Intermediate Macroeconomics Recitation 7

Topics: Solow model, midterm review

1 Solow model

Setup (6 equations, 6 unknowns):

$$
\begin{align}
Y_t &= A_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}\\
K_{t+1} &= K_t + I_t - \bar{d}K_t \tag{2}\\
L_{t+1} &= (1+n)L_t \tag{3}\\
S_t &= \bar{s}Y_t \tag{4}\\
Y_t &= C_t + I_t \tag{5}\\
S_t &= Y_t - C_t \tag{6}
\end{align}
$$

1. Production side (Cobb-Douglas)

   $$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

   Diminishing Returns to Capital $\rightarrow$ a nation cannot grow in the long run by accumulating capital; Constant return to scale.

   (a) A_t is exogeneous. For simplicity, assume $A_t = \bar{A}$

   (b) L_t grows at exogenously given rate by n (constant).

   (c) K_t accumulates from some given initial condition K_0.

   Note that there is no optimizing behavior like profit maximization. This is just for simplicity.

2. Consumers allocate income Y_t between consumption and saving. The savings rate is assumed to be an exogenous constant fraction of income, $S_t = \bar{s}Y_t$, and savings are transformed into investment.

   Again, no optimizing behavior (as in the production side) for simplicity.

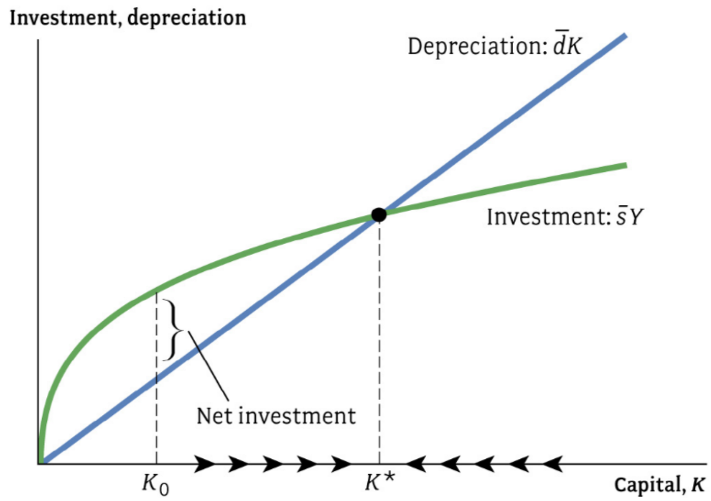
3. In equilibrium, the resource constraint (5) holds, linking the production and consumer sides. Combining the equation (5) and (6), we get $I_t = S_t$.

   $$I_t \overset{(5)}{=} Y_t - C_t \overset{(6)}{=} S_t$$

   Hence we determine I_t.

Understand: transition dynamics. What happens if the savings rate increases? The depreciation rate rises?

Figure 1: Solow Diagram



Solving the model

Since the labor force is growing over time, output is growing over time. It is useful to divide the equations by L_t, and rewrite the system of equations in terms of per capita quantities:

$$y_t = \bar{A}k_t^{\alpha} \tag{7}$$
$$k_{t+1}(1+n) = k_t + i_t - \bar{d}k_t \tag{8}$$
$$s_t = \bar{s}y_t \tag{9}$$
$$y_t = c_t + i_t \tag{10}$$
$$s_t = y_t - c_t \tag{11}$$

1. To solve, combine (10) and (11):

$$i_t = s_t \tag{12}$$

Substitute (7) into (9) and combine this with (12):

$$i_t \overset{(12)}{=} s_t \overset{(9)}{=} \bar{s}y_t \overset{(7)}{=} \bar{s}\bar{A}k_t^{\alpha} \tag{13}$$

Substituting (13) into (8), we get

$$k_{t+1}(1+n) \overset{(8)}{=} (1-\bar{d})k_t + i_t = (1-\bar{d})k_t + \bar{s}\bar{A}k_t^{\alpha} \tag{14}$$

This is a single equation that only involves the unknown k_t. Given initial condition k_0, we can solve for k_t. We just solved the dynamic equation! Note that we can get y_t given k_t using (7). Likewise get s_t given y_t using (9), c_t given s_t and y_t using (11), i_t given y_t and c_t using (10).

2. No long-run growth in per capita income. The steady state is exactly the situation in which per capita income is constant and hence k_t converges to the steady state value of *capital (per capita).*

3. Note that we can solve for the steady state. From equation (14),

$$\bar{k}(1+n) = (1-\bar{d})\bar{k} + \bar{s}\bar{A}\bar{k}^{\alpha} \tag{15}$$

$$\Rightarrow \bar{k} = \left(\frac{sA}{n+\bar{d}}\right)^{\frac{1}{1-a}} \tag{16}$$

   Solve for the other variables similarly. Observe $\bar{k}$ does not depend on the initial value k_0. $\bar{k}$ is the long run capital stock and k_t converges to $\bar{k}$ regardless of k_0. Therefore if countries have the same production technology(i.e, the same $A_t, a), \bar{d}, n$ and $\bar{s}$, then they will have the same capital per capita, k, and hence the same consumption per capital, c, in the long run!

What happens if the population growth rate rises or falls?

Transition dynamics

Subtract $k_t(1+n)$ from both of the left-hand side and the right hand-side of (8), and then divide the both side by $k_t(1+n)$.

$$g_{t+1} = \frac{k_{t+1}-k_t}{k_t} = \frac{1}{(1+n)}[\bar{s}Ak_t^{(\alpha-1)} - (n+\bar{d})] \tag{17}$$

Testable implications

1. Capital-output ratio is proportional to saving rate. In steady state, $\bar{d}K^* = \bar{s}Y^*$, so $\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}$. This holds reasonably well in the data (Jones, Figure 5.3).

2. Implies even larger role for TFP difference, since differences in saving rates cannot explain a large fraction of the differences in income across countries

$$\begin{aligned} \frac{y^{rich}}{y^{poor}} &= (\frac{A^{rich}}{A^{poor}})^{1/(1-\alpha)=3/2}(\frac{s^{rich}}{s^{poor}})^{\alpha/(1-\alpha)} \\ 108(observed) &\simeq 54(implied) \times 2(observed) \end{aligned}$$

   The Solow model implies large TFP differences.

3. No long run growth: due to diminishing returns, economy reaches steady state. So what drives long run growth?

4. Differences in growth rates should be related to initial wealth. This was generally true for OECD countries from 1960 to 2010 (Jones, Figure 5.8). However, it does not hold for most countries (Figure 5.9). If poor countries are at steady states (i.e. not growing) due to determinants (s, A), growth isn't expected.

2 Midterm review

1. Production and firms
   - Cobb-Douglas production function (properties of the model)
   - Firm's maximization problem (review constrained maximization in Recitation 1)
   - MPK & MPL
2. Labor leisure tradeoff
   - Household utility maximization problem (constrained maximization & calculus of variations method)
   - Labor market equilibrium (labor supply and demand)
   - Substitution effect vs. income effect of wage increase on labor supply
   - Labor supply and taxes
   - Keynes vs. Marx vs. Smith
   - TFP
3. Market efficiency
   - Notions of efficiency (pareto efficiency, exchange efficiency, production efficiency, efficient use of land)
   - First and Second welfare theorem
   - Market failures
   - The proper role of the government
4. Households' consumption savings decisions
   - Robinson Crusoe's utility maximization problem (Euler equation)
   - Permanent Income Hypothesis (PIH) & forward-looking behavior
   - Optimal vs. actual behavior (e.g. borrowing constraints, precautionary savings)
5. Regressions
   - Interpretation of regression model and table
   - Correlation vs causation
   - Experiments in Economics
6. Malthus Model
   - Labor demand & labor supply
   - Population dynamics & convergence to the steady state (graphical representation)

- Response to permanent/temporary technology/population shocks (graphical representatio)
- Why the world is no longer Malthusian

7. Industrial revolution

    - Review lecture slides

8. Solow Model and Innovation

    - Capital accumulation and growth
    - The set of key equations to solve the model
    - per capita form of the model
    - Steady state
    - Golden rule level of capital
    - Growth accounting
    - Solow model with population growth
    - Conditiona convergence
    - Emprical observations of the Solow model (know how to interpret the graphs)
    - Ideas