Econometric Methods Homework 8

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1 Problem 1

1. $E[C_p] = E[\hat{e}'\hat{e} + 2ks^2]$

$= E[\hat{e}'\hat{e} + 2k \cdot \frac{1}{n-k} \hat{e}'\hat{e}]$

$= E[\frac{n+k}{n-k} \hat{e}'\hat{e}]$

$\hat{e}'\hat{e} = Y'(I_n - P)Y$

$= (X\beta + e)' M (X\beta + e)$

$= e'Me \quad (\because MX = 0)$

$\because e|X \sim N(0, \sigma^2 I_n) \therefore Me \sim N(0, \sigma^2 M)$

$\Rightarrow E[\hat{e}'\hat{e}] = E[e'Me] \quad (\because MM = M \ \& \ M' = M)$

$= E[\|Me\|^2]$

$= \sigma^2 \mathrm{trace}(M)$

$= \sigma^2 (n-k)$

$E[C_p] = E[\frac{n+k}{n-k} \hat{e}'\hat{e}]$

$= \frac{n+k}{n-k} E[\hat{e}'\hat{e}]$

$= (n+k)\sigma^2$

$R := E[\|\hat{m} - m\|^2]$

$= E[e'Pe]$

$= \sigma^2 k$

$\therefore E[C_p] = R + n\sigma^2$ ※

2 Problem 2

2. $E[C_{1,p}] = E[\hat{e}_1'\hat{e}_1 + 2k_1 s^2]$

$= E[\hat{e}_1'\hat{e}_1 + 2k_1 \cdot \frac{1}{n-k}\hat{e}'\hat{e}]$

$= E[\hat{e}_1'\hat{e}_1] + \frac{2k_1}{n-k}E[\hat{e}'\hat{e}]$

$\hat{e}_1'\hat{e}_1 = (M_1Y)'(M_1Y)$, $M_1Y = M_1(X_1\beta_1 + X_2\beta_2 + e) = M_1X_2\beta_2 + M_1e$

$= (M_1X_2\beta_2 + M_1e)'(M_1X_2\beta_2 + M_1e)$

$= (\beta_2'X_2'M_1' + e'M_1')(M_1X_2\beta_2 + M_1e)$ $(\because M' = M \ \& \ MM = M)$

$= \beta_2'X_2'M_1X_2\beta_2 + e'M_1X_2\beta_2 + \beta_2'X_2'M_1e + e'M_1M_1e$

$\therefore E[\hat{e}_1'\hat{e}_1] = E[E[\hat{e}_1'\hat{e}_1|X]]$ $(\because E[e|X] = 0)$

$= E[\beta_2'X_2'M_1X_2\beta_2] + \underbrace{E[\|M_1e\|^2]}_{=\sigma^2 trace(M_1) = \sigma^2(n-k_1)}$ $e \sim (0, \sigma^2 I_n)$ $\therefore$ $M_1e \sim (0, \sigma^2 M_1)$

$= E[\beta_2'X_2'M_1X_2\beta_2] + \sigma^2(n-k_1)$

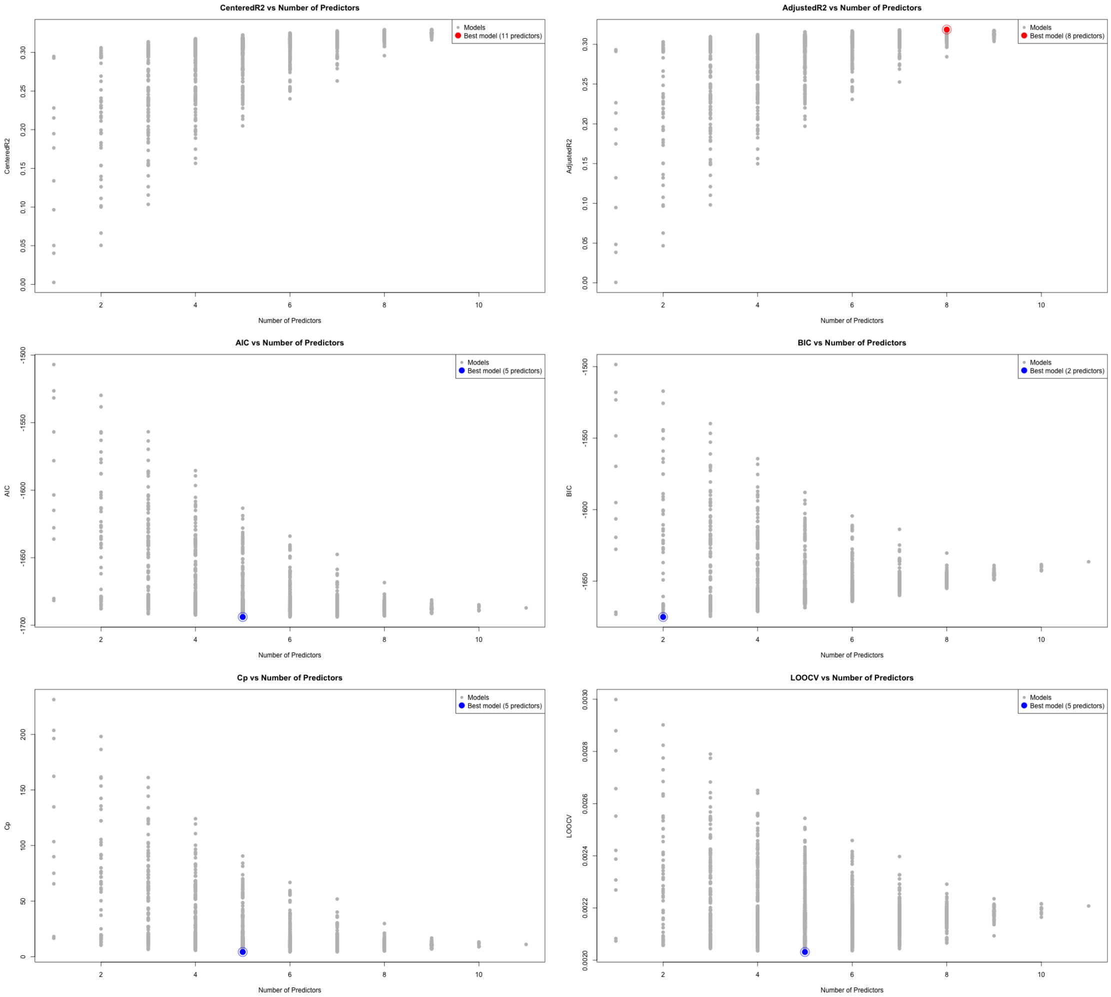
$\Rightarrow E[C_{1,p}] = E[\hat{e}_1'\hat{e}_1] + \frac{2k_1}{n-k}E[\hat{e}'\hat{e}]$ by #1 $E[\hat{e}'\hat{e}] = \sigma^2(n-k)$

$= E[\beta_2'X_2'M_1X_2\beta_2] + \sigma^2(n-k_1) + \frac{2k_1}{n-k}\cdot\sigma^2(n-k)$

$= E[\beta_2'X_2'M_1X_2\beta_2] + \sigma^2(n+k_1)$

$= R + n\sigma^2$ #

3 Problem 3

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Best model for CenteredR2 :
Number of predictors: 11
Predictors: ones, x_dfy, x_infl, x_svar, x_tms, x_tbl, x_dfy2, x_infl2, x_svar2, x_tms2, x_tbl2

Best model for AdjustedR2 :
Number of predictors: 8
Predictors: ones, x_dfy, x_svar, x_tms, x_infl2, x_svar2, x_tms2, x_tbl2

Best model for AIC :
Number of predictors: 5
Predictors: x_dfy, x_tms, x_tbl, x_infl2, x_tms2

Best model for BIC :
Number of predictors: 2
Predictors: x_dfy, x_dfy2

Best model for Cp :
Number of predictors: 5
Predictors: x_dfy, x_tms, x_tbl, x_infl2, x_tms2

Best model for LOOCV :
Number of predictors: 5
Predictors: x_dfy, x_tms, x_tbl, x_infl2, x_tms2
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GitHub Link

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