Intermediate Macroeconomics Recitation 6

Topics: Growth rate and growth series on a log scale, Malthus model (economics and equations, the steady-state, graphical solutions for population and wage dynamics after shocks).

1 The Malthus Model

1.1 Introduction

The Malthus model proposes a mechanism for why countries do not grow in per capita terms based upon the idea that whenever people get richer, birth rates rise until they return to subsistence levels. This can explain the following features:

- Why developed countries did not appear to grow from antiquity to the Industrial Revolution.
- Why some developing countries have high birth rates but do not grow in per capita terms.
- Why some developing countries believe that restricting birth rates will aid growth i.e. China, Vietnam.

1.2 Economics and equations in Malthus

1.2.1 Wage Condition

In the Malthus model, there are two equations. The first is the wage as a function of population, which comes from firms' optimal behavior:

$$w_t = (1-a)A_t\left(\frac{D}{HN_t}\right)^a \tag{1}$$

where D is land, H is hours, N_t is population, A_t is technology. Notice that this looks a lot like the firm maximisation condition for labor we derived earlier in the course ($w_t = (1-a)A_t(\frac{K_t}{L_t})^a$. The differences are that our 'capital' is now land (so it is fixed) and we add in an extra H term to represent human capital.

1.2.2 Population Condition

The second equation is the evolution of population as a function of the wage:

$$\frac{N_{t+1}}{N_t} = \left(\frac{w_t}{w^s}\right)\xi_t \tag{2}$$

where w^s is the subsistence wage (say, \$2 per day for food) and ξ_t is an exogenous shock to population. Note that this equation is given to you, it does not come from an economic behavior or optimality conditions.

Make sure you understand the economic intuition behind each of the two equations.

First, for the wage equation, suppose there is a plague, and half the population dies. The remaining population has more land to cultivate, and each farmer produces more than what he could produce before, so the wage increases. Conversely, suppose population increases: the new farmers have less land than their ancestors, they produce less with their share of land, and the wage falls. There are diminishing returns to labor.

Second, for the evolution of population, suppose that the wage increases. Then the farmers have more food for their children, which reduces infant mortality and increases population. Or suppose there is a plague in the form of a shock $\xi_t < 1$. Then population decreases.

We can rewrite eq. (2) without w_t to get an equation purely in terms of labor. Inputting w_t from eq. (1) into eq. (2) yields:

$$\frac{N_{t+1}}{N_t} = \left(\frac{(1-a)A_t \left(\frac{D}{HN_t}\right)^a}{w^s} \right) \xi_t, \tag{3}$$

We can simplify eq. (3) by replacing exogenous variables with a new variable ϕ:

$$N_{t+1} = \phi A_t N_t^{1-a} \xi_t \tag{4}$$

where:

$$\phi = \frac{1-a}{w^s} \left(\frac{D}{H}\right)^a \tag{5}$$

1.3 The steady-state in Malthus

1.3.1 Solving for the steady-state analytically

We look for the steady state of the model. The steady state are the values of the variables N_t, w_t in the long-term when there aren't any shocks. If there aren't any shocks then $\xi_t = 1$ (since there won't be any plagues), $A_t = \bar{A}$ (since productivity must be constant). We define the values that N_t, w_t will take in the long-term without shocks as $\bar{N}, \bar{w}$. Then eqs. (1) and (2) become:

$$\bar{w} = (1-a)\bar{A} \left(\frac{D}{H\bar{N}}\right)^a$$

$$1 = \left(\frac{\bar{w}}{w^s}\right)$$

We can simplify these equations to get:

$$\bar{N}^a = \frac{1-a}{\bar{w}} \bar{A} \left(\frac{D}{H}\right) \tag{6}$$

$$\bar{w} = w^s \tag{7}$$

Using the definition of ϕ (eq. (5)), we can rewrite section 1.3.2 as:

$$\bar{N}^a = \frac{1-a}{\bar{w}} \bar{A} \left(\frac{D}{H} \right) \tag{8}$$

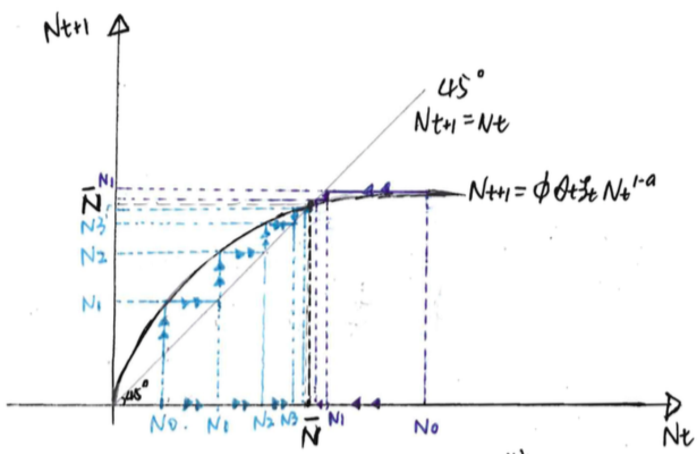
1.3.2 Solving for the steady-state graphically

To solve for the steady state graphically, we can plot eq. (4) i.e. N_t on the x-axis and N_{t+1} on the y-axis. We also add a 45 degree line. When eq. (4) meets the 45-degree line, we know that $N_{t+1} = N_t$ so (assuming there are no shocks), we will be in steady state.

What happens if we start at a point away from the steady state i.e. N_0?

- We put N_0 on the x-axis.
- We use eq. (4) to get N_1.
- We then reflect N_1 in the 45-degree line to put it on the x-axis.
- We can then use eq. (4) to get N_2.
- We can repeat this indefinitely.

We show this in section 1.3.2. We observe that we will always converge to the steady state if there are no shocks, even if we start away from the steady state.



1.4 Graphical solutions to the population and wage dynamics in response to shocks

We are interested in the *dynamics* of population N_t and real wage w_t after shocks to technology and to population in the economy, respectively. We solve graphically by answering the following questions:

(1) What is the *short-run* effect? What happens in the period of the shock?

(2) What changes in the population dynamics and real wage/population relationship? Will there be a *shift* of the curves, or a *movement along* the curves?

(3) What is the *long-run* effect? What changes in the steady state of population, $\bar{N}$, and real wage, $\bar{w}$?

Note that in section 1.3.1, we solved for the *steady-state wage* $\bar{w} = w^s$. Since the subsistence wage w^s is an *exogenous* variable, the steady-state wage also does not change in Malthus' model.

1.4.1 A permanent technology shock

A shock to technology at time t increases or decreases the productivity from A_{t-1} to A_t. If the shock is permanent, then productivity stays at the level of A_t for all future periods.

First, what happens to the population dynamics? Recall the equation that characterizes the evolution of population over time:

$$N_{t+1} = \phi A_t \xi_t N_t^{1-a}$$

Then, recall the equation in Malthus that comes from firms' optimal behavior and characterizes the relationship between real wage and population,

$$w_t = \frac{\phi w^s A_t}{N_t^a}$$

Graphically, an increase in technology ($A_{t+1} > A_t$) results in outward shifts of both the population dynamics curve and the the real wage/population curve as below:

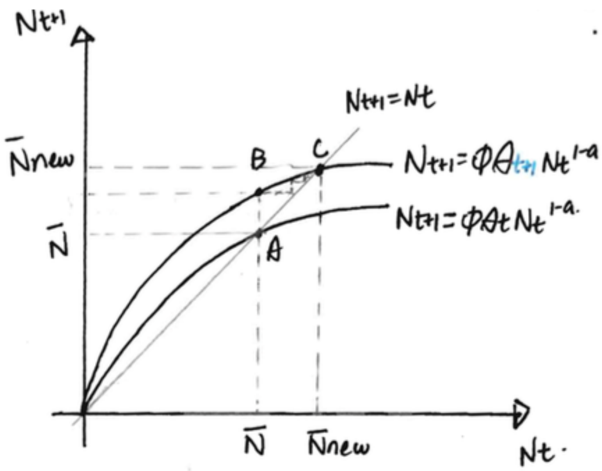


Figure 1

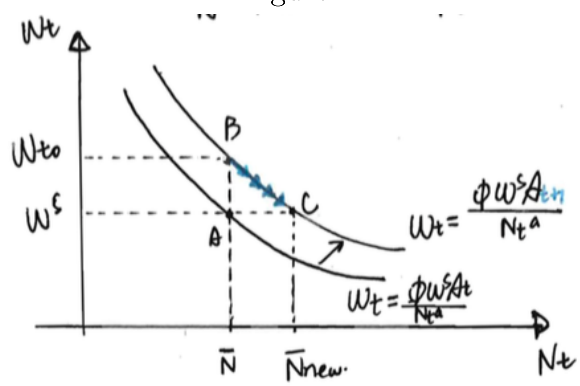


Figure 2

A to B: In the next period, N_{t+1} is predicted by the outer curve (B instead of A in Figure-1). An increase in technology increases the marginal product of labor in the short run. Wage increases at time t (A to B in Figure-2).

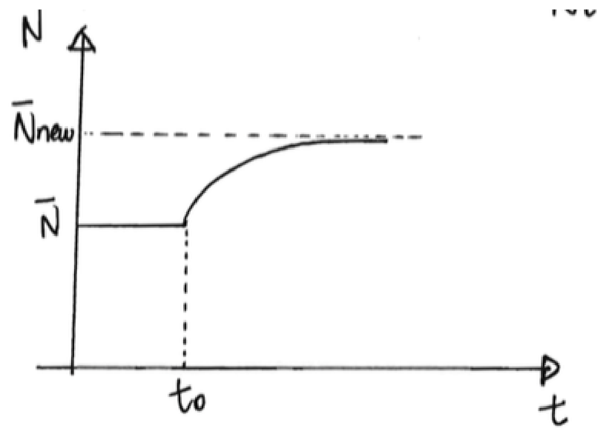
B to C: Population grows whenever wages are above subsistence ($w_t > w^s$). It keeps increasing until reaching a new, higher steady state (C in Figure-1). We can verify that the new steady state of population is higher ($\bar{N}_{new} > \bar{N}$) by taking the first-order derivative of $\bar{N}$ w.r.t to A_t. For $a \in (0, 1)$,

$$\frac{\partial \bar{N}}{\partial A_t} = \frac{1}{a}\phi^{\frac{1}{a}} A_t^{\left(\frac{1}{a}-1\right)} > 0$$

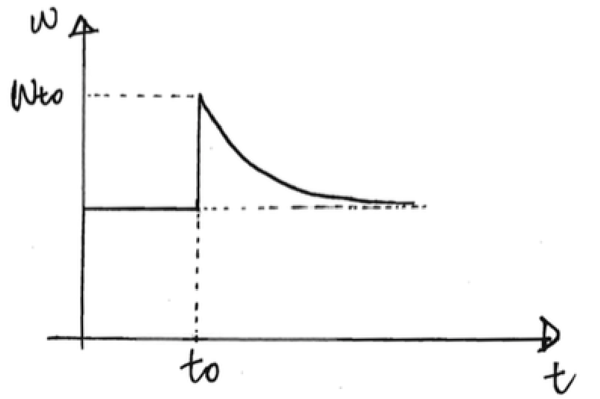
The steady-state wage does not change, as shown before, $\bar{w} = w^s$. The continuous increase in population until reaching the new steady state is a *movement along* the new curve in Figure-2. Since it is a downward-sloping curve, wage falls back down to the *subsistence wage* w^s as before the shock.

How do we plot the time series of population and real wage? In other words, how do they change with time after a permanent technology shock at time t_0?

Population gradually increases from $\bar{N}$ to $\bar{N}_{new}$, and once it has reached $\bar{N}_{new}$, it stays there for future periods. But how does the *slope* of the curve change with time? Since the curve for population dynamics is increasing and concave (Figure-1), its distance from the *45 degree* line decreases as time goes and as it gets closer to the new steady state. In other words, population increases at a decreasing rate. Thus, the time series is increasing and concave, as follows:



Wage spikes at time at t_0 to a high level in the short run, then it starts decreasing back to w^s. Since the w_t/N_t curve is decreasing and convex, even if population N_t increases at a constant rate, wage w_t still decreases at a decreasing rate. Now that we have shown N_t changes at a decreasing rate, wage certainly decreases at a decreasing rate. Thus, its time series is decreasing and convex.



1.4.2 A short-run shock to population (a "plague shock")

An exogenous shock to population at time t is captured by ξ_t in the population dynamics.

$$N_{t+1} = \phi A_t \xi_t N_t^{1-a}$$

When there is no shock, $\xi_t = 1$. With a shock, $\xi_t < 1$. *For one period only*, there is an inward shift of the population dynamics curve. There is no change in the real wage/population curve.

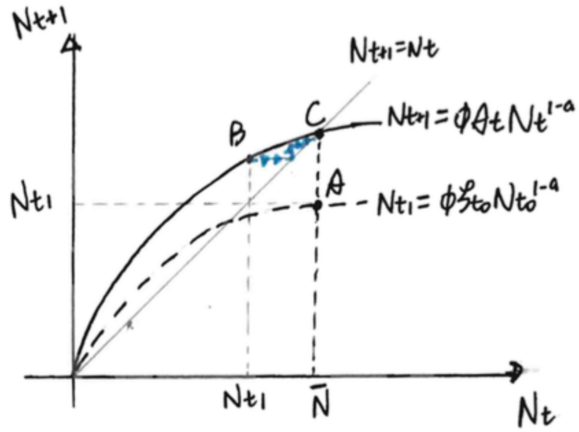


Figure 3

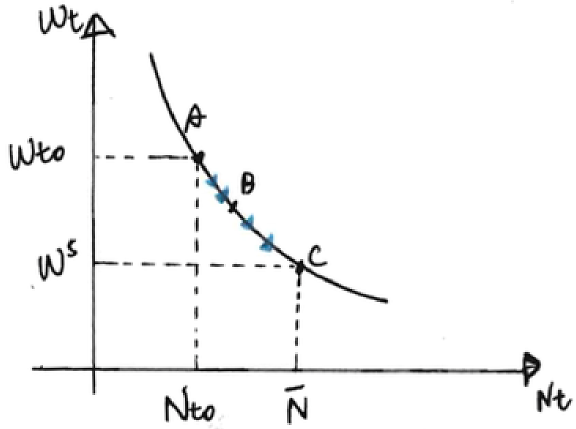


Figure 4

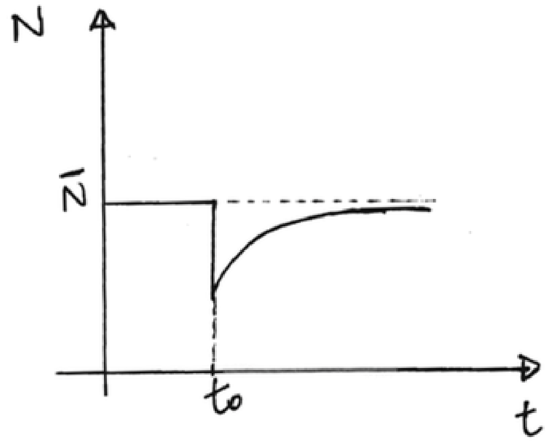
A: In this period, due to the plague, population drops directly to $A(\bar{N}, N_1)$ which lies on the lower curve (Figure-3). The survived population has *on average* more land to cultivate. Marginal product of labor, and thus wage, increases in the short run (Figure-4).

A to B: In the next period, N_{t+1} is again predicted by the intersection between the original population dynamics curve and the *45 degree* line (Figure-5). An population increases, marginal product of labor, and thus wage, starts to fall back down *along* the same curve (Figure-6).

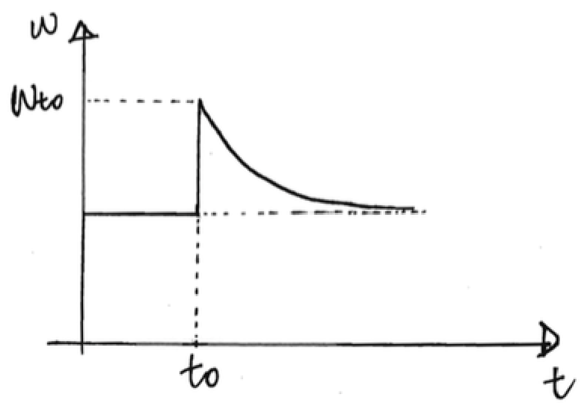
B to C: Population grows whenever wages are above subsistence ($w_t > w^s$). It keeps increasing until reaching the original steady state (C in Figure-5). Wage falls back to the original $\bar{w} = w^s$. In the long run, both population and wage are the same as before the shock.

How do we plot the time series of population and real wage?

Population falls in the period of shock t_0, but gradually increases back up until reaching $\bar{N}$ again. The analysis of the slope is the same as before. The time series is increasing and concave, as follows:



Wage spikes at time t_0 to a high level in the short run. Then it starts decreasing back to w^s. The analysis of shape is again the same. Thus, its time series is decreasing and convex from t_0 onwards.



1.5 Raising Per Capita Income

We saw in section 1.4.1 that even if there is a permanent increase in productivity, the wage will return to its subsistence level w^s so per capita income will stay the same in the long-term.

It is worth noting that Malthus actually wrote his model to argue against redistribution towards the poor for a similar reason. Within his model, redistribution towards the poor will not raise wages for the poor in the long-term. In the short-term, it will raise wages but this will lead to an increase in the population and then a subsequent fall back to the subsistence wage. And, without redistribution towards the poor, society may enjoy more of the goods that the rich can afford like cathedrals, palaces, monuments, art etc.

So what can we do to raise per capita income in the Malthus model? We see in eq. (7) that $w_t = w^s$ in the long-term. Therefore, the only way to raise per capita income is to raise w^s.

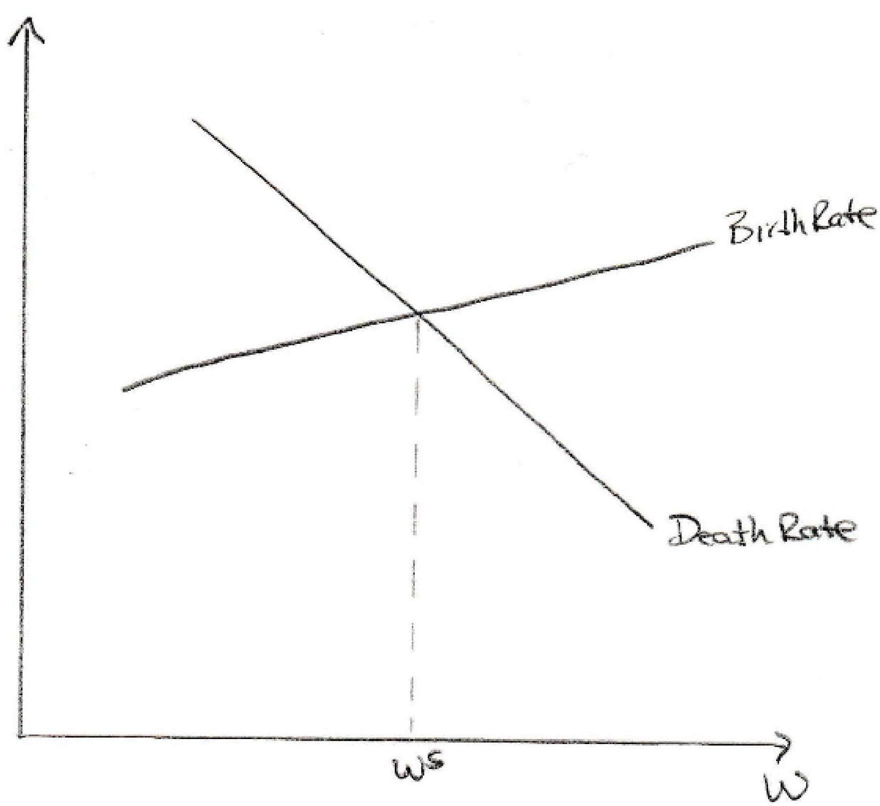
Let's consider where this w^s comes from. To do this, let's think more about how the population will grow. The population growth will depend upon how many people are born and die. How does this depend upon the wage?

- If the wage rises, the birth rate rises since people can afford more children.
- If the wage rises, the death rate falls since people can afford better healthcare and live for longer.

The point at which the rates intersect will be when population growth is constant (the number of people dying equals the number of people born). Since population growth is constant, we know that the wage at this point must be w^s.

We can represent this graphically:

Figure 5: Birth Rate and Death Rate Impact upon w^s

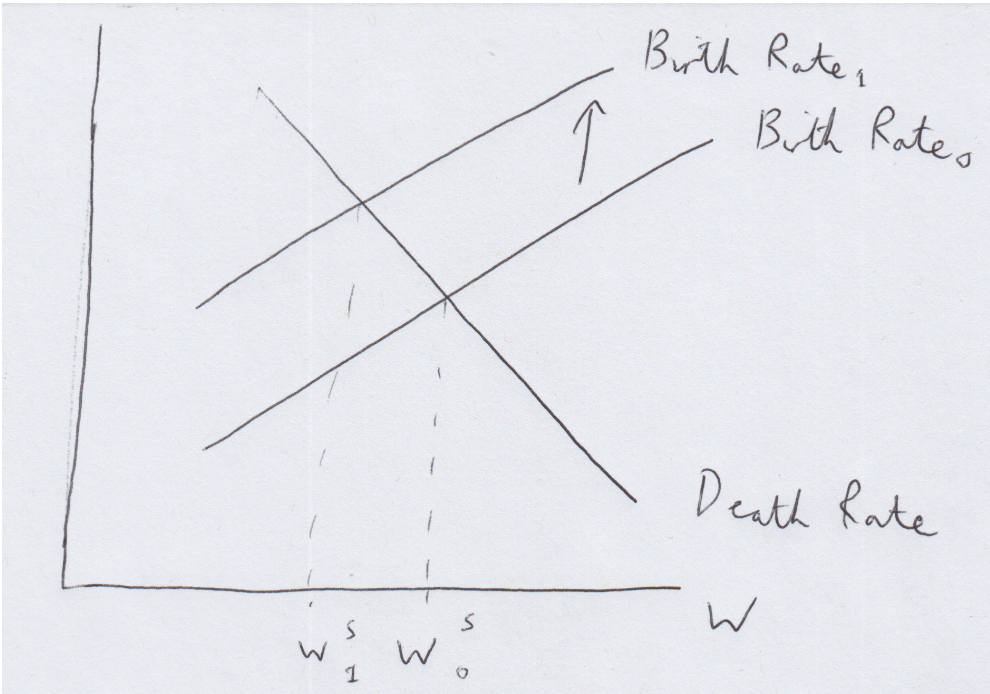


Since w^s is determined by the intersection of the birth rate and the death rate, we can change w^s by one of two methods:

- Change the birth rate.
- Change the death rate.

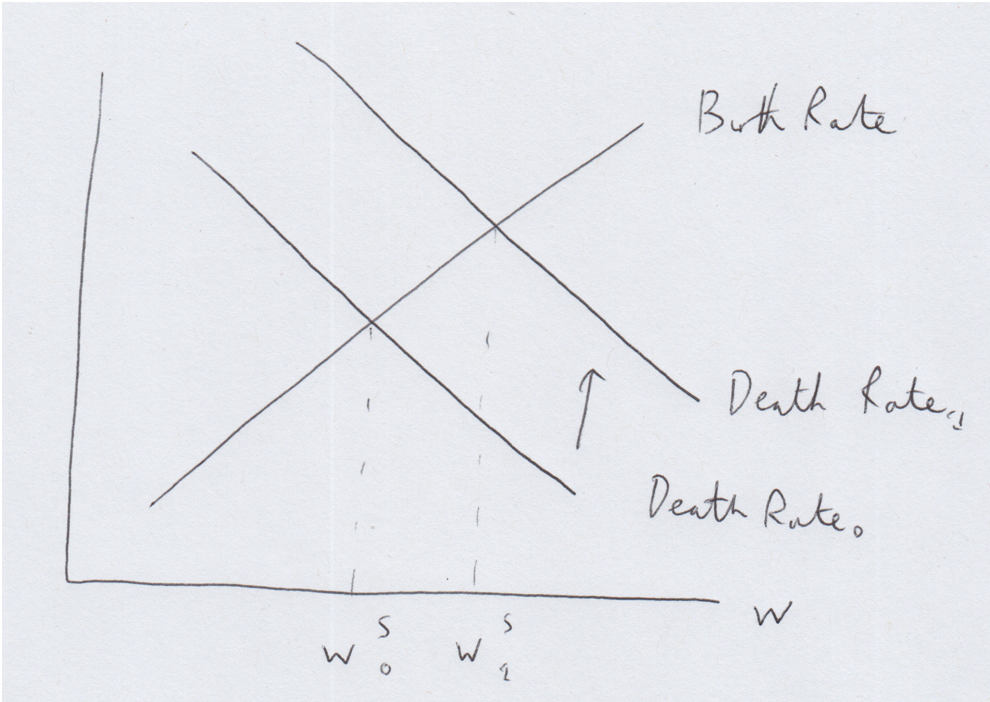
When the birth rate rises, the population growth is higher for any given wage. Therefore, for population growth to be zero, we must have that w^s falls. This is shown in the following graph:

Figure 6: Increase in Birth Rate



When the death rate rises, the population growth is lower for any given wage. Therefore, for population growth to be zero, we must have that w^s rises. This is shown in the following graph:

Figure 7: Increase in Death Rate



This implies that if a country has a relatively higher death rate or a relatively lower birth rate then that country will have a higher w^s and thus higher wages in general. In pre-industrial time:

- Japan was relatively clean compared to the squalor of Europe.
- There were a lot of wars in Europe.

This may explain why wages were relatively higher in pre-industrial times in Europe than in China or Japan.