Recitation 11

Topics: More game theory, currency attacks, bank bailouts

1 More Game Theory

Game theory is a formal way to study *interdependence*. It studies the *interaction* among a group of *rational agents* who behave *strategically*. A *game* specifies the following elements:

- Players: The group of rational agents who strategically interact
- Strategies: A player's strategy is a "program of play" that tells her what actions to take in response to *every possible strategy* that other players might use
- Payoffs: A number (assigned by an ordinal utility function) that indicates how much a player gains or loses from the choices made in the game

There are two main ways to represent a game, the *strategic* or *normal form*, and the *extensive form* (which will be introduced in later lectures to study dynamic games).

The *strategic representation* captures information about the players, strategies, and payoffs in a matrix. If there are two players, each with a finite number of strategies, the game can be represented by a two-dimensional matrix. Each of its cell gives the payoffs to both players for each combination of actions.

Note that for these games, we need to assume that it is *common knowledge* that players are rational. This means that "All players know that all players are rational, and all players know that all players know that all players are rational...etc."

Definition 1. *A (strongly) dominant strategy for a player gives him a (strictly) higher payoff, regardless of other players' choices.*

To demonstrate these concepts, here is an example.

Figure 1: Example 1

		Player 2		
		I	II	III
	A	5,1	2,0	10,-1
Player 1	B	6,0	4,1	3,-5
	C	7,4	3,3	1,2

1. Does Player 1 have a strictly dominated strategy? No.
2. Does Player 2 have a strictly dominated strategy? Yes: I strictly dominates III, and II weakly dominates III. What will player 2 do? Not play III. $\Rightarrow$ We can eliminate column III from the game.

Figure 2: After we eliminate the dominated strategy of player 2

		Player 2		
		I	II	III
	A	5,1	2,0	
Player 1	B	6,0	4,1	
	C	7,4	3,3	

Figure 3: After we eliminate the dominated strategy of player 1

		Player 2		
		I	II	III
	A			
Player 1	B	6,0	4,1	
	C	7,4	3,3	

3. Does Player 1 have a dominated strategy now? Yes: B and C both dominate A.
4. Now there are no more strategies we can eliminate, and we have to look for Nash equilibria.

Definition 2. *A Nash Equilibrium is a strategy combination where each player's strategy choice is a best response against his opponents' choices in that combination.*

Definition 3. *A best response for a player maximizes his payoff, taking all other players' choices as given.*

The underlined numbers in the matrix indicate best responses.

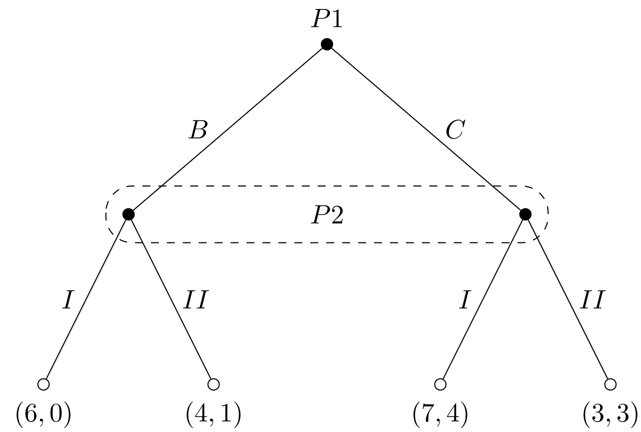
Figure 4: After we eliminate the dominated strategy of player 1

		Player 2	
		I	II
Player 1	B	6,0	$\underline{4}$, $\underline{1}$
	C	$\underline{7}$,$\underline{4}$	3,3

6. Which equilibrium is the social optimum? Does NE guarantee that?

In this game, we assumed that the players move *simultaneously*, or without knowledge of the other's action. This can also be written as an extensive form game, as in Figure 5.

Figure 5: Extensive form simultaneous game

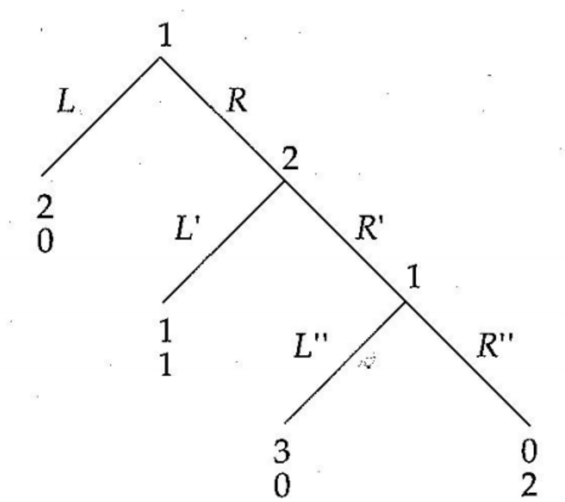


We can also have games in which players play sequentially. In this case, we can solve using backwards induction. In the example in lecture, the payoffs were as shown in Figure 6.

In this game, we solve by working backwards:

1. Player 1 prefers 3 to 0, so chooses L''
2. Player 2 prefers 1 to 0, so chooses L'
3. Player 1 prefers 2 to 1, so chooses L

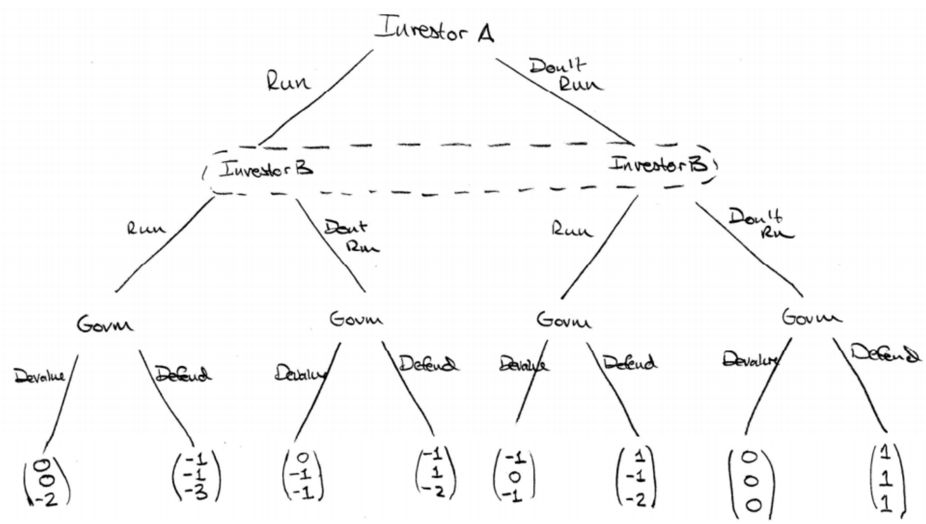
Figure 6: Extensive form sequential game



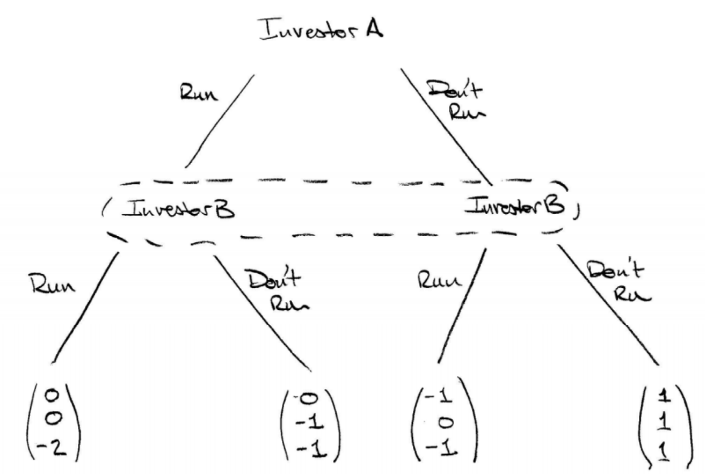
Definition 4. *A Nash Equilibrium is subgame perfect (SPNE) if the players strategies constitute a Nash Equilibrium in every subgame.*

2 Currency attack game

In this game, investors A and B act simultaneously, and then the government acts. In this game, the government will only defend the currency in the event that both investors don't run.



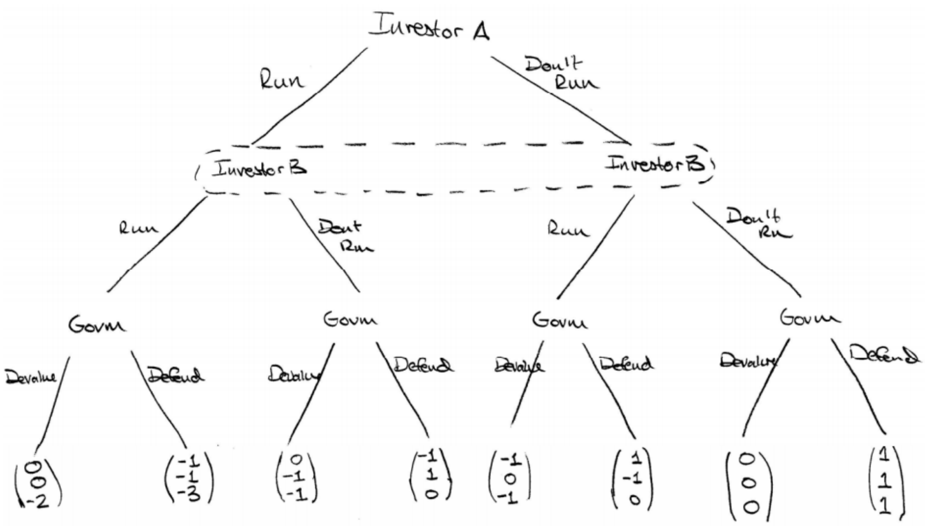
Since the government acts last, we can use backwards induction to simplify the game:



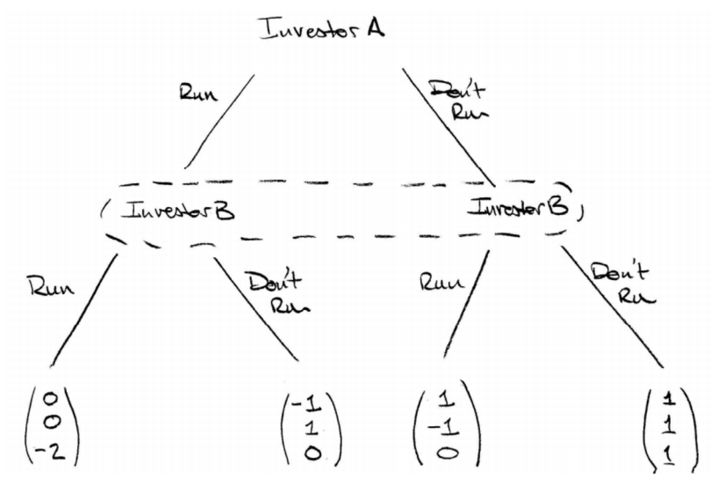
But there are two Nash Equilibria:

		Investor B	
		Run	Don't Run
Investor A	Run	$\underline{0},\underline{0}$	0,-1
	Don't Run	-1,0	$\underline{1},\underline{1}$

The government can force investors not to run by being "strong" in the sense of being able to defend at low cost (perhaps because it has lots of reserves). For example, this could be the case if the government is known to place a high value on maintaining the gold standard. To see this, we modify the payments for the government only: if only one investor runs, the government can defend at a low cost (0 instead of 2).



Again, we can use backwards induction to simplify:



Only one NE remains, because now both investors' dominant strategy is don't run:

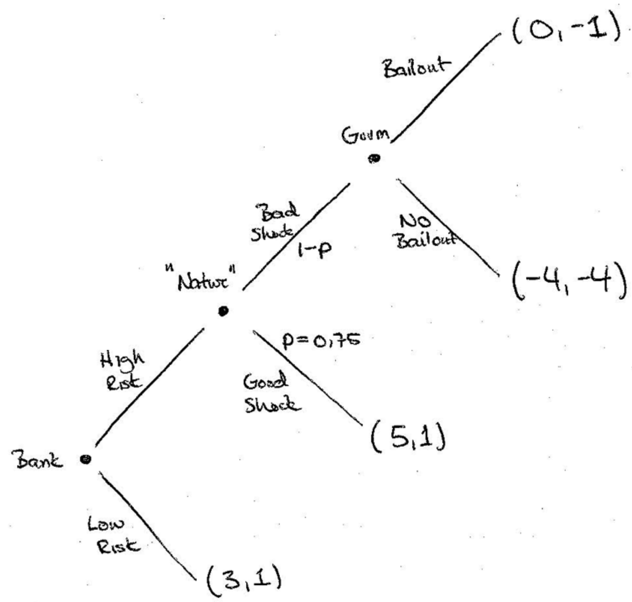
		Investor B	
		Run	Don't Run
Investor A	Run	0,0	-1,$\underline{1}$
	Don't Run	$\underline{1}$,-1	$\underline{1}$,$\underline{1}$

3 Bank bailout game

The bank bailout game has three players: the bank, "nature," and the government. However, only the bank and the government earn payoffs. If the bank gets a bad shock, the government will bail it out. However, the bank is taking an expectation of its payoffs when it decides whether to be high or low risk.

Conditional on the government bailing it out, the expected value of the high risk action is 0.75*5+0.25*0(because it expects the government to bail it out). This is 3.75, which is greater than 3, and so banks choose to be high risk. However, if the government could commit not not bail out the banks, they would choose to be low risk, because: $0.75 * 5 + 0.25 * -4 = 2.74 < 3$.

Figure 7: Bank bailout game



4 Monetary narrative of the Great Depression

Why are we talking about bank runs and bailouts in the first place? Recall that the monetary model linked output to an exogenously given amount of money:

$$\Delta \log M_t = \Delta \log P_t + \Delta \log Y_t \tag{1}$$

The determinants of money can further be understood as being driven by gold supplies, the gold cover ratio, and the bank money multiplier.

$$M = M_g \frac{M_b}{M_g} \frac{M}{M_b} = M_g \cdot \frac{1}{GCR} \cdot BMM \tag{2}$$

However, some of these underlying factors can be driven by coordination, as occurred in the Great Depression.