Econometric Methods Homework 3

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Environmental Variables Setup

```
Data <- read.csv("Equity_Premium.csv")
Matrix <- as.matrix(Data)
TimeMatrix <- Matrix[, 1, drop = FALSE]
Y <- Matrix[, 2, drop = FALSE]
X1 <- matrix(1, nrow = nrow(Y), ncol = ncol(1))
X2 <- Matrix[, 3, drop = FALSE]
X3 <- Matrix[, 4, drop = FALSE]
X4 <- Matrix[, 5, drop = FALSE]
X5 <- Matrix[, 6, drop = FALSE]
X6 <- Matrix[, 7, drop = FALSE]
X7 <- Matrix[, 8, drop = FALSE]
X <- cbind(X1, X2, X3, X4, X5, X6, X7)
```

Problem 1

a.

trace(P)

$= \text{trace}(X(X'X)^{-1}X')$
$= \text{trace}(X'X(X'X)^{-1})$
$= \text{trace}(I_k)$
$= \text{k}$
$\implies trace(P) = k$

trace(M)

$= \text{trace}(I_n - P)$
$= \text{trace}(I_n)$ - $\text{trace}(P)$
$= \text{n - k}$
$\implies trace(M) = n - k$

b.

P is positive semi-positive

Given a vector $c \neq 0$, $c'Pc = c'PPc = c'P'Pc = (cP)'cP = |cP|^2 \geq 0$. $\implies Q.E.D.$

M is positive semi-positive

Given a vector $c \neq 0$, $c'Mc = c'MMc = c'M'Mc = (cM)'cM = |cM|^2 \geq 0$. $\implies$ $Q.E.D.$

Problem 2

$\hat{\sigma}_Y^2$
$= \frac{1}{n}\Sigma_{i=1}^n (Y_i - \bar{Y})^2$
$= \frac{1}{n}\Sigma_{i=1}^n [(Y_i - \mu_Y) - (\bar{Y} - \mu_Y)]^2$
$= \frac{1}{n}[\Sigma_{i=1}^n (Y_i - \mu_Y)^2 - 2\Sigma_{i=1}^n (Y_i - \mu_Y)(\bar{Y} - \mu_Y) + \Sigma_{i=1}^n (\bar{Y} - \mu_Y)^2]$

$\therefore E[\hat{\sigma}_y^2]$
$= E[\frac{1}{n}[\Sigma_{i=1}^n (Y_i - \mu_Y)^2] - \frac{2}{n}E[Y_i - \mu_Y]E[\bar{Y} - \mu_Y] + E[\frac{1}{n}[\Sigma_{i=1}^n (\bar{Y} - \mu_Y)^2]]$
$= \sigma_Y^2 + \frac{1}{n}E[-2(\bar{Y} - \mu_Y)\Sigma_{i=1}^n (Y_i - \mu_Y) + \Sigma_{i=1}^n (\bar{Y} - \mu_Y)^2]$
$= \sigma_Y^2 + -E[(\bar{Y} - \mu_Y)^2]$
$= \sigma_Y^2 + -\frac{\sigma_Y^2}{n}$
$\implies$ Bias $= -\frac{\sigma_Y^2}{n}$

Problem 3

a.

```
Beta_X <- function(X){
  return(solve(t(X) %*% X) %*% (t(X) %*% Y))
}
print(Beta_X(X))
```

```
> print(Beta_X(X))
                y
        0.01079947
x_dfy   0.99791023
x_infl -0.55424292
x_svar -0.44950368
x_tms   0.23354924
x_tbl   0.14438939
x_dfr   0.06759062
```

b.

Because $k = 7$, select $k_1 = 3$ and $k_2 = 4$.

```
M2 <- rbind(t(cbind(X1, X2, X3)) %*% Y, t(cbind(X4, X5, X6, X7)) %*% Y)
M11 <- t(X_1) %*% X_1
M12 <- t(X_1) %*% X_2
M21 <- t(X_2) %*% X_1
M22 <- t(X_2) %*% X_2
M1 <- solve(cbind(rbind(M11, M21), rbind(M12, M22)))
print(M1 %*% M2)
```

```
> print(Beta)
                y
        0.01079947
x_dfy   0.99791023
x_infl -0.55424292
x_svar -0.44950368
x_tms   0.23354924
x_tbl   0.14438939
x_dfr   0.06759062
```

Problem 4

Centered $R^2 = \frac{RSS}{TSS}$

```
TSS <- 0
for(i in 1:504){
  TSS <- TSS + (Y[i] - mean(Y))^2
}
print(TSS)

Y_hat <- function(j, X){
  return(t(X[j]) %*% Beta[j])
}

X1i <- matrix(1, nrow = nrow(Y), ncol = 1)
X2i <- cbind(X1i, Matrix[, 3, drop = FALSE])
X3i <- cbind(X2i, Matrix[, 4, drop = FALSE])
X4i <- cbind(X3i, Matrix[, 5, drop = FALSE])
X5i <- cbind(X4i, Matrix[, 6, drop = FALSE])
X6i <- cbind(X5i, Matrix[, 7, drop = FALSE])
X7i <- cbind(X6i, Matrix[, 8, drop = FALSE])
XList <- list(X1i, X2i, X3i, X4i, X5i, X6i, X7i)

for(j in 1:7){
  RSS <- 0
  Y_hat <- XList[[j]] %*% Beta_X(XList[[j]])
  for(i in 1:504){
    RSS <- RSS + ((Y_hat[i]) - mean(Y))^2
  }
  print(paste("R-squared for X(i, ", j, ") = ", RSS/TSS, sep = ""))
}
```

```
[1] "R-squared for X(i, 1) = 4.72076745527166e-31"
[1] "R-squared for X(i, 2) = 0.0153708759317626"
[1] "R-squared for X(i, 3) = 0.01545068570891"
[1] "R-squared for X(i, 4) = 0.0215234499003107"
[1] "R-squared for X(i, 5) = 0.0225079938472702"
[1] "R-squared for X(i, 6) = 0.0300815401378616"
[1] "R-squared for X(i, 7) = 0.0305901113333621"
```

GitHub Link

EconometricMethods-homework3-b10901069