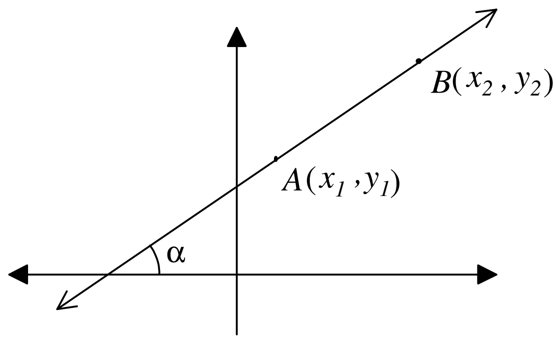
◈ *Inclination of a Line:*

The angle α $(0^\circ \le \alpha < 180^\circ)$ measure anti-clockwise from positive $x-$axis to the straight line l is called *inclination* of a line l.



◈ *Slope or Gradient of Line*

The slope m of the line l is defined by:

$$m = \tan\alpha$$

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be any two distinct points on the line l then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

See proof on book at page: 191

◈ *Note:*

l is horizontal, iff $m = 0$ $(\because \alpha = 0^\circ)$

l is vertical, iff $m = \infty$ i.e. m is not defined. $(\because \alpha = 90^\circ)$

If slope of $AB =$ slope of BC, then the points A, B and C are collinear i.e. lie on the same line.

◈ *Theorem*

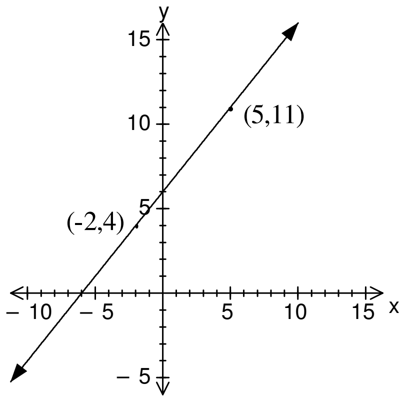
The two lines l_1 and l_2 with respective slopes m_1 and m_2 are

(i) Parallel iff $m_1 = m_2$

(ii) Perpendicular iff $m_1 m_2 = -1$ or $m_1 = -\dfrac{1}{m_2}$

◈ *Question # 1*

(i) $(-2,4)$; $(5,11)$

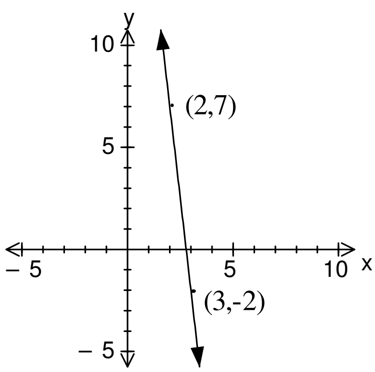


Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11-4}{5+2} = \dfrac{7}{7} = 1$

Since $\tan\alpha = m = 1$

$\Rightarrow \alpha = \tan^{-1}(1) = 45^\circ$

(ii) $(3,-2)$; $(2,7)$



Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7+2}{2-3} = \dfrac{9}{-1} = -9$

Since $\tan\alpha = m = -9$

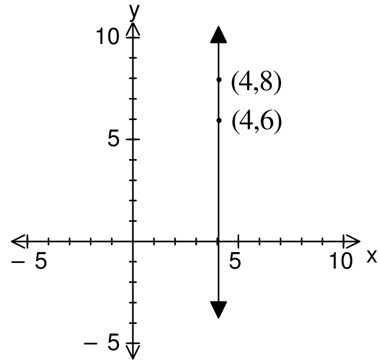
$\Rightarrow -\tan\alpha = 9 \Rightarrow \tan(180 - \alpha) = 9$

$\Rightarrow 180 - \alpha = \tan^{-1}(9)$

$\Rightarrow 180 - \alpha = 83^\circ 40'$

$\Rightarrow \alpha = 180 - 83^\circ 40' = 96^\circ 20'$

(ii) $(4,6)$; $(4,8)$



Slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-6}{4-4} = \dfrac{2}{0} = \infty$

Since $\tan\alpha = m = \infty$

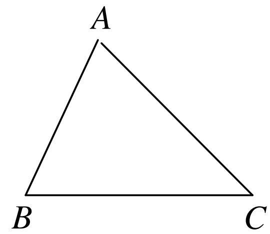
$\Rightarrow \alpha = \tan^{-1}(\infty)$

$= 90^\circ$

◈ *Question # 2*

Since $A(8,6)$, $B(-4,2)$ and $C(-2,-6)$ are vertices of triangle therefore

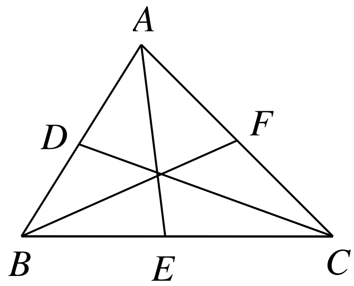
(i) Slope of side $AB = \dfrac{2-6}{-4-8} = \dfrac{-4}{-12} = \dfrac{1}{3}$



Slope of side $BC = \dfrac{-6-2}{-2+4} = \dfrac{-8}{2} = -4$

Slope of side $CA = \dfrac{6+6}{8+2} = \dfrac{12}{10} = \dfrac{6}{5}$

(ii) Let D, E and F are midpoints of sides AB, BC and CA respectively.
Then



Coordinate of $D = \left(\dfrac{8-4}{2}, \dfrac{6+2}{2}\right) = \left(\dfrac{4}{2}, \dfrac{8}{2}\right) = (2,4)$

Coordinate of $E = \left(\dfrac{-4-2}{2}, \dfrac{2-6}{2}\right) = \left(\dfrac{-6}{2}, \dfrac{-4}{2}\right) = (-3,-2)$

Coordinate of $F = \left(\dfrac{-2+8}{2}, \dfrac{-6+6}{2}\right) = \left(\dfrac{6}{2}, \dfrac{0}{2}\right) = (3,0)$

Hence Slope of median $AE = \dfrac{-2-6}{-3-8} = \dfrac{-8}{-11} = \dfrac{8}{11}$

Slope of median $BF = \dfrac{0-2}{3+4} = \dfrac{-2}{7}$

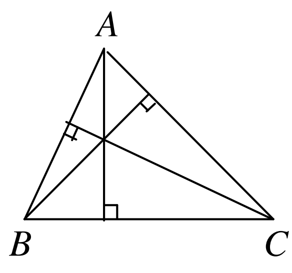
Slope of median $CD = \dfrac{4+6}{2+2} = \dfrac{10}{4} = \dfrac{5}{2}$

(iii) Since altitudes are perpendicular to the sides of a triangle therefore

Slope of altitude from vertex $A = \dfrac{-1}{\text{slope of side } BC} = \dfrac{-1}{-4} = \dfrac{1}{4}$

Slope of altitude from vertex $B = \dfrac{-1}{\text{slope of side } AC} = \dfrac{-1}{6/5} = -\dfrac{5}{6}$

Slope of altitude from vertex $C = \dfrac{-1}{\text{slope of side } AB} = \dfrac{-1}{1/3} = -3$



◈ *Question # 3*

(a) Let $A(-1,-3)$, $B(1,5)$ and $C(2,9)$ be given points

$$\text{Slope of } AB = \frac{5+3}{1+1} = \frac{8}{2} = 4$$

$$\text{Slope of } BC = \frac{9-5}{2-1} = \frac{4}{1} = 4$$

Since slope of $AB =$ slope of BC
Therefore A, B and C lie on the same line.

(b) & (c) *Do yourself as above*

(d) Let $A(a,2b), B(c,a+b)$ and $C(2c-a,2a)$ be given points.

$$\text{Slope of } AB = \frac{(a+b)-2b}{c-a} = \frac{a-b}{c-a}$$

$$\text{Slope of } BC = \frac{2a-(a+b)}{(2c-a)-c} = \frac{2a-a-b}{2c-a-c} = \frac{a-b}{c-a}$$

Since slope of $AB =$ slope of BC
Therefore A, B and C lie on the same line.

◈ *Question # 4*

Since $A(7,3)$, $B(k,-6)$, $C(-4,5)$ and $D(-6,4)$

$$\text{Therefore slope of } AB = m_1 = \frac{-6-3}{k-7} = \frac{-9}{k-7}$$

$$\text{Slope of } CD = m_2 = \frac{4-5}{-6+4} = \frac{-1}{-2} = \frac{1}{2}$$

(i) If AB and CD are parallel then $m_1 = m_2$

$$\Rightarrow \frac{-9}{k-7} = \frac{1}{2} \quad \Rightarrow -18 = k-7$$

$$\Rightarrow k = -18+7 \quad \Rightarrow \boxed{k=-11}$$

(ii) If AB and CD are perpendicular then $m_1 m_2 = -1$

$$\Rightarrow \left(\frac{-9}{k-7}\right)\left(\frac{1}{2}\right) = -1 \quad \Rightarrow -9 = -2(k-7)$$

$$\Rightarrow 9 = 2k-14 \quad \Rightarrow 2k = 9+14 = 23$$

$$\Rightarrow \boxed{k = \frac{23}{2}}$$

◈ *Question # 5*

Since $A(6,1), B(2,7)$ and $C(-6,-7)$ are vertices of triangle therefore

$$\text{Slope of } \overline{AB} = m_1 = \frac{7-1}{2-6} = \frac{6}{-4} = -\frac{3}{2}$$

$$\text{Slope of } \overline{BC} = m_2 = \frac{-7-7}{-6-2} = \frac{-12}{-8} = \frac{7}{4}$$

REMEMBER
The symbols
(i) $||$ stands for ‘parallel”
(ii) $\nparallel$ stands for “not parallel”
(iii) $\perp$ stands for “perpendicular”

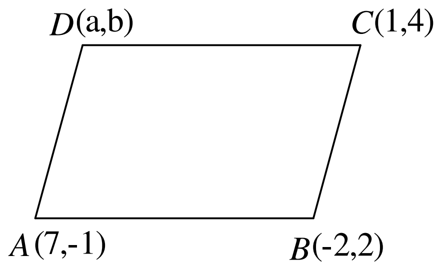
Slope of $\overline{CA} = m_3 = \dfrac{1+7}{6+6} = \dfrac{8}{12} = \dfrac{2}{3}$

Since $\quad m_1 m_3 = \left(-\dfrac{3}{2}\right)\left(\dfrac{2}{3}\right) = -1$

$\Rightarrow$ The triangle ABC is a right triangle with $m\angle A = 90^\circ$

◈ *Question # 6*

Let $D(a,b)$ be a fourth vertex of the parallelogram.



Slope of $\overline{AB} = \dfrac{2+1}{-2-7} = \dfrac{3}{-9} = -\dfrac{1}{3}$

Slope of $\overline{BC} = \dfrac{4-2}{1+2} = \dfrac{2}{3}$

Slope of $\overline{CD} = \dfrac{b-4}{a-1}$

Slope of $\overline{DA} = \dfrac{-1-b}{7-a}$

Since $ABCD$ is a parallelogram therefore

Slope of $\overline{AB} =$ Slope of $\overline{CD}$

$\Rightarrow -\dfrac{1}{3} = \dfrac{b-4}{a-1} \qquad \Rightarrow -(a-1) = 3(b-4)$

$\Rightarrow -a+1-3b+12 = 0 \qquad \Rightarrow -a-3b+13 = 0$ ....(i)

Also slope of $\overline{BC} =$ slope of $\overline{DA}$

$\Rightarrow \dfrac{2}{3} = \dfrac{-1-b}{7-a} \qquad \Rightarrow 2(7-a) = 3(-1-b) \qquad \Rightarrow 14-2a = -3-3b$

$\Rightarrow 14-2a+3+3b = 0 \qquad \Rightarrow -2a+3b+17 = 0$.............$(ii)$

Adding (i) and (ii)

$$\begin{array}{r} -a-3b+13=0 \\ -2a+3b+17=0 \\ \hline -3a \qquad +30=0 \end{array} \quad \Rightarrow 3a = 30 \quad \Rightarrow \boxed{a=10}$$

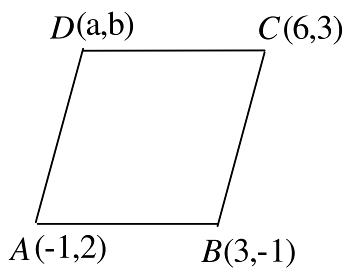
Putting value of a in (i)

$-10-3b+13 = 0 \quad \Rightarrow -3b+3 = 0 \quad \Rightarrow 3b = 3 \Rightarrow \boxed{b=1}$

Hence $D(10,1)$ is the fourth vertex of parallelogram.

◈ *Question # 7*

Let $D(a,b)$ be a fourth vertex of rhombus.



Slope of $\overline{AB} = \dfrac{-1-2}{3+1} = \dfrac{-3}{4}$

Slope of $\overline{BC} = \dfrac{3+1}{6-3} = \dfrac{4}{3}$

Slope of $\overline{CD} = \dfrac{b-3}{a-6}$

Slope of $\overline{DA} = \dfrac{2-b}{-1-a}$

Since $ABCD$ is a rhombus therefore

Slope of $\overline{AB} =$ Slope of $\overline{CD}$

$\Rightarrow \ -\dfrac{3}{4} = \dfrac{b-3}{a-6} \qquad \Rightarrow \ -3(a-6) = 4(b-3)$

$\Rightarrow \ -3a+18 = 4b-12 \qquad \Rightarrow \ -3a+18-4b+12 = 0$

$\Rightarrow \ -3a-4b+30 = 0 ....(i)$

Also slope of $\overline{BC} =$ slope of $\overline{DA}$

$\Rightarrow \ \dfrac{4}{3} = \dfrac{2-b}{-1-a} \qquad \Rightarrow \ 4(-1-a) = 3(2-b)$

$\Rightarrow \ -4-4a = 6-3b \qquad \Rightarrow \ -4-4a-6+3b = 0$

$\Rightarrow \ -4a+3b-10 = 0 ....(ii)$

$\times$ing eq. (i) by 3 and (ii) by 4 and adding.

$$\begin{array}{l} -9a \ -12b + 90 = 0 \\ \underline{-16a + 12b - 40 = 0} \\ -25a \qquad\quad + 50 = 0 \end{array} \quad \Rightarrow \ 25a = 50 \quad \Rightarrow \ \boxed{a = 2}$$

Putting value of a in (ii)

$-4(2)+3b-10 = 0 \ \Rightarrow \ 3b-18 = 0 \quad \Rightarrow \ 3b = 18 \ \Rightarrow \ \boxed{b = 6}$

Hence $D(2,6)$ is the fourth vertex of rhombus.

Now slope of diagonal $\overline{AC} = \dfrac{3-2}{6+1} = \dfrac{1}{7}$

Slope of diagonal $\overline{BD} = \dfrac{b-(-1)}{a-3} = \dfrac{6+1}{2-3} = \dfrac{7}{-1} = -7$

Since

(Slope of $\overline{AC}$)(Slope of $\overline{BD}$) = $\left(\dfrac{1}{7}\right)(-7) = -1$

$\Rightarrow$ Diagonals of a rhombus are $\perp$ to each other.

◈ *Question # 8*

(a) Slope of line joining $(1,-2)$ and $(2,4) = m_1 = \dfrac{4+2}{2-1} = \dfrac{6}{1} = 6$

Slope of line joining $(4,1)$ and $(-8,2) = m_2 = \dfrac{2-1}{-8-4} = \dfrac{1}{-12}$

Since $m_1 \neq m_2$

Also $m_1 m_2 = 6 \cdot \dfrac{1}{-12} = -\dfrac{1}{2} \neq -1$

$\Rightarrow$ lines are neither parallel nor perpendicular.

(b) *Do yourself as above.*

For notes and latest update visit **http://www.mathcity.org**

◈ *Equation of Straight Line:*

(i) Slope-intercept form

Equation of straight line with slope m and $y-$intercept c is given by:

$$\boxed{y = mx + c}$$

See proof on book at page 194

(ii) Point-slope form

Let m be a slope of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{y - y_1 = m(x - x_1)}$$

See proof on book at page 195

(iii) Symmetric form

Let α be an inclination of line and $A(x_1, y_1)$ be a point lies on a line then equation of line is given by:

$$\boxed{\frac{y - y_1}{\cos\alpha} = \frac{x - x_1}{\sin\alpha}}$$

See proof on book at page 195

(iv) Two-points form

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be points lie on a line then it's equation is given by:

$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)} \quad \text{or} \quad \boxed{y - y_2 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_2)} \quad \text{or} \quad \boxed{\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0}$$

See proof on book at page 196

(v) Two-intercept form

When a line intersect $x-$axis at $x = a$ and $y-$axis at $y = b$

i.e. $x-$intercept $= a$ and $y-$intercept $= b$, then equation of line is given by:

$$\boxed{\frac{x}{a} + \frac{y}{b} = 1}$$

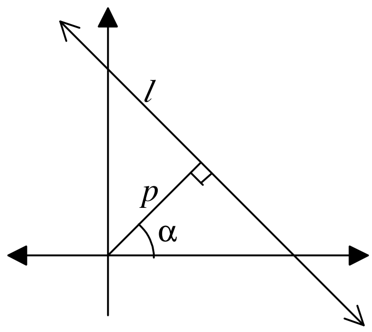
See proof on book at page 197

(vi) Normal form

Let p denoted length of perpendicular from the origin to the line and α is the angle of the perpendicular from +ive x-axis then equation of line is given by:

$$\boxed{x\cos\alpha + y\sin\alpha = p}$$

See proof on book at page 198



◈ *Question # 9*

(a) Since slope of horizontal line $= m = 0$

$$\& \ (x_1, y_1) = (7, -9)$$

therefore equation of line:

$$y-(-9)=0(x-7)$$
$$\Rightarrow\ x+9=0 \qquad \text{Answer}$$

(b) Since slope of vertical line $m=\infty=\dfrac{1}{0}$

$$\&\ (x_1,y_1)=(-5,3)$$

therefore required equation of line

$$y-3=\infty\big(x-(-5)\big)$$
$$\Rightarrow\ y-3=\frac{1}{0}(x+5) \qquad \Rightarrow\ 0(y-3)=1(x+5)$$
$$\Rightarrow\ x+5=0 \qquad \text{Answer}$$

(c) The line bisecting the first and third quadrant makes an angle of 45° with the $x-$axis therefore slope of line $=m=\tan 45^\circ=1$

Also it passes through origin $(0,0)$, so its equation

$$y-0=1(x-0) \quad \Rightarrow \quad y=x$$
$$\Rightarrow\ x-y=0 \qquad \text{Answer}$$

(d) The line bisecting the second and fourth quadrant makes an angle of 135° with $x-$axis therefore slope of line $=m=\tan 135^\circ=-1$

Also it passes through origin $(0,0)$, so its equation

$$y-0=-1(x-0) \quad \Rightarrow \quad y=-x$$
$$\Rightarrow\ x+y=0 \qquad \text{Answer}$$

◈ *Question # 10*

(a) $\because\ (x_1,y_1)=(-6,5)$

and slope of line $=m=7$

so required equation

$$y-5=7\big(x-(-6)\big)$$
$$\Rightarrow\ y-5=7(x+6) \qquad \Rightarrow\ y-5=7x+42$$
$$\Rightarrow\ 7x+42-y+5=0 \qquad \Rightarrow\ 7x-y+47=0 \qquad \text{Answer}$$

(b) *Do yourself as above.*

(c) $\because\ (x_1,y_1)=(-8,5)$

and slope of line $=m=\infty$

So required equation

$$y-5=\infty\big(x-(-8)\big)$$
$$\Rightarrow\ y-5=\frac{1}{0}(x+8) \qquad \Rightarrow\ 0(y-5)=1(x+8)$$
$$\Rightarrow\ x+8=0 \qquad \text{Answer}$$

(d) The line through $(-5,-3)$ and $(9,-1)$ is

$$y-(-3)=\frac{-1-(-3)}{9-(-5)}\big(x-(-5)\big) \qquad \Rightarrow\ y+3=\frac{2}{14}(x+5)$$

$$\Rightarrow\ y+3=\frac{1}{7}(x+5) \qquad \Rightarrow\ 7y+21=x+5$$

$$\Rightarrow\ x+5-7y-21=0 \qquad \Rightarrow\ x-7y-16=0 \qquad \text{Answer}$$

(e) $\because\ y-$intercept $=-7$

$\Rightarrow\ (0,-7)$ lies on a required line

Also slope $=m=-5$

So required equation

$$y-(-7)=-5(x-0)$$

$$\Rightarrow\ y+7=-5x \qquad \Rightarrow\ 5x+y+7=0 \qquad \text{Answer}$$

(f) $\because\ x-$intercept $=-9$

$\Rightarrow\ (-9,0)$ lies on a required line

Also slope $=m=4$

Therefore required line

$$y-0=4(x+9)$$

$$\Rightarrow\ y=4x+9 \qquad \Rightarrow\ 4x-y+9=0 \qquad \text{Answer}$$

(g) $x-$intercept $=a=-3$

$y-$intercept $=b=4$

Using two-intercept form of equation line

$$\frac{x}{a}+\frac{y}{b}=1 \qquad \Rightarrow\ \frac{x}{-3}+\frac{y}{4}=0$$

$$\Rightarrow\ 4x-3y=-12 \qquad \times\text{ing by } -12$$

$$\Rightarrow\ 4x-3y+12=0 \qquad \text{Answer}$$

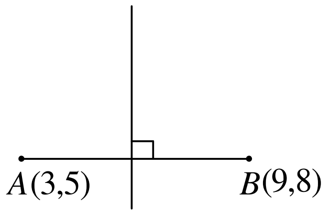
◈ *Question # 11*

Given points $A(3,5)$ and $B(9,8)$

Midpoint of $\overline{AB}=\left(\frac{3+9}{2},\frac{5+8}{2}\right)=\left(\frac{12}{2},\frac{13}{2}\right)=\left(6,\frac{13}{2}\right)$

Slope of $\overline{AB}=m=\frac{8-5}{9-3}=\frac{3}{6}=\frac{1}{2}$

Slope of line $\perp$ to $\overline{AB}=-\frac{1}{m}=-\frac{1}{1/2}=--2$



Now equation of $\perp$ bisector having slope -2 through $\left(6,\frac{13}{2}\right)$

$$\Rightarrow\ y-\frac{13}{2}=-2(x-6)$$

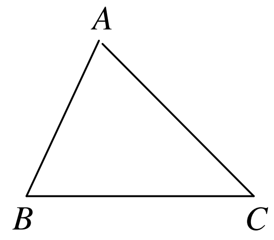
$$\Rightarrow\ y-\frac{13}{2}=-2x+12 \qquad \Rightarrow\ y-\frac{13}{2}+2x-12=0$$

$$\Rightarrow\ 2x+y-\frac{37}{2}=0 \qquad \Rightarrow\ 4x+2y-37=0 \qquad \text{Answer}$$

◈ *Question # 12*

Given vertices of triangle are $A(-3,2)$, $B(5,4)$ and $C(3,-8)$.

Equation of sides:



Slope of $\overline{AB} = m_1 = \dfrac{4-2}{5-(-3)} = \dfrac{2}{8} = \dfrac{1}{4}$

Slope of $\overline{BC} = m_2 = \dfrac{-8-4}{3-5} = \dfrac{-12}{-2} = 6$

Slope of $\overline{CA} = m_3 = \dfrac{2-(-8)}{-3-3} = \dfrac{10}{-6} = -\dfrac{5}{3}$

Now equation of side $\overline{AB}$ having slope $\dfrac{1}{4}$ passing through $A(-3,2)$

[You may take $B(5,4)$ instead of $A(-3,2)$]

$$y-2=\frac{1}{4}\big(x-(-3)\big) \quad\Rightarrow\quad 4y-8=x+3$$

$$\Rightarrow\quad x+3-4y+8=0 \quad\Rightarrow\quad \boxed{x-4y+11=0}$$

Equation of side $\overline{BC}$ having slope 6 passing through $B(5,4)$.

$$y-4=6(x-5) \quad\Rightarrow\quad y-4=6x-30$$

$$\Rightarrow\quad 6x-30-y+4=0 \quad\Rightarrow\quad \boxed{6x-y-26=0}$$

Equation of side $\overline{CA}$ having slope $-\dfrac{5}{3}$ passing through $C(3,-8)$

$$y-(-8)=-\frac{5}{3}(x-3) \quad\Rightarrow\quad 3(y+8)=-5(x-3)$$

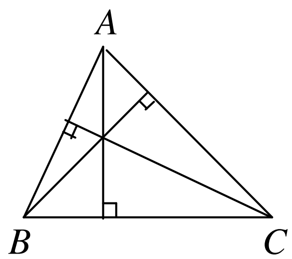
$$\Rightarrow\quad 3y+24=-5x+15 \quad\Rightarrow\quad 5x-15+3y+24=0$$

$$\Rightarrow\quad \boxed{5x+3y+9=0}$$

Equation of altitudes:

Since altitudes are perpendicular to the sides of triangle therefore

Slope of altitude on $\overline{AB} = -\dfrac{1}{m_1} = -\dfrac{1}{1/4} = -4$



Equation of altitude from $C(3,-8)$ having slope -4

$$y+8=-4(x-3) \quad\Rightarrow\quad y+8=-4x+12$$

$$\Rightarrow\quad 4x-12+y+8=0 \quad\Rightarrow\quad \boxed{4x+y-4=0}$$

Slope of altitude on $\overline{BC} = -\dfrac{1}{m_2} = -\dfrac{1}{6}$

Equation of altitude from $A(-3,2)$ having slope $-\dfrac{1}{6}$

$$y-2=-\frac{1}{6}(x+3) \quad\Rightarrow\quad 6y-12=-x-3$$

$$\Rightarrow\quad x+3+6y-12=0 \quad\Rightarrow\quad \boxed{x+6y-9=0}$$

Slope of altitude on $\overline{CA} = -\dfrac{1}{m_3} = -\dfrac{1}{-5/3} = \dfrac{3}{5}$

Equation of altitude from $B(5,4)$ having slope $\dfrac{3}{5}$

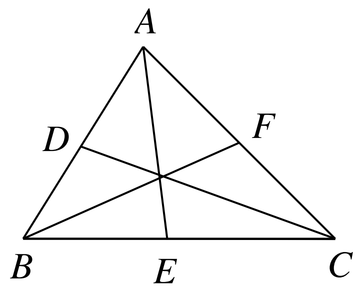
$$y-4=\frac{3}{5}(x-5) \quad \Rightarrow \quad 5y-20=3x-15$$

$$\Rightarrow \quad 3x-15-5y+20=0 \quad \Rightarrow \quad \boxed{3x-5y+5=0}$$

Equation of Medians:

Suppose D, E and F are midpoints of sides $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ respectively.

Then coordinate of $D=\left(\dfrac{-3+5}{2},\dfrac{2+4}{2}\right)=\left(\dfrac{2}{2},\dfrac{6}{2}\right)=(1,3)$



Coordinate of $E=\left(\dfrac{5+3}{2},\dfrac{4-8}{2}\right)=\left(\dfrac{8}{2},\dfrac{-4}{2}\right)=(4,-2)$

Coordinate of $F=\left(\dfrac{3-3}{2},\dfrac{-8+2}{2}\right)=\left(\dfrac{0}{2},\dfrac{-6}{2}\right)=(0,-3)$

Equation of median $\overline{AE}$ by two-point form

$$y-2=\frac{-2-2}{4-(-3)}(x-(-3))$$

$$\Rightarrow \quad y-2=\frac{-4}{7}(x+3) \quad \Rightarrow \quad 7y-14=-4x-12$$

$$\Rightarrow \quad 7y-14+4x+12=0 \quad \Rightarrow \quad \boxed{4x+7y-2=0}$$

Equation of median $\overline{BF}$ by two-point form

$$y-4=\frac{-3-4}{0-5}(x-5)$$

$$\Rightarrow \quad y-4=\frac{-7}{-5}(x-5) \quad \Rightarrow \quad -5y+20=-7x+35$$

$$\Rightarrow \quad -5y+20+7x-35=0 \quad \Rightarrow \quad \boxed{7x-5y-15=0}$$

Equation of median $\overline{CD}$ by two-point form

$$y-(-8)=\frac{3-(-8)}{1-3}(x-3)$$

$$\Rightarrow \quad y+8=\frac{11}{-2}(x-3) \quad \Rightarrow \quad -2y-16=11x-33$$

$$\Rightarrow \quad 11x-33+2y+16=0 \quad \Rightarrow \quad \boxed{11x+2y-17=0}$$

◈ *Question # 13*

Here $(x_1, y_1)=(-4,-6)$

Slope of given line $=m=\dfrac{-3}{2}$

$\because$ required line is $\perp$ to given line

$\therefore$ slope of required line $= -\dfrac{1}{m} = -\dfrac{1}{-3/2} = \dfrac{2}{3}$

Now equation of line having slope $\dfrac{2}{3}$ passing through $(-4,-6)$

$$y-(-6) = \frac{2}{3}\left(x-(-4)\right)$$

$$\Rightarrow\ 3(y+6) = 2(x+4) \qquad \Rightarrow\ 3y+18 = 2x+8$$

$$\Rightarrow\ 2x+8-3y-18 = 0 \qquad \Rightarrow\ 2x-3y-10 = 0 \quad \text{Answer}$$

◈ *Question # 14*

Here $(x_1, y_1) = (11,-5)$

Slope of given line $= m = -24$

$\because$ required line is $||$ to given line

$\therefore$ slope of required line $= m = -24$

Now equation of line having slope -24 passing through $(11,-5)$

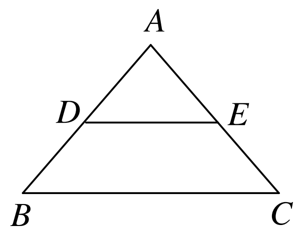
$$y-(-5) = -24(x-11)$$

$$\Rightarrow\ y+5 = -24x+264 \qquad \Rightarrow\ 24x-264+y+5 = 0$$

$$\Rightarrow\ 24x+y-259 = 0 \quad \text{Answer}$$

◈ *Question # 15*

Given vertices $A(-1,2)$, $B(6,3)$ and $C(2,-4)$



Since D and E are midpoints of sides $\overline{AB}$ and $\overline{AC}$ respectively.

Therefore coordinate of $D = \left(\dfrac{-1+6}{2}, \dfrac{2+3}{2}\right) = \left(\dfrac{5}{2}, \dfrac{5}{2}\right)$

Coordinate of $E = \left(\dfrac{-1+2}{2}, \dfrac{2-4}{2}\right) = \left(\dfrac{1}{2}, \dfrac{-2}{2}\right) = \left(\dfrac{1}{2}, -1\right)$

Now slope of $\overline{DE} = \dfrac{-1-5/2}{1/2-5/2} = \dfrac{-7/2}{-4/2} = \dfrac{7}{4}$

slope of $\overline{BC} = \dfrac{-4-3}{2-6} = \dfrac{-7}{-4} = \dfrac{7}{4}$

Since slope of $\overline{DE} =$ slope of $\overline{BC}$

Therefore $\overline{DE}$ is parallel to $\overline{BC}$.

Now $\quad \left|\overline{DE}\right| = \sqrt{\left(\dfrac{1}{2}-\dfrac{5}{2}\right)^2 + \left(-1-\dfrac{5}{2}\right)^2} = \sqrt{\left(-\dfrac{4}{2}\right)^2 + \left(-\dfrac{7}{2}\right)^2}$

$$= \sqrt{4+\frac{49}{4}} = \sqrt{\frac{65}{4}} = \frac{\sqrt{65}}{2} \ \ldots\ldots\ldots\ldots (i)$$

$$\left|\overline{BC}\right| = \sqrt{(2-6)^2 + (-4-3)^2} = \sqrt{(-4)^2 + (-7)^2}$$
$$= \sqrt{16+49} = \sqrt{65} \;\ldots\ldots\ldots\ldots\; (ii)$$

From (i) and (ii)

$$\left|\overline{DE}\right| = \frac{1}{2}\left|\overline{BC}\right| \qquad \text{Proved.}$$

◈ *Question # 16*

Let l denotes the number of litres of milk and p denotes the price of milk,

Then $(l_1, p_1) = (560, 12.50)$ & $(l_2, p_2) = (700, 12.00)$

Since graph of sale price and milk sold is a straight line

Therefore, from two point form, it's equation

$$p - p_1 = \frac{p_2 - p_1}{l_2 - l_1}(l - l_1)$$
$$\Rightarrow\; p - 12.50 = \frac{12.00 - 12.50}{700 - 560}(l - 560)$$
$$\Rightarrow\; p - 12.50 = \frac{-0.50}{140}(l - 560)$$
$$\Rightarrow\; 140p - 1750 = -0.50l + 280$$
$$\Rightarrow\; 140p - 1750 + 0.50l - 280 = 0$$
$$\Rightarrow\; 0.50l + 140p - 2030 = 0$$

ALTERNATIVE
You may use determinant form of two-point form to find an equation of line.

$$\begin{vmatrix} l & p & 1 \\ l_1 & p_1 & 1 \\ l_2 & p_2 & 1 \end{vmatrix} = 0$$

If $p = 12.25$

$$\Rightarrow\; 0.50l + 140(12.25) - 2030 = 0$$
$$\Rightarrow\; 0.50l + 1715 - 2030 = 0 \qquad \Rightarrow\; 0.50l - 315 = 0$$
$$\Rightarrow\; 0.50l = 315 \qquad \Rightarrow\; l = \frac{315}{0.50} = 630$$

Hence milkman can sell 630 litres milk at Rs. 12.25 per litre.

◈ *Question # 17*

Let p denotes population of Pakistan in million and t denotes year after 1961,

Then $(p_1, t_1) = (60, 1961)$ and $(p_2, t_2) = (95, 1981)$

Equation of line by two point form:

$$t - t_1 = \frac{t_2 - t_1}{p_2 - p_1}(p - p_1)$$
$$\Rightarrow\; t - 1961 = \frac{1981 - 1961}{95 - 60}(p - 60)$$
$$\Rightarrow\; t - 1961 = \frac{20}{35}(p - 60) \qquad \Rightarrow\; t - 1961 = \frac{4}{7}(p - 60)$$
$$\Rightarrow\; 7t - 13727 = 4p - 240 \qquad \Rightarrow\; 7t - 13727 + 240 = 4p$$
$$\Rightarrow\; 4p = 7t - 13487 \qquad \Rightarrow\; p = \frac{7}{4}t - \frac{13487}{4} \;\ldots\ldots\ldots\ldots\; (i)$$

This is the required equation which gives population in term of t.

(*a*) Put $t = 1947$ in eq. (i)

$$p = \frac{7}{4}(1947) - \frac{13487}{4} \quad = 3407.25 - 3371.75 \quad = 35.5$$

Hence population in 1947 is 35.5 millions.

(*b*) Put $t = 1997$ in eq. (i)

$$p = \frac{7}{4}(1997) - \frac{13487}{4} = 3494.75 - 3371.75 = 123$$

Hence population in 1997 is 123 millions.

◈ *Question # 18*

Let p denotes purchase price of house in millions and t denotes year then

$$(p_1, t_1) = (1, 1980) \quad \text{and} \quad (p_2, t_2) = (4, 1996)$$

Equation of line by two point form:

$$t - t_1 = \frac{t_2 - t_1}{p_2 - p_1}(p - p_1)$$

$$\Rightarrow \ t - 1980 = \frac{1996 - 1980}{4 - 1}(p - 1)$$

$$\Rightarrow \ t - 1980 = \frac{16}{3}(p - 1)$$

$$\Rightarrow \ 3t - 5940 = 16p - 16$$

$$\Rightarrow \ 3t - 5940 + 16 = 16p \quad \Rightarrow 16p = 3t - 5924$$

$$\Rightarrow \ p = \frac{3}{16}t - \frac{5924}{16} \quad \Rightarrow \ p = \frac{3}{16}t - \frac{1481}{4} \ \ldots\ldots\ldots\ldots (i)$$

> **ALTERNATIVE**
> You may use determinant form of two-point form to find an equation of line.
> $$\begin{vmatrix} p & t & 1 \\ p_1 & t_1 & 1 \\ p_2 & t_2 & 1 \end{vmatrix} = 0$$

This is the required equation which gives value of house in term of t.

Put $t = 1990$ in eq. (i)

$$p = \frac{3}{16}(1990) - \frac{1481}{4} \quad = 373.125 - 370.25 \quad = 2.875$$

Hence value of house in 1990 is 2.875 millions.

◈ *Question # 19*

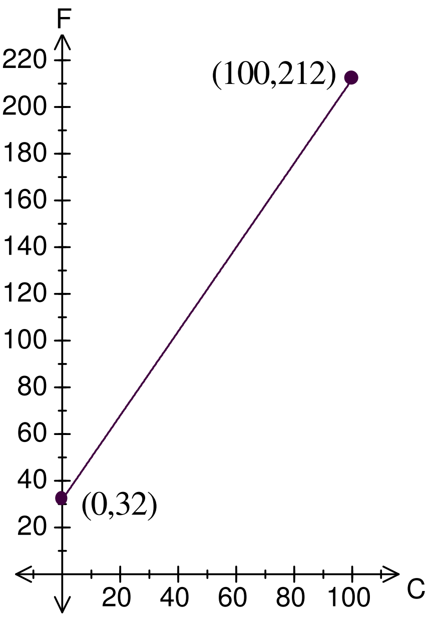
Since freezing point of water $= 0^\circ C \ = \ 32^\circ F$
and boiling point of water $= 100^\circ C \ = \ 212^\circ F$
therefore we have points $(C_1, F_1) = (0, 32)$ and $(C_2, F_2) = (100, 212)$

Equation of line by two point form

$$F - F_1 = \frac{F_2 - F_1}{C_2 - C_1}(C - C_1)$$

$$\Rightarrow \ F - 32 = \frac{212 - 32}{100 - 0}(C - 0)$$

$$\Rightarrow \ F - 32 = \frac{180}{100}C$$



Take scale 10ss = 20C and 10ss = 20F on x-axis and y-axis respectively to draw graph.

$$\Rightarrow\ F = \frac{9}{5}C + 32$$

◈ *Question # 20*

Let s denotes entry test score and y denotes year.

Then we have $(s_1, y_1) = (592, 1998)$ and $(s_2, y_2) = (564, 2002)$

By two point form of equation of line

$$y - y_1 = \frac{y_2 - y_1}{s_2 - s_1}(s - s_1)$$

$$\Rightarrow\ y - 1998 = \frac{2002 - 1998}{564 - 592}(s - 592) \quad \Rightarrow\ y - 1998 = \frac{4}{-28}(s - 592)$$

$$\Rightarrow\ y - 1998 = -\frac{1}{7}(s - 592) \quad \Rightarrow\ 7y - 13986 = -s + 592$$

$$\Rightarrow\ 7y - 13986 + s - 592 = 0 \quad \Rightarrow\ s + 7y - 14578 = 0$$

Put $y = 2006$ in (i)

$$s + 7(2006) - 14578 = 0 \quad \Rightarrow\ s + 14042 - 14578 = 0$$

$$\Rightarrow\ s - 536 = 0 \quad \Rightarrow\ s = 536$$

Hence in 2006 the average score will be 536.

◈ *Question # 21 (a)*

(i) - Slope-intercept form

$$\because\ 2x - 4y + 11 = 0$$

$$\Rightarrow\ 4y = 2x + 11 \quad \Rightarrow\ y = \frac{2x + 11}{4}$$

$$\Rightarrow\ y = \frac{1}{2}x + \frac{11}{4}$$

is the intercept form of equation of line with $m = \frac{1}{2}$ and $c = \frac{11}{4}$

(ii) - Two-intercept form

$$\because\ 2x - 4y + 11 = 0 \quad \Rightarrow\ 2x - 4y = -11$$

$$\Rightarrow\ \frac{2}{-11}x - \frac{4}{-11}y = 1 \quad \Rightarrow\ \frac{x}{-11/2} + \frac{y}{11/4} = 1$$

is the two-point form of equation of line with $a = -\frac{11}{2}$ and $b = \frac{11}{4}$.

(iii) - Normal form

$$\because\ 2x - 4y + 11 = 0 \quad \Rightarrow\ 2x - 4y = -11$$

Dividing above equation by $\sqrt{(2)^2 + (-4)^2} = \sqrt{20} = 2\sqrt{5}$

$$\frac{2x}{2\sqrt{5}} - \frac{4y}{2\sqrt{5}} = \frac{-11}{2\sqrt{5}} \quad \Rightarrow\ \frac{x}{\sqrt{5}} - \frac{2y}{\sqrt{5}} = \frac{-11}{2\sqrt{5}}$$

$$\Rightarrow\ -\frac{x}{\sqrt{5}} + \frac{2y}{\sqrt{5}} = \frac{11}{2\sqrt{5}} \qquad \times\text{ing by } -1.$$

Suppose $\cos\alpha = -\frac{1}{\sqrt{5}} < 0$ and $\sin\alpha = \frac{2}{\sqrt{5}} > 0$

$\Rightarrow$ α lies in 2nd quadrant and $\alpha = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right) = 116.57^\circ$

Hence the normal form is

$$x\cos\left(116.57^\circ\right) + y\sin\left(116.57^\circ\right) = \frac{11}{2\sqrt{5}}$$

And length of perpendicular from $(0,0)$ to line $= p = \frac{11}{2\sqrt{5}}$

◈ *Question # 21 (b)*

(i) - Slope-intercept form

$$\because\ 4x + 7y - 2 = 0$$

$$\Rightarrow\ 7y = -4x + 2 \qquad \Rightarrow\ y = \frac{-4x+2}{7}$$

$$\Rightarrow\ y = -\frac{4}{7}x + \frac{2}{7}$$

is the intercept form of equation of line with $m = -\frac{4}{7}$ and $c = \frac{2}{7}$

(ii) - Two-intercept form

$$\because\ 4x + 7y - 2 = 0 \qquad \Rightarrow\ 4x + 7y = 2$$

$$\Rightarrow\ 2x + \frac{7}{2}y = 1 \qquad \div\text{ing by } 2$$

$$\Rightarrow\ \frac{x}{1/2} + \frac{y}{2/7} = 1$$

is the two-point form of equation of line with $a = \frac{1}{2}$ and $b = \frac{2}{7}$.

(iii) - Normal form

$$\because\ 4x + 7y - 2 = 0$$

$$\Rightarrow\ 4x + 7y = 2$$

Dividing above equation by $\sqrt{(4)^2 + (7)^2} = \sqrt{16+49} = \sqrt{65}$

$$\Rightarrow\ \frac{4}{\sqrt{65}}x + \frac{7}{\sqrt{65}}y = \frac{2}{\sqrt{65}} \quad .$$

Suppose $\cos\alpha = \frac{4}{\sqrt{65}} > 0$ and $\sin\alpha = \frac{7}{\sqrt{65}} > 0$

$\Rightarrow$ α lies in first quadrant and $\alpha = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 60.26^\circ$

Hence the normal form is

$$x\cos\left(60.26^\circ\right) + y\sin\left(60.26^\circ\right) = \frac{2}{\sqrt{65}}$$

And length of perpendicular from $(0,0)$ to line $= p = \dfrac{2}{\sqrt{65}}$

◈ *Question # 21 (c)*

(i) - Slope-intercept form

$$\because\ 15y - 8x + 3 = 0$$

$$\Rightarrow\ 15y = 8x - 3 \qquad \Rightarrow\ y = \frac{8x-3}{15}$$

$$\Rightarrow\ y = \frac{8}{15}x - \frac{3}{15} \qquad \Rightarrow\ y = \frac{8}{15}x - \frac{1}{5}$$

is the intercept form of equation of line with $m = \dfrac{8}{15}$ and $c = -\dfrac{1}{5}$

(ii) - Two-intercept form

$$\because\ 15y - 8x + 3 = 0 \qquad \Rightarrow\ -8x + 15y = -3$$

$$\Rightarrow\ \frac{8x}{3} - 5y = 1 \qquad \Rightarrow\ \frac{x}{3/8} + \frac{y}{-1/5} = 1$$

is the two-point form of equation of line with $a = \dfrac{3}{8}$ and $b = -\dfrac{1}{5}$.

(iii) - Normal form

$$\because\ 15y - 8x + 3 = 0$$

$$\Rightarrow\ 8x - 15y = 3$$

Dividing above equation by $\sqrt{(8)^2 + (-15)^2} = \sqrt{64+225} = \sqrt{289} = 17$

$$\Rightarrow\ \frac{8}{17}x - \frac{15}{17}y = \frac{3}{17} \quad .$$

Suppose $\cos\alpha = \dfrac{8}{17} > 0$ and $\sin\alpha = -\dfrac{15}{17} < 0$

$\Rightarrow\ \alpha$ lies in 4th quadrant and $\alpha = \cos^{-1}\left(\dfrac{8}{17}\right) = 298.07^\circ$

$\alpha = \cos^{-1}\left(\dfrac{8}{17}\right)$ $= 61.93^\circ,\ 298.07^\circ$ Taking value that lies in 4th quadrant.

Hence the normal form is

$$x\cos\left(298.07^\circ\right) + y\sin\left(298.07^\circ\right) = \frac{3}{17}$$

And length of perpendicular from $(0,0)$ to line $= p = \dfrac{3}{17}$

◈ *General equation of the straight line*

A general equation of straight line (General linear equation) in two variable x and y is given by:

$$ax + by + c = 0$$

where a, b and c are constants and a and b are not simultaneously zero.

See proof on book at page: 199.

Note: Since $ax+by+c=0 \Rightarrow by=-ax-c \Rightarrow y=-\frac{a}{b}x-\frac{c}{b}$

Which is an intercept form of equation of line with slope $m=-\frac{a}{b}$ and $c=-\frac{c}{b}$

◈ *Question # 22*

(a) Let $l_1:\ 2x+y-3=0$

$l_2:\ 4x+2y+5=0$

Slope of $l_1 = m_1 = -\frac{2}{1} = -2$

Slope of $l_2 = m_2 = -\frac{4}{2} = -2$

Since $m_1 = m_2$ therefore l_1 and l_2 are parallel.

(b) Let $l_1:\ 3y=2x+5 \Rightarrow 2x-3y+5=0$

$l_2:\ 3x+2y-8=0$

Slope of $l_1 = m_1 = -\frac{2}{-3} = \frac{2}{3}$

Slope of $l_2 = m_2 = -\frac{3}{2} =$

Since $m_1m_2 = \left(\frac{2}{3}\right)\left(-\frac{3}{2}\right) = -1 \Rightarrow l_1$ and l_2 are perpendicular.

(c) Let $l_1:\ 4y+2x-1=0 \Rightarrow 2x+4y-1=0$

$l_2:\ x-2y-7=0$

Slope of $l_1 = m_1 = -\frac{2}{4} = -\frac{1}{2}$

Slope of $l_2 = m_2 = -\frac{1}{-2} = \frac{1}{2}$

Since $m_1 \neq m_2$ and $m_1m_2 = \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right) = -\frac{1}{4} \neq -1$

$\Rightarrow l_1$ and l_2 are neither parallel nor perpendicular.

(d) & (e) *Do yourself as above.*

◈ *Question # 23 (a)*

$l_1:\ 3x-4y+3=0$ ....(i)

$l_2:\ 3x-4y+7=0$ ....(ii)

We first convert l_1 and l_2 in normal form

$(i) \Rightarrow -3x+4y=3$

Dividing by $\sqrt{(-3)^2+(4)^2} = \sqrt{25} = 5$

$$-\frac{3}{5}x+\frac{4}{5}y=\frac{3}{5}$$

Let $\cos\alpha = -\frac{3}{5} < 0$ and $\sin\alpha = \frac{4}{5} > 0$

$\Rightarrow$ α lies in 2nd quadrant and $\alpha = \cos^{-1}\left(-\frac{3}{5}\right) = 126.87^\circ$

$$\Rightarrow\ x\cos(126.87) + y\sin(126.87) = \frac{3}{5}$$

Hence distance of l_1 form origin $= \frac{3}{5}$

Now (ii) $\Rightarrow$ $-3x + 4y = 7$

Dividing by $\sqrt{(-3)^2 + (4)^2} = \sqrt{25} = 5$

$$-\frac{3}{5}x + \frac{4}{5}y = \frac{7}{5}$$

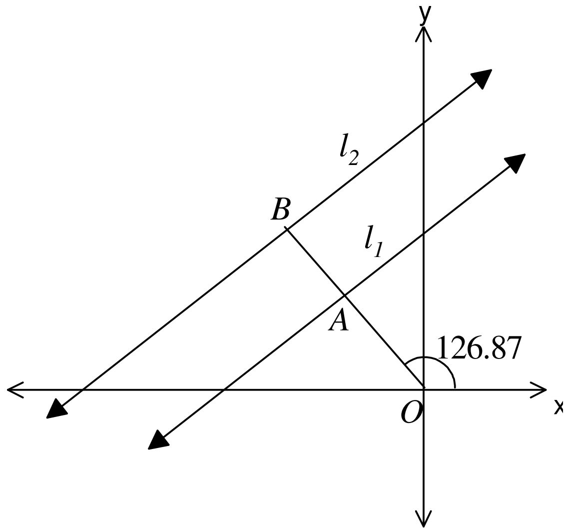
Let $\cos\alpha = -\frac{3}{5} < 0$ and $\sin\alpha = \frac{4}{5} > 0$

$\Rightarrow$ α lies in 1st quadrant

and $\alpha = \cos^{-1}\left(-\frac{3}{5}\right) = 126.87^\circ$

$$\Rightarrow\ x\cos(126.87) + y\sin(126.87) = 7/5$$

Hence distance of l_2 form origin $= 7/5$



From graph we see that both lines lie on the same side of origin therefore

Distance between lines $= |\overline{AB}| = |\overline{OB}| - |\overline{OA}| = \frac{7}{5} - \frac{3}{5} = \frac{4}{5}$

Let l_3 be a line parallel to l_1 and l_2, and lying midway between them. Then

Distance of l_3 from origin $= |\overline{OA}| + \frac{|\overrightarrow{AB}|}{2} = \frac{3}{5} + \frac{4/5}{2} = \frac{3}{5} + \frac{4}{10} = 1$

Hence equation of l_3

$$x\cos(126.87) + y\sin(126.87) = 1$$

$$\Rightarrow\ x\left(-\frac{3}{5}\right) + y\left(\frac{4}{5}\right) = 1 \quad \Rightarrow\ -3x + 4y = 5$$

$$\Rightarrow\ 3x - 4y + 5 = 0$$

◈ *Question # 23 (b)*

l_1: $12x + 5y - 6 = 0$.............(i)

l_2: $12x + 5y + 13 = 0$.............(ii)

We first convert l_1 and l_2 in normal form

$(i) \Rightarrow\ 12x + 5y = 6$

Dividing by $\sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$

$$\frac{12}{13}x + \frac{5}{13}y = \frac{6}{13}$$

Let $\cos\alpha = \frac{12}{13} > 0$ and $\sin\alpha = \frac{5}{13} > 0$

$\Rightarrow$ α lies in 1st quadrant and $\alpha = \cos^{-1}\left(\frac{12}{13}\right) = 22.62^\circ$

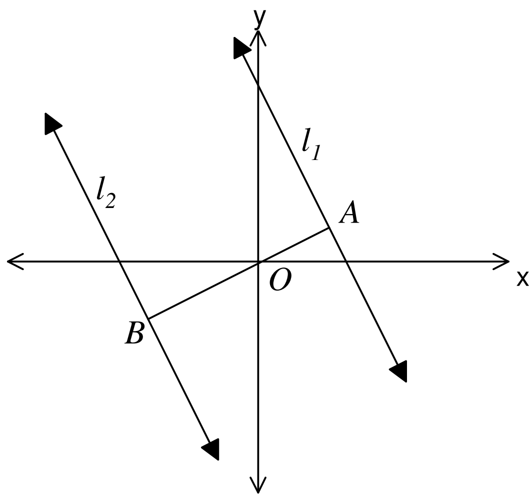
$$\Rightarrow \; x\cos(22.62) + y\sin(22.62) = \frac{6}{13}$$

Hence distance of l_1 form origin $= \frac{6}{13}$

Now $(ii) \Rightarrow -12x - 5y = 13$

Dividing by $\sqrt{(12)^2 + (5)^2} = \sqrt{169} = 13$

$$-\frac{12}{13}x - \frac{5}{13}y = 1$$



Let $\cos\alpha = -\frac{12}{13} < 0$ and $\sin\alpha = -\frac{5}{13} < 0$

$\Rightarrow$ α lies in 3rd quadrant

and $\alpha = \cos^{-1}\left(-\frac{12}{13}\right) = 202.62^\circ$

$$\Rightarrow \; x\cos(202.62) + y\sin(202.62) = 1$$

Hence distance of l_2 form origin $= 1$

From graph we see that lines lie on the opposite side of origin therefore

Distance between lines $= |\overline{AB}| = |\overline{OA}| + |\overline{OB}| = \frac{6}{13} + 1 = \frac{19}{13}$

Let l_3 be a line parallel to l_1 and l_2, and lying midway between them. Then

Distance of l_3 from origin $= |\overline{OB}| - \frac{|\overrightarrow{AB}|}{2} = 1 - \frac{19/13}{2} = 1 - \frac{19}{26} = \frac{7}{26}$

Hence equation of l_3

$$x\cos(202.62) + y\sin(202.62) = \frac{7}{26}$$

$$\Rightarrow \; x\left(-\frac{12}{13}\right) + y\left(-\frac{5}{13}\right) = \frac{7}{26} \qquad \Rightarrow -24x - 10y = 7$$

$$\Rightarrow \; 24x + 10y + 7 = 0$$

◈ *Question # 23 (c)*

Do yourself as Question # 23 (a)

◈ *Question # 24*

Let $l:\ 2x - 7y + 4 = 0$

Slope of $l = m = -\frac{2}{-7} = \frac{2}{7}$

Since required line is parallel to l

> **REMEMBER**
> If $l:\ ax + by + c = 0$
> then slope of $l = -\frac{a}{b}$

Therefore slope of required line $= m = \dfrac{2}{7}$

Now equation of line having slope $\dfrac{2}{7}$ passing through $(-4,7)$

$$y-7=\frac{2}{7}(x-(-4))$$
$$\Rightarrow\ 7(y-7)=2(x+4)$$
$$\Rightarrow\ 7y-49=2x+8 \quad \Rightarrow\ 2x+8-7y+49=0$$
$$\Rightarrow\ 2x-7y+57=0$$

Question # 25

Given: $A(-15,-18)$, $B(10,7)$ and $(5,8)$

Slope of $\overline{AB} = m = \dfrac{7-(-18)}{10-(-15)}$

$$=\frac{7+18}{10+15} = \frac{25}{25} = 1$$

Since required line is perpendicular to $\overline{AB}$

Therefore slope of required line $=-\dfrac{1}{m}=-\dfrac{1}{1}=-1$

Now equation of line having slope -1 through $(5,-8)$

$$y-(-8)=-1(x-5)$$
$$\Rightarrow\ y+8=-x+5$$
$$\Rightarrow\ x+y+8-5=0 \quad \Rightarrow\ x+y+3=0 \qquad \textit{Ans.}$$

Question # 26

Let l: $2x-y+3=0$

Slope of $l = m = -\dfrac{2}{-1} = 2$

Since required line is perpendicular to l

Therefore slope of required line $= -\dfrac{1}{m} = -\dfrac{1}{2}$

Let $y-$intercept of req. line $=c$

Then equation of req. line with slope $-\dfrac{1}{2}$ and $y-$intercept c

$$y=-\frac{1}{2}x+c \ \ldots\ldots\ldots\ldots (i)$$
$$\Rightarrow\ \frac{1}{2}x+y = c$$
$$\Rightarrow\ \frac{x}{2c}+\frac{y}{c} = 1$$

This is two intercept form of equation of line with

$x-$intercept $=2c$ and $y-$intercept $= c$

Since product of intercepts $=3$

$$\Rightarrow\ (c)(2c)=3 \qquad \Rightarrow\ 2c^2=3 \qquad \Rightarrow\ c^2=\frac{3}{2} \qquad \Rightarrow\ c=\pm\sqrt{\frac{3}{2}}$$

Putting in (i)

$$\Rightarrow\ y=-\frac{1}{2}x\pm\sqrt{\frac{3}{2}}$$

$$\Rightarrow\ \frac{1}{2}x+y\mp\sqrt{\frac{3}{2}}=0 \quad \Rightarrow\ \frac{1}{2}x+y\mp\sqrt{\frac{3\times 2}{2\times 2}}=0$$

$$\Rightarrow\ \frac{1}{2}x+y\mp\frac{\sqrt{6}}{2}=0$$

$\Rightarrow\ x+2y\mp\sqrt{6}=0$ are the required equations.

Question # 27

Let $A(1,4)$ be a given vertex and $B(x_1,y_1), C(x_2,y_2)$ and $D(x_3,y_3)$ are remaining vertices of parallelogram.

Since diagonals of parallelogram bisect at $(2,1)$ therefore

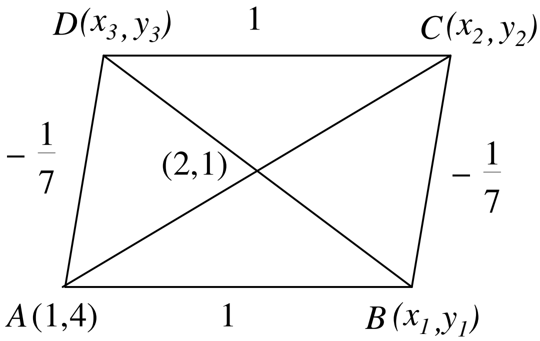
$$(2,1)=\left(\frac{1+x_2}{2},\frac{4+y_2}{2}\right)$$

$$\Rightarrow\ 2=\frac{1+x_2}{2} \quad \text{and} \quad 1=\frac{4+y_2}{2}$$

$$\Rightarrow\ 4=1+x_2 \quad , \quad 2=4+y_2$$

$$\Rightarrow\ x_2=4-1 \quad , \quad y_2=-4+2$$

$$\Rightarrow\ x_2=3 \quad , \quad y_2=-2$$



Hence $C(x_2,y_2)=C(3,-2)$

Now slope of $\overline{AB}=1$

$$\Rightarrow\ \frac{y_1-4}{x_1-1}=1 \quad \Rightarrow\ y_1-4=x_1-1$$

$$\Rightarrow\ x_1-y_1-1+4=0 \quad \Rightarrow\ x_1-y_1+3=0 \ \ldots\ldots\ldots\ldots\ldots\ (i)$$

Also slope of $\overline{BC}=-\frac{1}{7}$

$$\Rightarrow\ \frac{y_2-y_1}{x_2-x_1}=-\frac{1}{7} \quad \Rightarrow\ \frac{-2-y_1}{3-x_1}=-\frac{1}{7}$$

$$\Rightarrow\ -14-7y_1=-3+x_1 \quad \Rightarrow\ -3-x_1+14+7y_1=0$$

$$\Rightarrow\ x_1+7y_1+11=0 \ \ldots\ldots\ldots\ldots (ii)$$

Subtracting (i) and (ii)

$$\begin{array}{r} x_1-y_1+3=0 \\ \underline{\underset{-}{}x_1\underset{-}{+}7y_1\underset{-}{+}11=0} \\ -8y_1-8=0 \end{array}$$

$$\Rightarrow\ y_1+1=0 \quad \Rightarrow\ y_1=-1$$

Putting in (i)

$$x_1-(-1)+3 = 0 \quad \Rightarrow \quad x_1+4=0 \quad \Rightarrow \quad x_1=-4$$

$\Rightarrow \quad B(x_2,y_2) = B(-4,-1)$

Now E is midpoint of BD

$$\Rightarrow \quad (2,1) = \left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$$

$$= \left(\frac{-4+x_3}{2},\frac{-1+y_3}{2}\right)$$

$$\Rightarrow \quad 2 = \frac{-4+x_3}{2} \quad , \quad 1 = \frac{-1+y_3}{2}$$

$$\Rightarrow \quad 4 = -4+x_3 \quad , \quad 2 = -1+y_3$$

$$\Rightarrow \quad x_3 = 8 \quad , \quad y_3 = 3$$

$\Rightarrow \quad D(x_3,y_3) = D(8,3)$

Hence $(-4,-1)$, $(3,-2)$ and $D(8,3)$ are remaining vertex of $\|_{gram}$.

Position of the point with respect to line (Page 204)

Consider l: $ax+by+c = 0$ with $b>0$

Then point $P(x_1,y_1)$ lies

i) above the line l if $ax_1+by_1+c > 0$

ii) below the line l if $ax_1+by_1+c < 0$

Corollary 1 (Page 205)

The point $P(x_1,y_1)$ lies above the line if ax_1+by_1+c and b have the same sign and the point $P(x_1,y_1)$ lies below the line if ax_1+by_1+c and b have opposite signs.

Question # 28

(a) $\quad 2x-3y+6 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-2x+3y-6 = 0$$

Putting $(5,8)$ on L.H.S of above

$$-2(5)+3(8)+6 = -10+24-6 = 8 > 0$$

Hence $(5,8)$ lies above the line.

(b) Alternative Method

$$4x+3y-9 = 0$$ **Correction*

Putting $(-7,6)$ in L.H.S of given eq.

$$4(-7)+3(6)-9 = -28+18-9 = -19 \quad \ldots\ldots\ldots (i)$$

Since coefficient of y and expression (i) have opposite signs therefore $(-7,6)$ lies below the line.

Question # 29

(a) $2x-3y+6 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-2x+3y-6 = 0 \;\ldots\ldots\; (i)$$

Putting $(0,0)$ on L.H.S of (i)

$$-2(0)+3(0)-6 = -6 < 0$$

$\Rightarrow$ $(0,0)$ lies below the line.

Putting $(-4,7)$ on L.H.S of (i)

$$-2(-4)+3(7)-6 = 8+21-6 = 23 > 0$$

$\Rightarrow$ $(-4,7)$ lies above the line.

Hence $(0,0)$ and $(-4,7)$ lies on the opposite side of line.

(b) $3x-5y+8 = 0$

To make coefficient of y positive we multiply above eq. with -1.

$$-3x+5y-8 = 0 \;\ldots\ldots\; (i)$$

Putting $(2,3)$ on L.H.S of (i)

$$-3(2)+5(3)-8 = -6+15-8 = 1 > 0$$

$\Rightarrow$ $(2,3)$ lies above the line.

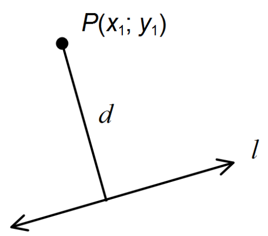
Putting $(-2,3)$ on L.H.S of (i)

$$\begin{aligned} -3(-2)+5(3)-8 &= 6+15-8 \\ &= 13 > 0 \end{aligned}$$

$\Rightarrow$ $(-2,3)$ lies above the line

Hence $(2,3)$ and $(-2,3)$ lies on the same side of line.

Perpendicular distance of $P(x_1,y_1)$ from line (Page 212)



The distance d from the point $P(x_1, y_1)$ to the line l, where l: $ax+by+c = 0$,

is given by: $$d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$$

Question # 30

$$l: \; 6x-4y+9 = 0$$

Let d denotes distance of $P(6,-1)$ from line l then

$$d = \frac{|6(6)-4(-1)+9|}{\sqrt{(6)^2+(-4)^2}} = \frac{|36+4+9|}{\sqrt{36+16}} = \frac{|49|}{\sqrt{52}} = \frac{49}{2\sqrt{13}}$$

Area of Triangular Region

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are vertices of triangle then

$$\text{Area of triangle} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

If A, B and C are collinear then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Question # 31 *Do yourself as below (Just find the area)*

Question # 32

Given: $A(2,3)$, $B(-1,1)$, $C(4,-5)$

$$\text{Area of } \Delta ABC = \begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 4 & -5 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\big(2(1+5) - 3(-1-4) + 1(5-4)\big)$$

$$= \frac{1}{2}(12+15+1) = \frac{1}{2}(28) = 14 \text{ sq. unit}$$

$\because$ Area of triangle $\neq 0$

$\Rightarrow$ A, B and C are not collinear.

Error Analyst		
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http://www.mathcity.org/error

Book: ***Exercise 4.3*** *(Page 215)*
Calculus and Analytic Geometry Mathematic 12
Punjab Textbook Board, Lahore.
Edition: May 2013.

Made by: ***Atiq ur Rehman*** *(Atiq@MathCity.org)*
Available online at http://www.MathCity.org *in PDF Format (Picture format to view online).*
Page Setup used A4
Printed: February 24, 2015.