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CHAPTER 4 – NORMAL CONIC PROJECTIONS

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CHAPTER 4 – NORMAL CONIC PROJECTIONS

We saw previously that constant and correct scale would be a desirable property on any navigational chart but, unfortunately, is generally unattainable in the transfer from a spherical to a flat surface. However, some charts exhibit scale that is 'nearly' constant and correct over significant areas. For instance, if we are prepared to accept scale deviation of up to 1%, the normal Mercator provides 'constant and correct scale' within about 8° of the equator, while the polar stereographic does the same from about latitude 78° to the pole. The transverse and oblique Mercator satisfy other specific requirements, but a need remains for a wide area chart to provide near constant scale over the remaining 70° of latitude between the equatorial and polar regions.

Johannes Lambert developed the chart that bears his name in the early 1770's, i.e. about 200 years after Mercator. His aim was to overcome the Mercator's excessive scale expansion at high latitudes, while retaining its orthomorphic properties. The Lambert's Conformal Conic Projection is now generally accepted as a standard wide area chart and is steadily replacing the normal Mercator for general navigation in low and mid-latitudes.

Figure 1: Johannes Lambert

Lambert's chart is based upon the simple geometric conic, subsequently modified as the orthomorphic conic with one standard parallel and, finally, as the orthomorphic conic with two standard parallels. We will trace the evolution of this important family of charts and show, in passing, that the orthomorphic conic with one standard parallel is really the general form of all the navigational charts in use today.

THE SIMPLE CONIC

A cone is a developable surface which, when cut along one side from base to apex and opened out, becomes a flat surface as shown in Figure 2.

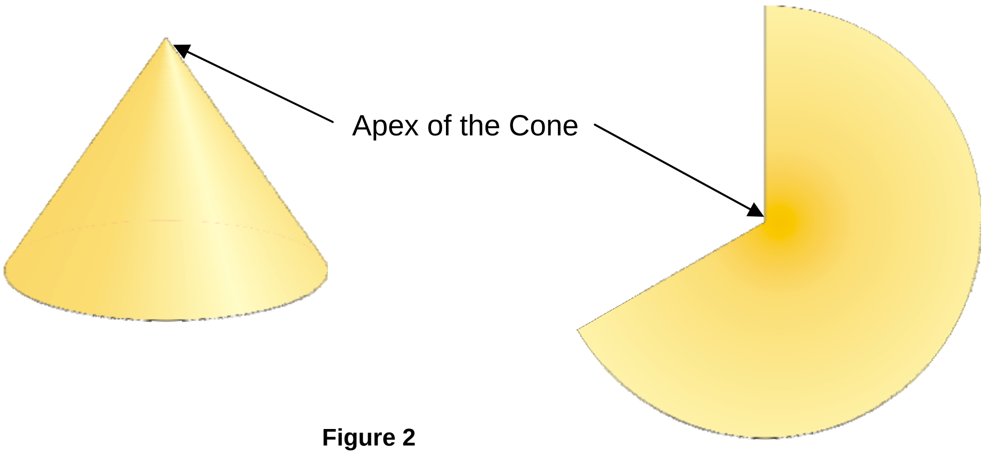


Figure 2

If the cone is placed over the reduced Earth as shown in Figure 3 so that the axes of both are coincident, we have the basis of the normal conic projection. (We will not consider non-normal conics in these notes).

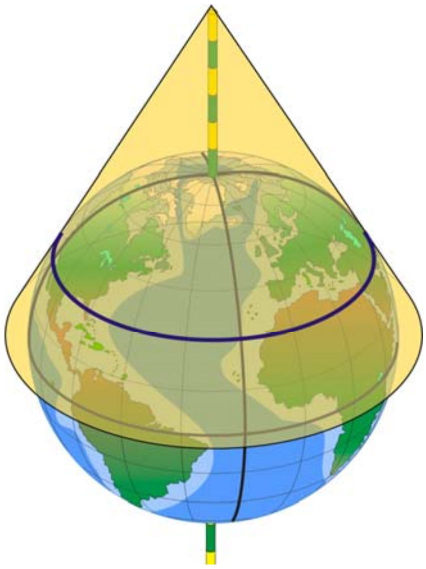
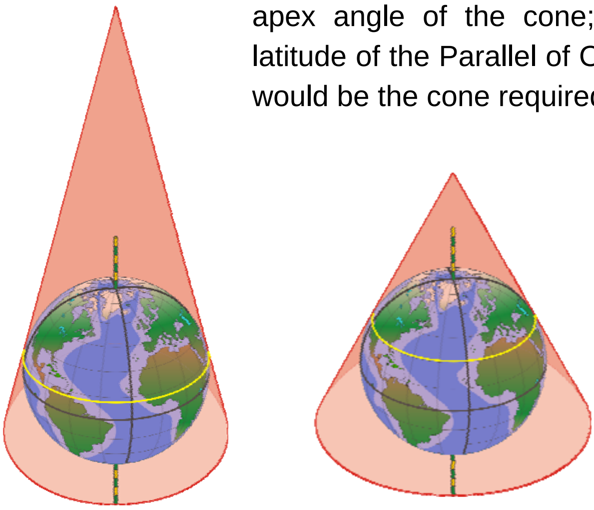


Figure 3

The cone is tangential to the reduced Earth around one parallel at latitude (λ_0), called the Parallel of Origin, and it is evident that we can change this parallel by changing the apex angle of the cone; the higher the latitude of the Parallel of Origin, the 'flatter' would be the cone required.



At the Parallel of Origin the cone and reduced Earth are in contact, so scale is constant and correct and chart convergency equals Earth convergency. The point of projection is the centre of the reduced Earth and the projection is normal, so the meridians appear as straight lines radiating from the apex of the cone (the pole). Hence, chart convergency is constant over the entire chart; it will exceed Earth convergency at latitudes lower than that of the Parallel of Origin, and be less than Earth convergency at higher latitudes.

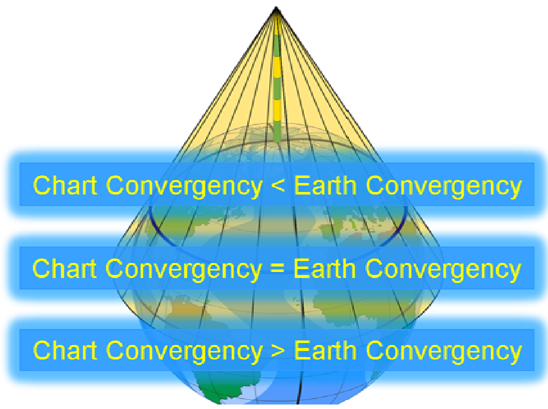


Figure 4

To change the shape of the cone and, hence, the parallel of origin, we vary the size of the segment that we remove from the original flat disc of paper. In Figure 5 we see the effect on the apex angle of the cones resulting from the removal of ¼, ½ and ¾ of the original disc.

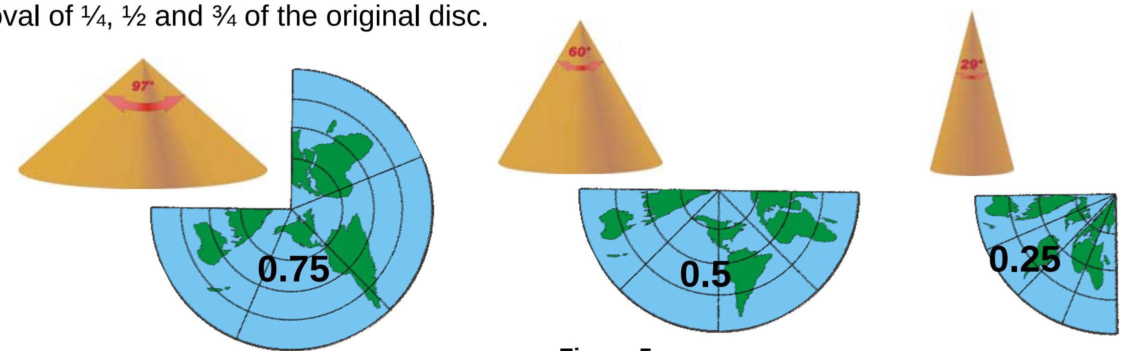


Figure 5

The percentage of the original disc that remains is called the 'Constant of the Cone' or 'Convergence Factor', and is usually denoted by the symbol 'η'. In Figure 5 we see cones with values of η of 0.75; 0.50; and 0.25 respectively. It is clear that the larger the value of η the flatter will be the cone, and the higher will be the latitude of the Parallel of Origin (PoO). The value of η is easily calculated by calculating sin PoO.

THE ORTHOMORPHIC CONIC WITH ONE STANDARD PARALLEL

A study of the Simple Conic Projection will find that within 10° of the parallel of origin the chart is "nearly orthomorphic" and through some minor mathematical adjustments a orthomorphic conic projection with one standard parallel is possible. This is however beyond the scope of this course.

A further mathematic examination of the scale deviation on a orthomorphic conic projection with one standard parallel will find that the within 8° of the Parallel of Origin the scale deviation is within 1%. This fact is not sensitive to the value of the Parallel of Origin. Its region of 'scale constancy' is similar to that of the normal Mercator, but it has the advantage that the region can be moved to cover any desired band of latitude simply by changing the Parallel of Origin as a result of changing the apex angle of the cone. However, the performance of the chart is improved further by making relatively minor modifications to produce the orthomorphic conic with two standard parallels, usually known as the Lambert's Conformal Conic Projection.

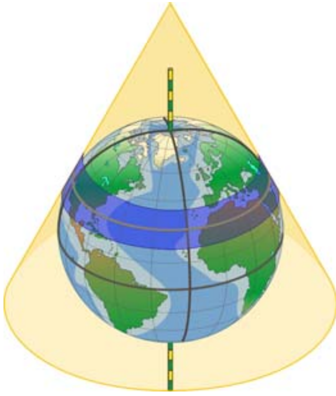


Figure 6

THE LAMBERT'S CONFORMAL PROJECTION

While the constant scale region of the orthomorphic conic can be moved at will it is always limited to about 16° of latitude. Lambert's major aim was to produce a near constant scale chart covering a large area, and the orthomorphic conic meets this requirement only marginally.

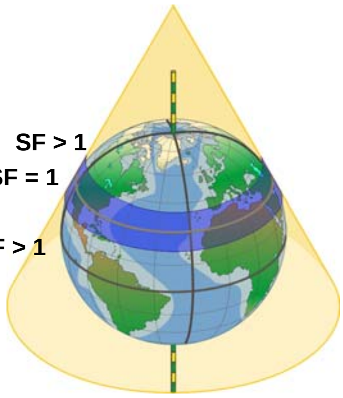


Figure 7

The relatively limited latitudinal coverage of the orthomorphic conic is due largely to its lack of negative scale deviation; the scale factor is 1.00 along the Parallel of Origin and increases to higher values at every other point on the chart.

If scale factors in the range 0.99 to 1.01 could be utilized, the region of 'scale constancy' could be increased. It is clear from purely geometric considerations that scale factors of less than 1.00 are not attainable while the cone contacts the reduced Earth only at the Parallel of Origin.

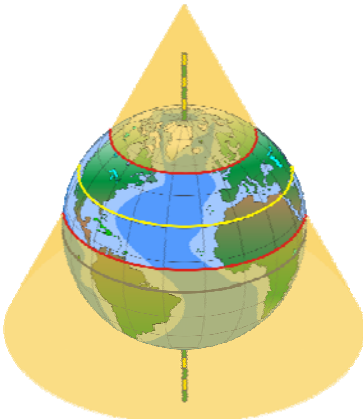


Figure 8

Instead, we must allow the cone to cut the surface of the reduced Earth, so that the area between the lines of contact is projected at reduced scale. Such a cone is called a 'secant cone', illustrated in Figure 8 and 9.

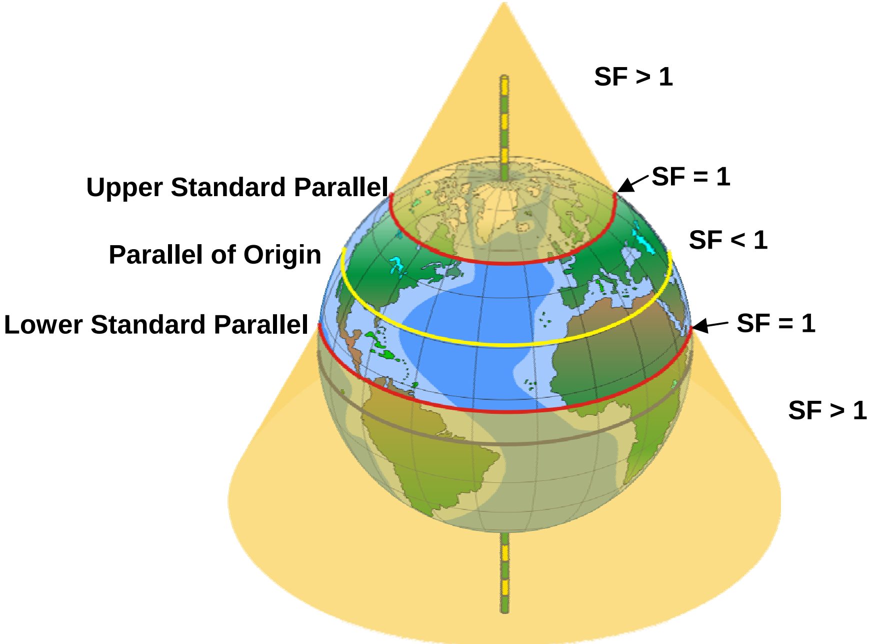


Figure 9

The secant cone cuts the surface of the reduced Earth at two Standard Parallels (SP), at both of which scale factor is 1. The Upper Standard Parallel is closer to the nearest Pole, and the Lower Standard Parallel is closer to the Equator. Between the standard parallels SF < 1, and outside the standard parallels SF > 1, as is the case with the normal conic. If the scale factor is restricted to values between 0.99 and 1.01, the chart will be considered a constant scale chart.

SCALE CHANGES AND $^{1}/_{6}$TH RULE

The rate at which scale changes between the SP's compared to the rate outside of the SP's is also of interest. In figure 10 it is clear that area A gets compressed onto the projection surface whereas area B is expanded. The amount at which area A is compressed is also much less than the amount of expansion suffered by area B even though both areas are the same distance from the SP.

The rate at which scale changes outside of the SP's are quicker than the rate of scale change between the SP's.

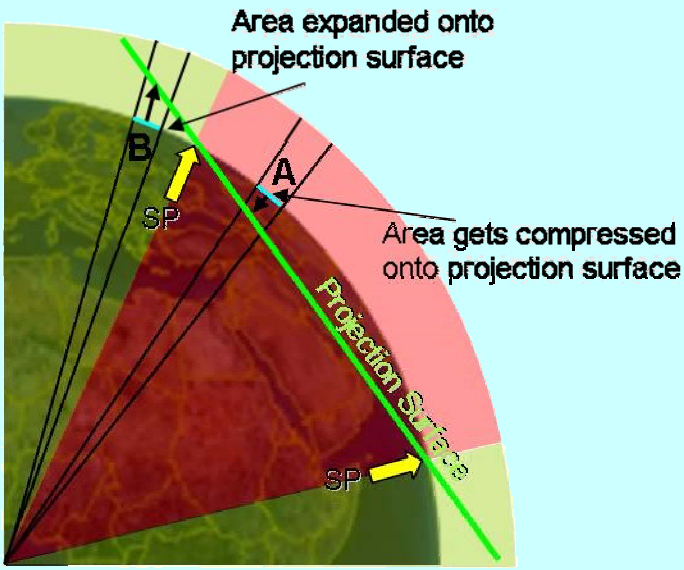


Figure 10

If there is a requirement to balance the scale expansion with the scale contraction the $^{1}/_{6}{}^{th}$ Rule is used.

The total coverage of the map north to south is 1½ times the standard parallel spacing where:

- $^{1}/_{6}{}^{th}$ of the coverage is to the Polar side of the Upper SP.
- $^{1}/_{6}{}^{th}$ of the coverage is to the Equatorial side of Lower SP.
- $^{4}/_{6}{}^{ths}$ are between the two SP's.

Example: If the SP's are 12° apart, the chart will extend 18° from North to South.

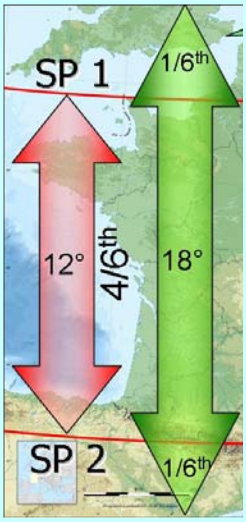


Figure 11

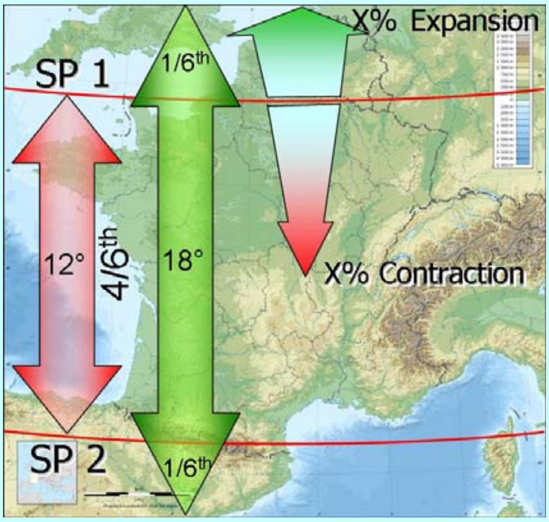


Figure 12

In this example the scale will contract by x% towards the PoO and it will expand by the same amount (x%) in approximately half the distance on the outside of the two SP's

Applying the $^{1}/_{6}{}^{th}$ Rule to a Lambert's chart only implies that the scale expansion at the very top and bottom of the chart is equal to the scale contraction at the approximate mid point between the SPs.

Applying the $^{1}/_{6}{}^{th}$ Rule only does not make a chart a constant scale chart.

To have a constant scale chart the $^{1}/_{6}{}^{th}$ Rule must be applied to a chart where the SPs are a maximum of 16° apart.

PROPERTIES OF THE LAMBERT'S CONFORMAL CONIC PROJECTION

APPEARANCE OF THE GRATICULE

The meridians are straight lines converging to the nearer pole with,

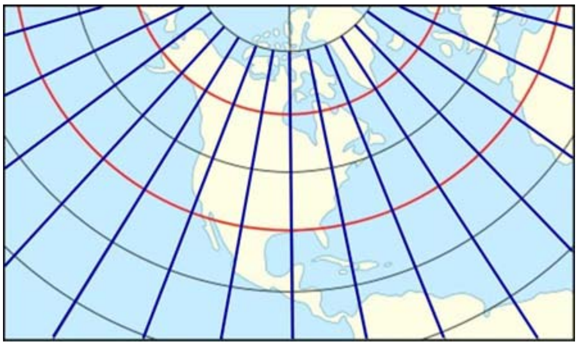
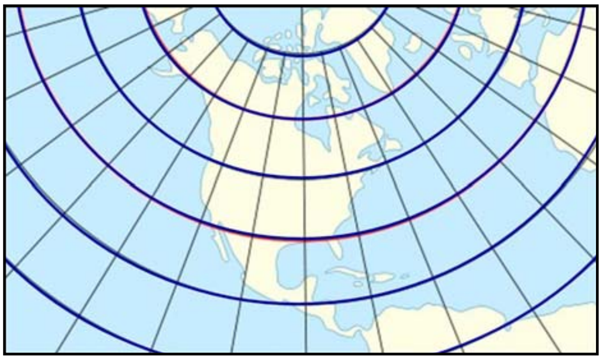


chart convergency = d long x η
= d long x sin P.o.O

η is the constant of the cone



The parallels are arcs of circles, nearly equally spaced, and all centred on the nearer pole. As the scale contracts between SP's and expands outside SP's, the parallels cannot be equally spaced.

In all cases, meridians and parallels intersect at right angles.

SCALE

Scale is correct at the two standard parallels. Scale contracts between the Standard Parallels reaching a minimum value at the Parallel of Origin, and expands continuously outside the Standard Parallels.

The total scale deviation, reflected in the value of scale factor at the Parallel of Origin, depends entirely upon the spacing of the standard parallels. If the spacing does not exceed about 16°, the scale deviation at the Parallel of Origin will be limited to about -1%, and the chart will exhibit 'constant scale' throughout. Lesser spacing results in smaller scale deviation and hence greater scale constancy.

To achieve the **best balance** of scale deviation over the chart 'the one-sixth rule' is frequently applied (as discussed above). If the standard parallels are 16 degrees apart, scale factors lie within the range 0.99 to 1.01, over a total coverage of 24 degrees of latitude.

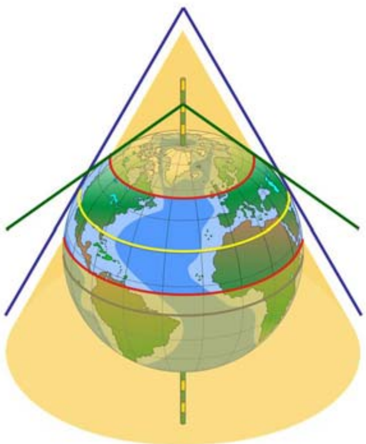
The WAC series of charts only covers 4° of latitude, so the scale deviation will be significantly less than 1%, and as such we utilise the charts as constant scale charts.

Irrespective of the area covered or the range of scale deviations, the QUOTED SCALE on all Lambert's Conformal Charts is the scale at the STANDARD PARALLELS.

ANGLES

The projection is orthomorphic, so angles and bearings accurately reflect the corresponding angles and bearings on the Earth.

CHART CONVERGENCY



The Parallel of Origin retains this property in the change from the tangential to the secant cone, because the parallel of origin remains the only line along which the direction of a meridian on the chart parallels the plane of the meridian passing through the centre of the reduced Earth.

Also all meridians are straight lines, therefore chart convergency between any two meridians is a constant over the entire latitude coverage of the chart.

Chart Convergency = d long x η = d long x sin PoO

Since Earth convergency = d long x sin latitude (λ),

Earth convergency = chart convergency

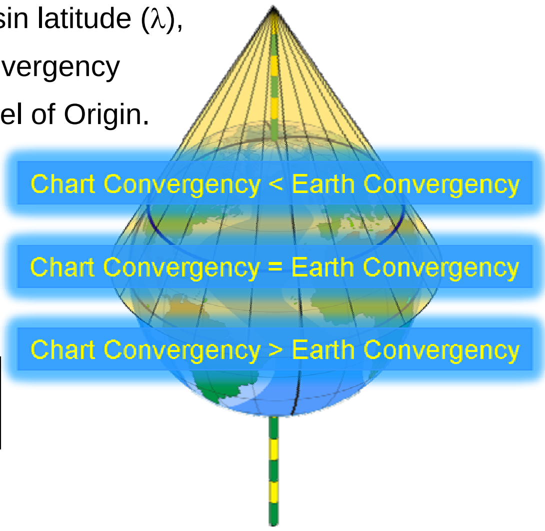
only when $\lambda = \lambda_0$, i.e. at the Parallel of Origin.

On the polar side of the Parallel of Origin,

Chart Convergency < Earth Convergency, and,

On the equatorial side,

Chart Convergency > Earth Convergency.

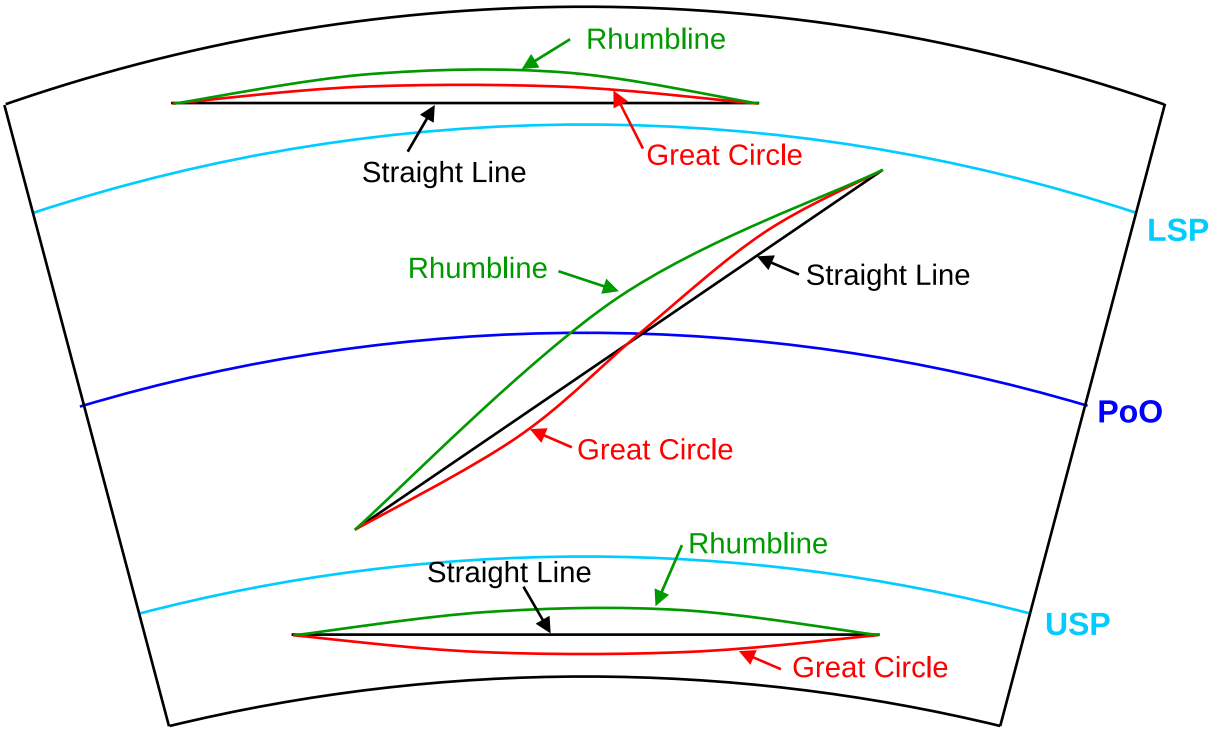


For this reason, chart convergency and Earth convergency are never equal at the standard parallels.

THE GREAT CIRCLE

The great circle drawn between points on the Parallel of Origin is a perfectly straight line. Great circles that cross the Parallel of Origin at right angles, i.e. meridians, are also straight lines. Other great circles appear as curves, always CONCAVE to the Parallel of Origin. Within the area of 'constant scale', i.e. about 24° of latitude coverage, provided the one-sixth rule is applied, the curvature of the great circle is relatively small, and a straight line approximation is used without significant error.

On most Lambert's charts, the great circle is assumed to be a straight line, but we should bear in mind that the actual great circle will be concave to the Parallel of Origin.



RHUMB LINES

Meridians are straight lines and parallels are arcs of circles centred on the pole, but all other rhumb lines are complex curves concave to the nearer pole. There is no simple means of plotting a rhumb line on a Lambert's chart. However, the straight line is approximately parallel to the rhumb line at the mid point; therefore, a close approximation to the rhumb line track is obtained by measuring the straight line track at the mid-meridian.

SHAPES AND AREAS

Although the Lambert's Conformal cannot be an equal area projection, the distortion of shapes and areas is small when compared with the Mercator. If the latitudinal coverage is limited to about 24°, distortion is minimized due to the near constant scale then achieved.

CONSTRUCTION

The normal conic projection provides an approximation to the orthomorphic conic with one standard parallel. The approximation is close near the parallel of origin but fails as displacement from the parallel of origin increases. The Lambert's Conformal Conic projection is no more than the orthomorphic conic with a scale reduction factor applied. The simple geometric projection is helpful in visualizing the basic characteristics of the charts, but the Lambert's Conformal is a mathematical projection that cannot be reproduced geometrically.

FIT

Charts of the same scale will fit in an E-W direction, but only if they have the same standard parallels. If the standard parallels differ, the scale deviations along the meridians will also differ, so the charts will not fit.

Lambert's charts of the same scale will not fit in the N-S direction. Because the parallels of origin and, hence, chart convergency then differ, the radii of the parallels at the N-S junction will be different.

ADVANTAGES AND DISADVANTAGES

ADVANTAGES

- Great circle is a straight line for practical purposes.
- VOR bearings easily plotted.
- Graduated ruler can be used to measure distance on constant scale maps where scale error is small.

DISADVANTAGES

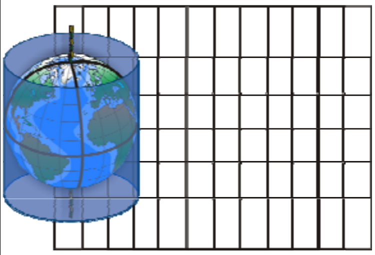
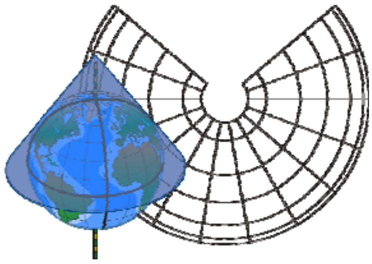
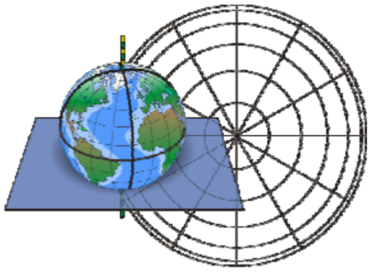
- If rhumbline tracks are required the straight line track must be measured at the mid-meridian.
- Graticule is not rectangular, plotting positions not as simple as on Mercator.
- Convergency has to be applied when plotting NDB position lines (using meridian transfer when plotting).

NAVIGATION USES

It is mainly used for navigational charts (also radio navigation charts) covering mid-latitudes between the area of constant scale of a Mercator and the Polar Stereographic. This is typically between about 8°N/S to about 75°N/S. It is ideally suited for mapping areas with a great east-west extent.

COMPARISON BETWEEN CHARTS

	MERCATOR	LAMBERTS	POLAR STEREOGRAPHIC
ORTHOMORPHIC	Yes	Yes	Yes
SCALE	Correct at Equator. Expands at Sec of Lat away from Equator contracts with Cos of Lat towards Equator. Scale Deviation within 1% up to 8° from Equator.	Correct and equal on the SPs. Contracted within the SPs (most contracted at PoO – halfway between SPs). Expanded outside SPs.	Correct at Pole (point of tangency). Expands away from pole. Scale Deviation within 1% from 78°N/S to the Pole. Scale can be calculated using ABBA (co-latitudes).
GRATICULE	Meridians are evenly spaced, straight parallel lines. Parallels are parallel straight lines with their spacing increasing with the secant of the latitude. Meridians and parallels cross each other at 90°.	Meridians are straight lines, radiating out from the nearer pole. Parallels are arcs of concentric circles, nearly equally spaced, centred at the pole. Meridians and parallels cross each other at 90°.	Meridians are straight lines radiating from the Pole. Parallels are unequally spaced concentric circles drawn from the Pole (distance from Pole determined by co-lat). Meridians and parallels cross each other at 90°.
SHAPES	Reasonably correct over small areas. Distortion over large areas, especially at high latitudes. Distortion increases away from Equator (circle of tangency – PoO).	Distortion is small when compared to Mercator. If latitudinal coverage limited to about 24°, distortion is minimised due to near constant scale achieved.	Distortion increases with increasing distance away from the Pole.
AREAS	Orthomorphic so cannot be equal area. Gross area distortion at higher latitudes.	Orthomorphic so cannot be equal area. Distortion is minimal (see shapes above).	Orthomorphic so cannot be equal area. Distortion increases away from Pole.
CHART CONVERGENCY	Constant across the chart. Correct at the Equator (zero as with the Earth). Chart convergency elsewhere is **<** Earth Convergency.	Constant across the chart. Correct at PoO. <u>Chart Convergence:</u> CC = d long x sin PoO = d long x n = d long x CCF (Chart Convergence Factor)	Constant across the chart. Correct at the Pole, elsewhere is **>** Earth Convergency Convergency = d long (n = 1).
RHUMBLINES	Straight lines everywhere on the chart. Only orthomorphic chart with this property.	Meridians are straight lines. All other rhumblines (i.e. parallels of latitude) are curves concave to the nearer Pole.	Curves concave to the nearer Pole (i.e. parallels of latitude).
GREAT CIRCLES	Equator and Meridians are straight lines (and are also rhumblines). All other great circles are curves concave to the Equator (PoO). Great Circles close to (8°N/S) and at the Equator are straight lines (chart convergency = Earth convergency).	Meridians are straight lines. At the PoO can be considered a straight line. At any other latitude, a curve concave to the PoO, still closer to Pole than rhumbline. In practice however the straight line is assumed to be the great circle.	Can be taken as straight lines at latitudes **>** 70°. At lower latitudes curves concave to the nearer Pole (PoO), but closer to Pole than rhumbline.

	MERCATOR	LAMBERTS	POLAR STEREOGRAPHIC
MAP APPEARANCE			
METHOD OF PRODUCTION	Mathematical.	Mathematical.	Geometric / Mathematical
POINT OF TANGENCY	Equator.	Two Standard Parallels.	Pole.
NAVIGATIONAL USES	Plotting charts up to about 75° Latitude. Topographical charts of equatorial areas (small scale distortion). Met synoptic charts (equatorial areas).	Navigational charts covering mid-latitudes between the area of constant scale of a Mercator and the Polar Stereographic (typically about 8°N/S to about 75°N/S). Radio Navigation Charts	Plotting and navigation in polar regions or high latitudes. Grid Navigation
ADVANTAGES	Rhumblines are straight lines. A/c compasses detect magnetic north, can easily steer rhumbline. Straight lines represent a constant true track. Graticule is rectangular so makes plotting easier.	Great circle is a straight line for practical purposes. VOR bearings easily plotted. Graduated ruler can be used to measure distance on constant scale maps where scale error is small.	Great circle is a straight line for practical purposes. Enables navigation in high latitudes.
DISADVANTAGE	Chart is not constant scale (distances must be measured against the lat scale, in small increments along the track). Not usable at latitudes >75° and poles cannot be projected. Radio waves follow great circles which cannot be directly plotted, Earth convergency must be applied.	If rhumbline tracks are required the straight line track must be measured at the mid-meridian. Graticule is not rectangular, plotting positions not as simple as on Mercator. Convergency has to be applied when plotting NDB position lines (using meridian transfer when plotting).	Added complication of grid navigation (all directions are South at North Pole and North at South Pole).