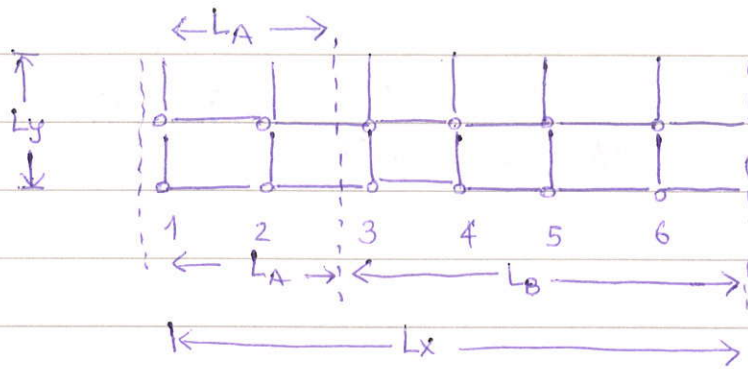1. 🖙 In this document the numerical implementation of Entanglement Entropy for a U(1) quantum link model. In the discussion below, we address the computation in a ladder system with the cut described as shown in fig below. The case for any other cut can be implemented the same manner.

The idea is the change L_A, and calculate the entanglement entropy as a function of L_A.

2. Let us denote by, nD = the total number of gauge invariant basis states in a given sector defined by winding numbers (W_x, W_y). Our discussion will never involve the winding numbers changing in real-time dynamics.

The gauge invariant basis states can then be represented as

$$\{|e\rangle\} = \{|e_k\rangle, \; k = 1, 2, \ldots, nD\} \qquad —(1)$$

A wavefn in this sector is then

$$|\psi\rangle = \sum_{k=1}^{nD} \Psi_k |e_k\rangle \qquad —(2)$$

Now, we have two partitions, $\{|e^{(A)}\rangle\}$ and $\{|e^{(B)}\rangle\}$ are the basis states that span these sections. Note that $\{|e^{(A)}\rangle\}$ and $\{|e^{(B)}\rangle\}$ are gauge invariant at all points in the bulk of the regions A and B respectively.

Only at the boundaries, they represent surface charges.

3. The calculation of $\{|e^A\rangle\}$ and $\{|e^B\rangle\}$ is also straightforward. Take each k in $\{|e_K\rangle\}$, and physically split up the systems into the links and sort them into $\{|e^A\rangle\}$ and $\{|e^B\rangle\}$ depending on whether they belong into the region A or B. Then remove the duplicates and obtain

$$\{|e^A\rangle\} = \{|e_{iA}^A\rangle, \quad iA = 1, \ldots, DA\} \qquad \Big\} — (3)$$
$$\{|e^B\rangle\} = \{|e_{iB}^B\rangle, \quad iB = 1, \ldots, DB\}$$

4. Given an eigenvector $|\psi\rangle$:

$$|\psi\rangle = \sum_K^{nD} \psi_K |e_K\rangle \qquad\qquad — 4.1$$

$$= \sum_{iA}^{DA} \sum_{iB}^{DB} X_{i_A i_B} |e_{iA}^A\rangle \otimes |e_{iB}^B\rangle \qquad — 4.2$$

$$= \sum_{\ell}^{n_X} X_\ell |\tilde{e}_\ell^{(A)}\rangle \otimes |\tilde{e}_\ell^{(B)}\rangle \qquad — 4.3$$

First, patch individual basis vectors, say the i_A from $\{|e^A\rangle\}$ and i_B from the list $\{|e^B\rangle\}$ and obtain the corresponding ~~elemen~~ matrix element $X(iA, iB)$. Clearly X is a rectangular matrix of dimensions ~~(XXXXX)~~. $DA \times DB$.

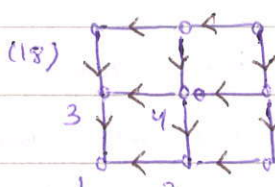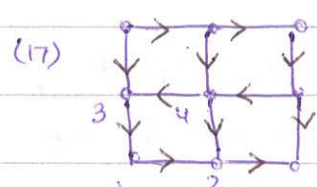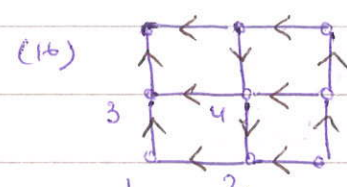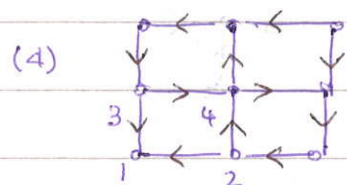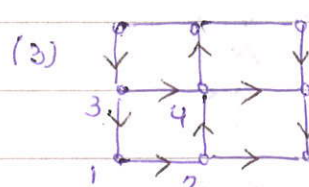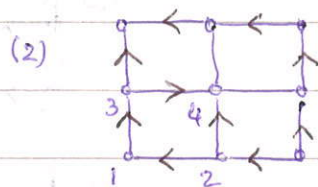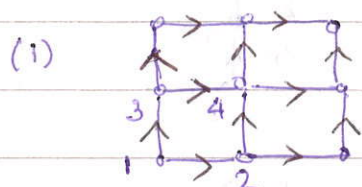From step (4.2) to (4.3), one can implement the Schmidt decomposition, which gives the ~~positive~~ {real and non-negative Schmidt values, X_ℓ, $\ell = 1, \ldots, n_X$, $X = \min\{DA, DB\}$.

The von-Neumann entropy for this subpartion of the state $|\psi\rangle$ is

$$S = -\sum_{i=1}^{X} |X_i|^2 \ln(|X_i|^2) \qquad\qquad — 4.4$$
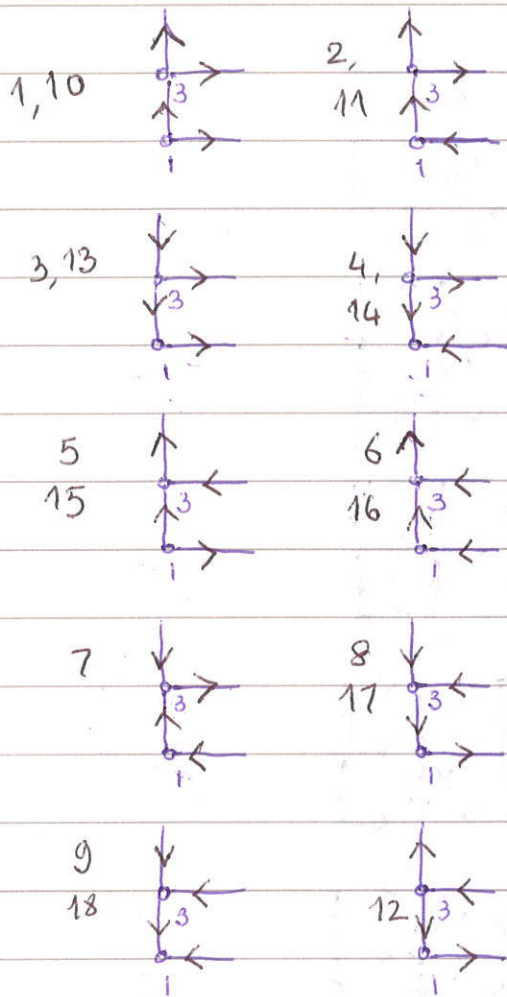
5. As a simple example, consider the case of the 2x2 lattice, which has 18 gauge invariant states.

These are the 18 states:
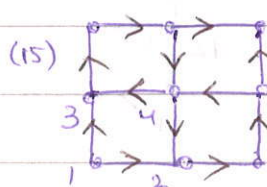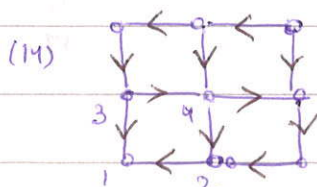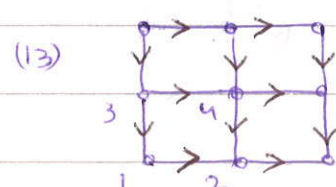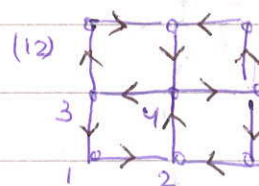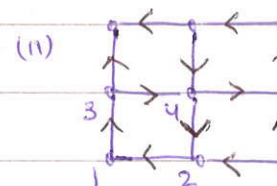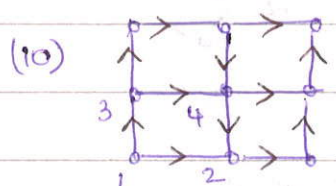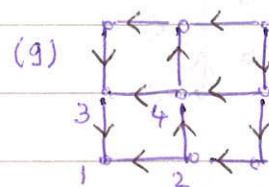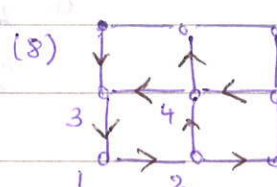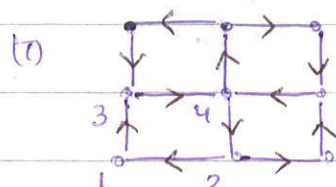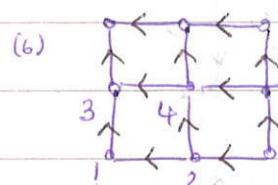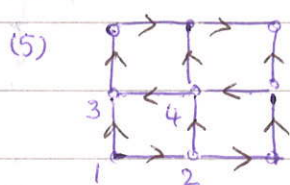
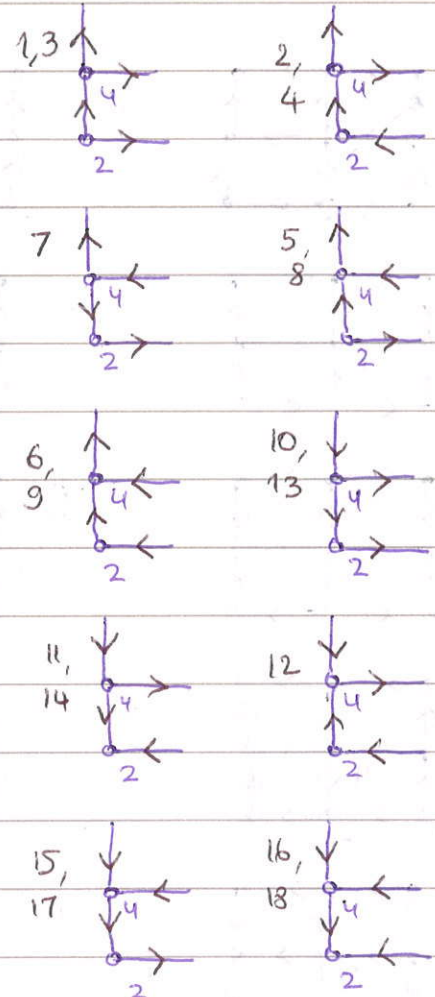On cutting the system ~~troor~~ through $l_A = 1$, we get the
following basis states in A & B:

A: B:

The example here shows how the splitting happens in
practise. The numbers show which of the ~~b~~ full basis
states these set in A & B correspond to.

As is evident, the basis in each partition is much lesser than
$2^4 = 16$, since ~~#~~ Gauss law partially rules out many states
(the info about Gauss law is inherited since the Gauge inv.
states of the full model ~~are~~ cut open to get these states).