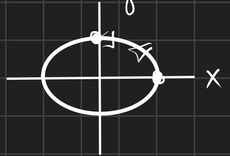
1) $\vec{r} : [0, 6\pi] \to \mathbb{R}^3$

$t \longmapsto (2\cos(t), 2\,\text{sen}(t), t)$

Si es un arco de Jordan pues la curva es inyectiva, es decir, no se intersecta.

2) Calcule. $\int_C [y\,\partial x + 2x\,\partial y]$ donde C es la curva $4x^2 + 9y^2 = 36$



$\boxed{f_{(x,y)} = (y, 2x)}$

$f(x(t), y(t))$

$C:$

$\begin{bmatrix} y = 2\,\text{sen}t \\ x = 3\cos t \end{bmatrix}$

$x(t) = r\cos(t)$;

$y(t) = r\sin(t)$

$C: 4x^2 + 9y^2 = 36 \to \left[\frac{x^2}{9} + \frac{y^2}{4} = 1\right]$

$\frac{1}{9} r^2\cos^2(t) + \frac{1}{4} \cdot r^2\sin^2(t) = 1$

$r^2\left(\frac{1}{9}\cos^2(t) + \frac{1}{4}\sin^2(t)\right) = 1$

$r^2(4(\cos^2(t) + \sin^2(t)) + 5\sin^2(t)) = 36$

$r^2[4 + 5\sin^2(t)] = 36$

$$\int_C f(x(t), y(t)) [\|\vec{r}\,'(t)\|]\,\partial t$$

$0 \leq t \leq 2\pi$

$f(x, y) = (y, 2x)$

$f(x(t), y(t)) = (2\sin t, 6\cos t)$.

$\|\vec{r}\,'\|$

$\vec{r}(t) = (3\cos t, 2\sin t)$

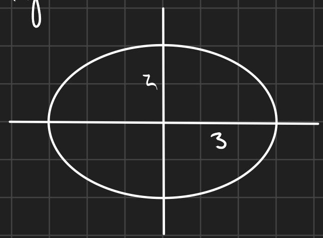
$$\frac{\partial \vec{r}}{\partial t} = (-3\sin t, 2\cos t).$$

$$\int_C \mathbf{F}(x(t), y(t)) \cdot r'(t)\,\partial t$$

2.- Calcule

$$\int_C y\,\partial x + 2x\,\partial y \quad \text{donde } C \text{ es la curva } 4x^2+9y^2=36$$

$C: 4x^2+9y^2=36$



① Parametrizar la curva C.

$x = r\cos t \to x = 3\cos t$ con $0 \leq t \leq 2\pi$
$y = r\sin t \quad y = 2\sin t$

$\vec{r}: \mathbb{R} \to \mathbb{R}^2$

$t \mapsto (3\cos t, 2\sin t)$

② Obtener función f.

$f(x,y) = (y, 2x) = (2\sin t, 6\cos t)$

$$\int_0^{2\pi} (2\sin t, 6\cos t)\cdot(-3\sin t, 2\cos t)$$

$$\int_0^{2\pi} (-6\sin^2 t + 12\cos^2 t)\,\partial t$$

$$6\int_0^{2\pi} (-\sin^2 t + 2\cos^2 t)\,\partial t \quad \times$$

$[\cos 2x = \cos^2 x - \sin^2 x]$

$\sin^2 = 1 - \text{sen}^2 - \cos 2x$

$2\sin^2 = 1 - \cos 2x$

$\sin^2 = \dfrac{1-\cos 2x}{2}$

$\cos^2 = \dfrac{1+\cos 2x}{2}$

$\sin(4\pi)$

$$-6\int_0^{2\pi} \sin^2 t\,\partial t + 12\int \cos^2 t\,\partial t$$

$$-\frac{6}{2}\int_0^{2\pi} 1-\cos(2t) + \frac{12}{2}\int_0^{2\pi} 1+\cos(2t)\,\partial t$$

$$-3\left[t + \frac{\sin(2t)}{2}\right]_0^{2\pi} + 6\left[t - \frac{\sin(2t)}{2}\right]_0^{2\pi}$$

$$-3(2\pi + 0) + 6(2\pi + 0)$$

$\cos 2x = \cos^2 - \text{sen}^2$

$-6\pi + 12\pi$

6π // :)

$12\cos^2 = 12(1-\text{sen}^2) - 6\,\text{sen}^2$

b) $\left[x e^{y} \partial x \right]$ en donde C es el arco de la curva $x = e^{y}$ desde $A = (1,0)$ y $B = (e,1)$

① Determinamos $f(x,y)$

$$f(x,y) = (x e^{y}, 0)$$

② Parametrización de la curva C

$$x = e^{t} \quad y = t \qquad (1,0) = (e^{t}, t) \quad (e,1) = (e^{t}, t) \quad \left[\begin{matrix} t = 0 \\ t = 1 \end{matrix} \right]$$

$$e^{t} = 1 \to t = 0$$

$$\int_0^1 (e^{2t}, 0) \cdot (e^{t}, 1)\, \partial t \qquad \left[e^{a} \cdot e^{b} = e^{a+b} \right]$$

③ Derivar r.

$$r(t) = (e^{t}, t) \qquad \frac{\partial r}{\partial t} = (e^{t}, 1)$$

$$\int_0^1 e^{3t}\, \partial t \quad \left[\frac{e^{3t}}{3} \right]_0^1 \quad \frac{e^{3}}{3} - \frac{1}{3} = \frac{e^{3} - 1}{3}$$

$$\int_C 3x^2 \partial x - y \partial y, \text{ donde } C: y = x^2 + 1 \,;\, A = (-1,2) \,;\, B = (2,5)$$

① $f(x,y) = (3x^2, -y)$; Función conservativa

Función potencial

② Parametrizar C.

↳ Fórmula

$$y = x^2 + 1 \,; \qquad \int_0^1 f(tx, ty) \cdot (x,y) \cdot \partial t$$

$$t^2 + 1 = t^2 + 1$$

$$\boxed{A(-1,2)} \boxed{B(2,5)}$$

$$y = t^2 + 1$$

$$x = t$$

$$\to \vec{r}(t) = (t, t^2 + 1)$$

$$\vec{r}\,'(t) = (1, 2t)$$

$$\int_{t_1 = -1}^{t_2 = 2} (3t^2, -t-1) \cdot (1, 2t)\, \partial t$$

$$(3(t)^2, -t^2 - 1)$$

$$A_x = t_1$$

$$(x,y) \to (t, t^2 + 1) \qquad B_x = t_2$$

$$A = (t, t^2 + 1)$$

$$(A_x, A_y) = (t, t^2 + 1)$$

$$\int_1$$

$$\int_{t_1=-1}^{t_2=2} (3t^2, -t^2-1)\cdot(1, 2t)\,\partial t$$

$$\int_{-1}^{2} (3t^2 - 2t^3 - 2t)\,\partial t$$

$$\left[t^3 - \frac{t^4}{2} - t^2 \right]_{-1}^{2}$$

$$\cancel{8-8}-4-\left(-1-\frac{1}{2}-1\right)$$

$$\left[-4+\frac{5}{2}\right]_{//}$$

$-1{,}5$!!

$$\vec{F}(x,y,z) = (y^2 z^3, 2xyz^3, 3xy^2z^2)$$

$\vec{r}(t) = (t, 1-2t^2, 1)$ con $t \in [0,1]$)

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2 z^2 \end{vmatrix} = 0$$

$\vec{F}(x,y,z) = xy^2z^3$.

$(1, -4t, 0)$

$F(b) - F(a)$

$\boxed{F(r(t)) - F(r(0))}$

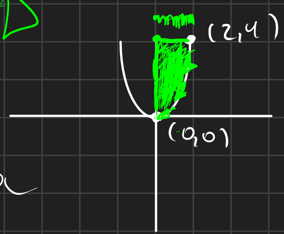
$t \cdot (1-2t^2)^2 \cdot 1^3 \qquad t(\quad 1-2 \qquad -1^2 = 1$

Ejercicio 1

Calcule $\oint_C x^2ydx + xy^3dy$, donde C consta del arco de parábola $y = x^2$ de $(0,0)$ a $(2,4)$, y de los segmentos de recta de $(2,4)$ a $(0,4)$ y de $(0,4)$ a $(0,0)$.

Teorema de Green

$$\iint_R \left(\frac{\partial n}{\partial x} - \frac{\partial m}{\partial y} \right) \partial A$$



Requisitos : $\oint$ → Indica que la integral es cerrada
→ Sentido anti horario

$\vec{r}(t)$

$\iint m\,\partial x + n\,\partial y$ (n? m?)

$\frac{\partial n}{\partial x} = y^3 \qquad \frac{\partial m}{\partial x} = x^2$

$$\int_0^2 \int_{x^2}^4 y^3 - x^2 \; \partial y \, \partial x$$

$$\int \left[\frac{y^4}{4} \right]_{x^2}^4 - x^2 \left[y \right]_{x^2}^4$$

$4^3 \qquad \frac{16 \cdot 4}{64}$

$$\int_0^2 \left(64 - \frac{x^8}{4} \right) - x^2 (4 - x^2)\, \partial x$$

$$\int_0^2 \left(64 - \frac{x^8}{4} \right) - 4x^2 + x^4 \Big)_0^2$$

$$\left(64x - \frac{x^9}{36} - \frac{4x^3}{3} + \frac{x^5}{5} \right)_0^2$$

$$128 - \frac{2^{9}}{9} - \frac{\cancel{2 \cdot 2 \cdot 2^3}\ 32}{3} + \frac{32}{5})$$

$32 \cdot 2 \cdot 2$

$32 \cdot 4$

128

$128 : 32 = 4$

$$128 - \frac{128}{9} - \frac{32}{3} + \frac{32}{5}$$

$$32\left(4 - \frac{4}{9} - \frac{1}{3} + \frac{1}{5}\right)$$

6

$$\frac{180 - 20 - 15 + 9}{45}$$

$4928 : 45 = 109{,}5$

42

$$32\left(\frac{154}{45}\right)$$