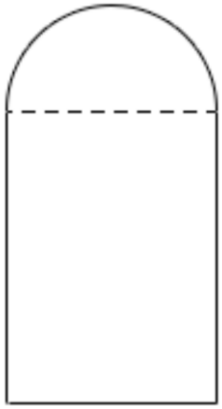
Optimum Design

Homework #4 (Due 05/28/2019)

Numerically solve the following problems by any commercial software or your own code. For example, you can use one of the Matlab optimization functions or the C code in Numerical Recipes in C.

1. A window is being built with the shape as the combination of a rectangle and a semicircle. If there is 10 meters of framing materials, what must the dimensions of the window be to let in the most light? (That is, maximize the area of the window with the rim of the window being equal to 10 meters.) Please solve this problem numerically instead of by hand calculation.



2. Solve the following muscle force distribution problem. The F_i values are to be solved. Values of A_i, M_i, and d_i are given below. (The solution and the background ideas can be found in the paper by R.T. Raikova and B.I. Prilutsky, 2001. Sensitivity of predicted muscle forces to parameters of the optimization-based human leg model revealed by analytical and numerical analyses. Journal of Biomechanics, 34, 1243-1255.)

$$\text{minimize } Z = \sum_{i=1}^{9} \left(\frac{F_i}{A_i}\right)^n \quad n = 2, \tag{1}$$

subject to:

$$f_1 = d_1F_1 - d_2F_2 - d_{3a}F_3 - M_1 = 0, \tag{2.1}$$

$$f_2 = -d_{3k}F_3 + d_4F_4 + d_{5k}F_5 - d_6F_6 - d_{7k}F_7 - M_2 = 0, \tag{2.2}$$

$$f_3 = d_{5h}F_5 - d_{7h}F_7 + d_8F_8 - d_9F_9 - M_3 = 0, \tag{2.3}$$

$$F_i \geqslant 0 \ (i = 1, 2, \ldots, 9), \tag{2.4}$$

The values of the constants are given below:
M1=4; M2=33; M3=31;
The values of d1, d2, d3a, d3k, d4, d5k, d5h, d6, d7k, d7h, d8, d9 are included in the following vector: d=[0.0298 0.044 0.044 0.0138 0.0329 0.0329 0.0279 0.025 0.025 0.0619 0.0317 0.0368];
The values of A_i are also included in the vector A=[11.5 92.5 44.3 98.1 20.1 6.1 45.5 31.0 44.3];

3. Solve the following problem. First use a linear programming method (e.g. simplex method, PAS method, …). Then use a nonlinear programming method (e.g. transform the problem into unconstrained optimization and use a nonlinear programming method of your choice).

Maximize $3x_1 + x_2 + 3x_3$ subject to

$$2x_1 + x_2 + x_3 \leqslant 2$$
$$x_1 + 2x_2 + 3x_3 \leqslant 5$$
$$2x_1 + 2x_2 + x_3 \leqslant 6$$
$$x_1 \geqslant 0,\ x_2 \geqslant 0,\ x_3 \geqslant 0.$$