QF4102 Assignment 1

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Q1 (ii)

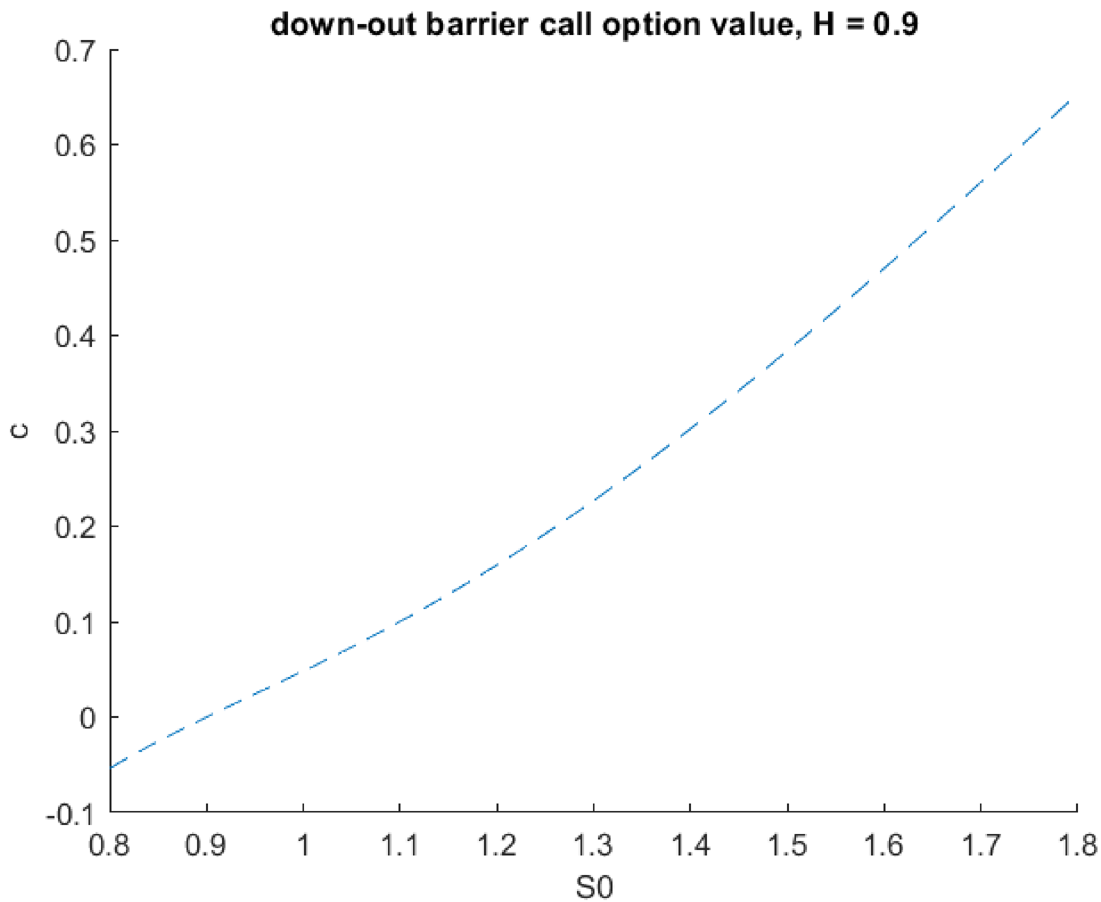

Figure 1: Price of Down-Out Barrier Option

(where $T = 1, X = 1.2, \sigma = 0.3, q = 0.01, r = 0.05$)

From Figure 1, the option value increases exponentially as S0 increases. This is because when S0 is higher, the likelihood of falling below barrier H for some 0 < t < N will be smaller and the payoff (S_T – X)+ will be larger. Another observation is that call option value $c < 0$ for $S0 < 0.9$. This reflects the down-out barrier option being extinguished (or with high likelihood that the barrier is being breached) when $S0$ reaches H.

~~The value < 0 for S0 < 0.9 because the formula, or the option, is only for 0.9 = H <= S0.~~

Q1 (iii)

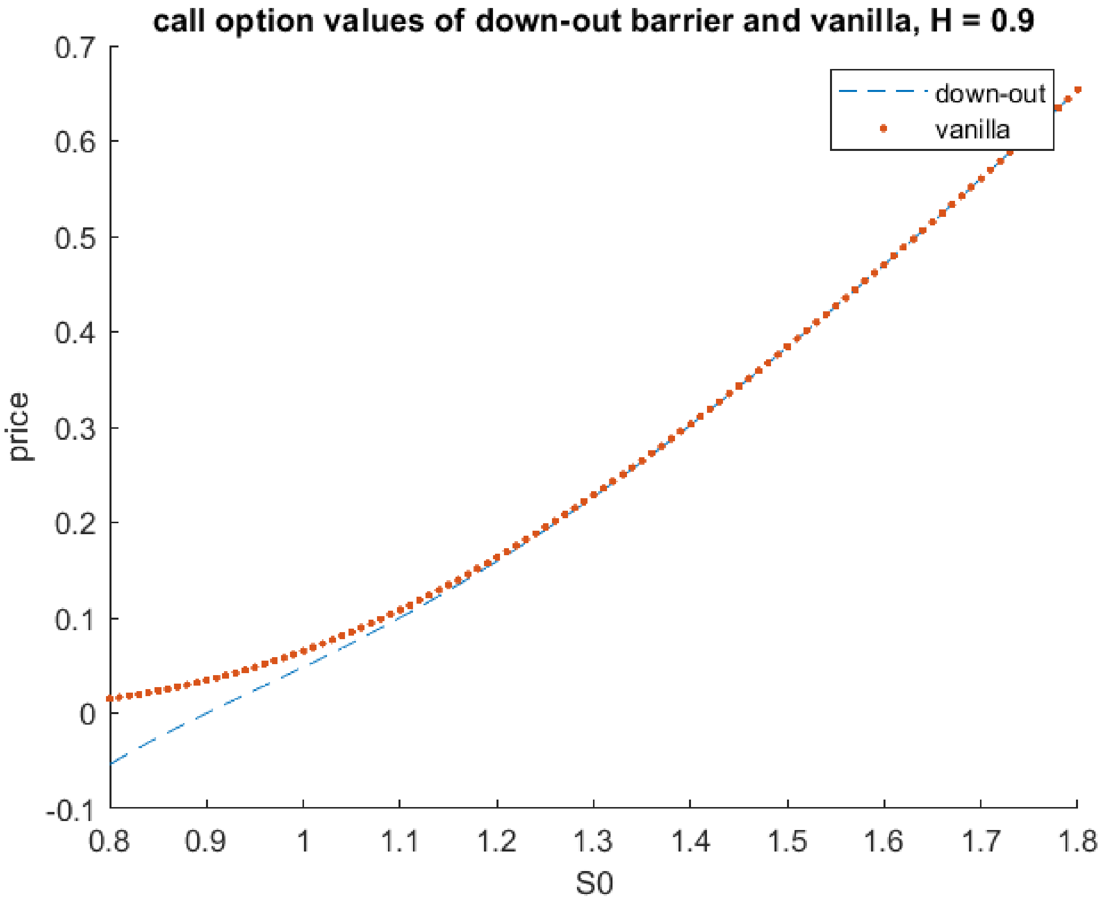

Figure 2: Price of Down-Out Barrier Option and Vanilla European Call

(where $T = 1, X = 1.2, \sigma = 0.3, q = 0.01, r = 0.05$)

The vanilla European call option value is always positive because the payoff is only determined at maturity and discounted after. There is always a non-zero chance for the stock to go above the strike X even when S0 < X (when the option is out-of-the-money). For example, even if the option is deep-out-of-the-money, a short squeeze may occur. Therefore, the difference between the down-out option and the vanilla European call option price is greater for smaller S0, more specifically, as S0 approaches $H = 0.9$ (see Figure 2). As S0 approach H, the down-out option will be more likely to be extinguished (knocked out). However, as S0 increases, the likelihood of the stock to go below barrier $H = 0.9$ will be smaller, so the price of down-out option will converge to the price of vanilla option.

Q1 (iv)

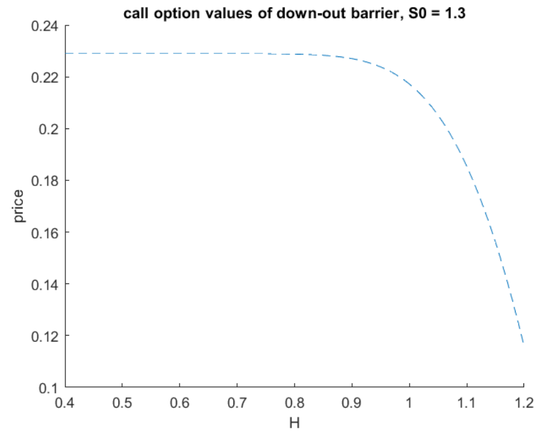

Figure 3: Value of Down-Out Barrier Option against H

As the barrier H increase, the down-out option value decreases. The option value initially decreases slightly for small change in H. However, when H > 0.9, it starts to plunge, the slope is decreasing. This is because as H approaches S0 = 1.3, the chance of St fall below H grow exponentially, reflecting the high likelihood for the down-out barrier option to be extinguished.

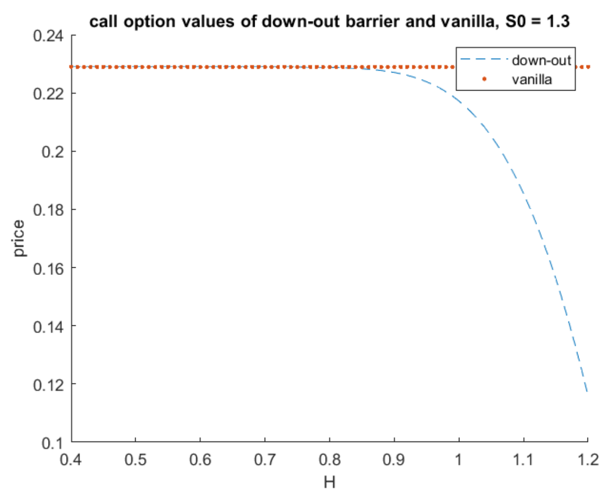

Figure 3: Value of Down-Out Barrier Option against H with Vanilla European Call Price

As H decreases, the price of down-out Barrier call option approaches vanilla European call price because if H is much smaller than S0, the barrier does not matter a lot since the likelihood of the down-out barrier option to be extinguished is very low, hence the price of the down-out Barrier option behaves like vanilla call option.

Q1 (v)

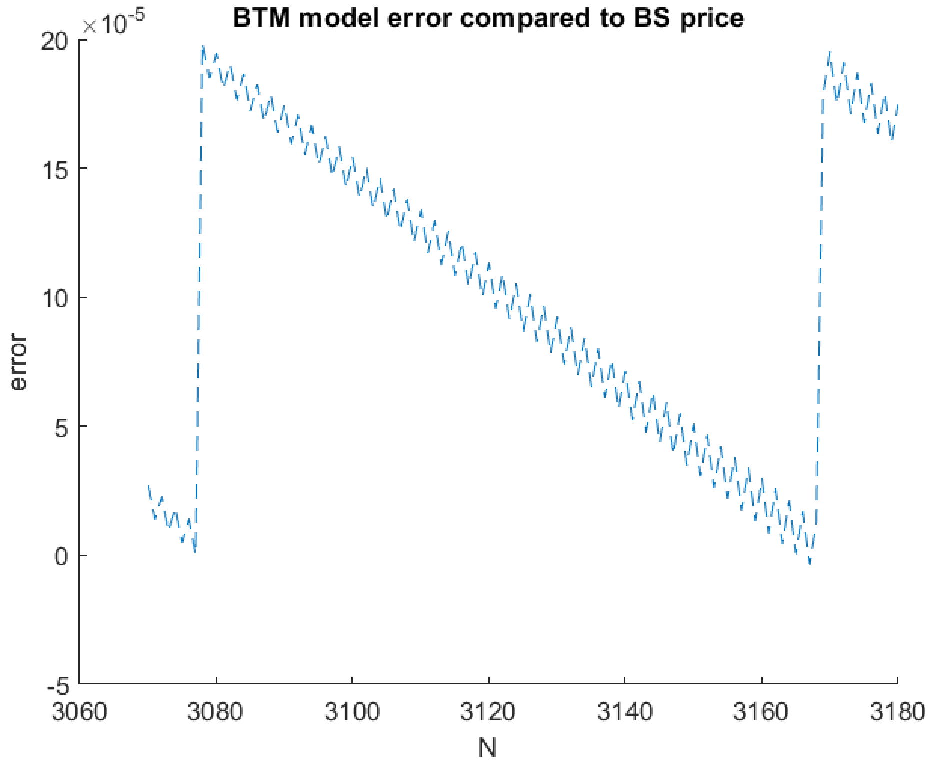

Figure 4: Absolute Errors vs N

The error is like a jump function. There are two jumps, one at around N = 3077, another one at 3167. The jumps are caused by the barrier because the discrete grid points may not always fit into the barrier. When the barrier is right above the grid, we can get the lowest error because the BTM model can capture the points that right below the barrier and make them extinguish. On the other hand, if the barrier is right below the grid point or exactly on the grid point, there will be a greater gap between the actual barrier H and the barrier set by the grid point. The binomial stock values need to be smaller than H by a significant distance for the option to be extinguished.

Q1 (vi)

The first jump is at N = 3077, second jump at N = 3167.

The local minima of the absolute error is at N = 3077 and N = 3166.

The N’s differ by 1 for the second jump which is understandable because the barrier may fit better to adjust better for smaller error.

N < T(i*sigma)^2 / (ln(H/S0))^2

3167 < (0.3i)^2 / 0.135221513

428.2465317 < (0.3i)^2

--> i < -68.98039 or i > 68.98 (rej.)

--> i = -68

3077 < (0.3i)^2 / 0.135221513

--> i < -67.99318557

--> i = -67

Q2(i)

NO. / / Date. / / /

$V = SH(\xi, t) = SW(x, t)$

$V_0 = SW(x_0, 0)$

Interpolation for not newly issued options

$x_0 = \ln\left(\frac{\min}{S}\right)$, min is the running min. S_t.

identify j and $j+1$ st. $j\sigma\sqrt{\Delta t} < x_0 < (j+1)\sigma\sqrt{\Delta t}$

calculate $V_0^j = SW(j\sigma\sqrt{\Delta t}, 0)$

& $V_0^{j+1} = SW(-(j+1)\sigma\sqrt{\Delta t}, 0)$

where $W(j\sigma\sqrt{\Delta t}, 0) = W_0^j$ & $W((j+1)\sigma\sqrt{\Delta t}, 0) = W_0^{j+1}$

can be calculated using backward iteration

for $n = N-1, N-2, \ldots 1, 0$

for $i = 0, 1, \ldots n$

$$W_n^{j,i} = e^{-r\Delta t}\left[p u W_{n+1}^{j,i} + (1-p) d W_{n+1}^{j, \min\{i-1, 0\}}\right]$$

where terminal condition

$$W_N^{j,i} = W(-(N-(j+i)\sigma\sqrt{\Delta t}, T)$$

$$= 1 - e^{-(N-(j+i))\sigma\sqrt{\Delta t}}$$

use the same method to obtain W_0^{j+1}, using interpolation

$$V_0 = S\left[\frac{(j+1)\sigma\sqrt{\Delta t} - x_0}{\sigma\sqrt{\Delta t}} W_0^j + \frac{x_0 - j\sigma\sqrt{\Delta t}}{\sigma\sqrt{\Delta t}} W_0^{j+1}\right] //$$

$W_1^{j,0} = W(-(0+1-j)\sigma\sqrt{\Delta t}, t_{0+1})$

$W_0^{j,i=0} = W(j\sigma\sqrt{\Delta t}, 0)$

$W_1^{j,i-1} = W(\min\{-(0+1-(j-1))\sigma\sqrt{\Delta t}, 0\}, t_{0+1})$

$W_{n+1}^{i} = W(-(n+1-i)\sigma\sqrt{\Delta t}, t_{n+1})$

$W_n^i = W(-(n-i)\sigma\sqrt{\Delta t}, t_n)$

$W_{n+1}^{\min\{i-1,0\}} = W\left(\min\{-(n-(i-1))\sigma\sqrt{\Delta t}, 0\}, t_{n+1}\right)$

$= W\left(\min\{-(n-i+1)\sigma\sqrt{\Delta t}, 0\}, t_{n+1}\right)$

$W_n^{j,i} = W(-(n-(j+i))\sigma\sqrt{\Delta t}, t_n)$

$((n+1)-(j+i-1))\sigma\sqrt{\Delta t} > 0$

$n+1 > (j+1)+i$

QF4102 Assignment 1

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2 i) Floating-strike lookback call

$V(S,m,T) = (S_T - m)^+ = S_T - m$

$m_{n+1} = \min\{S_{n+1}, m_n\}$

$m_{n+1}^{2k+1} = m_n$

$m_{n+1}^{2k} = \min\{S_{n+1}^{i}, m_n\}$

$H_n^i \to H(\xi^i d, t_{n+1}) = H(d^{n-i+1}, t_{n+1})$

$H_n^i \to H(\min\{\xi^i d, 1\}, t_{n+1})$
$= H(\min\{d^{n-i+1}, 1\}, t_{n+1})$
$= H(\min\{d^{n-(i-1)}, 1\}, t_{n+1})$

For floating-strike lookback call

$V(\lambda S, \lambda m, T) = (\lambda S_T - \lambda m)^+$
$= \lambda(S_T - m)$
$= \lambda V(S,m,T)$

$\therefore$ Take $\lambda = 1/S$

$\frac{1}{S} V(S,m,t) = V(1, \frac{m}{S}, t)$
$= H(\xi, t)$, $\xi = m/S \in [0,1]$

$\xi = \frac{m}{S} \nearrow \xi d$, $\searrow \begin{cases} 1, \text{ if } Sd < m \\ \xi u = \frac{m}{Sd}, Sd > m \end{cases}$

$H(\xi; T) = \frac{1}{S} V(S,m,t)$
$= \frac{1}{S}(S_T - m)$
$= 1 - \xi \geq 0$

$H(\xi, t-\Delta t) = \frac{1}{S} V(S, m, t-\Delta t)$
$= \frac{1}{S} e^{-r\Delta t}[pV(Su, m, t) + (1-p) V(Sd, \min\{Sd, m\}, t)]$
$= \frac{1}{S} e^{-r\Delta t}[p\, Su\, V(1, \frac{m}{Su}, t) + (1-p) Sd\, V(1, \min\{1, \frac{m}{Sd}\}, t)]$

$\xi_0 = 1$ $= e^{-r\Delta t}[pu\, H(\xi d, t) + (1-p) d\, H(\min\{\xi u, 1\}, t)]$

at t_n, possible state are $\{d^n, d^{n-1}, \ldots, d, 1\}$ (up / down)

above is for newly issued

$x = \ln \xi = \ln \frac{m}{S}$ $H(\xi, t) = H(e^{\ln \xi}, t) =: W(x,t)$, $\ln d = \ln e^{-\theta\sqrt{\Delta t}} = -\theta\sqrt{\Delta t}$

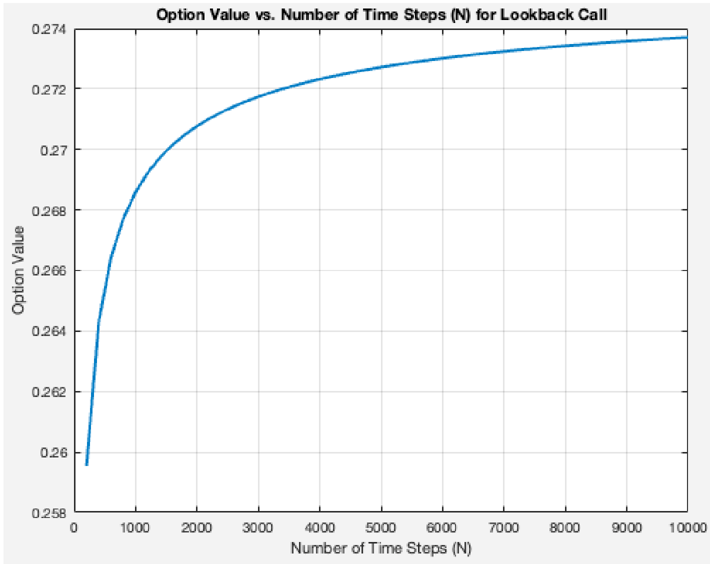
uniform grid : $\{d^n; d^{n-1}, \ldots, d, 1\} \longrightarrow \{n\ln d, (n-1)\ln d, \ldots, \ln d, 0\}$
$\xi^0, \xi^1, \ldots, \xi^{n-1}, \xi^n$ $\quad \| \quad \{-n\theta\sqrt{\Delta t}, -(n-1)\theta\sqrt{\Delta t}, \ldots, -\theta\sqrt{\Delta t}, 0\}$

$W_n^i := W(x_n^i = -(n-i)\theta\sqrt{\Delta t}, t_n)$

for $i = 0, 1, \ldots n$,
$n = 0, 1, \ldots N-1$ $W_n^i = e^{-r\Delta t}[pu\, W_{n+1}^i + (1-p) d\, W_{n+1}^{\min\{i-1, 0\}}]$

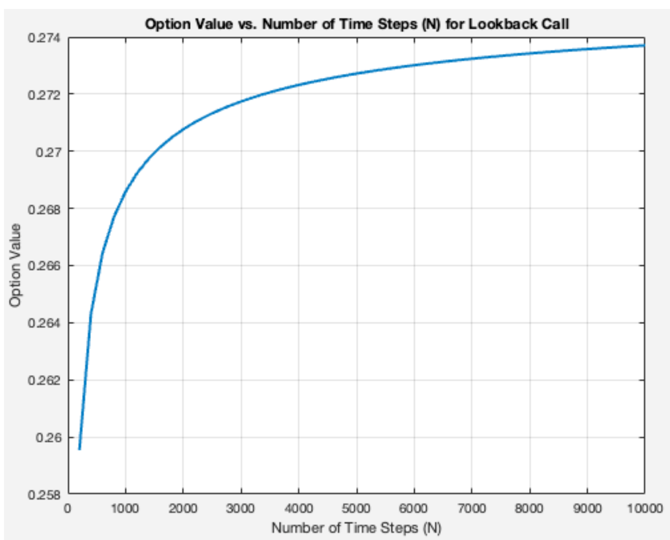
$W_N^i = W(x_N^i, T) = W(-(N-i)\theta\sqrt{\Delta t}, T) = H(e^{-(N-i)\theta\sqrt{\Delta t}}, T) = 1 - e^{-(N-i)\theta\sqrt{\Delta t}} \geq 0$

Q2(iii)



The option value increases (concave downward) and converges to a stable value of 0.273705 as N increases. This convergence is an accurate estimation because it is shown from the lectures that the BTM converges to the BSM as N increases.

Q2(iv)



The plot follows a similar trend with 2iii, where the option value increases concave downward and converges to a stable value of 0.273705 as N increases.

Comparison: The not newly issued convergence value is slightly higher than that of the options that are newly issued. This is because the current running minimum is lower than the underlier price, which reflects a higher likelihood of obtaining a higher payoff function as given by (S-m)^+.

Q3

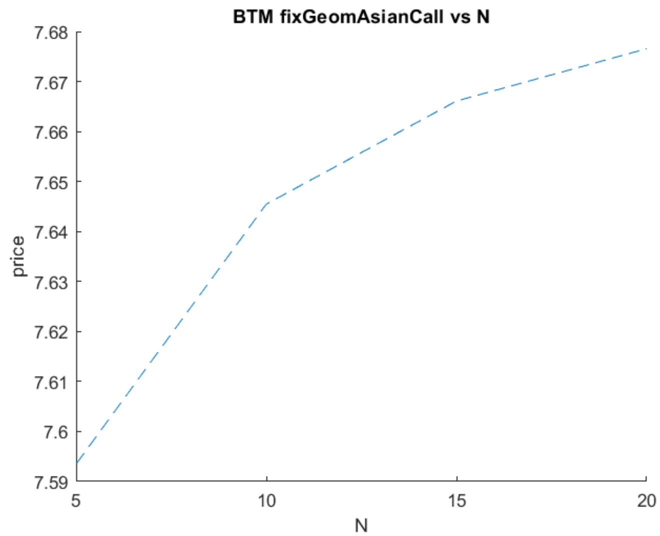


Figure 5: Price of Geometric Average Asian Call Option vs N

In general, the BTM price of the geometric average Asian call increases as N increases. The this occurs because averaging more stock prices reduces the influence of each individual stock price movement. Hence, the upward movement in option price becomes less sensitive to extreme stock price changes. As N tends to a large value, the option price will increase more gradually, forming a concave down curve (rate of increase slows and approaches a steady value), which is reflected in Figure 5.

~~This is because as N increases, the stock price can goes up in the exponential manner, while it will goes down relatively more slowly if we consider the absolute value instead of percentage. Thus, the upward price will push up the geometric average and hence the option price. The option price increases slowly, i.e. concave down, as it would approach a certain value for large N. (do u k how to explain on it)~~

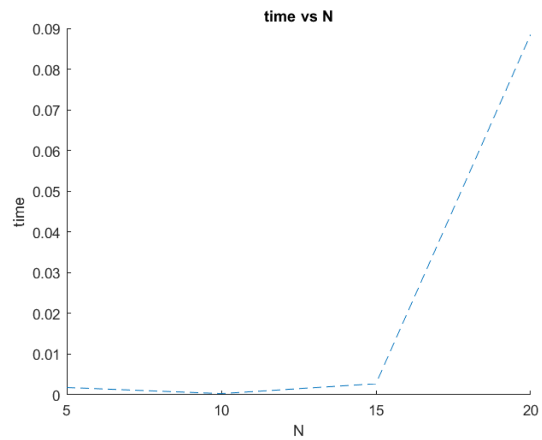


Figure 6: Time Complexity Analysis

Compute Time for N = 5, 10, 15, 20 is 0.0018, 0.0003, 0.0027, 0.0885 respectively.

The compute time is the shortest for N = 10 instead of N = 5 which is a surprising result. This may be due to some computational problem on MATLAB handling small values of N, instead of overall time complexity of algorithm. Such a difference between N = 5 and N = 10 is small, hence it does not matter a lot. However, differences in compute time between N = 20 being 32.8 times slower than the compute time when N = 15 is explained by the overall time complexity of the algorithm implemented in 3(i), which is $O(2^N)$. More explicitly, from N = 15 to N = 20, the number of nodes grew exponentially from 33,000 nodes to 1,050,000 nodes.

~~Also, the time is both small, so it does not matter so much. But for N = 20, the time grow exponentially. It is 32.8 times of the time required for N = 15. This is because the number of nodes grows exponentially, from 33,000 to 1,050,000. O(2^N) time complexity is very bad.~~