QF4102 Financial Modelling and Computation

Assignment 3

AY24/25 Semester 1

1. A European option on two underlying assets S_1 and S_2 has payoff given by

$$\varphi(S_{1T}, S_{2T}, X, H) = \begin{cases} (X - S_{1T})^+, & \text{if } (S_{2t}) < H \text{ for all } t < T, \\ (S_{2T} - S_{1T})^+, & \text{if } (S_{2t}) \geq H \text{ for some } t < T, \end{cases}$$

that is, this option grants the holder the right to sell a unit of the first underlying asset at a price that depends on the price path of $(S_{2t}), 0 \leq t < T$.

The current prices of the two underlying assets, and their parameters are given by

i	$S_{i,0}$	volatility	dividend yield
1	12	0.25	0.02
2	10	0.35	0.03

where the two assets are negatively correlated, with $\rho = -0.12$.

(i) Implement the Monte-Carlo method for estimating the two-asset option as a Matlab function

```
function v = MC_assm3(S0, X, r, T, sigma, rho, q, N, H, P)
```

where `S0`, `sigma` and `q` are vectors. This function would perform a single simulation run based on P number of samples.

Save this function as its own file.

(ii) With $X = 12$, $H = 14$, a 2 year time to maturity, and with the risk free rate at 4%, estimate the value of the option with 500 time periods, 30 simulation runs, each with 300 price-path bundles. Also compute the standard error of your estimate.

Repeat the above with $P = 3000$ and 30 000 price-path bundles, and compute their estimates and standard errors for each P.

Tabulate your results, and comment.

(iii) Suggest a suitable control variate which can be used to reduce the variance of the estimates. Write down the payoff function of this control variate, and explain the rationale of choosing it in your report.

(iv) Implement the Monte-Carlo approach with the control variate as a Matlab function. Use the header

```
function v = MC_assm3_CV(S0, X, r, T, sigma, rho, q, N, H, P)
```

and save this function as its own file.

Run this function, using the same grid of values in (ii). Tabulate your results, and comment.

2. Consider the pricing of an American vanilla call option $v(S,t)$.

   (i) Using the transformations $x = \ln(S/X)$, $\tau = T - t$, $v(S,t) = Xu(x,\tau)$, obtain a corresponding PDE model for the American option. State the solution domain and the initial condition of the model. *Hint: you may first assume that the option is European styled. Then, modify the transformed PDE model to incorporate the early-exercise feature.*

   (ii) Derive an $O(\Delta\tau+\Delta x)$ fully implicit scheme for the transformed PDE model. State the discretized grid, the FDE, the discretized initial and boundary conditions.

   (iii) Implement the fully implicit scheme in a Matlab function, using the projected SOR method for the forward time iterations. Use the header

   ```
   function v = FD_ids_amCall(S0,X,r,T,sigma,q,I,N,xmin,xmax,omega,eps)
   ```

   and use linear interpolation where necessary. Save the function as its own file.

   (iv) Use your function to estimate the value of an American vanilla option, whose underlying asset has a current price of \$1.231, a volatility of 0.2, and a continuous dividend yield of 4%. The risk-free interest rate is 3%, and the option has a strike price of \$1, with a 3 month time to expiry.

   Use a truncated grid of $x \in [-4, 4]$, with 1000 time periods, $I = 160$, a relaxation parameter of 1.3, and an error tolerance of 10^{-6}.

   (v) Repeat part (iv) with $I = 80, 160, \ldots, 800$. Plot the estimated values versus I, and comment on the plot obtained.

Submission

Your submission must be a .zip folder, containing the following:

- A pdf document consisting of all written responses, results, figures, and comments.
- Two Matlab scripts — one for each question — containing all the code that reproduces your results and figures.
  - Name these files `assm3_q1.m`, and `assm3_q2.m`.

  These scripts will be run during the grading process.
- Three Matlab function files, as specified in questions 1(i), 1(iv), and 2(iii).
- All supporting Matlab functions which are required for the execution of the above mentioned Matlab scripts and functions, such as the projected SOR method, or in the computations with a control variate.

Submit your .zip folder to Canvas by the due date.

Note: Plagiarism will not be tolerated. In the event of a violation of the academic integrity policy, all parties involved will be penalized severely, and referred for further action.