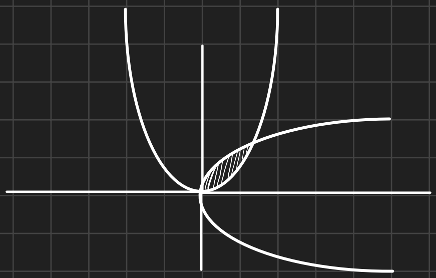
3.- Calcule la integral de línea (orientada positivamente)

$$\to \oint (y + e^{\sqrt{x}})\,\partial x + (2x + \cos(y^2))\,\partial y$$

donde C es la frontera de la región encerrada entre las parábolas $y = x^2$ y $x = y^2$

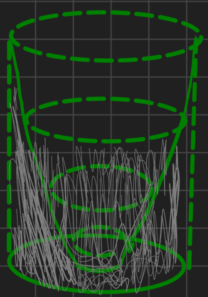


$C: y = x^2 \wedge x = y^2$

$$f(x,y) = \left(y + e^{\sqrt{x}},\ 2x + \cos(y^2) \right)$$

no podemos aplicar teorema de green pues la integral de línea está orientada positivamente.

53. Cilindro y paraboloide Calcule el volumen de la región acotada abajo por el plano $z = 0$, a los lados por el cilindro $x^2 + y^2 = 1$, y arriba por el paraboloide $z = x^2 + y^2$.



$x = r\cos(\theta)$; $y = r\sin(\theta)$; $z = z$

$J = r$

$$z = (r^2\cos^2(\theta)) + (r^2\sin^2(\theta))$$

$$z = (r^2(\cos^2 + \sin^2))$$

$\to z = r^2 \leftarrow$ Paraboloide

$x^2 + y^2 = 1$;

$r^2 = 1$

$\to r = 1 \leftarrow$ Cilindro

(r, θ, z)

$\downarrow$

$0 \le r \le 1$; $0 \le \theta \le 2\pi$; $0 \le z \le r^2$;

$$\int_0^1 \int_0^{2\pi} \int_0^{r^2} 1 \cdot r \,\partial z\,\partial\theta\,\partial r$$

$$\int_0^1 \int_0^{2\pi} [z]_0^{r^2} \cdot r\,\partial\theta \cdot \partial r$$

$$\int_0^1 \int_0^{2\pi} r^3 \, \partial\theta \, \partial r$$

$$\int_0^1 [\theta]_0^{2\pi} \cdot r^3 \cdot \partial r$$

$$\int_0^1 2\pi \cdot r^3 \, \partial r$$

$$2\pi \left[\frac{r^4}{4}\right]_0^1$$

$$2\pi \cdot \frac{1}{4}$$

$$\frac{\pi}{2}[u^3]$$

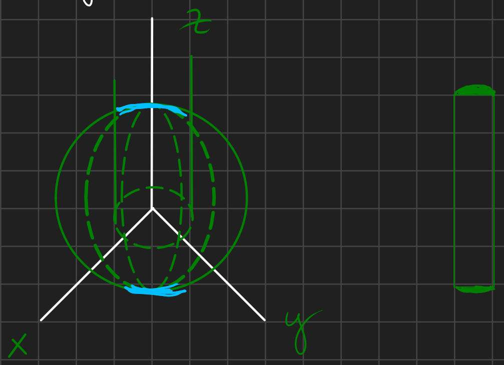
61. Cilindro y esfera Calcule el volumen de la región del cilindro sólido $x^2 + y^2 \leq 1$ cortada por la esfera $x^2 + y^2 + z^2 = 4$.

Por coordenadas esféricas

$$x = \rho \cdot \sin(\varphi) \cdot \cos(\theta) \;;\; y = \rho \sin(\varphi) \cdot \sin(\theta) \;;\; z = \rho \cos(\varphi)$$

$$J = \rho^2 \cdot \cos(\varphi)$$

∅



$x^2+y^2 \leq 1$

$$(\rho \sin(\varphi)\cos(\theta))^2 + (\rho \sin(\varphi)\sin(\theta))^2 \leq 1$$

$$\rho^2 \sin^2(\varphi)\cos^2(\theta) + \rho^2 \sin^2(\varphi)\sin^2(\theta) \leq 1$$

$$\rho^2 \sin^2(\varphi)\left(\cos^2(\theta) + \sin^2(\theta)\right) \leq 1$$

$$\rho^2 \sin^2(\varphi) \le 1 \quad ; \quad \sin^2(\varphi) \le \frac{1}{\rho^2} \quad ; \quad \sin(\varphi) \le \frac{1}{\rho}$$

$$\left[\quad \rho \le \frac{1}{\sin(\varphi)} \quad \right]$$

$$x = r\cos(\theta) \; ; \; y = r\sin(\theta) \; ;$$

$$x^2 + y^2 \le 1 \qquad x^2 + y^2 + z^2 = 4$$

$$r^2 \le 1 \qquad r^2 + z^2 = 4$$

$$r \le 1 \qquad z = \sqrt{4 - r^2}$$

$$0 \le \theta \le 2\pi$$

$$0 \le r \le 1$$

$$-\sqrt{4-r^2} \le z \le \sqrt{4-r^2}$$

$$\int_0^1 r \int_0^{2\pi} \int_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} z \quad \partial z \, \partial\theta \, \partial r$$

$$\int_0^1 r \int_0^{2\pi} \left[z \right]_{-\sqrt{4-r^2}}^{\sqrt{4-r^2}} \partial\theta \, \partial r$$

$$2 \int_0^1 r \sqrt{4-r^2} \int_0^{2\pi} \partial\theta \quad \partial r$$

$$2 \int_0^1 r \sqrt{4-r^2} \left[\theta \right]_0^{2\pi} \partial r$$

$$4\pi \int_0^1 r\sqrt{4-r^2} \, \partial r \qquad u = 4 - r^2$$

$$\partial u = -2r \, \partial r$$

$$0 \le r \le 1$$

$$-2\pi \int \sqrt{u} \, \partial u$$

$$r = 0$$

$$u = 4$$

$$-2\pi \int_4^3 \sqrt{u} \qquad \frac{2 x^{3/2}}{3}$$

$$r = 1$$

$$u = 3$$

$$-2\pi \cdot \left[\frac{2}{3} x^{3/2} \right]_4^3$$

$$-2\pi\left[\frac{2}{3}\,3^{3/2} - \frac{2}{3}\,4^{3/2}\right]$$

$$\frac{2}{3}\left(\sqrt{27} - 8\right)$$

$$\sqrt{9\cdot 3}$$

$$\frac{\cancel{3}\sqrt{3}}{\cancel{3}}$$

$$-2\pi\left(2(\sqrt{3}-8)\right)$$

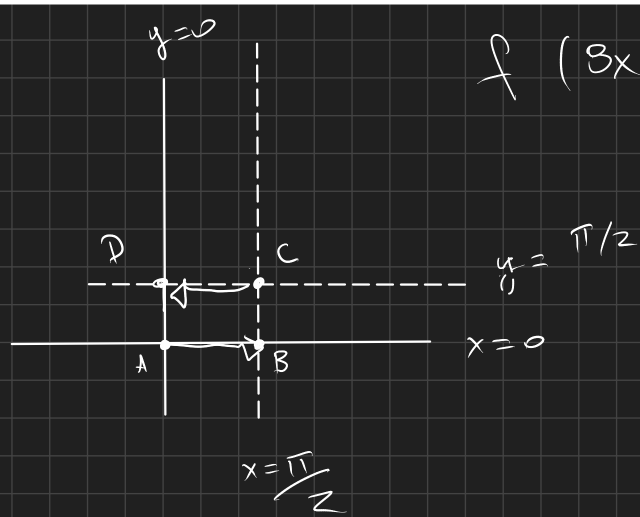
$$-4\pi\left(\sqrt{3}-8\right)$$

$$\frac{4\pi\left(8-3\sqrt{3}\right)}{3}$$

$\frac{\partial f}{\partial x}$ $\frac{\partial f}{\partial y}$

9. $\displaystyle\int_C 8x \operatorname{sen} y\, dx - 8y \cos x\, dy$

C es el cuadrado cortado en el primer cuadrante por las rectas $x = \pi/2$ y $y = \pi/2$.



$f\,(8x \operatorname{sen} y, -8y\cos x)$

$\vec{r}(t)$ $(0,0)$ $(\frac{\pi}{2}, 0)$

$A \to B$

$\vec{r}_1(t) = (t\,\frac{\pi}{2}, 0)$

$\vec{r}_2\left(\frac{\pi}{2}, t\cdot\frac{\pi}{2}\right)$

$\vec{r}_3\,\cancel{\left(t+\frac{\pi}{2}, \frac{\pi}{2}\right)}$

$\int F(r(t))\cdot r'(t)\,dt$

$t=1$ $x=0$

$t=0$ $x=\pi/2$

$$\vec{r}_3(t) = \left(t\left(-\frac{\pi}{2}\right) + \frac{\pi}{2}, \frac{\pi}{2} \right)$$

$(0, \frac{\pi}{2})$

$(0, 0)$

$$\vec{r}_4(t)\left(0, t\left(-\frac{\pi}{2}\right) + \frac{\pi}{2}\right)$$

$$r_i'(t) = \left(\frac{\pi}{2}, 0\right); \left(0, \frac{\pi}{2}\right) \left(-\frac{\pi}{2}, 0\right); \left(0, -\frac{\pi}{2}\right)$$

$$\sum_{i=1}^{4} \int F(r_i(t)) \cdot r_i'(t) \, \partial t$$

$$F(r_1(t))$$

$$\int 8\left(t\frac{\pi}{2}, 0\right) \cdot \left(\frac{\pi}{2}, 0\right) \partial t$$

$$\int_0^1 8\left(\frac{\pi}{2}\right)^2 t \, \partial t$$

$$\frac{8}{4} \cdot \pi^2 \cdot \left(\frac{t^2}{2}\right)\Big|_0^1 = \frac{8}{4} \cdot \pi^2 \cdot \frac{1}{2} = 1\pi^2$$

21. Encuentre la integral de línea de $f(x, y) = ye^{x^2}$ a lo largo de la curva $\mathbf{r}(t) = 4t\mathbf{i} - 3t\mathbf{j}$, $-1 \le t \le 2$.

$f(r(t)) = f((x,y))\underbrace{(x,y) \to (4t,-3t)}_{t(4,-3)}$

$x = 4t$
$y = -3t$

$f(r(t)) = -3t\left(e^{(4t)^2}\right) = -3te^{16t^2} \cdot \|r'(t)\|$

$\to r'(t) = (4,-3)$

$\sqrt{4^2+(-3)^2} = \sqrt{16+9}$

$\sqrt{25}$

5

$r(t) = (4t,-3t)$

$\int_{-1}^{2} -3te^{16t^2} \cdot 5\,\partial t$

$-15\int_{-1}^{2} te^{16t^2}\,\partial t$

$u = t^2$
$\partial u = 2t\,\partial t$

$\to u = 16t^2$
$\partial u = 32t\,\partial t$

$\frac{-15}{2} \cdot \int_{*}^{*} e^{16u}\,\partial$

$\to \frac{-15}{32}\int_{*}^{*} e^{u}\,\partial u$

$u_1 = 16(-1)^2 = 16$
$u_2 = 16(2)^2 = 64$

$\frac{-15}{32}\left[e^{u}\right]_{16}^{64}$

$\frac{-15}{32}\left[e^{64} - e^{16}\right]$ //

no
no
no

23. Evalúe $\int_C xy\,dx + (x + y)\,dy$ a lo largo de la curva $y = x^2$ desde $(-1, 1)$ hasta $(2, 4)$.

$f(xy, x+y)$

$P_1 \quad y = x^2 \quad P_2$

$(-1,1) \to (2,4)$

$\boxed{\begin{aligned} x &= t \\ y &= t^2 \end{aligned}}$

$r(t) = (t, t^2)$

$\mathbb{R} \to \mathbb{R}^2$

$t^2 = t^2$

$\int f(r(t)) \cdot r'(t)$

$\int (t^3, t + t^2) \cdot (1, 2t)\,\partial t$

$\int (2t^2 + 3t^3)\,\partial t = 2\left[\frac{t^3}{3}\right]_{-1}^{2} + 3\left[\frac{t^4}{4}\right]_{-1}^{2}$

$P_1 \to P_2$

$(-1,1) \to (2,4)$

(x, y)

$t_1 = -1$

$t_2 = 2$

(t, t^2)

$\frac{2}{3}\left[2^3 - (-1)^3\right] + \frac{3}{4}\left[2^4 - (-1)^4\right]$

$\boxed{6 + \frac{45}{4}}$

$\frac{24 + 45}{4} = \frac{69}{4}$

$6 + 11$

$\frac{45}{4}$

17^1

24. Evalúe

$$\int_C x^2\,dx + yz\,dy + (y^2/2)\,dz$$

a lo largo del segmento de recta C que une a (0, 0, 0) con (0, 3, 4).

$Bt + (1-t)A \quad C: t(0,3,4) + (1-t)(0,3,4)$

$$\begin{array}{ccc} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & yz & \frac{y^2}{2} \end{array} \qquad \frac{zy}{z} \cdot i - y\,i = 0$$

$$\left(x^2, yz, \frac{y^2}{2}\right)$$

$$\int_0^1 f(tx, ty, tz)\cdot(x,y,z)\cdot \partial t = \frac{x^3}{3} + \frac{y^2 \cdot z}{2}$$

$$\rightarrow F(x\,y\,z) = \frac{x^3}{3} + \frac{y^2 \cdot z}{2}$$

$$\frac{9 \cdot 4}{2} \quad 18$$