Optimal design of integrated decentralized constructed wetlands treatment systems under uncertainties

Abstract

Keywords:

1. Problem statement

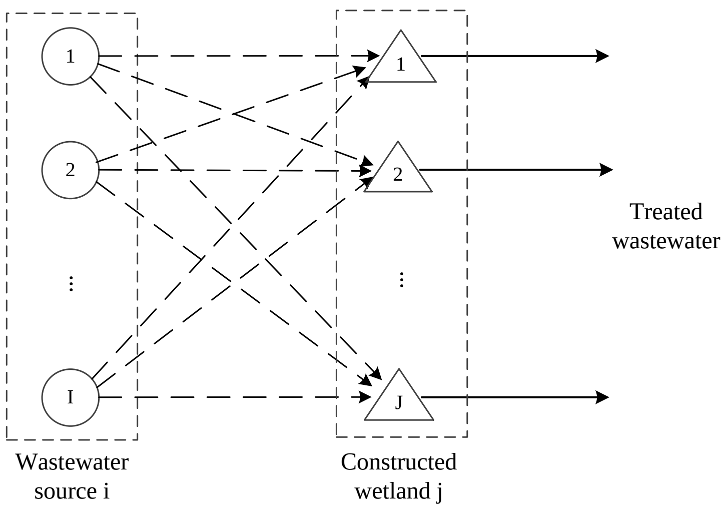

Figure 1: Schematic of an integrated decentralized CW treatment system network

We consider a general decentralized CWs treatment network as shown in Fig.1. Given I wastewater sources and J potential CW locations, wastewater is generated from the sources, treated in the CWs, and finally discharged or reused with regard to particular treatment purposes. Treatment targets are imposed for each CW and allowed to be specified according to the treatment purposes. For example, if a location commands the treated wastewater to be reused for landscape irrigation, water quality standards for irrigation are then enforced; if the CW is designed for pretreatment before discharging wastewater into the sewer system, treatment targets regarding with reference to discharge standards should be applied. However, due to the Hence, we aim at finding optimal planning solutions that determine the location and design of CWs, the interconnections between wastewater sources and CWs, and the corresponding allocated wastewater flows, such that the treatment targets could be satisfied as much as possible. Specifically, we make the following assumptions:

- Wastewater from each source $i = 1, ..., I$ can be allocated to all CWs and each CW $j = 1, ..., J$ can treat wastewater from all wastewater sources.

- For each CW j, K candidate design options $k = 1, ..., K$ could be selected. In particular, we use $k = 1$ to represent the choice of not constructing any CWs.
- A list of M pollutants $m = 1, ..., M$ and treatment targets $\tau_j^m, \forall j, m$ on all pollutant concentrations in the treated wastewater from each CW j are considered.

A basic list of model parameters and decision variables is provided in Table 1. Other notations would be introduced and defined as per required in the rest of the paper. The following constraints formulate the CW design option, wastewater allocation, pollutant concentration transformation, treatment targets and budget restriction:

Table 1: Notations of model parameters and decision variables

Indices	
i	index of wastewater sources, $i \in \{1, 2, ..., I\}$
j	index of potential CW sites, $j \in \{1, 2, ..., J\}$
m	index of all evaluated water pollutants, $m \in \{1, 2, ..., M\}$
k	index of CW design options, $k \in \{1, 2, ..., K\}$
Model parameters	
ε_i^m	concentration of pollutant m in the wastewater source i (mg/L)
$\varepsilon_{in,j}^m$	concentration of pollutant m in the influent of CW j (mg/L)
$\varepsilon_{out,j}^m$	concentration of pollutant m in the effluent of CW j (mg/L)
F_i	total wastewater flow generated by source i (L^3/T)
Q_{jk}	flow rate capacity of CW j in design option k (L^3/T)
A_{jk}	size area of CW j in design option k (L^2)
$c_{cw,jk}$	construction cost of CW j in design option k (\$)
τ_j^m	treatment target of pollutant m in CW j (mg/L)
d_{ij}	distance between wastewater source i and CW j (L)
c_s	unit construction cost of sewer lines per distance (\$/L)
Decision variables	
x_{ij}	binary variable, $x_{ij} = 1$ if sewer lines are constructed from wastewater source i to CW j and 0 otherwise
y_{jk}	binary variable, $y_{jk} = 1$ if the design option k is chosen for CW j and 0 otherwise. In particular, y_{j1} denotes the choice of not constructing any CWs in site j.
z_{ij}	wastewater flow assigned from source i to CW j

CW design option: For each CW, we assume that one specific design option k is chosen. In particular, any CW chooses the option $k = 1$ indicates that the CW is not constructed or open. Let y_{jk} be a binary variable that describes whether design option k is chosen for CW j, we can than have:

$$
\begin{aligned}
&\sum_{k=1}^{K} y_{jk} = 1, \quad \forall j \\
&y_{jk} \in \{0,1\}, \quad \forall j,k
\end{aligned}
\tag{1}
$$

Wastewater allocation: A binary decision variable x_{ij} is used to describe whether sewer lines are constructed between source i and CW j. If source i is connected with CW j by sewer lines, then x_{ij} takes value of 1; otherwise, x_{ij} is 0. Let z_{ij} denote the wastewater flow assigned from source i to CW j, we impose the constraints:

$$
\sum_{i=1}^{I} z_{ij} \le \sum_{k=1}^{K} Q_{jk} y_{jk}, \quad \forall j \tag{2}
$$

$$
\sum_{j=1}^{J} z_{ij} = F_i, \quad \forall i \tag{3}
$$

$$
0 \le z_{ij} \le F_i x_{ij}, \quad \forall i,j \tag{4}
$$

$$
x_{ij} \in \{0,1\}, \quad \forall i,j, \tag{5}
$$

where Q_{jk} is the flow rate capacity of CW j in design option k and F_i is the total wastewater flow generated by source i.

These constraints ensure that

- a CW can treat wastewater from any sources only if the CW is open and its total flow capacity is not exceeded. In other words, if a CW chooses option $k = 1$, Q_{j1} equals 0 and the right hand side of (2) would then equal 0, which allows no wastewater to be treated. On the other hand, if a CW chooses any other design option except $k = 1$, the right hand side of (2) equals the maximum flow capacity $Q_{jk} > 0$, which allows wastewater to be assigned.
- wastewater generated from each source must be fully treated by CWs.
- wastewater could be assigned from source i to CW j only if sewer lines are constructed between them.

Pollutant concentration transformation: Let ε_i^m be the concentration of pollutant m in the wastewater source i. Similarly, we denote the pollutant concentration in the influent and effluent of CW j by $\varepsilon_{in,j}^m$ and $\varepsilon_{out,j}^m$, respectively. Given an allocation strategy $\boldsymbol{z} \in \Re^{I\times J}$, $\varepsilon_{in,j}^m$ could be calculated as the average concentration in the mixed wastewater from different sources:

$$
\varepsilon_{in,j}^m \sum_{i=1}^{I} z_{ij} = \sum_{i=1}^{I} z_{ij} \varepsilon_i^m, \quad \forall j, m. \tag{6}
$$

To account for the relationship between pollutant concentrations in the influent $\varepsilon_{in,j}^m$ and effluent $\varepsilon_{out,j}^m$, we introduce the following general expression to predict the CW treatment performance:

$$
\varepsilon_{out,j}^m = \sum_{k=1}^{K} (a_{jk}^m \varepsilon_{in,j}^m + b_{jk}^m) y_{jk}, \quad \forall j, m, \tag{7}
$$

where a^m_{jk} and b^m_{jk} are design coefficients that depend on the specific CW design parameters in each option k and the type of pollutants. We can observe that once the design option y_{jk} is determined, the effluent concentration $\varepsilon^m_{out,j}$ is an affine function of the influent concentration $\varepsilon^m_{in,j}$. It is reasonable since regression equations are widely assumed in CW design models and even for more complex physical models (e.g., first-order k-C^* model), they also follow this affine format once the design parameters are fixed. Later in the application example, we would introduce specific design parameters (e.g., size area and flow rate) into the calculation of a^m_{jk} and b^m_{jk}. To substitute $\varepsilon^m_{in,j}$ in (7) with (6), we can then obtain the following expressions of effluent pollutant concentrations:

$$\varepsilon^m_{out,j} \sum_{i=1}^{I} z_{ij} = \sum_{k=1}^{K} a^m_{jk} y_{jk} \sum_{i=1}^{I} z_{ij} \varepsilon^m_i + \sum_{k=1}^{K} b^m_{jk} y_{jk} \sum_{i=1}^{I} z_{ij}, \quad \forall j, m. \tag{8}$$

Treatment targets: The constraints

$$\varepsilon^m_{out,j} \leq \tau^m_j, \quad \forall j, m \tag{9}$$

assure that all the pollutant concentrations in the effluent achieve the treatment targets τ^m_j.

Budget constraint: We only consider the fixed capital cost at the design phase and restrict them within a capital budget, denoted by B:

$$\sum_{i=1}^{I} \sum_{j=1}^{J} c_s d_{ij} x_{ij} + \sum_{j=1}^{J} \sum_{k=1}^{K} c_{cw,jk} y_{jk} \leq B, \tag{10}$$

where c_s is the unit construction cost of sewer lines per distance, $c_{cw,jk}$ is the construction cost of CW j in design option k and d_{ij} is the distance between source i and CW j. Hence, the left hand side of (10) denotes the sum of construction cost on sewer lines and CWs, which is required not to exceed the total capital budget B.

Consolidating the above, the resulting feasibility problem (Model F) can be formulated subject to the following constraints:

$$
\begin{aligned}
\text{Model F}: \quad \text{s.t.} \quad & \sum_{k=1}^{K}\sum_{i=1}^{I} a_{jk}^{m}\varepsilon_i^m y_{jk} z_{ij} + \sum_{k=1}^{K}\sum_{i=1}^{I} b_{jk}^{m} y_{jk} z_{ij} \le \tau_j^m \sum_{i=1}^{I} z_{ij}, \quad \forall j, m \\
& \sum_{i=1}^{I}\sum_{j=1}^{J} c_s d_{ij} x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk} y_{jk} \le B \\
& \sum_{k=1}^{K} y_{jk} = 1, \quad \forall j \\
& \sum_{i=1}^{I} z_{ij} \le \sum_{k=1}^{K} Q_{jk} y_{jk}, \quad \forall j \\
& \sum_{j=1}^{J} z_{ij} = F_i, \quad \forall i \\
& z_{ij} \le F_i x_{ij}, \quad \forall i, j \\
& x_{ij}, y_{jk} \in \{0,1\}, z_{ij} \ge 0, \quad \forall i, j, k.
\end{aligned} \tag{11}
$$

As can be seen, the feasibility problem in (11) is bilinear which involves the products of binary variables y_{jk} and continuous decision variables z_{ij}. We can also observe that z_{ij} are bounded in $[0, x_{ij}]$ such that the first bilinear constraint in (11) could be explicitly linearized with *McCormick's inequalities* (McCormick 1976). Replace $y_{jk}z_{ij}$ with θ_{ijk}, the bilinear constraint could be replaced with the following linear inequalities:

$$
\sum_{k=1}^{K}\sum_{i=1}^{I} a_{jk}^{m}\varepsilon_i^m \theta_{ijk} + \sum_{k=1}^{K}\sum_{i=1}^{I} b_{jk}^{m} \theta_{ijk} \le \tau_j^m \sum_{i=1}^{I} z_{ij}, \quad \forall j, m \tag{12}
$$

$$
\begin{pmatrix}
\theta_{ijk} \ge y_{jk}^L z_{ij} + z_{ij}^L y_{jk} - y_{jk}^L z_{ij}^L \\
\theta_{ijk} \ge y_{jk}^U z_{ij} + z_{ij}^U y_{jk} - y_{jk}^U z_{ij}^U \\
\theta_{ijk} \le y_{jk}^L z_{ij} + z_{ij}^U y_{jk} - y_{jk}^L z_{ij}^U \\
\theta_{ijk} \le y_{jk}^U z_{ij} + z_{ij}^L y_{jk} - y_{jk}^U z_{ij}^L
\end{pmatrix}, \quad \forall i, j, k, \tag{13}
$$

where y_{jk}^L and y_{jk}^U are lower and upper values of the design choice variable y_{jk}, and z_{ij}^L and z_{ij}^U are lower and upper bounds of allocation flow rates z_{ij}. The feasibility problem could then be reformulated by adding a sequence of linearized inequalities (13) as the following Mixed-Integer

Linear Programming (MILP) formulation (Model F-1):

$$\text{Model F-1 :}\quad \text{s.t.}\quad \begin{aligned} & \sum_{k=1}^{K}\sum_{i=1}^{I} a_{jk}^{m}\varepsilon_i^m\theta_{ijk} + \sum_{k=1}^{K}\sum_{i=1}^{I} b_{jk}^{m}\theta_{ijk} \le \tau_j^m \sum_{i=1}^{I} z_{ij}, \quad \forall j, m \\ & \theta_{ijk} \ge y_{jk}^L z_{ij} + z_{ij}^L y_{jk} - y_{jk}^L z_{ij}^L, \quad \forall i, j, k \\ & \theta_{ijk} \ge y_{jk}^U z_{ij} + z_{ij}^U y_{jk} - y_{jk}^U z_{ij}^U, \quad \forall i, j, k \\ & \theta_{ijk} \le y_{jk}^L z_{ij} + z_{ij}^U y_{jk} - y_{jk}^L z_{ij}^U, \quad \forall i, j, k \\ & \theta_{ijk} \le y_{jk}^U z_{ij} + z_{ij}^L y_{jk} - y_{jk}^U z_{ij}^L, \quad \forall i, j, k \\ & \sum_{i=1}^{I}\sum_{j=1}^{J} c_s d_{ij} x_{ij} + \sum_{j=1}^{J}\sum_{k=1}^{K} c_{cw,jk} y_{jk} \le B \\ & \sum_{k=1}^{K} y_{jk} = 1, \quad \forall j \\ & \sum_{i=1}^{I} z_{ij} \le \sum_{k=1}^{K} Q_{jk} y_{jk}, \quad \forall j \\ & \sum_{j=1}^{J} z_{ij} = F_i, \quad \forall i \\ & z_{ij} \le F_i x_{ij}, \quad \forall i, j \\ & x_{ij}, y_{jk} \in \{0,1\}, z_{ij} \ge 0, \quad \forall i, j, k. \end{aligned} \tag{14}$$

2. Conclusions

Appendix

References

McCormick, G. P. (1976), "Computability of global solutions to factorable nonconvex programs: Part IConvex underestimating problems," *Mathematical programming*, 10, 147–175.