Designing a Kalman Filter for a Maxon EC 8 mm Brushless DC Motor with Hall Sensors

1. Introduction

This report describes a Kalman filter design for estimating and controlling the state of a Maxon EC (Electronically Commutated) brushless DC motor with an 8 mm diameter. This small-diameter motor is often equipped with integrated Hall effect sensors for commutation and basic speed measurement. The high-speed, compact nature of this motor makes reliable state estimation crucial, especially when sensor resolution is limited.

Key objectives include:

- Accurate real-time estimation of rotor position and speed.
- Compensation for noise in Hall sensor signals.
- Robust feedback for motor control loops.

2. Motor and Hall Sensor Characteristics

2.1 Electrical Model

A simplified voltage equation for each phase can be written as:

$$V(t) = R\,i(t) + L\,\frac{di(t)}{dt} + e_{\text{back}}(t),$$

where

- $V(t)$ is the phase voltage,
- $i(t)$ is the phase current,
- R and L are the phase resistance and inductance,
- $e_{\text{back}}(t)$ is the back-EMF voltage proportional to rotor speed.

Typical Values (Approximate):

- $R \approx 5.5\,\Omega$,
- $L \approx 0.2\,\text{mH}$,
- Back-EMF constant $K_e \approx 2.5\,\frac{\text{mV}}{\text{rpm}}$,
- Torque constant $K_t \approx 2.4\,\frac{\text{mN}\cdot\text{m}}{\text{A}}$,
- *(Exact values depend on the specific EC 8 mm variant; consult the Maxon datasheet.)*

2.2 Mechanical Model

The motor's rotor dynamics:

$$J\,\frac{d\omega}{dt} = K_t\,i(t) - B\,\omega(t) - \tau_{\text{load}}(t),$$

where

- J is the rotor inertia,
- $\omega(t)$ is the angular velocity,
- B is the viscous friction coefficient,
- $\tau_{\text{load}}(t)$ is the load torque.

Typical Values:

- $J \approx 3 \times 10^{-7}\,\text{kg} \cdot \text{m}^2$ (small rotor inertia for 8 mm motor),
- $B \approx 1 \times 10^{-5}\,\text{N} \cdot \text{m} \cdot \text{s/rad}$ (estimated),
- *(Exact values from the motor's official datasheet.)*

2.3 Hall Effect Sensors

- **Resolution:** Typically 1–3 Hall sensors yield 6 or more electrical transitions per electrical revolution (commutation signals).
- **Noise/Uncertainty:** Mechanical tolerances can cause jitter or phase error in Hall signals.
- **Measurement Rate:** Hall transitions may be measured at microcontroller interrupt level. The motor speed must be inferred from time between transitions.

3. State-Space Model and Kalman Filter Setup

3.1 State Variables

Let the state vector be

$$x(t) = \begin{bmatrix} \theta(t) \\ \omega(t) \\ i(t) \end{bmatrix},$$

where

- θ [rad] is the estimated rotor angle,
- ω [rad/s] is the angular speed,
- i [A] is the phase or transformed current (e.g., in a dq reference frame).

Input $u(t)$ can be the applied phase voltage (or dq-axis voltages).

3.2 Discrete-Time Approximation

For a sampling time Δt, the model converts to:

$$x_{k+1} = A\,x_k + B\,u_k + w_k, \quad z_k = H\,x_k + v_k,$$

with w_k and v_k representing process and measurement noise. Noise covariances Q and R define the filter's trust in the model vs. measurements.

3.3 Measurement Vector

Hall sensors can directly provide coarse rotor angle increments or speed estimates. One approach is to measure:

$$z_k = \begin{bmatrix} \theta_{\text{Hall},k} \\ i_{\text{meas},k} \end{bmatrix},$$

where $\theta_{\text{Hall},k}$ is extracted from Hall transitions, and $i_{\text{meas},k}$ is current sensor data. Then

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

4. Kalman Filter Design Steps

4.1 Predict

$$\hat{x}_{k|k-1} = A\,\hat{x}_{k-1|k-1} + B\,u_{k-1},$$

$$P_{k|k-1} = A\,P_{k-1|k-1}\,A^\top + Q.$$

4.2 Update

$$K_k = P_{k|k-1}\,H^\top \big(H\,P_{k|k-1}\,H^\top + R\big)^{-1},$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k\big(z_k - H\,\hat{x}_{k|k-1}\big),$$

$$P_{k|k} = (I - K_k\,H)\,P_{k|k-1}.$$

5. Filter Tuning and Implementation

5.1 Process Noise (Q)

Set Q to account for:

- Load torque variations on the motor shaft.
- Uncertainty in phase inductance/resistance at different temperatures.
- Rotor friction modeling errors.

5.2 Measurement Noise (R)

- **Hall Sensor Noise**: Incorporate the quantization and possible jitter in Hall transitions.
- **Current Sensor Noise**: Determine from specifications (e.g., $\pm 2\%$ measurement error).
- Adjust R values if the filter either over-trusts or under-trusts measurements.

5.3 Sampling Time

Because Hall sensors give updates at discrete transitions, the effective sampling rate for the filter can vary with motor speed. A separate high-frequency loop (e.g., $> 1\,\text{kHz}$) can integrate and estimate speed. The KF can be updated either every fixed Δt or upon Hall transitions using event-based processing.

6. Practical Considerations

6.1 Limited Angle Resolution

Hall sensors offer coarse electrical angle increments (e.g., 6 or 12 transitions per electrical revolution). High mechanical speed can result in minimal resolution for advanced, high-precision servo tasks. The Kalman filter helps interpolate or smooth these coarse measurements.

6.2 Nonlinearities

A plain linear KF may still suffice if the operating range is near linear regions. For wider speed/flux range, consider using Extended Kalman Filter (EKF) or Unscented Kalman Filter (UKF) to handle back-EMF nonlinearities.

6.3 Real-Time Constraints

- Embedded microcontrollers must compute the Kalman update in $\approx \mu$s–ms timescales.
- Evaluate memory usage for state covariance matrices and matrix operations.

7. Example Flowchart

1. **Initialization**: Set $\hat{x}_{0|0}$, $P_{0|0}$, guess Q, R.
2. **Motor Voltage Input**: Apply u_k from higher-level controller (e.g., PID or FOC).
3. **KF Predict Step**: Compute $\hat{x}_{k|k-1}$, $P_{k|k-1}$.
4. **Measurement Acquisition**: Read Hall sensor transitions and current sensor data (forming z_k).
5. **KF Update Step**: Calculate K_k, correct state estimate $\hat{x}_{k|k}$.
6. **Feedback to Controller**: Use $(\hat{\theta}, \hat{\omega}, \hat{\imath})^\top$ in real-time control loop.
7. **Repeat**: Continue predict–update cycles.

8. Conclusion

Designing a Kalman filter for a small-diameter Maxon EC BLDC motor with integrated Hall sensors involves:

- Formulating an appropriate state model (angle, speed, current).
- Accommodating coarse angle measurements from Hall signals.
- Tuning process and measurement noise covariances to balance model fidelity with sensor noise.
- Implementing the filter on an embedded platform with careful consideration of real-time constraints.

The resultant system provides more stable angle and speed estimates, better utilization of limited Hall sensor resolution, and robust performance under load variations. When properly integrated with a motor controller, the Kalman filter significantly enhances precision and responsiveness of the EC 8 mm motor drive.

References

- Maxon Motor AG. "EC 8 mm DC Motor Specifications," `https://www.maxongroup.com/`
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