Econometric Methods Homework 7

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1 Problem 1

1.1 a

Lagrangian function : $\mathcal{L}(b,\lambda) = \frac{1}{2}SSE(b) + \lambda'(Rb-\theta_0)$

$$\begin{cases} \nabla_b \mathcal{L}(b,\lambda) = \frac{1}{2}\nabla_b SSE(b) + R'\lambda = -X'Y + X'Xb + R'\lambda \\ \nabla_\lambda \mathcal{L}(b,\lambda) = Rb - \theta_0 \end{cases}$$

$$\therefore \begin{cases} \nabla_b \mathcal{L}(\hat{\beta}_{cls}, \hat{\lambda}_{cls}) = -X'Y + X'X\hat{\beta}_{cls} + R'\hat{\lambda}_{cls} = 0 \\ \nabla_\lambda \mathcal{L}(\hat{\beta}_{cls}, \hat{\lambda}_{cls}) = R\hat{\beta}_{cls} - \theta_0 = 0 \end{cases}$$

$\Rightarrow R\hat{\beta}_{cls} = R\hat{\beta} - R(X'X)^{-1}R'\hat{\lambda}_{cls} = \theta_0$. $\hat{\lambda}_{cls} = [R(X'X)^{-1}R']^{-1}(R\hat{\beta}-\theta_0)$

$\hat{\beta}_{cls} = \hat{\beta} - (X'X)^{-1}R'\hat{\lambda}_{cls}$

$= \hat{\beta} - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\hat{\beta}-\theta_0)$ *

1.2 b

asymptotic distribution of $n^{1/2}(\hat{\beta}-\hat{\beta}_{cls}) = \sqrt{n}(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\hat{\beta}-\theta_0)$

for $\hat{\beta}$ under H_0, $\sqrt{n}(\hat{\beta}-\beta_0) \xrightarrow{d} N(0, V_\beta) = N(0, \sigma^2(X'X)^{-1})$

$\therefore \sqrt{n}(\hat{\beta}-\hat{\beta}_{cls}) \xrightarrow{d} N(0, \sigma^2(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R(X'X)^{-1}$ *

1.3 c

$\sqrt{n}(\hat{\beta}-\hat{\beta}_{cls}) \xrightarrow{d} N(0, \sigma^2(X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}R(X'X)^{-1})$ 可得 $\hat{V}_\beta$

$\dim(\theta) = q$

Wald statistic, for $\sqrt{n}(\theta-\theta_0) \xrightarrow{d} N(0, V_\theta)$ (under H_0), $W := n(\hat{\theta}-\theta_0)'\hat{V}_\theta^{-1}(\hat{\theta}-\theta_0)$

$\Rightarrow \hat{V}_\theta^{-1/2}\sqrt{n}(\hat{\theta}-\theta_0) \xrightarrow{d} N(0, I_q)$, $\hat{V}_\theta = R\hat{V}_\beta R' = \sigma^2[R(X'X)^{-1}R']$

$\therefore W = n(\hat{\theta}-\theta_0)[\sigma^2 R(X'X)^{-1}R']^{-1}(\hat{\theta}-\theta_0) = n(R\hat{\beta}-\theta_0)[\sigma^2 R(X'X)^{-1}R']^{-1}(R\hat{\beta}-\theta_0) \sim \chi^2(q)$ *

1.4 d

$$H_1: \overset{\hat{\theta}}{\underline{R\beta}} = \theta_0 + n^{-1/2} h, \quad h > 0 \qquad \text{for } \sqrt{n}(\theta - \theta_0) \xrightarrow{d} N(0, V_\theta)$$

$$W = n(\hat{\theta} - \theta_0)[\sigma^2 R(X'X)^{-1}R']^{-1}(\hat{\theta} - \theta_0) \sim \chi^2(q)$$

$$= n(n^{-1/2}h)[\sigma^2 R(X'X)^{-1}R']^{-1}(n^{-1/2}h) \sim \chi^2_q(h(\sigma^2 R(X'X)^{-1}R')h) \; *$$

2 Problem 2

2.1 a

```
> print(empirical_sizes)
 n_50 n_100 n_200 n_500
0.054 0.057 0.056 0.057
```

2.2 b

```
> print(empirical_powers)
 n_50 n_100 n_200 n_500
    1     1     1     1
```

2.3 c

```
> print(empiricalPowers)
      n_50 n_100 n_200 n_500
h_1  0.378 0.356 0.362 0.368
h_2  0.924 0.943 0.944 0.944
h_3  0.999 0.999 1.000 1.000
h_4  1.000 1.000 1.000 1.000
h_5  1.000 1.000 1.000 1.000
h_6  1.000 1.000 1.000 1.000
h_7  1.000 1.000 1.000 1.000
h_8  1.000 1.000 1.000 1.000
h_9  1.000 1.000 1.000 1.000
h_10 1.000 1.000 1.000 1.000
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