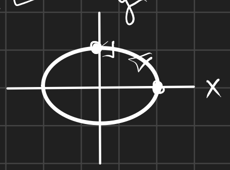
1) $\vec{r} : [0, 6\pi] \to \mathbb{R}^3$

$t \longmapsto (2\cos(t), 2\,\text{sen}(t), t)$

Si es un arco de Jordan pues la curva es inyectiva, es decir, no se intersecta.

2) Calcule. $\int_C [y\,\partial x + 2x\,\partial y]$ donde C es la curva $4x^2 + 9y^2 = 36$



$[f_{(x,y)} = (y, 2x)]$

$f(x(t), y(t))$

C:

$\begin{bmatrix} y = 2\,\text{sen}t \\ x = 3\cos t \end{bmatrix}$

$x(t) = r\cos(t)$;
$y(t) = r\sin(t)$

$C: 4x^2 + 9y^2 = 36 \to \left[\frac{x^2}{9} + \frac{y^2}{4} = 1\right]$

$\frac{1}{9} r^2\cos^2(t) + \frac{1}{4} \cdot r^2\sin^2(t) = 1$

$r^2\left(\frac{1}{9}\cos^2(t) + \frac{1}{4}\sin^2(t)\right) = 1$

$r^2(4(\cos^2(t) + \sin^2(t)) + 5\sin^2(t)) = 36$

$r^2[4 + 5\sin^2(t)] = 36$

$\int_C f(x(t), y(t)) [\|\vec{r}'(t)\|]\,\partial t$

$0 \leq t \leq 2\pi$

$f(x, y) = (y, 2x)$

$f(x(t), y(t)) = (2\sin t, 6\cos t).$

$\|\vec{r}'\|$

$\vec{r}(t) = (3\cos t, 2\sin t)$

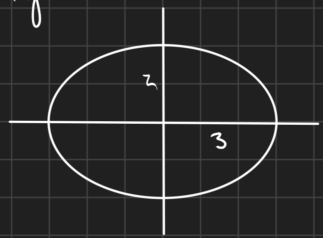
$\frac{\partial \vec{r}}{\partial t} = (-3\sin t, 2\cos t).$

$\int_C \mathbf{F}(x(t), y(t)) \cdot r'(t)\,\partial t$

2.- Calcule

$$\int_C y\,\partial x + 2x\,\partial y \quad \text{donde } C \text{ es la curva } 4x^2 + 9y^2 = 36$$

$C: 4x^2 + 9y^2 = 36$



① Parametrizar la curva C.

$x = r\cos t \rightarrow x = 3\cos t \quad \text{con} \quad 0 \leq t \leq 2\pi$
$y = r\sin t \qquad y = 2\sin t$

$\vec{r}: \mathbb{R} \rightarrow \mathbb{R}^2$

$t \mapsto (3\cos t, 2\sin t)$

② Obtener función f.

$f(x,y) = (y, 2x) = (2\sin t, 6\cos t)$

$$\int_0^{2\pi} (2\sin t, 6\cos t) \cdot (-3\sin t, 2\cos t)$$

$$\int_0^{2\pi} (-6\sin^2 t + 12\cos^2 t)\,\partial t$$

$$6\int_0^{2\pi} (-\sin^2 t + 2\cos^2 t)\,\partial t \qquad \times$$

$[\cos 2x = \cos^2 x - \sin^2 x]$

$\sin^2 = 1 - \text{sen}^2 - \cos 2x$

$2\sin^2 = 1 - \cos 2x$

$\sin^2 = \dfrac{1 - \cos 2x}{2}$

$\cos^2 = \dfrac{1 + \cos 2x}{2}$

$$-6\int_0^{2\pi} \sin^2 t\,\partial t + 12\int \cos^2 t\,\partial t$$

$$-\frac{6}{2}\int_0^{2\pi} 1 - \cos(2t) + \frac{12}{2}\int_0^{2\pi} 1 + \cos(2t)\,\partial t$$

$$-3\left[t + \frac{\sin(2t)}{2}\right]_0^{2\pi} + 6\left[t - \frac{\sin(2t)}{2}\right]_0^{2\pi}$$

$\sin(4\pi)$

$$-3(2\pi + 0) + 6(2\pi + 0)$$

$\cos 2x = \cos^2 - \text{sen}^2$

$$-6\pi + 12\pi$$

6π // :)

$12\cos^2 = 12(1 - \text{sen}^2) - 6\,\text{sen}^2$

b) $\left[x e^{y} \partial x \right]$ en donde C es el arco de la curva $x = e^{y}$ desde $A = (1,0)$ y $B = (e,1)$

① Determinamos $f(x,y)$

$$f(x,y) = \begin{pmatrix} x e^{y}, & 0 \end{pmatrix}$$

② Parametrización de la curva C

$$\begin{cases} x = e^{t} \\ y = t \end{cases} \qquad \begin{matrix} (1,0) = (e^{t}, t) \\ (e,1) = (e^{t}, t) \end{matrix} \quad \begin{bmatrix} t = 0 \\ t = 1 \end{bmatrix}$$

$$e^{t} = 1 \rightarrow t = 0$$

$$\int_0^1 (e^{2t}, 0) \cdot (e^{t}, 1)\, \partial t \qquad \left[e^{a} \cdot e^{b} = e^{a+b} \right]$$

③ Derivar r.

$$r(t) = (e^{t}, t) \qquad \frac{\partial r}{\partial t} = (e^{t}, 1)$$

$$\int_0^1 e^{3t}\, \partial t \quad \left[\frac{e^{3t}}{3} \right]_0^1 \quad \frac{e^{3}}{3} - \frac{1}{3} = \frac{e^{3} - 1}{3}$$

$$\int_C 3x^2 \partial x - y\, \partial y, \text{ donde } C: y = x^2 + 1;\ A = (-1,2);\ B = (2,5)$$

① $f(x,y) = (3x^2, -y)$; Función conservativa

Función potencial

↳ Formula

② Parametrizar C.

$$\int_0^1 f(tx, ty) \cdot (x,y) \cdot \partial t$$

$y = x^2 + 1$;

$t^2 + 1 = t^2 + 1$

$[A(-1,2)]\ [B(2,5)]$

$y = t^2 + 1$

$x = t$ $\rightarrow \vec{r}(t) = (t, t^2 + 1)$

$$\int_{t_1 = -1}^{t_2 = 2} (3t^2, -t-1) \cdot (1, 2t)\, \partial t$$

$\vec{r}\,'(t) = (1, 2t)$

$(3(t)^2, -t^2 - 1)$

$A_x = t_1$

$(x,y) \rightarrow (t, t^2 + 1)$ $\quad B_x = t_2$

$A = (t, t^2 + 1)$

$(A_x, A_y) = (t, t^2 + 1)$

$\int_1$

$$\int_{t_1=-1}^{t_2=2} (3t^2, -t^2-1) \cdot (1, 2t)\, \partial t$$

$$\int_{-1}^{2} \left(3t^2 - 2t^3 - 2t \right) \partial t$$

$$\left[t^3 - \frac{t^4}{2} - t^2 \right]_{-1}^{2}$$

$$\cancel{8-8}-4 - \left(-1 - \frac{1}{2} - 1 \right)$$

$$\left[-4 + \frac{5}{2} \right] //$$

$-1{,}5$!!

$$\vec{F}(x, y, z) = \left(y^2 z^3, 2xyz^3, 3xy^2z^2 \right)$$

$$\vec{r}(t) = (t, 1-2t^2, 1) \text{ con } t \in [0, 1]$$

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \\ y^2 z^3 & 2xyz^3 & 3xy^2 z^2 \end{vmatrix} = 0$$

$\vec{F}(x,y,z) = xy^2 z^3$

$(1, -4t, 0)$

$F(b) - F(a)$

$\boxed{F(r(t)) - F(r(0))}$

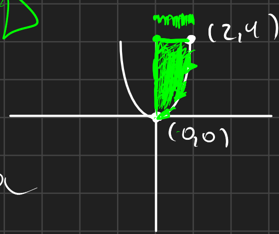
$t \cdot (1-2t^2)^2 \cdot 1^3 \quad t(\ 1-2 \quad -1^2 = 1$

Ejercicio 1

Calcule $\oint_C x^2y\,dx + xy^3\,dy$, donde C consta del arco de parábola $y = x^2$ de $(0,0)$ a $(2,4)$, y de los segmentos de recta de $(2,4)$ a $(0,4)$ y de $(0,4)$ a $(0,0)$.

Teorema de Green

$$\iint_R \left(\frac{\partial n}{\partial x} - \frac{\partial m}{\partial y}\right) \partial A$$



→ Requisitos: $\oint$ → Indica que la integral es cerrada

→ Sentido anti horario

$\vec{r}(t)$

$n?\quad m?$

$\iint m\,\partial x + n\,\partial y$

$\frac{\partial n}{\partial x} = y^3 \qquad \frac{\partial m}{\partial x} = x^2$

$$\int_0^2 \int_{x^2}^4 y^3 - x^2 \, \partial y\, \partial x$$

$$\int \left[\frac{y^4}{4}\right]_{x^2}^4 - x^2 \left[y\right]_{x^2}^4$$

$4^3 \quad \frac{16 \cdot 4^2}{64}$

$$\int_0^2 \left(64 - \frac{x^8}{4}\right) - x^2(4 - x^2)\,\partial x$$

$$\int_0^2 \left(64 - \frac{x^8}{4}\right) - 4x^2 + x^4 \Big)_0^2$$

$$\left(64x - \frac{x^9}{36} - \frac{4x^3}{3} + \frac{x^5}{5}\right)_0^2$$

$$128 - \frac{2^{9}}{9\cdot 4} - \frac{\cancel{2 \cdot 2 \cdot 2^3}\ 32}{3} + \frac{32}{5}\Big)$$

$32 \cdot 2 \cdot 2$

$32 \cdot 4$

128

$$128 - \frac{128}{9} - \frac{32}{3} + \frac{32}{5}$$

$128 : 32 = 4$

$$32\left(4 - \frac{4}{9} - \frac{1}{3} + \frac{1}{5}\right)$$

$$\frac{180 - 20 - 15 + 9}{45}$$

$$32\left(\frac{154}{45}\right)$$

$4928 : 45 = 109{,}5$

Ejercicio 4

Sea $a \in \mathbb{R}$ y el campo vectorial $\overrightarrow{F} : \mathbb{R}^3 \to \mathbb{R}^3$ definido por

$$\overrightarrow{F}(x, y, z) = yz\vec{i} + xz\vec{j} + \left(\frac{axy}{2} + 2z\right)\vec{k}$$

1. Determine el valor de $a \in \mathbb{R}$ para que $\overrightarrow{F}$ sea un campo conservativo.

$$\frac{\partial F}{\partial x} \quad \frac{\partial F}{\partial y} \quad \frac{\partial F}{\partial z}$$

$$F(x, y, z) = \left(yz, xz, \left(\frac{axy}{2} + 2z\right)\right)$$

$$i \quad j \quad k \qquad i\,\frac{\partial}{\partial y}\left(\frac{axy}{2} + 2z\right) + y\hat{j} + z\cdot\hat{k}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z} \qquad \left(\frac{ax}{2}\hat{i} + y\hat{j} + z\cdot\hat{k}\right) - \left(z\hat{k} + x\hat{i} + \frac{ay}{2}\hat{j}\right)$$

$$yz \quad xz \quad \frac{axy}{2} + 2z$$

$$\left|\left(\frac{ax}{2} - x\right)\hat{i} + \left(y - \frac{ay}{2}\right)\hat{j}\right|$$

$$\frac{ax}{2} - x = 0 \;\wedge\; y - \frac{ay}{2} = 0$$

$$ax - 2x = 0 \;\wedge\; 2y - ay = 0$$

$$x(a-2) = 0 \;\wedge\; y(2-a) = 0$$

$$x = 0 \vee a - 2 = 0 \qquad a = 2$$