**::::::::Desktop:UW CMYK Colour.eps**

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**Department of Electrical and Computer**

**Engineering**

**ECE 358: Computer Networks**

**Project 1: M/M/1 and M/M/1/K Queue**

**Simulation**

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# Question 1

|  |  |  |
| --- | --- | --- |
| **Run** | **Mean** | **Variance** |
| 1 | 0.013858672478097408 | 0.00017418973917540548 |
| 2 | 0.012303461910498893 | 0.00016470504001987526 |
| 3 | 0.013300150856802513 | 0.00017009661987798348 |
| 4 | 0.014624846865467396 | 0.00022944861550141895 |
| 5 | 0.013492076623601867 | 0.0001816752818396449 |
| **avg** | 0.013515842 | 0.000184023 |

The average mean is 0.013515842 while the average variance is 0.000184023.

This agrees with the expected values since the mean of the function is given by 1/𝜆, which in our case, with 𝜆 = 75, 1/ 𝜆 = 0.0133.

The variance also agrees with expected value since the expected value is 1/ 𝜆­2, which in our case is equal to 1.778\*10­-4.

# I. M/M/1 Queue

## Question 2

In our code, we have 5 defined functions that will be performing all the calculations. In the main function, we will be defining the variables needed and printing the final E(N) and P\_idle graphs.

Our 5 defined functions are **generateExponentialVar**(lamb), **populateArrays**(lamb, lambL, C, T), **createDES**(p\_a, p\_d, o), **queueProcessing**(eventTypes, DES, T), and **oneSimulation**(lamb, avg\_l, C, T).

**generateExponentialVar**(lamb) shown in Fig. 1 generates a random exponential value with the inverse method. The input of this function is the 𝜆 found in the equation. This function is used to generate the various random exponential values that are needed throughout the code. This includes inter-arrival time, inter-observation time, and packet length

Text

Description automatically generated

Figure 1. generateExponentialVar function

**populateArrays**(lamb, lambL, C, T) generates the multiples arrays that contain packet arrival times, packet departure times, as well observation times. Since this is an infinite queue, all these values are pre-generated and thus, this function is able to return packet departure times as well as the other two (which is not the case for finite queue).

In order to generate inter-arrival times, inter-observation times, as well as packet length, we specify the necessary lambda and run this through the previous function, **generateExponentialVar**. The arrival time that is appended to the arrival time array will then be a cumulative value of all the previously generated inter-arrival times. This logic also holds for observation times, with the difference being that the rate will be 5 times faster for observation times. A sample of the code used to generate arrival times is shown in Fig. 2 below.

Graphical user interface, text

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Figure 2. Sample code used to generate arrival times

Departure time is calculated differently, and since it uses information from both arrival times and packet length, it is calculated after the above while loops. Since there is one packet departure for every packet arrival, a for loop will iterate that many times. In the for loop a service time will be calculated each iteration based on the correlated random packet length. For the first arrival time, the departure will be directly correlated with arrival time plus the service time. For every packet after that, a check needs to be performed. If the current packet arrival time is greater than the previous departure time, then there is no queue, and the departure will be a simple sum of packet arrival time and packet service time. If the current packet arrives before the previous packet departs, that means there is a queue, and the packet needs to wait. This means the departure time will actually be the sum of the previous departure time and the current packet service time. The value that is calculated is then appended onto an array that will contain all the final departure times. The code that does this can be found in Fig. 3.

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Figure 3. Sample code used to generate the departure times

**createDES**(p\_a, p\_d, o) is the function used to create a combined list of Arrivals, Departures, and Observations, sorted by the time of the event. The inputs this function take are the 3 arrays that contain packet arrival times, packet departure times, and observation times. The outputs of this function are two arrays, one that contains the event type, and the other which contains the event time.

In Fig. 4, the logic behind the function is displayed. At step 1, we have the packet arrival time array, the packet departure time array, and the observation time array, all of which have length n. In step 2, the first value of all 3 arrays will be compared, in which the lowest value will get popped and added to a new array, DES. Depending on which array the value is popped from, a corresponding string (“Arrival”, “Departure”, or “Observation”) will be added to the second new array, eventTypes. This procedure will continue until the original 3 arrays are empty and DES contains all the time values. Fig. 5 shows some sample code of what happens when the lowest value is in the arrival time array.

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Figure 4. The logic behind createDES function

Text

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Figure 5. Sample code where arrival time array has the lowest value

The final function **queueProcessing**(eventTypes, DES, T) performs the calculations needed to find the E[N] and the PIDLE. The function requires the two arrays containing event types as well as event times, and the simulation time. It outputs the final E[N] and PIDLE.

**queueProcessing** will be iterating through eventTypes and DES simultaneously and if the DES value is greater than the simulation time, the process will stop. While iterating, depending on the event type value, NA, ND, and NO­ will be incrementing. When the event is an observation, an idle counter will also increment so that we can later calculate for PIDLE. During an observation, the number of packets in the queue will also be added to a total packet count which will be used to calculate E[N]. Finally, the function will actually calculate and return E[N] and PIDLE. Fig. 6 shows sample code of this function.

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Figure 6. Sample code of function queueProcessing

In terms of computing the performance metrics, E[N] was calculated as shown in Eq. 1 below, and PIDLE was calculated as shown by Eq. 2 below.

Equation 1. Formula used for calculating E[N] performance metric

Equation 2. Formula used for calculating PIDLE­ performance metric

In the **oneSimulation** function, it will be taking lamb, avg\_l, C and T as the inputs. This is where **populateArrays**, **createDES**, and **queueProcessing** are called. The final output is a printed value of E[N] and PIDLE.

Lastly, in the main function, the variables are initialized, and rho is then used to calculate the lambdas needed for the multiple iterations. The final outputs are 2 separate figures, one of E[N] vs rho and the other is of PIDLE vs rho.

The stability check was performed on T = 1000 and 2T = 2000. The variance in results between T and 2T are minimal enough that the time for simulation should be stable now.

|  |  |  |
| --- | --- | --- |
| **T** | **E[N]** | **PIDLE** |
| 1000 | 0.17313206296780262 | 0.852122932048538 |
| 2000 | 0.17571581703668546 | 0.8506730200039464 |

## Question 3

The packet length will follow an exponential distribution with an average of L = 2000 bits. Assume that C = 1Mbps. Use your simulator to obtain the following graphs. Provide comments on all your figures.

1. E[N], the average number of packets in the queue as a function of ρ (for 0.25 < ρ < 0.95, step size 0.1). Explain how you do that.
2. 2. PIDLE, the proportion of time the system is idle as a function of ρ, (for 0.25 < ρ < 0.95, step size 0.1). Explain how you do that.

[0.3353346181219163, 0.5376161641141187, 0.8201357053772789, 1.207436217050385, 1.8494997385843899, 2.9988873776568794, 5.673789933262058, 16.999984418625218]

[0.7495317985174286, 0.6502122066630717, 0.549254758266463, 0.4528076715532626, 0.35022917743068793, 0.2506493852841561, 0.15218134634495556, 0.05047986423148027]

## Question 4

For the same parameters, simulate for ρ=1.2. What do you observe? Explain.

# M/M/1/K Queue

## Question 5

## Question 6