Intermediate Macroeconomics Recitation 2

Topics: Empirical fit of the production model, TFP differences (Jones 4.3-4.5), review of labor supply by calculus of variations.

1 Analyzing the Production Model

We have solved for the general equilibrium of the production model in terms of Y, the total (aggregate) output. However, it is output *per capita* that determines a country's welfare.

Definition 1. *$y = \frac{Y}{L}$ is the output per person. $k = \frac{K}{L}$ is the capital per person.*

Then plugging the two new variables, y and k, back into the previous equilibrium,

$$y^* = \frac{Y^*}{L^*} = \frac{\bar{A}\bar{K}^{1/3}\bar{L}^{2/3}}{\bar{L}} = \bar{A}\bar{k}^{1/3}$$

Output per person (y) is higher when (1) productivity (A) is higher, and (2) capital per person (k) is higher. Note that increasing capital per person leads to **diminishing returns**.

1.1 Empirical Fit of the Production Model (Jones 4.3)

$$y^* = \bar{A}\bar{k}^{1/3}$$

This is an **empirical implication** of the model. **Development accounting** is using this equation to try to account for differences in output across countries.

Data that we need to check this implication:

(1) Output, Capital, Labor - can be measured

(2) Technology - hard to measure

Start with the assumption that all countries have the same level of technology (A normalized to 1). We compare the output per person of country x with that of the U.S.:

$$\bar{A}_x = \bar{A}_{US} = 1$$

$$\frac{y^*_x}{y^*_{US}} = \left(\frac{\bar{k}_x}{\bar{k}_{US}}\right)^{1/3}$$

Figure 1: The Model's Prediction for Per Capita GDP (U.S.=1). Note: assuming no difference in productivity, the model predicts smaller differences in income across countries than we observe.

Country	Observed capital per person, $\bar{k}$	Predicted per capita GDP $y = \bar{k}^{1/3}$	Observed per capita GDP
United States	1.000	1.000	1.000
Switzerland	1.287	1.088	0.870
Japan	1.173	1.055	0.713
Italy	0.927	0.975	0.672
Spain	0.908	0.968	0.733
United Kingdom	0.661	0.871	0.750
Brazil	0.134	0.512	0.225
China	0.127	0.502	0.183
South Africa	0.098	0.461	0.244
India	0.044	0.352	0.089
Burundi	0.003	0.149	0.015

TABLE 4.3 The Model's Prediction for Per Capita GDP (U.S. = 1)

Figure 2: Predicted Per Capita GDP in the Production Model

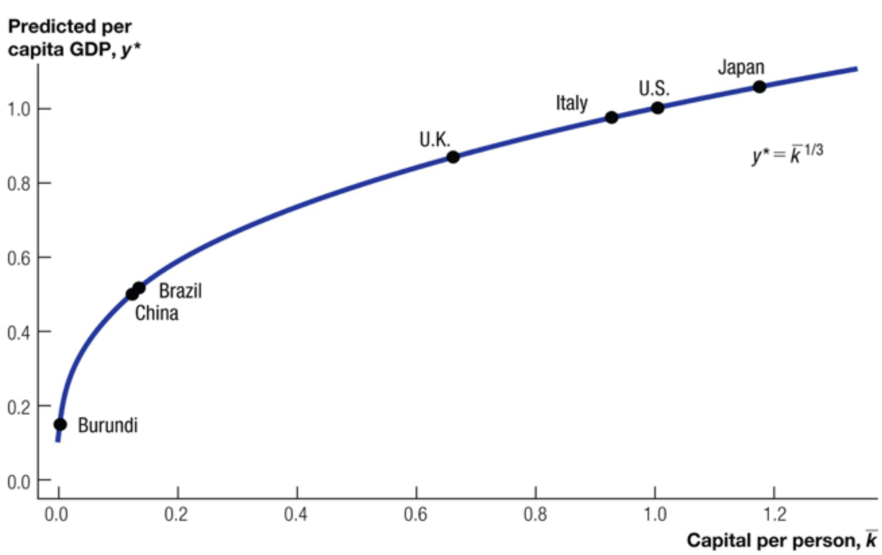

FIGURE 4.4 Predicted Per Capita GDP in the Production Model

Figure 3: The Model's Prediction for Per Capita GDP (U.S.=1). Note: if the model prediction is successful, the data points should line up close to the 45-degree line.

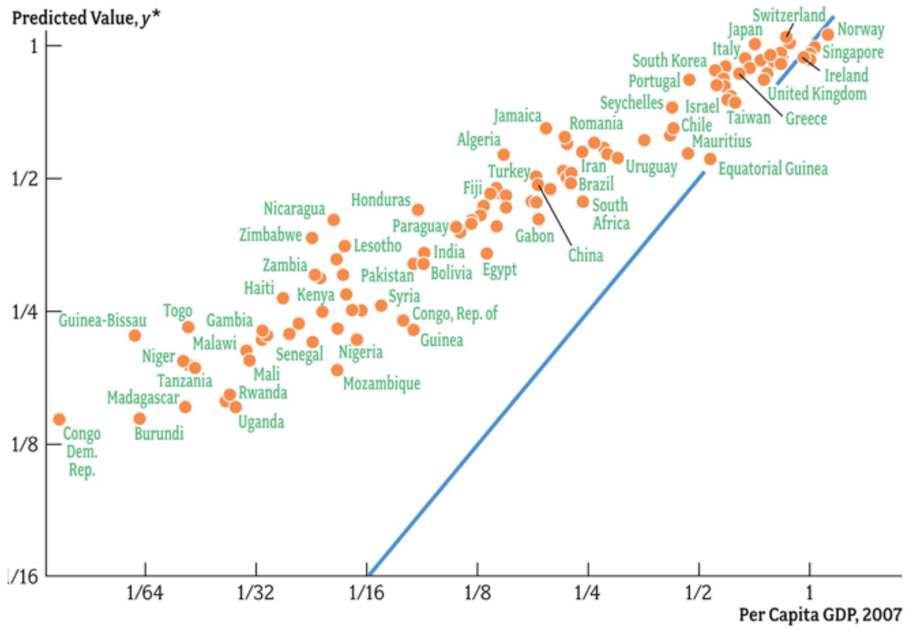

FIGURE 4.5 The Model's Prediction for Per Capita GDP (U.S. = 1)

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The model gets the direction right: Poor countries produce less than their capital stocks. However, it systematically predicts countries to be richer than they actually are.

Possible reasons for the bad fit:

(1) Mismeasurement in output, capital, labor

(2) Assumptions of model might be wrong: Functional form of production function; Skilled vs. unskilled labor; Some critical inputs might be missing (O-ring Theory) etc.

We also assumed that countries have the same level of technology. To improve the fit of the model, we should develop a model to adjust for the differences in technology among countries.

Optional: Why doesn't capital flow from rich to poor countries? (Jones 4.3 Case Study)

2 Total Factor Productivity (TFP) Differences

Technology can be broadly interpreted as the **efficiency of production**. One limitation is that we don not have independent measure of the TFP. However, we have data on y and k for each country. Assuming the model is correct, we can calculate the level of TFP for each country that makes the model fit exactly.

Taking the ratio of TFPs for two different countries allows a further break down on what technology consists of,

$$\frac{A_x}{A_{US}} = \left(\frac{\frac{y_x}{y_{US}}}{\frac{k_x}{k_{US}}} \right)^{1/3}$$

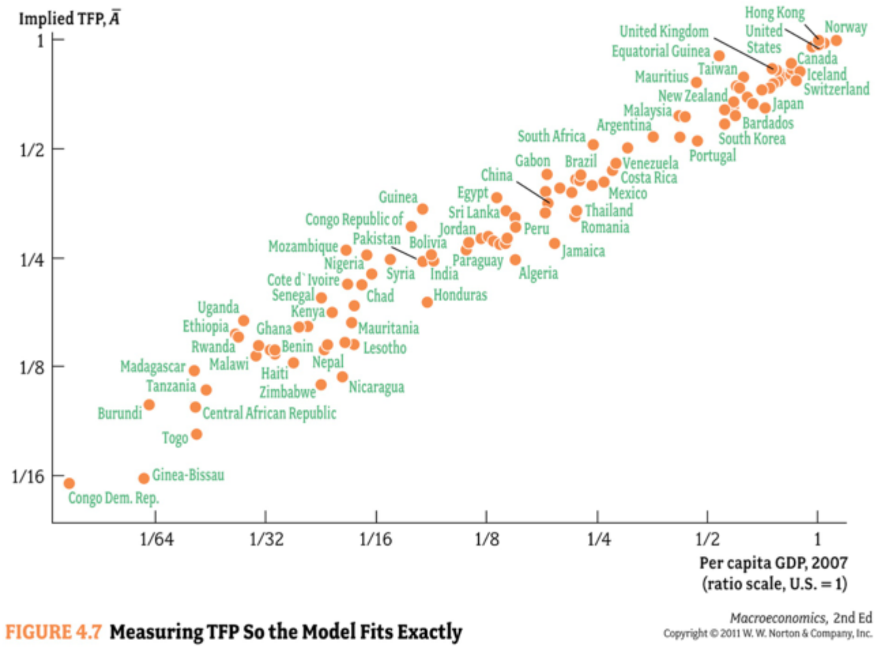
2.1 Taking the Model to Data (Jones 4.3)

Figure 4: Measuring TFP so the Model Fits Exactly. Note: in order for the model to match the data, poor countries must be very inefficient in production, i.e. they have low TFP.

Country	Per capita GDP (y)	$\bar{k}^{1/3}$	Implied TFP ($\bar{A}$)
United States	1.000	1.000	1.000
Switzerland	0.870	1.088	0.800
United Kingdom	0.750	0.871	0.861
Spain	0.733	0.968	0.757
Japan	0.713	1.055	0.676
Italy	0.672	0.975	0.689
South Africa	0.244	0.461	0.530
Brazil	0.225	0.512	0.439
China	0.183	0.502	0.365
India	0.089	0.352	0.253
Burundi	0.015	0.149	0.101

TABLE 4.4 Measuring TFP So the Model Fits Exactly

Figure 5: Measuring TFP so the Model Fits Exactly



Observations:

(1) TFP differences can be up to a factor of 10 among countries.

(2) Both differences in TFP and differences in capital per capita explain the differences in per capita GDP. We can do an accounting exercise to determine the relative importance of these two factors.

2.2 Explanations of TFP

(1) Human Capital (education)

(2) Technology

(3) Institutions

(4) Misallocation

3 Deriving Labor Supply by Calculus of Variations

(Refer to Lecture Slides) The household chooses H, the hours worked (labor supply), to maximize the following utility function,

$$U(wH^*) - V(H^*) \tag{1}$$

If the households adds or subtracts the labor supply by a small amount ϵ, the new utility function is:

$$U(w(H^* + \epsilon)) - V((H^* + \epsilon)) \tag{2}$$

If the maximum is achieved at H^*, then changing it to $H^* + \epsilon$ should not increase the value of the utility function,

$$(1) \geq (2) \tag{3}$$

Recall the Taylor series approximation,

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{1}{2}f''(x_0)(x - x_0)^2 + ...$$

Or:

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + o(x - x_0)$$

,where all terms after the second term are replace with an error function, $o(x - x_0)$. Then (2) can be rewritten as,

$$U(wH^*) - V(H^*) + (U'(wH^*)w - V'(H^*))\epsilon + o(\epsilon) \tag{4}$$

(1) $\geq$ (4) gives:

$$U(wH^*) - V(H^*) \geq U(wH^*) - V(H^*) + (U'(wH^*)w - V'(H^*))\epsilon + o(\epsilon)$$

$$\Rightarrow (U'(wH^*)w - V'(H^*))\epsilon \leq 0$$

This is true for all values of ϵ, so it must be that

$$U'(wH^*)w - V'(H^*) = 0 \tag{5}$$

$wH = C$, so rearranging gets us to the same supply curve that we derived previously,

$$\frac{V'(H^*)}{U'(C^*)} = w \tag{6}$$

The intuition behind (6) is that, say $\epsilon > 0$, the marginal benefit from increasing labor supply by ϵ is $U'(C^*)w\epsilon$, while the marginal cost is $V'(H^*)\epsilon$. At the optimum, the two terms have to be equal, which gives us (6).

What is the effect of a wage increase on labor supply? Review the **substitution effect** and the **income effect**.