# Intermediate Macroeconomics Recitation 4

Topics: Welfare theorems, present discounted value, consumption smoothing, Euler equations.

## 1 Welfare Theorems, Efficient and the Role of Government

### 1.1 First Welfare Theorem

In short: **competitive equilibria are pareto efficient.**

Necessary conditions for this to hold:

- Preferences are locally non satiable (agents always want to consume more when they can)
- Complete markets (can trade any good i.e. can buy a good that pays off when you cut your finger chopping a tomato on Tuesday.
- If households and firms act perfectly competitively (i.e. as price takers).
- Property rights over all goods and services are well defined and costlessly enforceable.

This can be generalized to economies with any number of goods, multiple periods (if all people exist at all times), and uncertainty (as long as information is "symmetric").

So does this mean governments should do nothing? Not necessarily. It might be Pareto efficient but this could involve one agent being very rich and everyone else being very poor.

### 1.2 Second Welfare Theorem

In short: **Redistribution can generate any Pareto Efficient outcomes in a competitive equilibrium.**

Second Welfare Theorem: with some initial distribution of wealth, any Pareto efficient allocation can be attained as an equilibrium of a competitive private ownership economy.

This does suggest there is a role for government after all. Governments might want to redistribute initial allocations of goods in order to obtain a more equitable outcome. Whether or not the government redistributes initially, the outcome is still Pareto efficient. However, redistribution can ensure we don't get one agent consuming everything.

## 1.3 Coase Theorem

If bargaining is costless and there are no wealth effects, the outcome of bargaining/contracting is Pareto efficient. Thus, even if there are externalities, we can still get a Pareto efficient outcome.

Idea: Suppose we are not at an efficient point. Then it is always in someone's interest to propose an efficient point, and describe how to get there in a way that leaves everybody at least as well off.

This maintains a large institutional requirement for government (courts), since government must exist to enforce property rights and contracts. "In Coase's view, contracts are a substitute for regulation. If potential externalities can be contracted around, no regulation is necessary. "

## 1.4 Market Failures

If conditions of the FWT or the Coase Theorem are not satisfied, markets will not yield an efficient outcome. We refer to these failures as market failures.

What are the main sources of such market failures?

1. Property rights are not well defined
2. Transaction costs are non zero
   - "From the efficiency perspective, the ubiquity of regulation is puzzling." Shleifer (2010) argues that extensive laws and regulation are a result of imperfect courts: by the failure of courts to solve contract and tort disputes cheaply, predictably, and impartially.
3. Imperfect competition (e.g. monopoly power, oligopoly, barriers to entry)
4. Public goods (especially non-rival goods, non-excludable goods)
5. Externalities (e.g. pollution, zoning; can be negative or positive)
6. Incomplete markets (e.g. insurance)
7. Imperfect information (e.g. asymmetric information, adverse selection, moral hazard)
8. Unemployment and other market disturbances
9. Rigidity of prices and wages
10. Government commitment problems
11. Search costs (e.g., hard to find a job)

See Stiglitz Chapter 4 for more detail on this.

### 1.5 What is the proper role of government?

Government policies are typically an attempt to do one of two things:

- Correct market failures to ensure that outcomes are efficient.
- Redistribute to attain a more equitable outcome.

# 2 Present Value

Present value gives us a value today of income received at different times.

Example 1: You know that if you invest 1\$ today, you will get $1 + r$\$ in a year. What is the present value of receiving 100\$ in a year? We need to find the income today, $x$, that would give us exactly 100\$ in a year. If we invest $x$ today, we will get $x(1+r)$ in a year. Therefore:

$$x(1 + r) = 100$$

So we see that the present value of 100\$ in a year is given by:

$$x = \frac{100}{1 + r}$$

Example 2: You know that if you invest 1\$ today, you will get $1 + r$\$ in one year. What is the present value of receiving 100\$ in two years? We need to find the income today, $x$, that would give us exactly 100\$ in two years. If we invest $x$ today, we will get $x(1 + r)$ next year and then $x(1 + r)^2$ in two years. Therefore:

$$x(1 + r)^2 = 100$$

So we see that the present value of 100\$ in two years is given by:

$$x = \frac{100}{(1 + r)^2}$$

Example 3: What is the present value of receiving 100\$ today? We know that if we receive money, we won't be able to invest it and make a return today. So the present value of receiving 100\$ today will be 100\$.

General formula: The return on investments between periods is $r$. The present value of $Y$ received $n$ periods from now is given by:

$$\frac{Y}{(1 + r)^n}$$

This comes from the fact that if receive $x$ income today and invest it for $n$ periods we get $x(1 + r)^n$ periods so if $x$ is the present value of $Y$, we have that:

$$x(1 + r)^n = Y$$

The net present value of receiving multiple income streams in different periods will just be the sum of their present values. For example, if we receive $Y_1$ today and $Y_2$ in the next period and the return on investments is $r$ then the present value is given by:

$$Y_1 + \frac{Y_2}{1+r}$$

# 3 Intertemporal Budget Constraint

We assume that agents live for two periods.

In period 1, agents receive income $Y_1$. They can use this to consume $C_1$ or save $B$ (note that $B$ can be negative in which case they are borrowing). Therefore:

$$C_1 + B \leq Y_1$$

In period 2, agents receive income $Y_2$ and the return on their savings (borrowings) $B(1+r)$. They can use this to consume (there is no point for them to save again since this is the last period of their lives). Therefore:

$$C_2 \leq Y_2 + B(1+r)$$

We typically imagine that consumers are nonsatiable i.e. they always want to consume more. In this case, it would never make sense for them not to use all of their income since then they could consume more. Thus, the budget constraints at each period must bind:

$$C_1 + B = Y_1$$

$$C_2 = Y_2 + B(1+r)$$

We want to write this a single budget constraint - an intertemporal (between period) budget constraint. To do this, note that we can write the first period budget constraint as:

$$B = Y_1 - C_1$$

We can then input the first period budget constraint into the second and cancel out $B$:

$$C_2 = Y_2 + (Y_1 - C_1)(1+r)$$

Simplifying:

$$\frac{C_2}{1+r} = \frac{Y_2}{1+r} + Y_1 - C_1$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

We see that the present value of the consumer's consumption must equal the present value of the consumer's income.

# 4 Consumption Saving choices

Suppose we have the following utility function for the consumer:

$$u(C_1) + \beta u(C_2)$$

The consumer faces the same budget constraints as in the previous section. Therefore, we get the following problem:

$$\max_{C_1, C_2} u(C_1) + \beta u(C_2)$$

s.t.

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

We can rewrite the intertemporal budget constraint as follows:

$$C_1(1+r) + C_2 = Y_1(1+r) + Y_2$$

$$C_2 = Y_1(1+r) + Y_2 - C_1(1+r)$$

We can then substitute out $C_2$ from the maximisation problem:

$$\max_{C_1} u(C_1) + \beta u(Y_1(1+r) + Y_2 - C_1(1+r))$$

We can now find the solution by taking FOCs with respect to $C_1$. We get:

$$u'(C_1) - \beta u'(C_2)(1+r) = 0$$

Therefore, we get the following condition which is called an Euler condition (a relationship between consumption in different periods):

$$u'(C_1) = \beta(1+r)u'(C_2)$$

The intuition for this is that if you save $\epsilon$ more units, you get $\epsilon(1+r)$ in the next period which give you marginal utility of $\beta u'(C_2)$ for each unit. So the marginal benefit is $\epsilon(1+r)\beta u'(C_2)$. In this case, you consume $\epsilon$ less units today which give you marginal utility of $u'(C_1)$ for each unit. So the marginal cost is $\epsilon u'(C_1)$. Setting these equal yields $u'(C_1) = \beta(1+r)u'(C_2)$.

Let's consider what happens when we specify a specific utility function:

$$u(C) = \log(C)$$

Then we get that:

$$u'(C) = \frac{1}{C}$$

Inputting this into the Euler condition yields:

$$\frac{1}{C_1} = \beta(1+r)\frac{1}{C_2}$$

Simplifying:

$$C_2 = \beta(1+r)C_1$$

We can input this into the intertemporal budget constraint to find a value for $C_1$ in terms of exogenous variables, not endogenous variables. Exogenous variables are variables that we do not choose i.e. $Y_1, Y_2, r$. Endogenous variables are variables that we choose i.e. $C_1, C_2$.

$$C_1 + \frac{\beta(1+r)C_1}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Simplifying:

$$C_1 + \beta C_1 = Y_1 + \frac{Y_2}{1+r}$$

$$C_1 = \frac{1}{1+\beta}(Y_1 + \frac{Y_2}{1+r})$$

Inputting this value for $C_1$ into the Euler condition yields:

$$C_2 = \beta(1+r)\frac{1}{1+\beta}(Y_1 + \frac{Y_2}{1+r})$$

$$C_2 = \frac{\beta}{1+\beta}(1+r)(Y_1 + \frac{Y_2}{1+r})$$

We see that as long as the present value of income stays the same (and the interest rate stays the same), when we receive income does not affect our choice of consumption.