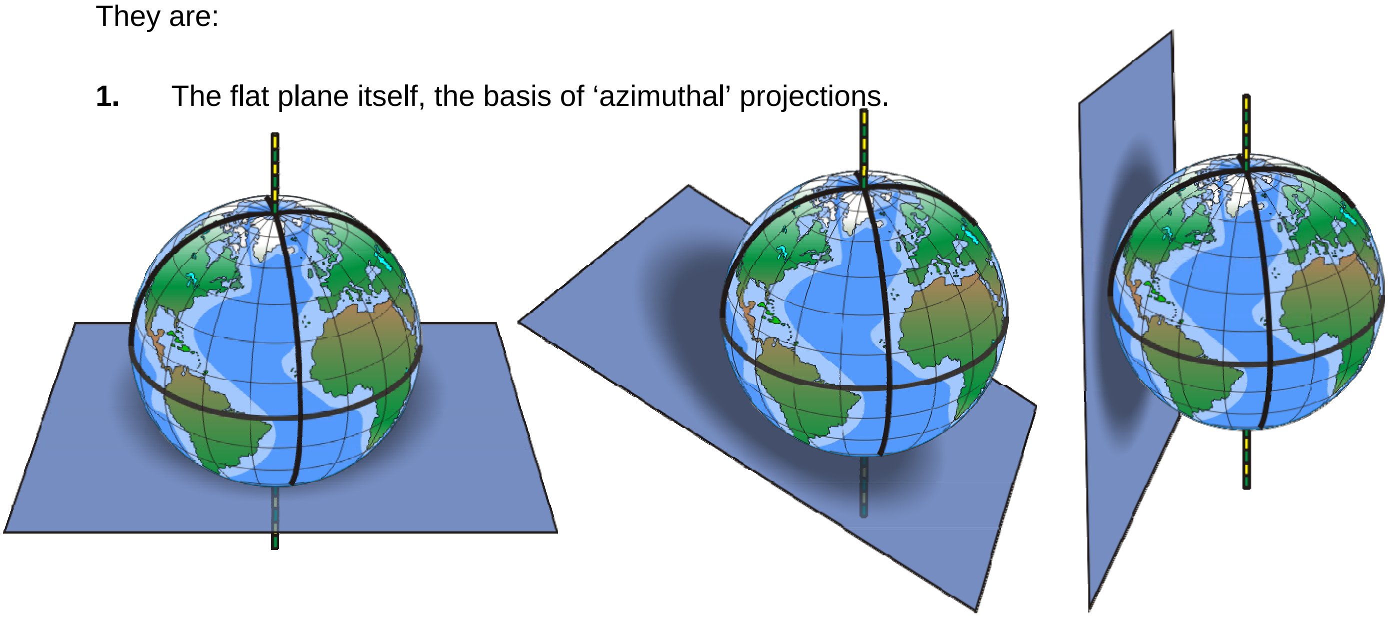
In this diagram the projection has been altered to produce a map with straight arcs of latitude. In doing so the light source at the centre of the reduced earth, the latitude line on the reduced earth and the image of the latitude line is no longer co-linear.

PROJECTION SURFACES

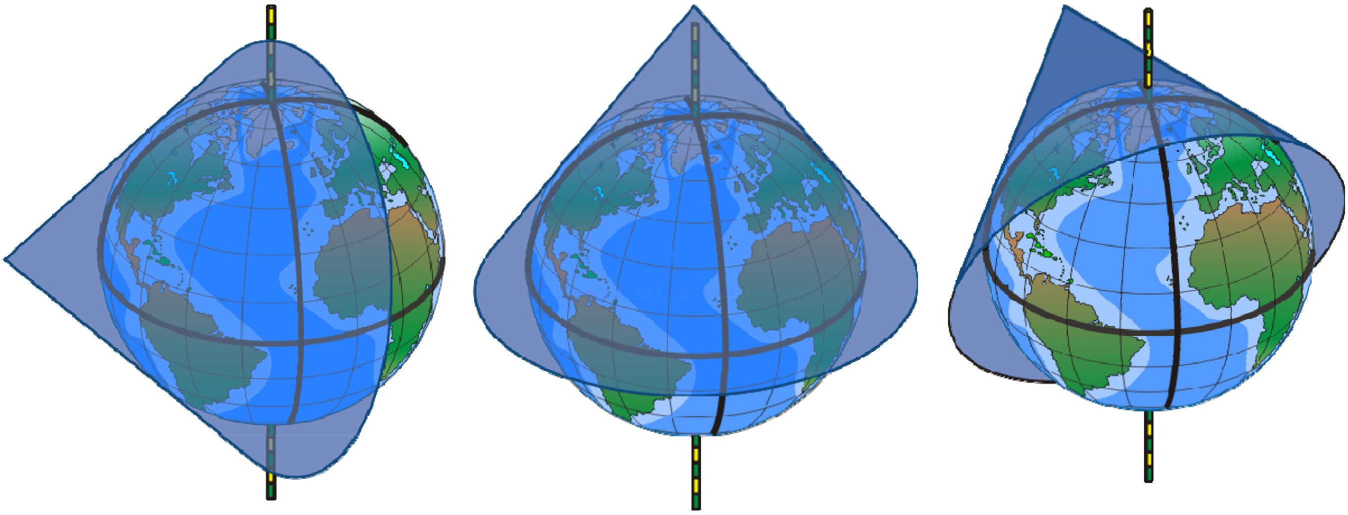
The screen on which the map is projected does not itself have to be a flat surface. It can be curved, but the curve must be able to be ‘unrolled’, without tearing or distortion, to form a perfectly flat surface. Such curved surfaces are said to be ‘Developable’ (into a flat plane).

In this discussion, we will consider just three surfaces on which a map might be projected. They are:

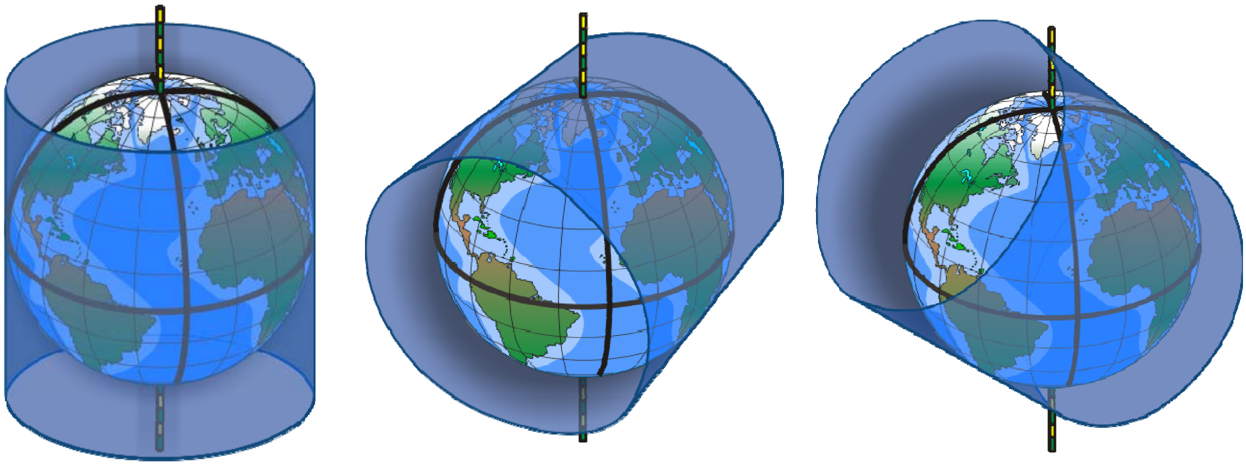
1. The flat plane itself, the basis of ‘azimuthal’ projections.



2. The cone, the basis of 'conic' projections.



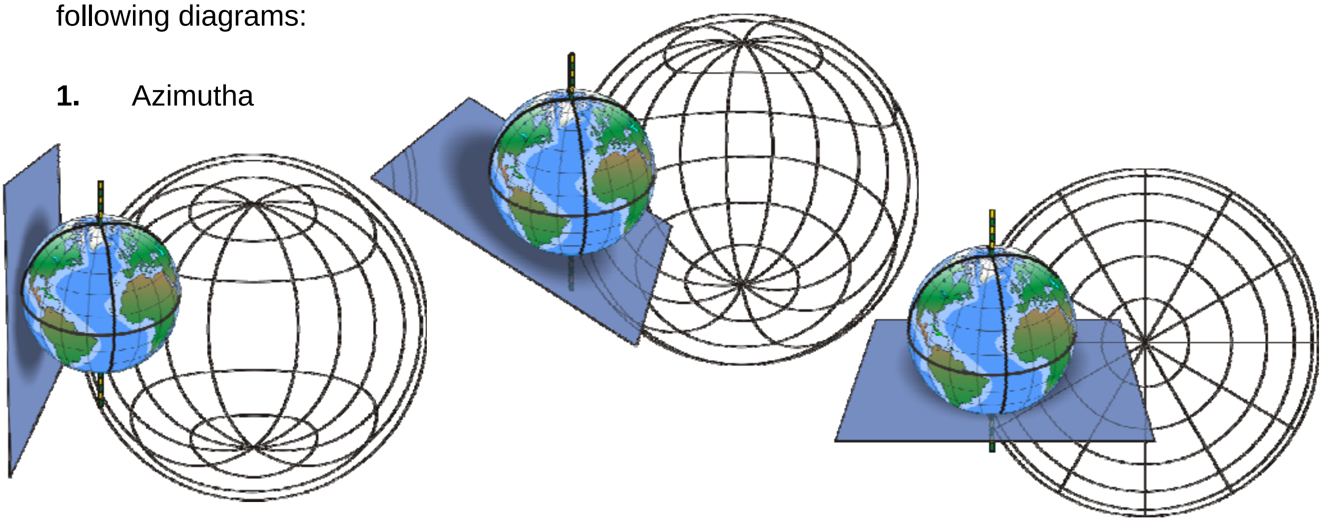
3. The cylinder, the basis of 'cylindrical' projections.



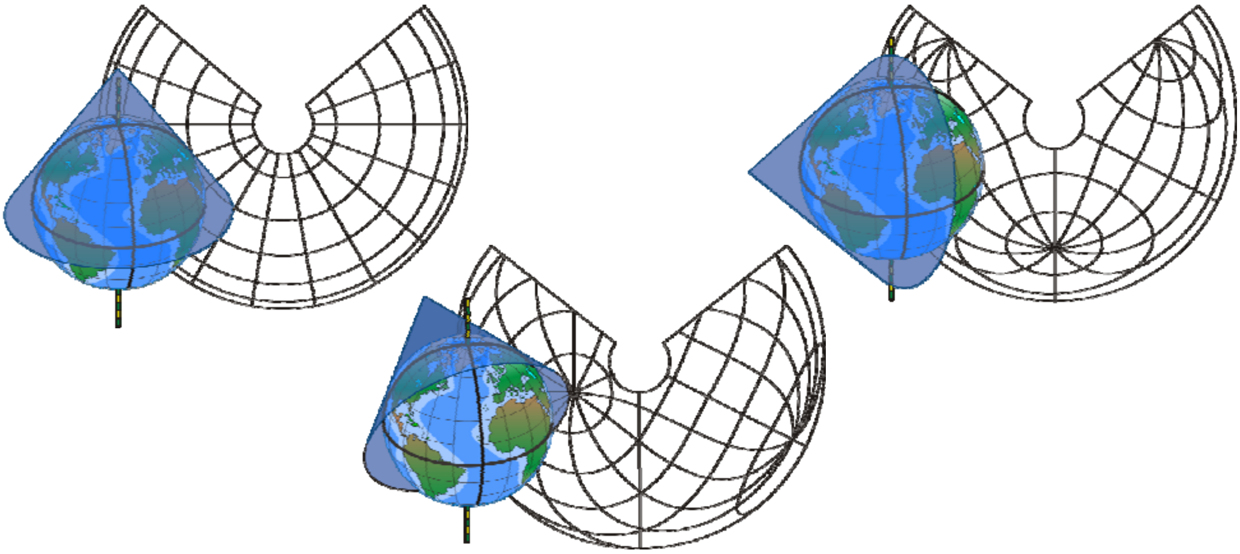
These three surfaces between them cover all the important aeronautical charts in use today.

The relationship between the reduced Earth and the finished map graticule is shown in the following diagrams:

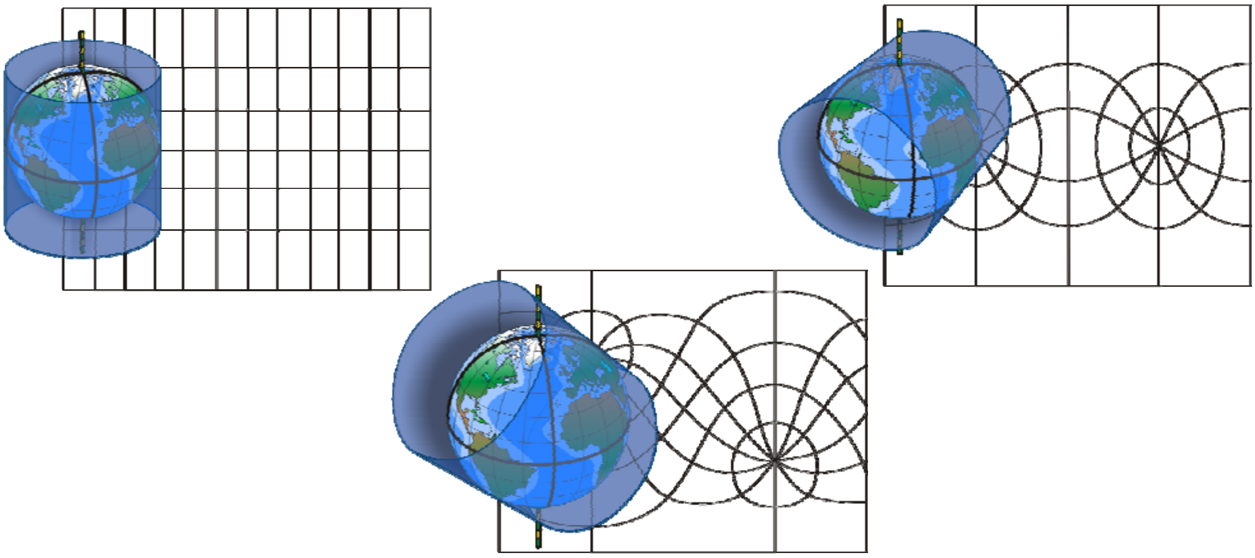
1. Azimutha



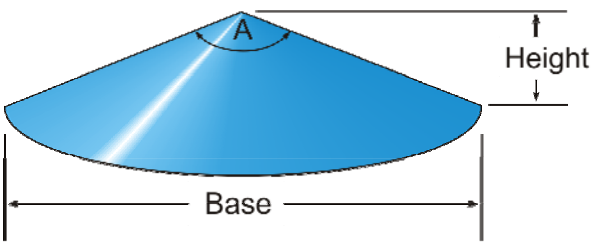
2. Conic

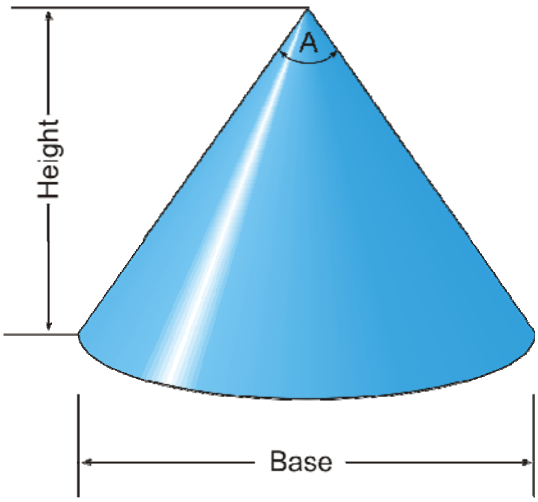


3. Cylindrical

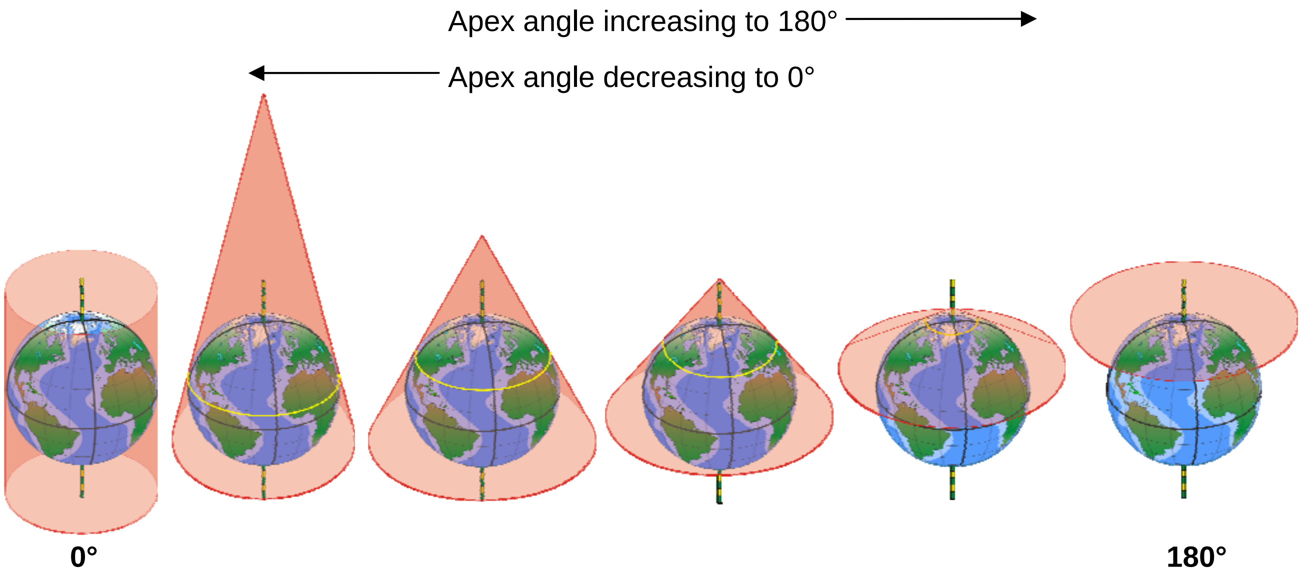


We shall see later that all these forms are really variations of the simple conic. The height of a cone in relation to its base is a function of the angle at its apex (A in the diagrams below)





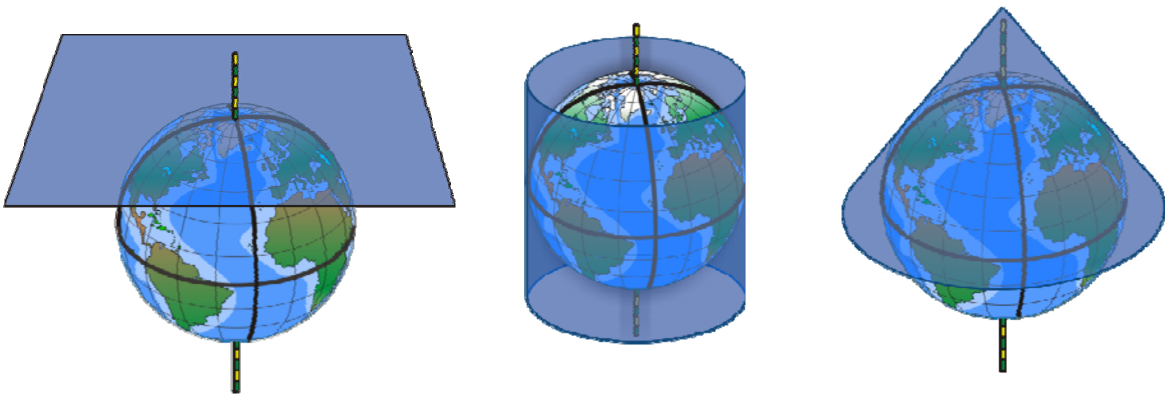
The limiting forms are the cylinder, in which the apex angle is zero - the sides of the 'cone' are parallel - and the flat plate, which is a cone with an apex angle of 180°.



Hence, despite the very marked differences in the appearance of the resulting projections, all have fundamental properties in common.

PROJECTION ALIGNMENT

The projections that are simplest mathematically and often of most use navigationally are 'Normal Projections'. A normal projection is one in which the axis of symmetry of the cone and the axis of the reduced Earth are coincident.

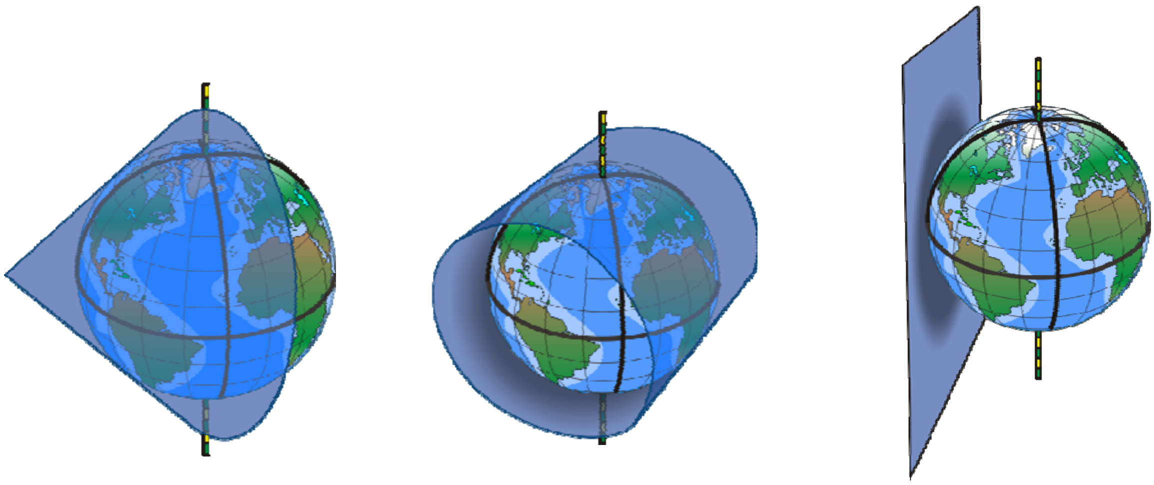


On all normal projections:

1. Meridians are projected as straight lines, and

2. The circle of tangency between the cone and the reduced Earth is a parallel of latitude.

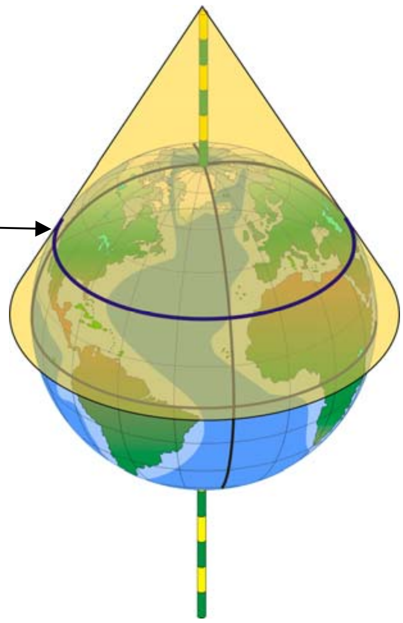
A projection that is not normal is said to be 'Oblique', and in the extreme case "Transverse".

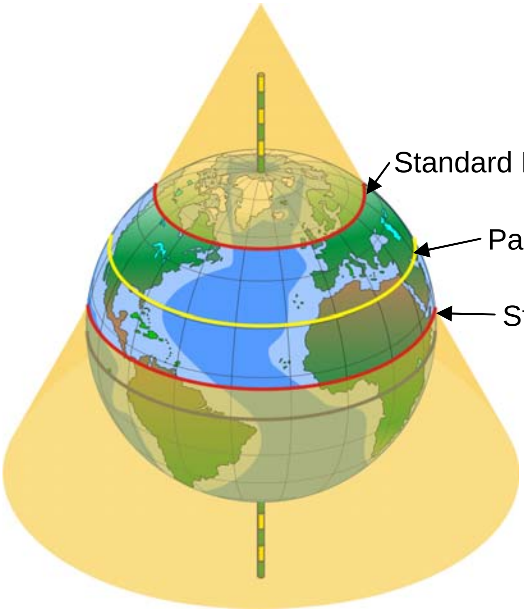


The selection of somewhat obscure developable surfaces is made necessary by the fact that, unfortunately, the sphere itself is not developable. A spherical surface, or part thereof, cannot be transformed into a flat plane without causing significant distortion and/or tearing of the original surface. Nor can the spherical surface be transferred into any other surface which is itself, developable, without similar distortion. Therefore, while it is clearly possible to make a developable surface that is tangential to the surface of the sphere at a single point, or along a line, it is not possible to make it perfectly tangential over any area of significant size. At the point or line of tangency, the image of objects on the reduced Earth will be reproduced faithfully, but everywhere else the image will be distorted in the transformation from the spherical to the developable surface. The study of these inevitable distortions and the means by which they are corrected or minimised, provides the major theme for our subsequent analysis.

PARALLEL OF ORIGIN AND STANDARD PARALLELS

The point where the projection surface touches the reduced earth is called the Parallel of Origin. The transfer of detail i.e. scale, convergency, shape, etc. is correct along this line.





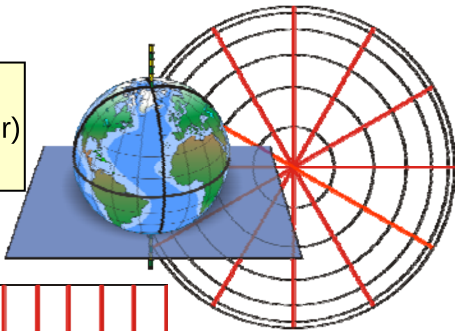
A secant surface cuts through the reduced earth in two places. The lines on the reduced earth where the projection surface and the reduced earth coincide are called the Standard Parallels. The transfer of detail i.e. scale, shape, etc. is correct along these lines.

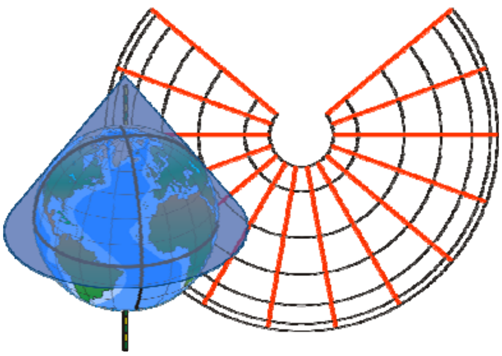
Approximately halfway between the Standard Parallels the projection surface and a tangent to the reduced earth will be parallel. This line is again called the Parallel of Origin and the chart convergency is derived from this point.

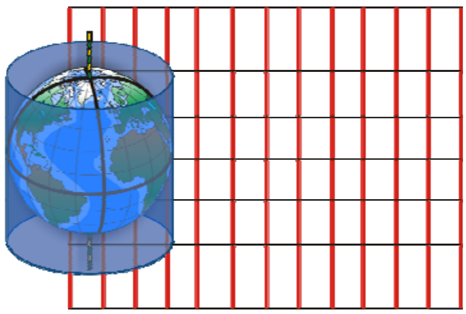
CHART CONVERGENCY

All meridians are straight lines on normal projections and convergency over the map will thus be constant, known as Chart Convergency.

Chart Convergency = d long x Sin PoO (Parallel of Origin)
= d long x CCF (Chart Convergency Factor)
= d long x n (Constant of the Cone)







PROPERTIES OF AN IDEAL MAP

Ideally, we would like a flat map to exhibit the following properties:

1. Scale constant and correct in every direction.

2. Shapes on the Earth accurately portrayed on the map.

3. The bearing measured between any two points on the map to reflect accurately the bearing between the same points on the Earth.

4. Equal areas on the Earth depicted as equal areas on the map.

5. The straight line on the map to have navigational utility.

6. Adjacent sheets of a series of maps to fit perfectly N-S and E-W.

We now examine these desirable properties in more detail.

SCALE

At the point or line of tangency between the developable surface and the sphere, the image of objects transferred from the reduced Earth to the paper will obviously retain correct, i.e. reduced Earth scale. But everywhere else, the distortion inherent in the transfer will cause the map scale to deviate from reduced Earth scale. Thus, in general, it is impossible to produce a flat map on which scale is everywhere constant and correct.

SCALE FACTOR

To describe this divergence from correct (reduced Earth) scale, we define the term 'Scale Factor':

$$\text{Scale Factor} = \frac{\text{Chart Scale at a point}}{\text{RE Scale at same point}}$$

or more usefully,

$$\text{Scale Factor} = \frac{\text{Chart Length}}{\text{Equivalent RE length}}$$

At any particular point, scale factor is a number like 1.12 but, more generally, it is defined mathematically, e.g:

'Scale factor = secant of the latitude'.

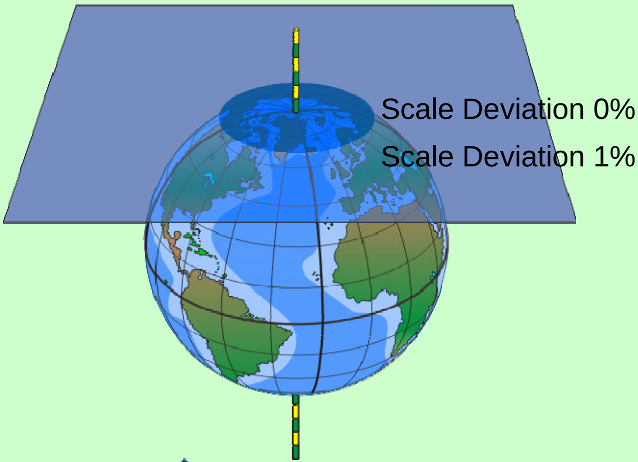
SCALE DEVIATION

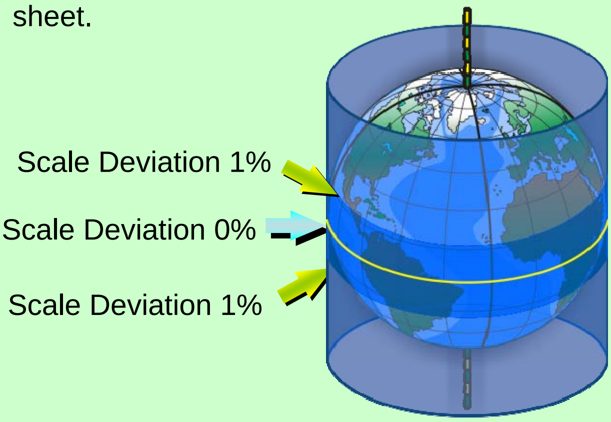
When chart scale equals RE scale, scale factor is unity. The amount by which it differs from unity in other areas is given by the expression 'Scale Deviation' (SD) where:

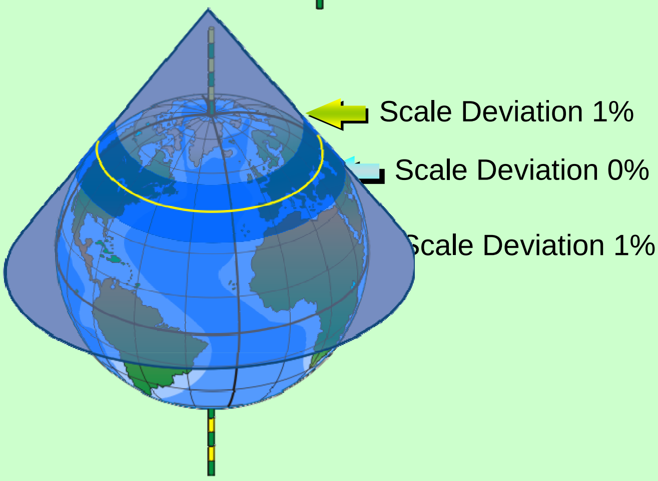
$$SD = (\text{Scale factor} - 1) \times 100\%$$

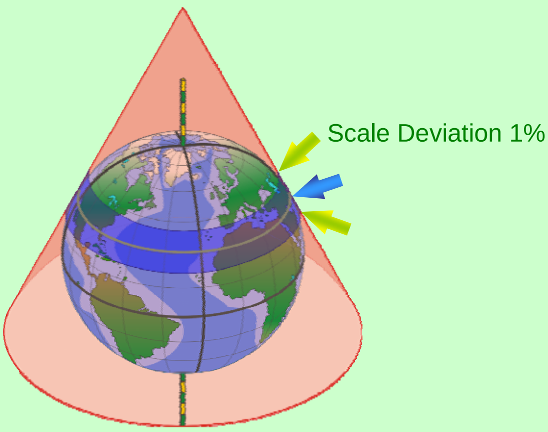
Note that SD can be either positive or negative.

All deviations from constant and correct scale cause difficulties in the accurate measurement of distance. However, for practical purposes it is generally accepted that such difficulties are negligible if the scale deviation is held between ±1%, i.e. scale factor within the range 0.99 - 1.01. On some projections it is possible to achieve this ideal over significant areas, and maps exhibiting this property are regarded as 'constant scale'. An example is the WAC, within the (relatively limited) coverage of each sheet.







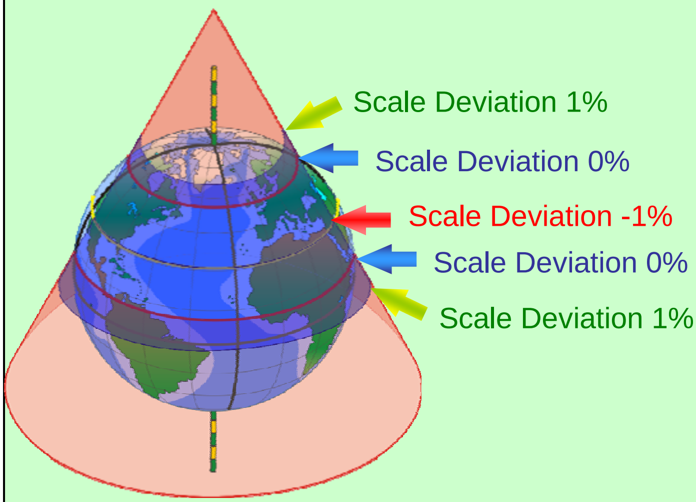


Coverage with only positive SD

When constant scale cannot be achieved - the usual situation with some projections, and with all projections if large areas of coverage are required - we should expect that the inevitable scale variations occur in a manner that is readily understood and easily managed.

The area of 'scale constancy' on a chart is sometimes extended, or moved to a more convenient position, by the application of a 'Scale Reduction Factor' (SRF). On charts for which scale deviation is always positive, i.e.

scale factor is always ≥1.00, we use only half the possible deviation of ±1%.



Coverage using both positive and negative SD

To utilize the -1%, we could multiply scale factors over the whole chart by a constant value of 0.99. This would introduce an area of scale contraction, in addition to the existing areas of scale expansion, thereby increasing the total area of coverage within which scale deviation is within the range ±1%. As we shall see, a scale reduction factor is applied to the simple conic in order to produce the important 'Lambert's conformal' projection. At this stage we need only note that SRF is a constant number, less than unity such that, over the whole chart:

Scale factor = Original scale factor x SRF

SHAPES

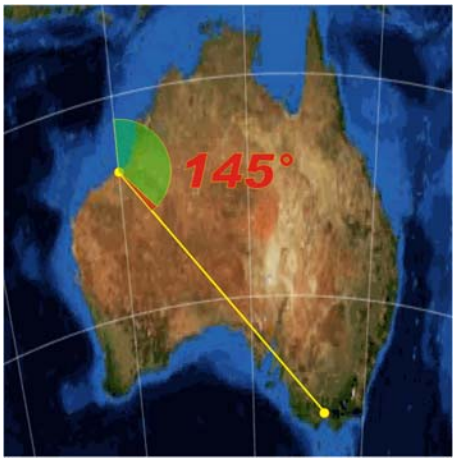
Shapes on the reduced Earth cannot, in general, be reproduced faithfully on the map, because the distortions that lead to scale variation will also distort shapes. We should, however, expect that the shapes of small features remain correct, at least within the area of coverage of greatest interest to the user.

This example shows a map that is useless in navigation but very suitable for depicting national population figures.

CORRECT ANGLES AND BEARINGS

Bearings measured between two points anywhere within the coverage of a chart should accurately reflect the bearings between the two points measured on the reduced Earth. Maps that possess this property are said to be 'Orthomorphic' or 'Conformal'. The two words are synonymous, meaning literally, 'correct form', but more accurately, 'showing angles correctly'. Orthomorphism is the most important single property of a navigational chart because, without it, it is not possible to measure tracks or bearings, and hence, not possible to navigate in any ordered fashion.

Orthomorphic Map showing correct track angle



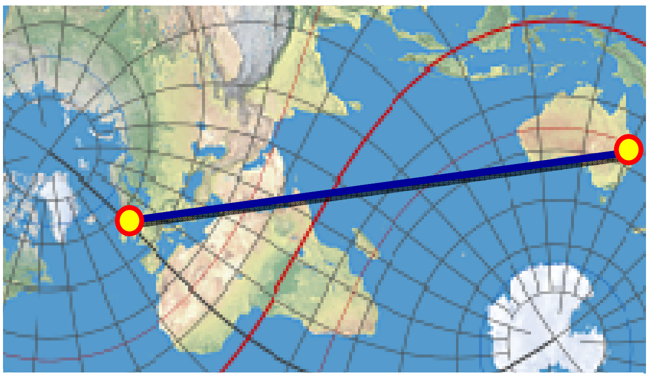
Non-orthomorphic Map showing incorrect track angle

The reduced Earth is, of course, orthomorphic, but retention of this vital navigational property is by no means assured in the transformation to a flat map. Hence navigational charts are restricted to the relatively small group of projections that are themselves orthomorphic, or can be made so by mathematical manipulation.

CORRECT AREAS

Equal areas on the reduced Earth should be reproduced as equal areas on the map. This can be achieved but, as we shall see, the property of equal area and the property of orthomorphism are mutually exclusive. Since orthomorphism is paramount, 'correct area' is a property that is never achieved on a navigational chart.

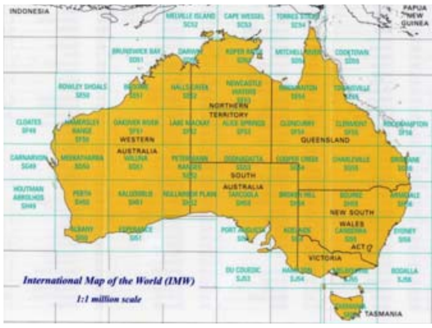
THE STRAIGHT LINE



Straight line track from London to Melbourne that approximates the great circle

It is usually not feasible to plot complex curves with precision on a map so aviators invariably plot tracks, position lines, etc, as straight lines, and navigate their aircraft along straight line tracks. Hence, it is important that the straight line between two points on the map represents a line on the Earth that has navigational significance. In particular, the straight line on the map should depict either the rhumb line or the great circle between the two points or, at least, provide a close approximation thereto.

FIT



Pilots of long range aircraft often fly distances that span the coverage of several maps during one flight. In addition, they frequently find that destinations or other areas of interest lie on the intersection of two or more map sheets. Hence, it is advantageous if adjoining sheets in a series of maps fit together in both the N-S and E-W directions, without gaps or distortion. Some map projections achieve this property either fully or partially, but such achievement is not universal.

SELECTION OF PROPERTIES

It is evident from the foregoing that it is not possible to devise a flat map on which all the ideal properties are represented. Indeed, some highly desirable properties, eg. constant and correct scale, can never be achieved. Others are attainable with varying degrees of difficulty, but normally not simultaneously. Hence, the production of a map to meet particular requirements must always involve a compromise in which desirable properties will probably have to be sacrificed to ensure that essential properties are retained. Some maps are more suitable than others for particular purposes, but there is no such thing as an 'ideal map' to cover all applications.

For navigational purposes, the overriding requirement is for orthomorphism. Such is the importance of this property that the next section is devoted solely to an analysis of its characteristics.

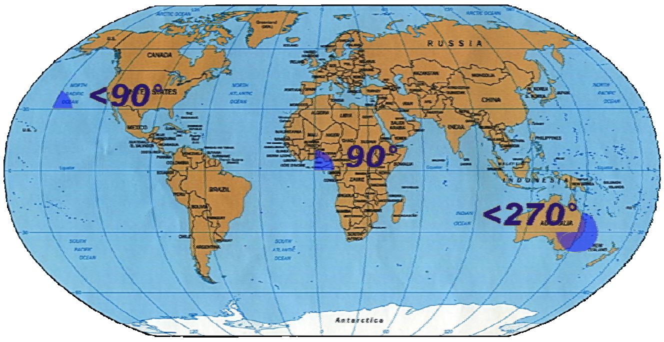
ORTHOMORPHISM

An orthomorphic or conformal map is defined as one on which the bearing between any two points accurately reflects the bearing between the same two points on the Earth.

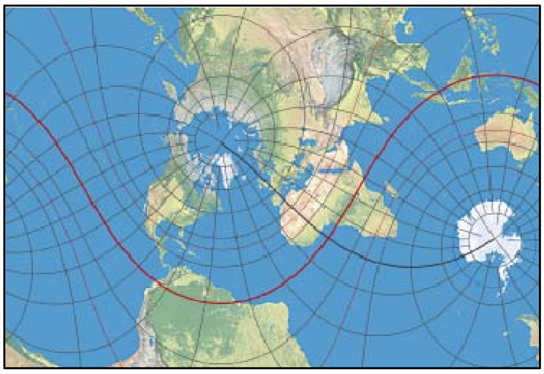
Orthomorphism is the property that enables us to plot and accurately measure tracks, bearings, wind vectors, position lines, etc on the map; in other words the property that enables an ordered system of navigation to exist.

The necessary and sufficient conditions for orthomorphism are twofold:

Firstly, the parallels and meridians depicted on the map must everywhere intersect at right angles, as they do on the Earth. The meridians define the directions 000-180, and the parallels define the 090-270 directions, so if the map is to have any utility for the measurement of bearings it is clear that these lines must be orthogonal. The need for this property is self evident, but a glance at any school atlas will reveal that many otherwise useful projections fail to achieve it.



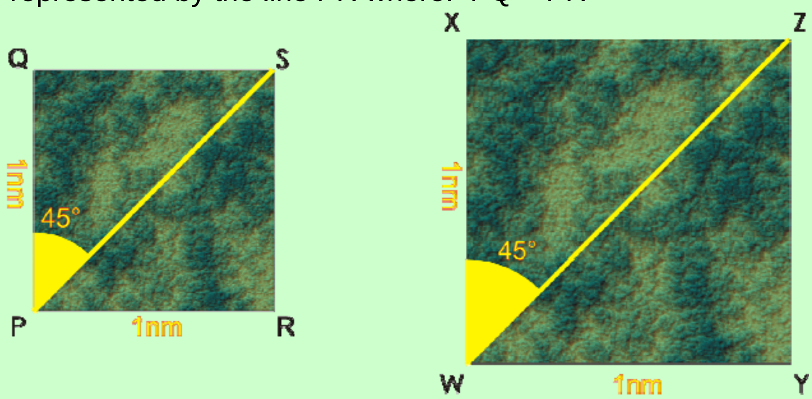
Orthogonal. In mathematics, two lines or curves are orthogonal if they are perpendicular at their point of intersection.



Incidentally, the requirement that parallels and meridians be orthogonal does not mean that they must be straight lines (refer the map along side); it is sufficient that their tangents at the point of intersection be orthogonal. On the Earth itself the meridians and parallels are, of course, not straight.

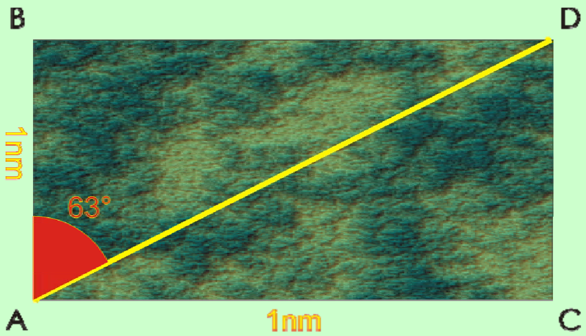
Secondly, about every point on the map, the scale for a short distance in every direction must be the same. This is not to suggest that the scale everywhere is equal; we have already seen that this is impossible to achieve on a flat map. Rather, we require the scale about a given point to be uniform in all directions, and if we move to a second point where there is a different scale, the new scale must also be uniform about the second point.

For instance, consider the point P on a map as shown in the figure below. If the scale is such that 1nm in the N-S direction is represented by the line PQ, then 1NM in the E-W direction should be represented by the line PR where: PQ = PR



The point S is then correctly placed along a true bearing of 045° from P, as it would appear on the Earth. Now consider another point W on the same map. At W the scale has increased, but the scales are still such that: WX = WY, and the point Z is correctly portrayed bearing 045° from W.

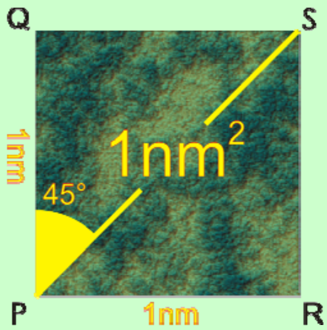
The scale variation described meets the requirement for orthomorphism. Now consider another map on which the point A is plotted.



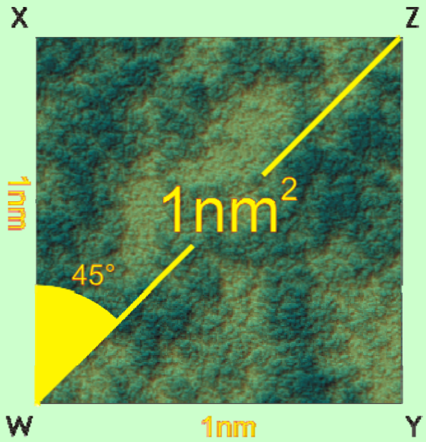
At point A the E-W scale is twice the N-S scale, so that: AC = 2AB

Since AB and AC both represent 1nm on the Earth, the bearing from A to D should be 045°, as in the previous example. However, measured on this map, the bearing would be about 063°. Such a map is not orthomorphic and, clearly, could not be used for navigation.

A similar analysis at the figures below demonstrates the incompatibility between the properties of equal area and orthomorphism. Consider an area of 1 square nm on the Earth at point P. The E-W scale equals the N-S scale at point P, and the bearing of S from P is correctly shown as 045 degrees.



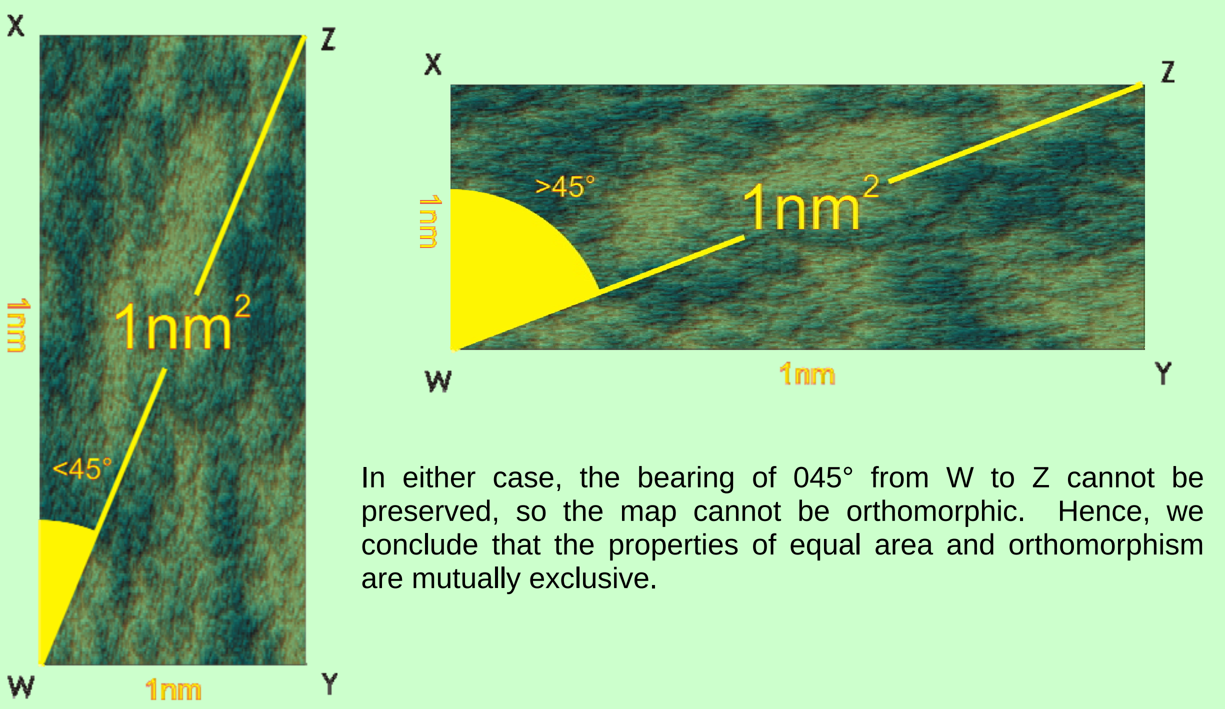
Area 'A' representing
1 nm^2



Area 'B' representing
1 nm^2

Elsewhere on the map at point W the scale increases so, to preserve orthomorphism, the E-W and N-S scales should expand equally as shown.

However, if the map is an equal area projection, the area of $1nm^2$ at point P must equal the area of $1nm^2$ at point W, a condition that clearly is not met on the map above. Given the unavoidable scale change, the equal area requirement could be achieved only if scale variation in the N-S and E-W directions were unequal, e.g. as in the figures below.



In either case, the bearing of 045° from W to Z cannot be preserved, so the map cannot be orthomorphic. Hence, we conclude that the properties of equal area and orthomorphism are mutually exclusive.

THE GREAT CIRCLE

We now present a general rule of the utmost importance to define the relationship between the great circle on the Earth and the straight line on an orthomorphic chart. Recall that the great circle lies in a flat plane that passes through the centre of the Earth, and that this plane cuts successive meridians - themselves great circles - at different angles. The difference at latitude λ is equal to convergency, and is given by:

Convergency = d long *Sin Latitude*,

or, if the successive intersections occur at different latitudes, by:

Convergency $\approx$ d long *Sin* (mean latitude)

Thus we see that Earth convergency changes as latitude changes, increasing from zero at the equator to d long at the poles.

On most of the commonly used navigational charts, the meridians are depicted as straight lines, so chart convergency, ie. the angle between successive meridians on the chart, is constant throughout the entire latitudinal coverage. In these circumstances it follows that chart convergency on a particular chart can equal Earth convergency at only one latitude, and that this is the latitude at which the developable surface was tangential to the reduced Earth. The parallel of latitude at which this occurs is called the ‘Parallel of Origin’ and can take any value between zero and 90 degrees north or south depending on the particular chart under consideration. It is normally represented by the symbol λ_0.

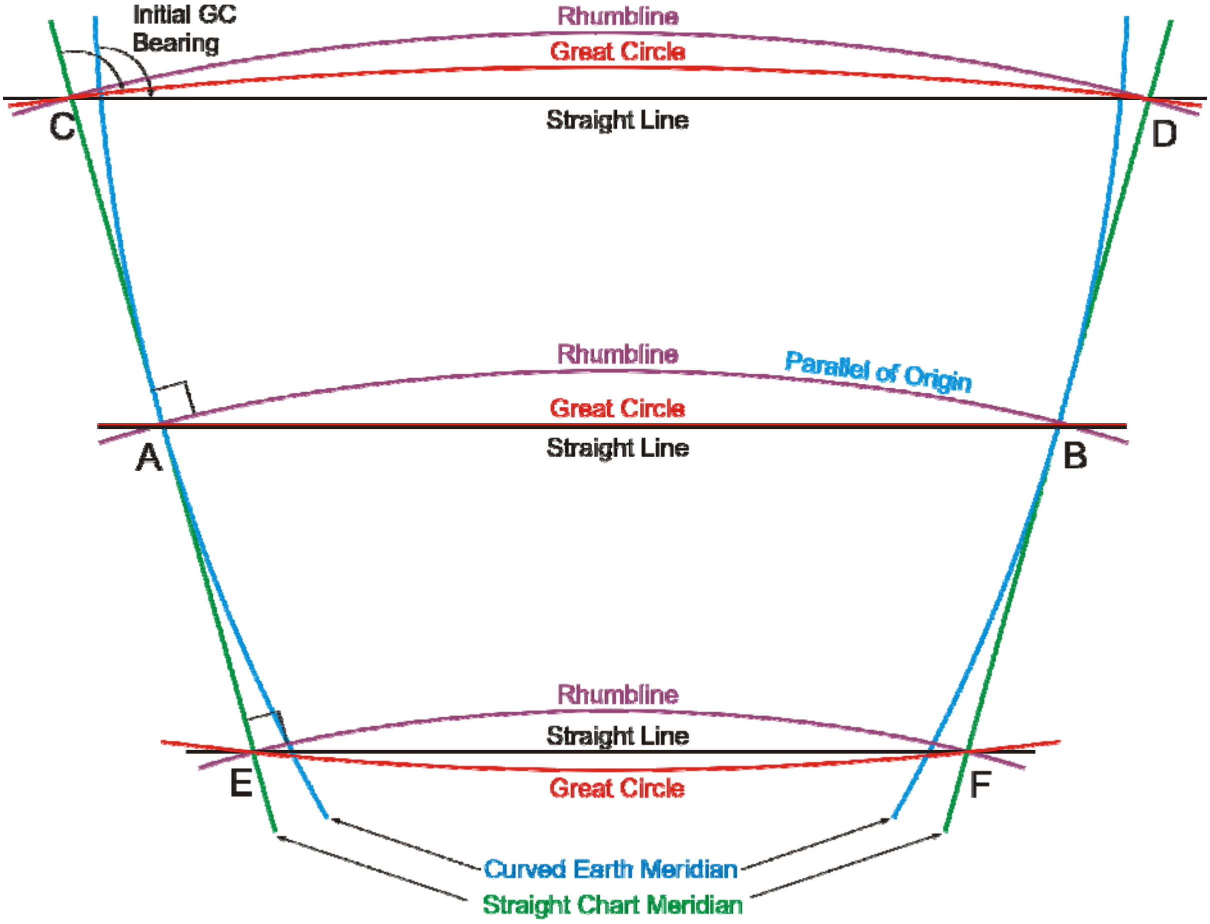
A straight line drawn on an orthomorphic chart between points on the parallel of origin will cut the chart meridians at the same angles at which the great circle intersects the meridians on the Earth. Hence, on an orthomorphic chart, the great circle is depicted as a perfectly straight line only where;

Chart convergency = Earth convergency

At all other latitudes, the great circle will be a curve concave to the parallel of origin, with the curvature increasing as the latitudinal displacement from the parallel of origin increases. (Note that, if the great circle cuts the parallel of origin at right angles, it will be portrayed as a straight line. The concept of concavity has no meaning in these circumstances).

Consider, for instance, the simple orthomorphic conic in the southern hemisphere, shown in the figure below.

Chart convergency equals Earth convergency at the parallel of origin. Towards the equator, Earth convergency decreases, so chart convergency is too large, and, towards the pole, Earth convergency increases so chart convergency is too small.



The straight line AB on the chart represents perfectly the great circle between A and B on the Earth. North of the parallel of origin at point C, the great circle cuts the Earth meridian at the angle shown, and the corresponding line on the chart must be slightly concave to the Parallel of Origin if it is to cut the chart meridians at the same angle. South of the parallel of origin at point E the same effect occurs, with the direction of curvature reversed to reflect the fact that the chart convergency is now in error in the opposite sense. The meridians, which are great circles cutting the parallel of origin at right angles, are straight lines.

On charts with small latitudinal coverage, the curvature of the great circle is insignificant. On such charts, it is reasonable to assume that the great circle is (approximately) a straight line throughout. But we must not forget that the straight line on the chart is not, in general, the same as the great circle on the Earth because:

Chart convergency = d long *Sin PoO*

is not, in general, the same as: Earth convergency = d long *Sin latitude*.

Therefore, the difference between the initial straight line bearing between two points shown on the chart, and the initial GC bearing between the same two points on the Earth will be different, dependent on how close the position is to the Parallel of Origin. Close to the Parallel of Origin the GC and SL are almost coincident, but the difference increases as the latitude moves further from the Parallel of Origin. **This will result in the Great Circle being concave to the Parallel of Origin.**

The most important point to retain from this discussion is that the relationship between the great circle and the straight line is the same on all orthomorphic charts. This, taken in conjunction with the fact that the rhumb line always lies closer to the equator than does the great circle, should simplify the problem of recalling the respective shapes of rhumb lines and great circles on the var