Recitation 10

Topics: Medieval Economy, Game Theory, Banking and Leverage

1 Medieval Economy

In our model of the medieval economy we have 3 main equations:

$$M_t\bar{V} = P_tY_t \tag{1}$$

$$Y_t = \bar{A}L_t \tag{2}$$

$$\frac{P_{t+1}}{P_t} = \left(\frac{L_t}{L^*}\right)^\theta \tag{3}$$

(1) is the Quantity Equation of Money, which tells us that in equilibrium, the quantity of money times its velocity has to equal the product of prices and real output.

(2) is the production function we are using, which does not have capital and uses labor as the only factor of production. It is hence linear in labor with slope equal to constant productivity $\bar{A}$.

(3) characterizes price adjustment in this economy: If labor demand is high relative to some "desired" labor supply L^*, prices go up to adjust for this excess demand. If demand is lower than the desired supply, prices go down to adjust for the lower demand. The parameter θ captures the speed of this price adjustment: The higher θ, the faster is the pass-through from excess demand in the labor market to higher prices.

1.1 Long Run

In the long run, this economy will reach a steady state. As previously, this is characterized by the variables not changing from period to period, i.e. $P_t = P_{t+1} = \bar{P}$. Together with equation (1), this implies:

$$\left(\frac{\bar{L}}{L^*}\right)^\theta = \frac{\bar{P}}{\bar{P}} = 1 \Rightarrow \bar{L} = L^*$$

This gives us the steady state value of output

$$\bar{Y} = \bar{A}\bar{L} = \bar{A}L^*$$

which in turn specifies the steady state level of prices:

$$\bar{P} = \frac{\bar{M}\bar{V}}{\bar{Y}}$$

From the above, we can see that in the long run, changes in the money supply do not affect output, as output is characterized by $\bar{Y} = \bar{A}L^*$, where the right-hand-side is unaffected by changes in M.

The only thing that changes in the long run in response to changes in the money supply is prices: Assume money supply changes from $\bar{M}$ to $\tilde{M}$. Then, **in the long run**, the price level will adjust from $\bar{P}$ to $\bar{P}'$ according to

$$\frac{\bar{P}'}{\bar{P}} = \frac{\frac{\tilde{M}\bar{V}}{\bar{Y}}}{\frac{\bar{M}\bar{V}}{\bar{Y}}} = \frac{\tilde{M}}{\bar{M}}$$

This result is called the **Classical Dichotomy** or **Long-run Monetary Neutrality**, where a change in money supply changes price proportionately, leaving output unaffected. The intuition behind this result is that only *relative* prices matter in the long run, i.e. real wage, real interest rate, etc., and the money supply only affects nominal variables.

However, this price adjustment is not taking place instantaneously in our model. Let's consider the short run dynamics of price and output as consequences of an increase in money supply next.

1.2 Short Run

Starting from the steady state, let's consider the effect of an increase in the money supply from $\bar{M}$ to $\tilde{M}$ at time t.

First, we re-write the 3 equations of the model in logs:

$$logM_t + log\bar{V} = logP_t + logY_t \tag{1}$$

$$logY_t = log\bar{A} + logL_t \tag{2}$$

$$logP_{t+1} - logP_t = \theta(logL_t - logL^*) \tag{3}$$

We can rewrite (2) to obtain $logL_t = logY_t - log\bar{A}$. Plugging this into (3) gives us

$$logP_{t+1} - logP_t = \theta(logY_t - log\bar{A} - log\bar{L})$$

Noting that $log\bar{A} + log\bar{L} = log\bar{Y}$, we are now left with two equations:

$$logM_t + log\bar{V} = logP_t + logY_t \tag{1}$$

$$logP_{t+1} - logP_t = \theta(logY_t - log\bar{Y}) \tag{2}$$

Now we are left with two endogenous variables: P_t and Y_t. M_t is determined exogenously and in our example is first equal to $\bar{M}$ and then changes to $\tilde{M}$. Let's change the consequences of this change in the period of the change and in the one right after it:

Period t:

Period t is the period in which the change in money supply takes place.

Since we started in the steady state, we know from equation (3) that

$$\frac{P_t}{P_{t-1}} = \frac{\bar{P}}{\bar{P}} = 1 \Rightarrow \Delta P_t = 0$$

Here, Δ is the change operator: $\Delta P_t = P_t - P_{t-1}$.

Equation (1) implies

$$logM_t + log\bar{V} = logP_t + logY_t \Rightarrow \Delta log\bar{M} + \Delta log\bar{V} = \Delta logP_t + \Delta logY_t$$

But we know from above that $\Delta logP_t = 0$ and since velocity is constant, $\Delta log\bar{V} = 0$ as well. Therefore:

$$\Delta log\tilde{M} = \Delta logY_t$$

Hence, in the period of the change in money supply, the entirety of the change goes towards changes in output and prices don't change at all! (Note that this is the opposite of the long run case: in the long run, changes in the money supply only change prices and have **no effect** on output)

Period t+1:

Period t+1 is the period right after the money supply was changed. Let's consider how this change now affects output and prices.

First, we use equation (2) to analyze the changes in prices in period t+1:

$$logP_{t+1} - logP_t = \theta(logY_t - log\bar{Y}) \Rightarrow \Delta logP_{t+1} = \theta(\Delta logY_t)$$

So changes in log prices in period t+1 equal the change in output in the previous period (since we were in the steady state in t-1, so $Y_{t-1} = \bar{Y}$. But from analyzing the period t, we know that $\Delta logY_t = \Delta log\tilde{M}$, and therefore:

$$\Delta logP_{t+1} = \theta(\Delta log\tilde{M})$$

Now that we know what happens to prices, let's analyze the changes in output. Starting from equation (1) written in changes as above, we observe:

$$\Delta log\bar{M} + \Delta log\bar{V} = \Delta logP_{t+1} + \Delta logY_{t+1} \Rightarrow \Delta logY_{t+1} = \Delta log\bar{M} + \Delta log\bar{V} - \Delta logP_{t+1}$$

Now we again amke use of the relationship of changes in prices and money supply obtained before: $\Delta logP_{t+1} = \theta(\Delta log\tilde{M})$. We can use this and the fact that $\bar{V}$ is constant so that $\Delta log\bar{V} = 0$ to simplify the right-hand-side of the above. We re-write:

$$\Delta logY_{t+1} = \Delta log\bar{M} + \Delta log\bar{V} - \Delta logP_{t+1} = \Delta log\tilde{M} - \theta(\Delta log\tilde{M}) = (1-\theta)\Delta log\tilde{M}$$

We now can see that not the entirety of the change in money supply goes towards higher output, but only $(1-\theta)$ times the magnitude of that change. The remaining $\theta\Delta log\tilde{M}$ goes towards changes in prices. This is why we say theta characterizes the speed of price adjustment: the higher theta, the more of the change in money supply leads to an adjustment of prices instead of output.

1.3 From Short- to Long-Run

Over time, more and more of the initial change in the money supply will lead to adjustment of prices, while output will converge back to its steady state. This relationship is represented in Figure 1, where we see money supply in red, prices in black and output in blue. In immediate reaction to the jump in the money supply from 1 to 2, output also doubles from 1 to 2. However, with some delay, prices start rising and slowly converge to their new steady state value of 2. With prices increasing, output falls until the steady state value of 1 is reached again.

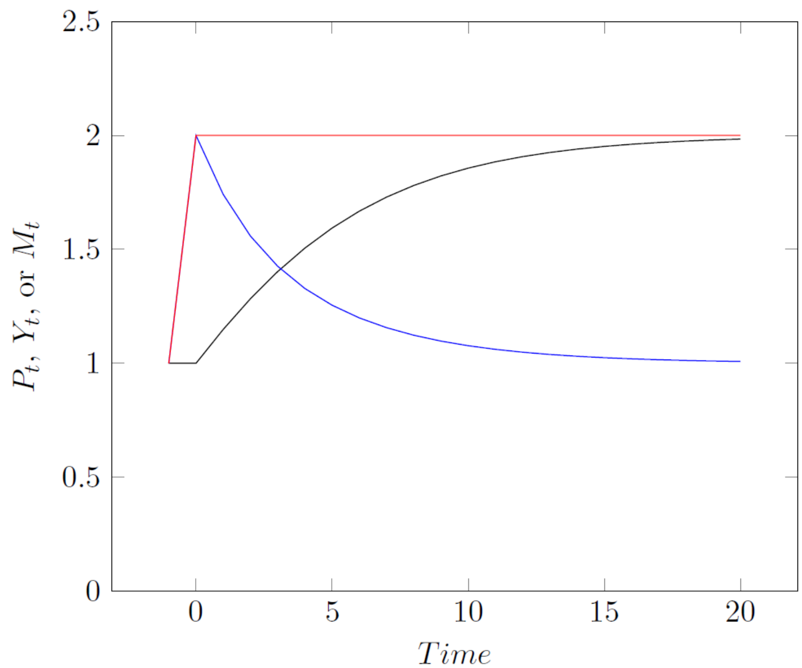

Figure 1: Time series of P_t Y_t and M_t. P_t in black, Y_t in blue, M_t in red.

How does the model help us understand prices in 19th century U.S.?

- "Greenbacks" were issued during the Civil War. Supply expanded and price level relative to gold doubled by the end of war
- Gold experienced huge deflation in the postwar period, partly because the supply of gold did not catch up with the increase in real output: $\Delta log P1 = \Delta log M1 - \Delta log Y1$

2 Game Theory

We introduce basic game-theoretic models to analyze the role of money and banking in the economy. Specifically, the Diamond-Dybvig Model is a classic model of bank runs that allows us to understand why self-fulfilling panics can arise among depositors.

2.1 Game Theory Basics

Game theory is a formal way to study *interdependence*. It studies the *interaction* among a group of *rational agents* who behave *strategically*. A *game* specifies the following elements:

- Players: The group of rational agents who strategically interact
- Strategies: A player's strategy is a "program of play" that tells her what actions to take in response to *every possible strategy* that other players might use
- Payoffs: A number (assigned by an ordinal utility function) that indicates how much a player gains or loses from the choices made in the game

There are two main ways to represent a game, the *strategic* or *normal form*, and the *extensive form* (which will be introduced in later lectures to study dynamic games).

The *strategic representation* captures information about the players, strategies, and payoffs in a matrix. If there are two players, each with a finite number of strategies, the game can be represented by a two-dimensional matrix. Each of its cell gives the payoffs to both players for each combination of actions. For example, in Prisoners' Dilemma,

		PRISONER 2	
		Confess	Lie
PRISONER 1	Confess	-8, -8	0, -10
	Lie	-10, 0	-1, -1

In this example, Prisoner 1's payoff appears as the first number of each pair, Prisoner 2's as the second. So, if both players confess, they each get a payoff of -8 (8 years in prison). This appears in the upper-left cell.

2.2 Nash Equilibria of a Game

Definition 1. *A best response for a player maximizes his payoff, taking all other players' choices as given.*

The four red circles in the matrix indicate best responses. For example, Prisoner 1's best response, given that Prisoner 2 confesses, is to confess, because by confessing he gets 8 years of prison instead of 10.

Definition 2. *A (strongly) dominant strategy for a player gives him a (strictly) higher payoff, regardless of other players' choices.*

In this case, Confess is a dominant strategy for Prisoner 1, because he gets a higher payoff both when Prisoner 2 confesses and lies. Note that this is a *symmetric game*, i.e. each player has exactly the same strategy set and the payoff functions are identical, so Confess is a dominant strategy for both players.

Definition 3. *A Nash Equilibrium is a strategy combination where each player's strategy choice is a best response against his opponents' choices in that combination.*

In other words, in a Nash Equilibrium, no player has incentive to deviate from his choice, given what everyone else is choosing. The combination (Confess, Confess) is a Nash Equilibrium. Note that a game can have multiple Nash Equilibria.

2.3 Diamond-Dybvig Model of Bank Runs

There is a bank and a large number of depositors. At t = 0, the bank takes deposits of \$1 from each depositor and invests them into a project. At t = 1, the depositor can withdraw. This requires the bank to call in the loan early, and yields a smaller return r, $0 < r < 1$. If the depositor chooses not to withdraw but wait until the project finishes, then at t = 2, his payoff is R, $R > 1$. The following is a strategic representation of this game reduced to two players, You and Everyone Else:

Everyone else / You	Withdraw	Not Withdraw
Withdraw	(r, r)	(1, 0)
Not Withdraw	(0, 1)	(R, R)

This is an example of a game with two Nash Equilibria (NE). The "good" NE yields R for every depositor, while the "bank run" NE yields r for every depositor.

Why is the bank run a **self-fulfilling prophecy** according to the model? Note that there is no dominant strategy in this game. Your *best response* is to choose what you *believe* everyone else chooses. Starting from the "good" NE, if there is a rumor that the bank is not performing well, you expect some depositors to withdraw. Even if the rumor is not true, when enough depositors believe others will withdraw, everyone deviates and rushes to withdraw money, leading to a bank run.

3 Banking and Leverage

3.1 A Bank's Balance Sheet

Definition 4. *A balance sheet is an accounting tool with assets on the left side and liabilities and net worth on the right side. When equity (net worth) is included, the two sides sum to the same value.*

An **asset** is something of value that is owned. For a bank, this can be loans made to consumers and businesses, financial investments, cash etc.

A **liability** is the amount that is owed. For a bank, this can be customers' deposits, short-term and long-term debts, etc.

Net worth, also called equity, or bank capital, is the difference between assets and liabilities.

Definition 5. *Leverage ratio is the ratio of total assets to net worth.*

TABLE 10.4

A Hypothetical Bank's Balance Sheet (billions of dollars)

Assets		Liabilities	
Loans	1,000	Deposits	1,000
Investments	900	Short-term debt	400
Cash and reserves	100	Long-term debt	400
Total assets	2,000	*Total liabilities*	1,800
		Equity (net worth)	200

In this example, the assets of the bank are 2,000 (billions of dollars). Liabilities are 1,800. It has a net worth of 2000 - 1800 = 200. Its leverage ratio is 2000 : 200 = 10 : 1.

What does this mean for the bank's risk? *Leverage increases the risk of insolvency.* The higher the leverage ratio, the more net worth falls as a result of a decrease in assets.

- Leverage ratio is x : 1. If assets of the bank fall by y%, net worth falls by xy%.
- Assets of the bank fall by y%, if the leverage ratio is $\frac{100}{y}$: 1 (or larger), then net worth is completely wiped out (falls by 100% or more).

In this example, if the assets of the bank fall by 10% or more, then net worth is depleted and the bank becomes **insolvent**.

3.2 Two Types of Banking Crises

A **liquidity crisis** for a bank is a shortage of liquid assets (cash) to meet short-term obligations, such as paying back depositors' deposits.

As a financial intermediary, the bank has illiquid assets (long-term lending for investment projects) and liquid liabilities (demand deposits), therefore it has a **maturity mismatch**.

In the Diamond-Dybvig Model of bank runs, even though assets are greater than liabilities (net worth is positive and the bank is solvent), a shift in the depositors' expectations can trigger deviations toward the undesirable Nash Equilibrium where they panic and rush to withdraw. In this scenario, the bank's net worth contracts acutely and even a solvent bank could fail.

A **solvency crisis** occurs if the value of assets is less than liabilities. In other words, net worth is depleted and the bank is bankrupt. No amount of short-term borrowing could change the situation. The depositors withdraw for a good reason.

This distinction has important policy implications. As the "lender of last resort, " the central bank can help banks recover from a liquidity crisis through short-term lending. However, a bank faced with insolvency requires a "bail out," which incurs a loss on the central bank.