Test Part 2

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1 Problem 1

Claim: $\{\, n : 28 \mid n\} \subseteq \{\, n : 4 \mid n\} \cap \{\, n : 14 \mid n\}$.

a) In order to prove $\{\, n : 28 \mid n\} \subseteq \{\, n : 4 \mid n\} \cap \{\, n : 14 \mid n\}$ we will show that *if* $28 \mid n$ *then* $4 \mid n$ and $14 \mid n$.

b) To prove *if* $28 \mid n$ *then* $4 \mid n$ and $14 \mid n$, we will assume $28 \mid n$. We want to show $4 \mid n$ and $14 \mid n$.

c) **Proof:** In order to show $\{n : 28 \mid n\} \subseteq \{n : 4 \mid n\} \cap \{n : 14 \mid n\}$, we will show the equivalent statement: *if* $28 \mid n$ *then* $4 \mid n$ and $14 \mid n$. To do this, we will assume $28 \mid n$; we want to show that $4 \mid n$ and $14 \mid n$.

	Mathematical Reasoning	**Reason this Statement is True (From the Approved List)**
	$28 \mid n$	By assumption
$\Rightarrow$	$n = 28c, c \in \mathbb{Z}$	By def of divisible by 28
$\Rightarrow$	$n = 28c$ and $n = 28c$, where $c \in \mathbb{Z}$	By def of n and by def of and
$\Rightarrow$	$n = 4 \cdot 7 \cdot c$ and $n = 14 \cdot 2 \cdot c$, where $c \in \mathbb{Z}$	By refactoring
$\Rightarrow$	$n = 4k$ and $n = 14l$, where $k, l \in \mathbb{Z}$	Since product of ints is int
$\Rightarrow$	n is divisible by 4 and n is divisible by 14	By def of divisible by 4 and 14
$\Rightarrow$	$4 \mid n$ and $14 \mid n$	By def of divisible by 4 and 14

2 Problem 2

Claim: Let $f, n, m \in \mathbb{Z}$. If $f \mid n$ and $f \nmid m$ then $f \nmid (n+m)$.

	Mathematical Reasoning	**Reason this Statement is True (From the Approved List)**
BWOC, suppose	There exists $f, n, m \in \mathbb{Z}$. such that $f \mid n$,$f \nmid m$ and $f \mid (n+m)$	Since this is the negation of the claim
$\Rightarrow$	$n = fc,\ (n+m) = fd, c, d \in \mathbb{Z}$	By def of divisible by f
$\Rightarrow$	$(n+m) - n = fd - fc, c, d \in \mathbb{Z}$	By substitution
$\Rightarrow$	$m = fd - fc, d, c \in \mathbb{Z}$	By algebra
$\Rightarrow$	$m = f(d-c), c, d \in \mathbb{Z}$	By factoring
$\Rightarrow$	$m = fv, v \in \mathbb{Z}$	Since difference of ints is int
$\Rightarrow$	m is divisible by f	By def of divisible by f
$\Rightarrow$	$f \mid m$	By def of divisible by f
$\Rightarrow$	Contradiction	Because we stated $f \nmid m$ and we showed that $f \mid n$, thus contradiction

Explanation: Since our negation of the claim was that there exists $f, n, m \in \mathbb{Z}$. such that $f \mid n$,$f \nmid m$ and $f \mid (n+m)$. We assumed that $f \nmid m$, this means that f does not divide m. However when trying to prove by contradiction, we showed that $f \mid m$ from our assumption. Thus we have proved our claim.

3 Problem 3

Claim: Let p, q be prime numbers with $p \neq q$ (i.e they are distinct). Then $\sqrt{p \cdot q}$ is irrational.

	Mathematical Reasoning	**Reason this Statement is True (From the Approved List)**
BWOC, suppose	$\sqrt{p \cdot q}$ is rational	Since this is the negation of our claim
$\Rightarrow$	$\sqrt{p \cdot q} = \frac{n}{d}$ where $n, d \in \mathbb{Z}, d \neq 0$, and n, d are in their lowest forms	By def of rational
$\Rightarrow$	$p \cdot q = \frac{n^2}{d^2}$	By squaring both sides
$\Rightarrow$	$d^2 p \cdot q = n^2$	By algebra
$\Rightarrow$	$p \cdot q \mid n^2$	By def of divisible by $p \cdot q$
$\Rightarrow$	$p \cdot q \mid n$	For $n \in \mathbb{Z}$, if n^2 is even then n is even, and if n^2 is odd then n is odd. If $p \cdot q \mid n^2$ then $p \cdot q \mid n$
$\Rightarrow$	$n = p \cdot q \cdot c$, where $c \in \mathbb{Z}$	By def of divisible by $p \cdot q$
$\Rightarrow$	$d^2 p \cdot q = c^2 \cdot q^2 \cdot p^2, c \in \mathbb{Z}$	By substitution
$\Rightarrow$	$p \cdot q \cdot c^2 = d^2, c \in \mathbb{Z}$	By algebra
$\Rightarrow$	$p \cdot q \mid d^2$	By def of divisible by $p \cdot q$
$\Rightarrow$	$p \cdot q \mid d$	For $n \in \mathbb{Z}$, if n^2 is even then n is even, and if n^2 is odd then n is odd. If $p \cdot q \mid n^2$ then $p \cdot q \mid n$
$\Rightarrow$	Contradiction	Because $p \cdot q$ cannot divide both n and d

Explanation: Since we assumed that that n, d are in their lowest forms, both n and d cannot be divided by $p \cdot q$. We know that p and q are distinct primes such that $(n, d) = 1$. In the proof we showed that $p \cdot q$ divides n and d, which is not true. Thus there is a contradiction.