Department of Electrical and Computer Engineering

ECE 358: Computer Networks

Project 1: M/M/1 and M/M/1/K Queue Simulation

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Contents

Question 1

Run	Mean	Variance
1	0.013858672478097408	0.00017418973917540548
2	0.012303461910498893	0.00016470504001987526
3	0.013300150856802513	0.00017009661987798348
4	0.014624846865467396	0.00022944861550141895
5	0.013492076623601867	0.00018167528183964 49
avg	0.013515842	0.000184023

The average mean is 0.013515842 while the average variance is 0.000184023.

This agrees with the expected values since the mean of the function is given by $1/\lambda$, which in our case, with $\lambda = 75$, $1/\lambda = 0.0133$.

The variance also agrees with expected value since the expected value is $1/\lambda^2$, which in our case is equal to $1.778*10^{-4}$.

I. M/M/1 Queue

Question 2

In our code, we have 5 defined functions that will be performing all the calculations. In the main function, we will be defining the variables needed and printing the final E(N) and P_idle graphs.

Our 5 defined functions are **generateExponentialVar**(lamb), **populateArrays**(lamb, lambL, C, T), **createDES**(p_a, p_d, o), **queueProcessing**(DES, T), and **oneSimulation**(lamb, avg_l, C, T).

generateExponentialVar(lamb) shown in Fig. 1 generates a random exponential value with the inverse method. The input of this function is the λ found in the equation. This function is used to generate the various random exponential values that are needed throughout the code. This includes inter-arrival time, inter-observation time, and packet length

```
def generateExponentialVar(lamb):
    # generate a random value between 0 and 1
    U = random.random()

    # using given equation, return a random exponential value
    return -(1 / lamb) * math.log(1 - U)
```

Figure 1. generateExponentialVar function

populateArrays(lamb, lambL, C, T) generates the multiples arrays that contain packet arrival times, packet departure times, as well observation times. Since this is an infinite queue, all these values are pre-generated and thus, this function is able to return packet departure times as well as the other two (which is not the case for finite queue).

In order to generate inter-arrival times, inter-observation times, as well as packet length, we specify the necessary lambda and run this through the previous function, **generateExponentialVar**. The arrival time that is appended to the arrival time array will then be a cumulative value of all the previously generated inter-arrival times. This logic also holds for observation times, with the difference being that the rate will be 5 times faster for observation times. A sample of the code used to generate arrival times is shown in Fig. 2 below.

```
# for a given number, generate that amount of arrivals and packet lengths
while arrival_time < T:

    arrival_time += generateExponentialVar(lamb)
    packet_arrivals.append(arrival_time)
```

Figure 2. Sample code used to generate arrival times

Departure time is calculated differently, and since it uses information from both arrival times and packet length, it is calculated after the above while loops. Since there is one packet departure for every packet arrival, a for loop will iterate that many times. In the for loop, a service time will be calculated each iteration based on the correlated random packet length. For the first arrival time, the departure will be directly correlated with arrival time plus the service time. For every packet after that, a check needs to be performed. If the current packet arrival time is greater than the previous departure time, then there is no queue, and the departure will be a simple sum of packet arrival time and packet service time. If the current packet arrives before the previous packet departs, that means there is a queue, and the packet needs to wait. This means the departure time will actually be the sum of the previous departure time and the current packet service time. The value that is calculated is then appended onto an array that will contain all the final departure times. The code that does this can be found in Fig. 3.

```
for i in range(len(packet_arrivals)):

    service_time = packet_length[i]/C

    a_i = packet_arrivals[i]

    if i > 0:
        d_i0 = packet_departure[i-1]

        if a_i > d_i0:
            departure = a_i + service_time
        else:
            departure = d_i0 + service_time
    else:
        departure = a_i + service_time

    packet_departure.append(departure)
```

Figure 3. Sample code used to generate the departure times

createDES(p_a, p_d, o) is the function used to create a combined list of Arrivals, Departures, and Observations, sorted by the time of the event. The inputs this function take are the 3 arrays that contain packet arrival times, packet departure times, and observation times. The output of this function is an array of dictionaries, where each dictionary represents one time point.

The logic of the **createDES** function is as follows. The code iterates through each of the 2 input arrays and adds a dictionary to the array with two keys. The two keys are “type” and “time”, where type corresponds with what type of event it was, and time is the numerical value found in the original array. The sample code for **createDES** can be found in Fig. 4.

```python
def createDES(p_a, p_d, o):
    # initialize DES array which is going to carry the dictionaries
    DES = []

    # for every value found in p_a
    for i in range(len(p_a)):
        # append a dictionary that contains Arrival as the event type, with it's corresponding value
        DES.append({"type": "Arrival", "time": p_a[i]})

    # the same thing that was done for p_a is done for p_d and o
    for i in range(len(p_d)):
        DES.append({"type": "Departure", "time": p_d[i]})

    for i in range(len(o)):
        DES.append({"type": "Observation", "time": o[i]})

    # sort the dictionaries in the array based on the time value in ascending order
    newDES = sorted(DES, key=lambda x: x["time"], reverse=False)

    # returns the sorted DES array
    return newDES
```

Figure 4. Sample code of createDES

The final function **queueProcessing**(eventTypes, DES, T) performs the calculations needed to find the E[N] and the P_{IDLE}. The function requires the two arrays containing event types as well as event times, and the simulation time. It outputs the final E[N] and P_{IDLE}.

queueProcessing will be iterating through eventTypes and DES simultaneously and if the DES value is greater than the simulation time, the process will stop. While iterating, depending on the event type value, N_A, N_D, and N_O will be incrementing. When the event is an observation, an idle counter will also increment so that we can later calculate for P_{IDLE}. During an observation, the number of packets in the queue will also be added to a total packet count which will be used to calculate E[N]. Finally, the function will actually calculate and return E[N] and P_{IDLE}. Fig. 5 shows sample code of this function.

```
# while the iteration counter is less than the length of DES,
# and the value of DES is less than simulation time, perform the following code
while i < len(DES) and DES[i] < T:
    eType = eventTypes[i]

    # based on event type, increment the correct counter
    if eType == "Arrival":
        N_a += 1
    elif eType == "Departure":
        N_d += 1
    else:
        N_o += 1

        # idle is incremented if there are no packets in queue, used to calculate P_IDLE
        if N_d == N_a:
            idle += 1

        # used for calculating E[N], cumulating packets in the queue at observation times
        total_packets += (N_a - N_d)
    i += 1

# calculating E[N] and P_IDLE
E_N = total_packets/N_o
P_IDLE = idle/N_o
```

Figure 5. Sample code of function queueProcessing

In terms of computing the performance metrics, E[N] was calculated as shown in Eq. 1 below, and P_{IDLE} was calculated as shown by Eq. 2 below.

$$E[N] = \frac{\sum_i N_{Ai} - N_{Di}}{N_{ofinal}}$$

Equation 1. Formula used for calculating E[N] performance metric

$$P_{IDLE} = \frac{counter_{IDLE}}{N_{ofinal}}$$

Equation 2. Formula used for calculating P_{IDLE} performance metric

In the **oneSimulation** function, it will be taking lamb, avg_l, C and T as the inputs. This is where **populateArrays**, **createDES**, and **queueProcessing** are called. The final output is a printed value of E[N] and P_{IDLE}.

Lastly, in the main function, the variables are initialized, and rho is then used to calculate the lambdas needed for the multiple iterations. The final outputs are 2 separate

figures, one of E[N] vs rho and the other is of P_{IDLE} vs rho. Fig. 6 below shows the for loop that runs through each iteration of rho

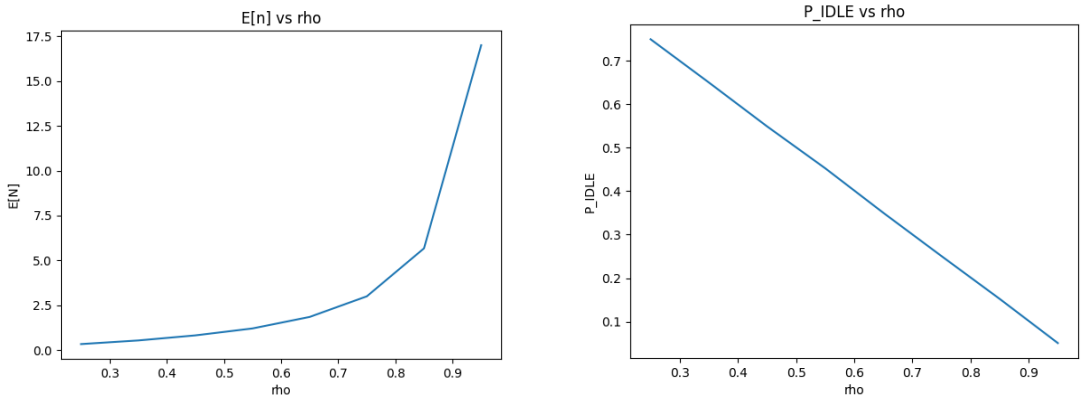
```
for i in range(25, 96, 10):
    rho = i/100  # utilization of queue
    lamb = (C*rho)/avg_l  # average number of packets generated / second

    E_N, P_IDLE = oneSimulation(lamb, avg_l, C, T)

    ENArray.append(E_N)
    PIDLEArray.append(P_IDLE)
    rhoArray.append(rho)
```

Figure 6. For loop that iterates through values of rho

The stability check was performed on T = 1000 and 2T = 2000, where lambda = 75. The variance in results between T and 2T are minimal enough that the time for simulation should be stable now.

T	E[N]	P_{IDLE}
1000	0.17313206296780262	0.852122932048538
2000	0.17571581703668546	0.8506730200039464

Question 3



The E[N] vs rho graph is an exponential curve while the P_{IDLE} vs rho is a linearly decreasing line.

The reason for the E[N] vs rho curve increasing is because as rho increases, there will be an increasing number of packets arriving per second. Since service time does not change, this means there will be a greater $N_A - N_D$ the greater the value of rho. It is exponential as

The reason for the P_{IDLE} vs rho line decreasing is because as rho increases, we can see that lambda also increases. As lambda increases, the number of packets arriving every second increases and leaves less time for the queue to be idle, thus resulting in decreasing P_{IDLE}. The reason it is linear is because rho is the utilization of the queue, and this is directly correlated with idle time as 100% - rho (utilization) = P_{IDLE}(idle time). Since rho is linear, P_{IDLE} should also be linear.

The T in this simulation was 1000, since 2T holds stability, as shown in the table below.

	E[N]		**P_{IDLE}**	
Rho	**T**	**2T**	**T**	**2T**
	0.3311	0.3325	0.7516	0.7507
	0.5349	0.5384	0.6507	0.6498
	0.8192	0.8168	0.5495	0.5499
	1.2261	1.2221	0.4491	0.4506
	1.8582	1.8513	0.3493	0.3505
	2.9937	3.0302	0.2489	0.2492
	5.7082	5.7180	0.1507	0.1490
	20.0646	18.7293	0.0502	0.0493

Question 4

For rho = 1.2, the E[N] is equal to 49483.3405 while the P_IDLE is equal to 2.9995220761492e-06. While initially these numbers seem to be extreme, it follows the trend set by the graphs of an exponential growth of E[N] and a linearly decreasing trend of P_{IDLE}. The queue at this point is essentially never idle, and as a result, the queue will accumulate more and more packages over time resulting in an E[N] that is exponentially growing towards infinity.

M/M/1/K Queue

Question 5

For the simulator built for the M/M/1/K Queue, it is very similar to simulator built for the M/M/1 Queue. It contains all the same functions that perform the same tasks. But the key difference appears in **populateArrays**. This is because, with a finite buffer size, the departure times for the packets cannot be pre-generated. Therefore, in the **populateArrays** function, the

departure times are calculated with a queue and then appended to an array. The function takes in an additional argument, *k*, which is used to define the size of the queue buffer.

```
for i in range(len(all_packet_arrivals)):

    # calculate the service time given a single packet length and C value
    service_time = packet_length[i] / C

    a_i = all_packet_arrivals[i]

    # process and add run through the departure queue until the arrival time is greater than the first
    # element of the queue or until the queue is empty
    while len(departure_queue) != 0 and departure_queue[0] <= a_i:
        departure = departure_queue.popleft()
        packet_departure.append(departure)
    # for every iteration that is not the first one, we will perform the following calculation

    # if the departure queue is full, skip the iteration and increment the total packets lost by 1
    if len(departure_queue) == k:
        total_lost += 1
        continue
    else:
        if i > 0 and len(departure_queue) != 0:
            # d_i0 is set as the last element in the departure queue
            d_i0 = departure_queue[-1]

            # if the current arrival time is greater than the last departure time in the queue,
            # that means we can just add the service time of the current package to arrival time
            # in order to determine the departure time of the package
            if a_i > d_i0:
                departure = a_i + service_time
            # otherwise, departure time of the current package is given by the sum of the
            # previous departure time and the current service time
            else:
                departure = d_i0 + service_time
        else:  # for the first iteration, it will be a simple addition of arrival time and service time
            departure = a_i + service_time
        departure_queue.append(departure)  # add the departure time to the queue
        actual_packet_arrivals.append(a_i)  # add the arrival time to successfully processed arrival times
    total_created = len(all_packet_arrivals)
```

Figure 7. Sample of code to generate and processes departure times in M/M/1/K simulator

All possible arrival times of a packet are generated through the same way as the M/M/1 simulator and stored into an array called *all_packet_arrivals.* Observation times are also generated the same way as the M/M/1 simulator. The function then iterates through all of the possible arrival times and generate a departure time through the service time. Once the departure time is generated, it will be added into the *departure_queue*. The departure queue is processed and ran through if the arrival time of the packet is greater than the first element of the queue. It will then append the time to an array that keeps track of successful departure times. The function will also append the corresponding arrival time to an array called

actual_packet_arrivals which keeps track of arrival times of packets that were not lost. Figure 7. Shows the code for the updated **populateArray** departure time calculations. A variable called *total_lost* and *total_created* are returned from the function additionally now for the purpose of calculating P_{LOSS}.

Functions **queueProcessing** and **oneSimulation** were slightly adjusted from the M/M/1 simulator. **queueProcessing** now has additional arguments, *total_lost* and *total_created_packets* which are arguments used for calculating the performance metric P_{LOSS} and the function also returns it with the variable *P_LOSS*. The P_{LOSS} of the simulation is calculated by dividing the total amount of packets lost with the total packets generated as shown in figure 8. The other performance metrics were calculated the same was as the M/M/1 simulator. **oneSimulation** has an additional argument, *K*, which is used to define the queue buffer size for **populateArray** to use.

```
# calculating E[N], P_IDLE, and P_LOSS
E_N = total_packets / N_o
P_IDLE = idle / N_o
P_LOSS = total_lost / total_created_packets
```

Figure 8. Calculations for the performance metrics

The other functions, **generateExponentialVar** and **createDES** were not changed from the M/M/1 simulator and perform the same tasks.

```
# initializing variables
avg_l = 2000  # average length of packets in bit
C = 10 ** 6  # transmission rate of output link in bits per second
T = 2000  # the total time for simulation
K = 10

rho = 1.2
lamb = (C * rho) / avg_l  # average number of packets generated / second
```

Figure 9. Initial variables used to test M/M/1/K simulator

The stability check for this simulator was performed with the variables shown in figure 8. T was increased from T=1000 to T=2000 and the results are stable and consistent as shown in the table below.

T	E[N]	P_{LOSS}
1000	6.725180043080399	0.19504449908807212
2000	6.7183604650368025	0.19296183132224523

Question 6

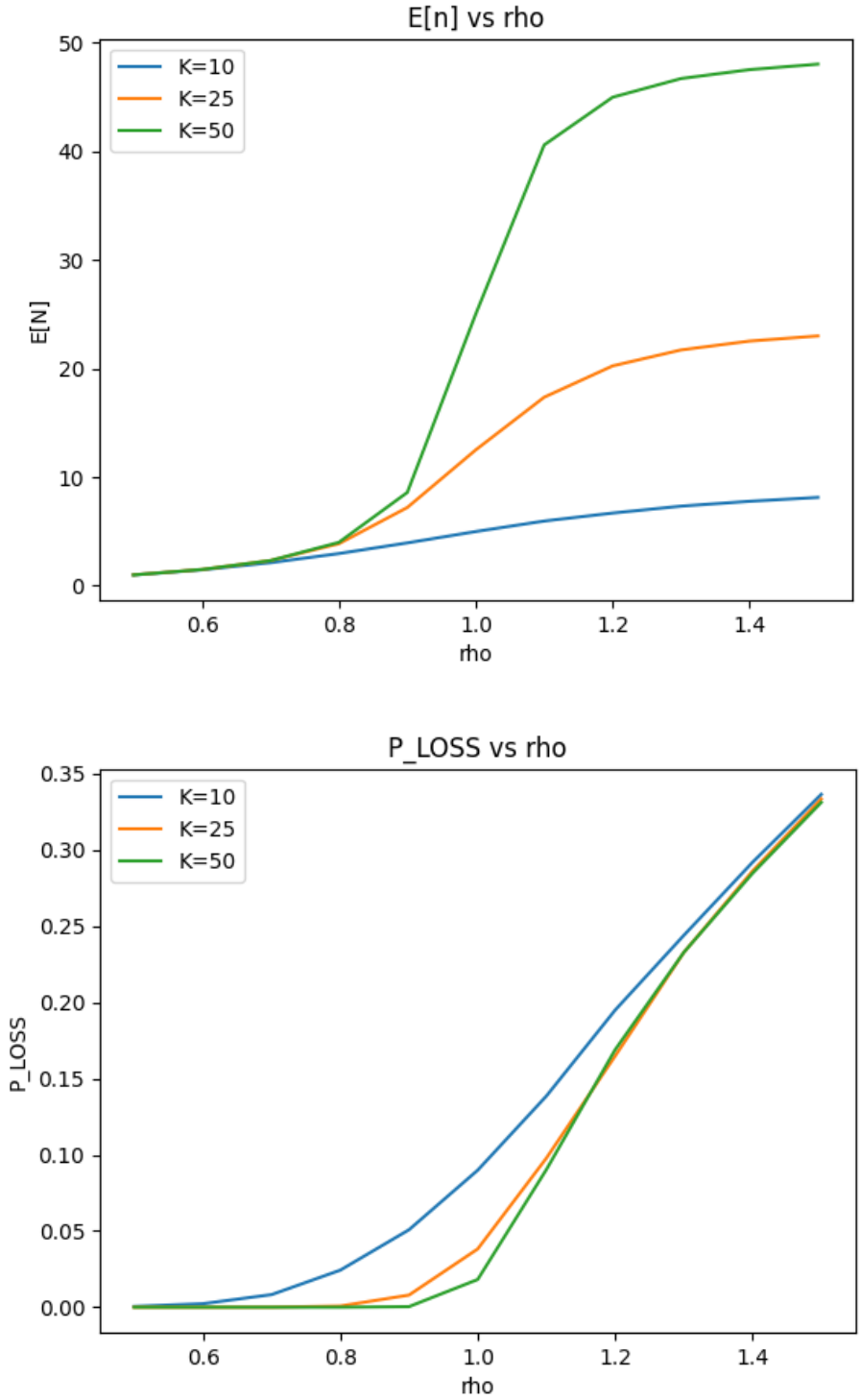


Figure 10. Graphs of E[n] and P_{LOSS} for the M/M/1/K simulator

The following tables show the stability check for when T goes from 1000 to 2000. All the differences in the values when T increases are negligible.

Table below is outlining the performance metrics for K = 10

Rho	E[n]		P_{LOSS}	
	T = 1000	T = 2000	T = 1000	T = 2000
0.5	0.996148766064	1.000798709905	0.000518636886096	0.0005263421223795
0.6	1.462591609541	1.475895346894	0.002593968855702	0.0024104287666694
0.7	2.120384719911	2.114989819331	0.008531268737813	0.0086573593982556
0.8	2.958124539010	2.970835215530	0.023287919618349	0.0239626483360349
0.9	3.942621381907	3.973827226545	0.049395972557546	0.0514628883758479
1.0	5.006034212330	4.998852564538	0.091325820868077	0.0914257494434219
1.1	5.933767721687	5.962419657107	0.139946656675781	0.1412683132577214
1.2	6.706252496537	6.700986850874	0.192388086932832	0.1909369624616349
1.3	7.309804234613	7.309515254800	0.243519043248470	0.2444113653183037
1.4	7.769015041824	7.777968256871	0.290985662146392	0.2925789744763649
1.5	8.121641704691	8.133312468951	0.337643991669619	0.3374924550890216

Table below is outlining the performance metrics for K = 25

Rho	E[n]		P_{LOSS}	
	T = 1000	T = 2000	T = 1000	T = 2000
0.5	1.0044561977967534	0.99715345480884	0.0	0.0
0.6	1.492643943532954	1.51204570039031	0.0	0.0
0.7	2.334307246853928	2.34181259451455	3.85492902647e-05	3.00051437389e-05
0.8	3.9542593387526943	3.89940124639118	0.0007975149534053	0.000671660552512
0.9	7.243439740768061	7.19255150449022	0.0079703577331129	0.007521279718227
1.0	12.490589499180327	12.4758754109112	0.0385504595409088	0.038000146038992
1.1	17.33176969984388	17.2774282335467	0.0986232476497247	0.097967438936617
1.2	20.212399412691166	20.2451944380955	0.1683797374382670	0.168549108036345
1.3	21.723717308405156	21.7085014917327	0.2320411554693028	0.231258499633861
1.4	22.51070658841379	22.5293378196685	0.2863348739270865	0.286386658492843
1.5	22.988852571450497	23.0103302040426	0.3330721985460015	0.334130518390246

Table below is outlining the performance metrics for K = 50

Rho	E[n]		P_{LOSS}	
	T = 1000	T = 2000	T = 1000	T = 2000
0.5	1.0005173396746756	1.00115651657687	0.0	0.0
0.6	1.488292532664094	1.51352062429729	0.0	0.0
0.7	2.359262475363478	2.35731015369139	0.0	0.0
0.8	3.9623881272799646	4.00965780996881	1.2513060506904082e-06	0.0
0.9	8.429612763052925	8.97072165956368	0.00047995652551564703	0.0007167550664
1.0	25.57334661174415	24.9286407680604	0.02035705638585461	0.0193486413869

1.1	40.17688001177438	40.3079659217776	0.08991470163805308	0.0909207538105
1.2	45.03651576346937	44.9535387135151	0.1660445864013643	0.1657787009989
1.3	46.677456722470154	46.6256808873909	0.23211153731803086	0.2299841543781
1.4	47.51144678111257	47.4960495082207	0.2859072132757028	0.2855645942959
1.5	48.0040840591172	47.9740095148468	0.33322695370953964	0.3328599928098

The results for the simulations are to be expected. In the first graph of figure 10, E[n] vs rho, when K = 10 the average number of packets in the buffer/queue is a smooth curve because the queue is only a size of 10 and therefore as rho increases the E[n] approaches 10. Once K is increased to 25, the average number of packets in the buffer/queue curve rises at a faster rate for larger values of rho and eventually approaches the max buffer size number of 25. And finally, when K = 50, the curve of E[n] rises exponentially for values of rho and approaches its max size. This is because, the average number of packets per second generated is derived from the rho value and as it rises, the packets generated per second rises while the service time is still the same. Therefore, at larger rho values the number of packets in the queue will rise too. And this also explains the curves for the P_{LOSS} vs rho graph. As the rho value increases, the number of packets that are lost from the queue being full will increase exponentially as well.