5. Dos paraboloides Calcule el volumen de la región acotada arriba por el paraboloide $z = 3 - x^2 - y^2$ y abajo por el paraboloide $z = 2x^2 + 2y^2$.

$x(t) = r\cos(t)$; $0 \le t \le 2\pi$

$y(t) = r\sin(t)$

$z = z$

$z = 3 - x^2 - y^2$

$z = 3 - (r\cos t)^2 - (r\sin t)^2$

$z = 3 - (r^2\cos^2 t + r^2\cos^2 t)$

$z = 3 - r^2(1)$

$z = 3 - r^2$

$z = 2x^2 + 2y^2$

$z = 2x^2 + 2y^2$

$z = 2(x^2 + y^2)$

$z = 2r^2$

$\int_0^{2\pi}\int_0^1\int_{2r^2}^{3-r^2} 1 \cdot r\,\partial z\,\partial r\,\partial\theta$

$\int\int r[z]_{2r^2}^{3-r^2}$

$\int_0^{2\pi}\int_0^1 r([3-r^2]-[2r^2])\,\partial r\,\partial\theta$

$\int\int r(3 - r^2 - 2r^2)\,\partial r\,\partial\theta$

$3 - x^2 - y^2 = 2x^2 + 2y^2$

$3 = 3x^2 + 3y^2 \quad / \cdot 1/3$

$1 = x^2 + y^2$

$\int_0^{2\pi}\int_0^1 r(3 - 3r^2)\,\partial r\,\partial\theta$

$3\int_0^{2\pi}\int_0^1 (r - r^3)\,\partial r\,\partial\theta$

$3\int_0^{2\pi}\left[\frac{r^2}{2} - \frac{r^4}{4}\right]_0^1 \partial\theta$

$\frac{3}{\cancel{4}} \cdot 2\pi = \frac{3}{2}\pi$

2. Calcular el volumen de la región de $\mathbb{R}^3$ acotada superiormente por la esfera $x^2 + y^2 + z^2 = 4$ e inferiormente por el plano $z = \sqrt{3}$.

$x(t) = \rho\sin(\varphi)\cdot\cos(\theta)$

$y(t) = \rho\sin(\varphi)\cdot\sin(\theta)$

$z(t) = \rho\cos(\varphi)$

$J = \rho^2\cos(\varphi)$

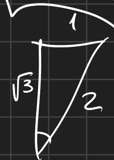
$0 \le \varphi \le \frac{\pi}{6}$

$0 \le \rho \le 2$!!

$0 \le \theta \le 2\pi$

$\boxed{\rho\cos(\varphi) = \sqrt{3}}$

$\rho = \frac{\sqrt{3}}{\cos(\varphi)}$



$0 \quad \frac{1}{2}$

$\sin(\pi/6)$

$\rho^2 = 4$

$\rho = 2$

$\pi/2$

$\pi/3$

$\pi/4$

$\pi/6$

$\to \Rightarrow x^2 + y^2 = 1$

$z(\sqrt{3}) = x^2 + y^2$

$(\rho^2\sin^2\varphi)\cdot\cancel{\cos^2\theta} +$

$(\rho^2\sin^2\varphi)\cancel{\sin^2\theta}$

$\int\sqrt{1-r^2}\,\partial r$

$\rho^2\sin^2(\varphi) = 1$

$\sin^2(\varphi) = \frac{1}{\rho^2} \quad // \sqrt{\ }$

$\sin(\varphi) = \frac{1}{\rho}$; $\rho = \frac{1}{\operatorname{sen}(\varphi)}$