Intermediate Macroeconomics Recitation 3

Topics: Indifference Curves, Notions of Efficiency, Marginal Rate of Substitution,

1 Indifference Curves

An indifference curve is a level curve of the utility function, so one indifference curves marks all the points where the level of utility received is the same. Mathematically, an indifference curve consists of all points (x, y) that satisfy

$$\bar{U} = U(x, y)$$

for some utility function $U(x, y)$ and some fixed level of utility $\bar{U}$.
This means that indifference curves never overlap, as each of them marks an unique level of utility. Usually we assume that it is always preferable to have more of a good, which means that the individual is better off being on a "higher" indifference curve (further up and to the right). Indifference curves are also usually convex and downward sloping. The negative slope of the indifference curve shows that to stay indifferent between two bundles of goods, in exchange for giving up a good one has to get more of another good instead. The convexity of the curve means that there is diminishing marginal utility - if you have a lot of one good, you are more likely to give up a lot of that good in exchange for some other good that you don't have as much of than if you have equal amounts of both goods.

Example 1. *Consider Robinson Crusoe on island where there are coconuts and shelter. We can use indifference curves to mark all combinations of coconuts and shelter that give Robinson Crusoe the same utility, so that he is indifferent between them.*

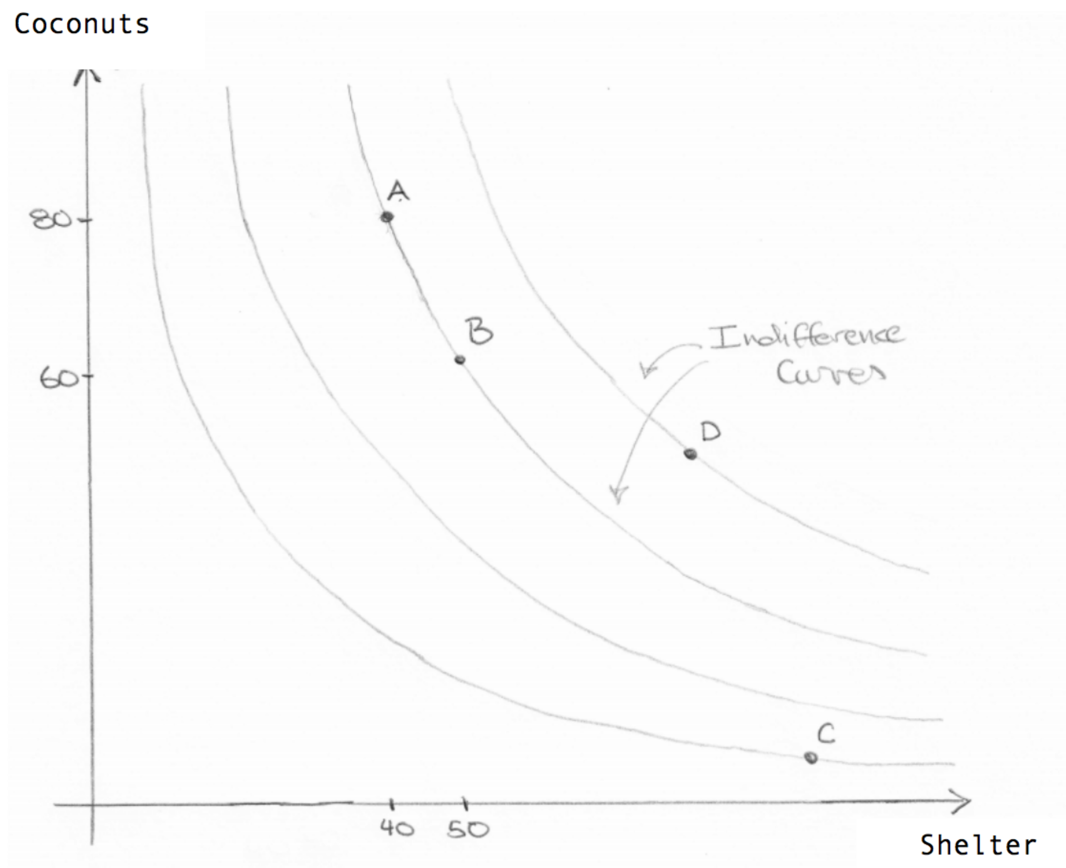

Figure 1: Indifference Curves

Here, Robinson Crusoe is indifferent between bundles A and B, since they are on the same indifference curve, whereas D is on a higher indifference curve and would give him more utility than A or B, and C is the least preferred of all options as it is on the lowest curve.

2 Marginal Rate of Substitution

As discussed before, the slope of an indifference curve is the rate at which an individual could exchange the two goods and still be on the same indifference curve, i.e. be indifferent between them. Since that rate is so important, for example in determining the optimal allocation as seen above, it has its own name: the Marginal Rate of Substitution (MRS).

2.1 Derivation

The MRS is by definition the amount of good y you have to receive in order to give up one unit of good x so that your utility is unchanged. The MRS is also the slope of the indifference curve, for goods x and y we can define it as the derivative of the indifference curve $\frac{dy}{dx}$. We express the MRS as a positive number but indifference curves slope downwards as discussed below, so:

$$MRS = -\frac{dy}{dx}.$$

We can show that these two definitions are equivalent:
First, remember that we defined an indifference curve as all points on a utility function that give the individual the constant utility $\bar{U}$ such that

$$\bar{U} = U(x, y).$$

That means that by definition an individual's utility cannot change when moving along the indifference curve. This means that the change in utility has to be zero since utility is constant along the indifference curve:

$$dU = 0 \tag{1}$$

Now, dU equals the change in utility generated from changing the amount of x by dx and the amount of y by dy, or mathematically:

$$dU = \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy \tag{2}$$

We can combine and rewrite (1) & (2):

$$\begin{aligned} 0 &= \frac{\partial U}{\partial x} dx + \frac{\partial U}{\partial y} dy \\ \frac{\partial U}{\partial y} dy &= -\frac{\partial U}{\partial x} dx \\ -\frac{dy}{dx} &= \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} \end{aligned}$$

So we have shown that

$$MRS = -\frac{dy}{dx} = \frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}},$$

or that the MRS equals the negative slope of the indifference curve, which is the same as the ratio of marginal utilities of receiving good x to the marginal utility of receiving y at point (x, y). Since indifference curves are not straight lines but convex, we know that the MRS varies along the indifference curve: The MRS is highest when x is low and y is high (because the marginal utility of receiving x is much higher than the marginal utility of y, since we have so much more of y than of x), so we are willing to give up a large amount of y in order to receive one x, which makes a really steep indifference curve or equivalently a large MRS.

3 Three Notions of Efficiency

3.1 Exchange Efficiency

Exchange efficiency can be thought of as the scenario where no mutually advantageous trade is possible.

We can investigate properties of the economy under exchange efficiency by considering a simple problem. Imagine that an agent gets utility from consuming two different goods x, y. They have W to spend and the prices of x, y are respectively P_x, P_y. Then the agent faces the following problem:

$$\max_{x,y} U(x, y)$$

s.t.

$$P_x x + P_y y \leq W$$

We can assume that agents always want to consume as much as possible so they get higher utility from consuming more (nonsatiable preferences). Then the budget constraint binds:

$$P_x x + P_y y = W$$

We can rewrite the budget constraint as:

$$y = \frac{W}{P_y} - \frac{P_x}{P_y} x$$

We can input the constraint into the utility function:

$$\max_{x} U(x, \frac{W}{P_y} - \frac{P_x}{P_y} x)$$

Taking FOC with respect to x:

$$\frac{\partial U}{\partial x} - \frac{\partial U}{\partial y} \frac{P_x}{P_y}$$

Rearranging:

$$-\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{P_x}{P_y}$$

We call the left-hand side the marginal rate of substitution. It is how many extra y the consumer will have to gain to maintain the same level of utility if they lose one x. We call the right-hand side the marginal rate of transformation. It is how many extra y the consumer can buy if they buy one less x. We see that for exchange efficiency to hold we must have that the marginal rate of substitution equals the marginal rate of transformation. This must be true for all consumers. If it is not then there would be gains from trading.

3.2 Production Efficiency

Production efficiency is the case in which the individual allocates the optimal amounts of inputs to produce the outputs that it can trade and consume. In the examples above we did not think about where the coconuts and shelter to be allocated came from, but in the full Robinson-Crusoe-model, Robinson and Friday use inputs (labor and land) to produce the goods they trade and consume. In the first recitation we talked about the production function and optimal levels of production. We saw that is was optimal for the firm to produce until the marginal revenue from producing another unit equals the marginal cost, which means that profits are maximized. A similar notion characterizes production efficiency, specifically that the marginal benefit of producing more of a good has to equal the marginal cost it creates.

In order to understand the conditions characterizing production efficiency, let's assume that Robinson can use his time to produce coconuts and shelter, as above. He uses land (N) and labor (L) to do so, and the production functions of coconuts (c) and shelter (s) are denoted $F_c = F_c(L, N)$ and $F_s = F_s(L, N)$ respectively. Robinson derives utility $U = U(c, s)$ from these coconuts and shelter.

Now, if Robinson wants uses ϵ more hours to produce coconuts, he produces $\frac{\partial F_c}{\partial L}\epsilon$ more coconuts, which give him additional utility of $\frac{\partial U}{\partial c}\frac{\partial F_c}{\partial L}\epsilon$. This is the marginal benefit from producing more coconuts.

However, producing more coconuts also carries a cost, as Robinson will have to give up producing shelter. Specifically, the decrease in shelter he produces by using ϵ more hours to produce coconuts equals $\frac{\partial F_s}{\partial L}\epsilon$, and the utility he looses from that equals $\frac{\partial U}{\partial s}\frac{\partial F_s}{\partial L}\epsilon$, which is the marginal cost from producing more coconuts.

In equilibrium, $MB = MC$. This is optimal because if marginal benefit is larger than marginal cost, Robinson will be better off by producing more coconuts, as he gets more out of having additional coconuts than what he loses by giving up the necessary amount of shelter. Likewise, if marginal cost was larger than marginal benefit, Robinson would produce less coconuts, increasing their marginal benefit and decreasing the marginal costs of giving up shelter until the two are equal. Hence, for efficient production:

$$MB = MC$$

$$\frac{\partial U}{\partial c}\frac{\partial F_c}{\partial L}\epsilon = \frac{\partial U}{\partial s}\frac{\partial F_s}{\partial L}\epsilon$$

$$\frac{\frac{\partial U}{\partial c}}{\frac{\partial U}{\partial s}} = \frac{\frac{\partial F_s}{\partial L}}{\frac{\partial F_c}{\partial L}}$$

Optimally, the ratio of marginal utilities is equal to the ratio or marginal production for both goods, or in other words, Robinson will allocate resources such that the marginal benefit of producing one more coconut equals the marginal cost of doing so.

Note that here we only discuss the optimal allocation of labor, but the idea of optimally allocating land follows the exact same logic.

3.3 Pareto Efficiency

An allocation is Pareto efficient if no one can attain a higher level of utility without decreasing someone else's utility.
In the Robinson Crusoe economy where Robinson and Friday use labor and land to produce coconuts and shelter, Pareto efficiency will be given if there is exchange efficiency, so if no trade can be made that makes Robinson better off without making Friday worse off (and vice versa), and production efficiency, which implies that neither of them can improve by adjusting its allocation of inputs to produce of coconuts of shelter. If both production and trade are efficient, this means that there is no other allocation of inputs or outputs that can make both Robinson and Friday better off, hence constituting Pareto efficiency.

If we want to be able to define whether an allocation of goods is "efficient", we need to define the preferences of the agents who receive those allocations. We do that by describing utility functions, which we can illustrate using indifference curves. An indifference curve is a line of different combinations of goods that an individual is exactly as happy to have - it is indifferent between them.