OPTIMAL DESIGN OF DECENTRALIZED CONSTRUCTED WETLAND TREATMENT SYSTEM UNDER UNCERTAINTIES

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Summary

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1. Introduction

1.1. Globalization and rapid urbanization

Today, more than half of the world's population, 3.5 billion people, are living in urban areas. By 2050, it is predicted that the proportion of people living in urban areas will rise to 6.5 billion, or 60 percent of the population (United Nations, 2014). This rapid progress in urbanization has been made possible due to advancements in technology and communications worldwide. However, such progress has been achieved at the cost of environmental deterioration, various types of pollution, widening economic disparity, climate change and more (Dalal-Clayton and Bass, 2002). The uncertainty surrounding the climate of the future has led to calls for sustainable development. Sustainable development intends for nations to be able to have positive social and economic development while reducing the environmental impact of the actions taken for such progress. Through various conferences and processes such as those arising following the Brundtland Report (1987), nations have since come to an agreement that development should be sustainable (Dalal-Clayton and Bass, 2002).

As the world becomes more and more urbanized, we are looking at more cities evolving into megacities, which are cities which have populations of more than 10 million. As of 2014, there are 28 megacities across the globe, and by 2050, there is expected to be 41 megacities in the world (United Nations, 2014). Although the proportion of people living in megacities only amounts to 6.7% of the global population, they use 9.3% of the world's electricity and produce 12.6% of the world's waste (Kennedy et al., 2015). Taking this into consideration together with the uncertain future of our environment, there is a need to plan the growth and development of future megacities so that they are sustainable.

1.2. City infrastructure

Traditionally, cities have the bulk of its infrastructure centralized in convenient areas within the city. This trend emerged in the second half of the 19^{th} century amid rising public health concerns, political control interests and capital accumulation (Gandy, 2004; Dingle, 2008). For example, in Singapore, solid waste is collected across the country and transported to one of the four waste-to-energy plants for disposal via incineration (National Environment Agency, 2016). These plants are all located in the southwest of the country. In Trondheim, the third most populous city in Norway (Statistics Norway, 2013), has two large police stations serving the entire city (Politiet, 2016). Having centralized infrastructure provides benefits that often come with economies of scale. In the case of solid waste management in Singapore, the capital cost of building a waste-to-energy plant does not increase significantly with the increased capacity of the plant, and the capital cost to build a plant that serves a smaller capacity is already very large, hence building a larger plant allows each dollar spent to be stretched further.

1.3. Wastewater management

In this project, we will focus on the wastewater management infrastructure of a city. As with other infrastructures described earlier, the wastewater treatment network in most cities today are centralized. Such treatment systems collect wastewater from households and are designed to handle large amounts of wastewater at a central location of the area it serves. With the implementation of centralized treatment systems in many countries, water pollution in those locations have been successfully controlled (Li et al., 2014). In the past few decades, there has been a paradigm shift in the approach to water management. It is moving towards a stronger focus on how to best allocate water for human needs (Gleick, 2000).

Figure 1: A centralized wastewater treatment plant in Singapore. It has a wastewater treatment capacity of 800,000 m^3/day. (ABB, 2009)

Typically, after treating the wastewater, centralized treatment systems redistribute the water back to the region for reuse or dispose the water into a water body. Additionally, centralized treatment systems require technology that are expensive, such as membrane bioreactors.

As urbanised areas grow in size and population, the amount of wastewater produced per day in an urban area increases. Hence, it is important to consider the water allocation and reuse within a region as it is costlier to bring water in from new sources (Gleick, 2000). In addition to the negative consequences of relying on a single location for wastewater treatment (Wilderer and Schreff, 2000; Bakir, 2001), a centralised wastewater management system itself becomes unwieldy and costly to build on a large scale (Wilderer and Schreff, 2000).

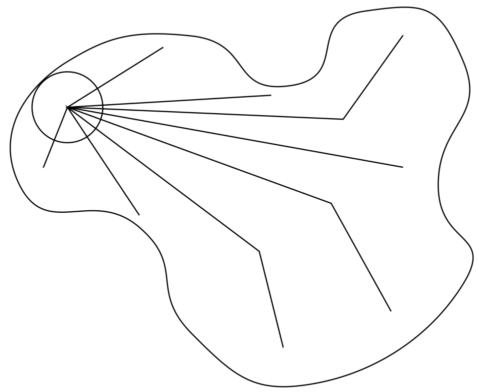


Figure 2: A centralized wastewater treatment network.

In order to tackle this, decentralized wastewater management systems as in Figure 3 have been proposed to reduce the distance from the wastewater source to the release point, cutting down on the cost of transporting wastewater to a dedicated facility (Otterpohl et al., 1997; Wilderer and Schreff, 2000; Bakir, 2001). A decentralized wastewater treatment network approach encourages wastewater to be treated near to the source for reuse in the same area. To serve an area of the same size as a centralized wastewater system, a decentralized wastewater treatment system needs to have a larger quantity of plants with smaller scale operations. Various case studies have conducted a cost-benefit analysis on the implementation of a centralised

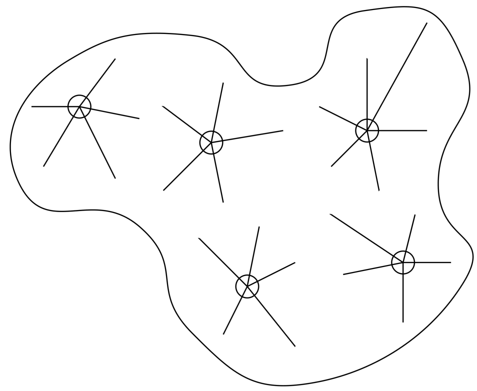


Figure 3: A decentralized wastewater treatment network.

wastewater management system against a decentralised one and have concluded that the decentralised wastewater management is generally cheaper to maintain (Prihandrijanti et al., 2008; Mobile Area Water & Sewer System et al., 2015).

1.4. Decentralized wastewater management

There are many advantages decentralized systems give over centralized ones. For one, a decentralized system has more treatment sites, which reduces the impact of the failure of a single treatment site. If a centralized treatment plant were to fail, the wastewater collected will accumulate and there is no backup plant nearby available to treat the water. On the other hand, if one treatment site of a decentralized system fails, the wastewater may be rerouted to other sites temporarily, ensuring that the system does not come to a standstill.

1.5. Constructed Wetlands

One approach to decentralised wastewater management is the constructed wetlands concept. Essentially, constructed wetlands aim to simulate real wetlands where water flows through and has its nutrients removed via biological processes. In the constructed version, wastewater flows through the wetland to provide the pollutants within as a nutrient source for plant absorption. The end product of wastewater going through these processes is water that meets the standards for release. The use of plants to treat the water allows the procedure of treating wastewater to be more hands off, thus requiring less maintenance. Thus, the costs involved in implementing constructed wetlands is relatively low as it uses the natural ability of plants to treat wastewater (Kadlec and Wallace, 2009; Mobile Area Water & Sewer System et al., 2015). However, the cost effectiveness of constructed wetlands is contingent on the following:

- Treatment sites are in optimal locations.
- Treatment sites are of a suitable size to handle the expected wastewater volume from the sources.

The size of the constructed treatment wetlands has to be big enough to serve the amount of wastewater produced, but not too big as the system may collapse due to a lack of nutrients for the plants towards the end of the wetlands. Additionally, the location of the constructed wetlands has to be taken into consideration to determine the shortest distance of pipes required to link the wastewater sources to the treatment area to balance the cost of transporting wastewater and building the site. (Mobile Area Water & Sewer System et al., 2015) In order to determine the ideal location and size of constructed wetlands in a defined area, a mathematical model has been developed to represent the problem. Solving this model will give the ideal location and size of constructed wetlands to be designed.

1.5.1. Horizontal sub-surface flow constructed wetlands

Over the years, implementation of more treatment wetlands in addition to higher water quality standards have encouraged studies to focus on establishing process design tools for the wetlands. Of all the types of wetlands that have been implemented, the horizontal sub-surface flow (HSSF) constructed wetlands is one of the most common.

Figure 4: A horizontal sub-surface flow constructed wetlands in Czech Republic. (Vymazal and Kröpfelová, 2008)

1.6. Objective

While constructed wetlands can essentially be implemented wherever there is space, the maximum effectiveness of using this decentralized wastewater treatment system is only achieved when it is located at an optimal distance to the areas which it serves. The performance of constructed wetlands also depends on whether the size is sufficient to handle the projected wastewater flow rate from the area. In addition, the components of wastewater is easily affected by the source and weather. Due to this natural variation, the pollutant composition in wastewater is not deterministic. The ideal configuration (location and size) of treatment sites from a deterministic optimization model may not perform well under such uncertainty. Hence, this project aims to show how a stochastic model may be used in this situation to improve overall performance of the network with the application of the formulated model in an area in Mobile, Alabama.

1.7. Significance

2. Methodology

2.1. Case study: a decentralized CWs system for municipal wastewater treatment

A case study approach is used to showcase the practical application of an optimization model for decentralized CW network to a real-life scenario. In this report, a case study of the optimal location and design size of a constructed wetlands treatment network in Mobile, Alabama.

2.1.1. Mobile, Alabama

With 412,992 people, Mobile County is the second most populated county within Alabama, a state in the United States. The population of Mobile County has been steadily increasing (Gauthier and United States Census Bureau, 2002). As populations grow, several fringe communities are created. In order to manage the wastewater produced by such communities, the traditional approach is to link these fringe communities up with long length large diameter pipes to transport all the wastewater produced to a single municipal wastewater treatment plant to be processed and then released into nearby water sources. In Mobile, the Mobile Area Water & Sewer System (MAWSS) serves such fringe communities and overall serves approximately 530 square kilometres in Mobile County (Mobile Area Water & Sewer System et al., 2015). MAWSS supports the region mostly through centralised wastewater treatment facilities. However, annual operations and maintenance costs for such centralised wastewater management systems have been shown to be costlier than decentralized ones. This is because decentralized treatment systems largely reduce the transport cost incurred and processes wastewater to be released very close to the community (Mobile Area Water & Sewer System et al., 2015). Thus, implementation of decentralized wastewater management systems such as constructed wetlands are being considered to reduce expenses in the long term.

Within Mobile, the constructed wetlands concept has been tested since 1997. Surrency (1997) designed and built a CW for urban stormwater management. His report recommended the implementation of constructed wetlands to replace wet ponds in managing stormwater. In 2005, MAWSS implemented a HSSF CW in Tricentennial Park in Mobile City as a trial to study the effectiveness of decentralized wastewater management systems in large urban sewer systems Mobile Area Water & Sewer System (2005).

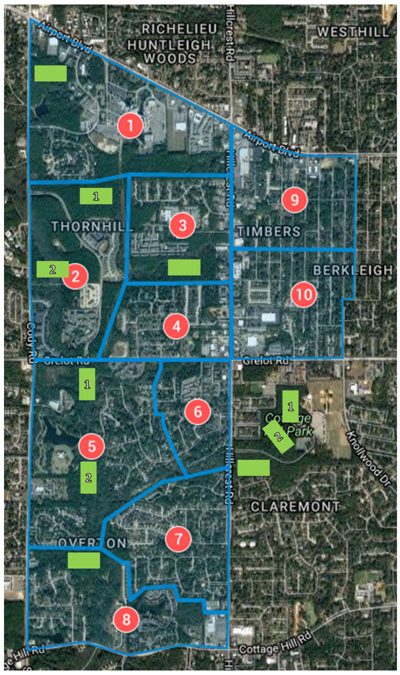


Figure 5: Case study area in Mobile, Alabama.

2.1.2. *Case Study Area*

The site chosen for the case study is near Mobile Regional Airport (Figure 5). It is a suburban area, mostly consisting of residences. As the objective of the project is to show how the optimization model can be applied, we have chosen a smaller area to keep the computation resources required reasonable.

3. Problem formulation

3.1. *Qualitative model description*

In general, a decentralized constructed wetlands system in a region can be represented as in Figure 6. Wastewater is generated from the sources I and routed to constructed wetlands, where it is treated and discharged or reused on site. Thus, given a set of wastewater sources I and a set of potential constructed wetlands locations J, the ultimate objective is to determine how to link the sources to the

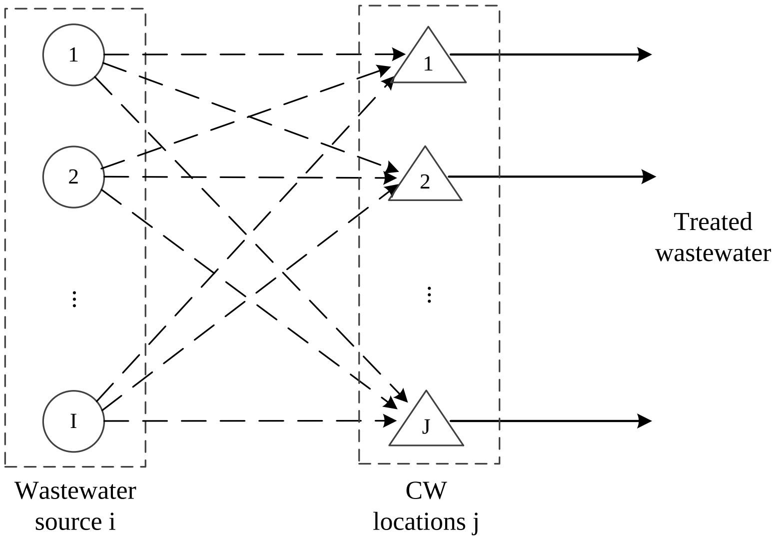


Figure 6: Superstructure of a decentralized CW treatment system network

constructed wetlands, where the wetlands should be located and the size of the wetlands at the site.

Besides this, in line with regulatory standards for treated water effluent, treatment targets are imposed on a set of pollutants M. This means that treated water leaving the constructed wetlands must have pollutant concentrations below specified levels. Depending on the intended use for treated water, treatment targets may be adjusted accordingly for each constructed wetlands site. This constraint directly affects the size of the constructed wetlands for a site as the size of the site determines the pollutant removal rate. It also indirectly affects the routing of the wastewater sources to the treatment sites.

Thus, a solution is deemed feasible only when it satisfies all the constraints. However, due to natural variation in the environment and pollutant concentration of the influent wastewater, constructed wetlands may not always effectively remove pollutants. In reality, it is also unlikely that the treated wastewater meets the targets all the time.

Additionally, cost is a major factor for decision makers to take into account hence the best feasible solution should be one that incurs the least cost. The cost consideration affects how the wastewater sources are routed to the treatment sites as the longer the distance, the higher the cost of constructing the pipe to link the source and site up. For the model, the following assumptions are made:

- wastewater from each source $i = 1, ..., I$ can be allocated to multiple constructed wetlands sites and each constructed wetlands can also treat wastewater from multiple wastewater sources. Sewer lines are needed to be constructed between wastewater sources and constructed wetlands.
- for any location $j = 1, ..., J$, if it is determined to construct a constructed wetlands, K design options $k = 1, ..., K$ could be selected. Otherwise, we use $k = 0$ to indicate that this location is not chosen to construct any constructed wetlands.
- a list of M pollutants $m = 1, ..., M$ are evaluated. Treatment target τ_j^m is set for each pollutant m and each potential site j.

3.2. Deterministic model

A basic list of model parameters and decision variables is provided in Table 1. Other notations would be introduced and defined as per required in the rest of the paper. Before the uncertainty component is addressed, we will consider a model where there are no stochastic components.

3.2.1. Objective Function

For the deterministic case, the objective of the problem is to minimize the total cost of implementing the system in a region.

$$min \sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij} c_s x_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} c_{cw,jk} y_{jk} \tag{3–1}$$

3.2.2. Constraints

The following constraints formulate the constructed wetlands design option, wastewater allocation between sources and constructed wetlands, pollutant removal performance and treatment target fulfillment:

Pollutant removal performance and treatment target fulfillment

First, we consider the influent of the constructed wetlands. The water that is entering the constructed wetlands should have the same amount of pollutants as all

Table 1: Notations of model parameters and decision variables

Indices	
i	index of wastewater sources, $i \in \{1, 2, ..., I\}$
j	index of potential CW locations, $j \in \{1, 2, ..., J\}$
m	index of evaluated water pollutants, $m \in \{1, 2, ..., M\}$
k	index of CW construction options, $k \in \{0, 1, 2, ..., K\}$
Model parameters	
ε_i^m	concentration of pollutant m in the wastewater source i (mg/m^3)
τ_j^m	treatment target above background concentration for pollutant m in CW site j (mg/m^3)
F_i	total wastewater flow generated by source i (m^3/d)
Q_{jk}	flow capacity of CW in option k for site j (m^3/d)
A_{jk}	area of CW in option k for site j (m^2)
$c_{cw,jk}$	construction cost of CW in design option k for site j ($)
d_{ij}	distance between wastewater source i and site j (m)
c_s	unit construction cost of sewer lines per distance ($/m)
Decision variables	
x_{ij}	binary variable, $x_{ij} = 1$ if sewer lines are constructed from wastewater source i to CW site j and 0 otherwise
y_{jk}	binary variable, $y_{jk} = 1$ if construction option k is chosen for site j and 0 otherwise. In particular, y_{j0} denotes the choice of not constructing any CWs in site j
z_{ij}	volume of wastewater flow assigned from wastewater source i to the CW in site j (m^3)

of the sources that are contributing wastewater to the site. So, the concentration of pollutant m in the influent at each potential CW site, $\varepsilon_{in,j}^m$ can be determined with Equation 3–2.

$$\varepsilon_{in,j}^m = \frac{\varepsilon_i^m \sum_{i=1}^I z_{ij}}{\sum_{i=1}^I z_{ij}} \qquad \forall j \tag{3–2}$$

To determine the pollutant concentration in the effluent, $\varepsilon_{out,j}^m$, we use a first-order $k - C^*$ model (Rousseau et al., 2004) to calculate the pollutant removal. The $k - C^*$ model uses a plug-flow reactor model to represent the reactions during the pollutant removal process as depicted in Figure 7. The treatment rate of a pollutant m in a HSSF CW with design option k at site j can be represented with a first-order $k - C^*$ model in the following equation:

$$\varepsilon_{out,j}^{m} - C_j^{m*} = e^{-\frac{A_{jk}}{Q_{jk}} k_A^m} [\varepsilon_{in,j}^{m} - C_j^{m*}] \qquad \forall j, k, m \qquad (3\text{–}3)$$

where k_A^m is the areal rate constant for pollutant m.

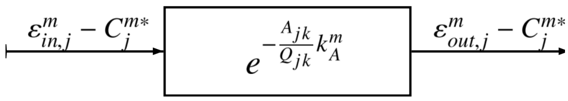


Figure 7: Pollutant treatment process

The areal rate constant k_A^m is determined empirically for each pollutant and varies depending on the region that the data is collected from. For the purposes of the case study, we will be using k_A^m values based on data from North America as collated by Vymazal and Kröpfelová (2008).

As the area A_{jk} and flow capacity Q_{jk} of the CWs are determined beforehand with the various design options, expansion of the right hand side of Equation 3–3 shows that the effluent pollutant concentration is a linear function of the influent pollutant concentration that follows the expression $ax + b$. To simplify the linear program definition, the following parameters will be used in place of the full form of the coefficient:

$$a_{jk}^{m} = e^{-\frac{A_{jk}}{Q_{jk}} k_A^m} \qquad \forall j, k, m$$

$$b_{jk}^{m} = C^* e^{-\frac{A_{jk}}{Q_{jk}} k_A^m} \qquad \forall j, k, m$$

$$\varepsilon_{out,j}^{m} - C_j^{m*} = a_{jk}^{m} \varepsilon_{in,j}^{m} + b_{jk}^{m} \qquad \forall j, k, m \qquad (3\text{–}4)$$

With the simplified parameters, Equation 3–3 can now be represented with Equation 3–4. After the wastewater is treated, the effluent is expected to meet pollutant concentration targets, τ_j^m. Since the treatment targets are set at a value above the background concentration, the constraint will be as follows:

$$\varepsilon_{out,j}^{m} - C_j^{m*} \leq \tau_j^m \qquad \forall j, m \qquad (3\text{–}5)$$

Combining Equations 3–2, 3–4 and 3–5, we will have a constraint that ensures the effluent pollutant concentration is under the treatment target. The nonlinearity in the constraint was resolved by introducing a Big M. In our model, the Big M is calculated using the

$$\sum_{i=1}^{I}(a_{jk}^{m}\varepsilon_{i}^{m}+b_{jk}^{m})z_{ij}-M_1(1-y_{jk})\leq\tau_{j}^{m}\sum_{i=1}^{I}z_{ij}\qquad\forall j,k,m\qquad(3\text{–}6)$$

Wastewater allocation from sources to sites

Let x_{ij} be a binary variable that describes the interconnection between the wastewater source i and potential contructed wetlands site j. When $x_{ij}=1$, there is a pipe constructed between wastewater source i and potential constructed wetlands site j. Wastewater is allowed to flow from source i to site j. Otherwise, when $x_{ij}=0$, there is no pipe connecting wastewater source i and potential constructed wetlands site j.

All the wastewater produced in the region studied has to be treated, so there should not be any excess wastewater volume. On top of that, the total volume of wastewater flowing into a constructed wetland cannot exceed the design capacity of the constructed wetland. If $y_{j0}=1$ for any j, there should not be any pipes connecting any wastewater source i to the respective potential constructed wetlands site j. Finally, there should only be wastewater sent to a site j if the wastewater source i and potential constructed wetlands site j is connected (i.e. $x_{ij}=1$).

If z_{ij} represents the volume of wastewater sent from source i to site j, F_i is the total wastewater flow from source i and Q_{jk} is the maximum flow capacity accepted by site j with design option k:

$$\sum_{i=1}^{I}z_{ij}\leq\sum_{k=0}^{K}Q_{jk}y_{jk},\qquad\forall j\qquad(3\text{–}7)$$

$$\sum_{j=1}^{J}z_{ij}=F_i,\qquad\forall i\qquad(3\text{–}8)$$

$$x_{ij}+y_{j0}\leq 1\qquad\forall i,j\qquad(3\text{–}9)$$

$$x_{ij}\in\{0,1\}\qquad\forall i,j\qquad(3\text{–}10)$$

$$z_{ij}\geq 0\qquad\forall i,j\qquad(3\text{–}11)$$

Constructed wetlands design options

Let y_{jk} be a binary variable that describes the design option k selected for the constructed wetlands site j. In the case where $k = 0$, there will be no constructed wetlands on the potential site j. Otherwise, any other values for k will correspond to the respective design options available. When $y_{jk} = 1$, it means that there will be a constructed wetlands of type k at site j.

The different design options result in different construction costs and pollutant removal capacities. At each potential CW site, there can only be one design option selected. Hence, the following constraints are introduced:

$$\sum_{k=0}^{K} y_{jk} = 1 \qquad \forall j \qquad (3\text{–}12)$$

$$y_{jk} \in \{0, 1\} \qquad \forall j, k \qquad (3\text{–}13)$$

3.2.3. Model

The following deterministic model is developed from the above constraints. It is a *Mixed-Integer Linear Programming* (MILP) problem (Model D):

$$\text{Model D :} \quad min \sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij} c_s x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk} y_{jk}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} (a_{jk}^{m} \varepsilon_i^{m} + b_{jk}^{m}) z_{ij} - M_1(1 - y_{jk}) \leq \tau_j^{m} \sum_{i=1}^{I} z_{ij} \qquad \forall j, k, m \qquad (1)$$

$$\sum_{i=1}^{I} z_{ij} \leq \sum_{k=0}^{K} Q_{jk} y_{jk} \qquad \forall j \qquad (2)$$

$$\sum_{j=1}^{J} z_{ij} = F_i \qquad \forall i \qquad (3)$$

$$z_{ij} \leq F_i x_{ij} \qquad \forall i, j \qquad (4)$$

$$x_{ij} + y_{j0} \leq 1 \qquad \forall i, j \qquad (5)$$

$$\sum_{k=0}^{K} y_{jk} = 1 \qquad \forall j \qquad (6)$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \qquad (7)$$

$$y_{jk} \in \{0, 1\} \qquad \forall j, k \qquad (8)$$

$$z_{ij} \geq 0 \qquad \forall i, j \qquad (9)$$

3.3. Two-stage stochastic model

In a real situation, the pollutant loading of wastewater is uncertain as they fluctuate depending on the actual weather, activities carried out for the day (laundry, showers, etc.) and other reasons. With this uncertainty, the CW network in the optimal solution for the deterministic model may not be able to perform well over a long period of time. If we take the uncertainty into consideration in the model before solving it, we may be able to attain a better optimal solution that is expected to meet the treatment standards more frequently.

We will use a two-stage stochastic model to incorporate the uncertainty into the model. The first-stage variables will be the construction options of the potential CW sites, y_{jk} and the connections between the wastewater sources and the potential CW sites, x_{ij}. The second-stage variable will be the volume of wastewater to be sent from the wastewater sources to the potential CW sites. In this section, the variable will be denoted as $z_{ij}(\tilde{\varepsilon})$ to show that it is dependent on the influent pollutant concentration. As we are considering a long run average, we will keep the total wastewater volume per day produced by each wastewater source i constant.

3.3.1. Objective function

In the stochastic model, we want to ensure that the best possible performance of the CW network is attained. Thus, the objective function will be to maximize the probability of the effluent pollutant concentrations meeting the treatment standards.

$$max\ \mathbb{P}(\tilde{\varepsilon}_{out} \leq \tau) \tag{3–14}$$

3.3.2. Constraints

Most of the constraints in the model are the same as the ones in Model D for wastewater allocation and the constructed wetlands design options. Since the pollutant treatment performance is evaluated in the objective function, the constraints are left in their original nonlinear expressions for now.

Cost

As the objective function is no longer to minimize the cost, a new constraint will be used to take the budget concerns into account. In this case, we will set the budget to be 5% above the total cost of the optimal solution from Model D (Equation 3–15). Allowing the cost to go above the minimum found in the deterministic model gives some leeway for the model to achieve better pollutant treatment performance at a slightly higher cost.

$$\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}c_s x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk}y_{jk} \leq B \tag{3–15}$$

3.3.3. Model

The two-stage stochastic model of the problem (Model S) is described following the above modifications from Model D.

$$\text{Model S}: \quad max\ \mathbb{P}(\tilde{\varepsilon}_{out} \leq \tau)$$

$$\text{s.t.} \quad \tilde{\varepsilon}^m_{in,j} = \frac{\sum_{i=1}^{I} z_{ij}(\tilde{\varepsilon})\tilde{\varepsilon}^m_i}{\sum_{i=1}^{I} z_{ij}(\tilde{\varepsilon})} \qquad \forall j, m \tag{1}$$

$$\tilde{\varepsilon}^m_{out,j} - C^{m*}_j = [\tilde{\varepsilon}_{in,j} - C^{m*}_j]\sum_{k=1}^{K} a^m_{jk}y_{jk} \qquad \forall j, m \tag{2}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}c_s x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk}y_{jk} \leq B \tag{3}$$

$$\sum_{i=1}^{I} z_{ij}(\tilde{\varepsilon}) \leq \sum_{k=1}^{K} Q_{jk}y_{jk} \qquad \forall j \tag{4}$$

$$\sum_{j=1}^{J} z_{ij}(\tilde{\varepsilon}) \leq F_i \qquad \forall i \tag{5}$$

$$z_{ij}(\tilde{\varepsilon}) \leq F_i x_{ij} \qquad \forall i, j \tag{6}$$

$$x_{ij} + y_{j0} \leq 1 \qquad \forall i, j \tag{7}$$

$$\sum_{k=0}^{K} y_{jk} = 1 \qquad \forall j \tag{8}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \tag{9}$$

$$y_{jk} \in \{0, 1\} \qquad \forall j, k \tag{10}$$

$$z_{ij}(\tilde{\varepsilon}) \geq 0 \qquad i, j \tag{11}$$

3.4. Sample average approximation

In order to evaluate the problem, the sample average approximation method will be used to form a deterministic equivalent of the stochastic model. Realizations of the uncertain parameter will replace the stochastic component. Each scenario to be evaluated will have a set of realizations of the concentration of each pollutant m from each wastewater source i. Let n represent a scenario to be evaluated ($n \in \{1, .., N\}$). The number of scenarios N to be evaluated is set to 100.

Estimating the probability of the effluent pollutant concentrations meeting the treatment targets can be done with two different methods. For the first method, each scenario is given a success variable - a binary variable that takes the value 1 if the scenario is successful and 0 otherwise. Any scenario n will be successful if the treatment target τ_j^m is met by all effluents and all pollutants. The objective function then aims to maximize the average of the success variable across all scenarios N. This method directly estimates the probability of the treatment target being met in the long run. Further elaboration on this method is in Section 3.4.1. The second method measures the mass of pollutants in excess of the treatment target. Each scenario has a variable that will be set to the largest excess mass of all the effluents for each pollutant m. When all effluents in a scenario n do not exceed any of the pollutant treatment targets τ_j^m, the new variables are set to 0. The objective function in this method aims to minimize the average amount by which the effluents exceed treatment targets. Section 3.4.2 covers this method in more detail.

Table 2: Notations of additional model parameters and decision variables

Model parameters	
ε_i^{mn}	concentration of pollutant m in the wastewater source i for scenario n (mg/m^3)
Decision variables	
z_{ij}^n	volume of wastewater flow assigned from wastewater source i to the CW in site j in scenario n (m^3)
γ^n	binary variable, $\gamma^n = 1$ if all pollutants in all effluents $\varepsilon_{out,j}^m n$ for scenario n meet the treatment target τ_j^m
λ_{ijk}^n	variable for the McCormick envelope for the bilinear term y_{jk} and z_{ij}^n

3.4.1. Maximizing average success

As described in the previous section, the objective of this method is to maximize the average success of all scenarios N. New parameters and decision variables in this model are defined in Table 2. This expanded model is defined in Model S1. The success variable γ^n is combined together with the two pollutant treatment equations 1 and 2 in Model S. The resulting constraint has two nonlinear terms which is resolved using the Big M method as well as using McCormick envelopes to relax the problem. The form of the revised constraint is in Equation 1 of Model S1.

$$\text{Model S1}: \quad max \ \frac{1}{N}\sum_{n=1}^{N}\gamma^n$$

$$\text{s.t.} \quad \sum_{k=1}^{K}\sum_{i=1}^{I}(a_{jk}^{m}\varepsilon_i^{mn} + b_{jk}^{m})\lambda_{ijk}^{n} - \tau_j^m \sum_{i=1}^{I} z_{ij}^{n} \leq M_2(1-\gamma^n) \quad \forall j,m,n \tag{1}$$

$$\sum_{i=1}^{I}\sum_{j=1}^{J} d_{ij}c_s x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk}y_{jk} \leq B \tag{2}$$

$$\sum_{i=1}^{I} z_{ij}^{n} \leq \sum_{k=0}^{K} Q_{jk}y_{jk} \quad \forall j,n \tag{3}$$

$$\sum_{j=1}^{J} z_{ij}^{n} = F_i \quad \forall i,n \tag{4}$$

$$z_{ij}^{n} \leq F_i x_{ij} \quad \forall i,j,n \tag{5}$$

$$x_{ij} + y_{j0} \leq 1 \quad \forall i,j \tag{6}$$

$$\sum_{k=0}^{K} y_{jk} = 1 \quad \forall j \tag{7}$$

$$\lambda_{ijk}^{n} \geq z_{ij}^{n} + z_{ij}^{nu}y_{jk} - z_{ij}^{nu} \quad \forall i,j,k,n \tag{8}$$

$$\lambda_{ijk}^{n} \geq 0 \quad \forall i,j,k,n \tag{9}$$

$$x_{ij} \in \{0,1\} \quad \forall i,j \tag{10}$$

$$y_{jk} \in \{0,1\} \quad \forall j,k \tag{11}$$

$$z_{ij}^{n} \geq 0 \quad \forall i,j,n \tag{12}$$

3.4.2. Minimizing pollutant shortfall

The objective of this method is to minimize the average mass by which the effluents exceed by for each pollutant. Let η^{mn} be the largest pollutant treatment shortfall mass for pollutant m in scenario n. As the shortfall amount for each type of pollutant cannot be simply summed and averaged together, the value of η^{mn} is normalized against the mass of each pollutant in the effluent if it were to just meet the treatment target in the objective function. The combination of the equations 1 and 2 in Model S with η^{mn} results in a constraint with a nonlinear term. This was replaced using the Big M method once again. The revised constraint is in Equation 1 in Model S2. Model S2 is the expanded model following this method to estimate $\mathbb{P}(\tilde{\varepsilon}_{out} \leq \tau)$.

$$\text{Model S2}: \quad min \; \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\eta^{mn}}{\tau_j^m \sum_{i=1}^{I} F_i}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} (a_{jk}^m \varepsilon_i^{mn} + b_{jk}^m - \tau_j^m) z_{ij}^n - M_3(1 - y_{jk}) \leq \eta^{mn} \qquad \forall j,k,m,n \tag{1}$$

$$\sum_{i=1}^{I} \sum_{j=1}^{J} d_{ij} c_s x_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} c_{cw,jk} y_{jk} \leq B \tag{2}$$

$$\sum_{i=1}^{I} z_{ij}^n \leq \sum_{k=0}^{K} Q_{jk} y_{jk} \qquad \forall j,n \tag{3}$$

$$\sum_{j=1}^{J} z_{ij}^n = F_i \qquad \forall i,n \tag{4}$$

$$z_{ij}^n \leq F_i x_{ij} \qquad \forall i,j,n \tag{5}$$

$$x_{ij} + y_{j0} \leq 1 \qquad \forall i,j \tag{6}$$

$$\sum_{k=0}^{K} y_{jk} = 1 \qquad \forall j \tag{7}$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \tag{8}$$

$$y_{jk} \in \{0,1\} \qquad \forall j,k \tag{9}$$

$$z_{ij}^n \geq 0 \qquad \forall i,j,n \tag{10}$$

$$\eta^{mn} \geq 0 \qquad \forall n \tag{11}$$

3.4.3. Scenario construction

The uncertainty of the concentration of pollutant m from wastewater source i is treated as a random variable with a *known* distribution. It is assumed that the pollutant loading across different sources are independent. That is, for each scenario, a realization of the distribution is determined independently for each source i and for each pollutant m. In this project, the realizations are generated using Microsoft Excel.

4. Case study

The site has been divided into 10 blocks based on six census tract blocks. The blocks are represented with blue outlines in Figure 5. Geographic information about the region listed in Table 3 was also retrieved from the census (United States Census Bureau, 2016). For each block, we have assumed a point source for the wastewater (see red circles).

Table 3: Geographical information about the 10 blocks in Mobile, Alabama. Information retrieved from Google Maps and United States Census Bureau (2016).

Block i	Latitude	Longitude	Area (km^2)	Population	Wastewater Production F_i (m^3/day)
1	30.678	-88.199	1.41	818	181.43
2	30.667	-88.204	1.06	818	181.43
3	30.671	-88.195	0.725	817	181.21
4	30.664	-88.195	0.593	817	181.21
5	30.655	-88.202	1.47	850	188.53
6	30.657	-88.194	0.487	849	188.31
7	30.648	-88.195	0.847	992	220.03
8	30.642	-88.200	0.986	991	219.8
9	30.679	-88.185	0.837	1104	244.87
10	30.666	-88.185	0.840	1104	244.87

Ten potential sites for the constructed wetlands were identified from Google Maps and represented in Figure 5 as green rectangles. These areas are uninhabited and thus the construction of wetlands there will impact few residents negatively. A point location is used to represent each of these sites. There are three potential sites that are outside of the area boundary, which was left included The coordinates of the potential sites are stated in Table 6. The distances between the wastewater sources and potential sites were calculated as well and stated in Table 10.

Table 4: Geographical information about the 10 potential CW sites in Mobile, Alabama. Information retrieved from Google Maps.

Potential CW Site j	Latitude	Longitude
1	30.682	-88.206
2-1	30.673	-88.202
2-2	30.668	-88.205
3	30.668	-88.196
5-1	30.659	-88.203
5-2	30.652	-88.203
8	30.647	-88.203
11-1	30.658	-88.186
11-2	30.656	-88.187
12	30.654	-88.189

4.1. Pollutants

Once again, to keep the model manageable, we have narrowed down the number of pollutant indicators to three, the Biological Oxygen on Demand over 5 days (BOD_5), Total Suspended Solids (TSS) and Total Nitrogen (TN). The pollutants have been selected based on the potential impact on the ecosystem if these substances were not removed and the wastewater was released into water bodies. In this case study, we will set the treatment target to be the same regardless of the intended use for the treated water. The minimum and maximum concentrations provided by Gross (2005) are used with the assumption of a Normal distribution for each pollutant. For the deterministic model, the pollutant concentration is taken to be the average of the maximum and minimum values.

Table 5: Pollutant influent concentrations and treatment targets. Information retrieved from Gross (2005) and Alabama Department of Environmental Management (2015).

Indicator	Influent concentration ε^m (mg/l)			Treatment target τ^m
m	Minimum	Maximum	Average	(mg/ml)
BOD_5	155	330	242.5	30
TSS	155	286	220.5	30
TN	26	75	50.5	10

4.1.1. Impact of selected pollutants on the environment

Biological oxygen demand over 5 days

BOD_5 measures the quantity of oxygen consumed by the aerobic biological activity of microorganisms in the water over 5 days. This gives an indication of the amount of biological content in the water. High BOD_5 values indicates that there are a lot of microorganisms present in the water, and their presence in the wastewater can deplete the dissolved oxygen in waters. This creates anoxic conditions in the water which is detrimental to aquatic life. (United States Environmental Protection Agency, 2002)

Total suspended solids

TSS measures the amount of solids that are suspended in the water. Accumulation of suspended solids leads to the development of sludge deposits. In surface water, this contributes to toxicity and can block sunlight. This harms aquatic plants as access to sunlight will be restricted. (United States Environmental Protection Agency, 2002)

Total nitrogen

TN measures the amount of organic and inorganic nitrogen that resides in the water. Nitrogen is a nutrient for plants, thus if it is present in excess in water, it may cause eutrophication and hypoxic conditions. In surface waters, the algae that proliferates as a result of abundant nitrogen will prevent sunlight from reaching aquatic plants at the bottom of water bodies such as lakes and coastal embayments. Hypoxic conditions and lack of sunlight can severely harm aquatic life. (United States Environmental Protection Agency, 2002)

4.2. Design options

The design of the constructed wetlands will be represented by its treatment capacity. That is, the volume of wastewater in a day that the constructed wetlands can treat and meet treatment standards for. In reality, constructed wetlands will not be built to serve the exact amount of water that is allocated, rather they will have a few set designs with defined capacities. Hence, four design options for the constructed wetlands with treatment capacities between $450m^3$ and $950m^3$ were determined prior to solving the optimization model. The four design options and their respective construction costs, area and volume flow capacities can be found in Table 11. The formula for estimating the cost based on area can be found in Kadlec and Wallace (2009). To keep the cost involved consistent with the pipe construction cost, the values have been revised to 2013 prices using the Engineering News-Record Construction Cost Index (CCI).

Table 6: Selected design options for the CWs.

Design Option k	Flow Capacity Q_k (m^3/day)	Area A_k (m^2)	Cost $c_{cw,k}$ ($'000)
1	450	7000	606
2	650	10000	780
3	800	12500	912
4	950	15000	1037

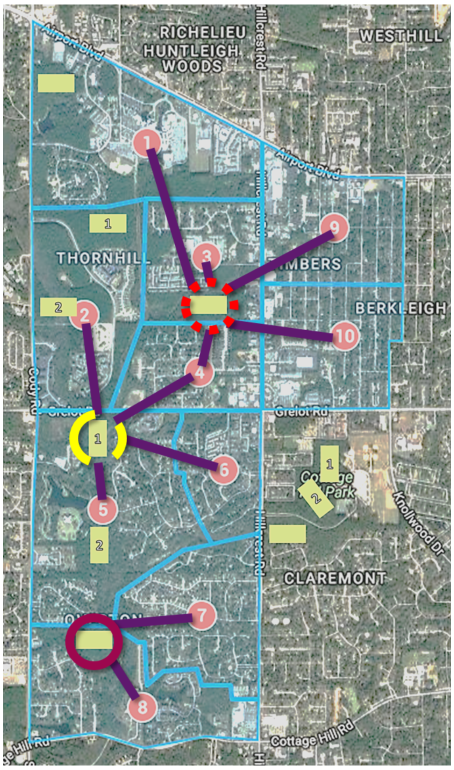
4.3. Sewer pipe construction cost

Based on Kadlec and Wallace (2009), the normal size of sewer pipes for treatment wetlands vary from $10 - 30cm$ diameter pipes. Hence, the sewer pipe construction cost is estimated to be about $US\$134/m$ (United States Environmental Protection Agency, 2000) after revising to latest prices using the CCI.

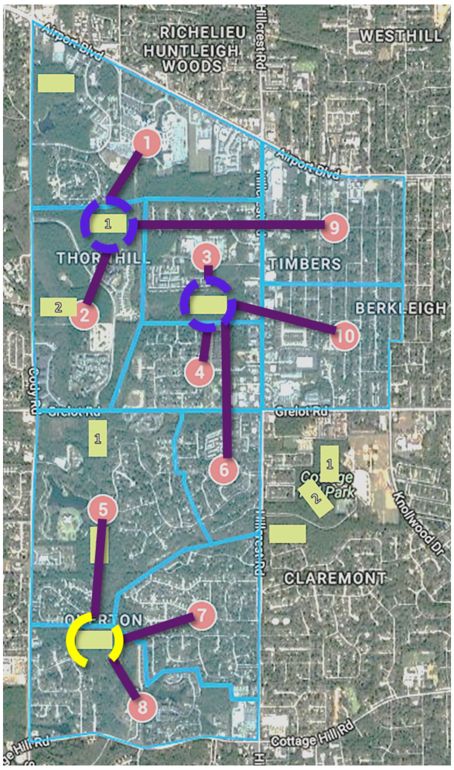
The above information provides a good set-up to determine optimal locations for constructed wetlands to be built within Mobile, Alabama.

5. Results

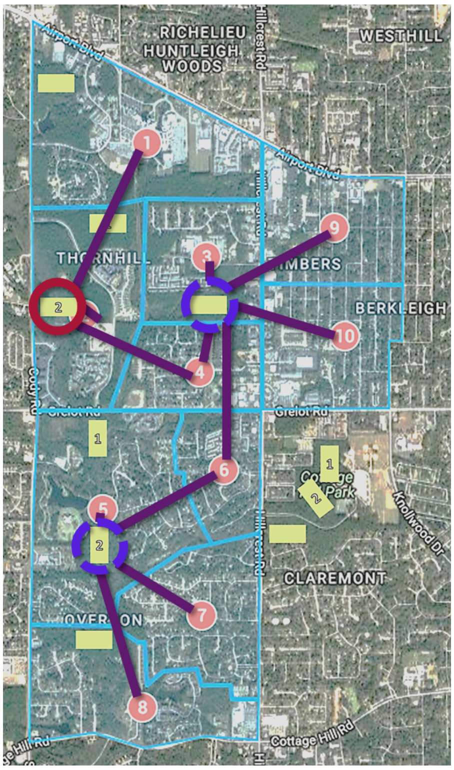
The models were solved using CPLEX 12.6.2. The networks from the optimal solutions of all the models is in Figure 8. In general, all three solutions opted to build 3 CWs to serve the whole area, although the CW design options selected were different.



(a) CW network
from Model D



(b) CW network
from Model S1



(c) CW network
from Model S2

Figure 8: CW networks proposed from the optimal solutions of all models

5.1. *Deterministic model, Model D*

The total cost of the optimal solution is \$3.849 million. Hence, the budget for the stochastic models will be set to \$4.042 million.

Table 7: Model D optimal solution: Details of selected CWs

CW Site	Design Option	Flow Capacity	Visual on Figure 8a
3	4	950	dotted red
5-1	2	650	dashed yellow
8	1	450	solid maroon

5.2. *Stochastic model: maximizing average success, Model S1*

The average shortfall in the optimal solution is - and the total cost of the optimal solution is -.

Table 8: Model S2 optimal solution: Details of selected CWs

CW Site	Design Option	Flow Capacity	Visual on Figure 8b
-			dotted red
-			dashed yellow
-			solid maroon

5.3. *Stochastic model: minimizing pollutant shortfall, Model S2*

The average shortfall in the optimal solution is 0.00190 and the total cost of the optimal solution is \$4.037 million. Optimality gap between the linear optimal solution and the integer feasible solution is $1.14 \times 10^{-14}\%$.

Table 9: Model S2 optimal solution: Details of selected CWs

CW Site	Design Option	Flow Capacity	Visual on Figure 8c
2-2	1	450	solid maroon
3	3	800	dashed blue
5-2	3	800	dashed blue

6. Computational study

The different networks will be evaluated for its performance by out-of-sample testing. A random sample of $N = 1000$ was generated and both the average success and average shortfall will be measured for the three networks.

6.0.1. Out-of-sample performance

6.0.2. Increasing budget

7. Discussion

8. Conclusion

8.1. Further Directions

Proceeding from here, the next step would be to find a solution for the deterministic model, which will be done using MATLAB. After which, work will commence on developing the probabilistic model and running it.

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Appendix

Table 10: Distance in km between the wastewater sources and potential constructed wetlands sites.

i/j	1	2-1	2-2	3	5-1	5-2	8	11-1	11-2	12
1	0.751	0.683	1.275	1.211	2.158	2.873	3.537	2.599	2.728	2.909
2	1.595	0.612	0.113	0.765	0.922	1.65	2.307	2.016	2.047	2.096
3	1.583	0.735	1.022	0.383	1.541	2.186	2.835	1.701	1.831	2.022
4	2.232	1.198	1.018	0.448	0.889	1.418	2.045	1.117	1.173	1.285
5	2.972	1.966	1.445	1.549	0.456	0.274	0.937	1.619	1.495	1.319
6	2.927	1.882	1.578	1.154	0.918	1.024	1.528	0.734	0.655	0.627
7	3.848	2.802	2.369	2.165	1.419	0.856	0.799	1.385	1.172	0.85
8	4.396	3.375	2.872	2.833	1.882	1.162	0.587	2.153	1.937	1.612
9	2.198	1.617	1.945	1.152	2.274	2.792	3.382	1.67	1.872	2.162
10	2.679	1.846	1.947	1.09	1.916	2.276	2.798	0.916	1.126	1.433

Table 11: Construction cost of constructed wetlands with four different design options as well as projected area and flow capacity.

k	q	a	cost
1	450	7000	606
2	650	10000	780
3	800	12500	912
4	950	15000	1037