

# Econometric Methods Homework 11

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## Problem 1

prove that under  $H_0$ ,  $\sqrt{T}\hat{\rho}(k) \xrightarrow{d} N(0,1)$ , then  $T\hat{\rho}^2(k) \xrightarrow{d} \chi^2(1)$  is proved  
lag-k autocorrelation coefficient  $\rho(k) := \text{corr}(y_t, y_{t-k}) = \frac{\text{cov}(y_t, y_{t-k})}{\sigma^2}$

under  $H_0: \rho(k) = 0$

$$\hat{\rho}(k) = \frac{1}{T-k} \sum_{t=k+1}^T \left( \frac{y_t - \bar{y}}{\hat{\sigma}} \right) \left( \frac{y_{t-k} - \bar{y}}{\hat{\sigma}} \right), \quad \hat{\sigma}^2 = \frac{1}{T} \sum_{i=1}^T (y_i - \bar{y})^2, \quad \hat{\sigma} \rightarrow \sigma \text{ as } T \rightarrow \infty$$

$$\therefore \hat{\rho}(k) \sim \frac{1}{T} \cdot \frac{1}{\sigma^2} \sum_{t=k+1}^T (y_t - \bar{y})(y_{t-k} - \bar{y}) \text{ as } T \rightarrow \infty$$

$\Rightarrow$  According to CLT,  $\frac{1}{T} \sum_{t=k+1}^T (y_t - \bar{y})(y_{t-k} - \bar{y})$  has mean = 0

and has variance =  $\sigma^4/T \Rightarrow \sigma^4/T \times \frac{1}{\sigma^2} = \sigma^2/T$

$\Rightarrow$  under  $H_0$ ,  $\sqrt{T}\hat{\rho}(k) \xrightarrow{d} N(0,1) \therefore T\hat{\rho}^2(k) \xrightarrow{d} \chi^2(1) *$

## Problem 2

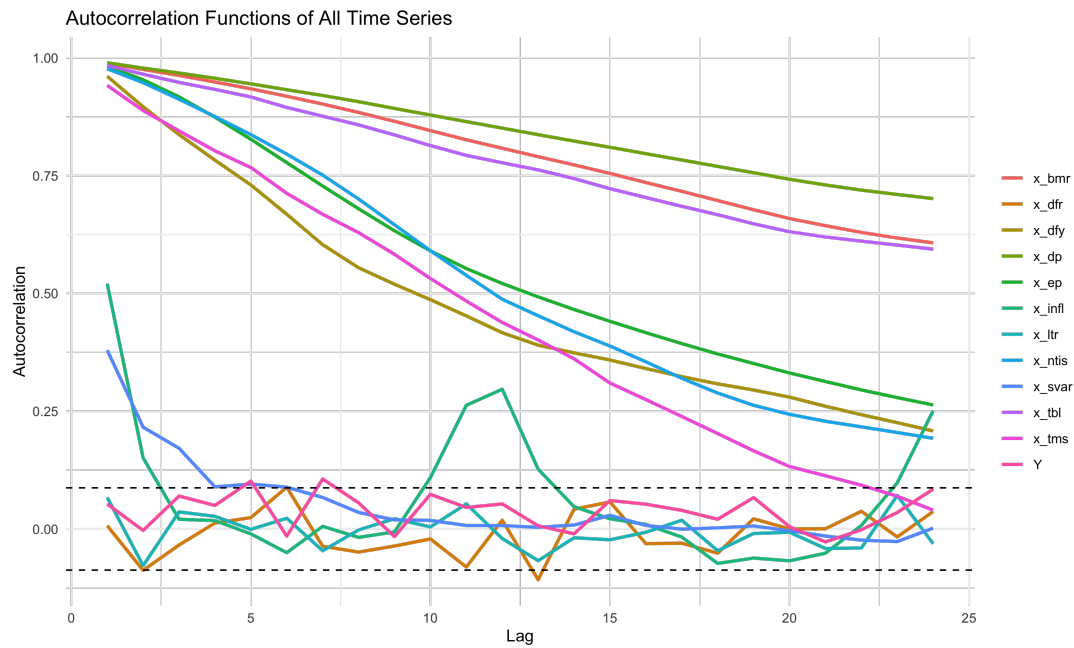
According to #1, we have  $T\hat{\rho}^2(k) \xrightarrow{d} \chi^2(1) \forall k < T$

For  $\{y_t\}_{t=1}^T$  is an IID sequence, there's no correlation between  $y_t$  and  $y_{t-k}$  for different  $k$

$\therefore T\hat{\rho}^2(k)$  are asymptotic independent for different  $k$

$$Q(m) = T \sum_{k=1}^m \hat{\rho}^2(k) = \sum_{k=1}^m \chi^2(1) = \chi^2(m) \text{ as } T \rightarrow \infty *$$

## Problem 3



Series	Lag_12_P_Value	Lag_24_P_Value	Q_12	Q_24
x_dfy	0.00000000	0.000000e+00	2810.909115	3376.77507
x_infl	0.00000000	0.000000e+00	235.120420	289.65187
x_svar	0.00000000	3.330669e-16	126.842991	128.12640
x_tms	0.00000000	0.000000e+00	3035.224545	3354.72766
x_tbl	0.00000000	0.000000e+00	4742.989144	7443.11281
x_dfr	0.23714455	2.804238e-01	15.079000	27.52858
x_dp	0.00000000	0.000000e+00	5172.397798	8723.78726
x_ltr	0.65434920	7.850608e-01	9.561726	18.36284
x_ep	0.00000000	0.000000e+00	3572.369411	4422.18379
x_bmr	0.00000000	0.000000e+00	4973.900401	7891.05780
x_ntis	0.00000000	0.000000e+00	3595.622964	4174.72253
Y	0.02750482	8.120110e-02	23.026142	34.19858

## GitHub Link

EconometricMethods-homework11-b10901069