A2 Mechanics
Paper 4

Week 1

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How was your summer?

PART1

第一部分

Course Overview

Syllabus Overview

Content overview

Content section	Assessment component	Topics included
4 Mechanics	Paper 4	4.1 Forces and equilibrium 4.2 Kinematics of motion in a straight line 4.3 Momentum 4.4 Newton's laws of motion 4.5 Energy, work and power

List of formulae and statistical tables (MF19)

Structure

There are six components that can be combined in specific ways (please see below for details):

Paper 1: Pure Mathematics 1

Paper 2: Pure Mathematics 2

Paper 3: Pure Mathematics 3

Paper 4: Mechanics

Paper 5: Probability & Statistics 1

Paper 6: Probability & Statistics 2

All AS Level candidates take two written papers.

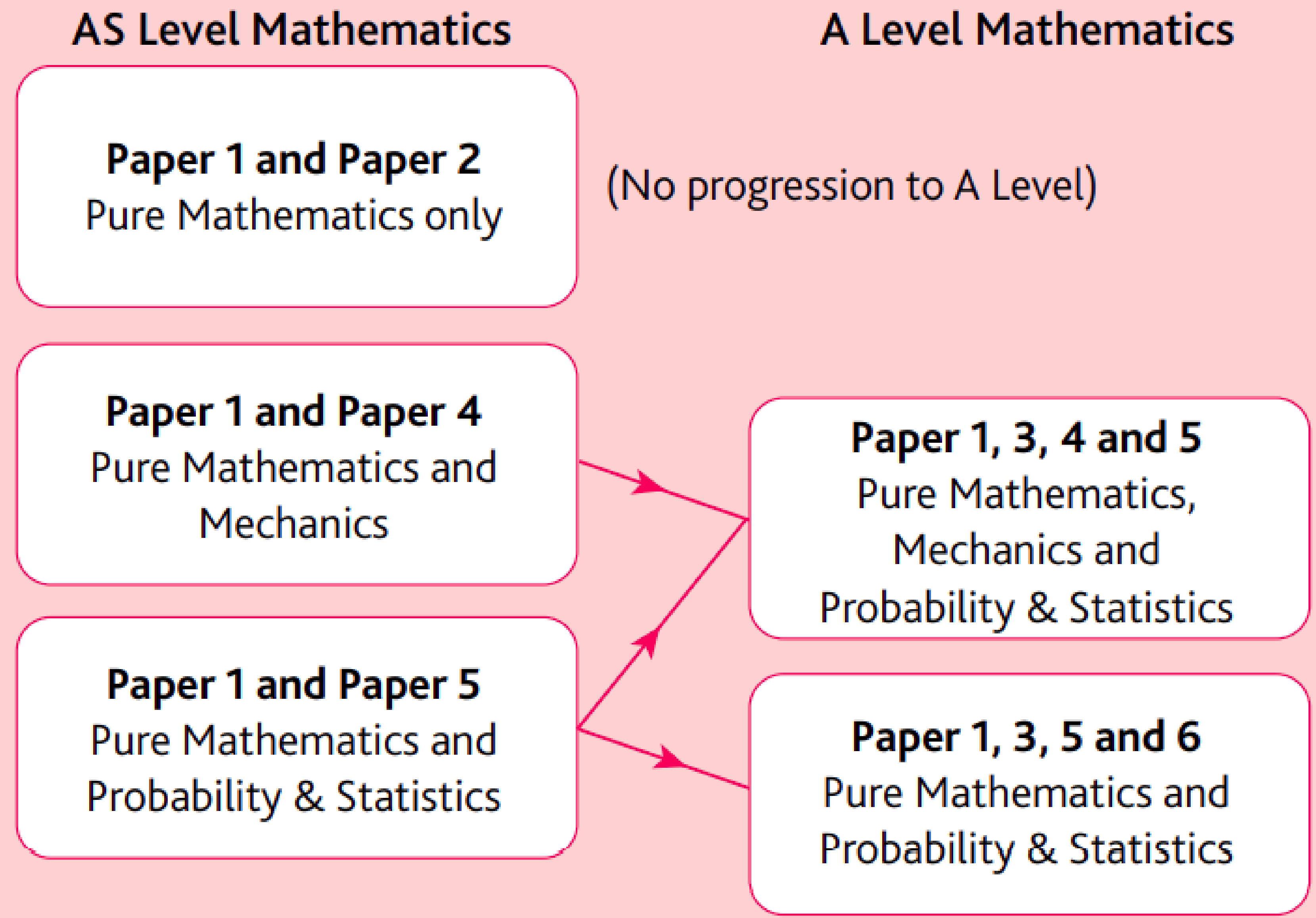
All A Level candidates take four written papers.

A Level Mathematics

The Cambridge International A Level Mathematics qualification offers two different options:

- Pure Mathematics, Mechanics and Probability & Statistics (Papers 1, 3, 4 and 5) **or**
- Pure Mathematics and Probability & Statistics (Papers 1, 3, 5 and 6).

Structure of AS Level and A Level Mathematics



Assessment overview

Paper 4

Mechanics 1 hour 15 minutes

50 marks

6 to 8 structured questions based on the Mechanics subject content

Written examination

Externally assessed

40% of the AS Level

20% of the A Level

Offered as part of AS Level or A Level

Cambridge International AS Level candidate grades March 2022

Cumulative world totals grades a - e

These figures may differ from those of March 2021 series because the population of candidates making entries was different. The awarding standard in March 2022 was aligned to that of equivalent qualifications taken in England in June 2021.

	% achieving a	% achieving b or above	% achieving c or above	% achieving d or above	% achieving e or above	% candidates ungraded
Accounting	31.9	44.3	56.8	66.7	80.9	19.1
Biology	34.2	51.1	66.1	80.1	89.9	10.1
Business	27.8	44.0	57.5	70.7	79.7	20.3
Chemistry	32.7	46.4	63.1	75.9	88.4	11.6
Economics	29.9	44.0	58.7	68.8	80.4	19.6
English Language	20.9	33.7	48.2	65.0	77.5	22.5
General Paper	23.7	46.8	72.5	87.3	96.5	3.5
Information Technology	19.4	30.2	37.9	55.6	63.4	36.6
Mathematics	32.2	47.4	63.4	76.0	86.0	14.0
Physics	35.7	53.3	68.6	82.7	92.3	7.7
Psychology	37.4	52.2	66.2	76.3	86.0	14.0

4 Mechanics (for Paper 4)

Questions set will be mainly numerical, and will aim to test mechanical principles without involving difficult algebra or trigonometry. However, candidates should be familiar in particular with the following trigonometrical results:

$$\sin(90^\circ - \theta) \equiv \cos\theta,\ \cos(90^\circ - \theta) \equiv \sin\theta,\ \tan\theta \equiv \frac{\sin\theta}{\cos\theta},\ \sin^2\theta + \cos^2\theta \equiv 1.$$

Knowledge of algebraic methods from the content for Paper 1: Pure Mathematics 1 is assumed.

This content list refers to the equilibrium or motion of a 'particle'. Examination questions may involve extended bodies in a 'realistic' context, but these extended bodies should be treated as particles, so any force acting on them is modelled as acting at a single point.

Vector notation will not be used in the question papers.

4.1 Forces and equilibrium

Candidates should be able to:	Notes and examples
• identify the forces acting in a given situation	e.g. by drawing a force diagram.
• understand the vector nature of force, and find and use components and resultants	Calculations are always required, not approximate solutions by scale drawing.
• use the principle that, when a particle is in equilibrium, the vector sum of the forces acting is zero, or equivalently, that the sum of the components in any direction is zero	Solutions by resolving are usually expected, but equivalent methods (e.g. triangle of forces, Lami's Theorem, where suitable) are also acceptable; these other methods are not required knowledge, and will not be referred to in questions.
• understand that a contact force between two surfaces can be represented by two components, the normal component and the frictional component	
• use the model of a 'smooth' contact, and understand the limitations of this model	
• understand the concepts of limiting friction and limiting equilibrium, recall the definition of coefficient of friction, and use the relationship $F = \mu R$ or $F \leqslant \mu R$, as appropriate	Terminology such as 'about to slip' may be used to mean 'in limiting equilibrium' in questions.
• use Newton's third law.	e.g. the force exerted by a particle on the ground is equal and opposite to the force exerted by the ground on the particle.

4.2 Kinematics of motion in a straight line

Candidates should be able to:	Notes and examples
• understand the concepts of distance and speed as scalar quantities, and of displacement, velocity and acceleration as vector quantities	Restricted to motion in one dimension only. The term 'deceleration' may sometimes be used in the context of decreasing speed.
• sketch and interpret displacement–time graphs and velocity–time graphs, and in particular appreciate that – the area under a velocity–time graph represents displacement, – the gradient of a displacement–time graph represents velocity, – the gradient of a velocity–time graph represents acceleration	
• use differentiation and integration with respect to time to solve simple problems concerning displacement, velocity and acceleration	Calculus required is restricted to techniques from the content for Paper 1: Pure Mathematics 1.
• use appropriate formulae for motion with constant acceleration in a straight line.	Questions may involve setting up more than one equation, using information about the motion of different particles.

4.3 Momentum

Candidates should be able to:	Notes and examples
• use the definition of linear momentum and show understanding of its vector nature	For motion in one dimension only.
• use conservation of linear momentum to solve problems that may be modelled as the direct impact of two bodies.	Including direct impact of two bodies where the bodies coalesce on impact. Knowledge of impulse and the coefficient of restitution is not required.

4.4 Newton's laws of motion

Candidates should be able to:	Notes and examples
• apply Newton's laws of motion to the linear motion of a particle of constant mass moving under the action of constant forces, which may include friction, tension in an inextensible string and thrust in a connecting rod	If any other forces resisting motion are to be considered (e.g. air resistance) this will be indicated in the question.
• use the relationship between mass and weight	$W = mg$. In this component, questions are mainly numerical, and use of the approximate numerical value 10 ($\mathrm{m\,s^{-2}}$) for g is expected.
• solve simple problems which may be modelled as the motion of a particle moving vertically or on an inclined plane with constant acceleration	Including, for example, motion of a particle on a rough plane where the acceleration while moving up the plane is different from the acceleration while moving down the plane.
• solve simple problems which may be modelled as the motion of connected particles.	e.g. particles connected by a light inextensible string passing over a smooth pulley, or a car towing a trailer by means of either a light rope or a light rigid tow-bar.

4.5 Energy, work and power

Candidates should be able to:	Notes and examples
• understand the concept of the work done by a force, and calculate the work done by a constant force when its point of application undergoes a displacement not necessarily parallel to the force	$W = Fd\cos\theta$; Use of the scalar product is not required.
• understand the concepts of gravitational potential energy and kinetic energy, and use appropriate formulae	
• understand and use the relationship between the change in energy of a system and the work done by the external forces, and use in appropriate cases the principle of conservation of energy	Including cases where the motion may not be linear (e.g. a child on a smooth curved 'slide'), where only overall energy changes need to be considered.
• use the definition of power as the rate at which a force does work, and use the relationship between power, force and velocity for a force acting in the direction of motion	Including calculation of (average) power as $\frac{\text{Work done}}{\text{Time taken}}$. $P = Fv$.
• solve problems involving, for example, the instantaneous acceleration of a car moving on a hill against a resistance.	

A Level Mechanics

1. Motion in a straight line
2. The constant acceleration formulae
3. Forces and Newton's laws of motion
4. Applying Newton's second law along a line
5. Vectors
6. Forces in equilibrium and resultant forces
7. General motion in a staright line
8. A model for friction
9. Energy, work and power
10. Momentum

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1. Velocity and acceleration
2. Force and motion in one dimension
3. Forces in two dimensions
4. Friction
5. Connected particles
6. General motion in a straight line
7. Momentum
8. Work and energy
9. The work-energy principle and power

List of formulae and statistical tables (MF19)

MECHANICS

Uniformly accelerated motion

$$v = u + at\,, \qquad s = \tfrac{1}{2}(u+v)t\,, \qquad s = ut + \tfrac{1}{2}at^2\,, \qquad v^2 = u^2 + 2as$$

FURTHER MECHANICS

Motion of a projectile

Equation of trajectory is:

$$y = x\tan\theta - \frac{gx^2}{2V^2\cos^2\theta}$$

Elastic strings and springs

$$T = \frac{\lambda x}{l}\,, \qquad E = \frac{\lambda x^2}{2l}$$

Motion in a circle

For uniform circular motion, the acceleration is directed towards the centre and has magnitude

$$\omega^2 r \qquad \text{or} \qquad \frac{v^2}{r}$$

Centres of mass of uniform bodies

Triangular lamina: $\frac{2}{3}$ along median from vertex

Solid hemisphere of radius r: $\frac{3}{8}r$ from centre

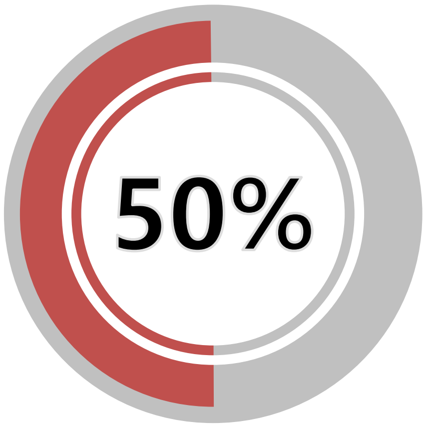
Hemispherical shell of radius r: $\frac{1}{2}r$ from centre

Circular arc of radius r and angle 2α: $\dfrac{r\sin\alpha}{\alpha}$ from centre

Circular sector of radius r and angle 2α: $\dfrac{2r\sin\alpha}{3\alpha}$ from centre

Solid cone or pyramid of height h: $\frac{3}{4}h$ from vertex

Grades



Test scores

Daily performance

- Attendance
- Participation
- Homework
- Quiz

PART2

第二部分

Chapter 1
Velocity and acceleration

Chapter 1
Velocity and acceleration

In this chapter you will learn how to:

- work with scalar and vector quantities for distance and speed
- use equations of constant acceleration
- sketch and read displacement–time graphs and velocity–time graphs
- solve problems with multiple stages of motion.

PREREQUISITE KNOWLEDGE

Where it comes from	What you should be able to do	Check your skills
IGCSE® / O Level Mathematics	Solve quadratics by factorising or using the quadratic formula.	**1** Solve the following equations. **a** $x^2 - 2x - 15 = 0$ **b** $2x^2 + x - 3 = 0$ **c** $3x^2 - 5x - 7 = 0$
IGCSE / O Level Mathematics	Solve linear simultaneous equations.	**2** Solve the following pairs of simultaneous equations. **a** $2x + 3y = 8$ and $5x - 2y = 1$ **b** $3x + 2y = 9$ and $y = 4x - 1$

What is Mechanics about?

How far should the driver of a car stay behind another car to be able to stop safely in an emergency? How long should the fuse on a firework be so the firework goes off at the highest point? How quickly should you roll a ball so it stops as near as possible to a target? How strong does a building have to be to survive a hurricane? Mechanics is the study of questions such as these. By modelling situations mathematically and making suitable assumptions you can find answers to these questions.

In this chapter, you will study the motion of objects and learn how to work out where an object is and how it is moving at different times. This area of Mechanics is known as 'dynamics'. Solving problems with objects that do not move is called 'statics'; you will study this later in the course.

Vectors and Scalars

Example

Mass

Ammount of matter

Scalar: 150 Kg

Weight (Force)

Attraction of an ammount of matter to the Earth

Vector: 1500 N

1.1 Displacement and velocity

Distance: scalar; measure the total length of path travelled
Displacement: vector; gives the location of an object relative to a fixed reference point or *origin*.

- to define displacement you need to first define positive direction.

KEY POINT 1.1

Displacement is a measure of location from a fixed origin or starting point. It is a vector and so has both magnitude and direction. If you take displacement in a given direction to be positive, then displacement in the opposite direction is negative.

Since you will be working in only one dimension, you will often refer to the displacement as just a number, with positive meaning a displacement in one direction from the origin and negative meaning a displacement in the other direction. Sometimes the direction and origin will be stated in the problem. In other cases, you will need to choose these yourself. In many cases the origin will simply be the starting position of an object and the positive direction will be the direction the object is moving initially.

1.1 Displacement and velocity

Speed: scalar

Velocity: vector

- define positive direction.

For an object moving at constant speed, if you know the distance travelled in a given time you can work out the speed of the object.

KEY POINT 1.2

For an object moving at constant speed:

$$\text{speed} = \frac{\text{distance covered}}{\text{time taken}}$$

This is valid only for objects moving at constant speed. For objects moving at non-constant speed you can consider the average speed.

KEY POINT 1.3

$$\text{average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

1.1 Displacement and velocity

Velocity measures how quickly the displacement of an object changes. You can write an equation similar to the one for speed.

KEY POINT 1.4

For an object moving at constant velocity:

$$\text{velocity} = \frac{\text{change in displacement}}{\text{time taken}}$$

As with speed, for objects moving at non-constant velocity you can consider the average velocity.

KEY POINT 1.5

$$\text{average velocity} = \frac{\text{net change in displacement}}{\text{total time taken}}$$

TIP

We use vertical lines to indicate magnitude of a vector.

So, $\text{speed} = |\text{velocity}|$

1.1 Displacement and velocity

We can rearrange the equation for velocity to deduce that for an object moving at constant velocity v for time t, the change in displacement s (in the same direction as the velocity) is given by:

$$s = vt$$

The standard units used for distance and displacement are metres (m) and for time are seconds (s). Therefore, the units for speed and velocity are metres per second (usually written in mathematics and science as $\mathrm{m\,s^{-1}}$, although you may also come across the notation m/s). These units are those specified by the *Système Internationale* (SI), which defines the system of units used by scientists all over the world. Other commonly used units for speed include kilometres per hour (km/h) and miles per hour (mph).

1.1 Displacement and velocity

A car travels 9 km in 15 minutes at constant speed. Find its speed in $m\,s^{-1}$.

$9\,km = 9000\,m$ and

$15\text{ minutes} = 900\,s$

Convert to units required for the answer, which are SI units.

$s = vt$

so $9000 = 900v$

$v = 10\,m\,s^{-1}$

Substitute into the equation for displacement and solve.

1.1 Displacement and velocity

A cyclist travels at $5\,\text{m}\,\text{s}^{-1}$ for 30 s then turns back, travelling at $3\,\text{m}\,\text{s}^{-1}$ for 10 s. Find her displacement in the original direction of motion from her starting position.

$s = vt$

So $s_1 = 5 \times 30$

$= 150$

and $s_2 = -3 \times 10$

$= -30$

Separate the two stages of the journey.

Remember travelling back means a negative velocity and a negative displacement.

So the total displacement is

$s = 150 + (-30)$

$= 120\,\text{m}$

1.1 Displacement and velocity

A cyclist spends some of his journey going downhill at $15\,\mathrm{m\,s^{-1}}$ and the rest of the time going uphill at $5\,\mathrm{m\,s^{-1}}$. In 1 minute he travels 540 m. Find how long he spent going downhill.

Let t be the amount of time spent going downhill.

Define the variable.

Then $60 - t$ is the amount of time spent going uphill.

Write an expression for the time spent travelling uphill.

$$\begin{aligned} \text{Total distance} = 15t + 5(60 - t) &= 540 \\ 15t + 300 - 5t &= 540 \\ 10t &= 240 \\ t &= 24\,\mathrm{s} \end{aligned}$$

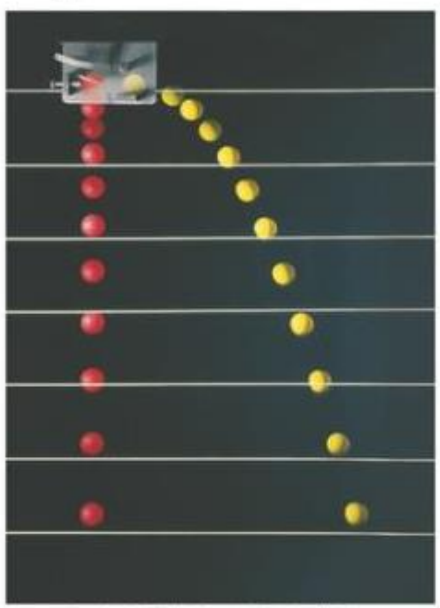
Vectors & Scalars

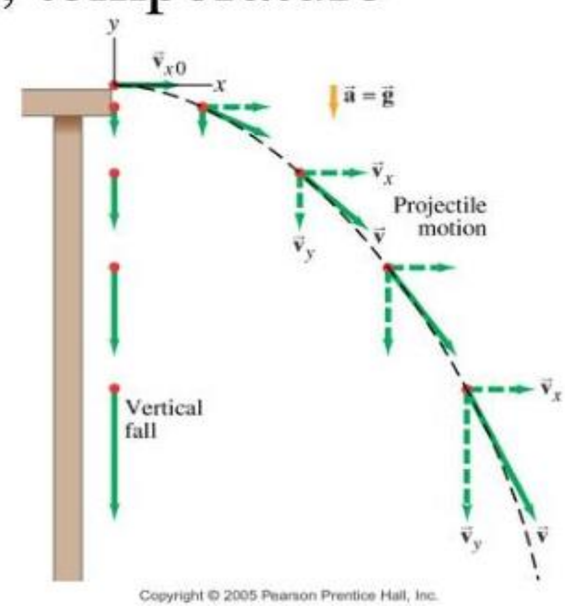
•A **vector** has magnitude as well as direction.

•Examples: displacement, velocity, acceleration, force, momentum

•A **scalar** has only magnitude

•Examples: time, mass, temperature

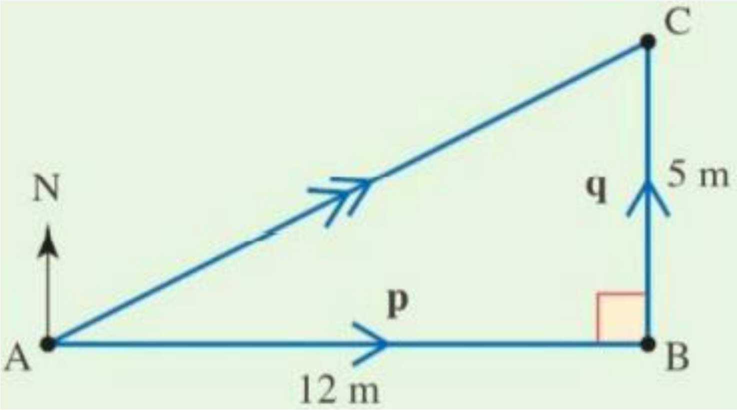




1.1 Displacement and velocity

› If you walk 12 m east and then 5 m north, how far and in what direction will you be from your starting point?

You write the vectors as $\overrightarrow{AB}$ and $\overrightarrow{BC}$ The arrow above the letters is very important as it indicates the direction of the vector. $\overrightarrow{AB}$ means from A to B. $\overrightarrow{AB}$ and $\overrightarrow{BC}$ are examples of **displacement vectors**. Their lengths represent the magnitude of the displacements.



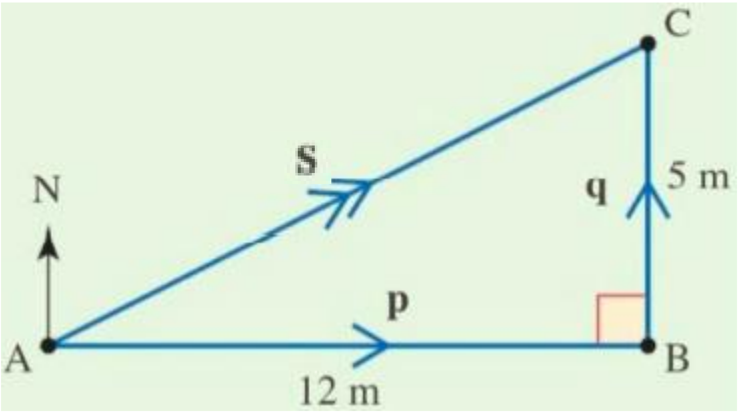
The magnitudes of **p** and **q** are then shown as |**p**| and |**q**| or *p* and *q* (in italics). These are scalar quantities.

$p = 12$

$q = 5$

$s = \sqrt{12^2 + 5^2}$

1.1 Displacement and velocity



The combined effect of the two displacements $\overrightarrow{AB}$ (= **p**) and $\overrightarrow{BC}$ (= **q**) is $\overrightarrow{AC}$ and this is called the **resultant vector**. It is marked with two arrows to distinguish it from **p** and **q**. The process of combining vectors in this way is called **vector addition**. You can write $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ or $\mathbf{p} + \mathbf{q} = \mathbf{s}$.

You can use Pythagoras' theorem and trigonometry to calculate the resultant.

In triangle ABC $\quad AC = \sqrt{12^2 + 5^2} = 13$

and $\quad \tan\alpha = \frac{12}{5}$

$\alpha = 67°$ (to the nearest degree)

The distance from the starting point is 13 m and the direction is 067°.

1.1 Displacement and velocity

- A bird is caught in a wind blowing east at $12\,\mathrm{m\,s^{-1}}$ and flies so that its speed would be $5\,\mathrm{m\,s^{-1}}$ north in still air. What is its actual velocity?
- A sledge is being pulled by two children with forces of 12 N east and 5 N north. What single force would have the same effect?

Homework: EXERCISE 1A

4 The speed of light is $3.00 \times 10^8\ \mathrm{m\,s^{-1}}$ to 3 significant figures. The average distance between the Earth and the Sun is 150 million km to 3 significant figures. Find how long it takes for light from the Sun to reach the Earth on average. Give the answer in minutes and seconds.

6 A runner runs at $5\ \mathrm{m\,s^{-1}}$ for $7\ \mathrm{s}$ before increasing the pace to $7\ \mathrm{m\,s^{-1}}$ for the next $13\ \mathrm{s}$.

a Find her average speed.

b What assumptions have been made to answer the question?

7 A remote control car travels forwards at $6\ \mathrm{m\,s^{-1}}$ in Drive and backwards at $3\ \mathrm{m\,s^{-1}}$ in Reverse. The car travels for $10\ \mathrm{s}$ in Drive before travelling for $5\ \mathrm{s}$ in Reverse.

a Find its displacement from its starting point.

b Find its average velocity in the direction in which it started driving forwards.

c Find its average speed.

11 Two cars are racing over the same distance. They start at the same time, but one finishes $8\ \mathrm{s}$ before the other. The faster one averaged $45\ \mathrm{m\,s^{-1}}$ and the slower one averaged $44\ \mathrm{m\,s^{-1}}$. Find the length of the race.

12 Two air hockey pucks are $2\ \mathrm{m}$ apart. One is struck and moves directly towards the other at $1.3\ \mathrm{m\,s^{-1}}$. The other is struck $0.2\ \mathrm{s}$ later and moves directly towards the first at $1.7\ \mathrm{m\,s^{-1}}$. Find how far the first puck has moved when the collision occurs and how long it has been moving for.

1.2 Acceleration

Velocity is not the only measure of the motion of an object. It is useful to know if, and how, the velocity is changing. We use **acceleration** to measure how quickly velocity is changing.

KEY POINT 1.6

For an object moving at constant acceleration,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{time taken}}$$

If an object has constant acceleration a, initial velocity u and it reaches final velocity v in time t, then

$$a = \frac{v - u}{t}$$

where u, v and a are all measured in the same direction.

EXPLORE 1.2

If the initial velocity is negative, what effect would a positive acceleration have on the car? Would it be moving more quickly or less quickly?

What effect would a negative acceleration have on the car in this situation? Would it be moving more quickly or less quickly?

When the acceleration is constant, the average velocity is simply the average of the initial and final velocities, which is given by the formula $\frac{1}{2}(u+v)$. This can be used to find displacements using the equation for average velocity from Key Point 1.5.

KEY POINT 1.7

If an object has constant acceleration a, initial velocity u and it reaches final velocity v in time t, then the displacement s is given by

$$s = \frac{1}{2}(u+v)t$$

A parachutist falls from rest to $49\,\mathrm{m\,s^{-1}}$ over $5\,\mathrm{s}$. Find her acceleration.

$$\begin{aligned} a &= \frac{v-u}{t} \\ &= \frac{49-0}{5} \\ &= 9.8\,\mathrm{m\,s^{-2}} \end{aligned}$$

A tractor accelerates from $5\,\mathrm{m\,s^{-1}}$ to $9\,\mathrm{m\,s^{-1}}$ at $0.5\,\mathrm{m\,s^{-2}}$. Find the distance covered by the tractor over this time.

$$a = \frac{v-u}{t}$$

$$\text{So } 0.5 = \frac{9-5}{t}$$

$$0.5t = 4$$

$$t = 8\,\mathrm{s}$$

Substitute into $a = \dfrac{v-u}{t}$ first to find t.

$$s = \frac{1}{2}(u+v)t$$

$$= \frac{1}{2}(5+9)\times 8$$

$$= 56\,\mathrm{m}$$

Substitute into $s = \dfrac{1}{2}(u+v)t$ to find s.

EXERCISE 1B

1 A car accelerates from $4\,\text{m s}^{-1}$ to $10\,\text{m s}^{-1}$ in $3\,\text{s}$ at constant acceleration. Find its acceleration.

2 A car accelerates from rest to $10\,\text{m s}^{-1}$ in $4\,\text{s}$ at constant acceleration. Find its acceleration.

3 A car accelerates from $3\,\text{m s}^{-1}$ at an acceleration of $6\,\text{m s}^{-2}$. Find the time taken to reach $12\,\text{m s}^{-1}$.

4 An aeroplane accelerates at a constant rate of $3\,\text{m s}^{-2}$ for $5\,\text{s}$ from an initial velocity of $4\,\text{m s}^{-1}$. Find its final velocity.

5 A speedboat accelerates at a constant rate of $1.5\,\text{m s}^{-2}$ for $4\,\text{s}$, reaching a final velocity of $9\,\text{m s}^{-1}$. Find its initial velocity.

6 A car decelerates at a constant rate of $2\,\text{m s}^{-2}$ for $3\,\text{s}$, finishing at a velocity of $8\,\text{m s}^{-1}$. Find its initial velocity.

7 A car accelerates from an initial velocity of $4\,\text{m s}^{-1}$ to a final velocity of $8\,\text{m s}^{-1}$ at a constant rate of $0.5\,\text{m s}^{-2}$. Find the car's displacement in that time.

Homework 1B:

9 A wagon is accelerating down a hill at constant acceleration. It took 1 s more to accelerate from a velocity of $1\,m\,s^{-1}$ to a velocity of $5\,m\,s^{-1}$ than it took to accelerate from rest to a velocity of $1\,m\,s^{-1}$. Find the acceleration.

10 A driver sees a turning 100 m ahead. She lets her car slow at constant deceleration of $0.4\,m\,s^{-2}$ and arrives at the turning 10 s later. Find the velocity she is travelling at when she reaches the turning.

11 A cyclist is travelling at a velocity of $10\,m\,s^{-1}$ when he reaches the top of a slope, which is 80 m long. There is a bend at the bottom of the slope, which it would be dangerous to go round any faster than $11\,m\,s^{-1}$. Because of gravity, if he did not pedal or brake he would accelerate down the slope at $0.1\,m\,s^{-2}$. To go as fast as possible but still reach the bottom at a safe speed should the cyclist brake, do nothing or pedal?

1.3 Equations of constant acceleration

There are five equations relating the five variables s, u, v, a and t. Each equation relates four of the five variables.

KEY POINT 1.8

For an object travelling with constant acceleration a, for time t, with initial velocity u, final velocity v and change of displacement s, we have

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

These equations are often referred to as the suvat equations.

1.3 Equations of constant acceleration

a A go-kart travels down a slope of length 70 m. It is given a push and starts moving at an initial velocity of $3\,\text{m}\,\text{s}^{-1}$ and accelerates at a constant rate of $2\,\text{m}\,\text{s}^{-2}$. Find its velocity at the bottom of the slope.

b Find the time taken for the go-kart to reach the bottom of the slope.

a The time, t, is unknown.

The final velocity, v, is unknown.

$s = 70$

$u = 3$

$a = 2$

$$v^2 = u^2 + 2as$$
$$= 3^2 + 2 \times 2 \times 70$$
$$= 289$$
$$v = \pm 17$$

We know that $v > 3$.

$v = 17\,\text{m}\,\text{s}^{-1}$

b
$$s = ut + \frac{1}{2}at^2$$
$$70 = 3t + \frac{1}{2} \times 2t^2$$
$$t^2 + 3t - 70 = 0$$
$$(t + 10)(t - 7) = 0$$

$t = -10$ or $t = 7$

We know that $t > 0$ so $t = 7$ s.

1.3 Equations of constant acceleration

A trolley has a constant acceleration. After 2 s it has travelled 8 m and after another 2 s it has travelled a further 20 m. Find its acceleration.

Let the initial speed be v_1.

Let the speed after 2 s be v_2. $s = 8$ $t = 2$

Let the speed after 4 s be v_3. $u = v_1$ $v = v_2$

Acceleration, a, is unknown.

$$s = vt - \frac{1}{2}at^2$$

$$8 = 2v_2 - \frac{1}{2} \times a \times 2^2$$

$$2v_2 - 2a = 8$$

$$s = ut - \frac{1}{2}at^2$$

$$20 = 2v_2 + \frac{1}{2} \times a \times 2^2$$

$$2v_2 + 2a = 20$$

$$4v_2 = 28$$

$$v_2 = 7\text{ m s}^{-1}$$

So $a = 3\text{ m s}^{-2}$.

or

$$s = ut + \frac{1}{2}at^2$$

$$8 = 2v_1 + \frac{1}{2} \times a \times 2^2$$

$$28 = 4v_1 + \frac{1}{2} \times a \times 4^2$$

giving $v_1 = 1\text{ m s}^{-1}$.

So $a = 3\text{ m s}^{-2}$.

1.3 Equations of constant acceleration

1 For each part, assuming constant acceleration, write down the equation relating the four variables in the question and use it to find the missing variable.

a Find s when $a = 3\,\text{m}\,\text{s}^{-2}$, $u = 2\,\text{m}\,\text{s}^{-1}$ and $t = 4\,\text{s}$.

b Find s when $a = 2\,\text{m}\,\text{s}^{-2}$, $v = 17\,\text{m}\,\text{s}^{-1}$ and $t = 8\,\text{s}$.

c Find a when $s = 40\,\text{m}$, $u = 3\,\text{m}\,\text{s}^{-1}$ and $t = 5\,\text{s}$.

d Find a when $s = 28\,\text{m}$, $v = 13\,\text{m}\,\text{s}^{-1}$ and $t = 4\,\text{s}$.

e Find a when $s = 24\,\text{m}$, $u = 2\,\text{m}\,\text{s}^{-1}$ and $v = 14\,\text{m}\,\text{s}^{-1}$.

f Find u when $s = 45\,\text{m}$, $a = 1.5\,\text{m}\,\text{s}^{-2}$ and $t = 6\,\text{s}$.

g Find v when $s = 24\,\text{m}$, $a = -2.5\,\text{m}\,\text{s}^{-2}$ and $t = 4\,\text{s}$.

h Find s when $a = 0.75\,\text{m}\,\text{s}^{-2}$, $u = 2\,\text{m}\,\text{s}^{-1}$ and $v = 5\,\text{m}\,\text{s}^{-1}$.

Homework 1C:

6 A car is travelling at a velocity of $20\,\mathrm{m\,s^{-1}}$ when the driver sees the traffic lights ahead change to red. He decelerates at a constant rate of $4\,\mathrm{m\,s^{-2}}$ and comes to a stop at the lights. Find how far away from the lights the driver started braking.

7 An aeroplane accelerates at a constant rate along a runway from rest until taking off at a velocity of $60\,\mathrm{m\,s^{-1}}$. The runway is 400 m long. Find the acceleration of the aeroplane.

11 In a game of curling, competitors slide stones over the ice at a target 38 m away. A stone is released directly towards the target with velocity $4.8\,\mathrm{m\,s^{-1}}$ and decelerates at a constant rate of $0.3\,\mathrm{m\,s^{-2}}$. Find how far from the target the stone comes to rest.

12 A golf ball is struck 10 m from a hole and is rolling towards the hole. It has an initial velocity of $2.4\,\mathrm{m\,s^{-1}}$ when struck and decelerates at a constant rate of $0.3\,\mathrm{m\,s^{-2}}$. Does the ball reach the hole?

13 A driverless car registers that the traffic lights change to amber 40 m ahead. The amber light is a 2 s warning before turning red. The car is travelling at $17\,\mathrm{m\,s^{-1}}$ and can accelerate at $4\,\mathrm{m\,s^{-2}}$ or brake safely at $8\,\mathrm{m\,s^{-2}}$. What options does the car have?

KEY POINT 1.8

For an object travelling with constant acceleration a, for time t, with initial velocity u, final velocity v and change of displacement s, we have

$$v = u + at$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

These equations are often referred to as the suvat equations.

TIP

In general, these equations are only valid if the acceleration is constant.

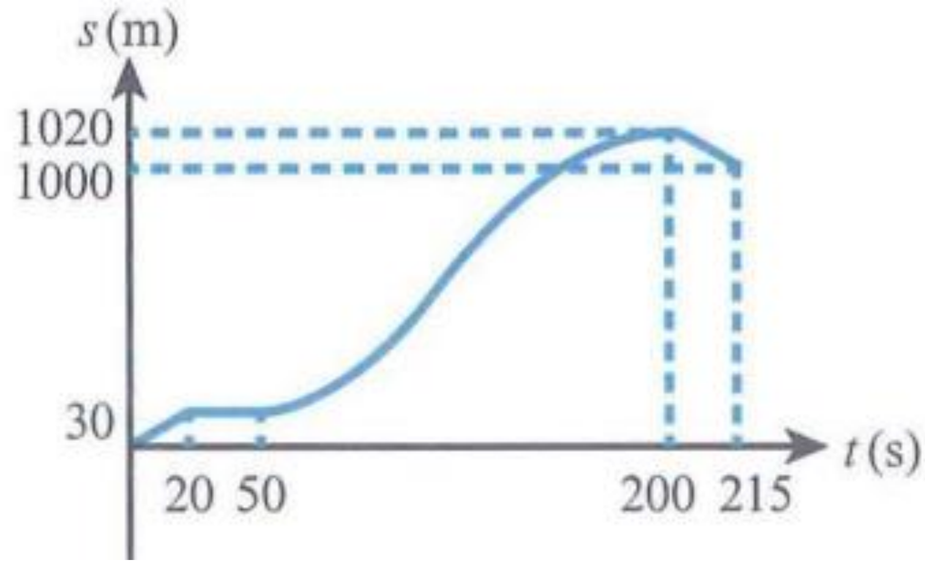
The first two equations in Key Point 1.8 are $v = u + at$ and $s = \frac{1}{2}(u + v)t$. You can use these to derive the other equations.

a By substituting for v in the second equation, derive $s = ut + \frac{1}{2}at^2$.

b Derive the remaining two equations, $s = vt - \frac{1}{2}at^2$ and $v^2 = u^2 + 2as$, from the original two equations.

1.4 Displacement–time graphs and multi-stage problems

Imagine the following scenario. A girl is meeting a friend 1 km down a straight road. She takes 20 s to walk 30 m along the road to a bus stop. Then she waits 30 s for a bus, which takes her to a bus stop 20 m past her friend. The bus does not stop to pick anyone else up or drop them off. The journey takes 150 s. The girl takes 15 s to walk the 20 m back to meet her friend.

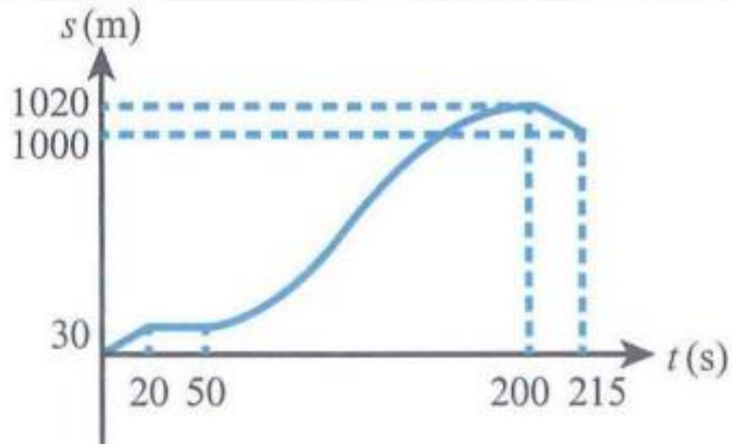


- Always show time on the x-axis and displacement on y-axis
- When graph is horizontal, the displacement is __________.
- The steepness of the line indicates ______________________.
- A straight line on a s-t graph indicates _____________.
- A curved line indicates ____________.
- Speed is the _________ of the gradient; velocity includes direction.
- How do we show negative displacement?

1.4 Displacement–time graphs and multi-stage problems

KEY POINT 1.9

The gradient of a displacement–time graph is equal to the velocity of the object.



When sketching the motion graph,

- Show clearly the shape of the graph – line or curve
- Show key points: intercept on vertical/horizontal axis
- if there’s more than one stage, indicate the time and displacement at the change in motion.

1.4 Displacement–time graphs and multi-stage problems

A cyclist is travelling at a velocity of $15\,\text{m s}^{-1}$ when he passes a junction. He then decelerates at a constant rate of $0.6\,\text{m s}^{-2}$ until coming to rest. A second cyclist travels at a constant velocity of $20\,\text{m s}^{-1}$ and passes the junction 4 s after the first cyclist. Find the time at which the second cyclist passes the first and the displacement from the junction when that happens.

Let t_1 be the time from the first cyclist reaching the junction and t_2 be the time from the second cyclist reaching the junction.

Let s_1 be the displacement of the first cyclist from the junction and s_2 be the displacement of the second cyclist from the junction.

$$s = ut + \frac{1}{2}at^2$$

$$s_1 = 15t_1 + \frac{1}{2} \times (-0.6)t_1^2$$

$$s = vt$$

$$s_2 = 20t_2$$

$$t_2 = t_1 - 4$$

So $s_2 = 20(t_1 - 4)$

$$s_1 = s_2$$

$$15t_1 - 0.3t_1^2 = 20(t_1 - 4)$$

$$3t_1^2 + 50t_1 - 800 = 0$$

$$(3t_1 + 80)(t_1 - 10) = 0$$

$$t_1 = 10, -\frac{80}{3}$$

So $t_1 = 10\,\text{s}$.

$$s_1 = 15 \times 10 + \frac{1}{2} \times (-0.6) \times 10^2$$

$$= 120\,\text{m}$$

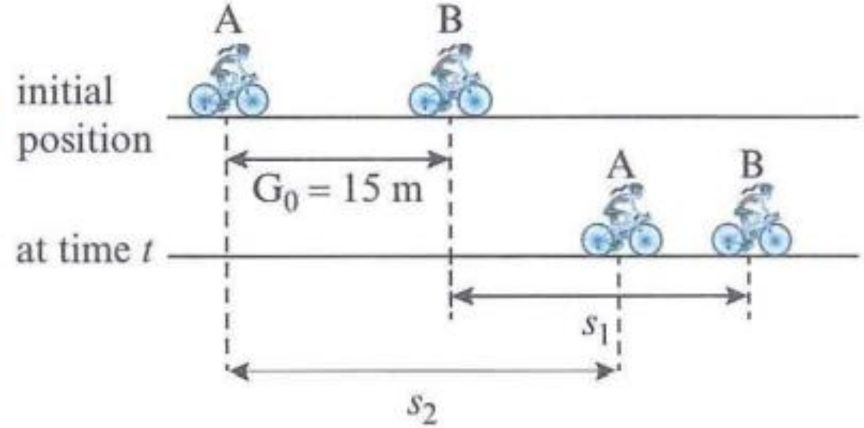
Or $s_2 = 20(10 - 4)$

$$= 120\,\text{m}$$

1.4 Displacement–time graphs and multi-stage problems

Cyclist A is travelling at $16\,m\,s^{-1}$ when she sees cyclist B 15 m ahead travelling at a constant velocity of $10\,m\,s^{-1}$. Cyclist A then slows at $1.5\,m\,s^{-2}$. Find the minimum gap between the cyclists.

Let t be the time measured from when the cyclists are 15 m apart and let the gap between the cyclists at time t be $G(t)$.



$$G(t) = G_0 + s_1 - s_2$$

$$G(t) = 15 + 10t - (16t - 0.75t^2)$$

$$G(t) = 0.75(t-4)^2 + 3$$

Minimum gap is 3 m at 4 s.

Complete the square to find the minimum gap and the time at which it occurs.

Or $16 - 1.5t = 10$

$t = 4$

$G(t) = 15 + 10t - (16t - 0.75t^2) = 3$ m

Alternatively, the closest distance is when the cyclists travel at the same speed because once the cyclist behind slows down the gap will increase again.

1.4 Displacement–time graphs and multi-stage problems

EXERCISE 1D

1 Sketch the displacement–time graphs from the information given. In each case, consider north to be the positive direction and home to be the point from which displacement is measured.

a Bob leaves his home and heads north at a constant speed of $3\,\mathrm{m\,s^{-1}}$ for $10\,\mathrm{s}$.

b Jenny is $30\,\mathrm{m}$ north of home and walks at a constant speed of $1.5\,\mathrm{m\,s^{-1}}$ until reaching home.

c Ryo is sitting still at a point $10\,\mathrm{m}$ south of his home.

d Nina is $300\,\mathrm{m}$ north of her home. She drives south at a constant speed of $10\,\mathrm{m\,s^{-1}}$, passing her home, until she has travelled a total of $500\,\mathrm{m}$.

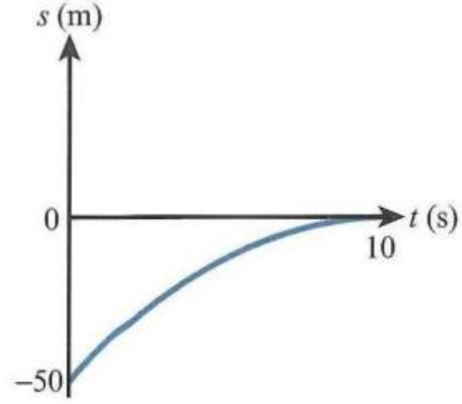
7 Two trains travel on parallel tracks that are $5\,\mathrm{km}$ long. One starts at the southern end, travelling north at a constant speed of $25\,\mathrm{m\,s^{-1}}$. The second train starts at the northern end $40\,\mathrm{s}$ later, travelling south at a constant speed of $15\,\mathrm{m\,s^{-1}}$.

a Sketch the two displacement–time graphs on the same set of axes.

b Find the time for which the first train has been moving and the distance the first train has travelled when the trains pass each other.

Homework 1D:

5 The sketch shows a displacement–time graph of a car slowing down with constant acceleration before coming to rest at a set of traffic lights.



a The equation of the displacement–time graph can be written in the form $s = p(t-q)^2 + r$. Using the two points marked and the fact that the car is stationary at $t = 10$, find p, q and r.

b By comparison with the equation $s = s_0 + ut + \frac{1}{2}at^2$, find the initial speed and acceleration of the car.

6 Two cars drive along the same highway. One car starts at junction 1, travelling north at a constant speed of $30\,\text{m s}^{-1}$. The second car starts at junction 2, which is 3 km north of junction 1, travelling south at a constant speed of $20\,\text{m s}^{-1}$.

a Sketch the two displacement–time graphs on the same set of axes.

b Find the equations of the two displacement–time graphs.

c Solve the equations to find the time at which the cars pass each other and, hence, find the distance from junction 1 at which they pass.

14 Swimmers going down a waterslide 30 m long push themselves off with an initial speed of between $1\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$. They accelerate down with constant acceleration $0.8\,\text{m s}^{-2}$ for the first 20 m before more water is added and the acceleration is $1\,\text{m s}^{-2}$ for the last 10 m of the slide. For safety there must be at least 5 s between swimmers arriving at the bottom of the slide. Find the minimum whole number of seconds between swimmers being allowed to start the slide.