ASTROPHYSICS
Junior Honours
Lab Handbook

2015/16

Spectroscopy
Astronomical Exercises

CONTENTS

Edited expanded and revised by PWJL Brand. contributions (Quasar Redshift, revision of Astrometry) by JS Dunlop. Based on a programme developed by MJ Smyth and MT Brück from laboratory exercises devised by EA Baker. Thanks to Starlink Project of PPARC for software and documentation, and to J. Carder for his spectral atlas.

Updated : 27/10/2012 by R. McLure
Updated : 13/01/2013 by A. Ferguson
Updated : 06/01/2015 by D. McLeod, M. Lam
Updated : 20/09/2015 by D. McLeod, M. Lam

Astrophysics Laboratory Timetable 2015-16

Mon/Thu	17 Sep	24 Sep	1 Oct	8 Oct	15 Oct	22 Oct	29 Oct	5 Nov	12 Nov	12 Jan	19 Jan	26 Jan	2 Feb	9 Feb	16 Feb	23 Feb	2 Mar	9 Mar
		25 Sep	2 Oct	9 Oct	16 Oct	23 Oct	30 Oct	6 Nov		15 Jan	22 Jan	29 Jan	5 Feb	12 Feb	19 Feb	26 Feb	5 Mar	12 Mar
Group 1	Spectroscopy									Computing								
2			Computing										Spec			troscopy		
3					Spectroscopy					Computing								

Group 1	Spectroscopy	17 Sep - 15 Oct	Computing	12 Jan - 29 Jan
Group 2	Computing	1 Oct - 16 Oct	Spectroscopy	2 Feb - 5 Mar
Group 3	Spectroscopy	16 Oct - 12 Nov	Computing	12 Jan - 29 Jan

GROUP 1	**GROUP 2**	**GROUP 3**
(1) David Behrendt	(1) Themis Rallis	(1) Aaron Bradley
(2) Evan Cornforth	(2) Gavin Prior	(2) Alexander Falk
(3) Jindra Gensior	(3) Eleanor Spring	(3) Emily Filmer
(4) Tara Bruendl	(4) Alonso Riuz Blanco	
(5) Peter McLagan	(5) Benjamin Sidi	
(6) Craig Atkins	(6) Calum Turner	

Report Hand-in dates : Hand-in dates for each group are 2 weeks after the completion of the individual lab module in question. All reports must be handed-in to the IfA teaching office by 5pm on the following dates:

29th October :	Group 1 spectroscopy report hand-in date
30th October :	Group 2 computing report hand-in date
26th November :	Group 3 spectroscopy report hand-in date
12th February :	Group 1 computing report hand-in date
26th February :	Group 2 spectroscopy report hand-in date
12th March :	Group 3 computing report hand-in date

Sadegh Khochfar
December 2014

FORMAT FOR REPORTS

1. The report should supply enough detail to make clear to someone like yourself what the purpose and methods are, how they were achieved, and what the results are, with diagrams as required.

2. The data should be displayed (in graphs and tables for instance) in enough detail to allow confirmation of your conclusions.

3. The fundamental empirical basis of scientific fact is a number PLUS associated error. Without the error (usually quoted as a standard deviation or error on the mean, with the hidden – and sometimes unwarranted – assumption that the measurement error is normally distributed) it is impossible to assess the odds that the hypothesis that you are advancing is true. *It is very important in these projects that you analyse the measurement (and any other) errors properly.*

4. The length is arbitrary, but as a guide, the *Astronomical Exercises* can be reported in **five to six pages**, perhaps with detailed lists of data as an appendix. The *Spectroscopy* report might occupy **ten to fifteen pages**, including graphs and diagrams, perhaps with a numerical appendix as for the *AE* report.

5. Relevant references, acknowledgements, and attributions should be listed at the end.

6. The report should be produced by word processing, and be a neat and professional document, sensibly bound (perform the following thought experiments: (1) if your report were dropped from a height of two metres onto a hard floor, would it disintegrate? (2) you are the assessor: do you need a tin-opener to start reading this report? Both answers better be *no*).

7. If working in tandem, note the name of your lab partner on the front of the book.

8. **Deadlines** are three term weeks after the end of the relevant slot.

ACCESS TO LABORATORIES AND CLASS LIBRARY

For reasons of Health and Safety, and for security it will not be possible for you to remain in the laboratories after 5:00pm. The Library is available as study space. Arrangements to borrow books may be made with the librarian (Karen Moran, karen.moran@stfc.ac.uk; Rm. R11), who will also provide information about the library's collection and its borrowing. If you choose to remain in the Observatory after 5:00pm (to work in the Library, for example) you **MUST** sign the out-of-hours attendance sheet at the gate-lodge.

SPECTROSCOPY

THE AIMS OF THE SPECTROSCOPY AND ASTRONOMICAL EXERCISES MODULES :

- **Introduce the tasks faced by astronomers making observations**
- **Give hands-on experience of astrophysics taught in lecture courses.**
- **Generate understanding of the statistical nature of physical knowledge, and how to deal with it.**

SAFETY ISSUES & CARE OF EQUIPMENT

1. Ensure equipment is properly connected to mains electricity, from final item (*e.g.* lamp) to wall socket, before switching on. Switch off before changing connections. At the end of each work period, ensure that power is switched off at the wall.
2. In particular, do not use the spectral lamp power supply without a lamp installed, as very high voltages are generated.
3. Do not handle the spectral lamps directly, as they may be very hot; and your hands leave deposits that may etch the glass and destroy the lamp.
4. Do not touch optical surfaces. In particular, NEVER touch the face of a grating.
5. Always close down the laptop computers in an orderly fashion.
6. TLC with everything.

Spectroscopy lab equipment

An important part of this module of the laboratory course is to finish where you started, at least as far as equipment is concerned. At the start of the module, you should have your own bench with a complete inventory of equipment, listed below. At the finish, it should be neatly returned to this state, with all mains switches off and mains plugs removed from sockets, and *checked with a demonstrator before departure*.

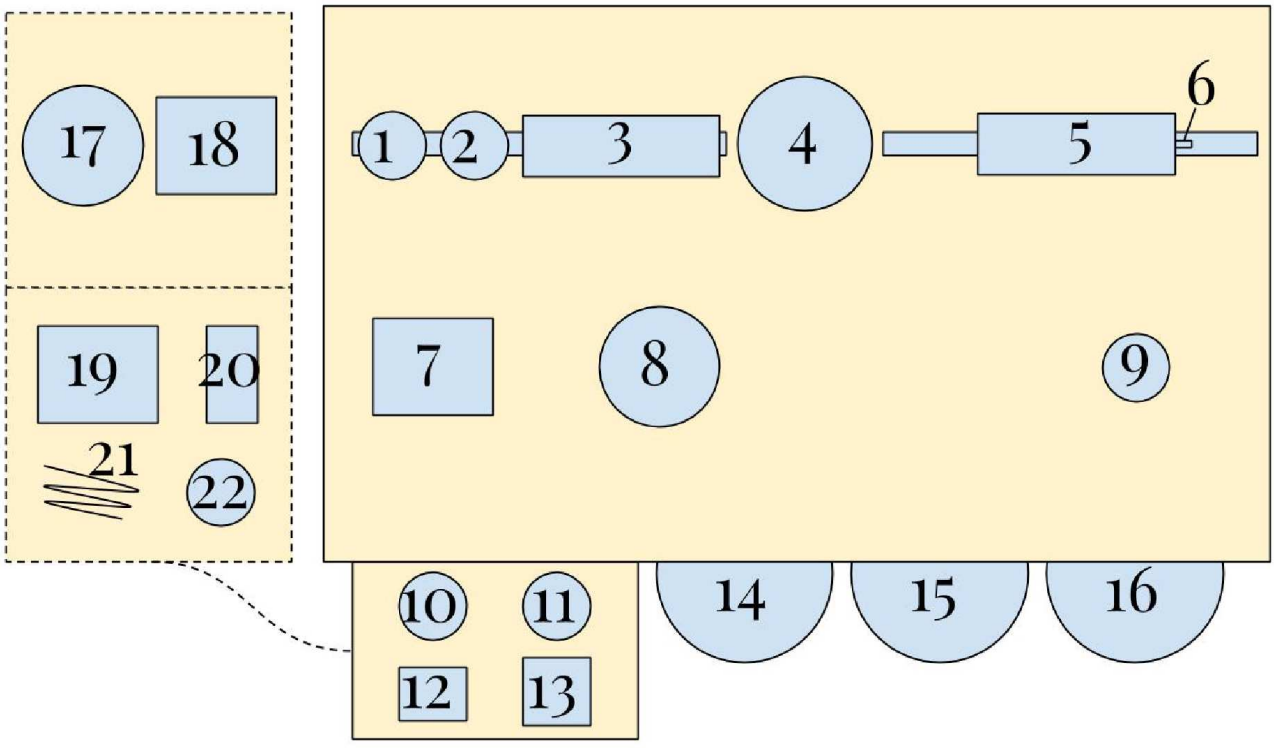
Bench top :
1. He spectrum lamp (*v. expensive!*) in spectrum lamp power supply
2. 12V lamp
3. collimator
4. prism and table, covered by lid, on turntable
5. telescope
6. eyepiece with cross-wire
7. 10V power supply
8. safelight with filter
9. torch

Drawer :
10. lens
11. 2 inch filter holder
12. spirit level
13. filters and lens tissues

Under bench :
14. grating Big Lid
15. prism Big Lid
16. lab stool

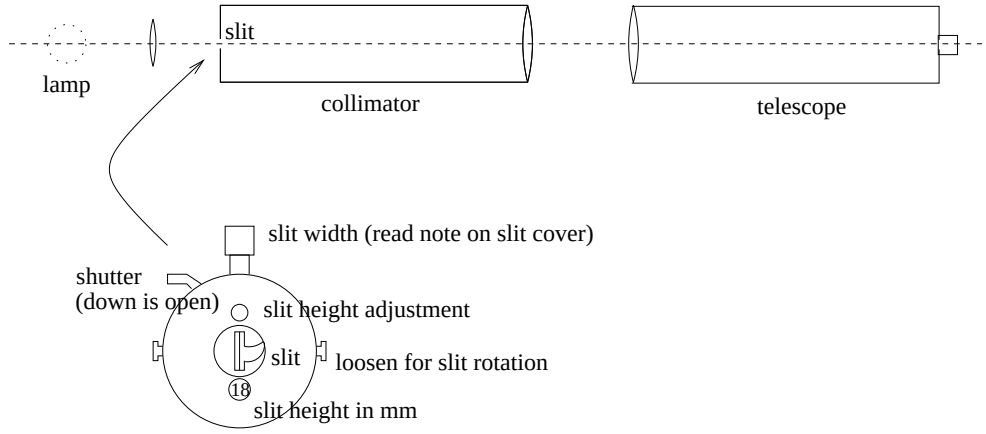
Cupboard :
17. grating and table, covered by lid
18. black cloth
19. laptop computer
20. Power supply
21. lan cable
22. USB camera plus lens

Side bench (shared) :
23. allen keys
24. unknown elements



1. ADJUSTMENT OF PRISM SPECTROMETER

Levelling the Spectrometer



1. Check that you know how a prism spectrometer works.

2. Set the collimator parallel to the front of the bench (where you can lock it to the bench). Level the collimator with a spirit level. Make sure it is at the same height as the opening in the prism table cover (otherwise it won't fit!).

3. Fully open the slit and use a lens to focus a white light source on the slit. Set the telescope 180° from the collimator and adjust its height to receive beam from collimator. Level the telescope[1] with the spirit level. Reduce the brightness of the beam and shorten the slit before looking through the eyepiece in the plate at the back of the telescope. Adjust the level using the screw on the saddle at the back of the telescope until the image of the slit is centred on the exit slot. Check that the collars on the saddle stems are set and locked with Allen screws.

4. Adjust and level the prism table, as follows. Place the prism on table. Replace the white light with a helium lamp. Take great care with the transformer, which develops a high open-circuit voltage: make sure that the lamp is inserted before you switch on the transformer. Find the minimum deviation (See Appendix B) of the prism and bring the telescope round to this position. Shorten the slit and re-level the telescope[1] with the fine screw (the bench may not be perfectly levelled). Check the levelling of the prism table[1] by making sure that the spectrum remains at the same height when the prism is *rotated through the spectrum on both sides of minimum deviation*. Small adjustments to the prism table levelling screws may be needed to make this perfect. For levelling, the shorter the slit, the more critically you can make this adjustment. The crosshair provides a good horizon.

5. If the spectral lines are not vertical (i.e., not at right angles to dispersion), check the slit head, which is slightly rotatable (black knobs to the sides of the slit mount at the end of the collimator tube). Make the slit long for this adjustment.

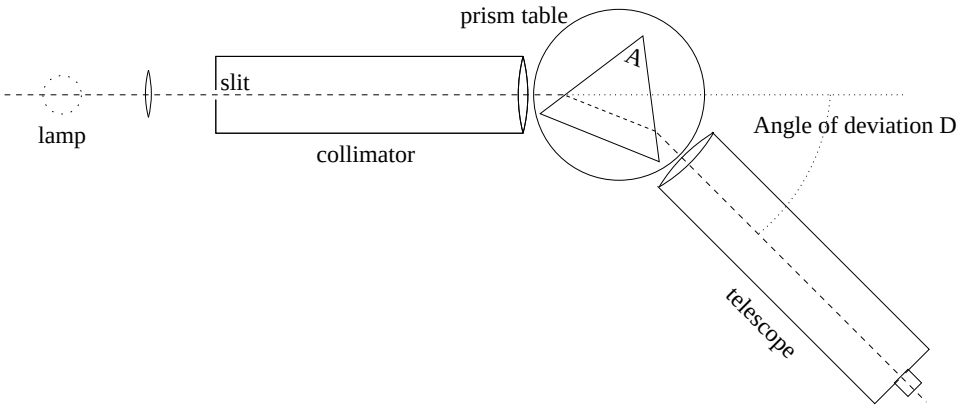
6. Focus the collimator and telescope if necessary (see Appendix B).

[1] The bench itself may not be levelled. You want the set up to be parallel to your bench.

7. Make sure that you understand why there is a minimum deviation, and check that the angle of minimum deviation D_{min}, which occurs when light travels through the prism parallel to its base, is given in terms of prism angle A and refractive index n by

$$\sin\left(\frac{A + D_{min}}{2}\right) = n \sin\left(\frac{A}{2}\right)$$

Calculate the refractive index n for your prism and **estimate the error in** n. *Repeat the process for at least five emission lines to measure how the refractive index varies as a function of wavelength.* You should include a graph of refractive index versus wavelength in your report.



2. ADJUSTMENT OF GRATING SPECTROMETER

Levelling the spectrometer

1. Replace the *prism-and-its-table* by the grating. Be very careful NEVER to touch the face of the grating[2]. The grating is to be used in the **first order**, which is blazed at an angle of about 8°.

2. Move the collimator and telescope to the front positions on the bench (marked out by screw holes). Screw the collimator to the bench. Level the grating, for a start with the three screws on its table, using the spirit level.

3. Set up the white light plus lens and use a very wide slit to give plenty of light. Begin adjustment by considering the grating as a mirror. Find the zeroth order (*i.e.* direct reflection); place the grating at *right angles* to the collimator and send the reflected light back on itself. Cover one half of the collimator lens with a piece of white paper: the reflected beam of the exposed half should coincide with the covered half. If it does not, tip the grating by using the front levelling foot.

4. Move the telescope out of the way (if necessary) and rotate the grating to throw the light from it on the laboratory wall. The zeroth order and side spectra will be seen. By patient adjustment of level of grating the zeroth order will be made to move in a horizontal plane *as the grating is rotated*. The side spectra may not, however, lie in the same plane as the zeroth order, because the grating rulings may not be vertical.

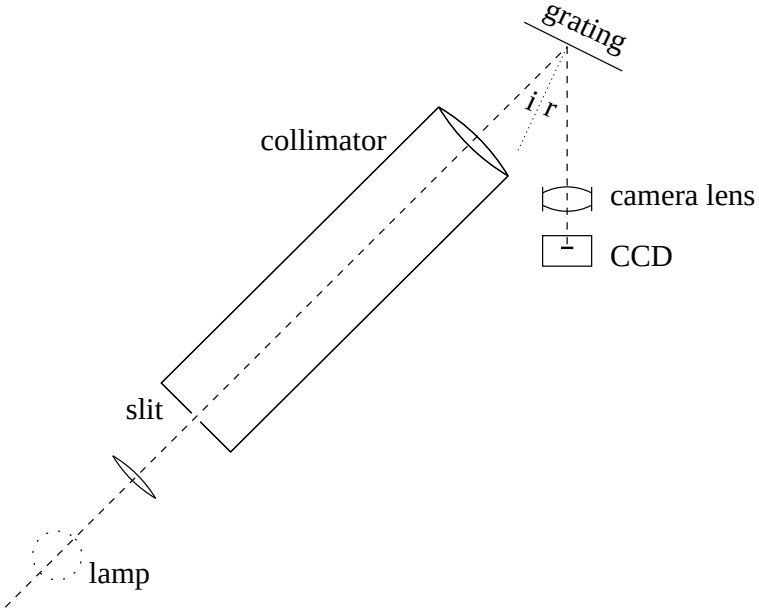
5. To adjust this (make these changes very carefully, very little at a time), the grating is rotatable in its cell by screws at the top (loosen one screw; tighten the other). When the rulings are vertical the first orders are at the same height as the zeroth order (though this does not remain so at the higher orders). Return the telescope to its position and centre the zeroth order (vertically) at the eyepiece by re-adjusting the level of the telescope (the fine adjustment screw ought to be sufficient since the instrument was already levelled in the prism position). Make the slit very short and get the zeroth order on the horizontal cross-wire. Rotate the grating and make final adjustments to get the side spectra at the same vertical height as the zeroth order.

6. Change to the helium lamp. Check that the slit is vertical. Place the horizontal cross-wire along the direction of dispersion (unfortunately the cross-wires are not rotatable so you may have to unscrew the eyepiece a little to achieve this). Now make the slit very long. Place a spectrum line at the cross-wire: it should coincide with the vertical wire. If not, rotate the slit head in the collimator until it is.

7. Clamp the grating table in such a way that *the entire first order spectrum can be brought into view by using the screw on the grating table. (Choose the first order that you can see* **more** *of the spectrum at one time.)*

[2] If you have, don't try to clean it yourself, let your demonstrator know!

Setting up the 25mm CCD camera system



1. Set the yellow line of helium in the middle of the telescope eyepiece. (Optional: Place the metal cover over the spectrum tube which will reduce reflection of the table and contaminating the experiments of your neighbouring groups. Place the Big Lid over the grating, with the collimator lens inserted into its hole. This may may reduce lights from other groups illuminating onto your grating mirror.)

2. Replace the telescope with the 25mm lens CCD camera as follows. Loosen the saddle bases, and slide the telescope along the optical bench rearward from the grating table. *Take great care as the structure is very unbalanced.* Lock the saddles onto the bench again, and transfer the CCD camera to the front of the optical bench. Now transfer the telescope to the section of optical bench from where the CCD camera was taken, and screw it securely to the bench.

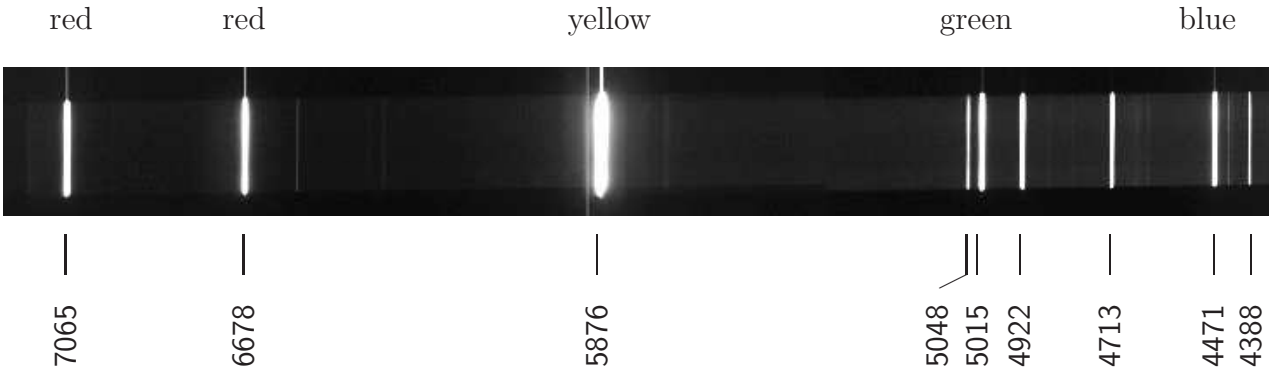
3. Check that the height of the CCD camera is correct, and that it is aligned with the optical bench. Move it as close to the grating as possible, and lock it onto the bench. (Ask the demonstrators for a piece black cloth to shield the light from other groups if needed. Sometimes the *unknown light source* could be the light from your lamp that did not go through the collimator but falls onto the grating mirror.)

4. Boot the laptop computer, and login with the passwd `slabuser`. Once you are logged in, plug-in the CCD camera USB cable (a green light should light up on the back of the camera).

5. The CCD camera is controlled via the software called FLYCAPTURE which is installed on the laptop, ask your demonstrator to show you how the software works. It is important to ensure that the CCD camera is set-up to read-out **16-bit images** and that the exposure time is set to an appropriate value. If everything is set-up properly, on the laptop display you should be able to see continually updating images from the camera.

6. By adjusting the (1) aperture and (2) the focus rings on the camera lens (and the width of the entrance slit on the collimator if necessary), find the settings required to produce a sharp image of the helium spectrum.

7. **You should remember to comment in your report on the significance of the aperture/focus ring settings which produce the best image quality.**



Emission spectrum of a helium discharge tube. The colours you would be able to detect with your eyes are marked above the spectrum. The laboratory wavelengths of the various emission lines are marked below the spectrum.

3. IDENTIFYING UNKNOWN ELEMENTS

The aim of this experiment is to identify the emission line spectra of a series of different gas discharge tubes by measuring the wavelengths of the observed emission lines and comparing them to accurate laboratory values.

1. The first stage of the process is to have a reference image of the spectrum of a known element, from which we can wavelength calibrate our spectra. Wavelength calibration means fitting a simple function which transforms between wavelength and measured x-pixel position on the camera image.

2. For this experiment you are going to use the helium spectrum as the your wavelength reference spectrum. Consequently, the first stage is to take a reference image of the helium spectrum which contains as many emission lines as possible. To do this, you should rotate the diffraction grating to place the bright yellow helium line in roughly the centre of the camera image (as shown above).

*** **If you have moved the grating mirror or the camera, you have to redo everything starting from the next line.**

3. Once you are happy with your helium spectrum (remember to check that it is not saturated) save a number of images of it. The image will be saved as a `pgm` file **(Important: the filenames should be less than 15 characters long and contain no spaces and special characters)**.

4. Now you are ready to change the helium tube for discharge tubes with 'unknown' elements in them.

   Note 1. SWITCH OFF the lamp power supply before changing lamps
   Note 2. DO NOT TOUCH the lamps (for your safety – they're hot; for their safety – you damage the glass). The **demonstrators** *will swap the lamps.*

5. When changing from the helium tube to a new element(s), DO NOT ADJUST THE POSITION OF THE CAMERA OR THE DIFFRACTION GRATING. If these settings are changed the wavelength calibration image you have obtained of the helium spectrum will be useless and you have to start from *** again for a new set of helium images for wavelength calibration.

6. Take images of the spectra of as many different gas discharge tubes as time permit. Remember, to get good quality images of the spectra of different elements you may well need to adjust the entrance slit, camera aperture and camera exposure time settings. Your demonstrator will be able to show you how to adjust the exposure time settings if necessary. Before obtaining your final images of the spectrum of each element, make sure that you are not saturating the camera (ask your demonstrator if you are not sure about this). Once you are happy with your images of the various spectra, save them as `pgm` files on the laptop.

Remember that if some of the emission lines are faint, it is possible to to take several images and then add them together to boost the signal-to-noise ratio (100 images should be sufficient). Ask your demonstrator(s) to show you how this is done in IRAF later in the analysis stage.

4. COLOUR FILTERS

The aim is to produce a graph of transmission versus wavelength for *at least three* colour filters. This is a task that is routinely done by astronomers when they set up new colour photometric systems, or design filters to image just one spectral emission line, or design blocking filters for spectrometers on telescopes.

1. It is easiest to start with the green filter. Set up the continuum light source (bulb), and insert the filter in front of the slit. Seek a position and exposure/intensity setting that shows (a) no saturation of the CCD for filtered or unfiltered light, (b) full wavelength coverage of the part of the spectrum transmitted by the filter[3].

   If you have moved the grating or camera since doing the previous experiment, you need to take another helium image for wavelength calibration purposes. If you haven't changed the spectrograph set-up, then your previous wavelength calibration should still be valid.

2. Without altering any other part of the set-up, make and save *unsaturated* images of the continuum source with and without the green filter (use the same exposure time for each). **Remember that if the signal-to-noise you obtain for the transmission curve is too low, it is possible to take several images and add them up to improve the quality.**

3. Also without altering any of the settings, make and save 'dark' frames by covering the camera lens with the lens cap before taking another exposure of the same duration. The dark frame is a measurement of the instrument noise produced in the absence of any signal.

[3]You can check if the image is saturating by clicking on the ''`horizontal line cut`'' icon and examining the count levels in the image rows

5. WAVELENGTH CALIBRATION

Once you have all of the necessary images saved on the laptop you will need to copy them onto a USB memory stick and transfer them to your account on the blackford server, as follows:

1. Log-in to one of the blackford thin clients.
2. Insert the memory stick into the USB port located on the side of the thin client you are logged into.
3. A USB icon should appear on the desktop, allowing you to then drag your files across to your home space (if in doubt, ask one of the demonstrators to help).

The files that have been transferred onto the server are in the (.pgm) format - these need to be converted to (.fits). A program for converting the files, along with instructions on its use, can be found at the following link: http://www.roe.ac.uk/∼mlam/teaching.html

Note: Saving plots for your report

For the purposes of writing your final report you may want to include plots of the various stages of the reduction process. In IRAF you can generate postscript copies of the plots generated by *splot, identify* etc by typing:

```
:.snap eps
```

in the appropriate window. This will produce a postscript '.eps' copy of the IRAF window in your directory which can be viewed with ghostview (gv) and then printed.

You are going to analyse the spectra using a set of software packages called IRAF (Image Reduction and Analysis Facility[4]). If you have already completed the Computer Exercises then you should already be familiar with IRAF, and the process of wavelength calibration. If not, don't worry, full instructions are provided here as well.

In your home directory you will find a directory called `iraf`. Change into the `iraf` directory (i.e. `cd iraf`) and then type:

```
xgterm -sb &
```

and from within the new terminal which appears, type: `cl`

This will start IRAF which should then display a list of available packages. The list of packages that these exercises will use are listed in the Appendix A. These should be typed into the *login.cl* file so that you can access them without manually loading them in each time you access IRAF - the demonstrator should show you how to do this.

Now type `cd ..` to move up a directory, back to where your data are located [5]. To display the various spectra you are going to use a software package called GAIA. For example, if your Helium calibration spectrum is called `heliumspectrum.fits`, if you type:

[4] http://iraf.noao.edu/

[5] to exit IRAF at any time simply type `logout`

```
gaia heliumspectrum.fits &
```

in an xterm (not the IRAF window), your helium spectrum should appear in GAIA. For images taken with the CCD camera, the dispersion direction of the spectra is horizontal, with blue wavelengths to the right and red wavelengths to the left. The basic operation of GAIA is all "point and click" and is fairly intuitive (online help is available by clicking on the HELP tab in the top right corner of the GAIA window). You should spend some time familiarizing yourself with how GAIA operates, learning how to zoom in and out, change the auto-cut levels etc. If you are having difficulties, ask your lab demonstrator to show you how the basic features work.

At this stage it is necessary to convert the two-dimensional Helium image into a one dimensional spectrum. First, work out how many pixels wide your two-dimensional Helium spectrum is (should be 1280 pixels). Next, work out what the middle y-pixel value of the Helium spectrum is. As an example, if the middle y-pixel value is 480 (and the spectrum is 1280 pixels wide) you would then type:

```
imcopy heliumspectrum.fits[1:1280,480:480] helium_trim.fits
```

which will produce a new, one dimensional spectrum running from x=1 to x=1280, but only one pixel in height. This is a spectrum of flux versus x-pixel value which is suitable for further analysis. You can take a first look at your one-dimensional spectrum using the IRAF task *splot* (spectra-plot). At the command prompt type:

```
splot helium_trim.fits
```

If *splot* asks you for a dispersion direction, type `1 <return>`. As well as simply displaying spectra, the *splot* task allows you to perform many basic analysis tasks (ask the demonstrator to show you how it works). Note that if you need to close *splot* for any reason, it is imperative that you press "q" within the window first. The command line prompt should then reappear and it is safe to close the *splot* window - if you do not do this, IRAF will crash and you will have to start over.

The next stage in the process is to perform the wavelength calibration. At present, the one-dimensional Helium spectrum is simply flux versus x-pixel number, and we need to convert this into a one-dimensional spectrum of flux versus wavelength. The wavelength calibration is performed using the IRAF task *identify*. At the command prompt, type:

```
identify helium_trim.fits
```

Which should open a new window displaying the one-dimensional Helium spectrum. In order to make the identification of the arc lines easier, it may be worthwhile extending the new window horizontally (ask the demonstrator how to do this).

To identify the lines, click on the *identify* window to activate it, place the cursor over the arc line of interest and press the "m" key to mark the line. At this point you will be asked to enter the wavelength of the arc line. Repeat this process until you have identified the $\simeq 9$ Helium lines which should be visible in your spectrum.

Once you have identified the lines you can press "f" to fit the dispersion solution. The *identify* window will update to show you the fit, at which point you can delete outlier points with the

"d" key and re-fit the solution by pressing "f" once again. **Once you are happy with the quality of the fit (all points within $\simeq 1$Å) you should first note down the r.m.s of your wavelength solution (displayed in `xterm window`) because this is the typical uncertainty in the transformation between pixel coordinates and wavelength, which you will require later.** Next you should press "q" twice to exit *identify*, and then confirm (in the `iraf` window) that you want to write the dispersion solution to the database.

To let IRAF know that you want to associate this dispersion solution with your one-dimensional spectrum, the IRAF package *hedit* (header edit) can be used. To add the keyword "REFSPEC1", type:

```
hedit helium_trim.fits fields=REFSPEC1 value=helium_trim.fits add=yes
```

And finally, to apply the wavelength calibration to your spectrum, type:

```
dispcor helium_trim.fits helium_trim_wave.fits
```

The task *dispcor* reads the keyword you have just added, pointing to the solution for `helium_trim.fits`, and applies this to the image. You can now check that your wavelength solution is sensible by displaying your wavelength calibrated Helium spectrum using `splot`:

```
splot helium_trim_wave.fits
```

and by measuring the wavelengths of the emission lines using the cursor and by pressing the space bar.

5a. Identifying the unknown elements

To do this you should first process each of your different spectral images into one-dimesional spectra as outlined above. Following that you can apply your wavelength calibration using `hedit` and `dispcor`, again as outlined above. Once you have produced wavelength calibrated spectra for each element, you can measure the wavelengths of any emission lines and compare them with standard laboratory values[6].

Note that you need to estimate the errors in your result to assess the wavelength matches that you find. You can present the results in your laboratory report in either a tabular or graphical format (see GnuPlot tutorial in Appendix A for information on producing simple graphs).

[6] `http://physics.nist.gov/PhysRefData/ASD/lines_form.html`

5b. Calibrating colour filters

1. After transferring your images to blackford, you should first subtract the dark frame from the filtered and unfiltered images using the `imarith` task in IRAF:

   For example: `imarith cont_spectrum.fits - dark_spectrum.fits cont_dark.fits`

2. After dark subtraction you should produce one-dimensional, wavelength calibrated, spectra of the filtered and unfiltered images following the procedure from the previous experiment.

3. Finally, by dividing the two spectra (i.e. filtered/unfiltered using IMARITH) you can produce a curve of the filter transmission with wavelength. You should be able to display this transmission curve using SPLOT and include the plot in your final report (see Appendix A).

4. Once you have successfully produced the transmission curve for the green filter, the final task is to produce identical curves for the orange/yellow and blue filters. The transmission curves for all three filters should be included in your final report. In the report you should comment on the different sources of uncertainty in your derivation of the filter curves, and how these could in principle be reduced.

For the yellow and blue filters you may need to adjust the exposure times to obtain images of the filtered spectrum with sufficient signal to analyse. In which case you will need to remember to take separate dark frames of the appropriate exposure time. Moreover, when you divide the filtered by the unfiltered spectrum to produce the transmission curve, you must first correct for any difference in exposure time (using `imarith`*).*

ASTRONOMICAL EXERCISES

CCD photometry

1. Introduction

This exercise involves the analysis of two images of a cluster of galaxies which have been obtained using a $v-$filter ($\lambda_{centre} = 5500$Å) and an $i-$filter ($\lambda_{centre} = 8000$Å). The first stage in the process is to use images of a standard star taken through the $v-$filter and $i-$filter at two different airmasses (elevations) to compute the correction due to atmospheric absorption and to calculate the zero-point necessary to convert instrumental magnitudes onto a standard magnitude system. The second stage is to accurately measure the $v-$ and $i-$magnitudes of the galaxies in the cluster images and to compute their $(v-i)$ colours. Based on their $(v-i)$ colours the aim of the exercise is to determine which of the galaxies are likely cluster members, and which are simply foreground galaxies which just happen to lie along the same line of sight.

Copy from `/home/sadeghk/astrolab_files` the following six files into your home directory:

```
cluster_v.sdf
cluster_i.sdf
standard_v_a.sdf
standard_v_b.sdf
standard_i_a.sdf
standard_i_b.sdf
```

The first two files are the images of the galaxy cluster taken through the $v-$ and $i-$filters (.sdf simply indicates that the images are stored in a particular image format). The other four files are images of the same standard star taken through the $v-$ and $i-$filters at two different values of airmass; $\sec\theta = 1.0$ (a) and $\sec\theta = 1.5$ (b). Your first task is to use the observations of the standard star to work out the absorption of the atmosphere in the $v-$ and $i-$filters as a function of airmass ($\sec\theta$).

2. Standard star analysis

All of the photometry measurements in this exercise are going to be performed using a software package called GAIA which allows you to display, manipulate and analyse two dimensional astronomical images. The first thing to try is to display one of the standard star images so, for example, type:

```
gaia standard_i_a.sdf &
```

which should load a GAIA window showing an image of the standard star (see Fig 1). The basic operation of GAIA is all "point and click" and is fairly intuitive (online help is available by clicking on the HELP tab in the top right corner of the GAIA window). You should spend some time familiarizing yourself with how GAIA operates, learning how to zoom in and out, change the autocut levels etc. If you are having difficulties, ask your lab demonstrator to show you how the basic features work.

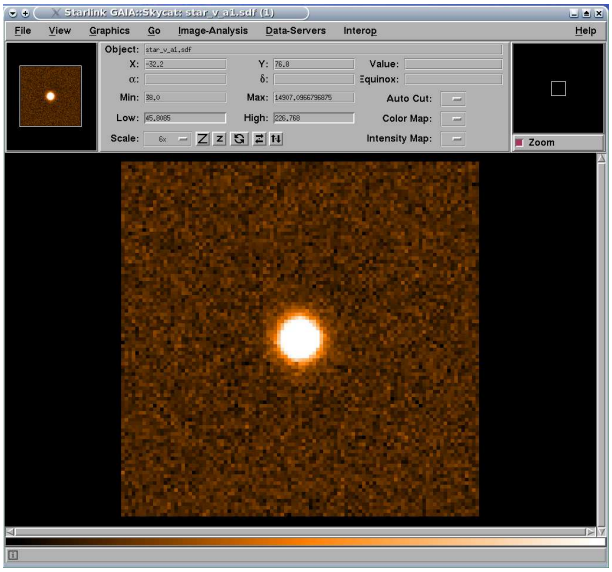


Figure 1: GAIA window displaying the image of a standard star.

3. Instrumental magnitudes

The first measurements you need to make are to determine the instrumental magnitudes of the standard stars. Instrumental magnitudes are simply $m = -2.5 \log_{10} C$; where C is the data-counts from the object, which are proportional to the object's flux[7].

When you have one of the standard star images displayed in GAIA, select the "Image-Analysis" tab located at the top of the GAIA window, and then select "Aperture photometry" and "Results in magnitudes" from the sub-menus which appear. Before measuring any magnitudes it is necessary to set the zero-point parameter (top of aperture photometry window) to zero. This tells GAIA that we don't know the proper zero-point and that we are therefore calculating instrumental magnitudes.

To measure a magnitude, click the "define object aperture" tab and then, with the mouse, drag out a circular aperture which fully encloses the flux of the standard star. GAIA will then automatically define an annulus enclosed by two yellow circles, within which it will measure the data counts coming from the sky background (see Fig 2). Now click on the "calculate results" tab to calculate the magnitude of the star within your chosen aperture. GAIA will now calculate the instrumental magnitude and the appropriate magnitude error. GAIA will also report the "Sum in aperture", which is the data counts within your aperture coming from the star (i.e. total data counts minus the sky background. You should check that the magnitude value is simply $-2.5 \log_{10} C$; where C is the "Sum in aperture"). Note down the data counts, magnitude and magnitude error. Repeat this process for all four standard star images. **To obtain the most accurate results it is best to employ the same sized apertures on each standard star image.**

[7]instrumental magnitudes are so-called because they are specific to a particular instrument/telescope combination

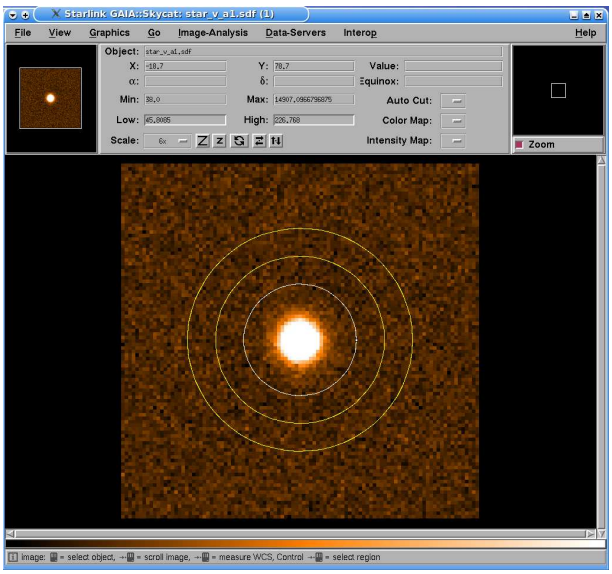


Figure 2: GAIA window displaying the image of a standard star with a circular aperture overlaid (inner circle). The annulus defined by the two outer circles is used by GAIA to measure the data counts coming from the sky background.

In your report you should demonstrate with an example that you understand how GAIA calculates the magnitude errors (see notes at the end of this section).

4. Correcting for atmospheric extinction

At any airmass ($\sec\theta$) the instrumental $v-$magnitude is:

$$m_v(\text{at airmass } \sec\theta) = m_v(0) + e_v \sec\theta$$

where $m_v(0)$ is the $v-$filter instrumental magnitude that would be obtained if observing from above the atmosphere. The parameter e_v is referred to as the extinction coefficient, and describes how much the observed instrumental magnitude is affected by atmospheric absorption as a function of airmass ($\sec\theta$).

The key to determining the extinction coefficient is to observe the *same* star at two different airmasses. You should be able to show that for two stars observed with the $v-$filter at two different airmasses, it follows from the above equation that:

$$e_v = \frac{m_v(\text{at airmass } \sec\theta_2) - m_v(\text{at airmass } \sec\theta_1)}{\sec\theta_2 - \sec\theta_1}$$

and similarly for e_i. Use this equation to calculate the values of e_v and e_i and estimate the corresponding errors (you can assume that the values of $\sec\theta$ carry negligible uncertainty).

5. Converting from instrumental to standard magnitudes

Armed with knowledge of the atmospheric absorption as a function of airmass it is possible to convert observed magnitudes to "above atmosphere" values. It is known from previous observations that on the standard magnitude system the standard star has above atmosphere magnitudes of $v = 17.700 \pm 0.005$ and $i = 17.500 \pm 0.005$. The $v-$filter and $i-$filter images of the galaxy cluster were obtained at zenith ($\sec\theta = 1$), the same airmass as

`star_v_a.sdf and star_i_a.sdf`. Use these two pieces of information to calculate the zero points necessary to convert from instrumental magnitudes to above atmosphere magnitudes on the standard magnitude system for the cluster galaxy images; i.e find ZP:

$$m_{stan} = -2.5 \log_{10} C + ZP$$

where m_{stan} is the above atmosphere magnitude of the standard star.

6. Determining cluster membership

Now that you have calculated the appropriate zero points for the cluster galaxy images we are in a position to measure magnitudes and $(v-i)$ colours of the potential cluster galaxies on the standard magnitude system. The procedure is very straightforward, simply display the $v-$filter cluster image in GAIA and measure the magnitudes, magnitude errors and (x, y) positions of each galaxy. Repeat the process for the $i-$filter image and produce a table which lists the (x, y) position, $v-$mag, $v-$mag error, $i-$mag, $i-$mag error, $(v-i)$ and $(v-i)$ error for each galaxy.

When making these measurements you should think about the importance of aperture-size choice on the $v-$ and $i-$filter images.

Once you have your table of results, the next stage is to plot $(v-i)$ colour versus $i-$magnitude. You are entirely free to produce this plot using any software you are familiar with, but for convenience a help sheet has been included in Appendix A describing how to produce suitable plots with a software package called Gnuplot[8].

It is known from previous observations that galaxies which lie at the centre of clusters tend to be massive and largely populated by old stars which formed many Gyrs ago. As a result of their old age, galaxies at the centre of clusters tend to have a red $(v-i)$ colours; i.e $(v-i) \geq 1.5$. Consequently, any galaxy significantly bluer than $(v-i) = 1.5$ is likely to be an unrelated foreground galaxy. Use this information to decide which of the galaxies are cluster members, and which are foreground interlopers.

Re-plot the $(v-i)$ versus $i-$magnitude diagram for just those galaxies which are likely cluster members. Does there appear to be a relationship between $(v-i)$ colour and $i-$magnitude? Due to the way galaxies evolve in cluster environments, it is known observationally that there is a relationship between the brightness of a cluster galaxy and it's colour, with brighter galaxies having redder colours (the so-called "red sequence"). Using Gnuplot (or any other software you choose) fit a function of the form:

$$(v-i) = a \times i-\text{mag} + b$$

to your data and produce a final plot of $(v-i)$ versus $i-$magnitude with your best-fitting function overplotted. Does your data display a convincing red sequence?

[8] http://www.gnuplot.info/

Quasar Redshift

1. Introduction

The aim of this Astronomical Exercise is to lead you through the process of reduction, calibration and analysis of a real optical spectrum of a quasar in order that you can determine its redshift. Redshift is an important quantity in astronomy because it allows us to determine the cosmological distance to an object, and therefore its luminosity. While the redshift, z, of the quasar is the ultimate objective of this process, along the way you should gain a greatly improved understanding of the key reduction processes of flat-fielding, sky subtraction, and wavelength calibration.

In your home directory there should be three two-dimensional spectra. These spectra are in files called

- `quasar.fits` - the spectrum of the target quasar.
- `flatfield.fits` - an exposure of the inside of the dome illuminated by a tungsten lamp, which provides a continuous spectrum of use for mapping the sensitivity variation of the detector.
- `arc.fits` - a spectrum of the Copper/Neon/Argon arc lamp (which is located in the spectrograph). This provides emission lines at *known* wavelengths (see attached spectral atlas), thus enabling accurate wavelength calibration of the target spectrum.

2. Reduction Package

You are going to analyse the spectra using a set of software packages called IRAF (Image Reduction and Analysis Facility[9]). In your home directory you will find a directory called `iraf`. Change into this directory (i.e. `cd iraf`) and then type:

```
xgterm -sb &
```

and from within the new terminal which appears, type:

```
cl
```

This will start IRAF which should then display a list of available packages. The list of packages that these exercises will use are listed in Appendix A. These should be typed into the *login.cl* file so that you can access them without manually loading them in each time you access IRAF - the demonstrator should show you how to do this. To exit IRAF simply type `logout`.

3. Displaying the spectra

The first thing to do is to display the three spectra, which is done with the GAIA software package you used previously for the CCD photometry exercise. From an xterm (not the IRAF window), type:

[9] http://iraf.noao.edu/

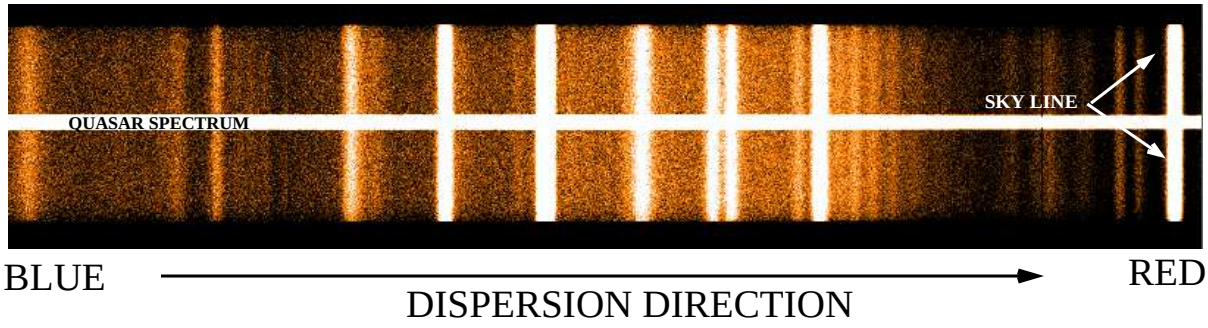


Figure 3: Two-dimensional quasar spectrum as displayed by GAIA

```
gaia quasar.fits &
```

which should display the two-dimensional quasar spectrum in a GAIA window. When displayed by GAIA the two-dimensional quasar spectrum should look something like Fig 3, with the quasar spectrum itself appearing as a bright horizontal line superimposed on the night-sky emission lines. The dispersion direction of the spectrum is horizontal, with blue wavelengths to the left and red wavelengths to the right.

Display the other two spectra in new GAIA windows and familiarize yourself with them. The arc spectrum should consist of a series of bright vertical emission lines. These emission lines have known wavelengths, and will be used to wavelength calibrate the quasar spectrum (i.e. make the conversion between pixel coordinates and wavelength). The flatfield spectrum should look fairly uniform (i.e pixel values $\simeq 1$) and is basically a map of the different sensitivity of the CCD pixels. By dividing the quasar spectrum by the flatfield it is possible to correct for this difference in sensitivity with wavelength.

4. Flatfielding

The first task is to flatfield the quasar spectrum, which simply involves dividing the quasar spectrum by the flatfield spectrum. To do this we use a IRAF task called *imarith*. From the IRAF command prompt type:

```
imarith quasar.fits / flatfield.fits quasarflat.fits
```

This should produce a new file called ‘‘`quasar_flat.fits`’’ (you can choose any name you like of course) in your home directory. To use *imarith* to perform other arithmetic operations you simply replace `/` by `+-*`.

5. Sky subtraction

At present the quasar spectrum contains the signal from the quasar itself *plus* the spectrum of the night sky (including the bright sky lines). The next stage of the reduction process is to remove the night sky emission from the quasar spectrum, a process known as “sky subtraction”. To perform the sky subtraction you are going to use an IRAF task called *background*. This task works by fitting a polynomial function to the sky signal in each column of the two-dimensional spectrum and then subtracting it. However, there is a slight complication here, because along the middle of the two-dimensional spectrum the quasar spectrum is superimposed on top of the sky spectrum, and we don’t want to include this

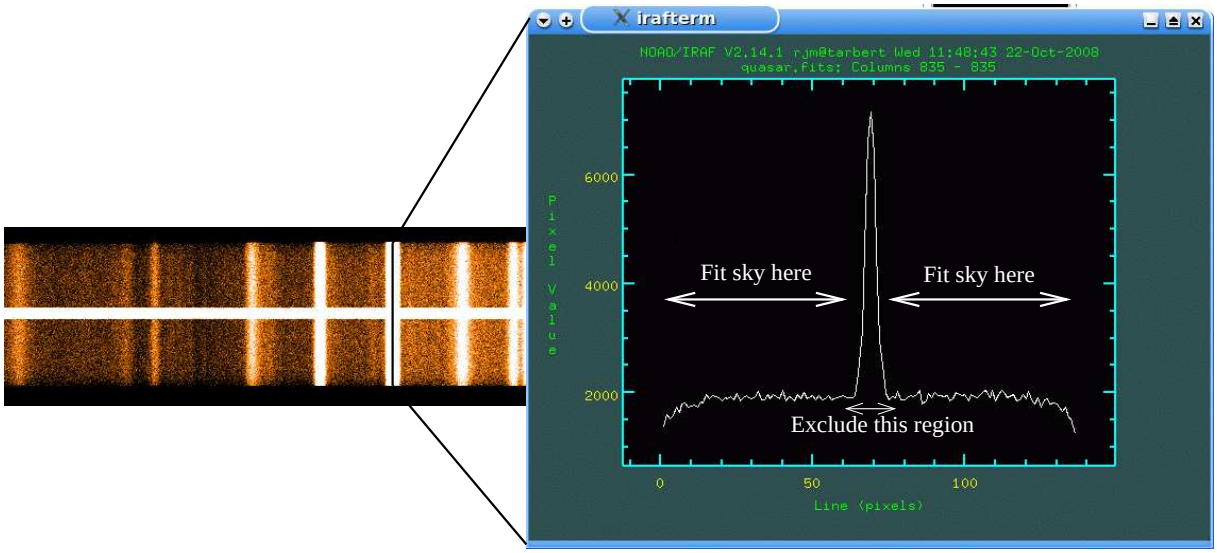


Figure 4: Illustration of how the IRAF task *background* subtracts the sky background from 2D spectra by fitting either side of the object spectrum.

region in our sky fit. This problem is illustrated in Fig 4, which shows a plot of the flux in one column of the two-dimensional quasar spectrum. So, the first task is to determine which range of y-pixel values in the two-dimensional quasar spectrum are dominated by sky. Using GAIA you should note down four y-pixel values which describe the two regions of the two-dimensional spectrum which are dominated by sky (i.e. $y_1 \rightarrow y_2$ and $y_3 \rightarrow y_4$).

Once you've done this you can subtract the sky-background by typing:

`background quasarflat.fits quasarskysub.fits axis=2 sample=` $y_1 : y_2,\ y_3 : y_4$

Make sure that you now have a new file in your home directory which is the flatfielded quasar spectrum with the sky background removed.

6. Extracting the quasar spectrum

The next stage of the reduction process is to turn the two-dimensional quasar spectrum into a one-dimensional spectrum of flux versus x-pixel. This process is know as *extracting* the spectrum and is performed with an IRAF task called *apsum*. At the command prompt, type:

```
apsum quasarskysub.fits
```

(If you are asked how many apertures to identify, simply type: 1 <return>. If you are asked to edit/trace apertures or clobber the existing output image, type 'no', but if asked to extract the aperture spectra or write to database, type 'yes'.)

This will produce a file called "quasarskysub.ms.fits" which is your one-dimensional spectrum. To extract the one-dimensional spectrum *apsum* performs the following operation: at each x-pixel position in the two-dimensional spectrum, *apsum* detects which y-pixels are dominated by the quasar spectrum and simply adds the flux in those pixels together. The final result of this process is a one-dimensional spectrum of flux versus x-pixel value. The range of y-pixels dominated by the quasar spectrum is referred to as the extraction *aperture*, which explains why the task is called *apsum*.

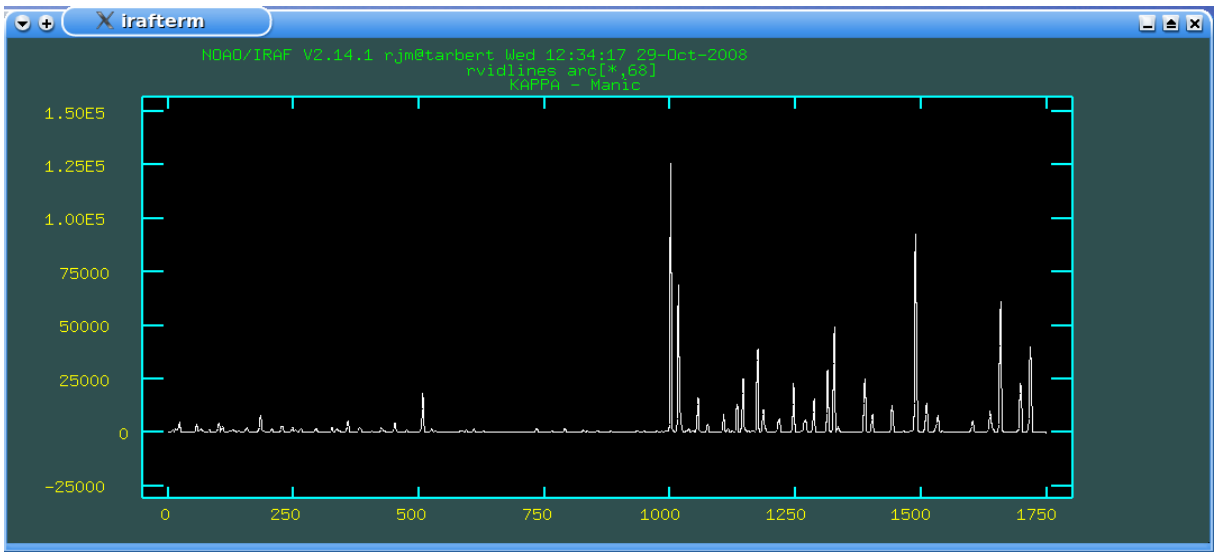


Figure 5: IRAF window generated by *identify* showing the arc spectrum

At this stage you can take a first look at your one-dimensional quasar spectrum using the IRAF task *splot* (spectra-plot). At the command prompt type:

```
splot quasarskysub.ms.fits
```

As well as simply displaying spectra, the *splot* task allows you to perform many basic analysis tasks (ask the demonstrator to show you how it works). Note that if you need to close *splot* for any reason, it is imperative that you press "q" within the window first. The command line prompt should then reappear and it is safe to close the *splot* window - if you do not do this IRAF will crash and you will have to start over.

7. Wavelength calibration

The next stage in the reduction process is to perform the wavelength calibration. At present, the one-dimensional spectrum of the quasar is simply flux versus x-pixel number, and we need to convert this into a one-dimensional spectrum of flux versus wavelength. To achieve this we have to analyse a so-called "arc spectrum", which is a spectrum of a gas discharge tube containing elements[10] which produce emission lines at known laboratory wavelengths. By studying the arc spectrum it is therefore possible to perform the transformation between x-pixels and wavelength. The wavelength calibration is performed using the IRAF task *identify*. At the command prompt, type:

```
identify arc.fits
```

Which should open a new window showing the arc spectrum (see Fig 5). In order to make the identification of the arc lines easier, it may be worthwhile extending the new window horizontally (ask the demonstrator how to do this).

Now for the tricky bit. You need to compare this arc spectrum with that contained in the *CCD Atlas of Helium/Neon/Argon Spectra* (see Appendix B [11]), in order that you can

[10] in this case helium, neon and argon

[11] note: the arc line at 6965.4300 Å is mislabelled

identify $\simeq 20$ lines of known wavelength, preferably scattered reasonably well through the spectral range. It is a good idea to print out your arc spectrum, so that you can write wavelengths on it for those lines that you believe you have identified. As a clue, the bright line at pixel number 500 is the 5015.675 Å line which comes before a region of much weaker emission lines, and the bright line at pixel number 1000 is the 5852.4878Å line which comes at the red end of this emission-line "desert".

To identify the lines, click on the *identify* window to activate it, place the cursor over the arc line of interest and press the "m" key to mark the line. At this point you will be asked to enter the wavelength of the arc line. Repeat this process until you have identified $\simeq 20$ arc lines. Once you have identified enough arc lines you can press "f" to fit the dispersion solution. The *identify* window will update to show you the fit, at which point you can delete outlier points with the "d" key and re-fit the solution by pressing "f" once again. Once you are happy with the quality of the fit (all points within $\simeq 1$Å) press "q" twice to exit *identify*, and then confirm that you want to write the dispersion solution to the database.

To let IRAF know that you want to associate this dispersion solution with your one-dimensional quasar spectrum, the IRAF package *hedit* (header edit) can be used. To add the keyword "REFSPEC1", type:

```
hedit quasarskysub.ms.fits fields=REFSPEC1 value=arc.fits add=yes
```

And finally, to apply the wavelength calibration to your quasar spectrum, type:

```
dispcor quasarskysub.ms.fits quasarskysub_wave.ms.fits
```

The task *dispcor* reads the keyword you have just added, pointing to the solution for `arc.fits`, and applies this to the image.

8. Redshift determination

You can now view your final wavelength calibrated spectrum by typing:

```
splot quasarskysub_wave.ms.fits
```

Using *splot* note down the wavelengths of any obvious emission lines in the quasar spectrum using the cursor and by pressing the space bar.

Redshift z is defined by the equation

$$\lambda_{observed} = (1 + z)\lambda_{emitted}$$

and so if you can work out which emission lines these actually are, then it is trivial to calculate the redshift. Moreover, you will know if you have identified the emission lines correctly because then the calculation of redshift from each emission line will give the same value of z to an accuracy of typically 3 decimal places.

A list of the brightest emission lines seen in active galaxies and quasars is given below, along with their laboratory wavelengths. Only a few of these lines are ever visible in the optical

spectrum of a given quasar, and precisely which ones are visible obviously depends on its actual redshift.

By a process of elimination, work out which of these lines are visible in your reduced spectrum, and hence calculate the redshift of the quasar, quoting an accuracy based on the line-by-line variation in z. **It is important that you clearly demonstrate in your report how you determined the quasar redshift.**

9. Saving plots for your report

For the purposes of writing your final report you may want to include plots of the various stages of the reduction process.

Within IRAF you can generate postscript copies of the plots generated by *splot, identify* etc. by typing:

```
:.snap eps
```

in the appropriate window. This will produce a postscript ".eps" copy of the IRAF window in your directory which can be viewed with ghostview (gv) and then printed.

Table: Prominent UV/optical emission lines in quasars

Lyman$-\alpha$	1215Å
NV	1240Å
CIV	1549Å
CIII	1909Å
MgII	2799Å
HeI	3189Å
[NeV]	3426Å
[OII]	3728Å
[NeIII]	3870Å
[NeIII]	3961Å
H-δ	4103Å
H-γ	4346Å
H-β	4861Å
[OIII]	4959Å
[OIII]	5007Å
[OI]	6300Å
H-α	6563Å
[SII]	6724Å

Astrometry

Astrometry is the determination of accurate spherical coordinates, Right Ascension (α) and Declination (δ), for an object whose position was previously either unknown or poorly established. As well as simply establishing the position of astronomical sources, astrometry can be used to measure the proper motion of objects such as asteroids or 'nearby' stars, provided astronomical images at suitably different epochs are available.

In fact, asteroids move sufficiently quickly that they produce a trail across a long exposure CCD image. Measurement of the length of this trail can thus be used to establish the proper motion of the asteroid.

The aim of this exercise is to measure the proper motion of the 13th-magnitude asteroid (595) Polyxena from its trailed image on a 90-minute CCD exposure and hence to estimate its orbital radius from the Sun.

1. The position of the beginning and end of the asteroid trail needs to be established by setting up a coordinate frame (known as a plate solution) fitted to the measured x, y positions of stars of known α and δ which can be seen on the CCD image around the position of Polyxena. The first thing you therefore need to do is to load the `polyxena.fits` CCD image into GAIA and measure the x, y coordinates of the 25 stars shown in Fig. 6. The stars are numbered, and their equatorial coordinates are provided in Table 1, so make sure you note down which x, y coordinate refers to which star.

2. You should now have all the information necessary to make a final list containing 5 columns: star number, α, δ, x, y. Based on this list you can now use GAIA to perform the coordinate transformation (ask your demonstrator to help with this). Using the grid of 25 stars, it should be possible to produce a plate solution with a rms of $\simeq 0.5$ arcseconds. When you have an good solution, accept the fit and save the `polyxena.fits` image under a new filename (e.g. `polyxena-astrom.fits`). Reload this new image into GAIA and carefully measure the (α, δ) coordinates of the start and end of the asteroid trail (which can be seen near the centre of the field of view).

3. Based on the coordinates of the start and end of the asteroid trail (and the 90 minute exposure time), it is now possible to calculate the angular velocity of the asteroid (**think about the cos(δ) factor!**). Assuming circular motion for the asteroid's orbital path around the Sun, and knowing that the field was close to opposition at the time of observation (see Appendix C), compute its distance from the Sun in Astronomical Units. As indicated by the diagram, you can assume that the asteroid is orbiting the Sun in the same direction as the Earth.

Star	RA	Dec
1	14:52:49.429	−28:54:51.13
2	14:52:40.927	−28:56:37.55
3	14:51:52.970	−28:58:56.30
4	14:51:51.318	−28:59:00.21
5	14:52:14.225	−29:02:41.80
6	14:52:06.541	−29:03:31.15
7	14:51:44.912	−29:02:41.68
8	14:52:02.075	−29:05:46.92
9	14:52:05.628	−29:11:02.29
10	14:52:02.219	−29:12:39.20
11	14:52:35.570	−29:09:41.13
12	14:53:04.314	−29:05:43.35
13	14:53:07.746	−29:04:42.56
14	14:51:00.138	−28:58:57.20
15	14:50:53.009	−29:00:13.11
16	14:51:07.926	−29:12:56.19
17	14:52:56.981	−28:59:47.82
18	14:52:50.848	−29:01:58.89
19	14:52:48.953	−29:03:11.91
20	14:52:44.232	−29:03:59.56
21	14:52:42.356	−29:05:36.85
22	14:51:25.420	−29:08:48.12
23	14:51:16.278	−29:06:31.77
24	14:51:10.221	−29:08:07.60
25	14:51:07.936	−29:06:44.75

Table 1: List of equatorial coordinates for the 25 stars highlighted in Fig. 6

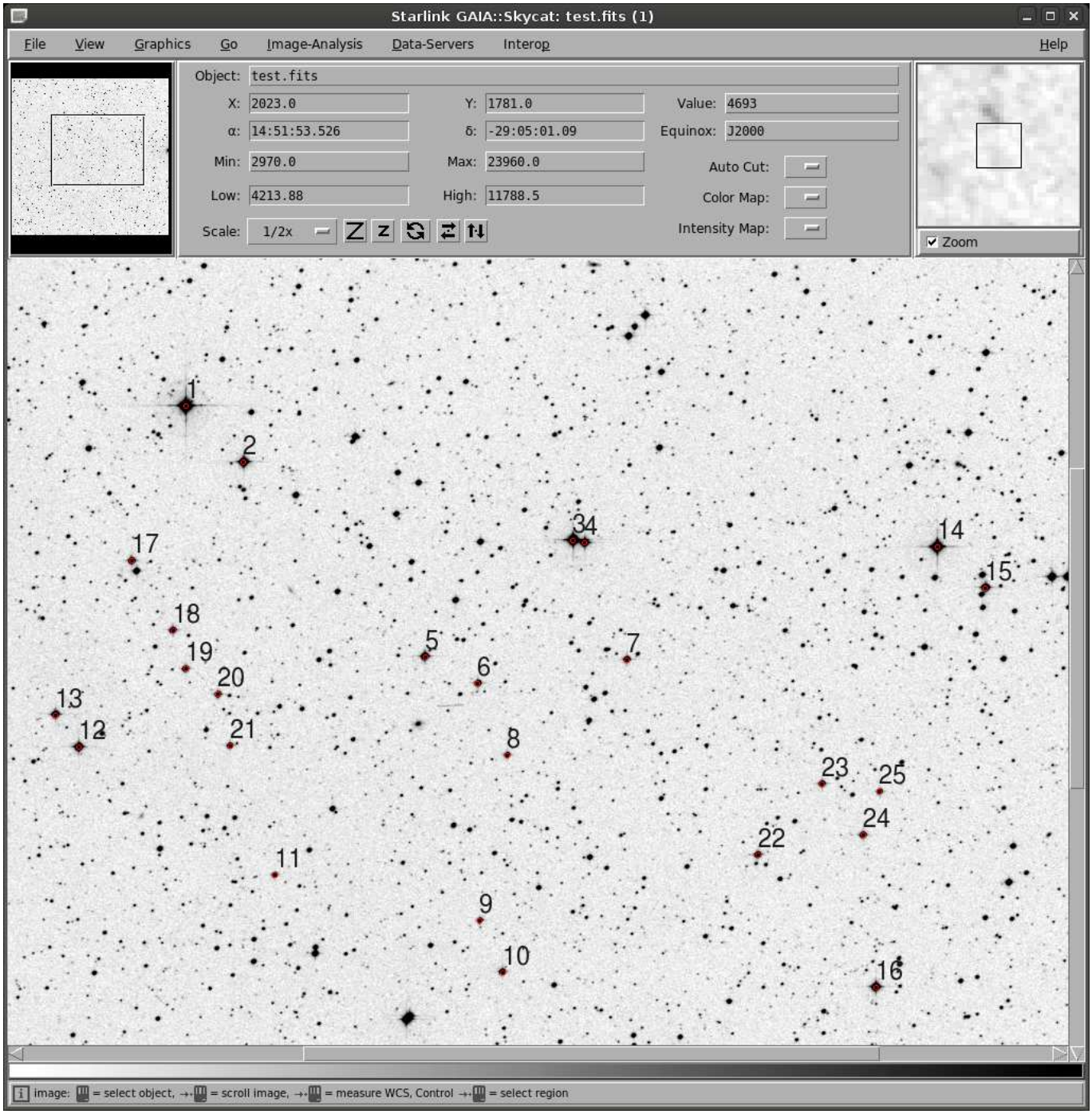


Figure 6: Screenshot of the polyxena.fits image showing the locations of the 25 stars which can be used to define the equatorial coordinate system.

Appendix A - General Computing

1. Basic Unix Commands

Command	Description
cd	Change location to home directory
cd *dirname*	Change location to directory *dirname*
cd ..	Move up one directory in the tree
pwd	Print current directory
ls	List files in current directory
ls -l	List files in detail
ls -h	List files in kB, MB, GB etc.
ls -a	List files including hidden files
ls -t	List files ordered by time
more *f1*	Show contents of *f1* on the screen
mkdir *dirname*	Create new directory *dirname*
rmdir *dirname*	Remove directory *dirname*
mv *f1 dirname*	Move file *f1* to directory *dirname*
mv *f1 f2*	Rename file *f1* as *f2*
cp *f1 dirname*	Copy file *f1* to directory *dirname*
cp *f1 f2*	Copy file *f1* to new file *f2*
rm *f1*	Remove file *f1*
exit	Close current terminal window
passwd	Change password

2. Useful software packages

For working with ascii data files on the Unix system it is probably easiest to use the basic text editor package: *emacs*. To open a new file with emacs, simply type at the command prompt:

```
emacs myfile.txt (return)
```

For viewing and printing postscript files, use the software package *gv*. At the command prompt type:

```
gv (return)
```

For taking screenshots use the software package *gimp*. At the command prompt type:

```
gimp (return)
```

Then go to the drop-down menu `File:Create:Screenshot`.

For plotting graphs and fitting functions the easiest package to use if probably *gnuplot*. On the next page there is a brief tutorial on how do use gnuplot.

3. IRAF packages and user initialisation

In order to use IRAF, the user has to type a series of commands in the terminal - as done step by step below:

```
cd $HOME
mkdir iraf ; cd iraf
mkiraf :  terminal type:  xgterm
mkdir dev ; cd dev
mknod imt1i p
mknod imt1o p
chmod 600 imt1[io]
```

The packages that are required for the various computing exercises are listed below. These are needed to run some of the tasks, and should be included in the *login.cl* file for IRAF, to save you manually calling them every time you run IRAF. After opening the *login.cl* file, look for the line

List any packages you want loaded at login time, ONE PER LINE.

There should be four default packages already included:

```
images   # General image operators
plot     # Graphics tasks
dataio   # Data conversions, import export
lists    # List processing
```

Append the followings to the list:

```
noao        # A package developed by the National Optical Astronomy Observatories
onedspec    # One dimensional spectral red & analysis package
twodspec    # Two dimensional spectral red & analysis package
imred       # Image reductions package
generic     # Generic image reductions tools
apextract   # Aperture Extraction Package
```

To access the manual of each of the packages, type help package-name.

4. Gnuplot

This is designed to be a very brief tutorial on using the plotting package Gnuplot. More information can be found at: `http://www.gnuplot.info/`, and a more detailed tutorial can be found at: `http://www.duke.edu/~hpgavin/gnuplot.html`.

Start Gnuplot by simply typing:

```
gnuplot -mono (return)
```

on the command line. In your home directory I have included a file called `toydata.dat` to illustrate how the basic features of Gnuplot work. This file contains three columns: x, y & y error. To simply plot x versus y, type:

```
gnuplot> plot "toydata.dat"
```

Gnuplot should produce a plot of x versus y in a new window, and will automatically choose appropriate ranges for the x-axis and y-axis. To label the axes and to add a title type:

```
gnuplot> set xlabel "x"
gnuplot> set ylabel "y"
gnuplot> set title "toydata plot"
```

If you now type:

```
gnuplot> plot "toydata.dat"
```

The graph should be correctly labelled. To plot the data including the errors on the y variable you can type:

```
gnuplot> plot "toydata.dat" with yerrorbars
```

To fit a function to your data-set you can type:

```
gnuplot> f1(x)=a*x+b
```

which defines a new linear function (gnuplot can fit much more complicated functions if necessary). And then type:

```
gnuplot> fit f1(x) "toydata.dat" using 1:2 via a,b
```

which tells Gnuplot to fit the function to columns 1 & 2 of "toydata.dat", determining the best-fitting values of parameters a & b. To then produce a combined plot of your data and the best-fitting function you can type:

```
gnuplot> plot "toydata.dat" with yerrorbars, f1(x)
```

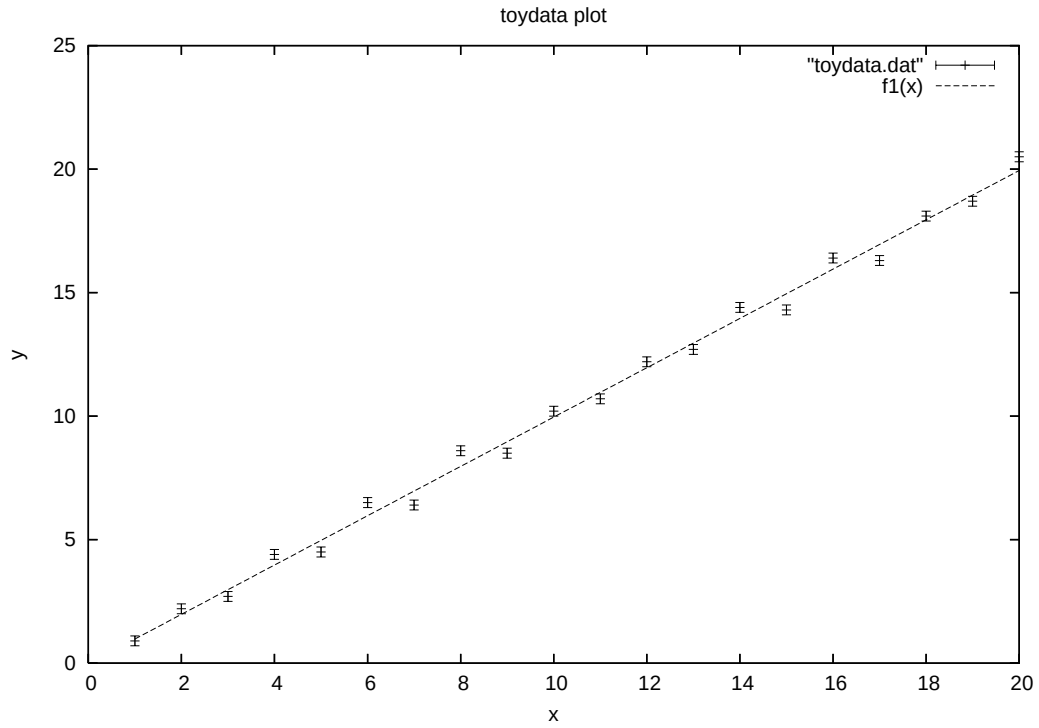


Figure 7: Example graph produced by gnuplot based on the datafile "toydata.dat".

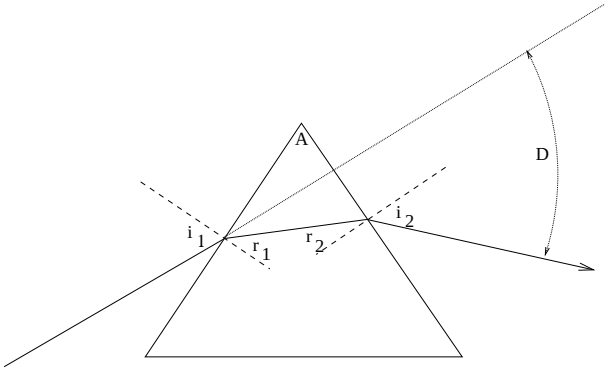
At the end of this process the graph in the Gnuplot window should look something like Fig 1. If you wanted to save your graph as a postscript file (suitable for printing) you could type:

```
gnuplot> set terminal postscript landscape
gnuplot> set output "toydataplot.ps"
gnuplot> plot "toydata.dat" with yerrorbars, f1(x)
gnuplot> set terminal x11
```

which should produce a postscript file called "toydataplot.ps". The final command returns the default output to the normal Gnuplot window. To exit Gnuplot type `CNTR-D`.

Appendix B - Spectroscopy

1. Minimum Deviation Angle



The diagram shows a ray passing though a prism with refractive index n and prism angle A. The ray is deviated by angle D. From geometry it is clear that:

$$(90^\circ - r_1) + A + (90^\circ - r_2) = 180^\circ \Rightarrow A = r_1 + r_2 \tag{1}$$

and that the angle of deviation D is given by:

$$D = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - A \tag{2}$$

The minimum deviation occurs when:

$$\frac{dD}{di_1} = 0 \Rightarrow \frac{di_2}{di_1} = -1 \tag{3}$$

A simple geometrical argument demonstrates that the minimum deviation must happen when $i_1 = i_2$ (i.e. when the light ray passes through the prism parallel to the base). If this were not the case, we could simply reverse the direction of the light path to prove that there are two distinct angles of minimum deviation, which is not true.

At minimum deviation ($i_1 = i_2$) we can see from equation 2 that: $i_1 = \frac{D_{min}+A}{2}$
And from Snell's law ($\sin i_1 = n \sin r_1$), at minimum deviation ($r_1 = r_2 = A/2$) we have:
$\sin i_1 = n \sin \frac{A}{2}$ which in turn means that:

$$n \sin \frac{A}{2} = \sin \frac{D_{min} + A}{2} \tag{4}$$

This equation allows us to calculate the refractive index of the prism.

2. Schuster's method

We have just demonstrated that $di_2 = -di_1$ at the minimum deviation. However, at an arbitrary angle of incidence, a small change in incidence angle i_1 gives rise to a change in emergence angle i_2 which can be written

$$\mathrm{d}i_2 = -K\,\mathrm{d}i_1$$

where K can be calculated from Snell's law (e.g. $\sin i = n \sin r$) which, at each surface, gives

$$\mathrm{d}i = n(\cos r / \cos i)\mathrm{d}r.$$

Since $r_1 + r_2 = A$ (a constant) $\mathrm{d}r_1$ = -$\mathrm{d}r_2$ and hence

$$K = \left(\frac{\cos r_2}{\cos i_2}\right) / \left(\frac{\cos r_1}{\cos i_1}\right) = \sqrt{\frac{1 - \mu^2 \sec^2 r_1}{1 - \mu^2 \sec^2 r_2}},$$

To prove this result requires some algebra. You will need to use Snell's law, the identity $sin^2\theta + cos^2\theta = 1$ and make the substitution: $\mu^2 = 1 - 1/n^2$. Hence if $i_2 < i_1$ then, since $r_2 < r_1$ and $\sec r_2 < \sec r_1$, it follows that $K < 1$. This means that $|di_2| < |di_1|$. And conversely, if $i_2 > i_1$ then $K > 1$ and $|di_2| > |di_1|$.

Thus the result: a small change in incidence angle i_1 makes a smaller change in emergence angle i_2 if the emergence angle is smaller than the incidence angle (and bigger if bigger).

By applying this principle we can focus the spectrometer. The basic idea is that you find the minimum deviation of the prism (at which point $i_1 = i_2$). You then move the prism away from minimum deviation, such that $i_2 > i_1$ and re-acquire the emission line with the telescope (position 1). From the above argument, at this position a small change in i_1 will result in a larger change in i_2 - which means that any defocus in the telescope will be exaggerated (which is why it is best to focus the telescope in this position). In the second prism position where you can see the emission line $i_2 < i_1$. This means that any defocus in the collimator will be exaggerated at this position (which is why we focus the collimator in this position).

Applying the principle to the spectrometer we see the following.

When all the light rays incident on the prism are parallel then all are deviated by the same amount and emerge parallel, and so the focus position of the telescope remains the same whatever the deviation, and the image remains sharp moving from one side of minimum deviation to the other.

However, if the incident beam is convergent or divergent on the prism (set so that the emergent rays are more nearly normal, *i.e.* average incidence angle is greater than average emergence angle), then the range of angles of emergent rays is *less*: they are more parallel as they leave the prism and enter the telescope. In this case, the telescope should be focussed as its focus position will be more nearly at the focal plane. Moving to the other prism position at which the line is visible, the collimator beam is the more parallel, so the collimator should be focussed in the beam which – since it is more parallel than is the telescope beam – will

bring the focus position of the collimator even closer to its focal plane. Alternating positions and focussing the more parallel beam will converge to the situation in which each beam is parallel, and the system is focussed for that wavelength.

Focussing the collimator by Schuster's method[12]

1. Set the prism at minimum deviation on one spectrum line, e.g., green (see Helium picture and tables). Close the slit to fairly narrow width. Rotate the prism table away from minimum deviation, increasing angle of *incidence* on prism; we'll call this prism position 1'. Then move the telescope to find the line.

2. Focus the telescope for a sharp line (first focus eyepiece (push/pull) on cross-wire, then focus spectrum line on cross-wire).

3. Keep telescope fixed in the same position. Rotate prism table in the direction opposite to 1. above, thus *decreasing* angle of incidence on prism, until the image of the same line again appears central in the eyepiece. Focus the collimator. To focus the collimator while viewing through the eyepiece get your lab partner to focus while you view through the slit. Now return the prism to its position 1.

4. Repeat processes (2) and (3), and continue until no further improvement is possible. Note collimator scale reading.

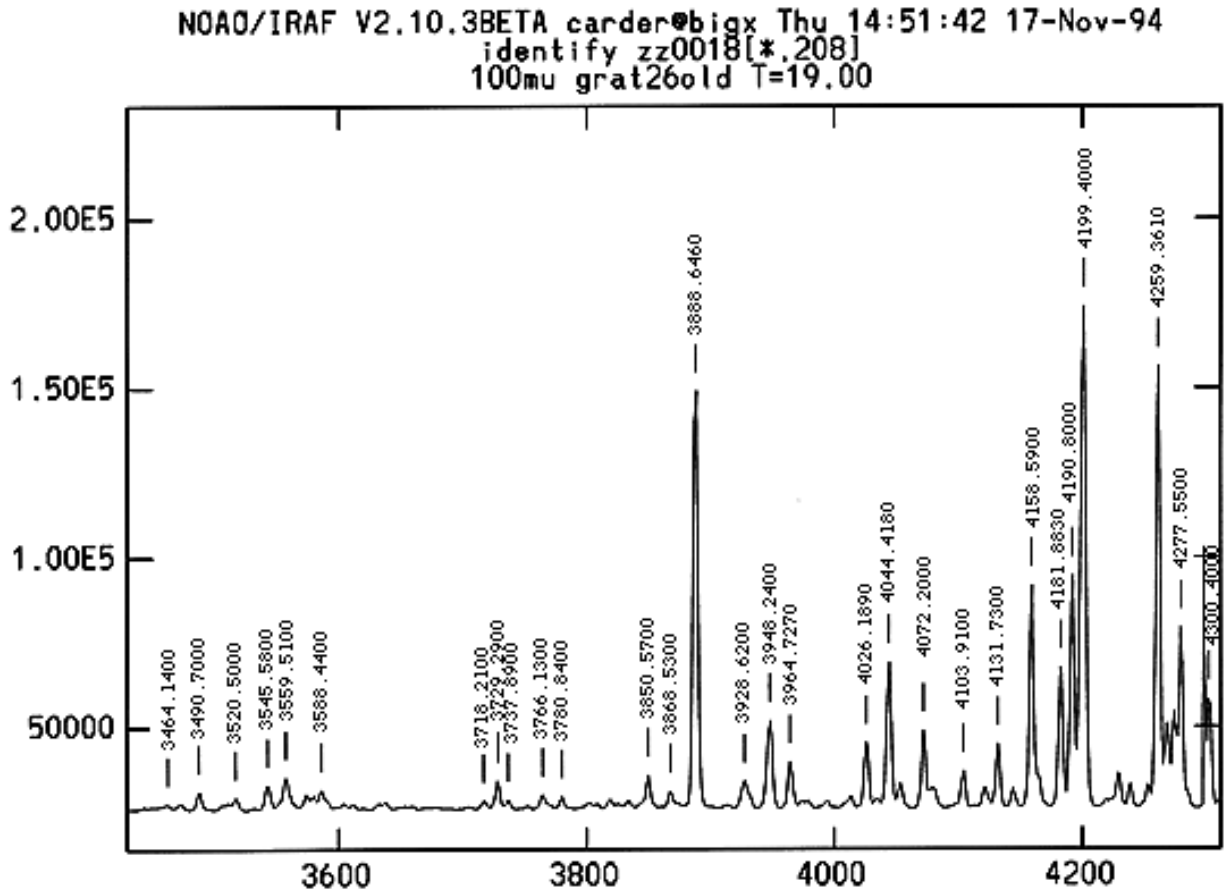
5. Repeat the process with other spectrum lines and draw up a table and graph relating collimator scale reading to wavelength. **Estimate and explain the sources of the errors in the measurements, and explain any trends.**

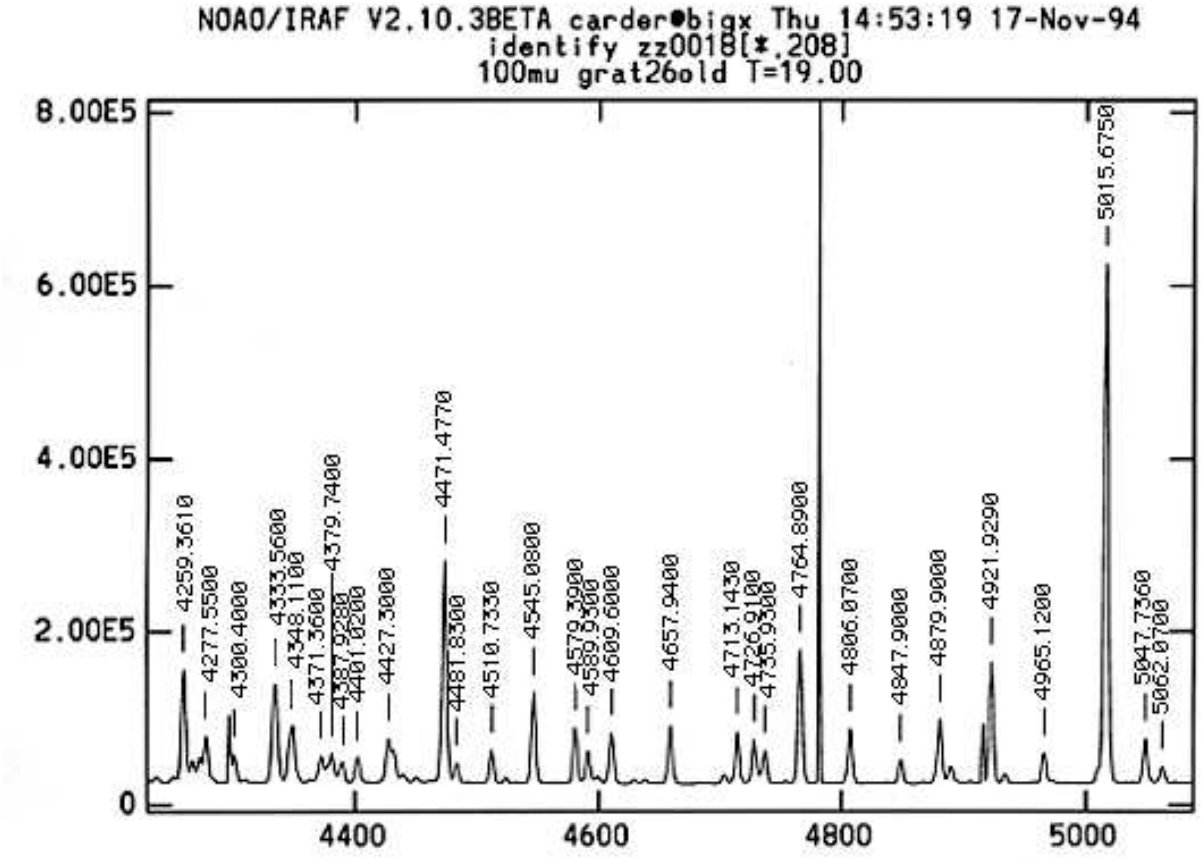
6. Choose the collimator position which you think is the best average position for the wavelength range you intend to use, and fix it permanently.

[12]SCHUSTER, A., 1879, 'An easy Method for Adjusting the Collimator of a Spectroscope', *Phil. Mag.* **7**, 95

3. A CCD atlas of Helium/Neon/Argon spectra

E. Carder (April 17, 1996)

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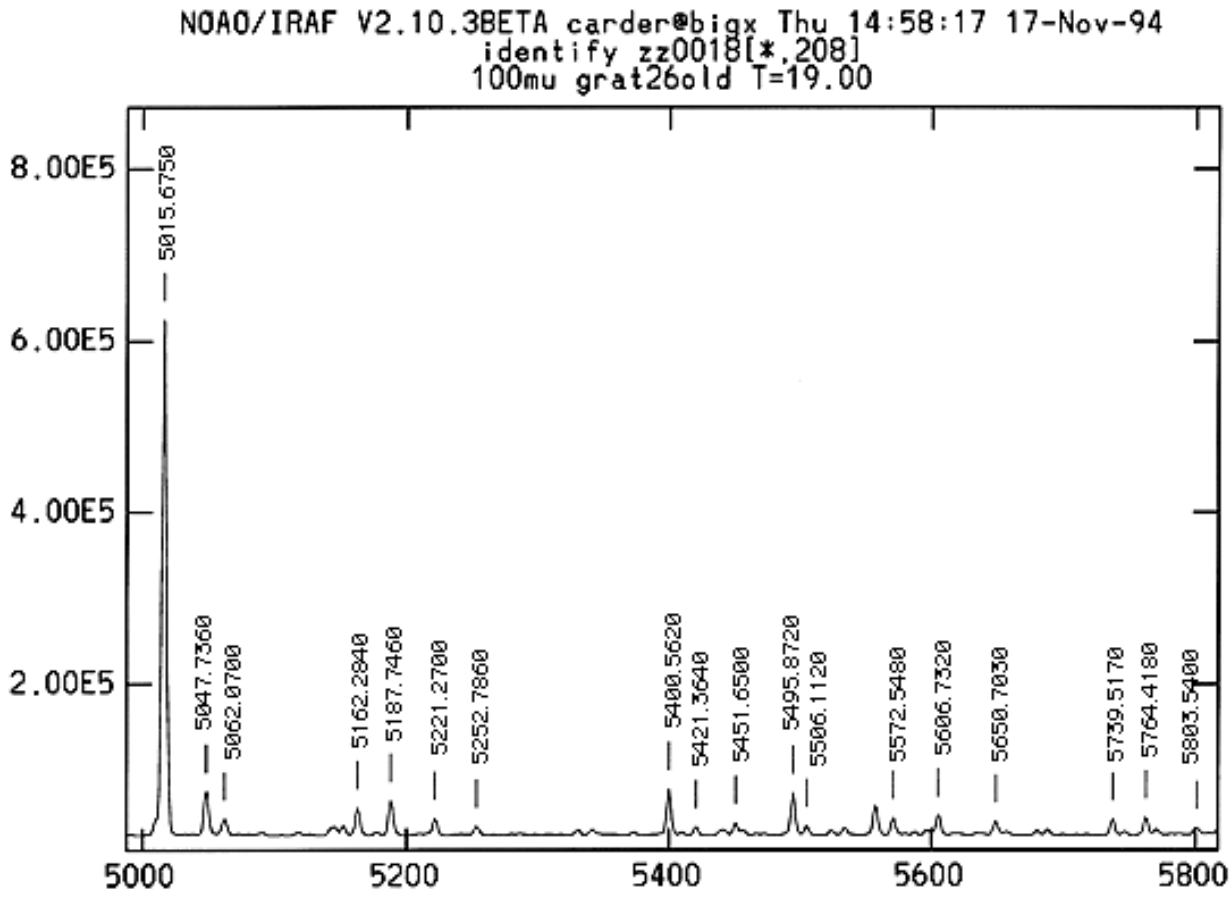
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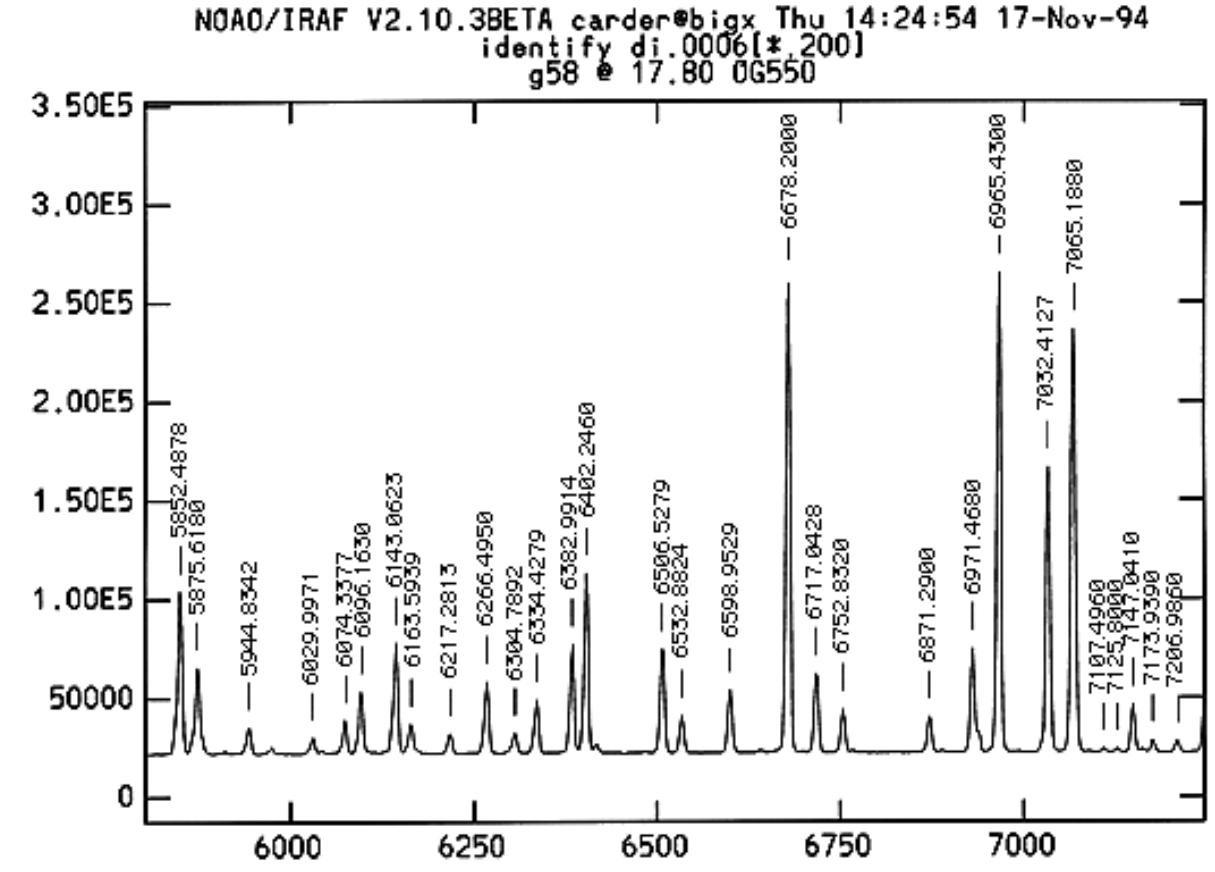
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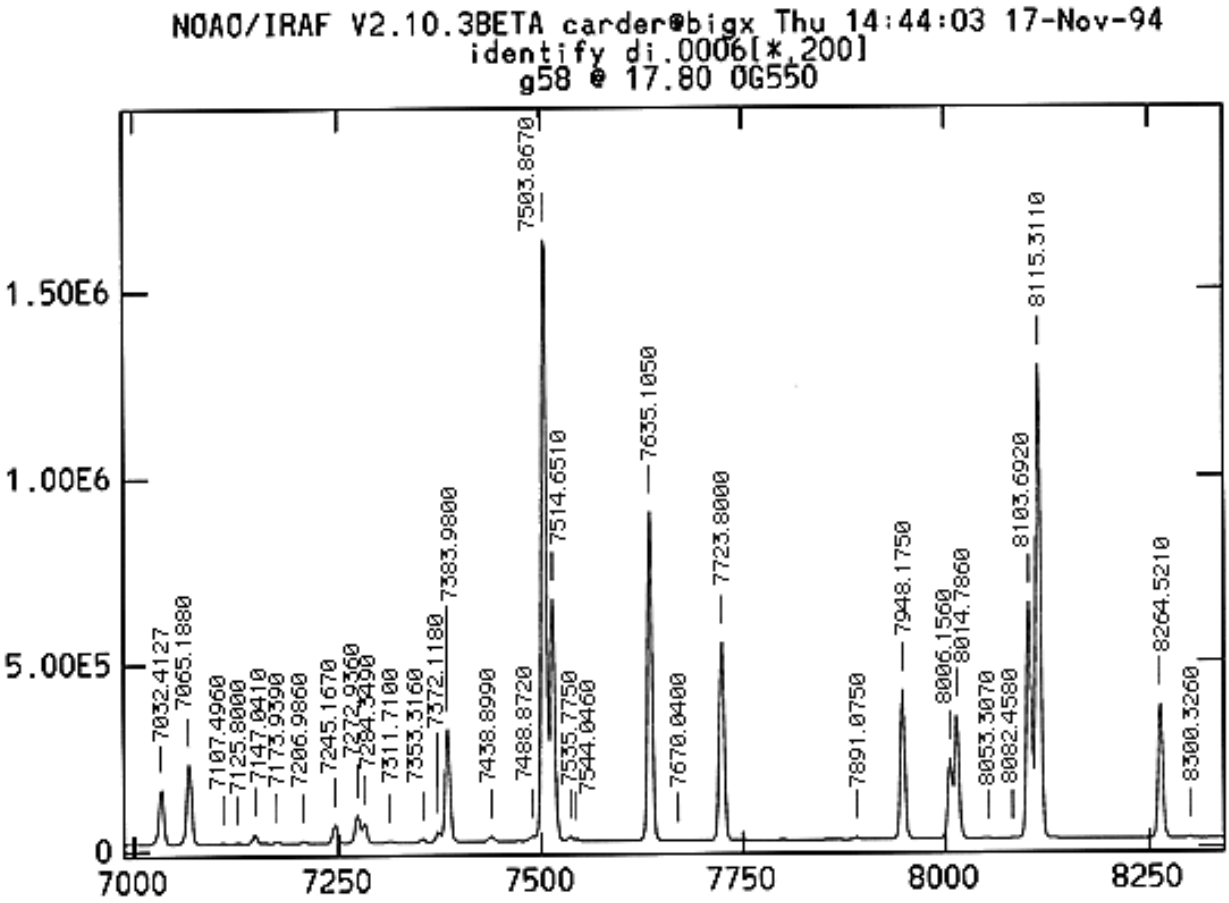
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Appendix C - Astronomical Exercises

1. Error Propagation

Addition/Subtraction: If $f = a \pm b$, then σ_f is given by:

$$\sigma_f^2 = \sigma_a^2 + \sigma_b^2$$

Multiplication/Division: if $f = ab$ or $f = a/b$, then σ_f is given by:

$$\left(\frac{\sigma_f}{f}\right)^2 = \left(\frac{\sigma_a}{a}\right)^2 + \left(\frac{\sigma_b}{b}\right)^2$$

Raising to a power: If $f = a^c$, where c is a constant, then σ_f is:

$$\frac{\sigma_f}{f} = c\left(\frac{\sigma_a}{a}\right)$$

Logarithms: If $f = \log_e(a)$ then σ_f is given by:

$$\frac{\sigma_f}{f} = \frac{1}{a}$$

Remember that: $\log_e(a) = \log_e(10) \times \log_{10}(a)$

2. Calculation of magnitude errors

The definition of magnitude is:

$$m = -2.5 \times log_{10}C \tag{5}$$

where C is the object counts within the aperture. This can be re-written as:

$$m = -2.5/ln(10) \times lnC \tag{6}$$

which can be differentiated to give:

$$dm = -2.5/ln(10) \times (\delta C/C) \tag{7}$$

This is the equation that GAIA uses to calculate the magnitude error (dm) in terms of the error on the object counts within the aperture (δC).

You should remember that the total counts within the aperture (t) are a combination of the counts from the object (C) and the counts from the sky (s):

$$t = C + s \tag{8}$$

where s is the mean sky counts multiplied by the number of pixels within the aperture. Therefore, the final error on the objects counts is the sum in quadrature of the error on the total counts and the error on the sky counts:

$$\delta C = \sqrt{(}\delta t^2 + \delta s^2) \tag{9}$$

Moreover, due to the fact that we are dealing with Poisson/counting errors, we can also assume that:

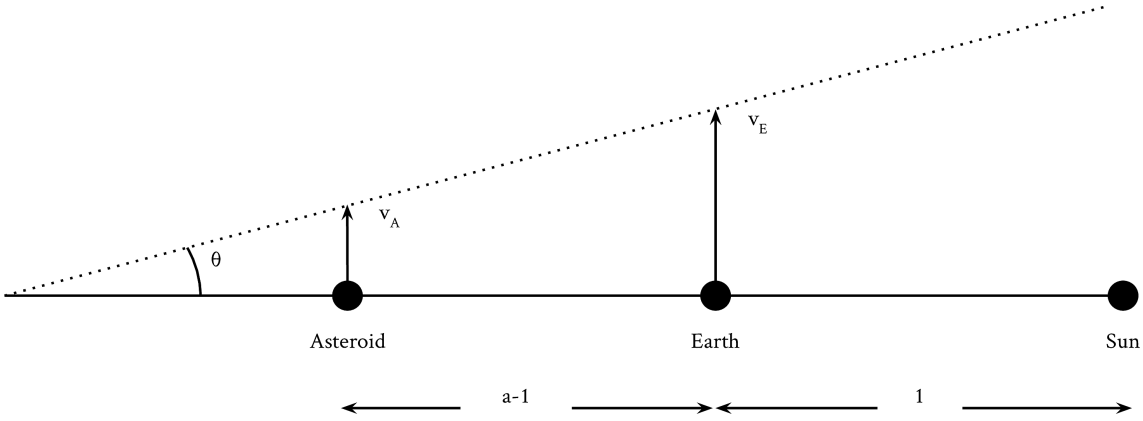
$$\delta t = \sqrt{(}t) \text{ and } \delta s = \sqrt{(}s) \tag{10}$$

which means that the error on the object counts (δC) is simply:

$$\delta C = \sqrt{(}t + s) \tag{11}$$

substituting this back into Eqn 7 then allows you to calculate the magnitude error (dm).

3. Solving Polyxena's motion to give its distance from Sun



Over time t, the asteroid has moved by an angle θ, so

$$D_A = v_A t$$
$$D_E = v_E t.$$

Subtracting one by the other, we have

$$D_E - D_A = (v_E - v_A)t$$

By trigonometry, we also know that

$$\theta = \frac{D_E - D_A}{a-1},$$

hence

$$\theta = \frac{(v_E - v_A)t}{a-1}$$

Assuming the Earth and the asteroid follow two concentric circular orbits, by equating the Gravitational force to the centripetal force for the Earth,

$$\frac{GM_\odot M_E}{r_E^2} = \frac{M_E v_E^2}{r_E};$$

and for the asteroid,

$$\frac{GM_\odot M_A}{r_A^2} = \frac{M_A v_A^2}{r_A}$$

Solving these two relations gives

$$v_A = \frac{v_E}{\sqrt{r_A}} = \frac{v_E}{a}.$$

Hence,

$$\theta = \frac{v_E(1 - \frac{1}{\sqrt{a}})t}{a-1}$$
$$\frac{\theta}{v_E t} = \frac{1}{a-1}\frac{\sqrt{a}-1}{\sqrt{a}}$$
$$= \frac{1}{(\sqrt{a}+1)(\sqrt{a}-1)}\frac{\sqrt{a}-1}{\sqrt{a}}$$
$$\sqrt{a} = \frac{-1 \pm \sqrt{1 + \frac{4v_E t}{\theta}}}{2}.$$